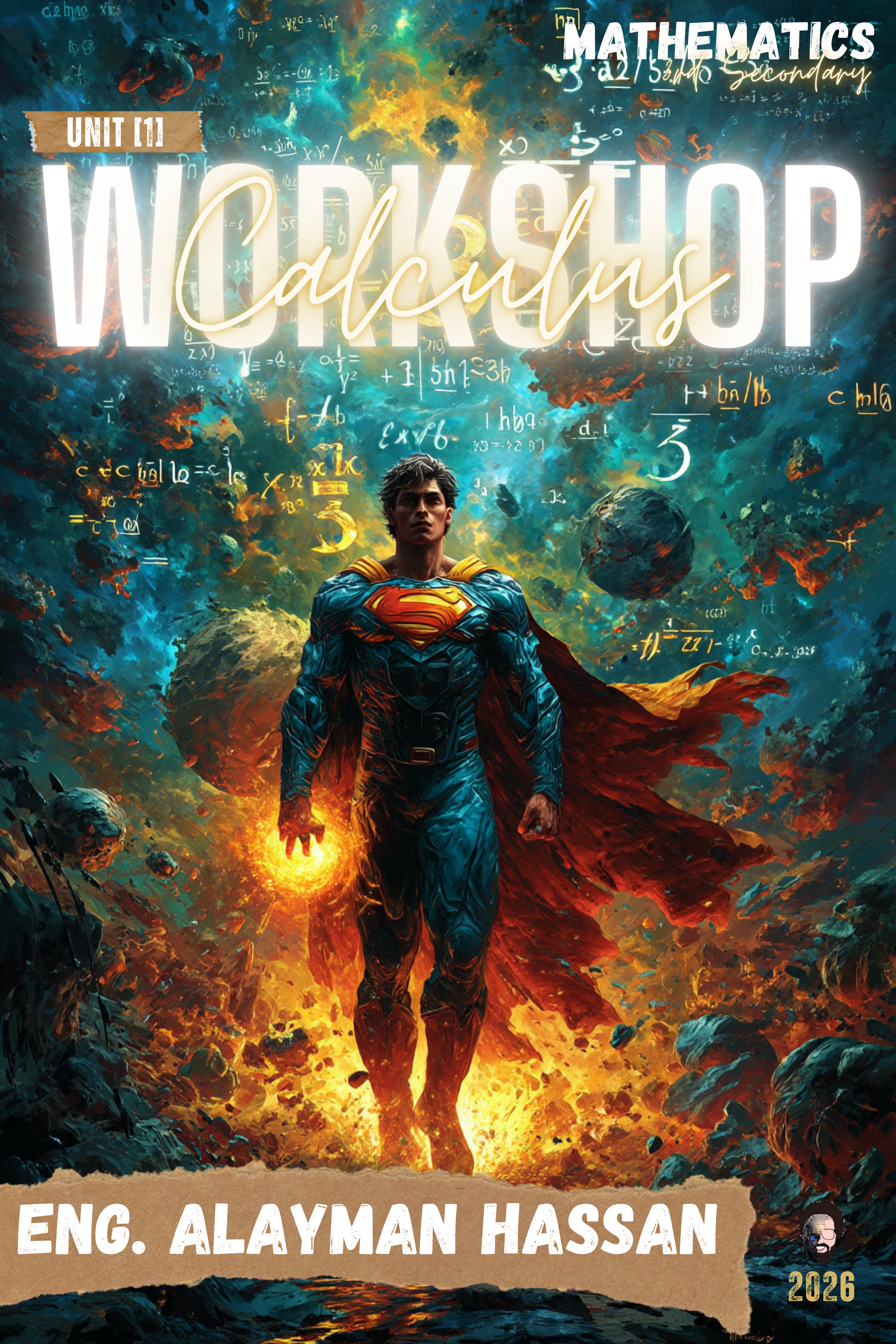
MATHEMATICS

3rd Secondary

UNIT [1]

WORKSHOP Calculus



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2026

Workshop Unit [1] - Calculus

Level (1)

(1) If $f(x) = e^{3x}$, then $f'(x) =$ ....

[a] e^{3x} [b] $3e^{3x}$ [c] $9e^{3x}$ [d] $3e^{2x}$

$e^{3x} \cdot 3$

$\frac{dy}{dt} \times \frac{dt}{dx}$

(2) If $x = 5t + 3$, $y = 16t^2 + 9$ then $\frac{dy}{dx} =$ .... at $t = 5$

[a] 8 [b] 32 [c] 16 [d] 4

$\frac{dx}{dt} = 5$, $\frac{dy}{dt} = 32t$

(3) If $y = 3\tan x - \sec^2 x$ then at $x = \frac{3\pi}{4}$ $\frac{dy}{dx}$ equals ....

[a] 12 [b] 13 [c] 10 [d] 14

$3\sec^2 x - 2\sec x \sec x \tan x$

(4) **Slope of tangent of $y = 1 + \sqrt{2}\csc x + \cot x$ at $\left(\frac{\pi}{4}, 4\right)$ equals ……….**

a 1 **b** 2 **c** 3 **d** −4

$\sqrt{2}$ csc cot + −csc

(5) **If the radius length of a circle increases at a rate $\frac{1}{\pi}$ cm./sec. the circumference of the circle increases at a rate of ……… cm./sec.**

a $\frac{2}{\pi}$ **b** 2 **c** π **d** 2π

$\frac{dr}{dt} = \frac{1}{\pi}$

(6) **If $f(x) = 4\sec^2 x$ then $f'\left(\frac{\pi}{4}\right) = ………$**

a −12 **b** 12 **c** 14 **d** 16

8 sec(x) sec tan

(7) **The radius length of a circle increases at a rate of 2 cm/min and its area at a rate of $20\ \pi\ \text{cm}^2/\text{min}$, then the radius length of the circle at this moment equals ……… cm**

[a] 3 [b] 4 [c] 5 [d] 6

$\frac{dr}{dt} = 2$, $\frac{dA}{dt} = 2\pi$

(8) **If the slope of the tangent to the curve $y = f(x)$ at a point equals $\frac{1}{2}$ and the x −coordinate of this point decreases at rate 3 units/sec. then the rate of change of its y −coordinate equals ……… unit/sec.**

[a] $-\frac{1}{6}$ [b] $-\frac{3}{2}$ [c] $\frac{1}{6}$ [d] $\frac{3}{2}$

(9) **If $y = \cos^2 x - \sin^2 x$, then $\frac{d^2y}{dx^2} =$ ………**

[a] $4\sin 2x$ [b] $4\cos 2x$ [c] $-4\cos 2x$ [d] $-4\sin 2x$

(10) If $x = 2t^2 + 3$, $y = \sqrt{t^3}$ then $\frac{dy}{dx} = \dots\dots\dots$ when $t = 1$

a) 8 b) 3 c) $\frac{8}{3}$ d) $\frac{3}{8}$

(11) $x = \sqrt{3t - 2}$, $y = \sqrt{4t + 1}$ find $\frac{dy}{dx} = \dots\dots\dots$ at $t = 2$

a) $\frac{1}{3}$ b) $\frac{18}{9}$ c) $\frac{4}{9}$ d) $\frac{7}{9}$

$\frac{dx}{dt} = \frac{3}{2\sqrt{3t-2}}$

$\frac{dy}{dt} = \frac{4}{2\sqrt{4t+1}}$

(12) If $y = \sin^2 x - \cos^2 x$ then $\frac{dy}{dx} = \dots\dots\dots$

a) $-\cos 2x$ b) $2\cos 2x$ c) $-2\sin 2x$ d) $2\sin 2x$

(13) If $\frac{dy}{dx} = \sqrt{1 - y^2}$ then $\frac{d^2y}{dx^2} = \dots\dots\dots$

a $-2\,y$ b $-y$ c y d $\frac{-y}{\sqrt{1 - y^2}}$

(14) A circular segment in which the length of its radius of its circle is 10 cm and its center angle measured x^{rad} and change at rate $3^{rad}/min$ find the rate of the increase of its area at $x = 60^{\circ}$

a 9 b 75 c 25 d 19

(15) If $y = \sec x + \tan x$ then $\frac{y'}{y} = \dots\dots\dots$

a $\cos x$ b $\sec x$ c $\csc x$ d $\sin x$

(16) A spherical balloon inflated so that its diameter is increasing at a rate of 2 cm/min then when its diameter length 10 cm the volume increasing at rate ……… cm^3/min

a 25π b 75π c 10π d 100π

(17) Find the slope of the tangent to the curve $\cos\sqrt{\pi\, y} = 3\,x + 1$ at $\left(-\frac{1}{3}, \frac{\pi}{4}\right)$

a -3 b 3 c 19 d 9

(18) If $x^3 + y^3 = 9$ then $\frac{dy}{dx}$ at the point $(1, 2) =$ ………

a 3 b -3 c 4 d -4

(19) If the rate of change in volume of a sphere equals the rate of change of its radius, then $r =$ ……… length unit

a 1 b $\sqrt{2\pi}$ c $\frac{1}{\sqrt{2\pi}}$ d $\frac{1}{2\sqrt{\pi}}$

(20) A right circular cone, if both the length of its base radius and its height increase by rate $\frac{1}{2}$ cm/sec then the rate of change of its volume at the moment its radius base length 6 cm and its height 9 cm

a 20π b 24π c 21π d 4

(21) A rectangle its length twice its width if its length increases by rate 2 cm/sec then when its length 10 cm the rate of increase of its area ………

a 10 b 30 c 20 d 40

(22) Which of the following functions satisfies the relation $\frac{d^3y}{dx^3} = y$?

[a] $\frac{1}{12}(x+1)^4$ [b] $\sin x$ [c] e^x [d] $\frac{x}{x-1}$

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(23) If $\ln x + x\,e^y = 1$, $\frac{dx}{dt} = 5$ at $x = 1$, $y = 0$ then $\frac{dy}{dt}$

[a] 10 [b] 12 [c] -10 [d] -12

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(24) If $x^2 + y^2 = 2\,x\,y$, then $y'' = \ldots\ldots\ldots$

[a] zero [b] 1 [c] 2 [d] -1

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Level (2)

(25) A point (x, y) moves on a circle centered at $(0, 0)$ and radius 5 units when the point on $(3, 4)$, $\frac{dx}{dt} = -2$ unit/sec then $\frac{dy}{dt} =$ ……… unit/sec

a $-\frac{8}{3}$ b $-\frac{2}{3}$ c $-\frac{3}{5}$ d $\frac{3}{2}$

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(26) The rate of change of the volume of a sphere with respect to its radius when its radius is 2 cm is ………

a 8π b 16π c 6π d 4π

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(27) If the perimeter of a lamina in a square – form increase by 0.4 cm/sec and its surface area increase by $6\ cm^2/sec$ then the length of the lamina edge at that moment equals …….. cm

a 30 b 40 c 50 d 60

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(28) **Circle of perimeter p cm if the radius length decreases at rate $0.1\ cm/sec$, then the rate of change of its area $=$ ……… cm^2/sec**

[a] -0.2 p   [b] -0.1 p   [c] 0.1 p   [d] 0.2 p

(29) **If $y = 2\sin^2 x - 5\cos^2 x$ then $\frac{d^2y}{dx^2} =$ ………**

[a] $7\sin 2x$ [b] $14\cos 2x$ [c] $14\sin 2x$ [d] $7\cos 2x$

(30) **If $y = 3^z$, $z = \sin t$, $t = \ln x$ then $\frac{dy}{dx}$ at $x = 1$ is ……...**

[a] $3\ln 3$ [b] $\ln 3$ [c] $2\ln 3$ [d] 1

(31) If $y = A\sin^2 x + B\cos^2 x$ then $\frac{2\,y'}{y''} = \ldots\ldots\ldots$

[a] $\tan 2\,x$ [b] $\cot 2\,x$ [c] $\sin 2\,x$ [d] $\sec 2\,x$

(32) If $f\,(x) = \sec x$ then $f''\left(\frac{\pi}{3}\right) = \ldots\ldots\ldots$

[a] 12 [b] 13 [c] 14 [d] 15

(33) The slope of the tangent to the curve $\cos\sqrt{\pi\,y} = 3\,x + 1$ at $\left(-\frac{1}{3}, \frac{\pi}{4}\right)$ ………

[a] 3 [b] -3 [c] 4 [d] -4

(34) **If $x = \tan y$ then $\frac{dy}{dx} = \ldots\ldots\ldots$**

a $1 + x^2$ b $\frac{1}{1 + x^2}$ c $\frac{1}{\sqrt{1 + x^2}}$ d $\sqrt{1 + x^2}$

(35) **The tangent to the curve of the function $y = e^{\cos x}$ at $x = \frac{\pi}{2}$ makes an angle with the positive direction of $x - axis$ of measure ………**

a $\frac{\pi}{2}$ b $\frac{3\pi}{4}$ c $\frac{5\pi}{6}$ d π

(36) **The slope of tangent to the curve of the function $y = \ln\ (\tan x)$ at $x = \frac{\pi}{4}$ equals …….**

a -1 b 1 c 0 d 2

(37) If $y = e^{n\,x}$ then $\frac{d^n y}{dx^n} = \ldots\ldots\ldots$

[a] $n\,e^x$ [b] $n^n e^x$ [c] $n^n e^{n\,x}$ [d] $n^n e^{n\,x-1}$

(38) Slope of the normal of the function $f\,(x) = 2\ln\sec x$ at $x = \frac{\pi}{4}$ is ....

[a] -2 [b] $-\frac{1}{2}$ [c] $\frac{1}{2}$ [d] 2

(39) $\frac{d}{dx}\left(e^{ln(x^{10})}\right) = \ldots\ldots\ldots$

[a] $e^{ln(x^{10})} \times 10$ [b] $e^{ln(x^{10})} \times 10x^9$ [c] $10\,x^9$ [d] $e^{ln(x^{10})} \times x^9$

(40) If $y = \ln\ (\sec x + \tan x)$ then, $\frac{dy}{dx} = \dots\dots\dots$

[a] $\tan x$ [b] $\sec x$ [c] $\tan^2 x$ [d] $\csc x$

(41) If $f\ (x) = 5^{x^2}$ then $f'\ (1) = \dots\dots\dots$

[a] $\ln 5$ [b] $15 \ln 5$ [c] $3 \ln 5$ [d] $5 \ln 5$

(42) The equation of the tangent to the curve of the function $f\ (x) = e^{2\,x+1}$ at the point $\left(-\frac{1}{2}, 1\right)$ is $\dots\dots\dots$

[a] $2\,y = x + 1$ [b] $y = 2\,x + 2$ [c] $y = 2\,x - 3$ [d] $2\,y = 3\,x + 1$

(43) If $y = ln(e^{\sin^2 x})$ then $\frac{dy}{dx} = \dots\dots\dots$

a $\cos 2x$ b $2\cos 2x$ c $\sin 2x$ d $\sec 2x$

(44) The rate of change of $\sqrt{x^2+16}$ with respect to $\frac{x}{x-1}$ at $x = 3$ equal ....

a -2.4 b 2.4 c 3.7 d 1.75

(45) A triangle metal fine lamina expands regularly such that the base length equals three times its height. If the rate of increasing of its area is $.27\ \text{cm}^2./\text{sec}$, then the rate of change of its height at the moment when the lamina's height equal 9 cm. is .... cm./sec.

a 1 b $\frac{1}{2}$ c 0.01 d 0.1

(46) **The rate of change of e^{x^3} with respect to $\ln x$ equals ....**

$\boxed{a}$ $3\,x^2\,e^{x^3} + 3\,x^2$ $\boxed{b}$ e^{x^3} $\boxed{c}$ $3\,x^3\,e^{x^3}$ $\boxed{d}$ $3\,x^2\,e^{x^3}$

(47) **If $f(x) = (\sin x)^x$, then $\frac{dy}{dx} = \ldots\ldots$**

$\boxed{a}$ $y\,(\ln \sin x + x \cot x)$ $\boxed{b}$ $\sin x \times \ln \sin x$

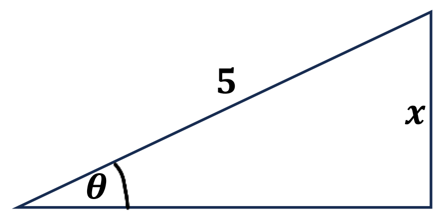
$\boxed{c}$ $x\,(\sin x)^{x-1}\,(\cos x)\ \ln \sin x$ $\boxed{d}$ $x\,(\sin x)^{x-1}\ \cos x$

Level (3)

(49) A point A (x, y) moves on the curve: $y = x^2 + 1$ such that its $x -$ coordinate increases at rate 4 length units/sec. If S is the distance between the point A and the point B $(0, 1)$, then the rate of change of S at $x = 3$ equal .... length unit/sec.

a $\frac{38\sqrt{10}}{5}$ b $\frac{25\sqrt{2}}{7}$ c $\frac{17\sqrt{3}}{2}$ d $\frac{32\sqrt{5}}{3}$

(50) In the triangle shown, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?



a 3 b 4 c 9 d 12

(51) **Find the rate of change of the area of a square with respect to its perimeter when its perimeter equals 24 cm**

[a] 1 [b] 3 [c] 10 [d] 13

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(52) **A point moves on the curve: $y^2 = 25 - x^2$ such that: $\frac{dx}{dt} = \frac{1}{2\,x + 3}$, then $\frac{dy}{dt}$ at the point $(-3\,,4)$ equals ....**

[a] $-\frac{1}{4}$ [b] $\frac{1}{4}$ [c] $\frac{1}{9}$ [d] 9

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(53) **If $\sin x = x\,y$, then: $x^2\,(y + y'') + 2\,\cos x =$ ....**

[a] $4\,y$ [b] $5\,x^2$ [c] y^2 [d] $2\,y$

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(54) **Water is poured into an empty cylindrical container of radius 10 cm. and height 60 cm. at the rate of 30 π $cm^3./sec.$, then the rate at which the height of the water rises in the container = ………**

$\boxed{a}$ **3 cm./sec.** $\boxed{b}$ **3 m./sec.** $\boxed{c}$ **0.003 m./sec.** $\boxed{d}$ $\frac{1}{3}$ **cm./sec.**

(55) **A balloon rises up vertically with constant speed 15 m./sec. , when the balloon at height 90 m. , a car passes on the ground just below the balloon and remains moving in a straight line with constant speed 25 m./sec. , then the rate of increasing of distance between the car and the balloon after 2 seconds from the moment that the car was below the balloon = ……… m./sec.**

$\boxed{a}$ **130** $\boxed{b}$ $\frac{3.5}{13}$ $\boxed{c}$ $\frac{450}{13}$ $\boxed{d}$ $\frac{305}{13}$

(56) If $y = \ln z$, $z = e^{3\,t}$, $t = \sin^2 x$, then $\frac{dy}{dx} = \text{........}$ at $x = \frac{\pi}{3}$

a $\frac{3}{2}$ b $\frac{-3}{2}$ c $\frac{-3\sqrt{3}}{2}$ d $\frac{3\sqrt{3}}{2}$

(57) If $f(2\,x + 3) = 4\,x^2 + 1$, then $f'\,(1) = \text{........}$

a -7 b -6 c -5 d -4

(58) *Essay:* The length of the legs of the right-angle of a right-angled triangle at a moment , are 6 cm. and 30 cm. If the length of the first leg increases at a rate of $\frac{1}{3}$ cm./min. and the length of the second leg decreases at a rate of 1 cm./min. , Find :

(a) The rate of increase in the area of the triangle after 3 minutes.

(b) The time at which the increase of the area of the triangle stops.

(59) If $y = \frac{3x-5}{x-2}$, then at $x = 1$, $\frac{d^3y}{dx^3}$ equals ....

$\boxed{a}$ -12 $\boxed{b}$ -6 $\boxed{c}$ 6 $\boxed{d}$ 12

(60) *Essay :* If $\sin y + \cos 2x = 0$ Prove that : $\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 \tan y = 4\cos 2x \sec y$