디지털논리회로

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(Problem Solutions of Chapter 5)



1. 진리표를 이용하여 두 함수가 같음을 증명

① $F = \overline{XYZ}$, $G = \overline{X} + \overline{Y} + \overline{Z}$

X	Y	Z	XYZ	식 <i>F</i>	\overline{X}	\overline{Y}	\overline{Z}	식 <i>G</i>
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	0	0	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	0	1
1	1	0	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0

(2) $F = X\overline{Y} + Y\overline{Z} + \overline{X}Z$, $G = \overline{X}Y + \overline{Y}Z + X\overline{Z}$

X	Y	Z	$X\overline{Y}$	$Y\overline{Z}$	$\overline{X}Z$	식 <i>F</i>	$\overline{\overline{X}}Y$	$\overline{Y}Z$	$X\overline{Z}$	식 <i>G</i>
0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	0	1	0	1
0	1	0	0	1	0	1	1	0	0	1
0	1	1	0	0	1	1	1	0	0	1
1	0	0	1	0	0	1	0	0	1	1
1	0	1	1	0	0	1	0	1	0	1
1	1	0	0	1	0	1	0	0	1	1
1	1	1	0	0	0	0	0	0	0	0

a	b	c	$\frac{\overline{a}\overline{c}}{a}$	$\frac{1}{a}c$	bc	식 <i>f</i>	$\frac{\overline{a}}{a} + c$	$\overline{a} + b + \overline{c}$	식 g
0	0	0	1	0	0	1	1	1	1
0	0	1	0	1	0	1	1	1	1
0	1	0	1	0	0	1	1	1	1
0	1	1	0	1	1	1	1	1	1
1	0	0	0	0	0	0	0	1	0
1	0	1	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	1	0
1	1	1	0	0	1	1	1	1	1

a	b	c	$\frac{-}{ac}$	bc	$a\overline{b}$	식 <i>f</i>	$\bar{b}\bar{c}$	ac	$\overline{a}b$	식 g
0	0	0	1	0	0	1	1	0	0	1
0	0	1	0	0	0	0	0	0	0	0
0	1	0	1	0	0	1	0	0	1	1
0	1	1	0	1	0	1	0	0	1	1
1	0	0	0	0	1	1	1	0	0	1
1	0	1	0	0	1	1	0	1	0	1
1	1	0	0	0	0	0	0	0	0	0
1	1	1	0	1	0	1	0	1	0	1

 $(5) \quad f = ab + ac + \overline{a}bd, \quad g = bd + a\overline{b}c + ab\overline{d}$

a	b	c	d	ab	ac	$\overline{a}bd$	식 f	bd	$a\overline{b}c$	$ab\overline{d}$	식 g
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	0	1	1	1	0	0	1
0	1	1	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	1	1	1	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	0	0
1	0	1	0	0	1	1	1	0	1	0	1
1	0	1	1	0	1	1	1	0	1	0	1
1	1	0	0	1	0	1	1	0	0	1	1
1	1	0	1	1	0	1	1	1	0	0	1
1	1	1	0	1	1	1	1	0	0	1	1
1	1	1	1	1	1	1	1	1	0	0	1

2. 불 대수를 이용한 논리식 간소화

- ① $1 \cdot (A+B+C) = A+B+C$
- (2) A+B+C+1=1
- \bigcirc *ABC*+1=1
- $(\overline{A} \cdot 1) + A = \overline{A} + A = 1$
- (5) $(\overline{A}AB+D)(CE\overline{E}+\overline{D})=D\overline{D}=0$
- (6) $(A+0)(C+D\overline{D}) = AC$
- (7) $(A+\overline{A})B+(C+\overline{C})B=B+B=B$
- $(8) \quad \overline{\overline{A}} = \overline{A} = A A = A$
- 9 $AB\overline{A} + (B\overline{B}C + C) + C(D+1) = 0 + C + C = C$
- (10) $(A+B)+(\overline{A}+\overline{B})=(A+\overline{A})+(B+\overline{B})=1+1=1$

3. 불 대수를 이용한 논리식 간소화

- ① (AB+CD)+(A+C)=(A+AB)+(C+CD)=A+C
- ② A(AB+C) = AAB+AC = AB+AC
- $\overline{A}B + AB + \overline{A}\overline{B} = (\overline{A}B + \overline{A}\overline{B}) + (\overline{A}B + AB) = \overline{A} + B$
- (5) $AB + A\overline{B}C = ABC + AB\overline{C} + A\overline{B}C = ABC + AB\overline{C} + ABC + A\overline{B}C = AB + AC$
- $(\overline{)} \quad A\overline{B}\overline{C} + A\overline{B}C + AB\overline{C} + ABC = A\overline{B}(\overline{C} + C) + AB(\overline{C} + C) = A\overline{B} + AB = A(\overline{B} + B) = A$
- $(A\overline{B}(C+BD) + \overline{A}\overline{B})C = (A\overline{B}C + A\overline{B}BD + \overline{A}\overline{B})C$ $= (A\overline{B}C + \overline{A}\overline{B})C = A\overline{B}CC + \overline{A}\overline{B}C$ $= A\overline{B}C + \overline{A}\overline{B}C = \overline{B}C(A + \overline{A}) = \overline{B}C$

4. 불 대수를 이용한 증명

- $\underbrace{\overline{X}}_{Y} + XY + \overline{X}Y = \overline{X} + Y \\ \overline{X}_{Y} + XY + \overline{X}Y = \overline{X}(\overline{Y} + Y) + XY = \overline{X} + XY = (\overline{X} + X)(\overline{X} + Y) = \overline{X} + Y$
- ② $\overline{X}Y + X\overline{Y} + XY + \overline{X}\overline{Y} = 1$ $(\overline{X} + X)Y + (X + \overline{X})\overline{Y} = 1 \cdot Y + 1 \cdot \overline{Y} = Y + \overline{Y} = 1$
- $\begin{array}{ll} \ \overline{X} + XY + X\overline{Z} + X\overline{Y}\overline{Z} = \overline{X} + Y + \overline{Z} \\ & \overline{X} + XY + X\overline{Z} + X\overline{Y}\overline{Z} = \overline{X}(Y+1) + XY + X\overline{Z}(1+\overline{Y}) = \overline{X}\underline{Y} + \overline{X} + XY + X\overline{Z} \\ & = \overline{X} + (\overline{X} + X)\underline{Y} + X\overline{Z} = \overline{X} + Y + X\overline{Z} = \overline{X}(\overline{Z} + 1) + Y + X\overline{Z} = \overline{X}\overline{Z} + \overline{X} + Y + X\overline{Z} \\ & = \overline{X} + Y + (\overline{X} + X)\overline{Z} = \overline{X} + Y + \overline{Z} \end{aligned}$

5. 불 대수를 이용한 간략화

- ① $F = XYZ + \overline{X}Y + XY\overline{Z}$ $F = XYZ + \overline{X}Y + XY\overline{Z} = XY(Z + \overline{Z}) + \overline{X}Y = XY + \overline{X}Y = (X + \overline{X})Y = Y$
- ② $F = \overline{X}YZ + XZ$ $F = \overline{X}YZ + XZ(Y+1) = \overline{X}YZ + XYZ + XZ = (\overline{X}+X)YZ + XZ = XZ + YZ$
- $3 F = (\overline{X+Y})(\overline{X}+\overline{Y})$ $F = \overline{XY}(\overline{X}+\overline{Y}) = \overline{XY}+\overline{XY}=\overline{XY}$

- $\begin{array}{ll} \textcircled{6} & F = \overline{A} \, \overline{C} + ABC + A \, \overline{C} \\ & F = (\overline{A} + A) \, \overline{C} + ABC = \overline{C} + \underline{ABC} = \overline{C} (\underline{AB} + 1) + \underline{ABC} \\ & = ABC + \overline{C} + ABC = AB(\overline{C} + C) + \overline{C} = AB + \overline{C} \\ \end{array}$

6. 드모르간의 정리

$$F = XY + \overline{X}\overline{Y} + \overline{Y}Z$$

$$F = \underbrace{XY + \overline{X}\overline{Y} + \overline{Y}Z}_{XY + \overline{X}\overline{Y} + \overline{Y}Z} = \overline{XY + \overline{X}\overline{Y} + \overline{Y}Z}_{XY \cdot \overline{X}\overline{Y} \cdot \overline{\overline{Y}Z}} = \underbrace{(XY - \overline{X}\overline{Y}) \cdot (XY - \overline{Y})}_{XY \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y}} = \underbrace{(XY - \overline{Y}) \cdot (XY - \overline{Y}) \cdot (XY - \overline{Y})}_{XY \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y}}_{XY \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y}} = \underbrace{X \cdot \overline{Y}}_{XY \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y}}_{XY \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y}}_{XY \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y} \cdot \overline{Y}}_{XY \cdot \overline{Y}}_{XY \cdot \overline{Y} \cdot \overline{Y}}_{XY \cdot \overline{Y}}$$

7. 논리식의 부정

①
$$F = X\overline{Y} + \overline{X}Y$$

 $\overline{F} = \overline{X}\overline{Y} + \overline{X}Y = (\overline{X} + Y) \cdot (X + \overline{Y})$

②
$$F = (A\overline{B} + C)\overline{D} + E$$

 $\overline{F} = \overline{(A\overline{B} + C)\overline{D} + E} = ((\overline{A} + B)\overline{C} + D)\overline{E}$

$$\overline{F} = (A + \overline{B} + C)(\overline{A} + \overline{C})(A + B)$$

$$\overline{F} = \overline{(A + \overline{B} + C)(\overline{A} + \overline{C})(A + B)} = \overline{A}B\overline{C} + AC + \overline{A}B$$

$$\begin{array}{l} \text{(§)} \quad f = ab\overline{d} + \overline{b}\,\overline{c} + \overline{a}cd + \overline{a}b\overline{c}d \\ \\ \overline{f} = \overline{ab\overline{d}} + \overline{b}\,\overline{c} + \overline{a}cd + \overline{a}b\overline{c}d = (\overline{a} + \overline{b} + d)(b + c)(a + \overline{c} + \overline{d})(a + \overline{b} + c + \overline{d}) \end{array}$$

$$f = (a + \overline{b} + c)(\overline{a} + b + c)(a + \overline{b} + \overline{c})$$

$$\overline{f} = \overline{(a + \overline{b} + c)(\overline{a} + b + c)(a + \overline{b} + \overline{c})} = \overline{abc} + a\overline{bc} + \overline{abc}$$

8. 최소항과 최대항

①
$$F = (XY+Z)(Y+XZ)$$

 $F = (XY+Z)(Y+XZ) = XY+XYZ+YZ+XZ$
 $= XY(Z+Z)+XYZ+(X+X)YZ+X(Y+Y)Z$
 $= XYZ+XYZ+XYZ+XYZ$

 $Minterm = \Sigma m(3,5,6,7)$

기호	X	Y	Z	F
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	0
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	1
m_7	1	1	1	1

$$Maxterm = \Pi M(0,1,2,4) = (X+Y+Z)(X+Y+\overline{Z})(X+\overline{Y}+Z)(\overline{X}+Y+Z)$$

②
$$F = (\overline{A} + B)(\overline{B} + C)$$

$$F = (\overline{A} + B)(\overline{B} + C) = \overline{A}\overline{B} + \overline{A}C + B\overline{B} + BC$$

$$= \overline{A}\overline{B}C + \overline{A$$

 $Minterm = \Sigma m(0,1,3,7)$

디지털논리회로 Solution of Chapter 5

기호	X	Y	Z	F
m_0	0	0	0	1
m_1	0	0	1	1
m_2	0	1	0	0
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	0
m_6	1	1	0	0
m_7	1	1	1	1

 $Maxterm = \Pi M(2,4,5,6) = (A + \overline{B} + C)(\overline{A} + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)$

$$(3) F = \overline{Y}Z + WX\overline{Y} + WX\overline{Z} + \overline{W}\overline{X}Z$$

$$F = (\underbrace{W + \overline{W}}(X + \overline{X}) \overline{YZ} + \underbrace{WX} \overline{Y}(Z + \overline{Z}) + \underbrace{WX} (\underline{Y} + \overline{Y}) \overline{Z} + \underline{WX} (\underline{Y} + \overline{Y}) Z$$

$$= \underline{WX} \underline{YZ} + \underline{WX} \underline{YZ} +$$

 $Minterm = \Sigma m(1,3,5,9,12,13,14)$

기호	WXYZ	F	기호	WXYZ	F
m_0	0000	0	m_8	1000	0
m_1	0001	1	m_9	1001	1
m_2	0010	0	m_{10}	1010	0
m_3	0011	1	m_{11}	1011	0
m_4	0100	0	m_{12}	1100	1
m_5	0101	1	m_{13}	1101	1
m_6	0110	0	m_{14}	1110	1
m_7	0111	0	m_{15}	1111	0

$$\begin{array}{ll} \mathit{Maxterm} &= \mathit{\Pi} \mathit{M}(0,\!2,\!4,\!6,\!7,\!8,\!10,\!11,\!15) \\ &= (\mathit{W} \!+\! \mathit{X} \!+\! \mathit{Y} \!+\! \mathit{Z})(\mathit{W} \!+\! \mathit{X} \!+\! \mathit{Y} \!+\! \mathit{Z})}{(\mathit{W} \!+\! \mathit{X} \!+\! \mathit{Y} \!+\! \mathit{Z})(\mathit{W} \!+\! \mathit{X} \!+\! \mathit{Y} \!+\! \mathit{Z})(\mathit{W} \!+\! \mathit{X} \!+\! \mathit{Y} \!+\! \mathit{Z})} \end{array}$$

$$\bigcirc$$
 $F = A + \overline{B}C$

$$F = A + \overline{B}C = A(B + \overline{B})(C + \overline{C}) + (A + \overline{A})\overline{B}C = \overline{A}\overline{B}C + A\overline{B}\overline{C} + A\overline{B}C + AB\overline{C} + AB\overline{C} + ABC$$
$$Minterm = \Sigma m(1,4,5,6,7)$$

기호	A	B	C	F
m_0	0	0	0	0
m_1	0	0	1	1
m_2	0	1	0	0
m_3	0	1	1	0
m_4	1	0	0	1
m_5	1	0	1	1
m_6	1	1	0	1
m_7	1	1	1	1

$$Maxterm = \Pi M(0,2,3) = (A+B+C)(A+\overline{B}+C)(A+\overline{B}+\overline{C})$$

(5)
$$f = \overline{a} \overline{c} \overline{d} + \overline{a} \overline{c} \overline{d} + bc$$

$$\begin{split} f &= \overline{a}\,\overline{c}\,\overline{d} + \overline{a}\,\overline{c}\,\overline{d} + bc = \overline{a}(b+\overline{b})\,\overline{c}\,\overline{d} + \overline{a}(b+\overline{b})\,\overline{c}\,\overline{d} + (a+\overline{a})bc(d+\overline{d}) \\ &= \overline{a}\,\overline{b}\,\overline{c}\,\overline{d} + \overline{a}\,\overline{b}\,\overline{c}\,\overline{d} + \overline{a}\,\overline{b}\,\overline{c}\,\overline{d} + \overline{a}\,\overline{b}\,\overline{c}\,\overline{d} + \overline{a}\,bc\,\overline{d} + abc\,\overline{d} + abc\,\overline{d} + abc\,\overline{d} + abc\,\overline{d} \end{split}$$

 $Minterm = \Sigma m (0,2,4,6,7,14,15)$

기호	abcd	F	기호	abcd	F
m_0	0000	1	m_8	1000	0
m_1	0001	0	m_9	1001	0
m_2	0010	1	m_{10}	1010	0
m_3	0011	0	m_{11}	1011	0
m_4	0100	1	m_{12}	1100	0
m_5	0101	0	m_{13}	1101	0
m_6	0110	1	m_{14}	1110	1
m_7	0111	1	m_{15}	1111	1

$$\begin{array}{ll} \mathit{Maxterm} &= \mathit{\Pi} \mathit{M}(1, 3, 5, 8, 9, 10, 11, 12, 13) \\ &= (a + b + c + \overline{d})(a + b + \overline{c} + \overline{d})(a + \overline{b} + c + \overline{d})(\overline{a} + b + c + d)(\overline{a} + b + c + \overline{d}) \\ &\qquad (\overline{a} + b + \overline{c} + d)(\overline{a} + b + \overline{c} + \overline{d})(\overline{a} + \overline{b} + c + d)(\overline{a} + \overline{b} + c + \overline{d}) \end{array}$$

 $6 \quad f = wx\overline{y} + x\overline{y}z + w\overline{x}\overline{z}$

$$\begin{array}{l} f = wx\overline{y} + x\overline{y}z + w\overline{x}\overline{z} = wx\overline{y}(z + \overline{z}) + (w + \overline{w})x\overline{y}z + w\overline{x}(y + \overline{y})\overline{z} \\ = wx\overline{y}\overline{z} + wx\overline{y}z + \overline{w}x\overline{y}z + wx\overline{y}z + wx\overline{y}z + wx\overline{y}z \end{array}$$

기호	WXYZ	F	기호	WXYZ	F
m_0	0000	0	m_8	1000	1
m_1	0001	0	m_9	1001	0
m_2	0010	0	m_{10}	1010	1
m_3	0011	0	m_{11}	1011	0
m_4	0100	0	m_{12}	1100	1
m_5	0101	1	m_{13}	1101	1
m_6	0110	0	m_{14}	1110	0
m_7	0111	0	m_{15}	1111	0

$$Minterm \stackrel{\triangle}{\hookrightarrow} = wx\overline{y}\overline{z} + wx\overline{y}\overline{z} + \overline{w}x\overline{y}\overline{z} + \overline{w}x\overline{y}\overline{z} + \overline{w}x\overline{y}\overline{z} + \overline{w}x\overline{y}\overline{z} = \Sigma m(5,8,10,12,13)$$

$$\begin{array}{l} \mathit{Maxterm} \ \ \stackrel{\triangleleft}{\hookrightarrow} = \mathit{\Pi} \mathit{M}(0,1,2,3,4,6,7,9,11,14,15) \\ = (w+x+y+z)(w+x+y+\overline{z})(w+x+\overline{y}+z)(w+x+\overline{y}+\overline{z})(w+\overline{x}+y+z) \\ (w+\overline{x}+\overline{y}+z)(w+\overline{x}+\overline{y}+\overline{z})(\overline{w}+x+y+\overline{z})(\overline{w}+x+\overline{y}+\overline{z})(\overline{w}+\overline{x}+\overline{y}+z) \\ (\overline{w}+\overline{x}+\overline{y}+\overline{z}) \end{array}$$

9. 최소항과 최대항

① E^{\square} Minterm : $\overline{X}\overline{Y}\overline{Z}$, $\overline{X}\overline{Y}Z$, $\overline{X}Y\overline{Z}$

F \supseteq | Minterm : $\overline{X}Y\overline{Z}$, $\overline{X}YZ$, $XY\overline{Z}$, XYZ

 E^{\square} Maxterm : $X + \overline{Y} + \overline{Z}$, $\overline{X} + Y + Z$, $\overline{X} + Y + \overline{Z}$, $\overline{X} + \overline{Y} + Z$, $\overline{X} + \overline{Y} + \overline{Z}$

Fall Maxterm : X+Y+Z, $X+Y+\overline{Z}$, X+Y+Z, X+Y+Z

② \overline{E} 의 Minterm : $\overline{X}YZ$, $X\overline{Y}Z$, $X\overline{Y}Z$, $XY\overline{Z}$, XYZ

 \overline{F} \supseteq | Minterm : \overline{XYZ} , \overline{XYZ} , \overline{XYZ} , \overline{XYZ}

 $(3) E = \overline{X}\overline{Y}\overline{Z} + \overline{X}\overline{Y}Z + \overline{X}Y\overline{Z}$

 $F = \overline{X}Y\overline{Z} + \overline{X}YZ + XY\overline{Z} + XYZ$

F = Y

10. 논리회로의 논리식 표현

 $\bigcirc X = \overline{A} + B$

② X = AB + B

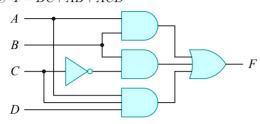
3 X = (A+B) + AB

(4) $X = (A + B)(\overline{B} + C)$

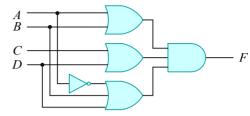
$$(5) X = \overline{A} + \overline{A}B + AC$$

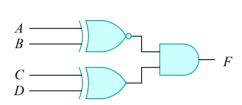
11. 논리식의 논리회로 표현

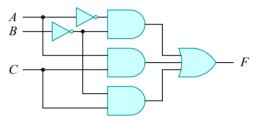
① $F = B\overline{C} + AB + ACD$



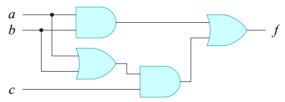
② $F = (A+B)(C+D)(\overline{A}+B+D)$



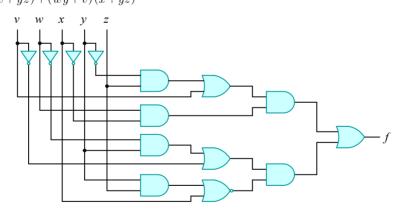




 $(5) \quad f = ab + c(a+b)$

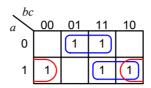


 $(6) f = w\overline{x}(v + \overline{y}z) + (\overline{w}y + \overline{v})\overline{(x + yz)}$



12. 최소항 및 최대항

- ① $f(a,b,c) = \Sigma m(1,3,4,6,7)$
- ② $f(a,b,c) = \overline{abc} + \overline{abc} + \overline{abc} + a\overline{bc} + ab\overline{c} + abc$
- $(3) f(a,b,c) = ab + \bar{ac} + \bar{ac}$
- (4) $\overline{f}(a,b,c) = \overline{\Sigma m(1,3,4,6,7)} = \Sigma m(0,2,5)$



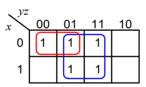
13. SOP 표현 및 간소화

① $f(x,y,z) = \Sigma m(0,1,3,5,7)$

$X \ Y \ Z$	f
0 0 0	1
0 0 1	1
0 1 0	0
0 1 1	1
100	0
1 0 1	1
1 1 0	0
1 1 1	1

②
$$f(x,y,z) = \overline{x}\overline{y}z + \overline{x}\overline{y}z + \overline{x}yz + \overline{x}yz + xyz$$

$$(3) f(x,y,z) = \overline{x} \overline{y} + z$$



4
$$\overline{f}(x,y,z) = \overline{\Sigma m(0,1,3,5,7)} = \Sigma m(2,4,6)$$

14. 최소항 표현

①
$$f(a,b,c) = a\overline{b} + \overline{b}\overline{c} = a\overline{b}c + a\overline{b}\overline{c} + a\overline{b}\overline{c} + \overline{a}\overline{b}\overline{c} = a\overline{b}c + a\overline{b}\overline{c} + \overline{a}\overline{b}\overline{c}$$

$$(2) f(x,y,z) = \overline{x} + yz + \overline{y}\overline{z} = \overline{x}(\underline{y} + \overline{y})(z + \overline{z}) + (x + \overline{x})yz + (x + \overline{x})\overline{y}\overline{z}$$

$$= \overline{x}\overline{y}\overline{z} + \overline{x}\overline{y}z + \overline{x}y\overline{z} + xyz + xy\overline{z} + xyz$$

$$(3) f(a,b,c,d) = a\overline{b}c + bd + \overline{a}\overline{d}$$

$$= a\overline{b}c(d+\overline{d}) + (a+\overline{a})b(c+\overline{c})d + \overline{a}(b+\overline{b})(c+\overline{c})\overline{d}$$

$$= \overline{a}\overline{b}\overline{c}\overline{d} + \overline{a}\overline{b}$$