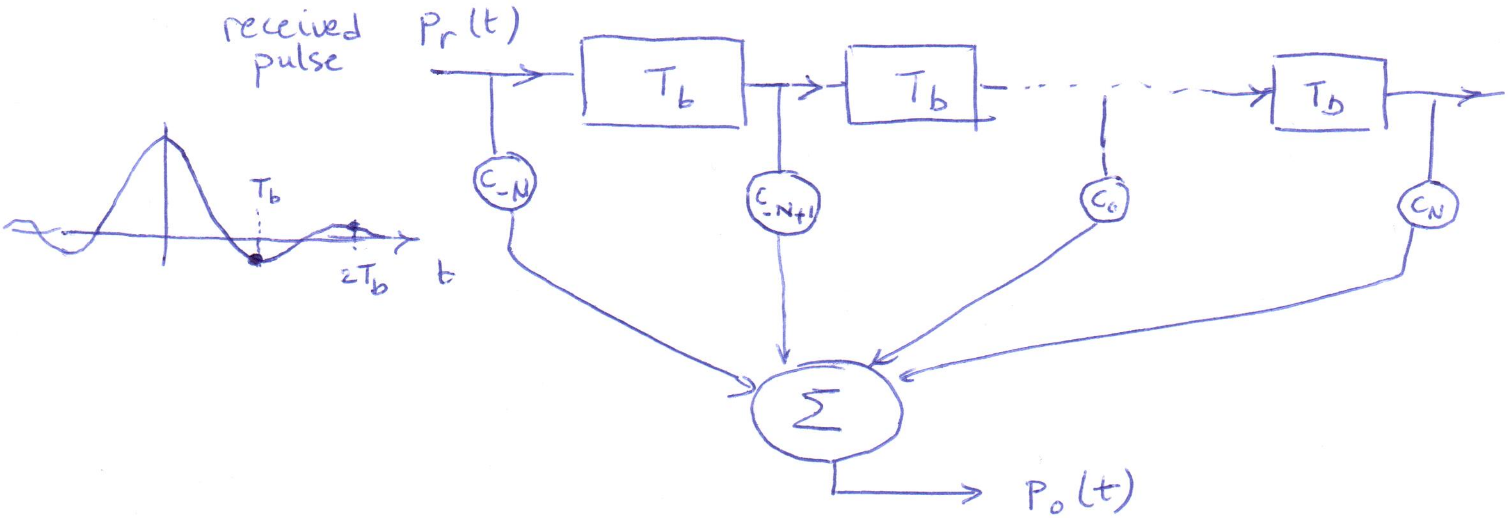
Lecture 14

Equalizers

Zero_ Forcing Equalizer (ZFE)

Eliminates or minimize ISI with neighboring pulses at sampling instants only on the Rx side. Use digital filters to force the o/p pulse to have zeros at sampling times (nT_b)



we need $P_o(t)$ to satisfy Nyquist's or the controlled ISI criteria by adjusting tap gains.

$$P_o(t) = \sum_{n=-N}^{N} c_n P_r(t - nT_b)$$

At $t = kT_b$:

$$P_o(kT_b) = \sum_{n=-N}^{N} c_n P_r(kT_b - nT_b) \qquad n = 0, \pm 1, \pm 2, \ldots$$

We use the shorthand notation

$$P_o(k) \triangleq P_o(kT_b)$$
$$P_r(k) \triangleq P_r(kT_b)$$

$$P_o(k) = \sum_{n=-N}^{N} \underbrace{c_n}_{2N+1 \text{ variables}} P_r(k-n) \qquad k = -N, \dots, N$$

Nyquist criterion requires $P_o(k) = 0,\ k \neq 0$ and $P_o(k) = 1,\ k = 0$

$$P_o(k) = \begin{cases} 1 & k = 0 \\ 0 & k = \pm 1, \pm 2, \dots \pm N \end{cases}$$

specify $2N+1$ equs

This will ensure that a pulse has zero ISI at sampling instants with N preceding and N succeeding pulses

$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} P_r(0) & P_r(-1) & \cdots & P_r(-2N) \\ P_r(1) & P_r(0) & \cdots & P_r(-2N+1) \\ \vdots & & & \\ \vdots & & & \\ P_r(2N) & P_r(2N-1) & \cdots & P_r(0) \end{bmatrix} \begin{bmatrix} c_{-N} \\ c_{-N+1} \\ \vdots \\ c_0 \\ c_1 \\ \vdots \\ c_N \end{bmatrix}$$

In matrix form,

$$\underline{P_o} = \mathbb{P}_r \, \underline{c}$$

P_r : $(2N+1) \times (2N+1)$ matrix
Toeplitz matrix

$\underline{c}$: $N \times 1$ vector of coeffs

$$\Rightarrow \quad \underline{c} = \mathbb{P}_r^{-1} \, \underline{P_o}$$

<u>Example</u> : $P_r(0) = 1$, $P_r(1) = -0.3$, $P_r(2) = 0.1$, $P_r(-1) = -0.2$, $P_r(-2) = 0.05$

Design a 3-tap $(N=1)$ equalizer.

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -0.2 & 0.05 \\ -0.3 & 1 & -0.2 \\ 0.1 & -0.3 & 1 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} \quad \Rightarrow \quad \underline{c} \checkmark$$

Remark: There will be residual ISI for $k \in \{0, \pm 1\}$

for example, $$p_o(k) = \sum_{n=-N}^{N} c_n p_r(k-n)$$

$$p_o(-3) = 0.01, \quad p_o(-2) = 0.0145, \quad p_o(2) = 0.0176 \ldots$$

To achieve zero ISI we need an infinite number of taps.

MMSE equalizer: Minimize the mean-square difference between the o/p of the equalizer $p_o(k)$ and the desired ISI response. It doesn't force the samples to zero at $2N$ points.

Consider a window $[-K, K]$:

$$MSE(\underline{c}) = \frac{1}{2K+1} \sum \left(p_o(k) - \delta(k)\right)^2$$

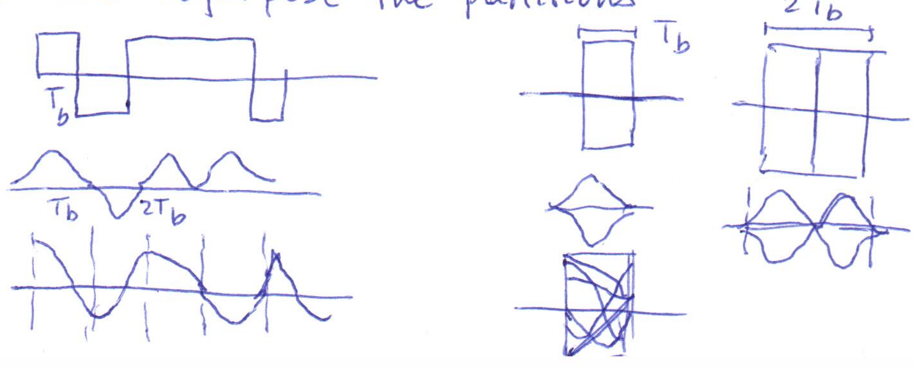
where $p_o(k) = \sum_n c_n p_r(k-n)$, $\quad \delta(k) = \begin{cases} 1 & k=0 \\ 0 & \text{o.w.} \end{cases}$

Hence, we minimize the MSE, i.e. solve

$$\min_{\underline{c}} MSE(\underline{c})$$

Eye diagram on oscilloscope gives a measure of ISI. We cut the signal every T_b and superpose the partitions

Ex:



Completely open eye (infinite BW)

zero ISI

Distortive channel

eye not fully open $ISI \neq 0$