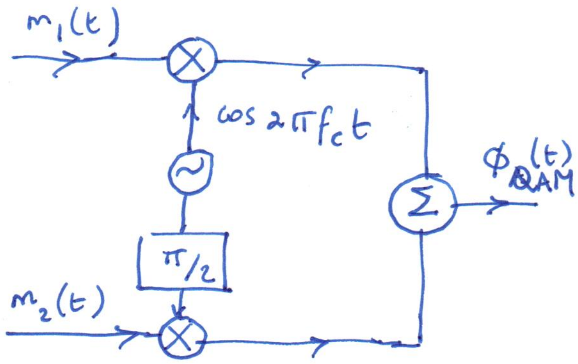
Lecture 17

IV) Quadrature Amplitude Modulation (QAM)

The idea is to transmit signals on carriers with the same frequency but with $\frac{\pi}{2}$ phase shift (quadrature phase)



$m_1(t)$ and $m_2(t)$ are baseband binary polar pulse sequences.

Using QAM, we transmit both signals on the same channel, thus doubling the transmission rate.

Since both signals are PSK, this is also known as QPSK.

M-ary digital carrier modulation:

We can generalize binary modulation by employing M-ary signaling, to get M-level ASK, M-freq. FSK, M-phase PSK.

Next, we talk about these different schemes.

① M-ary ASK

Recall, in binary ASK, '0' is transmitted as $\phi(t) = 0$ and '1' as $\phi(t) = A\cos\omega_c t$ (WLOG we assume square pulses).

In M-ary ASK, we use $\phi(t) = 0, A\cos\omega_c t, 2A\cos\omega_c t, \ldots (M-1)A\cos\omega_c t$ to transmit M different symbols ($\log_2 M$ bits at a time).

Compared to binary, we can use same BW, M^2 times the power. Demodulation can be done using an envelope detector.

An example of a M-ary ASK signal is shown below:

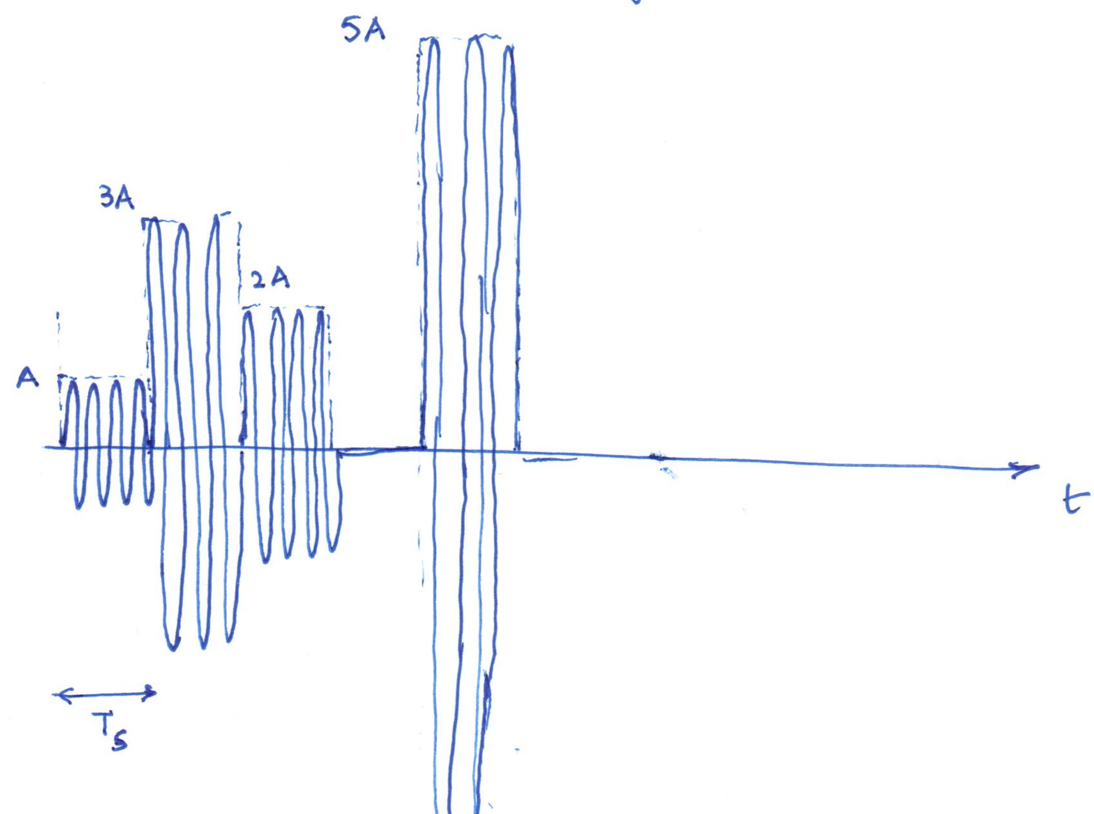
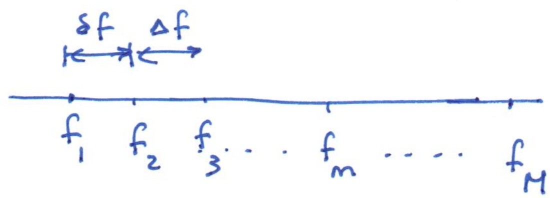


Figure: M.ASK signal.

② M-ary FSK and orthogonal signaling

A Sinusoid set $\{A\cos(2\pi f_i t),\ i=1,\dots,M\}$ is used to transmit $\log_2 M$ bits at a time. Assuming equal frequency separation,

$$f_m = f_1 + (m-1)\,\delta f \qquad \text{where } \delta f \text{ is the frequency increment.}$$



If δf is large $\Rightarrow$ large BW

If δf is small $\Rightarrow$ smaller BW but more error prone.

One possibility to address this tradeoff is to use an orthogonal set of FSK signals, i.e.

$$(1)\text{-----}\quad \boxed{\int_0^{T_s} A^2\cos(2\pi f_m t)\cos(2\pi f_n t)\,dt = 0 \qquad, m\neq n}$$

Hence, the goal is to solve the optimization problem

$$\min\ \delta f \qquad \text{subject to constraint (1)}$$

Noting that $\displaystyle\int_0^{T_s} A^2\cos(2\pi f_m t)\cos(2\pi f_n t)\,dt$

$$= \frac{A^2}{2}\left[\underbrace{\int_0^{T_s}\cos\big(2\pi(f_n+f_m)t\big)\,dt}_{\text{Zero}} + \int_0^{T_s}\cos 2\pi(f_m-f_n)t\ dt\right]$$

$$= \frac{A^2}{2}\ \frac{\sin 2\pi(f_m-f_n)T_s}{2\pi(f_m-f_n)},$$

we require that $\sin 2\pi(f_m-f_n)T_s = 0$ for $m\neq n$

Since $f_m = f_1 + (m-1)\delta f$, we get that

$$\sin\left(2\pi (m-n)\, \delta f\, T_s\right) = 0 \quad , \quad m \neq n$$

$\Rightarrow$ The least $\delta f = \dfrac{1}{2T_s}$ Hz

This yields a scheme known as the minimum shift FSK (MFSK)

<u>Note</u>: M-ary FSK requires M-times the BW of binary signaling since it's an <u>M-ary orthogonal scheme</u>. Hence, rate increases by a factor of $\log_2 M$ at the expense of M-fold the transmission BW increase. But power can be independent of M (compare to M-ary ASK)

③ M-ary PSK

$$\phi_{PSK} = \sqrt{\frac{2}{T_s}}\, A\cos(\omega_c t + \theta_m) \qquad m = 1, 2, \ldots, M$$

$$= \sqrt{\frac{2}{T_s}}\, A\left(\cos\theta_m \cos\omega_c t - \sin\theta_m \sin\omega_c t\right) \qquad , 0 \leq t \leq T_s$$

$$= a_m \sqrt{\frac{2}{T_s}} \cos\omega_c t + b_m \sqrt{\frac{2}{T_s}} \sin\omega_c t$$

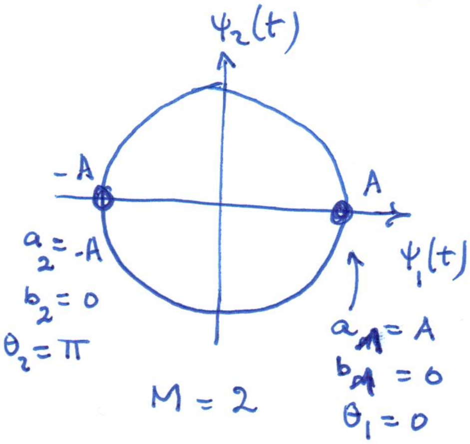
where $\boxed{a_m \triangleq A\cos\theta_m}$, $\boxed{b_m \triangleq A\sin\theta_m}$ $\qquad m = 1, \ldots, M$

Define, $\psi_1(t) = \sqrt{\frac{2}{T_s}} \cos\omega_c t$, $\psi_2(t) = \sqrt{\frac{2}{T_s}} \sin\omega_c t$

as an orthonormal basis (functions are orthogonal and have unit energy)

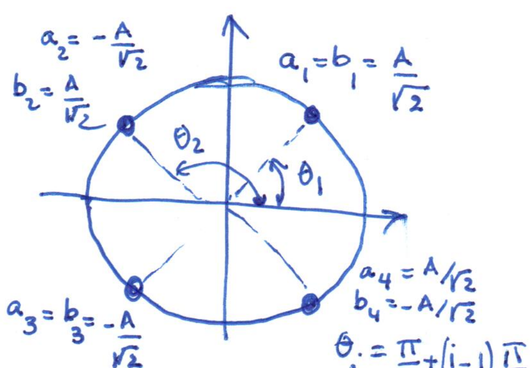
$$\int_0^{T_s} \psi_i(t)\, \psi_j(t)\, dt = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \qquad i, j \in \{1, 2\}$$

Note: $a_m^2 + b_m^2 = A^2$ = constant. This yields the geometric representations (constellation diagram) shown.



M = 2

BPSK



$\theta_i = \frac{\pi}{4} + (i-1)\frac{\pi}{2}$

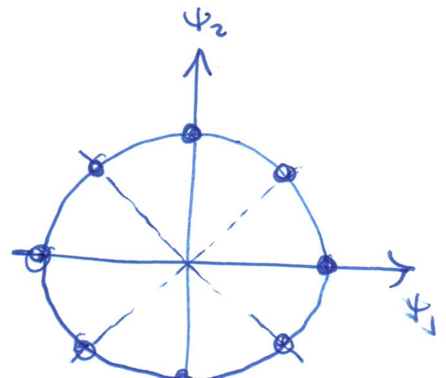
$= i\frac{\pi}{2} - \frac{\pi}{4}$

$i = 1, \ldots, 4$

M = 4

QPSK

4-PSK



$\theta_i = (i-1)\frac{\pi}{4}$

M = 8

8-PSK

$$\boxed{\theta_m = \theta_0 + \frac{2\pi}{M}(m-1)}$$

$m = 1, 2, \ldots M$

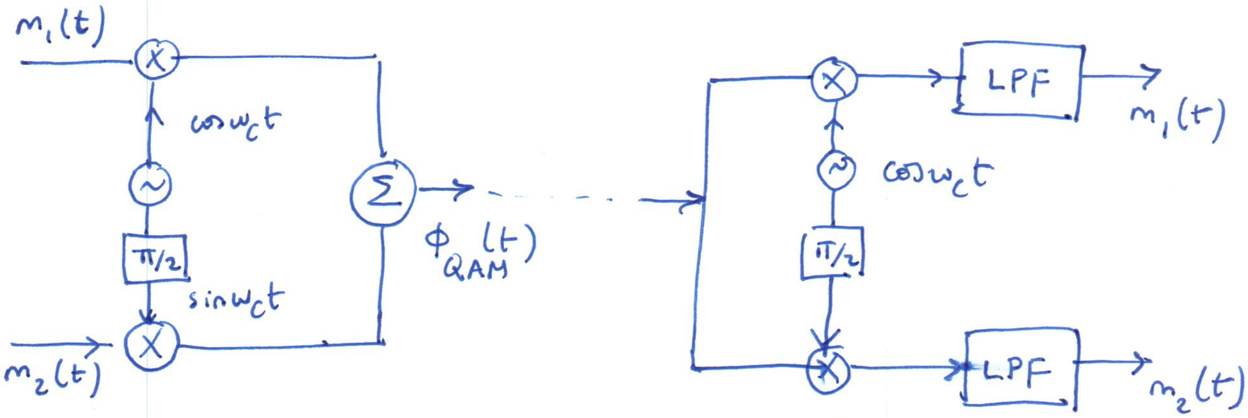
M-ary QAM

$$\phi_{QAM}(t) = (a_i \cos \omega_c t + b_i \sin \omega_c t)\, p_i(t) \quad , \quad 0 \le t \le T_s,\ i = 1 \dots M$$

where $p_i(t)$ is a properly shaped baseband pulse (e.g. rectangular)

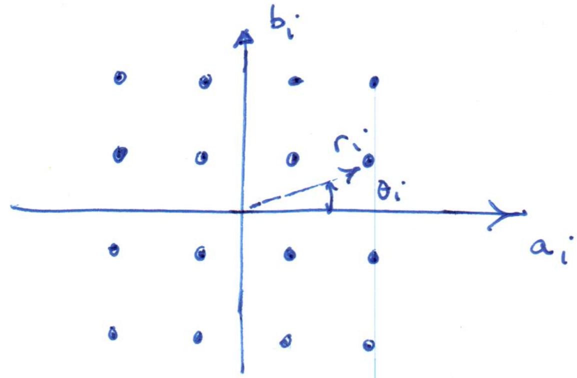
$$\Rightarrow \quad \phi(t) = r_i \cos(\omega_c t - \theta_i)\, p_i(t),$$

where $r_i = \sqrt{a_i^2 + b_i^2}$ $\quad \theta_i = +\tan^{-1} \frac{b_i}{a_i}$



$m_1(t)$, $m_2(t)$ $\quad \sqrt{M}$-ary baseband signals

$m_1(t) = a_i\, p_i(t) \qquad m_2(t) = b_i\, p_i(t)$



16 possible signals

$\log_2 16 = 4$ bits/symbol

16-QAM