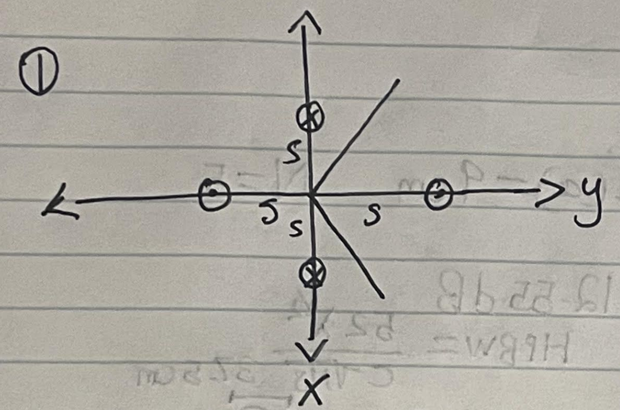
①



$\psi_{\hat{y}\hat{r}} = \hat{a}_r \cdot \hat{a}_y = \sin\theta \sin\phi$

$\psi_{-\hat{x}\hat{r}} = \hat{a}_r \cdot (-1)\hat{a}_x = -\cos\theta \sin\phi$

$\psi_{-\hat{y}\hat{r}} = \hat{a}_r \cdot (-1)\hat{a}_y = -\sin\theta \sin\phi$

$\psi_{\hat{x}\hat{r}} = \hat{a}_r \cdot \hat{a}_x = \cos\theta \sin\phi$

(a) $S = \lambda/2 \rightarrow ks = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$ $\quad |k\ell| = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \pi/2$

$AF = -e^{j\pi\cos\theta\sin\phi} - e^{-j\pi\cos\theta\sin\phi} + e^{j\pi\sin\theta\sin\phi} + e^{-j\pi\sin\theta\sin\phi}$

$AF = -2\cos(\pi\cos\theta\sin\phi) + 2\cos(\pi\sin\theta\sin\phi)$

$E_{dipole}(\theta,\phi) = C[\cos(\pi/2 \cos\theta) - \cos(\pi/2)] / \sin\theta$

$E_t(\theta,\phi) = 2j\eta \frac{I_0 e^{-jkr}}{2\pi r} \left(\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right)\left(\cos(\pi\sin\theta\sin\phi) - \cos(\pi\cos\theta\sin\phi)\right)$

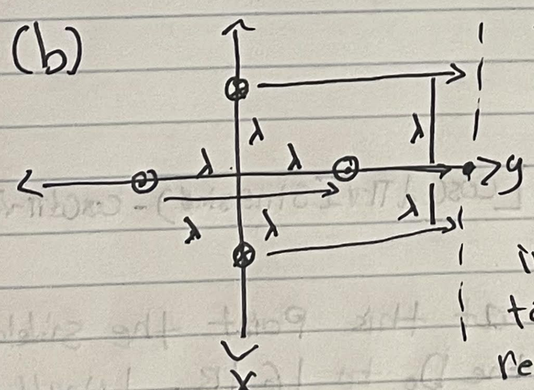
E-Plane: xz $\quad$ H-Plane: xy $\rightarrow$ E-Plane: $\phi = 0, \theta = [0,\pi]$ $\quad$ H-Plane: $\phi = [0, 2\pi)$, $\theta = 90°$

$P_{rad} = \frac{1}{2}\oint Re\{E \times H^*\} \cdot \hat{a}_r \hat{r} \sin\theta \, d\theta d\phi \cong 52.9565$

$U_{max} = \pi/2$

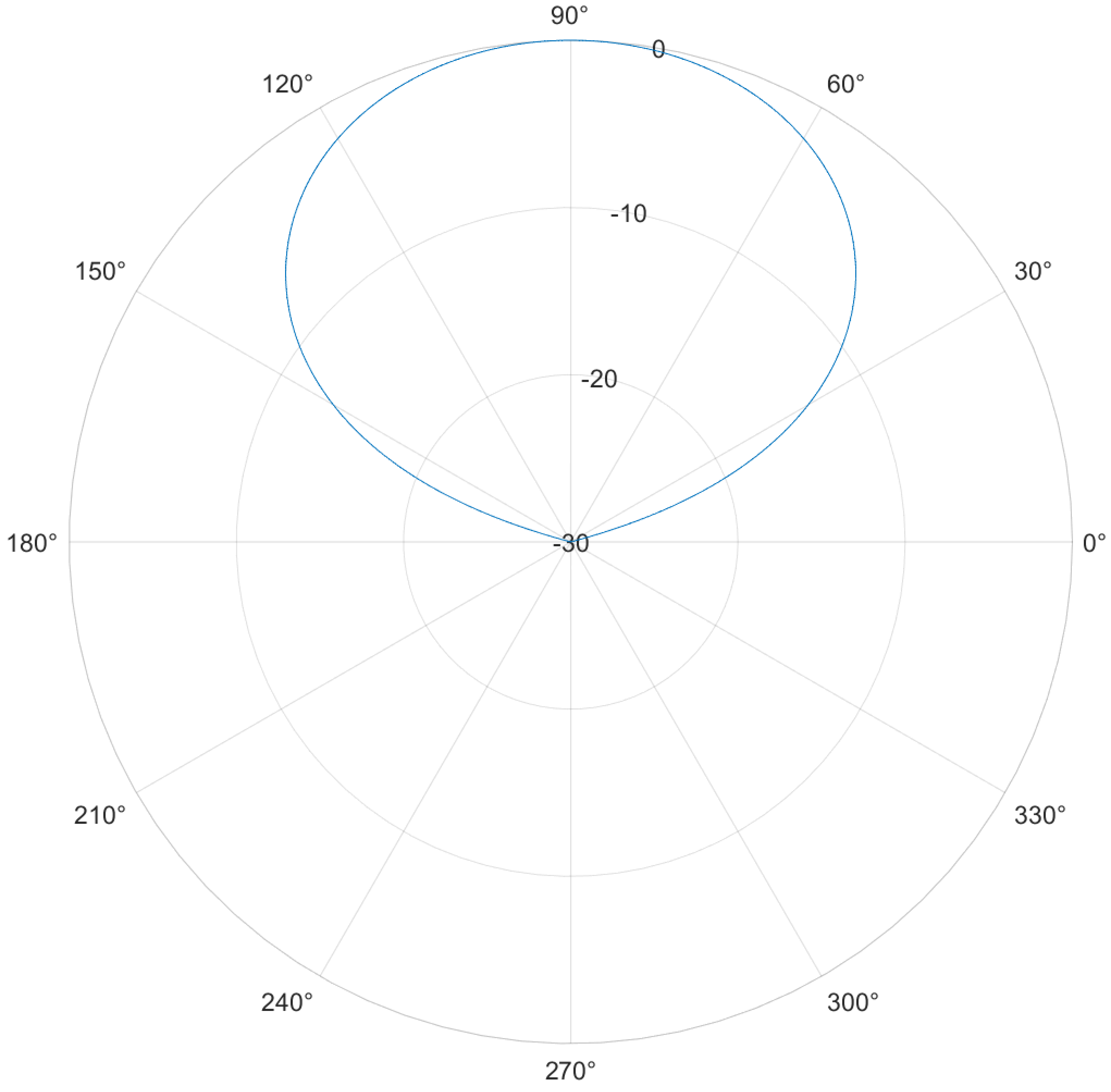
$D_0 = \frac{4\pi U_{max}}{P_{rad}} \cong \frac{4\pi(\pi/2)}{52.9565} = 3.7967$ $\quad D_0(dB) = 5.79\,dB$

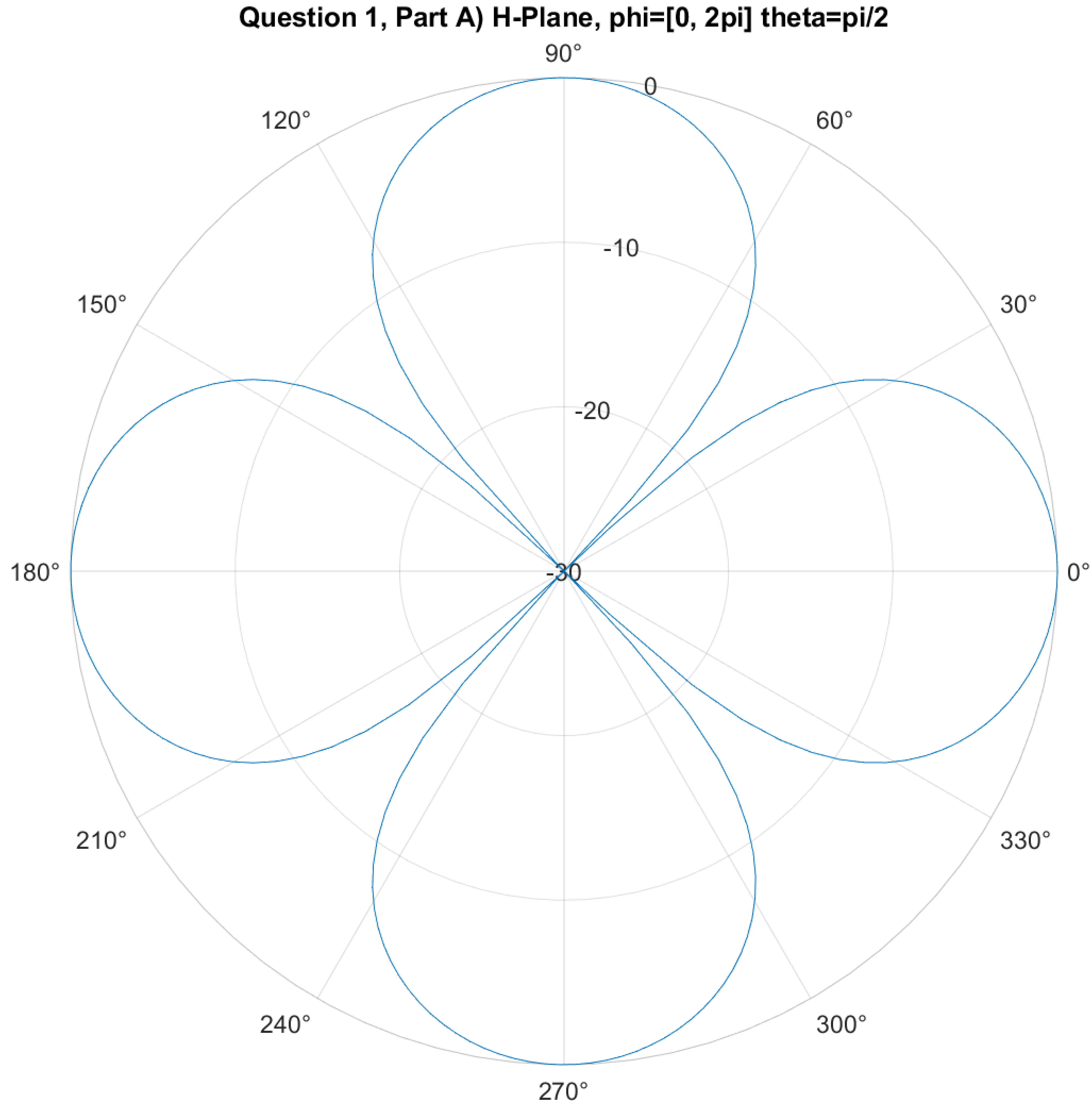
(b)

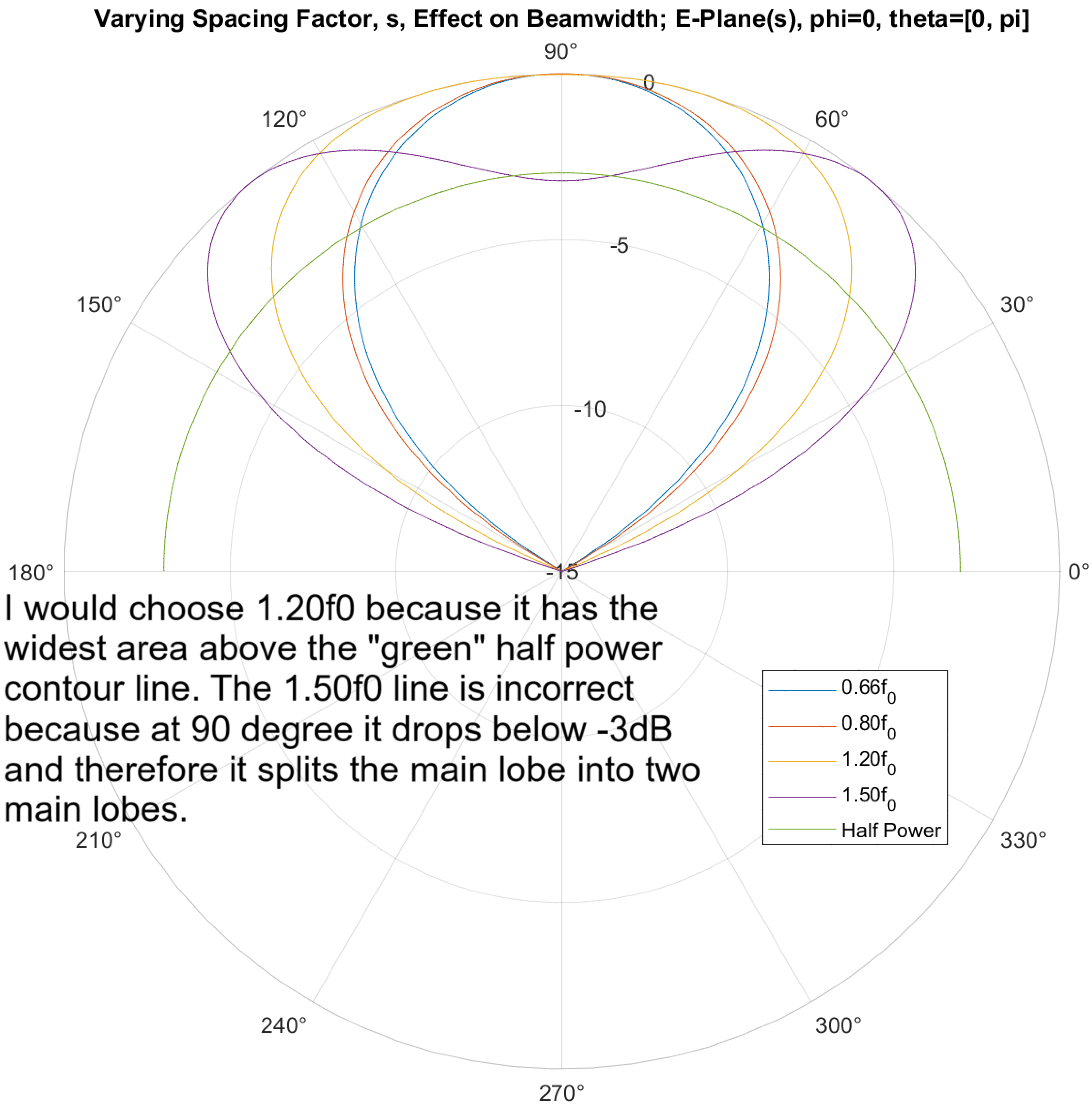


The array facter is equal 0 when y=0 therefore there is no radiation. This is because in the far field, the phane waves radiated by the source and the images have a path phase differene due to the spacing factor, s (s=λ), which result's in a 360° phase shift for all 3 images when y=0 (or x=0). Since two of the image have opposing magnitude to the other two sources, they will destructively interfer at this point.

Question 1, Part A) E-Plane, phi=0, theta=[0, pi]




Question 1, Part A) H-Plane, phi=[0, 2pi] theta=pi/2
90°
0
120°
60°
-10
150°
30°
-20
180°
-30
0°
210°
330°
240°
300°
270°



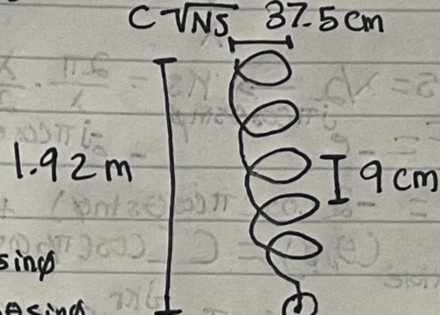
Varying Spacing Factor, s, Effect on Beamwidth; E-Plane(s), phi=0, theta=[0, pi]
90°
120°
60°
150°
30°
180°
0°
210°
330°
240°
300°
270°
0
-5
-10
-15
0.66f0
0.80f0
1.20f0
1.50f0
Half Power
I would choose 1.20f0 because it has the widest area above the "green" half power contour line. The 1.50f0 line is incorrect because at 90 degree it drops below -3dB and therefore it splits the main lobe into two main lobes.

② $\lambda_0 = \dfrac{3\times10^8}{800\,MHz} = 37.5\text{ cm}$

(a) $C=\lambda \quad \alpha = 13.5° \quad S = C\tan(\alpha) = 9\text{ cm} \quad N=5$

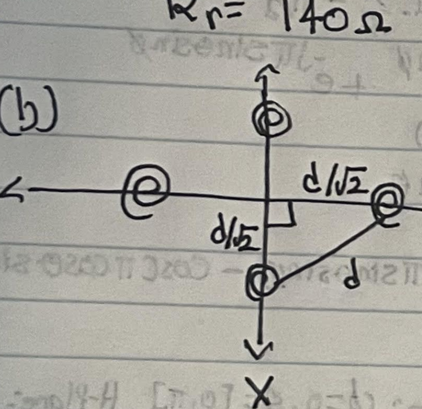
$$D_0 = 10\log_{10}\left[15N\frac{C^2S}{\lambda^3}\right] = 12.55\text{ dB}$$

$$L_0 = N\sqrt{C^2+S^2} = 1.92\text{ m} \qquad HPBW = \frac{52\lambda^{3/2}}{C\sqrt{NS}}$$



$R_r = 140\,\Omega$

(b)



$\psi_1 = \sin\theta\sin\phi$

$\psi_2 = -\cos\theta\sin\phi$

$\psi_3 = -\sin\theta\sin\phi$

$\psi_4 = \cos\theta\sin\phi$

$$\frac{2\pi}{\lambda}\cdot\lambda d\sqrt{2} = d\pi\sqrt{2}$$

$$AF = 2\left[\cos(d\pi\sqrt{2}\sin\theta\sin\phi) - \cos(d\pi\sqrt{2}\cos\theta\sin\phi)\right]$$

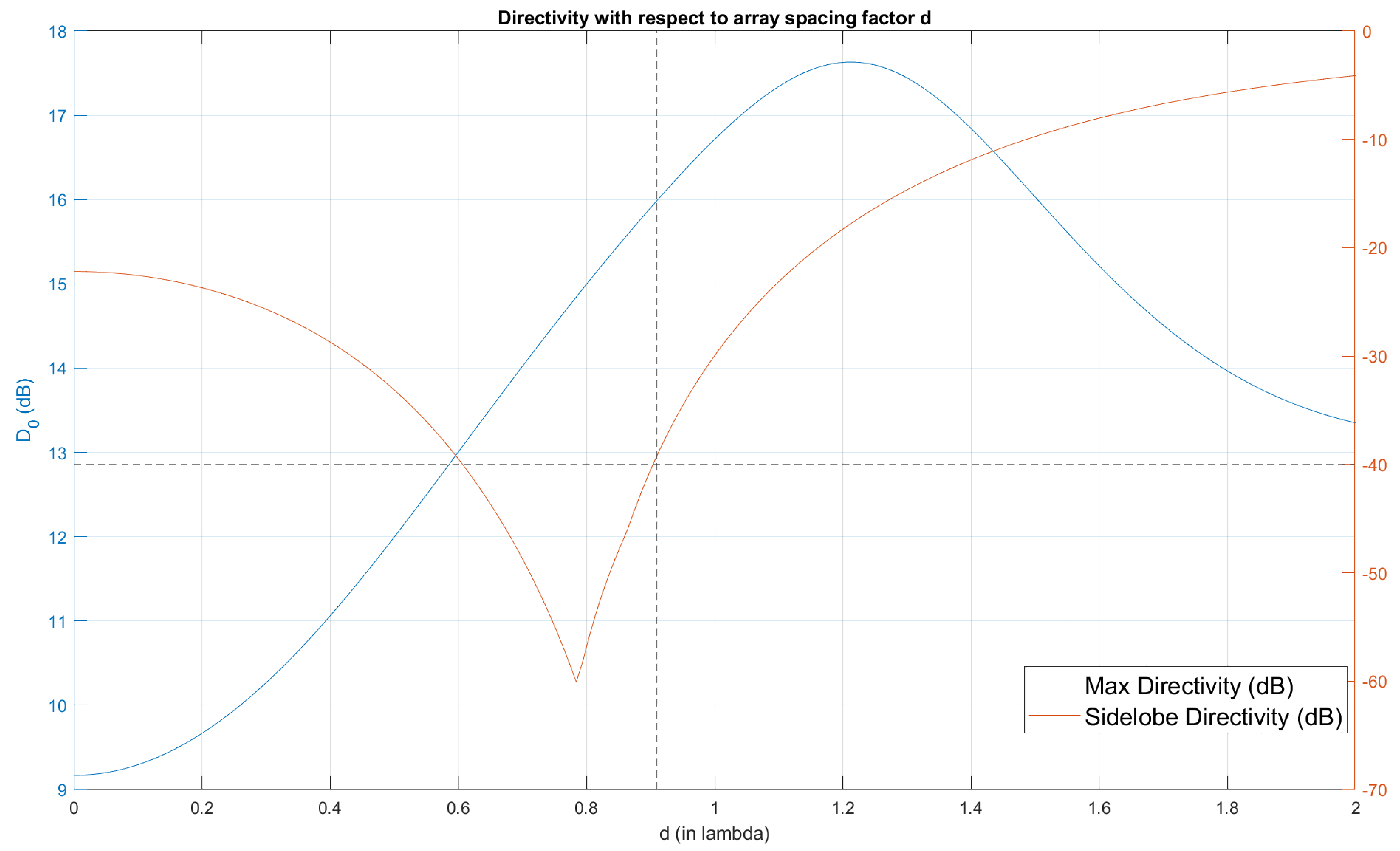
Element factor:

$$\psi = k_0\left(S\cos\theta - \frac{L_0}{p}\right) = 1.508\cos(\theta) - 7.792$$

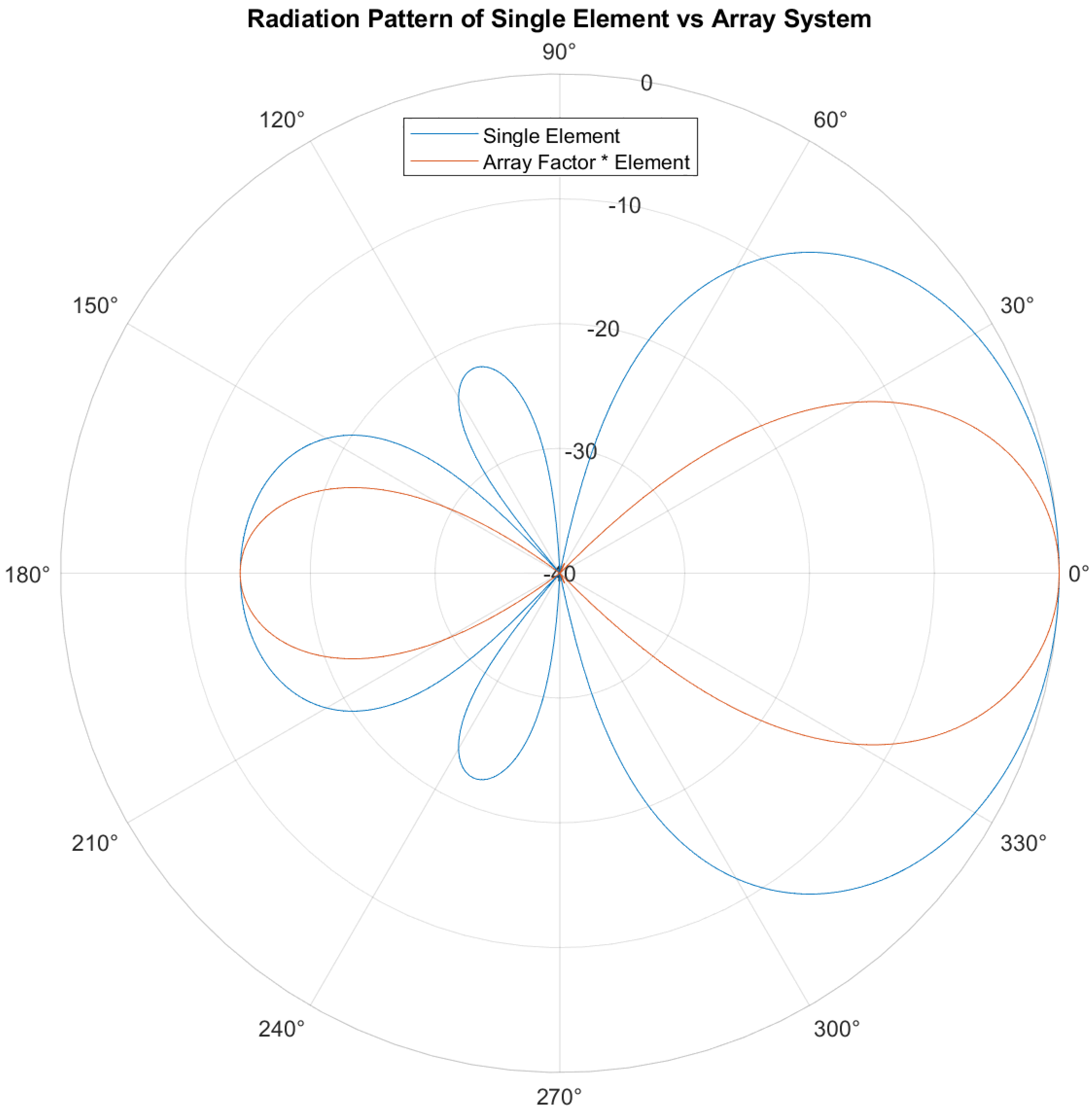
$$p = \frac{L_0/\lambda_0}{S/\lambda_0 + 1} = 0.8293$$

$$E_t(\theta,\phi) = 2\sin\left(\frac{\pi}{2N}\right)\cos\theta\,\frac{\sin[(N/2)\psi]}{\sin(\psi/2)}\left[\cos(d\pi\sqrt{2}\sin\theta\sin\phi) - \cos(d\pi\sqrt{2}\cos\theta\sin\phi)\right]$$

I would choose $d = 0.91\lambda$ because at this point the sidelobe level is -40 dB while increasing the D_0 to 16 dB. I would not choose ~~$\lambda = 1.2$~~ $d = 1.2\lambda$ because it introduces grating lobes.



Directivity with respect to array spacing factor d
D_0 (dB)
18
17
16
15
14
13
12
11
10
9
0
-10
-20
-30
-40
-50
-60
-70
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
d (in lambda)
Max Directivity (dB)
Sidelobe Directivity (dB)



Radiation Pattern of Single Element vs Array System
Single Element
Array Factor * Element
90°
60°
30°
0°
330°
300°
270°
240°
210°
180°
150°
120°
0
-10
-20
-30
-40

(3)

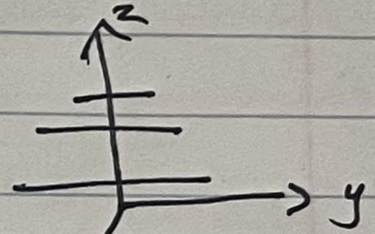
$$AF = 0.9e^{-j0.4\pi + \gamma} + 1 + 0.7e^{j0.6\pi\cos\theta + \alpha} + 0.5e^{j\pi\cos\theta + \beta}$$

$$\alpha = -0.6\pi \quad \beta = -\pi \quad \gamma = 0.4\pi$$

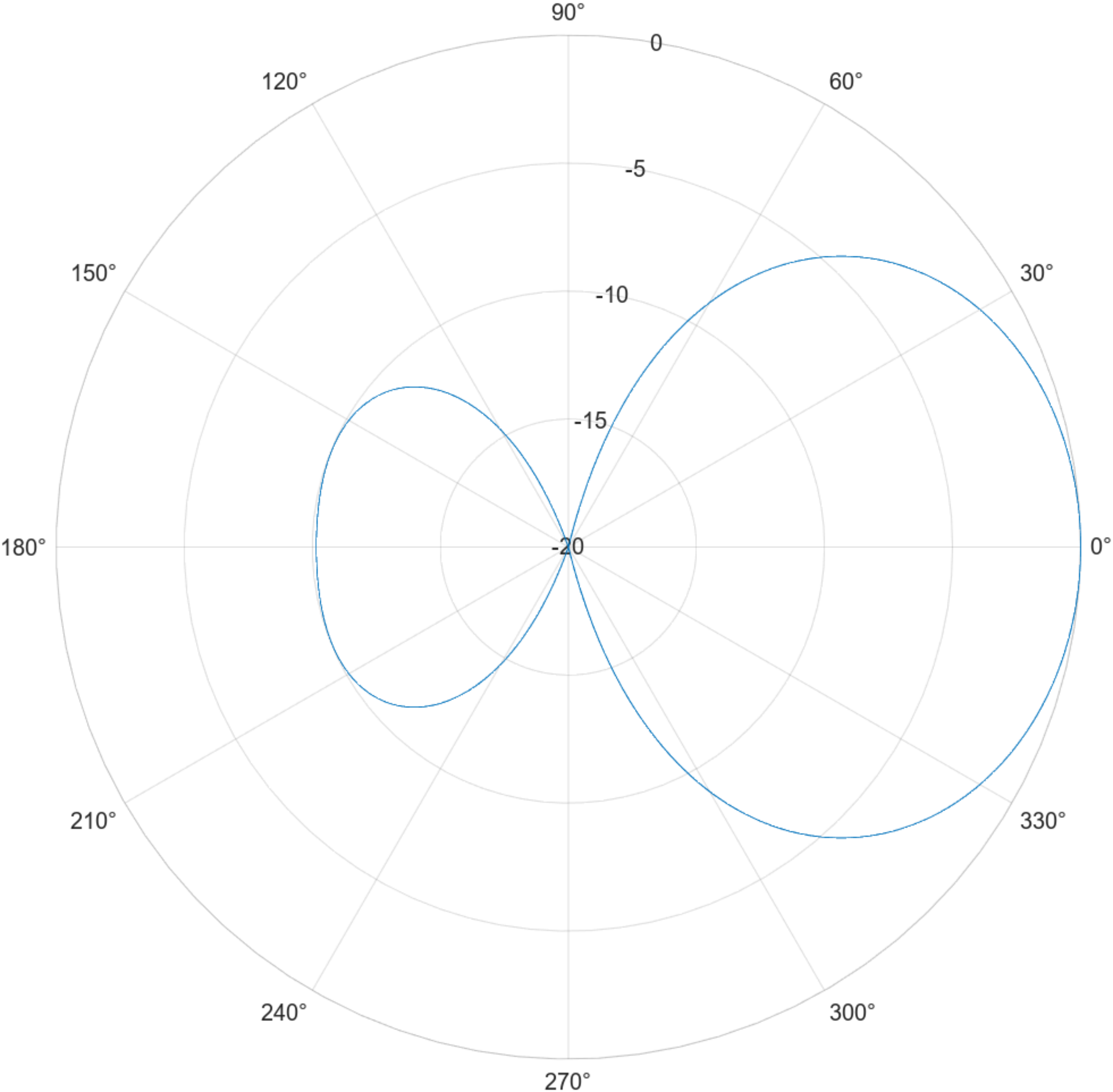
Element factor $= \cos\theta$

$D_0 = 6.4 \Rightarrow D_0(dB) = 8.1\,dB$ Front-to-back ratio $= 10.4$

E-Plane: xy Plane H-Plane: xz Plane



E Plane Radiation Pattern $D_0 = 6.42$ --- $D_0(dB) = 8.07$



H-Plane Radiation Pattern D_0 = 6.42 --- D_0(dB) = 8.07