$$f(u) = \sum_{(u,v) \in E} f_{uv} + \sum_{(u,v) \in V \times V} g_{uv}$$

↓ Hooke's Law
Force exerted by Edge (Springs)

↓ Coulomb's Law
Force exerted by 2 nodes on each other
(Force between 2 charges)

Hooke's Law:

① $F = -Kx$

$x = L - b$

② $F = -K(L-b)$

③ $F = -KL + Kb$

K: Constant
x: extension
L: spring length
b: base length of spring at force = 0

Coulomb's law:

$$F = K\frac{q_1 q_2}{R^2}$$

$q_1 = q_2 \Rightarrow$ we assume all nodes have same electric charge

K: Coulomb's constant
q1: charge 1
q2: charge 2
r: distance of separation

$$F = \frac{Kq^2}{r^2}$$

Some Notations:

~~The euclidean dis~~

$d(p,q)$: Euclidean distance between ~~points~~ Nodes $\{p,q\}$

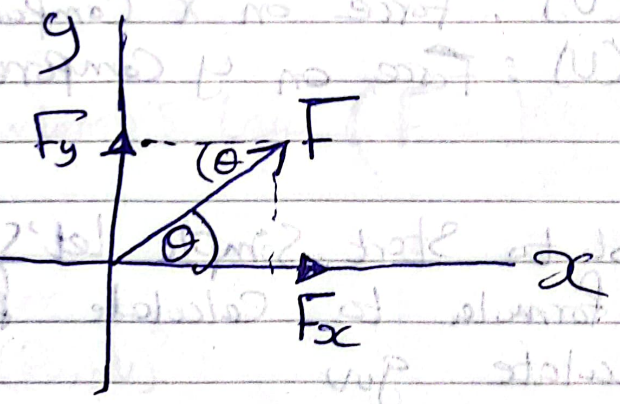
~~$P_v = (x_v, y_v)$~~

$P_v = (x_v, y_v)$: Position of Node V

Some Physics Intros:

① Lets talk about Force ~~resol~~ resolutions, Simply force resolution is Splitting a force into 2 or more Components Such that the effect of these force Components have the Same exact effect as the initial Force

We are mainly focusing on resolving a force into 2 Components which are
- Force on X-Component / axis
- Force on Y-Component / axis



a) $\cos\theta = \frac{adj}{hypo}$ $adj = F_x$, $hypo = F$

$\cos\theta = \frac{F_x}{F}$

$$\boxed{\begin{aligned} F_x &= F\cos\theta \\ F_y &= F\sin\theta \end{aligned}}$$

b) $\sin\theta = \frac{opp}{hypo}$

$opp = F_y$
$hypo = F$

$\sin\theta = \frac{F_y}{F}$

Now lets return back to our original Formula which Computes the Force on a Node

$$f(v) = \sum_{(u,v) \in E} f_{uv} + \sum_{(u,v) \in V \times V} g_{uv}$$

⇒ Here ~~used~~ what we want to do is Compute the Force on a Node in terms of 2 Components the X-Component and the Y-Component

$f_x(v)$: Force on X Component
$f_y(v)$: Force on y Component

⇒ First to Start Simple let's define a formula to Calculate f_{uv} and to Calculate g_{uv}

① $f_{uv} \Rightarrow$ Hooke's Law Force $(F = -Kx)$

$(F = -K(l-b))$

① $f_{uv} = K_{uv}(d(p_u, p_v) - l_{uv})$

K_{uv}: Spring Constant aka Stifness of the Spring

$d(p_u, p_v)$: Length of the Spring /Edge

l_{uv}: natural (zero energy) length of the Spring

② $g_{uv} \Rightarrow$ Coulomb's Law $\left(F = \frac{Kq^2}{r^2}\right)$

$$g_{uv} = \frac{K_{uv}}{d(p_u, p_v)^2}$$

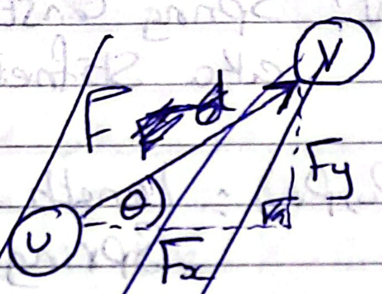
K_{uv}: The strength of the electric repulsion between U and V

$K_{uv} = Kq^2$

$d(p_u, p_v)^2$: The distance of Separation between U and V

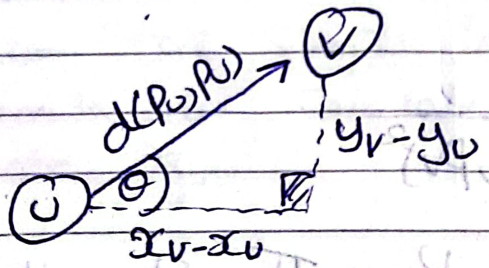
⇒ Now that we have defined the formulas for f_{uv} and g_{uv} lets shift our focus on Computing the forces in terms of 2 Components X and Y

~~recall~~



$$\sin\theta = \frac{F_y}{F} \qquad \cos\theta = \frac{F_x}{F}$$

$$F_y = F\sin\theta \qquad F_x = F\cos\theta$$



$$\cos\theta = \frac{x_v - x_u}{d(P_u, P_v)}$$

$$\sin\theta = \frac{y_v - y_u}{d(P_u, P_v)}$$

⇒ recall that we quickly went over force ~~resol~~ resoulotions and showed that

$$F_x = F\cos\theta \quad \text{and} \quad F_y = F\sin\theta$$

we have just found formulas for $\cos\theta$ and $\sin\theta$

$$\cos\theta = \frac{x_v - x_u}{d(P_u, P_v)}$$

$$\sin\theta = \frac{y_v - y_u}{d(P_u, P_v)}$$

this implies that

$$F_x = F\left(\frac{x_v - x_u}{d(P_u, P_v)}\right)$$

$$F_y = F\left(\frac{y_v - y_u}{d(P_u, P_v)}\right)$$

⇒ Finally we can conclude the following

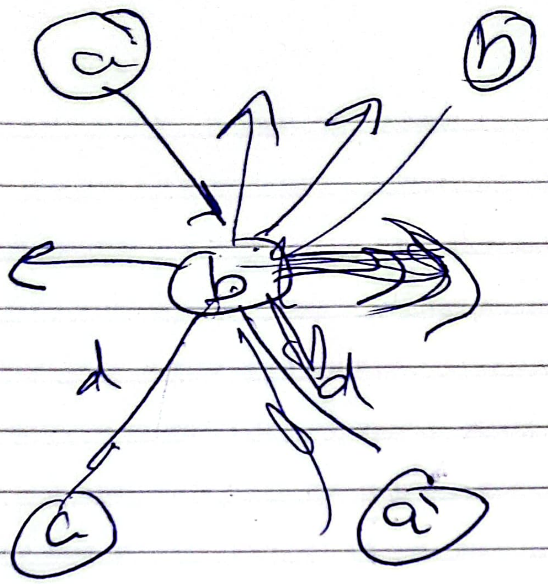
$\cancel{f_x(v) = f_{uv}}$

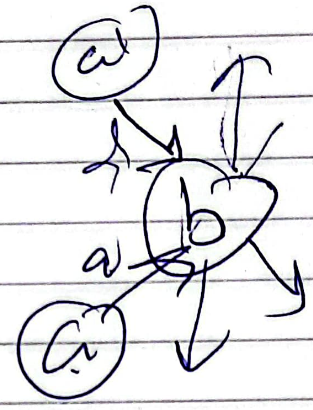
$$f_x(v) = \sum_{(u,v)\in E} f_{uv}\cos\theta + \sum_{(u,v)\in V\times V} g_{uv}\cos\theta$$

if we expand this formula we will have the following: ~~Formula~~

⇒ you can apply the same rules for the y-component

$$f_x(v) = \sum_{(u,v)\in E} k_{uv}\left(d(p_u,p_v) - l_{uv}\right)\left(\frac{x_v - x_u}{d(p_u,p_v)}\right) + \sum_{(u,v)\in V\times V} \frac{k_{uv}}{d(p_u,p_v)^2}\cdot\frac{x_v - x_u}{d(p_u,p_v)}$$





$$f \quad \frac{b_x - a_x}{d}$$

$$f \quad \frac{b_y - a_y}{d} \qquad \underline{b_y - a_y}$$