HW 2

1. $EF \propto \sin\theta$

$$AF = 1 + e^{jkd\cos\theta} + e^{-jkd\cos\theta}$$

$$= 1 + 2\cos(kd\cos\theta)$$

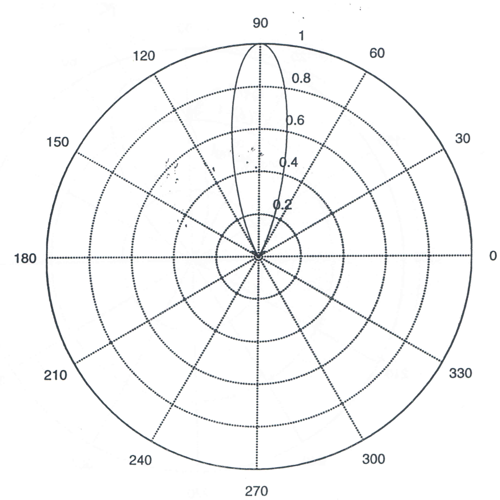
$d = 0.6\lambda$

$kd = \frac{2\pi}{\lambda} \cdot 0.6\lambda = 1.2\pi$

$$\text{Pattern} \propto \sin^2\theta\left[1 + 2\cos(1.2\pi\cos\theta)\right]^2$$

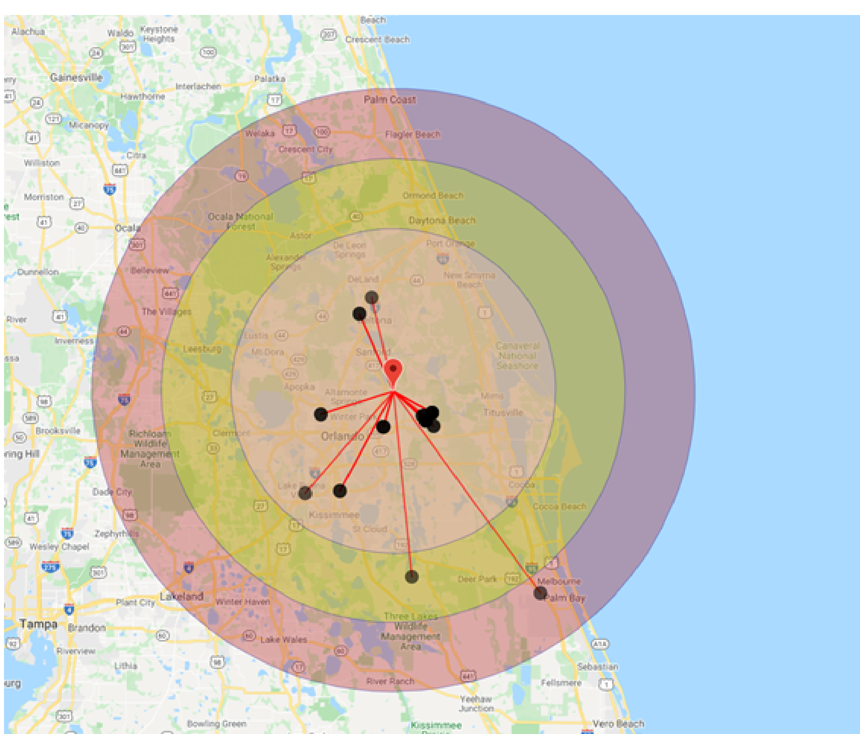
See Plot:

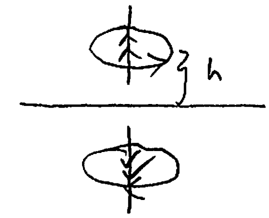
$$D = \frac{4\pi U_{max}}{P_{rad}} = 3.9$$



2. If $\frac{S}{N}$ ratio is sufficient, you need a less-directive antenna to receive more channels.

See TV stations around UCF:





3. (a) $E_\phi = \eta \dfrac{(ka)^2 I_o e^{-jkr}}{4r} \sin\theta$

$$|AF| = |e^{-jkh\cos\theta} - e^{+jkh\cos\theta}|$$

$$= |2j \sin(kh\cos\theta)|$$

$$E_{\phi,\,total} = \eta \frac{(ka)^2 I_o e^{-jkr}}{4r} \sin\theta \cdot 2\sin(kh\cos\theta) \quad \begin{Bmatrix} \text{above} \\ \text{the ground} \end{Bmatrix}$$

(b) $h = \lambda \qquad k \cdot h = \frac{2\pi}{\lambda} \cdot \lambda = 2\pi$

$$\sin\theta \cdot 2\sin(2\pi\cos\theta) = 0$$

$\sin\theta = 0 \Rightarrow \theta = 0^\circ$

$\sin(2\pi\cos\theta) = 0 \qquad 2\pi\cos\theta = n\pi, \quad n = 0, 1, 2$

$n=1 \qquad 2\pi\cos\theta = \pi \qquad \cos\theta = \frac{1}{2} \qquad \theta = 60^\circ$

$n=2 \qquad 2\pi\cos\theta = 2\pi \qquad \cos\theta = 1 \qquad \theta = 0^\circ$

$n=0 \qquad 2\pi\cos\theta = 0 \qquad \cos\theta = 0 \qquad \theta = 90^\circ$

Therefore, nulls occure at $0^\circ, 60^\circ, 90^\circ$

(c) $\theta = 60^\circ \qquad \sin\theta \cdot \sin(kh\cos\theta) = 0$

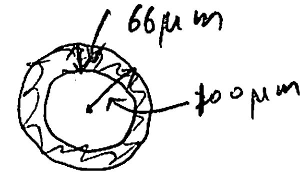
$$\sin\left[kh\tfrac{1}{2}\right] = 0$$

$$\frac{kh}{2} = n\pi, \quad n = 0, 1, 2, \cdots$$

$$\therefore \frac{\cancel{2}\cancel{\pi} h}{\cancel{2}\lambda} = n\cancel{\pi} \Rightarrow \underbrace{h = n\lambda}_{\text{nonzero}}, \quad n = 1, 2, 3, \cdots$$

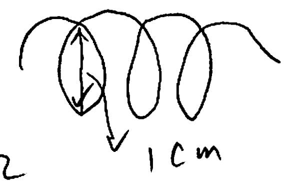
4.

skin depth @ 1 MHz $\quad \delta = \dfrac{1}{\sqrt{\pi f \mu \sigma}} = 66\,\mu m$



$A_{wire} = \pi \times (100\mu m)^2 - \pi\left[(100 - 60)\mu m\right]^2$

$= 2.6 \times 10^{-8}\ m^2$



$R_s = \dfrac{l}{\sigma A_{wire}} = \dfrac{\overset{(1cm)}{\pi \times 0.01} \times \overset{(\#\text{ of turn})}{100}}{5.8 \times 10^7 \times 2.6 \times 10^{-8}} = 2.1\ \Omega$

$R_r = 20\pi^2 \left(\dfrac{\pi \cdot 0.01}{\lambda}\right)^4 \times 100^2 \cdot \mu_r^2 \qquad \lambda = \dfrac{c}{f} = \dfrac{3 \times 10^8}{10^6} = 300m$

$= 2.4\ \mu\Omega$

$\eta_{loop} = \dfrac{R_r}{R_r + R_s} = \dfrac{2.4\mu\Omega}{2.4\mu\Omega + 2.1\ \Omega} \approx \dfrac{2.4\mu\Omega}{2.1\Omega} \approx 1.1 \times 10^{-6}$

Monopole at 1 MHz . 98 MHz $\quad \lambda @ 98MHz \approx \dfrac{c}{100MHz} = 3m$

$l = \dfrac{\lambda @ 98MHz}{4} = 0.75m$

@ 1MHz $\quad \dfrac{l}{\lambda @ 1MHz} = \dfrac{0.75}{300} = 2.5 \times 10^{-3}$

$R_s = \dfrac{l}{\sigma A_{wire}} = \dfrac{0.75}{5.8 \times 10^7 \times 2.6 \times 10^{-8}} = 0.5\ \Omega$

$R_r = \underset{\downarrow \text{ monopole}}{\dfrac{1}{2}}\ \underset{80}{20}\ \pi^2 \left(\dfrac{l}{\lambda}\right)^2 = 10 \times \pi^2 \times \left(\dfrac{0.75}{300}\right)^2 = \underset{24.8}{6.2} \times 10^{-4}\ \Omega$

$\eta_{monopole} = \dfrac{R_r}{R_r + R_s} = \dfrac{\overset{24.8}{6.2} \times 10^{-4}}{\underset{24.8}{6.2} \times 10^{-4} + 0.5} \approx \dfrac{\overset{24.8}{6.2} \times 10^{-4}}{0.5} = \overset{49.6}{1.2} \times 10^{-\overset{4}{3}}$ =0.5%

1000 times larger.
500

5. $EF \propto \sin\theta \ \hat{\theta}$

$$AF = 2j \underbrace{\sin(kh \sin\theta \sin\phi)}_{\text{Ground plane}} \underbrace{2\cos(kh \sin\theta \cos\phi)}_{\text{Array in } x\text{-direction}}$$

$h = \frac{\lambda}{4}$ $\quad kh = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$

Total pattern $\propto \left[\sin\theta \cdot \sin\left(\frac{\pi}{2}\sin\theta\sin\phi\right)\cos\left(\frac{\pi}{2}\sin\theta\cos\phi\right)\right]^2 \hat{\theta}$

E plane : Y-z plane $\phi = 90^\circ$

pattern $\propto \left[\sin\theta \ \sin\left(\frac{\pi}{2}\sin\theta\right)\right]^2$ $\quad \theta \in [0, \pi]$

H plane $\quad$ x-y plane $\theta = 90^\circ$

pattern $\propto \left[\sin\left(\frac{\pi}{2}\sin\phi\right)\cos\left(\frac{\pi}{2}\cos\phi\right)\right]^2$ $\quad \phi \in [0, 2\pi]$