HW5

1. Calculate:
   a. The resonant length and self-impedance of a narrow slot (w/λ=0.016 and negligible thickness) at the first and second resonance. For the second resonance, you need to approximate the input impedance of a 1-λ-long dipole.

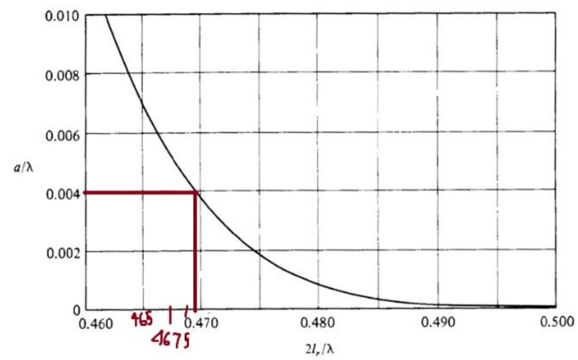


Fig. 7.13 Resonant Length versus Radius for Center-Fed Cylindrical Dipoles

Optimal Length for first resonance.

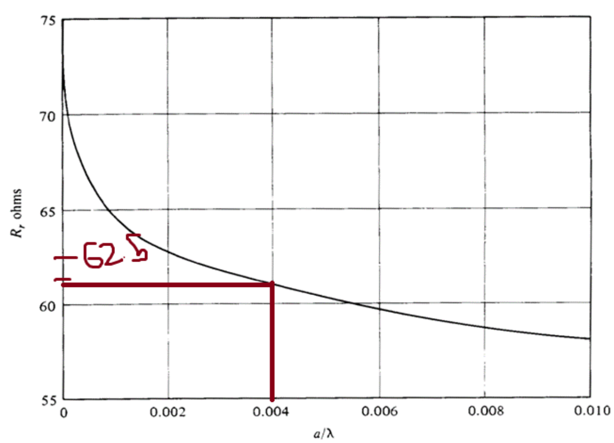
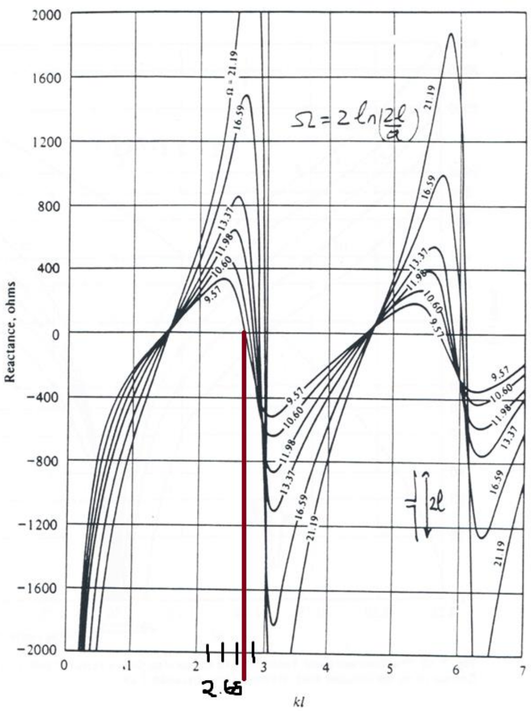


Fig. 7.14 Resonant Resistance versus Radius for Center-Fed Cylindrical Dipoles

Input Impedance for first resonance.



Optimal Length for Second Resonance.

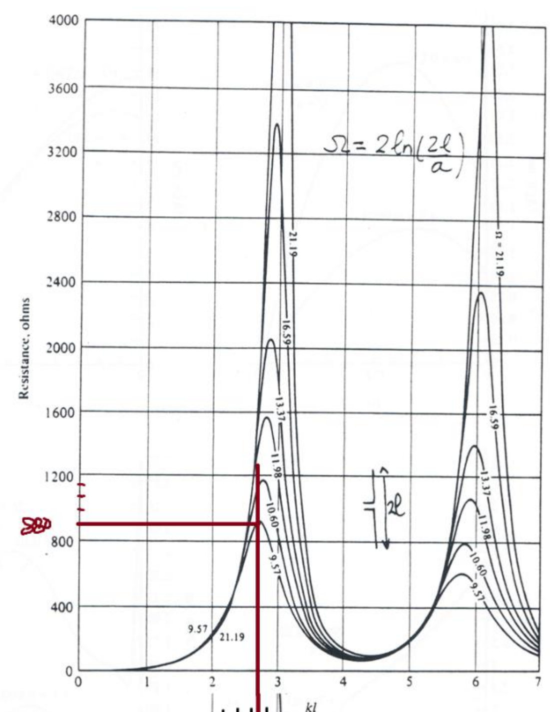


Fig. 7.15a Hallén's Curves of I... a Center-Fed Cylindrical Dipole versus kl and Ω (Reprinted from E. Hallé... oratory Report No. 46, 1946, Courtesy of Harvard University.)

Input Impedance for Second Resonance.

HW Q1

(1)

Effective Radius: $a_e = 0.25a$ $W/\lambda = 0016$

$a_e = 0.004\lambda$

first resonace around $\lambda/2$ and Second resonace around λ

$\lambda/2$

(See Plot)

Resonance length: 0.469λ

$Z \cong 61\Omega$

↓

Bocker Relationship

$Z_s Z_c = \eta^2/4$

$Z_c \cong 582\Omega$

λ

$k\ell = \frac{2\pi}{\lambda} \cdot \lambda \cdot 1/2 = \pi$

$\Omega = 2\ln\left(\frac{2\ell}{a}\right) = 11.04$

$R \approx 880\Omega$

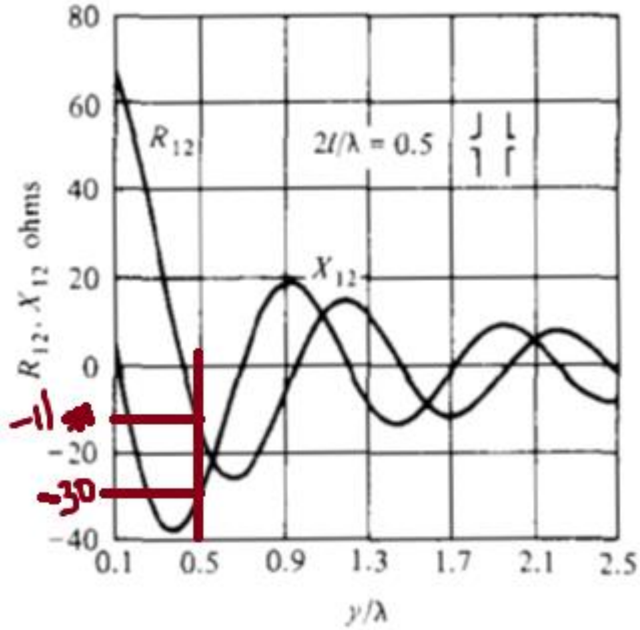
Resonance length: $k\ell = 2.65$

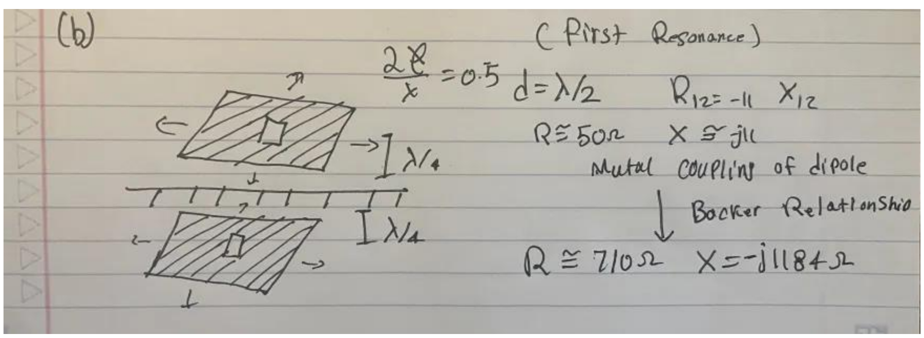
$2\ell = 0.8435\lambda$

↓ Bocker Relationship

$Z_c \cong 40.37\Omega$

b. The mutual impedance and new input impedance of the slot antenna at first resonance when placed λ/4 away from a ground plane. Do it using (1) image theory and slot antenna and (2) the complete dual and replacing all metals with air and all airs with metal. Prove that they are the same.





2. Consider the patch antenna with thickness t<<λ, dielectric constant ε_r, and dimension (a, b). for (m, n)=(0,1), and ε_r=2.2 and 10.2 (two cases),
   a. Calculate the dimensions of the antenna to resonate at 1.57 GHz. Choose b=1.4a.
   b. Draw the magnetic current and electric field distribution on the faces x=0, x=a, y=0, y=b. For one case only.

HW 5 (2)

(2) $f = 1.57\,GHz$ $\varepsilon_{eff} \cong \varepsilon_r$ because $h \ll \lambda$

(a)

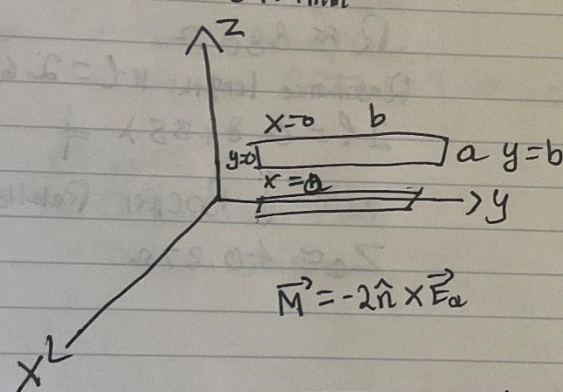
$$f = \frac{1}{2L_{eff}\sqrt{\varepsilon_{reff}}\sqrt{\mu_0\varepsilon_0}} \Longrightarrow \frac{1.57\,GHz}{3\times10^8} = \frac{1}{2L_{eff}\sqrt{\varepsilon_{reff}}}$$

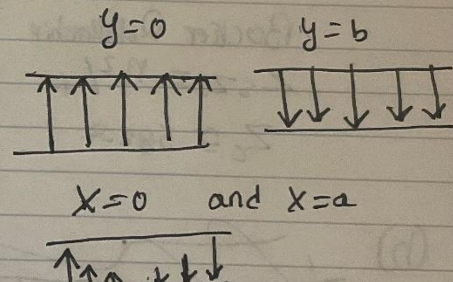
$$\frac{1}{L_{eff}} = \frac{2\sqrt{\varepsilon_{reff}}\ 1.57\,GHz}{3\times10^8}$$

$W = h/1.4$

$\varepsilon_r = 2.2$
$W = 46\,mm$
$L = 64.4\,mm$

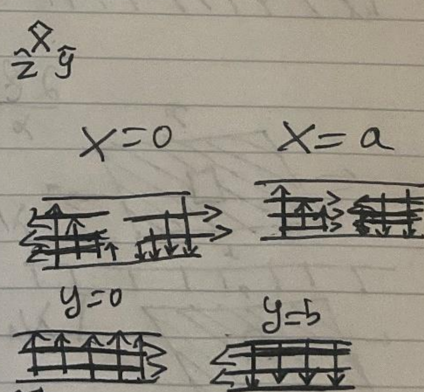
$\varepsilon_r = 10.2$
$W = 21.4\,mm$
$L = 29.9\,mm$





$\vec{M} = -2\hat{n}\times\vec{E}_a$

$\boxed{\text{Magnetic current}}$



$y=0$
$\hat{n} = -\hat{y}$ $\vec{E}_a = \hat{z}$
$\hat{M} = -(-\hat{y}\times\hat{z}) = \hat{x}$

$y=b$
$\hat{n} = \hat{y}$
$\hat{M} = -\hat{x}$

$x=0$ $x=b$ $\hat{E}_a = [\hat{z}, -\hat{z}]$
$\hat{n} = -\hat{x}$ $\hat{n} = \hat{x}$
$\hat{M}_0 = [-\hat{y}, \hat{y}]$ $\hat{M}_a = [\hat{y}, -\hat{y}]$
~~$\hat{M} = [$~~

c. Calculate and plot the E, H, and phi=52 degree patterns by considering the magnetic currents on the faces y=0, y=b (linear polar). Calculate the directivity. Why the directivity of the antenna with ε_r=2.2 is higher? Which antenna is better for GPS application?

I would choose a higher directivity for GPS application because it has wide enough beamwidth to cover the sky. We need a higher directivity to use less power to communicate with satellites. However, the directivity cannot be too high since we cannot assume that the antenna will be pointed at the GPS satellites very accurately. Also, we must communicate with at least 4 GPS satellites.

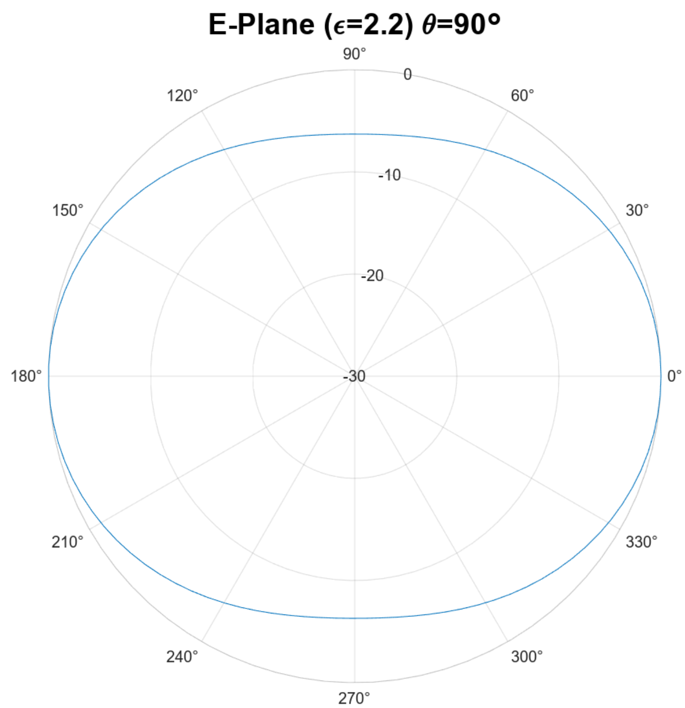
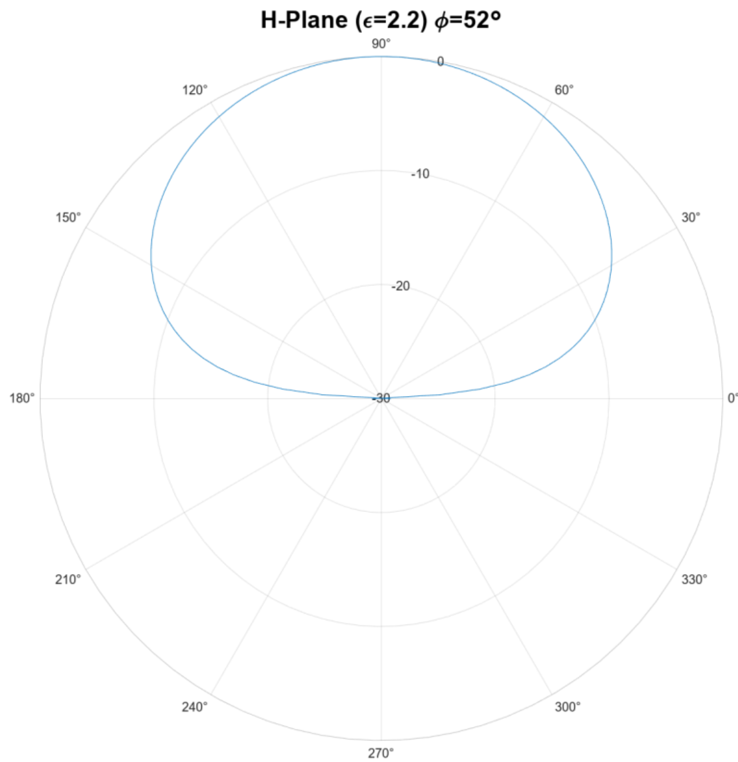
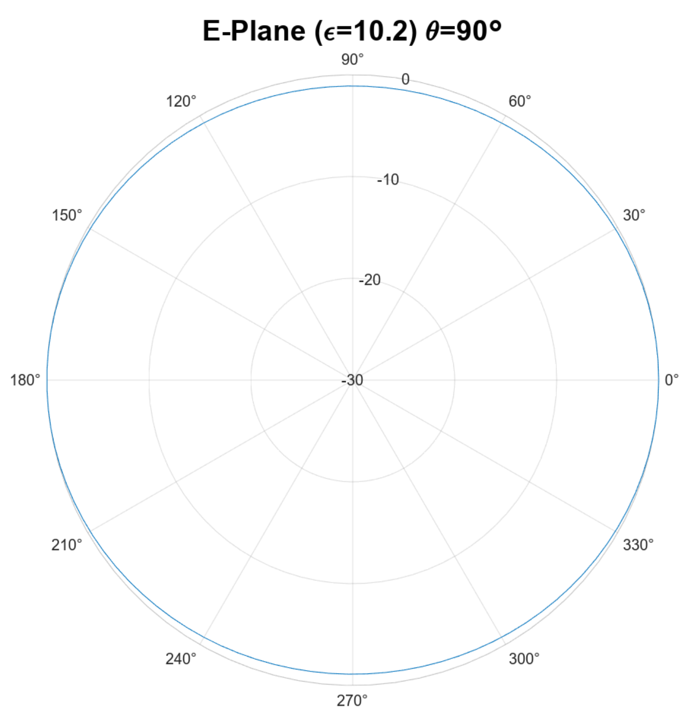
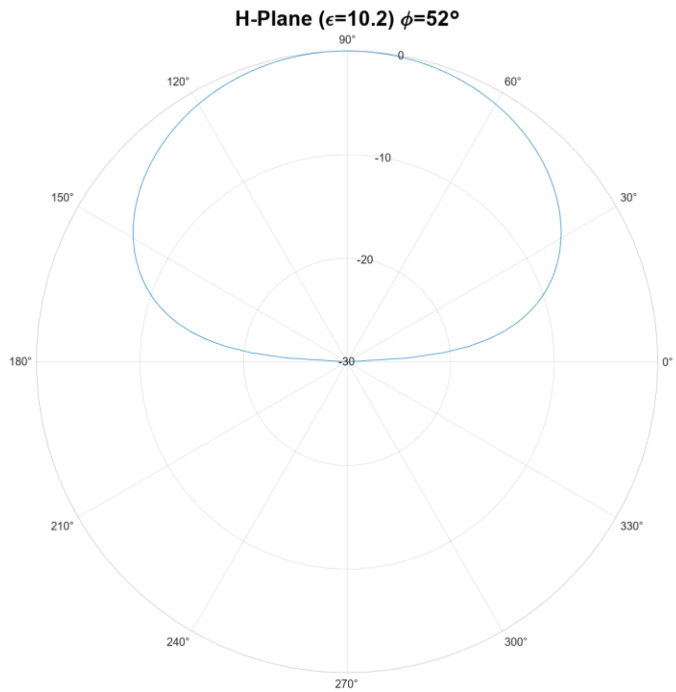


Figure 1


H-Plane (ϵ=2.2) ϕ=52°
90°
60°
30°
0°
330°
300°
270°
240°
210°
180°
150°
120°
0
-10
-20
-30

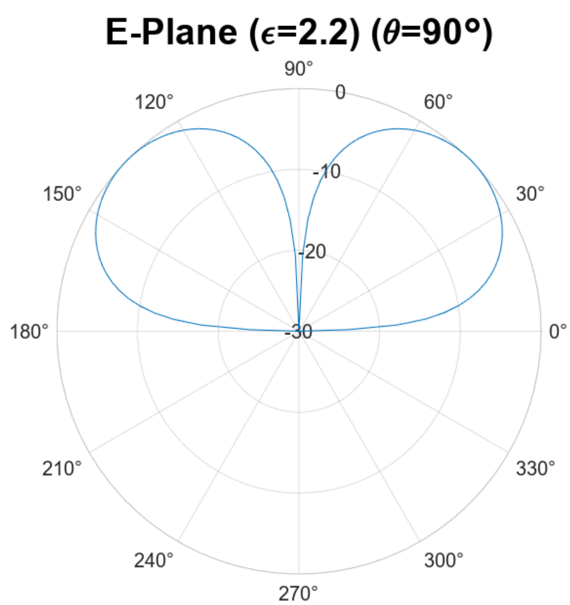
1.92 dBi


E-Plane (ϵ=10.2) θ=90°
90°
60°
30°
0°
330°
300°
270°
240°
210°
180°
150°
120°
0
-10
-20
-30

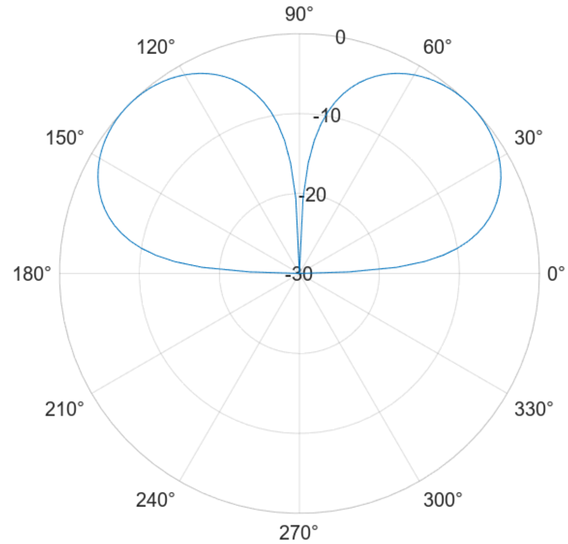


Directivity = 1.8 dBi

d. Calculate and plot the E, H, and phi=52 degree patterns by considering the magnetic currents on the faces x=0, x=a (linear polar). Compare with the co-pol. Pattern in (3). Make sure they are scaled to the same constant so that you can compare their relative magnitude. What is the cross-pol. Level in dB for the two cases?



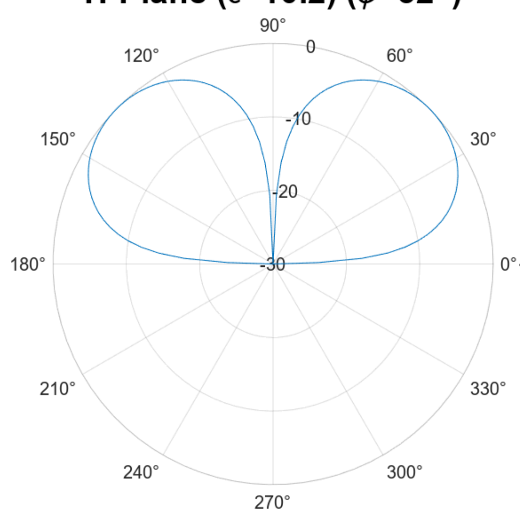
H-Plane (ϵ=2.2) (ϕ=90°)



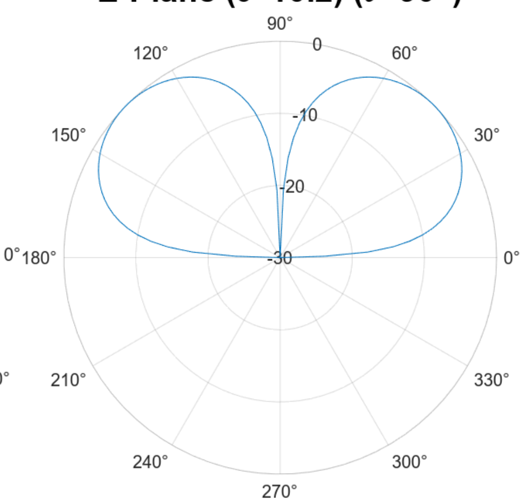
Directivity = 6.98 dBi

- The plots in part D are the cross pols for part C, unfortunately I ran out of time so I couldn't plot them together. I would normalize the plots of part D with the plots of part C magnitude to determine the cross-pol level. From there I would determine the dBi level of the cross pol.

H-Plane (ϵ=10.2) (ϕ=52°)



E-Plane (ϵ=10.2) (θ=90°)



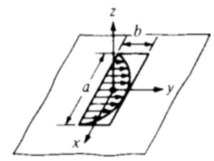
3. For a horn antenna with TE_{20} mode on the x-axis with a=8λ (x-axis) and b=5λ (y-axis):
    a. Calculate the far field pattern. Plot the E and H plane patterns together on a rectangular, -30dB-0dB range, for 0<θ<90°.
    b. Find the peak radiation angle. Calculate the directivity and aperture efficiency at the peak radiation angle.

Far-zone fields

$$X = \frac{ka}{2} \sin\theta \cos\phi$$

$$Y = \frac{kb}{2} \sin\theta \sin\phi$$

$$C = j\frac{abkE_0 e^{-jkr}}{2\pi r}$$



$$\mathbf{M}_s = \begin{cases} -2\hat{\mathbf{n}} \times \mathbf{E}_a & \left.\begin{array}{l} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{array}\right\} \\ 0 & \text{elsewhere} \end{cases}$$

$$\mathbf{J}_s = 0 \qquad \text{everywhere}$$

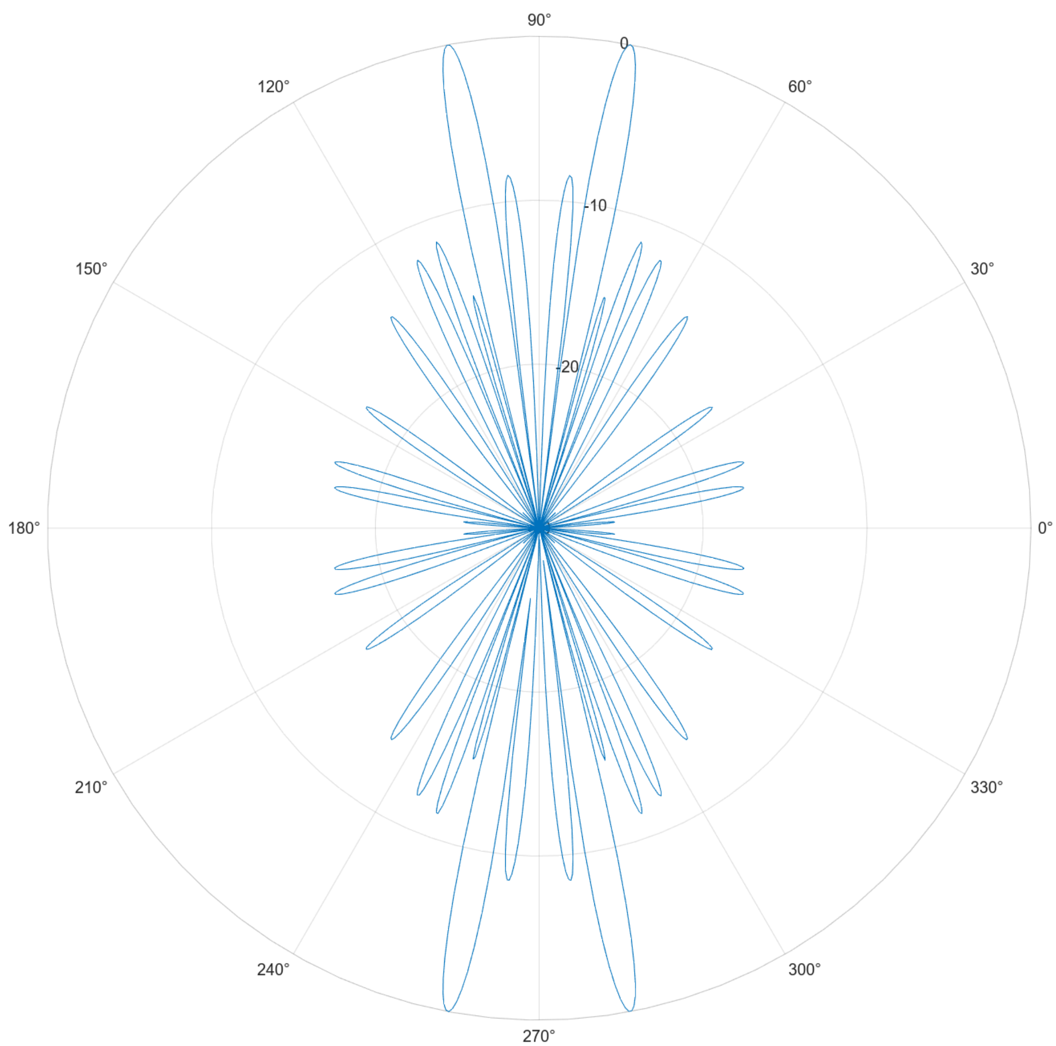
$$E_r = H_r = 0$$

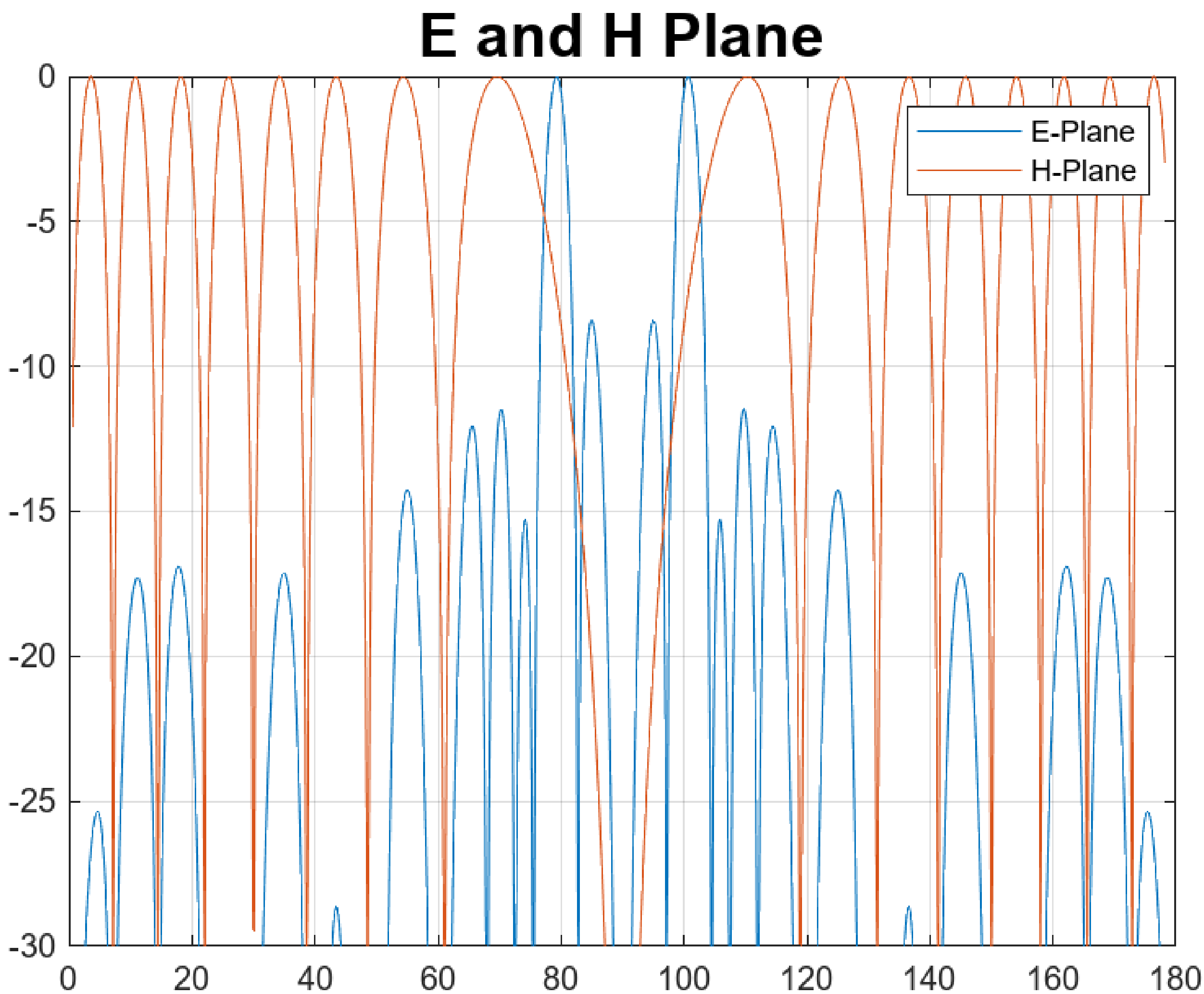
$$E_\theta = -\frac{\pi}{2} C \sin\phi \frac{\cos X}{(X)^2 - \left(\frac{\pi}{2}\right)^2} \frac{\sin Y}{Y}$$

$$E_\phi = -\frac{\pi}{2} C \cos\theta \cos\phi \frac{\cos X}{(X)^2 - \left(\frac{\pi}{2}\right)^2} \frac{\sin Y}{Y}$$

$$H_\theta = -E_\phi/\eta$$

$$H_\phi = E_\theta/\eta$$



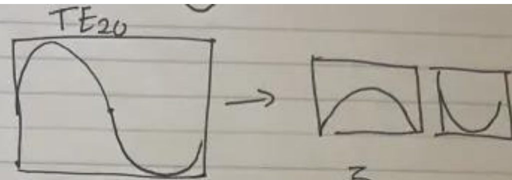


$$\boxed{\frac{8}{\pi^2}\left[4\pi\left(\frac{ab}{\lambda^2}\right)\right] = 0.81\left[4\pi\left(\frac{ab}{\lambda^2}\right)\right]}$$

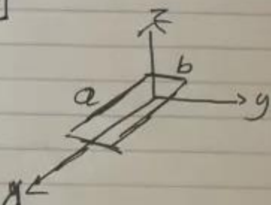
The peak radiation angle are the peaks in E and H plane plot.

(3) (a)

TE20



$a = 8\lambda \quad b = 5\lambda$



AF : $d = a/2 = 4\lambda$

$AF = e^{jkd\cos\psi_x} - e^{-jkd\cos\psi}$

$\psi = \hat{a}_r \cdot \hat{x} = \sin\theta\cos\phi$

$AF = e^{-j4\lambda \cdot \frac{2\pi}{\lambda} \cdot \sin\theta\cos\phi} - e^{-j4\lambda \cdot \frac{2\pi}{\lambda}\sin\theta\cos\phi}$

$AF = e^{-j8\pi\sin\theta\cos\phi} - e^{j8\pi\sin\theta\cos\phi} = 2\cos(8\pi\sin\theta\cos\phi)$

Directivity: $\frac{8}{\pi^2}\left[4\pi\left(\frac{ab}{\lambda^2}\right)\right] = 0.81\left[4\pi\left(\frac{a/b}{\lambda^2}\right)\right.$

$= 0.81(4\pi \cdot \cancel{40}\ 20)$

$= 251$ (linear

$= 24$ dBi

Directivity (AF) : 3 dBi

Total $D_0 = 3 + 24 \cancel{F}\ 27$ dBi

$A_{em} = \frac{\lambda^2}{4\pi} \cdot D_0 \quad A_{eff} = \frac{A_{em}}{ab} = \frac{\lambda^2}{4\pi} D_0 \cdot \frac{1}{40\lambda^2} = \frac{D_0}{160\pi}$

$A_{eff} = 99.7\%$

```
clear all
close all
clc
syms theta phi lambda
a=5*lambda;
b=8*lambda;
k=(2*pi)/lambda;

X(theta, phi) = (k*a/2) * sin(theta) * cos(phi);
Y(theta, phi) = (k*b/2) * sin(theta) * sin(phi);
%C = 1j * a * b * k / (2*pi);

E_theta(theta, phi) = sin(pi/2) * ...
                        ( cos(  X(theta, phi)  ) / ( (X(theta, phi)) ^ 2 - (pi/2)^2 ) ) * ...
                        (  sin(Y(theta, phi)) / Y(theta, phi) );

E_phi(theta, phi) = cos(theta) * cos(phi) * ...
                    (  cos(X(theta, phi)) / ( (X(theta, phi))^2 - (pi/2)^2  )  ) * ...
                    (  sin(Y(theta, phi)) / Y(theta, phi) );

AF(theta, phi) = 2j*sin(8*pi*sin(theta)*cos(phi));
E(theta, phi) = sqrt( abs(E_theta(theta, phi))^2 + abs(E_phi(theta, phi))^2 ) * AF(theta, phi);
E1 = matlabFunction(E);
% E-Plane
phi1 = linspace(0.01, .99*pi, 1000);
theta1 = zeros(1, length(phi1)) + deg2rad(90);
EPlane = E1(theta1, phi1);
EPlane = abs(EPlane) ./ max(abs(EPlane));
EPlane = abs(EPlane) .^ 2;
EPlane = 10 .* log10(EPlane);
EPlane(EPlane < -31) = -31;
plot(rad2deg(phi1), EPlane, "DisplayName", "E-Plane");
ylim([-30, 0]);
hold on;

% H-Plane
theta1 = linspace(0.01, .99*pi, 1000);
phi1 = zeros(1, length(theta1)) + deg2rad(90);
HPlane = E1(theta1, phi1);
HPlane = abs(HPlane) ./ max(abs(HPlane));
HPlane = abs(HPlane) .^ 2;
HPlane = 10 .* log10(HPlane);
HPlane(HPlane < -31) = -31;
plot(rad2deg(theta1), HPlane, "DisplayName", "H-Plane");
ylim([-30, 0]);
grid on
title("E and H Plane", "FontSize", 18);
legend();

D0 = 10 * log10(compute_directivity(AF));
```

```matlab
function [theta_max, phi_max, U_max] = findUmax(U_func)
    if nargin(U_func) == 2
        % Define a nested function to be minimized
        neg_U_func = @(x) -U_func(x(1), x(2));
        % Initial guess for theta and phi
        x0 = [0, 0];
        % Perform the optimization
        [x_max, U_max] = fminsearch(neg_U_func, x0);
        % Extract theta_max and phi_max
        theta_max = x_max(1);
        phi_max = x_max(2);
        U_max = -U_max;
    elseif nargin(U_func) == 1
        % Define a nested function to be minimized
        neg_U_func = @(x) -U_func(x(1));
        % Perform the optimization
        options = optimset('Display','iter');
        %  Bug in MATLAB
        xValue = fminbnd(neg_U_func, -pi, pi, options);
        % Extract theta_max and phi_max
        theta_max = xValue;
        U_max = U_func(xValue);
        phi_max = 0;
    else
        error('Incorrect number of input arguments.');
    end
end

function D0 = compute_directivity(field)
    if nargin(matlabFunction(field)) == 2
        syms theta phi
        U_inten = matlabFunction(abs(field) .^ 2);
        Prad = matlabFunction(U_inten(theta, phi) * sin(theta), 'Vars', {theta, phi});
        Prad_value = integral2(Prad, 0, pi, 0, 2*pi);
        [~, ~, Umax] = findUmax(U_inten);

        D0 = (4*pi*Umax)/Prad_value;
    elseif nargin(matlabFunction(field)) == 1
        syms theta
        U_inten = matlabFunction(abs(field) .^ 2);
        Prad = matlabFunction(U_inten(theta) * sin(theta), 'Vars', {theta});
        Prad_value = 2*pi*integral(Prad, 0, pi);
        [~, ~, Umax] = findUmax(U_inten);

        D0 = (4*pi*Umax)/Prad_value;
    else
        error('Incorrect number of input arguments.');
    end
end
```

4. Take a rectangular aperture with |x|<a/2 and |y|<b/2, (a=6λ and b=8λ). surrounded by an infinite ground-plane. The electric field across the aperture is given by

$$\bar{E} = \hat{x}E_0 e^{-j\alpha x^2} = \hat{x}E_0(\cos(\alpha x^2) - j\sin(\alpha x^2)) \approx \hat{x}E_0(1 - j\alpha x^2) \text{ for } \alpha x^2 \ll 1$$

   a. Calculate the E and H plane patterns. Plot the E-plane in phase, quadrature and total patterns for $\alpha(a/2)^2 = \pi/4$, on a rectangular, -30dB-0dB range, for 0<θ<90°. (hint: calculate $f(k_x, k_y)$ first)

   b. Calculate the aperture efficiency. Use the coupling formula.

In case you need it: $$\int_{-a/2}^{+a/2} x^2 e^{-jkx}\,dx = \frac{-d^2}{dk^2}\left[\frac{a\sin\left(k\frac{a}{2}\right)}{\left(k\frac{a}{2}\right)}\right]$$

HW 5

(4)

$$\vec{E} = \hat{x}\,(1 - j\alpha x^2)$$

$$f_x = \int_{-a/2}^{a/2}\int_{-b/2}^{b/2} \vec{E}\, e^{jk(x\sin\theta\cos\phi)}\, e^{jk(y\sin\theta\sin\phi)}\, dx\,dy$$

$$= -j\alpha \int_s x^2\, e^{jk(x\sin\theta\cos\phi)}\, e^{jk(y\sin\theta\sin\phi)}\, dx\,dy$$

$$\int_{-a/2}^{a/2} x^2 e^{-jkx}\,dx = -\frac{d^2}{dk^2}\left[a\,\frac{\sin(ka/2)}{k(a/2)}\right]$$

$$X = \frac{ka}{2}\sin\theta\cos\phi \quad Y = k\,b/2\,\sin\theta\sin\phi$$

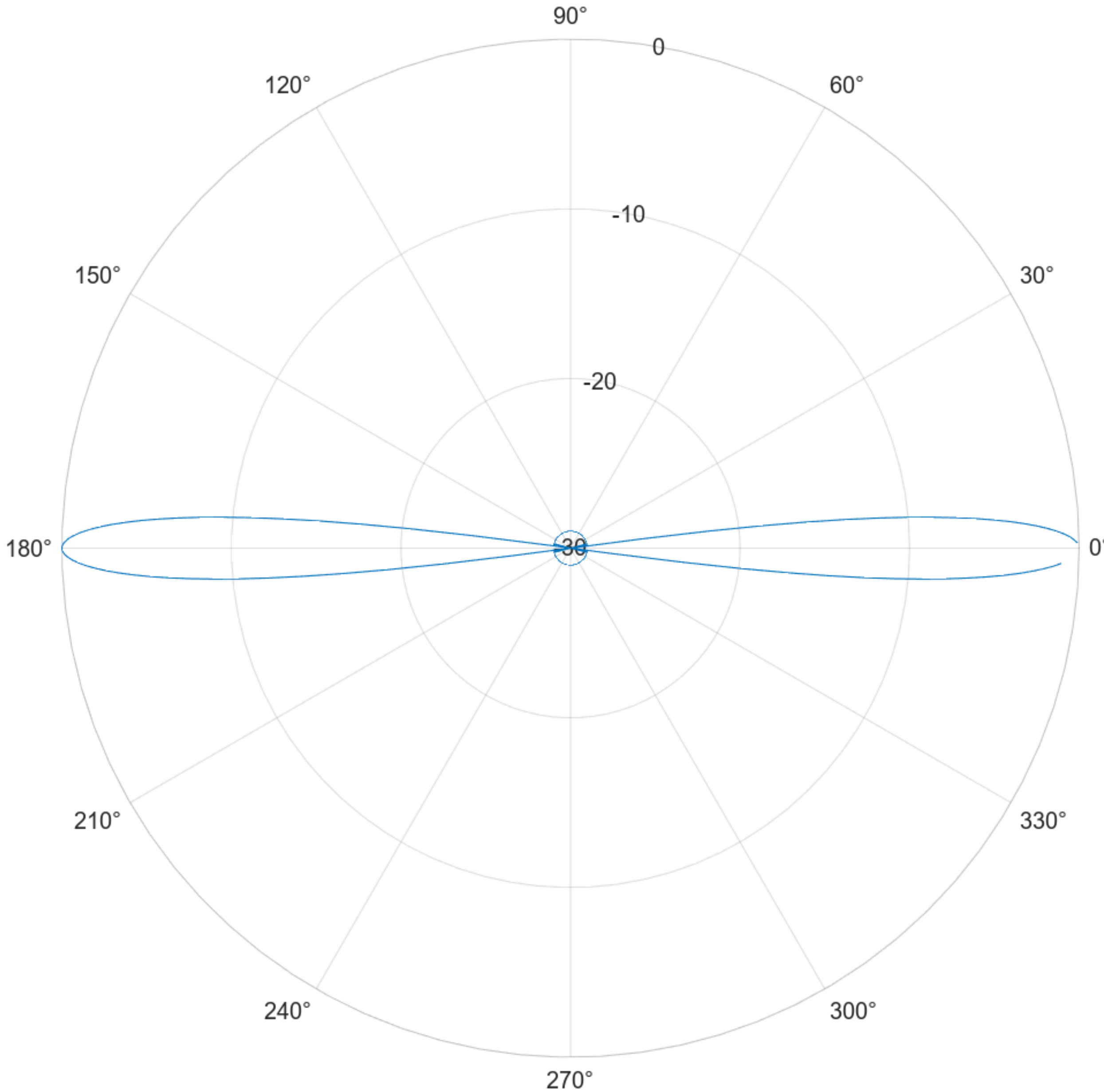
$$f_x = 2ab\,\mathrm{sinc}(X)\,\mathrm{sinc}(Y) + 2\alpha jab\,\mathrm{sinc}(Y)\;\frac{1}{k^2 X}\left((2 - X^2)\sin(X) - 2X\cos(X)\right)$$

$$E_\theta \,\alpha\, jf_x \cdot \sin\phi$$

$$E_\phi \,\alpha\, jf_x \cos\theta\cos\phi$$

H-Plane: $\theta = 0^\circ$ x-y Plane

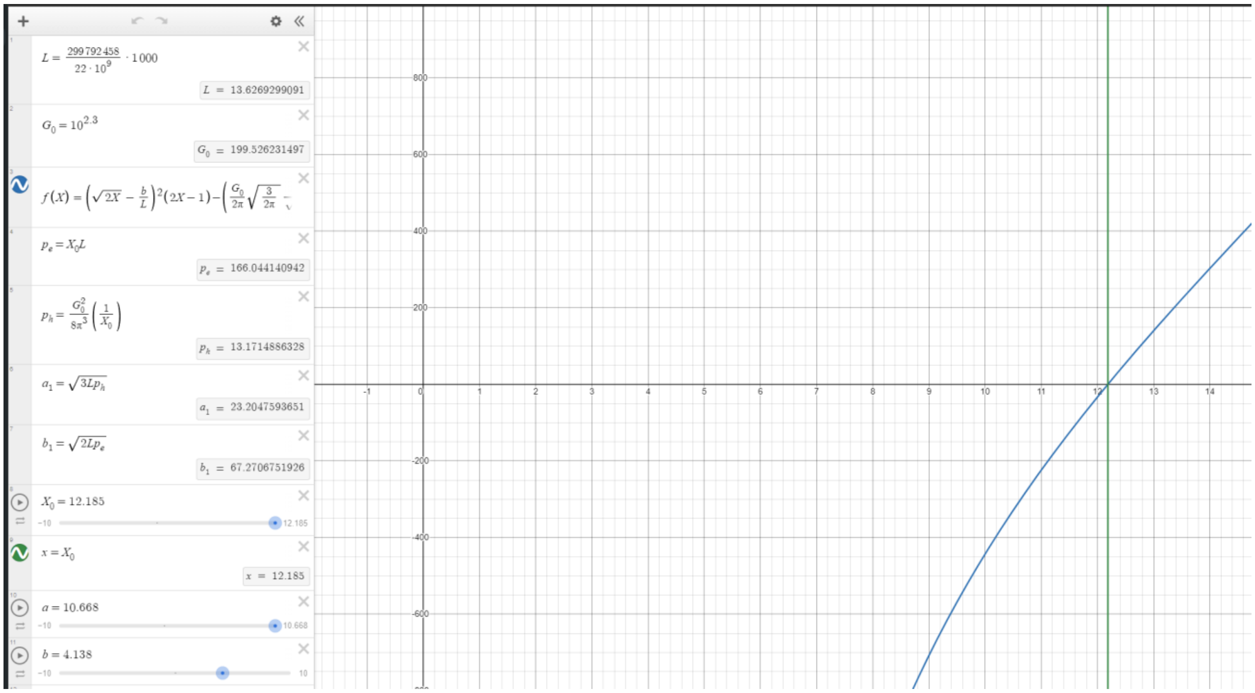
E-Plane: y-z Plane



5. Design a 23-dBi standard gain horn antenna for 18-26 GHz operations (K band). The feeding waveguide is WR42 (find the waveguide dimensions on internet). The gain is specified at 22 GHz. Plot the gain and aperture efficiency at 14-32 GHz. What is the phase error across the x and y direction at 18 and 26 GHz, respectively? To what maximum frequency you would like to use this horn, What is the limiting factor?

WR42 Specifications

Recommended Frequency Band:	18.00 to 26.50 GHz
Cutoff Frequency of Lowest Order Mode:	14.051 GHz
Cutoff Frequency of Upper Mode:	28.102 GHz
Dimension:	0.42 Inches [10.668 mm] x 0.17 Inches [4.318 mm]



(Design Graph)

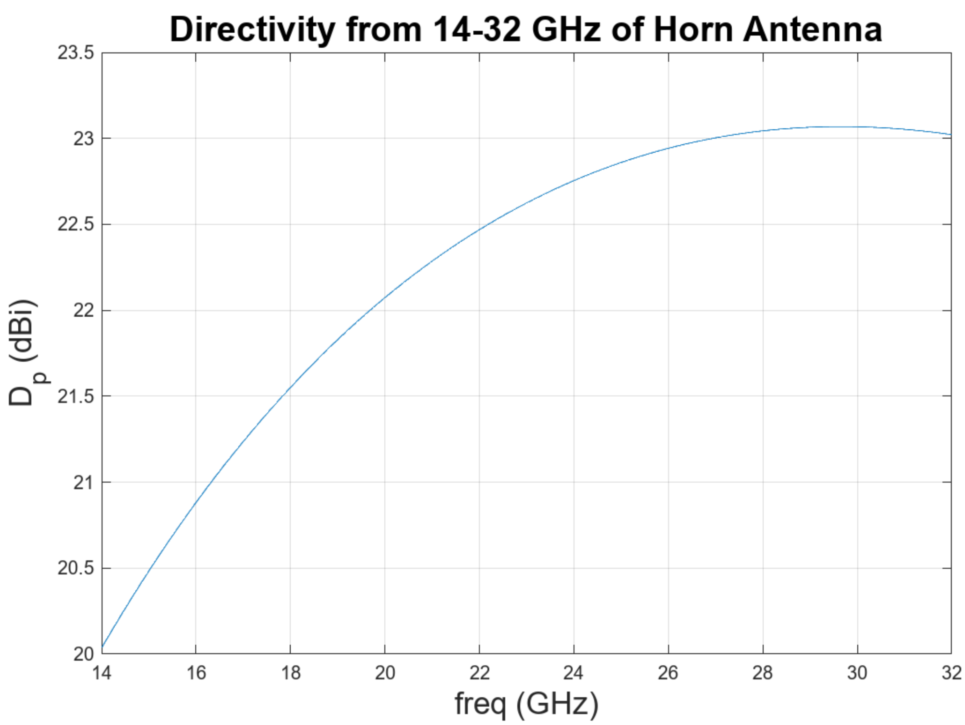
[HornAntenna | Desmos]

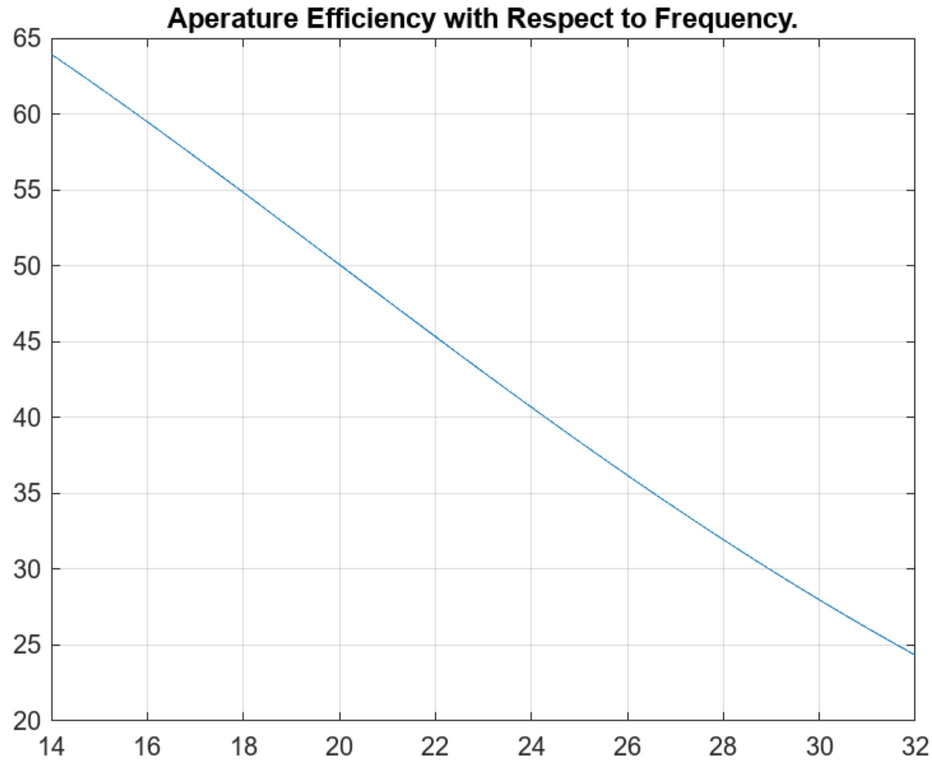
$$\boxed{\begin{aligned} D_E &= \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{64 a \rho_1}{\pi \lambda b_1} |F(t)|^2 \\ &= \frac{64 a \rho_1}{\pi \lambda b_1} \left[C^2 \left(\frac{b_1}{\sqrt{2\lambda\rho_1}} \right) + S^2 \left(\frac{b_1}{\sqrt{2\lambda\rho_1}} \right) \right] \end{aligned}}$$

$$\boxed{D_p = \frac{\pi \lambda^2}{32ab} D_E D_H}$$

$$\boxed{D_H = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi b\rho_2}{a_1\lambda} \times \{[C(u) - C(v)]^2 + [S(u) - S(v)]^2\}}$$

$$\boxed{\begin{aligned} u &= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{\lambda\rho_2}}{a_1} + \frac{a_1}{\sqrt{\lambda\rho_2}}\right) \\ v &= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{\lambda\rho_2}}{a_1} - \frac{a_1}{\sqrt{\lambda\rho_2}}\right) \end{aligned}}$$






```
clear all
close all
clc

syms x f lambda X

% Dimenions are in mm
a = 10.668*1e-3;
b = 4.138*1e-3;
f0 = 22e9;
G0_dbi = 23;
G0 = 10^(G0_dbi/10);
Lambda(f) = (3e8/f);% * 1e3; % lambda in mm from frequency in Hz
a1 = 85.66*1e-3;%23.20;
b1 = 67.27*1e-3;%67.27;
pe = 152.599655472*1e-3;
ph = 152.599655472*1e-3;%13.17;

C(x) = fresnelc(x);
S(x) = fresnels(x);

DE(lambda) = ((64*a*pe)/(pi*lambda*b1)) * ( (C(b1/sqrt(2*lambda*pe)))^2 + (S(b1/sqrt(2*lambda*pe)))^2 );

u(lambda) = (1/sqrt(2)) * ( sqrt(lambda*ph)/a1 + a1/sqrt(lambda*ph) );
v(lambda) = (1/sqrt(2)) * ( sqrt(lambda*ph)/a1 - a1/sqrt(lambda*ph) );
DH(lambda) = (4*pi*b*ph)/(a1*lambda) * ( (C(u(lambda)) - C(v(lambda)))^2 + (S(u(lambda)) - S(v(lambda)))^2 );
DP(lambda) = (pi*lambda^2)/(32*a*b) * DE(lambda) * DH(lambda);
DP(lambda) = matlabFunction(DP);

f1 = linspace(14e9, 32e9, 1000);
lam1 = eval(Lambda(f1));

Dp1 = eval(DP(lam1));
plot(f1/1e9, 10.0.*log10(Dp1));
xlabel("freq (GHz)", "FontSize", 16);
ylabel("D_{p} (dBi)", "FontSize", 16);
title("Directivity from 14-32 GHz of Horn Antenna", "FontSize", 18);
grid on;


%% Aperature Efficieincy
figure
Aem = lam1 .^ 2 ./ (4 * pi) .* Dp1;
ApertureEff = Aem ./ (a1 * b1);
plot(f1/1e9, ApertureEff*100)
grid on;
title("Aperature Efficiency with Respect to Frequency.");
```

~~The maximum frequency~~

The operational frequency range is determined by 2 factors

(1) Multiple Modes: At higher frequencies multiple modes could exist.

(2) Aperature eff and VSWR: The horn must maintain a good aperature eff and VSWR level otherwise it would not be a good design.

I would operate this horn from 14 GHz to about 26.5 GHz

14 GHz: 20 dB seems good enough and highest eff.

26.5 GHz: Gain is good but at this point the eff drops below 35%. I think 35% would be the lowest eff I should operate this horn at otherwise I should consider a different design.

HW 5

(5) Horn dimensions:

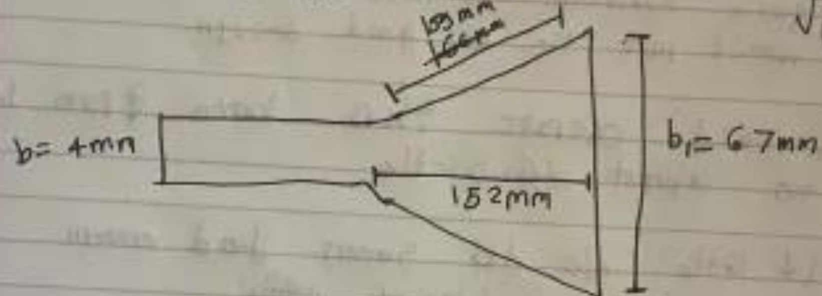
$a = 10mm$ $b = 4mm$ ~~$b_1 = 85mm$ a_1~~

$a_1 = 85mm$ $b_1 = 67mm$

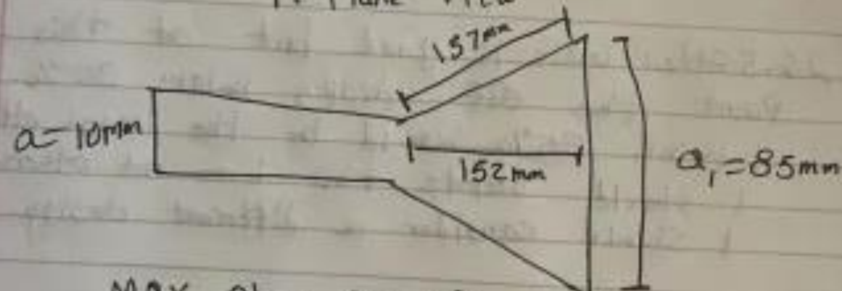
$P_e = P_n = 152mm$

E-Plane View

$\sqrt{(67/2)^2 + 152^2} =$



H-Plane View



Max Phase Error ($\hat{x}$)

$$X_\theta = \frac{a_1^2}{8\lambda P_n}$$

$X_\theta(18\text{ GHz}) = 20°$

$X_\theta(26\text{ GHz}) = 29°$

Max Phase Error ($\hat{y}$)

$$y_\theta = \frac{b_1^2}{8\lambda P_e}$$

$y_\theta(18\text{ GHz}) = 12°$

$y_\theta(26\text{ GHz}) = 18°$