Lecture 15

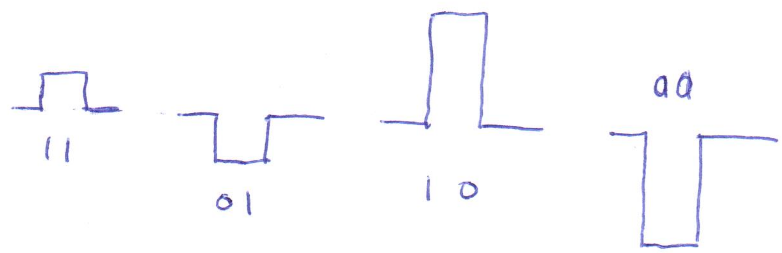
M-ary based signaling

Q. How to increase rate? One way would be to reduce T_b, but this will require more BW. Alternatively, we can allow each pulse to carry multiple bits and increase the number of symbols.

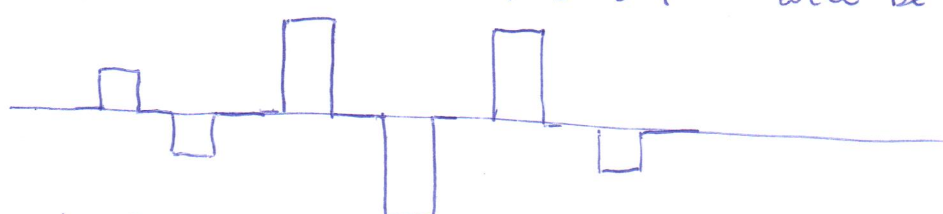
<u>Example</u>: $M = 4$ symbols, 4-ary

a sequence of 2 bits can be transmitted by one of the 4 symbols.

For example, in PAM, each symbol is a pulse with a different amplitude as shown below. PAM: Pulse Amplitude Modulation



For example, the sequence $\overset{\frown}{11}\ \overset{\frown}{01}\ \overset{\frown}{10}\ \overset{\frown}{00}\ \overset{\frown}{10}\ \overset{\frown}{01}$ will be transmitted as



Each symbol occupies a duration T_s.

To transmit a sequence of n bits, we need $\left(\frac{n}{2}\right)$ 4-ary pulses

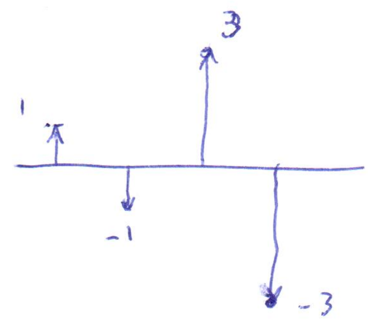
Information in one M-ary symbol $= \log_2 M$ bits. Hence, we can increase the information rate by increasing M. The BW is independent of M but the price is more power (c.f. noise immunity which requires keeping the same separation)

Ex: Determine the PSD of the 4-ary signaling above when '1' and '0' are equally likely.

Solution:

a_k for 4-ary line code (corresponding to 4 different combinations of 2 message bits) are:

$$a_k = \begin{cases} -3 & \rightarrow 00 \\ -1 & \rightarrow 01 \\ 1 & \rightarrow 10 \\ 3 & \rightarrow 11 \end{cases}$$



note $d_{min}(\text{binary}) = 1-(-1) = 2$

$d_{min}(4\text{ ary}) = 2$ (1 is the min. separation between symbols)

Since a_k's are all equally likely,

$$R_0 = \overline{a_k^2} = \frac{1}{4}(1+1+9+9) = 5$$

$$R_n = \overline{a_k a_{k+n}}$$

$$= 0$$

a_{k+n} \ a_n	-3	-1	1	3
-3	9	3	-3	-9
-1	3	1	-1	-3
1	-3	-1	1	3
3	-9	-3	3	9

$$\Rightarrow S_y(f) = \frac{R_0}{T_s}|P(f)|^2 = \frac{5}{T_s}|P(f)|^2$$

Same PSD of binary polar signaling but uses 5 times the original power. BW is the same.

M-ary communication

$I_M = \log_2 M$ information transmitted by an M-ary symbol.

- **Multiamplitude schemes**

Can increase information rate by increasing M. Since BW depends only on pulse rate and not on pulse amplitudes, the BW is independent of M.

But the power increases with M to keep same noise immunity.

$$\text{pulse rate} = \text{Baud rate} = \frac{1}{T_s}$$

Example 4-ary 160 Kbps

Baud rate = 80 Ksymbols/s

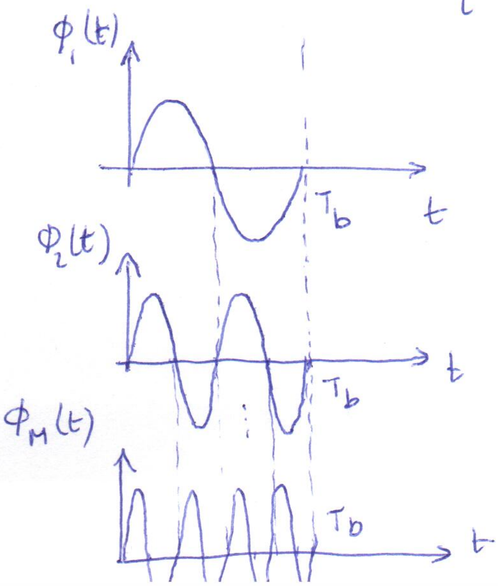
- **Orthogonal schemes**

We can also use other pulse shapes. For example, we can use M orthogonal pulses $\phi_1(t), \phi_2(t), \dots \phi_M(t)$, i.e.

$$\int_0^{T_b} \phi_i(t)\,\phi_j(t)\,dt = \begin{cases} c & i = j \\ 0 & i \neq j \end{cases}$$

e.g.

$$\phi_k(t) = \begin{cases} \sin\left(\frac{2\pi t k}{T_b}\right) & 0 < t < T_b \\ 0 & \text{o.w.} \end{cases} \qquad k = 1, 2, \dots M$$



$$\sin\left(\frac{2\pi k t}{T_b}\right) \quad [0, T_b]$$

The BW will be that of the highest frequency pulse, i.e. $\frac{M}{T_b}$

<u>Note</u> BW of orthogonal scheme M-ary = M x BW of binary scheme

Hence, the rate increases by $\log_2 M$ at the expense of BW increase by a factor of M. But power here is independent of M.

$$\text{SNR} \xleftrightarrow{\text{exchange}} \text{BW}$$

Increase factors

	Multiamplitude	Orthogonal
rate	$\log_2 M$	$\log_2 M$
BW	Indep. of M	M
SNR (power)	M^2	Indep. of M

Telephone lines (BW is limited)

Satellite comm (power limited)