HW 1

① The Direct Broadcast Satellite (DBS) is placed 36,000 km above the sea level at the geostationary orbit. The reciever on earth is a 12" (30cm) dish antenna with an aperture efficiency of 80%. The frequency of operation is 11.7–12.2 GHz (500 MHz BW)

(a) Calculate the gain of the reciver antenna in dB.

$\lambda = \frac{c}{11.95\,GHz} = 2.5\,cm$ $\quad A_{eff} = \frac{\lambda^2}{4\pi} G \quad e_a = \frac{A_{eff}}{A_{phy}} \quad A_{phy} = (15cm)^2\pi$

$A_{phy} = 0.0225\,\pi\,m^2$

$$G_r = \frac{4\pi A_{phy}\, e_a}{\lambda^2} = 1127.43$$

$$\text{Gain} = 30.5\,dB$$

(b) Calculate the space loss factor between the satellite and receiver.

$$\text{Space loss factor} = \left(\frac{\lambda}{4\pi R}\right)^2 = -205\,dB$$

(C) If the gain of the transmit antenna is 33 dB. What is the size of the transmit antenna if its aperture effeciency is 85%?

$$33\,dB \Rightarrow \frac{10^{3.3}\lambda^2}{e_a(4\pi)} = \text{Area of dish} = 11.768\ cm^2$$

$$\pi r^2 = 11.768\ cm^2 \Rightarrow r = \sqrt{\frac{11.768\,cm^2}{\pi}} = D/2 \Rightarrow D = 3.87\,cm$$

What is the footprint of the beam from the transmit antenna with a gain of 43 dB on the satellite in order to cover the entire U.S.A.?

$D_0 = e_{cd} G_0 \qquad D_0 = 33.71\ dB$ or $\frac{10^{3.3}}{0.85} \qquad D_0 \cong \frac{4\pi}{\Omega_A} = \frac{4\pi}{\Theta_{r_1}\Theta_{r_2}}$

$\sqrt{\Omega_A} = 0.074\,rad$ or $4.19°$

$h = 36{,}000\,km \qquad \tan(\theta/2) = \frac{r}{h}$

$r = 1317.6\,km$

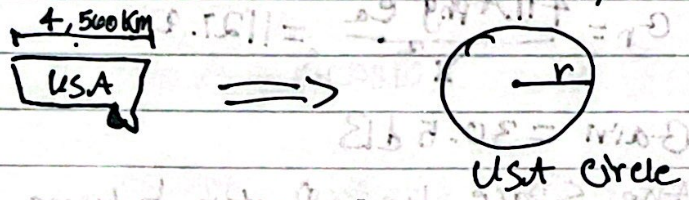
$\text{Footprint} = \pi r^2 = 5.48\times10^{12}\ m^2$

(d.) Can we use transmit antenna with a gain of 43 dB on the satellite in order to cover the entire U.S.A.?

$$G_0 = 43\ dB \Rightarrow D_0 = 10^{4.3}/0.85$$

$$D_0 \approx \frac{4\pi}{\Omega_A} = \frac{4\pi}{\Theta_{r_1}\Theta_{r_2}} \qquad \Theta_{r} = \Theta_{r_2} = \sqrt{\frac{4\pi}{D_0}} = 1.32567^\circ$$

Google: USA width = 2,800 miles or 4,500 km



USA radius = 2250 km

$h = 36,000$ km

$\tan(\theta/2) = \frac{r}{h} \Rightarrow r = 416.5$ km

NO, the satellite cannot cover all of USA because it's radius is smaller than USA.

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2. A transmit antenna produces a maximum electric field intensity at the far field in a certain direction given by: $|\vec{E}| = \frac{90}{I} e^{j\omega t} e^{-jkr}$ (V/m) Where I is the peak value of the antenna current. The input resistance of this lossless antenna is 50Ω. Find the maximum effective aperture of the antenna.

$P_{rad} = \frac{I^2 R}{2}$ & $W_{dir} = \frac{|E|^2}{2\eta}$ $U = r^2 W_{rad}$

$D = \frac{4\pi U}{P_{rad}} = \frac{r^2 90^2 I^2 (4\pi)}{\cancel{2} r^2 \eta} \times \frac{\cancel{2}}{I^2 R} = \frac{32,400\pi}{\cancel{2} R\eta}$

$\eta = 120\pi$ (air / vaccum) $\Rightarrow D = \cancel{2.7}\ 5.4$

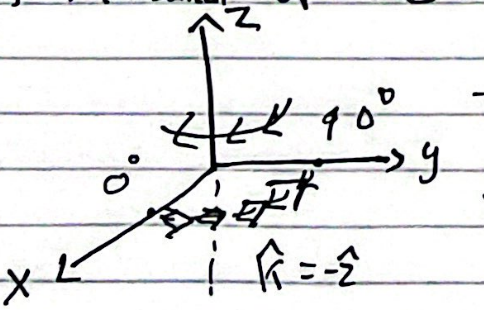
lossless $\rightarrow e_{cd} = 1$ and matched $|1 - |\Gamma|^2| = 0$

$G = D$

$A_e = D\frac{\lambda^2}{4\pi} = \cancel{\frac{0.675\lambda^2}{\pi}}\ \frac{1.35\lambda^2}{\pi}$ or $A_e = \cancel{\frac{81\lambda^2}{\eta}}$

3. The electric field of a uniform plane wave traveling along the negati[ve] z-direction is given by $\vec{E} = (\hat{x} + j\hat{y}) E_0 e^{jkz}$ and is incident wave ~~is given by~~ upon a receiving antenna placed at the origin and whose radiated electric fie[ld] towards the incident wave is given by $\vec{E_a} = (\hat{x} + 2\hat{y}) E_1 \frac{e^{-jkr}}{r}$ where E_0 and E_1 are co[nstants]

(a) Polarization of the incident wave?



~~RHCP~~
~~LHCP~~
RHCP

(b) Polarization of antenna?

~~Linear, because the magnitudes of $\hat{x}$ and $\hat{y}$ components are equal and the PI~~

Linear because the phase difference between $\hat{x}$ and $\hat{y}$ is 0°

(C) Loss due to polarization mismatch?

$PLF = |\hat{P}_a \cdot \hat{P}_b|^2 = \left|\frac{(\hat{x} + j\hat{y})E_0}{E_0\sqrt{2}} \cdot \frac{(\hat{x} + 2\hat{y})E_1}{E_1\sqrt{5}}\right|^2$

$|\vec{P_a}| = E_0\sqrt{2}$

$|\vec{P_b}| = E_1\sqrt{5}$

$= \left|\frac{1}{\sqrt{2}\sqrt{5}} + \frac{2j}{\sqrt{10}}\right|^2 = \left[\sqrt{\frac{1}{10} + \frac{4}{10}}\right]^2 = 1/2 \Rightarrow -3dB$

HW 1

(4) Consider the current density, $\vec{J} = (\alpha\hat{x} + j\beta\hat{y})\,\delta(x)\,\delta(y)\,\delta(z)\,e^{j\omega t}$ where α and Beta are real constants.

a. Find the far-field power density.

$$A = \frac{\mu}{4\pi}\int_V \vec{J}\,\frac{e^{-jkR}}{R}\,dv$$

Since $\vec{J}$ is point source, Integration is not required.

$$A = \frac{\mu}{4\pi}(\alpha\hat{x} + j\beta\hat{y})\frac{e^{-jkR}}{R}$$

$r = R$ (Point source)

$r = \sqrt{x^2+y^2+z^2}$

$E_\theta \approx -j\omega A_\theta$ and $E_\phi \approx -j\omega A_\phi$ ($E_r \approx 0$ at far-field)

$$A_\theta = \frac{\mu e^{-jkr}}{4\pi r}(\alpha\cos\theta\cos\phi + j\beta\cos\theta\sin\phi)$$

$$A_\phi = \frac{\mu e^{-jkr}}{4\pi r}(-\alpha\sin\phi + j\beta\cos\phi)$$

$$E_\phi = \frac{-j\omega\mu e^{-jkr}}{4\pi r}(-\alpha\sin\phi + j\beta\cos\phi)$$

$$E_\theta = \frac{-j\omega\mu e^{-jkr}}{4\pi r}(\alpha\cos\theta\cos\phi + j\beta\cos\theta\sin\phi)$$

$$P_{rad} = \oint\!\!\oint_\Omega U\,d\Omega = \int_0^{2\pi}\!\!\int_0^{\pi} U\sin(\theta)\,d\theta\,d\phi$$

$$U \simeq \frac{r^2}{2\eta}\left[|E_\theta(\theta,\phi)|^2 + |E_\phi(\theta,\phi)|^2\right]$$

$$U = \frac{\cancel{r^2}}{2\eta}\Big[\frac{\omega^2}{(4\pi r)^2}(\alpha^2\cos^2\theta\cos^2\phi + \beta^2\cos^2\theta\sin^2\phi) + \frac{\omega^2}{(4\pi r)^2}(\alpha^2\sin^2\phi + \beta^2\cos^2\phi)\Big]$$

$\cancel{P_{rad} = \frac{\omega^2}{2\eta}}$

$$P_{rad} = \frac{\omega^2}{32\eta\pi^2}\int_0^{2\pi}\!\!\int_0^{\pi}[\alpha^2(\cos^2\theta\cos^2\phi + \sin^2\phi) + \beta^2(\cancel{\cos^2\theta\sin^2\phi + \cos^2\phi})]\sin\theta\,d\theta\,d\phi$$

$$P_{rad} = \frac{\omega^2}{32\eta\pi^2}\left[\frac{8\pi\alpha^2}{3} + \frac{8\pi\beta^2}{3}\right] = \frac{\omega^2(\alpha^2+\beta^2)}{\cancel{32}\,12\pi\eta}$$

[Wolfram Alpha :)

$$D = \frac{U}{U_0}$$

$$U_0 = \frac{P_{rad}}{4\pi} = \frac{\omega^2(\alpha^2+\beta^2)}{48\pi^2\eta}$$

$$U_0 = \frac{\omega^2(\alpha^2+\beta^2)}{48\pi^2\eta}$$

$\cancel{U=}$

$$D = \frac{\omega^2}{32\eta\pi^2}\left[\alpha^2\underbrace{(\cos^2\theta\cos^2\phi + \sin^2\phi)}_{1} + \beta^2\underbrace{(\cos^2\theta\sin^2\phi + \cos^2\phi)}_{1}\right] \times \frac{48\pi^2\eta}{\omega^2(\alpha^2+\beta^2)}$$

$$\max D_0 = \frac{48}{32(\alpha^2+\beta^2)}\left[\alpha^2+\beta^2\right] = \frac{3}{2\cancel{(\alpha^2+\beta^2)}}\cancel{[\alpha^2+\beta^2]}$$

$$\max D_0 = 3/2$$

According to 3D graph the antenna becomes most isotropic when $\alpha=\beta$ and as α and β approach 0.

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5. Consider the magnetic current density $\vec{M} = (\alpha\hat{x} + \beta\hat{y})\delta(z)$, $-\frac{L_x}{2} \leq x \leq \frac{L_x}{2}$, $-\frac{L_y}{2} \leq y \leq \frac{L_y}{2}$ and $L_x, L_y \ll \lambda$ where α and β are complex constants.

a. Calculate the far field pattern for $\alpha = 1$ and $\beta = 2$ and Identify field at $(\theta = 90, \phi = 0)$, $(\theta = 90, \phi = 90)$ and $(\theta = 0)$

$$F = \frac{\varepsilon}{4\pi} \int_V \vec{M} \frac{e^{-jkR}}{R} dv \implies \vec{F} = \frac{\varepsilon e^{-jkr}}{4\pi r} [\alpha\hat{x} + \beta\hat{y}]$$

(Point source)

$E_r \approx 0$; $E_\theta \approx -j\omega\eta F_\phi$ $\quad E_\phi = j\omega\eta F_\theta$

$$F_\phi = \frac{\varepsilon e^{-jkr}}{4\pi r} [-\alpha \sin\phi + \beta\cos\phi] \quad F_\theta = \frac{\varepsilon e^{-jkr}}{4\pi r} [\alpha\cos\theta\cos\phi + \beta\cos\theta\sin\phi]$$

$$E_\theta \approx \frac{-j\omega\eta\varepsilon e^{-jkr}}{4\pi r} [-\alpha\sin\phi + \beta\cos\phi] \quad E_\phi = \frac{j\omega\eta\varepsilon e^{-jkr}}{4\pi r} [\alpha\cos\theta\cos\phi + \beta\cos\theta\sin\phi]$$

$[\theta = 90, \phi = 0]$ $\quad E_r \approx 0 \quad E_\theta \approx \frac{-\beta j\omega\eta\varepsilon e^{-jkr}}{4\pi r} \quad E_\phi = 0$

$[\theta = 90, \phi = 90]$ $\quad E_r \approx 0 \quad E_\theta \approx \frac{\alpha j\omega\eta\varepsilon e^{-jkr}}{4\pi r} \quad E_\phi = 0$

$[\theta = 0]$ $\quad E_r \approx 0$

$$E_\theta \approx -\frac{j\omega\eta\varepsilon e^{-jkr}}{4\pi r} [-\alpha\sin\phi + \beta)$$

$$E_\phi = \frac{j\omega\eta\varepsilon e^{-jkr}}{4\pi r} [\alpha\cos\phi + \beta\sin\phi]$$

a)

$[\theta = 90][\phi = 90]$

$$E_\theta = \frac{j\omega\eta\varepsilon e^{-jkr}}{4\pi r}$$

$E_\phi = E_r = 0$

$[\theta = 0]$

$$E_\theta = \frac{-j\omega\eta\varepsilon e^{-jkr}}{4\pi r} [-\sin\phi + 2]$$

$$E_\phi = \frac{j\omega\eta\varepsilon e^{-jkr}}{4\pi r} [\cos\phi + 2\sin\phi]$$

(b) Choose α and β to yield Circular Polarization at $(\theta=0)$

$$E_\theta = \frac{-j\omega\eta\varepsilon e^{-jkr}}{4\pi r}\left[-\alpha\sin\phi + \beta\cos\phi\right] \qquad [\theta=0]$$

$$E_\phi = \frac{j\omega\eta\varepsilon e^{-jkr}}{4\pi r}\left[\alpha\cos\theta\cos\phi + \beta\cos\theta\sin\phi\right]$$

$\downarrow \theta=0$

E Components:

$\theta \overset{\alpha}{=} -\alpha\sin\phi + \beta\cos\phi \qquad \phi \overset{\alpha}{=} \alpha\cos\phi + \beta$

When alpha=1 and Beta=0

$\underbrace{\theta \overset{\alpha}{=} -\sin\phi \qquad \phi \overset{\alpha}{=} \cos\phi}_{90^\circ \text{ difference and same magnitude}}$

(C) Choose α and β to yield Circular Polarization $(\theta=90^\circ, \phi=0)$

$$E_\theta = \frac{-j\omega\eta\varepsilon e^{-jkr}}{4\pi r}\left[-\alpha\sin\phi + \beta\cos\phi\right]$$

$$E_\phi = \frac{j\omega\eta\varepsilon e^{-jkr}}{4\pi r}\left[\alpha\cancel{\cos\theta\cos\phi} + \beta\cancel{\cos\theta\sin\phi}\right]$$

$\downarrow$

E components:

$\theta \overset{\alpha}{=} \beta\cancel{\cos\phi} \qquad \phi \overset{\alpha}{=} \alpha$

$\underbrace{\alpha = j \qquad \beta = 1}_{90^\circ \text{ and same magnitude}}$

This multiplied by $e^{j\omega t}$ so they will oscillate but with 90° Phase difference and same magnitude.