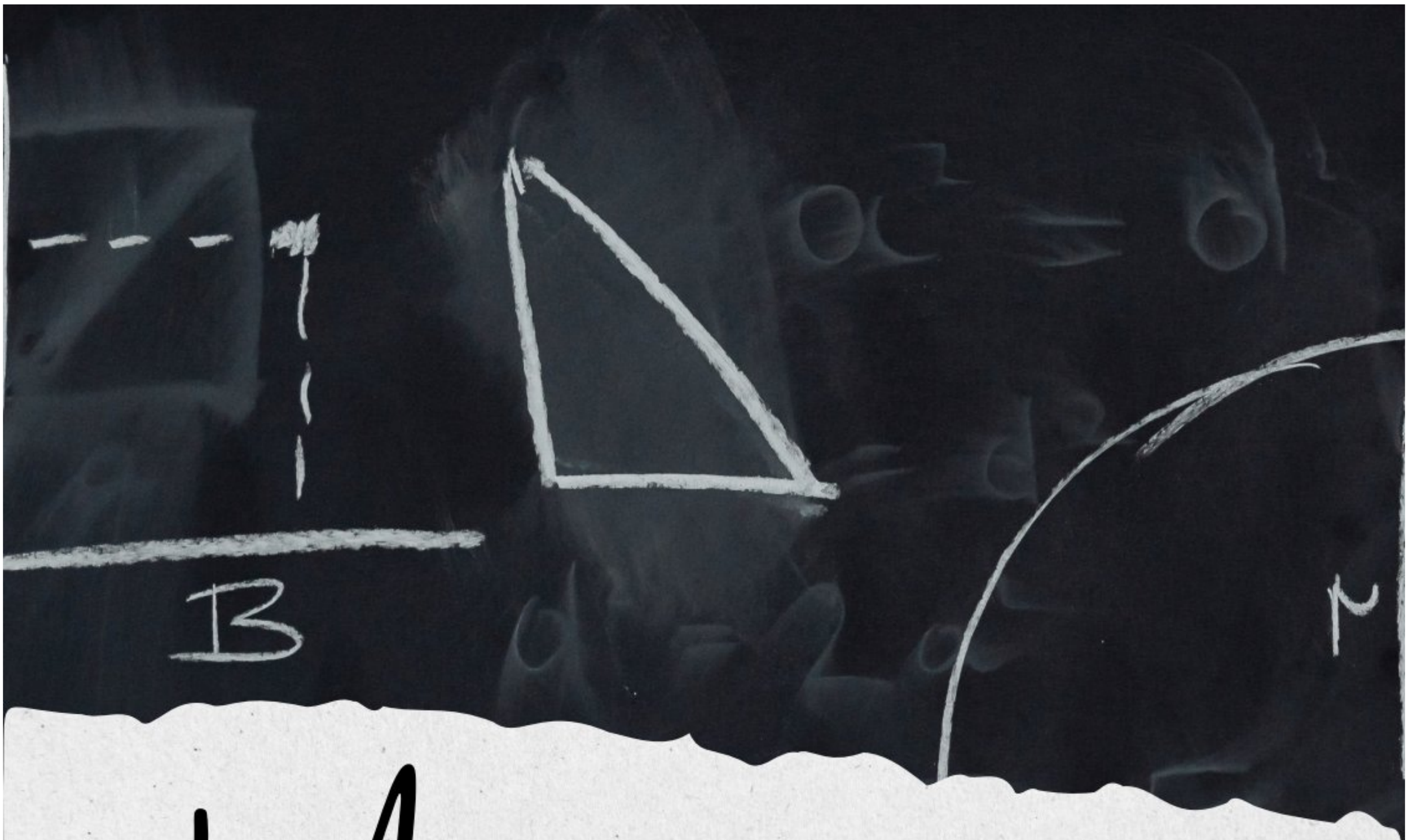


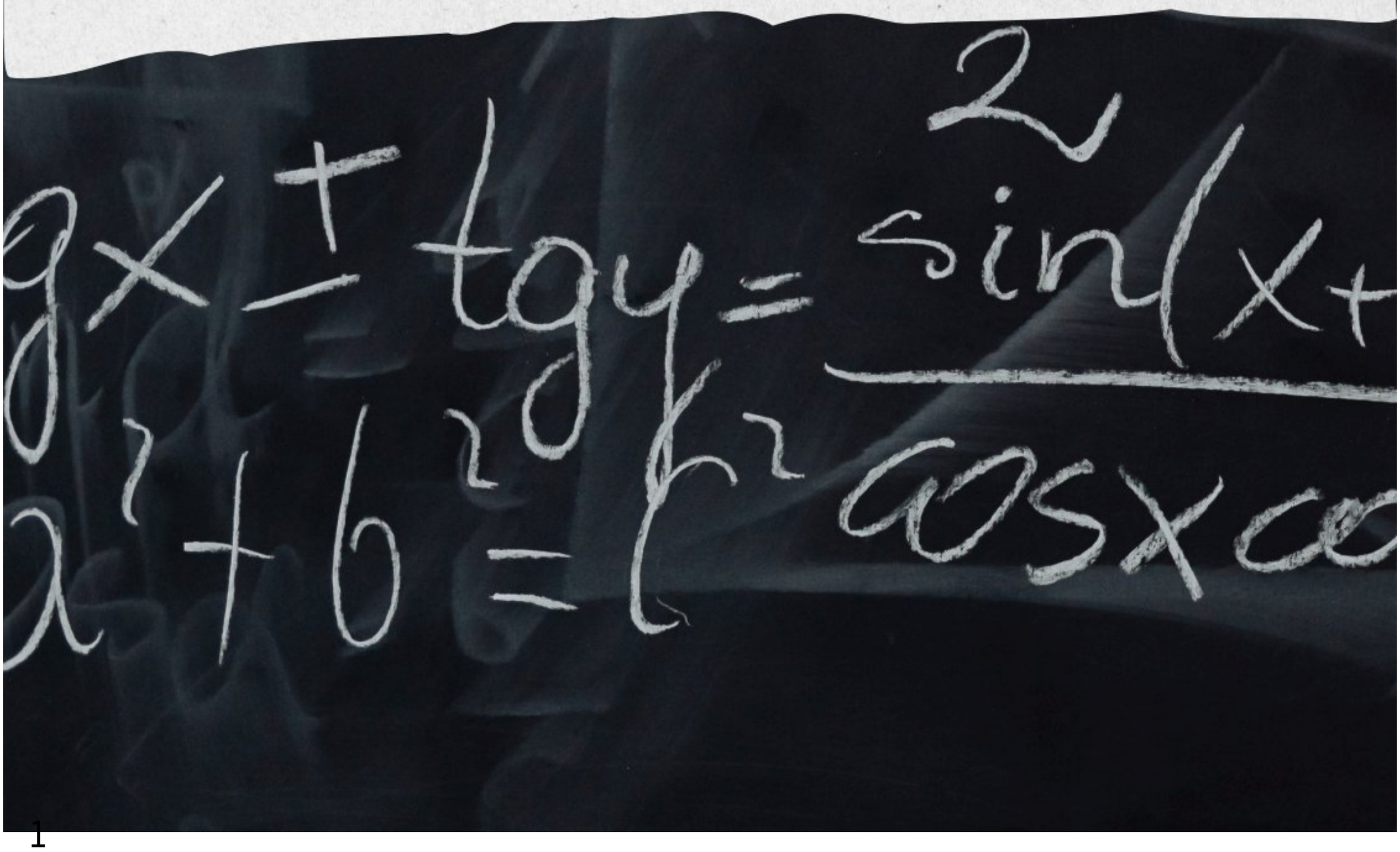
Mathematics

--------Ahmed sultan --------

REVISION

Unit2

CALCULUS



Q(1)If $F(X) = -2X^3 + 6X^2 - 3$ find

(1) Critical points (2) increase and decrease intervals

(3) local maximum and local minimum value

(4) concave up and down intervals (5) inflection point

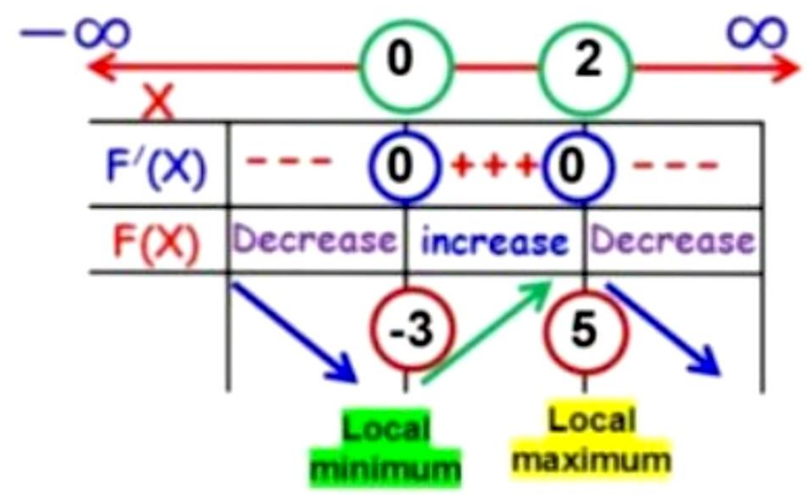
(6) sketch

$F^{/}(X) = -6X^2 + 12X$

$\therefore -6X^2 + 12X = 0$

$\therefore -6X\,(X-2) = 0$

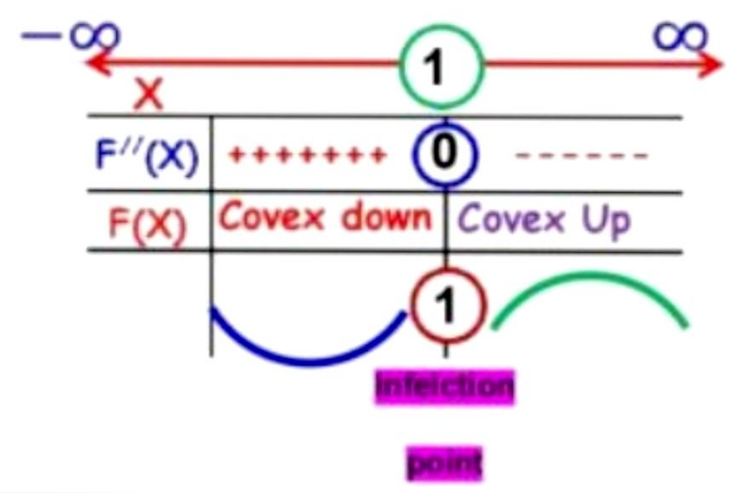
$\therefore \begin{cases} X = 0 & \therefore Y = -3 \\ X = 2 & \therefore Y = 5 \end{cases}$



Decrease in $]2, \infty[$, $]-\infty, 0[$ increase in $]0, 2[$

The point (2,5) is local maximum point

The point (0,-3) is local minimum point



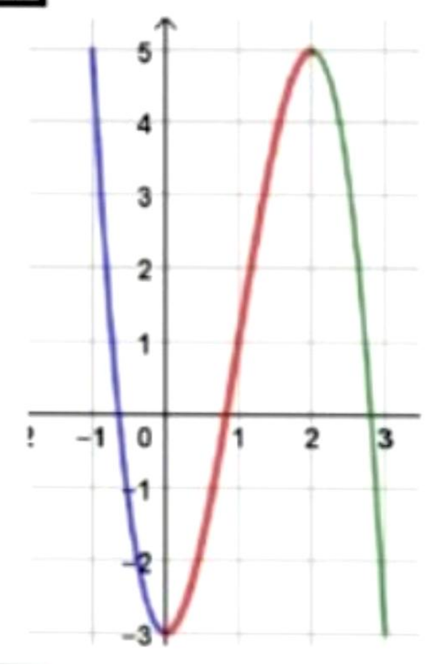
<u>Second derivative</u> : $F^{//}(X) = -12X + 12$

$\therefore -12X + 12 = 0 \quad \therefore -12X = -12$

$\therefore X = 1 \quad \therefore Y = 1$

$\boxed{\text{Convex downwards in }]-\infty, 1[}$ $\boxed{\text{Convex upwards in }]1, \infty[}$

The point (1,1) is called inflection point



Q(6) The sum of two positive integers is 10 find the two numbers if their product is to be maximum

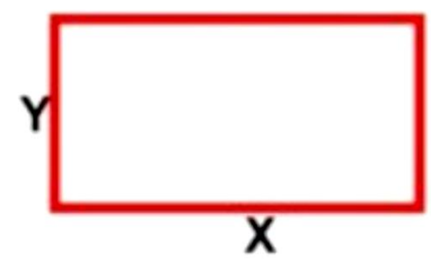
Let the two numbers X and 10 − X

$F(X) = X(10 - X) = 10X - X^2 \qquad \therefore\ F^{/}(X) = 10 - 2X = 0$

$\therefore\ 2X = 10 \quad \therefore\ X = 5 \quad \therefore$ the two numbers 5, 5

$F^{//}(X) = -2 \quad \therefore\ F^{//}(X) < 0$

Q(7) The perimeter of a rectangle is 30cm find the length of its dimensions such that its surface area is to be maximum



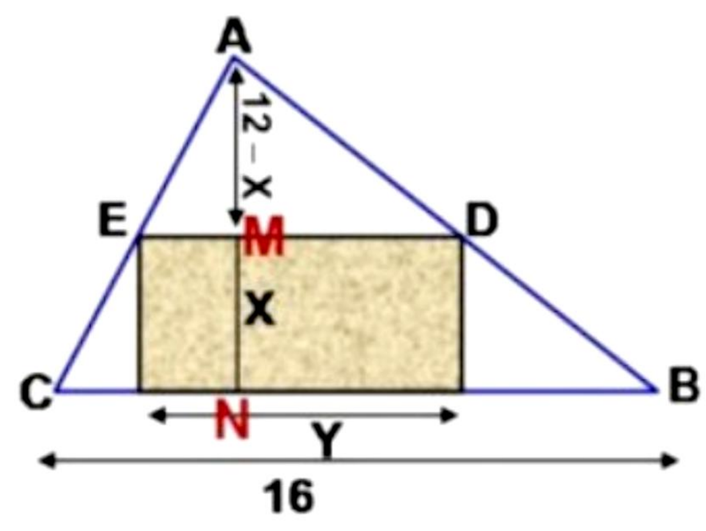
$2(X + Y) = 30 \quad \therefore\ X + Y = 15 \quad \therefore\ Y = 15 - X$

$F(X) = XY = X(15 - X) = 15X - X^2$

$\therefore\ F^{/}(X) = 15 - 2X = 0 \quad \therefore\ 2X = 15 \quad \therefore\ X = 7.5$

$\therefore\ F^{//}(X) = -2 \quad \therefore\ F^{//}(7.5) < 0$

Q(8) Find the two dimensions of a rectangle which has the biggest area and it can be drawn inside a triangle whose base length is 16cm and its height is 12 cm such that one of its sides lies on the base of the triangle and the two vertices of the opposite side lie on the two other sides of the triangle .



$$\frac{ED}{BC} = \frac{AM}{AN}$$

$$\frac{Y}{16} = \frac{12 - X}{12} \quad \therefore\ 12Y = 16(12 - X)$$

$$\therefore\ Y = \frac{16}{12}(12 - X) \qquad \boxed{\therefore\ Y = \frac{4}{3}(12 - X)}$$

$$F(X) = XY = X\left(\frac{4}{3}(12 - X)\right) = 16X - \frac{4}{3}X^2$$

$$F^{/}(X) = 16 - \frac{8}{3}X = 0 \quad \therefore\ X = 6$$

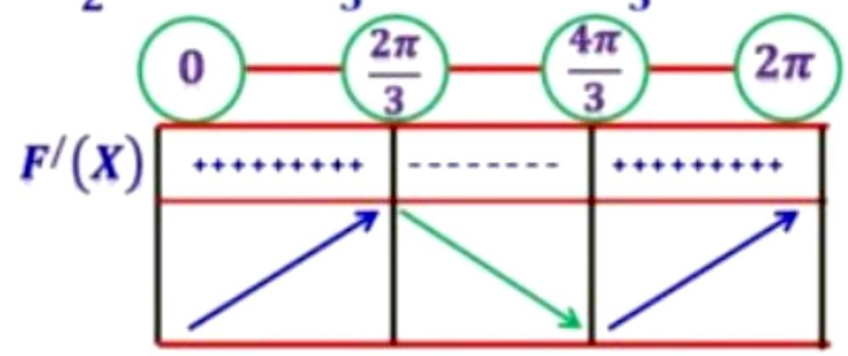
$$F^{//}(X) = -\frac{8}{3} < 0 \quad \text{Maximum area} = 6 \times 8 = 48cm^2$$

Q(9) The function $F(X) = X + 2\sin X$ on $0 < X < 2\pi$

Find the interval of decrease

$F^{/}(X) = 1 + 2\cos X = 0 \quad \therefore \cos X = -\frac{1}{2} \quad \therefore \theta = \frac{2\pi}{3} \text{ or } \theta = \frac{4\pi}{3}$

$\therefore \text{decrease} \left]\frac{2\pi}{3}, \frac{4\pi}{3}\right[$



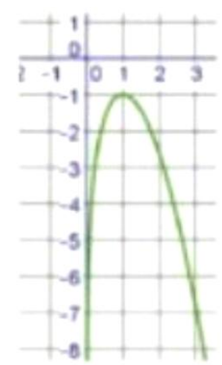
(b) The function $F(X) = 2\ln X - X^2$

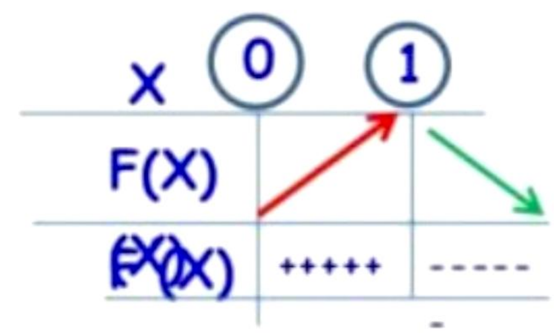
Find the interval of decrease

Domain $]0, \infty[$

$F^{/}(X) = \frac{2}{X} - 2X = \frac{2 - 2X^2}{X} = \therefore X = 1$

Increase $]0, 1[$, decrease $]1, \infty[$





Q(10) A rectangle - like playground ends in two semi-circles. If the perimeter of the playground is 420 m, find the maximum area of the playground.

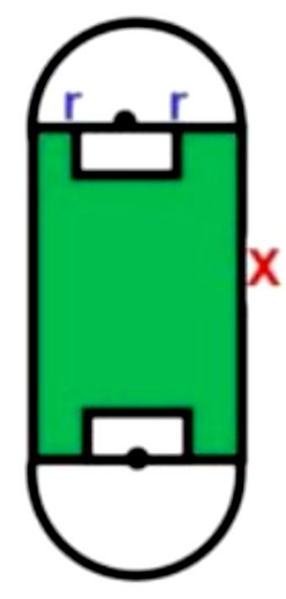
$P = 2X + 2\pi r = 420 \quad \therefore X + \pi r = 210 \quad \therefore \boxed{X = 210 - \pi r}$

$A = \pi r^2 + 2rX = \pi r^2 + 2r(210 - \pi r)$

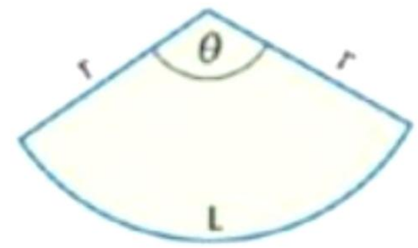
$A = \pi r^2 + 420r - 2\pi r^2 = 420r - \pi r^2$

$\frac{dA}{dr} = 420 - 2\pi r = 0 \quad \therefore 2\pi r = 420 \quad \therefore r = \frac{210}{\pi}$

$\frac{d^2A}{dr^2} = -4\pi < 0$



Q(11) If the perimeter of a circular sector is 30 cm and its area is as maximum as possible , find the radius length of its circle.

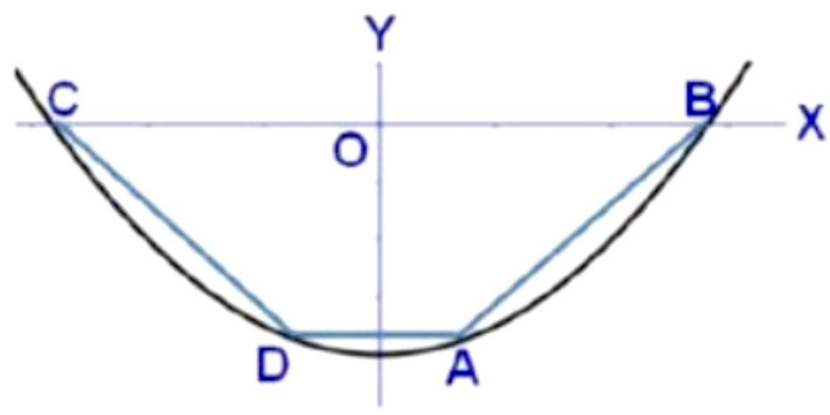


$$2r + L = 30 \quad \therefore L = 30 - 2r$$

$$A = \frac{1}{2}Lr \quad \therefore A = \frac{1}{2} \times r(30 - 2r) = 15r - r^2$$

$$\frac{dA}{dr} = 15 - 2r = 0 \quad \therefore r = 7.$$

Q(12) Find the area of largest trapezium drawn indide curve of the function Two of its verices lies on the curve $F(X) = X^2 - 9$ and the other two On the X-axis



$$X^2 - 9 = 0 \quad \therefore \begin{cases} X = 3 & \therefore B(3,0) \\ X = -3 & \therefore B(-3,0) \end{cases}$$

$$A = \frac{1}{2}(6 + 2X) \times Y$$

$$A = \frac{1}{2}(6 + 2X) \times (X^2 - 9)$$

$$A = (3 + X)(X^2 - 9) = 3X^2 - 27 + X^3 - 9X$$

$$\frac{dA}{dX} = 6X + 3X^2 - 9 = 0 \quad \therefore X = 1$$

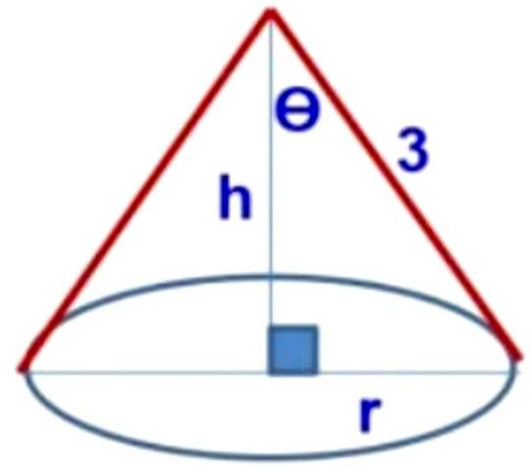
Q(13) If the curve : $Y = (2X - a)^3 + 4$ has an inflection Point at $X = 5$ Then a=.....

①5 ②10 ③15 ④20

$$\frac{dY}{dX} = 3(2X - a)^2 \quad \therefore \frac{d^2Y}{dX^2} = 12(2X - a) = 0 \ when\ X = 5$$

$$2 \times 5 - a = 0 \quad \therefore a = 10$$

Q(14) Find the maximum volume of a right circular cone having slant height 3m



$h = 3\cos\theta \quad , r = 3\sin\theta$

$V = \frac{1}{3}\pi r^2 \times h = \frac{\pi}{3}(9\sin^2\theta)(3\cos\theta)$

$\frac{dV}{d\theta} = 9\pi(2\sin\theta\cos\theta \times \cos\theta - \sin\theta \times \sin^2\theta)$

$\therefore \sin\theta(2\cos^2\theta - \sin^2\theta) \quad \therefore \frac{\sin\theta}{\cos\theta} = \sqrt{2}$

$\therefore \tan\theta = \sqrt{2} \quad \therefore \sin\theta = \sqrt{\frac{2}{3}}$

$\cos\theta = \frac{1}{\sqrt{3}} \quad \therefore V = 2\sqrt{3}\pi$

Q(15) Identify the convexity intervals downwards and the inflection points (if existed) to the curve of the function f where $F(X) = X^2 - \frac{8}{X}$

$F^{/}(X) = 2X + \frac{8}{X^2} \Rightarrow F^{//}(X) = 2 - \frac{16}{X^3} = 0 \quad \therefore \frac{16}{X^3} = 2 \quad \therefore X^3 = 8 \quad \therefore X = 2$

Convex upward $]0,2[$, convex downward $]-\infty, 0[$, $]2, \infty[$ inflection point (2,0)

Q(16) Find the absolute maximum values of the function $F(X) = Xe^{-X}$, $X \in [0, 2]$

$F^{/}(X) = e^{-X} - Xe^{-X} = 0 \quad \therefore e^{-X}(1 - X) = 0 \quad \therefore X = 1$

$\because 1 \in [0, 2]$

$$\begin{cases} F(0) = 0 \\ F(1) = e^{-1} \\ F(2) = e^{-2} \end{cases} \quad \therefore \textit{Absolute maximum} = e^{-1}$$

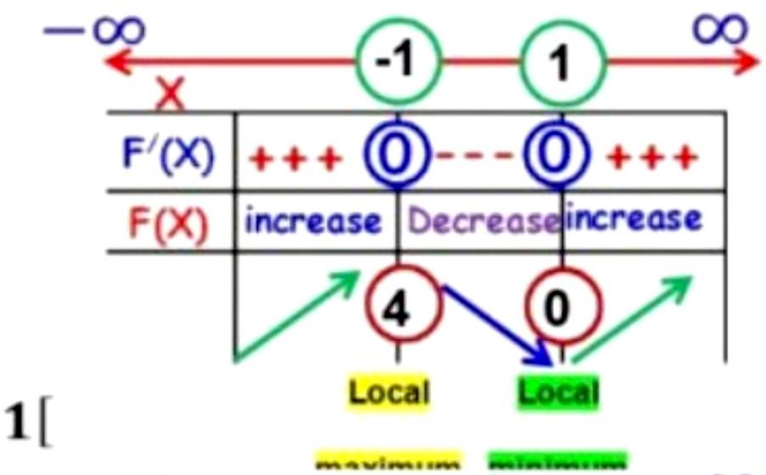
Q(17)If $F(X) = X^3 - 3X + 2$find

(1) Critical points (2) increase and decrease intervals

(3) local maximum and local minimum value

(4) concave up and down intervals (5) inflection point

(6) sketch



First derivative : $F^{/}(X) = 3X^2 - 3$

$\therefore 3X^2 - 3 = 3(X^2 - 1) = 0$

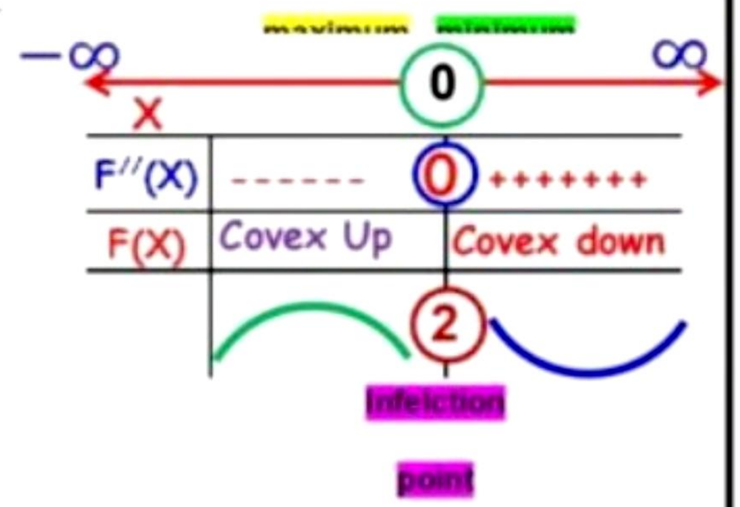
$\therefore (X - 1)(X + 1) = 0$

$\therefore \begin{cases} X = 1 & \therefore Y = 1^3 - 3(1) + 2 = 0 \\ X = -1 & \therefore Y = (-1)^3 - 3(-1) + 2 = 4 \end{cases}$

Increase in $]1, \infty[$, $]-\infty, -1[$ decrease in $]-1, 1[$

The critical point (1,0) is local minimum

The critical point (-1,4) is local maximum



Second derivative : $F^{//}(X) = 6X$

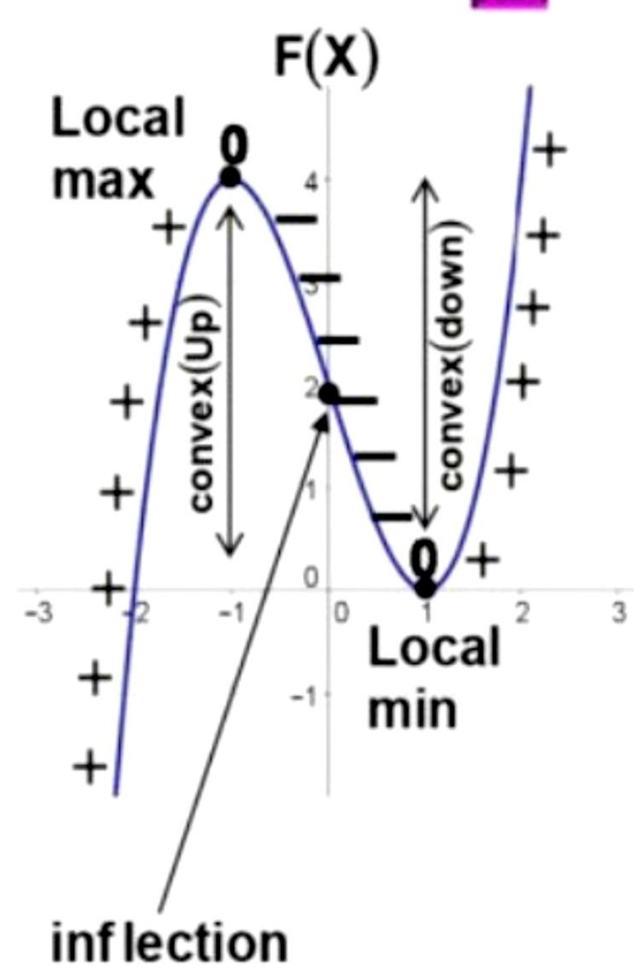
$\therefore 6X = 0 \quad \therefore X = 0 \quad F^{//}(X) = \begin{cases} F^{//}(-1) = -6L.max \\ F^{//}(1) = 6L.min \end{cases}$

$\therefore Y = 0^3 - 3(0) + 2 = 2$

$\boxed{\text{Convex upwards in }]-\infty, 0[}$

$\boxed{\text{Convex downwards in }]0, \infty[}$

The point (0,2) is called inflection point



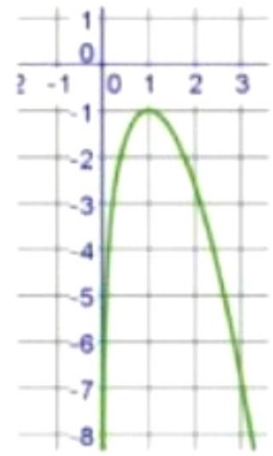
Q(18) The function $F(X) = 2\ln X - X^2$ decrease on .....

① $]0, 1[$ ② $]1, \infty[$ ③ $]-1, \infty[$ ④ $]-\infty, \infty[$

Domain $]0, \infty[$

$$F^{/}(X) = \frac{2}{X} - 2X = \frac{2 - 2X^2}{X} = \therefore X = 1$$

Increase $]0, 1[$, decrease $]1, \infty[$



X		0		1	
F(X)			↗		↘
F/(X)			+++++		-----

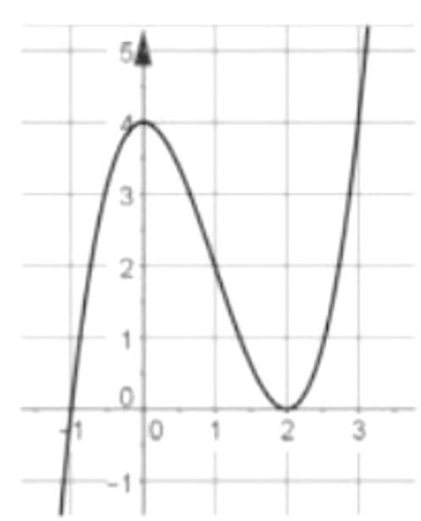
Q(19) Find the values of a and b such that the curve $Y = X^3 + aX^2 + bX$ has an inflection point at $(2, 2)$

$$\frac{dY}{dX} = 3X^2 + 2aX + b \quad \therefore \frac{d^2Y}{dX^2} = 6X + 2a = 0 \text{ when } X = 2$$

$\therefore 2a = -12 \;\; \therefore a = -6$

$2 = 2^3 - 6 \times 2^2 + 2b \;\; \therefore b = 9$

Q(20) If $F(X) = X^3 + aX^2 + bX + 4$ where a and b are two constants, Find the two values of a and b if the function f has a local minimum value when x = 2 and an inflection point when x = 1, then sketch the curve of the function f



$F^{/}(X) = 3X^2 + 2aX + b$, $F^{//}(X) = 6X + 2a$

When X=2 $\therefore 3 \times 2^2 + 2a \times 2 + b = 0$

$\therefore 4a + b = -12 \rightarrow (1)$ When X=1 $\therefore 6 + 2a = 0$

$\therefore 2a = -6 \;\; \therefore a = -3 \;\; \therefore b = 0$

$\therefore F(X) = X^3 - 3X^2 + 4 = 0$

$\Rightarrow F^{/}(X) = 3X^2 - 6X = 0 \;\; \therefore X = 0 \;\; or \; X = 2$

Critical points $(0, 4)$, $(2, 0)$ $F^{//}(X) = 6X - 6 \;\; \therefore X = 1 \;\; \therefore (1, 2) inflection\ point$

Q(21) Find the dimensions of a rectangle piece of land bounded by a river from one side and of area $2048m^2$ such that the length of a fence surrounding its three remaining sides is minimum

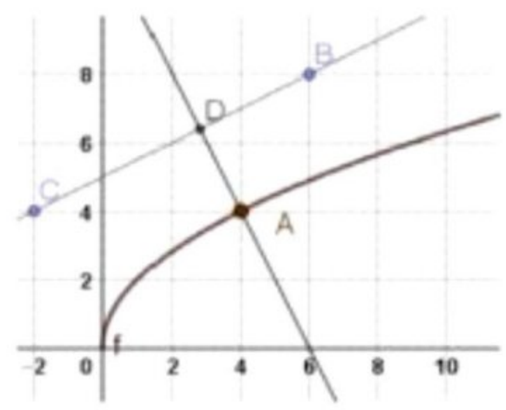
Let the two dimensions of the fence X and Y

$XY = 2048 \therefore Y = \frac{2048}{X}$

$P = X + 2Y = X + 2 \times \frac{2048}{X} = X + 4096X^{-1} \quad \therefore \frac{dP}{dX} = 1 - 4096X^{-2} = 0$

$\frac{4096}{X^2} = 1 \quad \therefore X^2 = 4096 \quad \therefore X = 64$

Q(22) Find the shortest distance between the straight line $X - 2Y + 10 = 0$ and the curve $Y^2 = 4X$



$length\ of\ perpendicular\ from (X_1, Y_1)$

To the line whose equation $aX + bY + c = 0$

$\frac{|aX_1 + bY_1 + c|}{\sqrt{a^2 + b^2}}$

The point on the curve is $(X, Y) = (X, \sqrt{4X}) = (X, 2\sqrt{X})$

$\therefore F(X) = \frac{X - 2 \times 2\sqrt{X} + 10}{\sqrt{1^2 + (-2)^2}} = \frac{X - 4\sqrt{X} + 10}{\sqrt{5}}$

$F^{/}(X) = \frac{1}{\sqrt{5}}\left(1 - \frac{4}{2\sqrt{X}}\right) = 0 \quad \therefore \frac{4}{2\sqrt{X}} = 1 \quad \therefore 2\sqrt{X} = 4 \quad \therefore X = 4$

Q(23) The function $F(X) = \frac{1}{3}X^3 + 3X^2 - 7X + 3$ has an inflection point at X=....

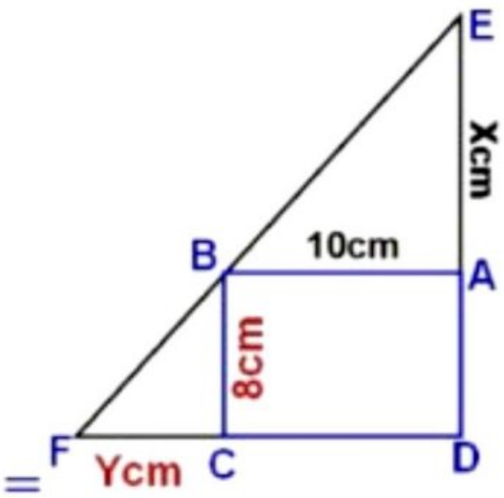
(a) -3 (b) 3 (c) -1 (d) 1

$F^{/}(X) = X^2 + 6X - 7 \quad \therefore F^{//}(X) = 2X + 6$

The inflection point when $F^{//}(X) = 0$

$2X + 6 = 0 \quad \therefore X = -3$

Q(24) ABCD is is a rectangle with AB=10cm and BC=8cm length of AE=Xcm And CF =Ycm Find the maximum area of triangle EDF



$$\frac{10}{Y} = \frac{X}{8} \quad \therefore Y = \frac{80}{X}$$

$$A = \frac{1}{2} \times (Y + 10) \times (8 + X)$$

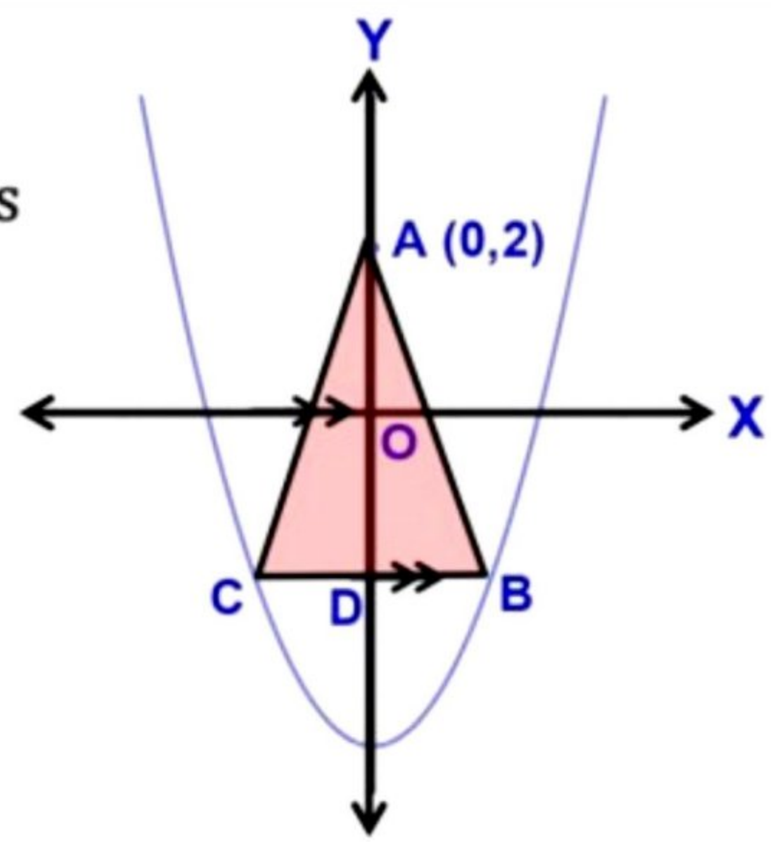
$$= \frac{1}{2} \times \left(\frac{80}{X} + 10\right)(8 + X) = \left(\frac{40}{X} + 5\right)(8 + X)$$

$$\frac{dA}{dX} = -80X^{-2} \times (8 + X) + \left(\frac{40}{X} + 5\right) = 0 \quad \times X^2$$

$$-80(8 + X) + 40X + 5X^2 = 0 \quad \therefore \quad 5X^2 - 40X - 640 =$$

Q(25) The opposite figure :

Represent the curve of the quadratic function $F(X) = X^2 - 4$ If the coordinates of $A(0, 2)$, the two points B,C lies on The curve of the function such that : $\overleftrightarrow{BC}$//to the X axis **Find** the coordinates of the point B which makes The area of ΔABC greatest as possible



$$A = \frac{1}{2} BC \times AD = \frac{1}{2} \times BC \times (AO + OD)$$

$$A = \frac{1}{2} \times (2X) \times (2 - Y)$$

$$A = \frac{1}{2} \times (2X) \times \left(2 - X^2 + 4\right) = X\left(-X^2 + 6\right) = -X^3 + 6X$$

$$\frac{dA}{dX} = -3X^2 + 6 = 0 \quad \therefore X^2 = 2 \quad \therefore X = \pm\sqrt{2}$$

Q(26) If the function $F(X) = X^3 - aX^2 + 1$ has a critical point At $X = 2$ then $a = \cdots$

(a) 3 (b) 2 (c) 1 (d) 0

$F^{/}(X) = 3X^2 - 2aX$

Critical point when $F^{/}(X) = 0$ when $X = 2$

$\therefore 3 \times 2^2 - 2a \times 2 = 0 \quad \therefore 12 - 4a = 0 \quad \therefore 4a = 12 \quad \therefore a = 3$

The answer no(a)

Q(27) If the function $F(X) = X^3 + aX^2 + 5X + 1$ has an inflection point At $X = -1$ then $a = \cdots$

(a) -1 (b) 1 (c) 3 (d) 2

$F^{/}(X) = 3X^2 + 2aX + 5 \quad \therefore F^{//}(X) = 6X + 2a$

$6X + 2a = 0 \ when\ X = -1$

$-6 + 2a = 0 \quad \therefore a = 3$

Q(28) Find the values of a and b such that the curve $Y = X^3 + aX^2 + bX$ has an inflection point at $(2, 2)$

$\frac{dY}{dX} = 3X^2 + 2aX + b \quad \therefore \frac{d^2Y}{dX^2} = 6X + 2a = 0 \ when\ X = 2$

$\therefore 2a = -12 \quad \therefore a = -6$

$2 = 2^3 - 6 \times 2^2 + 2b \quad \therefore b = 9$

Q(29) Find the absolute extreme value of the function F Where $F(X) = 2X^2e^X$, $X \in [-3, 1]$

$F^{/}(X) = 2(2Xe^X + e^XX^2) = 0 \quad \therefore 2Xe^X(2 + X) = 0 \quad \therefore \begin{cases} X = 0 \\ X = -2 \end{cases}$

Notice that 0,-2$\in [-3, 1]$

$$\begin{cases} F(-3) = 18e^{-3} \\ F(-2) = 8e^{-2} \\ F(0) = 0 \\ F(1) = 2e \end{cases} \quad \therefore max = 2e \ , min = 0$$

Q(30) <u>Answer only one of the following two questions:</u>

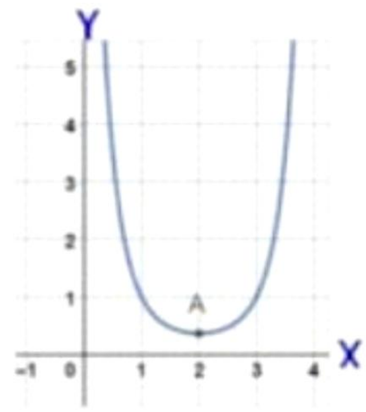
(a) Find the local maximum and minimum values (if existed) of the function $F(X) = e^{X^2-4X+5}$

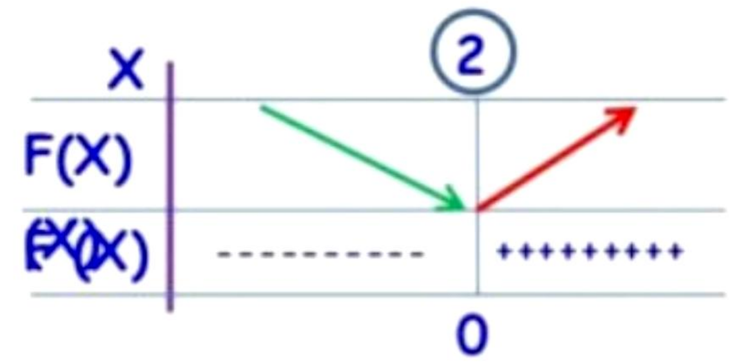
$F^{/}(X) = (2X-4)e^{X^2-4X+5}$

$\therefore 2X - 4 = 0 \quad \therefore X = 2$

$F(2) = e^{-1}$

local minimum value





(b) If $F(X) = 15X + 6X^2 - X^3$ find the convexity intervals upwards and downwards and the inflection points (if existed) to the curve of the function f

$F^{/}(X) = 15 + 12X - 3X^2$

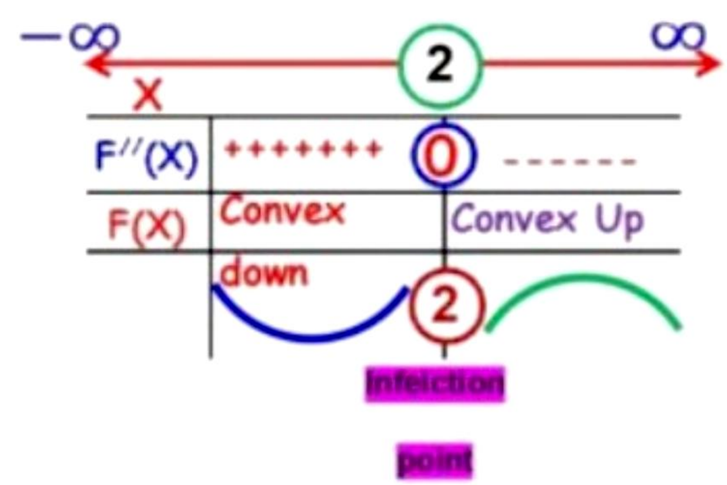
$F^{//}(X) = 12 - 6X = 0$

$\therefore X = 2$

Convex up in $]2, \infty[$

Convex down in $]-\infty, 2[$

Inflection point $(2, 46)$



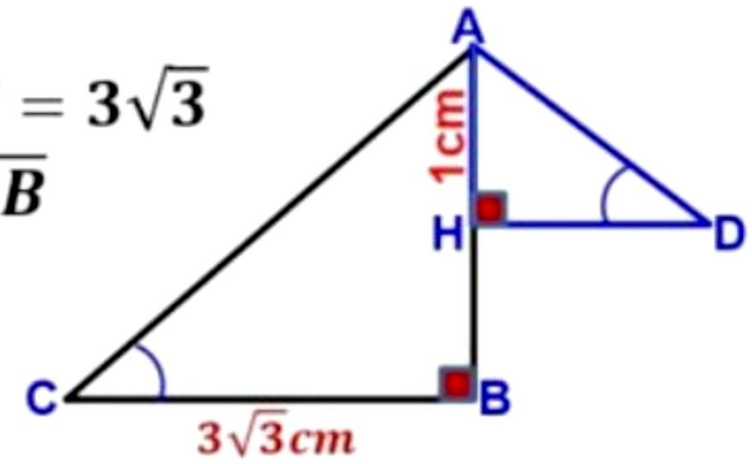
Q(31) The opposite figure :

$\overline{DH} \perp \overline{AB}$, $\overline{AB} \perp \overline{BC}$, $AH = 1cm$, $BC = 3\sqrt{3}$

$m(\angle D) = m(\angle C)$ Find the length of $\overline{HB}$

Which makes $AD + AC$

Is minimum as possible



In ΔAHD: $sin\theta = \frac{AH}{AD} = \frac{1}{AD}$ $\therefore AD = \frac{1}{sin\theta} \rightarrow (1)$

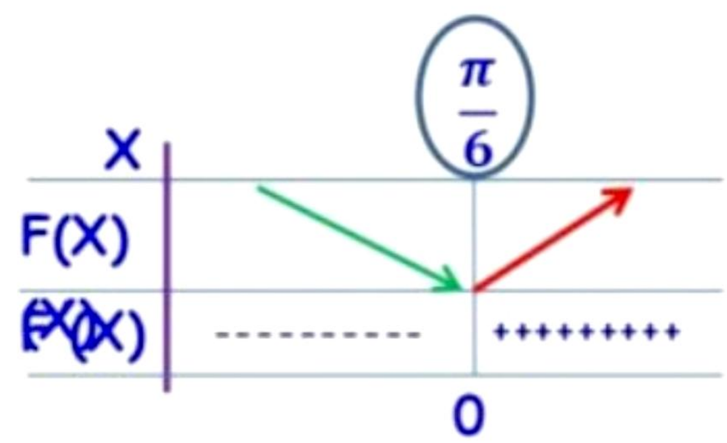
In ΔAHD: $cos\theta = \frac{BC}{AC} = \frac{1}{AD}$ $\therefore AC = \frac{3\sqrt{3}}{cos\theta} \rightarrow (2)$

$$F(\theta) = AD + AC = \frac{1}{sin\theta} + \frac{3\sqrt{3}}{cos\theta}$$

$$F^{/}(\theta) = \frac{-cos\theta}{sin^2\theta} + \frac{3\sqrt{3}sin\theta}{cos^2\theta} = \frac{3\sqrt{3}sin^3\theta - cos^3\theta}{sin^2\theta cos^2\theta} = 0$$

$\therefore 3\sqrt{3}sin^3\theta - cos^2\theta = 0$ $\therefore tan^3\theta = \frac{1}{3\sqrt{3}}$

$\therefore tan\theta = \frac{1}{\sqrt{3}}$ $\therefore \theta = \frac{\pi}{6}$



In ΔABC $tan\theta = \frac{AB}{BC}$ $\therefore AB = 2cm$

Q(32) If $Y = 2X - 8$ then the minimum value of the product (XY)= ....

① -16 ② -8 ③ -4 ④ 2

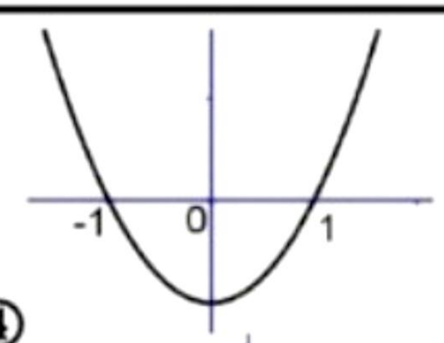
$F(X) = X(2X - 8) = 2X^2 - 8X \quad \therefore F^{/}(X) = 4X - 8 = 0$
$\therefore X = 2 \quad \therefore F(2) = 8 - 16 = -8$

Q(33) If the function $F(X) = X^2 e^{KX}$ has a critical point at $X = \frac{2}{3}$ then the value of K equals

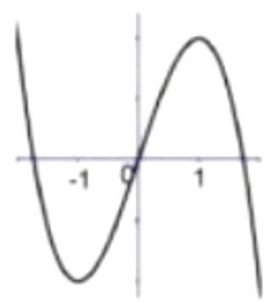
① -3 ② 0 ③ $-\frac{3}{2}$ ④ $-\frac{1}{3}$

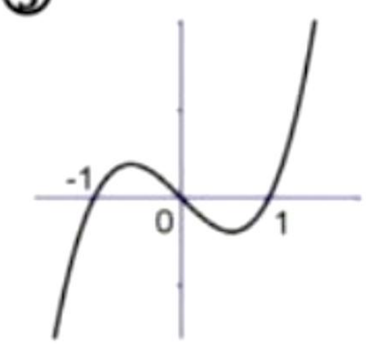
$F^{/}(X) = 2Xe^{KX} + KX^2 e^{KX} = Xe^{KX}(2 + KX)$
$2 + KX = 0 \; when \; X = \frac{2}{3} \quad \therefore 2 + K\left(\frac{2}{3}\right) = 0 \quad \therefore K = -3$

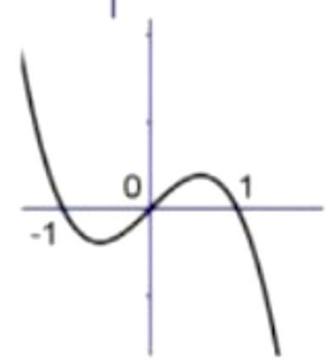
Q(34) The graph of the derivative of f(X) is shown in the figure above. Which of the following could be the graph of f(X)



①

②


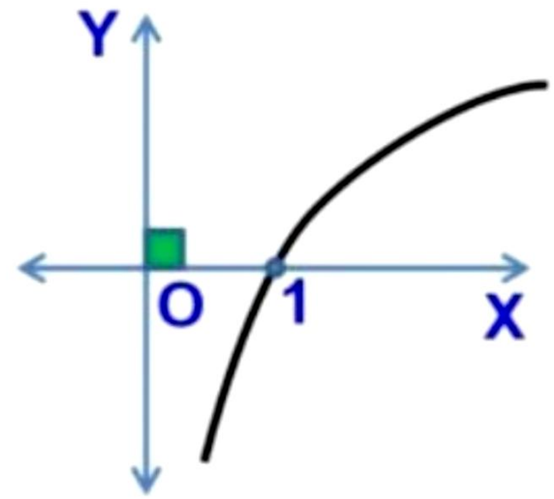
③


④


Q(35) If $F(X) = -X^3 + 6X^2 + 2X + 1$ find the greatest value of tangent to this curve

$\mathbf{m} = -3\mathbf{X}^2 + 12\mathbf{X} + 2$ this is he function of the slope
$\frac{\mathbf{dm}}{\mathbf{dX}} = -6\mathbf{X} + 12 = 0 \quad \therefore \mathbf{X} = 2$
Maximum slope if $-3(2)^2 + 12 \times 2 + 2 = 14$

Q(36) The graph of $F(X)$ is shown in the opposite figure.
Which of the following is true?

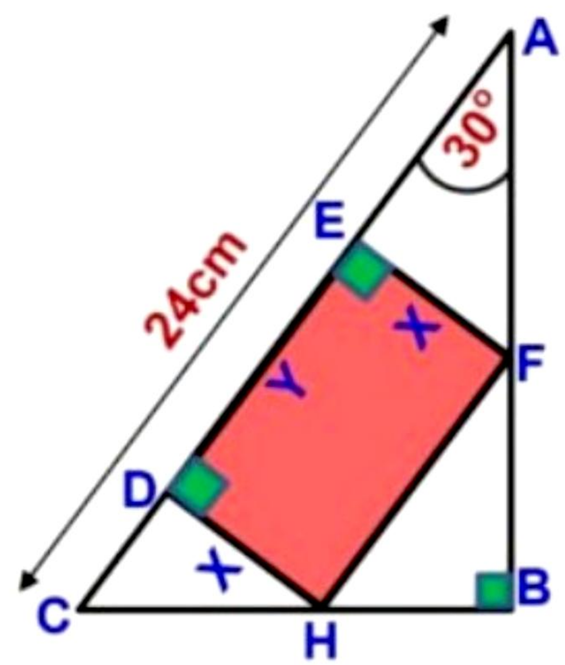


① $F(1) < F^{/}(1) < F^{//}(1)$
② $F(1) < F^{//}(1) < F^{/}(1)$
③ $F^{/}(1) < F(1) < F^{//}(1)$
④ $F^{//}(1) < F(1) < F^{/}(1)$

$F(1) = 0 \quad , f^{/}(1) = + \quad , f^{//}(1) = -$

④ $F^{//}(1) < F(1) < F^{/}(1)$

Q(37) In the opposite figure : Find the largest area of the rectangle Drawn indide the triangle whose hypotenous length 24cm



$$tan30^o = \frac{X}{AE} \quad \therefore AE = \sqrt{3}X$$

$$, tan60^o = \frac{X}{CD} \quad \therefore CD = \frac{\sqrt{3}}{3}X$$

$$\sqrt{3}X + \frac{\sqrt{3}}{3}X + Y = 24$$

$$\therefore \frac{4\sqrt{3}}{3}X + Y = 24 \quad \therefore Y = 24 - \frac{4\sqrt{3}}{3}X$$

$$A = XY = X\left(24 - \frac{4\sqrt{3}}{3}X\right) = 24X - \frac{4\sqrt{3}}{3}X^2$$

$$\frac{dA}{dX} = 24 - \frac{8\sqrt{3}}{3}X = 0 \quad \therefore X = 3\sqrt{3},$$

$$A = 24 \times 3\sqrt{3} - \frac{4\sqrt{3}}{3}\left(3\sqrt{3}\right)^2 = 36\sqrt{3}cm^2$$

Q(38) The function $F(X) = e^{8X-X^2}$ increase in ....

① $]-\infty, 4[$ ② $]-\infty, -4[$ ③ $]4, \infty[$ ④ $]-\infty, -4[$

$F^{/}(X) = (8 - 2X)e^{8X-X^2} = 0$

increase in $]-\infty, -4[$

Q(39) A glass cylindrical can with metal cover its volume $96\pi m^3$ if the cost of $1m^2$ of glass is 2L.E and its twice the cost of $1m^2$ of the metal then the height of the cylinder minimize the cost price is

$V = 96\pi = \pi r^2 \times h \quad \therefore h = \frac{96}{r^2} = 96r^{-2}$

$A = 2\pi r \times h \times 2K + \pi r^2 \times K + \pi r^2 \times 2K$

$\therefore A = 4K\pi r \times 96r^{-2} + 3K\pi r^2 = \pi K(384r^{-1} + 3r^2)$

$\frac{dA}{dr} = 2\pi(-384r^{-2} + 6r) = 0 \quad \therefore \frac{384}{r^2} = 6r \quad \therefore 6r^3 = 384 \quad \therefore r = 4$

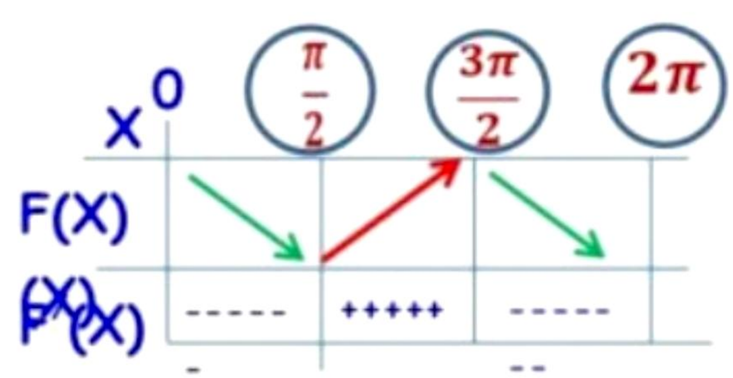
$h = \frac{96}{16} = 6m$

Q(40) The function $F(X) = 1 - \sin X$, $X \in]0, 2\pi[$ increasing on .....

① $]0, \frac{\pi}{2}[$ ② $]\frac{\pi}{2}, \frac{3\pi}{2}[$ ③ $]\frac{2\pi}{2}, 2\pi[$ ④ $]\frac{\pi}{2}, 2\pi[$

$F^{/}(X) = -\cos X = 0 \quad \therefore X = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$

Increase $]\frac{\pi}{2}, \frac{3\pi}{2}[$



Q(41) Find the extreme values of the function

$$F(X) = \sin X + \cos X \text{ on } [0, 2\pi]$$

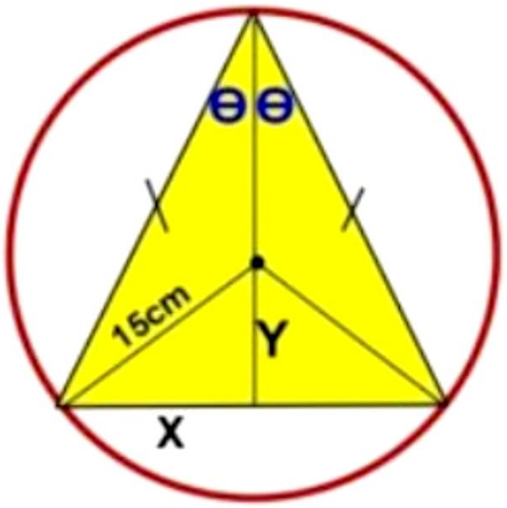
$F'(X) = \cos X - \sin X = 0 \quad \therefore \sin X = \cos X \quad \therefore \dfrac{\sin X}{\cos X} = 1 \therefore \tan X = 1$

$\therefore X = 45^o$ or $X = 225^o$

$F(0) = 1 \ , F(45^o) = \sqrt{2} \ , F(225^o) = -\sqrt{2} \ , F(360^o) = 1$

Absolute maximum $= \sqrt{2}$, Absolute minimum $= -\sqrt{2}$

Q(42) Find the dimensions of the isosceles triangle of <u>maximum</u> area that can be inscribed in a circle of radius 15cm



$$A = 2 \times \frac{1}{2} \times 15 \times 15 \times \sin(180^o - 2\theta) +$$

$$\times \frac{1}{2} \times 15 \times 15 \times \sin(4\theta)$$

$$A = 225\sin 2\theta + \frac{1}{2} \times 225\sin 4\theta$$

$$\frac{dA}{d\theta} = 450\cos 2\theta + 450\cos 4\theta = 0$$

$$\therefore \cos 2\theta + \cos 4\theta = 0$$

$$\cos 2\theta + 2\cos^2 2\theta - 1 = 0 \quad \therefore 2\cos^2 2\theta + \cos 2\theta - 1 = 0$$

$$\therefore \begin{cases} \cos 2\theta = 1 \quad \therefore 2\theta = 0 \\ \cos 2\theta = \dfrac{1}{2} \therefore 2\theta = 60^o \quad \therefore \theta = 30^o \end{cases}$$

$$A = 225\sin 60^o + \frac{1}{2} \times 225\sin 120^o = \frac{225\sqrt{3}}{2} + \frac{225\sqrt{3}}{4} = \frac{675\sqrt{3}}{4} = \frac{3\sqrt{3}}{4} r^2$$

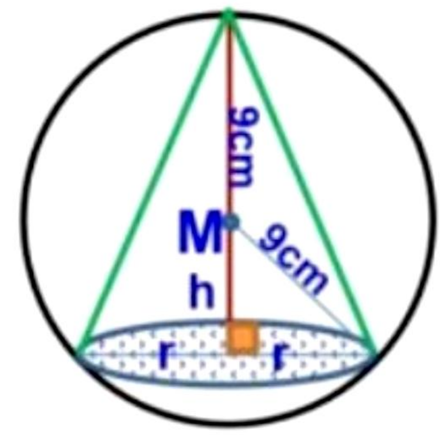
Q(43) Find the values of a and b such that the curve $Y = X^3 + aX^2 + bX$ has an inflection point at $(2, 2)$

$$\frac{dY}{dX} = 3X^2 + 2aX + b \quad \therefore \frac{d^2Y}{dX^2} = 6X + 2a = 0 \text{ when } X = 2$$

$$\therefore 2a = -12 \ \therefore a = -6$$

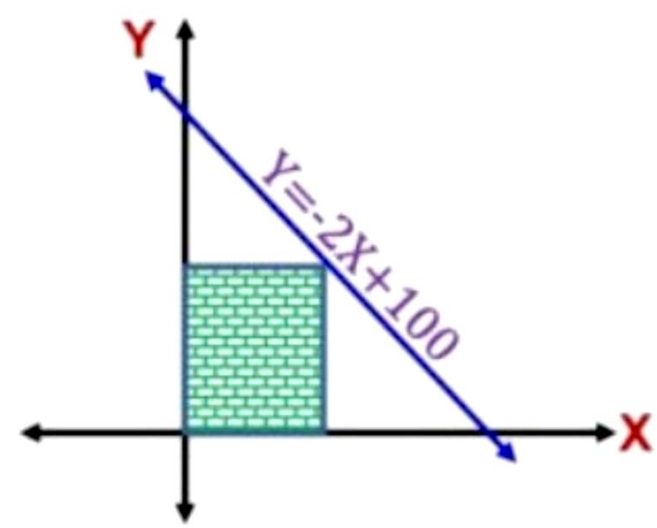
$$2 = 2^3 - 6 \times 2^2 + 2b \quad \therefore b = 9$$

Q(44) Find the largest volume of a cone can be put inside A sphere of radius 9cm



$$r^2 + h^2 = 81 \quad \therefore r^2 = 81 - h^2$$
$$V = \frac{1}{3}\pi r^2(h+9) = \frac{1}{3}\pi(81 - h^2)(h+9)$$
$$V = \frac{1}{3}\pi(81h + 729 - h^3 - 9h^2)$$
$$\frac{dV}{dh} = \frac{1}{3}\pi(81 - 3h^2 - 18h) = 0$$
$$\therefore h^2 + 6h - 27 = 0 \quad \therefore h = 3$$

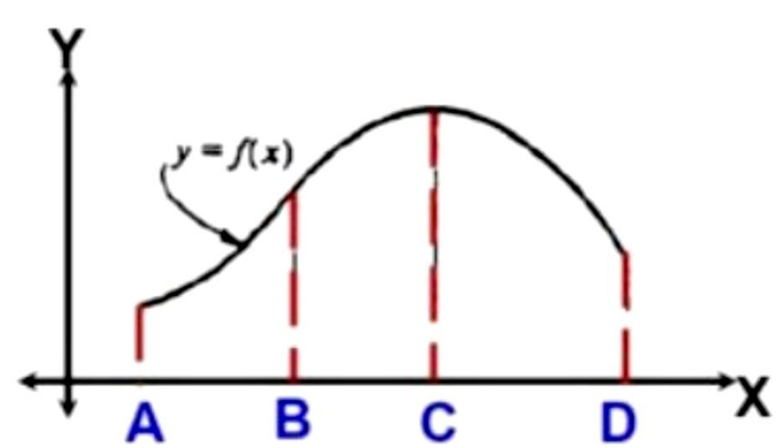
(45) in the opposite figure Find the maximum area of the rectangle



$$A = XY = X(-2X + 100) = -2X^2 + 100X$$
$$\frac{dY}{dX} = -4X + 100 = 0 \quad \therefore X = \frac{100}{4} = 25$$
$$\therefore Y = -2 \times 25 + 100 = 50$$
$$\frac{d^2Y}{dX^2} = -4X < 0$$
$$A = XY = 25 \times 50 = 1250$$

Q(46) The graph of $Y = F(X)$ is shown in the figure above. On which of the following intervals are $\frac{dY}{dX} > 0$, $\frac{d^2Y}{dX^2} < 0$

① $X \in]A, B[$
② $X \in]B, C[$
③ $X \in]C, D[$
④ $X \in]B, D[$



②$X \in]B, C[$

Q(47) find The absolute extreme values of the function f : $F(X) = X^2e^X$ the interval [-1,3] is

$F^/(X) = 2Xe^X + X^2e^X = 0 \quad \therefore Xe^X(2+X) = 0 \quad \therefore X = 0 \ or \ X = -2$

Critical at $X = 0$

$F(0) = 0 \quad , \ F(-1) = e^{-1} \ , \ F(3) = 9e^3$

absolute extreme= $9e^3$ at X=3

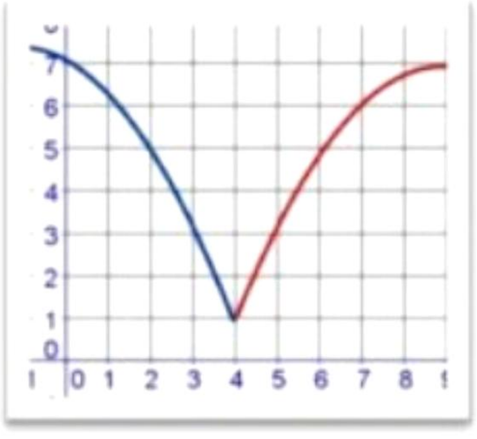
Q(48) The volume of a bacterial culture placed in a food medium grows in regard to the relation $F(n) = 2000 + \frac{5000t}{100+t^2}$ where time t is measured in hours, identify the maximum value of the culture volume.

$$F^/(n) = \frac{5000 \times (100+t^2) - 2t \times 5000t}{(100+t^2)^2} = 0$$

$\therefore 100 + t^2 - 2t^2 = 0 \quad \therefore t^2 = 100 \quad \therefore t = 10$

Q(49) The opposite figure represent a graph of a function $F(X)$, $F(4) = 1$ which of the follwing is true

①	$F^/(X) < 0, F^{//}(X) > 0 \ when X < 4, F^/(X) > 0, F^{//}(X) > 0 \ when X > 4$
②	$F^/(X) < 0, F^{//}(X) < 0 \ when X < 4, F^/(X) > 0, F^{//}(X) < 0 \ when X > 4$
③	$F^/(X) < 0, F^{//}(X) < 0 \ when X < 4, F^/(X) > 0, F^{//}(X) > 0 \ when X > 4$
④	$F^/(X) < 0, F^{//}(X) > 0 \ when X < 4, F^/(X) > 0, F^{//}(X) < 0 \ when > 4$



Q(50) Right prism of volume $2cm^3$ its base is equilateral triangle find the length of its base which make the total surface area minimum

$V = base\ area \times height =$

$\frac{1}{2}X \times X\sin 60^o \times Z = \frac{\sqrt{3}}{4}X^2Z = 2 \quad \therefore Z = \frac{8}{\sqrt{3}X^2}$

Area$=2 \times base\ area + lateral\ area$

$A = 2 \times \frac{\sqrt{3}}{4}X^2 + 3ZX = \frac{\sqrt{3}}{2}X^2 + 3X \times \frac{8}{\sqrt{3}X^2} = \frac{\sqrt{3}}{2}X^2 + \frac{8\sqrt{3}}{X}$

$\frac{dA}{dX} = \sqrt{3}X - \frac{8\sqrt{3}}{X^2} = 0 \quad \therefore X = 2$

Q(51) The opposite figure represent a graph of a function $F(X)$, $F(4) = 1$ which of the follwing is true

① $F^/(X) < 0, F^{//}(X) > 0\ when X < 4, F^/(X) > 0, F^{//}(X) > 0\ when X > 4$

② $F^/(X) < 0, F^{//}(X) < 0\ when X < 4, F^/(X) > 0, F^{//}(X) < 0\ when X > 4$

③ $F^/(X) < 0, F^{//}(X) < 0\ when X < 4, F^/(X) > 0, F^{//}(X) > 0\ when X > 4$

④ $F^/(X) < 0, F^{//}(X) > 0\ when X < 4, F^/(X) > 0, F^{//}(X) < 0\ when X > 4$

④

Q(52) The function $F(X) = Xe^{X(1-X)}$ is ...

①increasing on R ②Decreasing on R

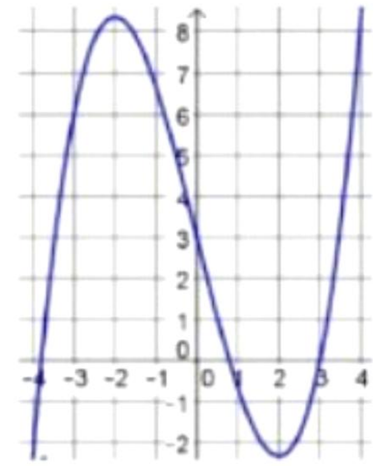
③increasing on $\left]-\frac{1}{2}, 1\right[$ ④Decreasing on $\left]-\frac{1}{2}, 1\right[$

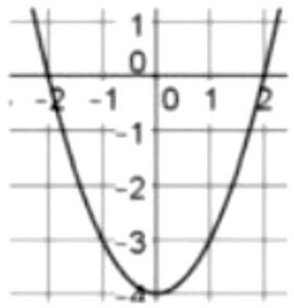
$F^/(X) = e^{X(1-X)} + X(1-2X)e^{X(1-X)} = e^{X(1-X)}(1 + X - 2X^2)$

$e^{X(1-X)}$ is always positive $(1 + X - 2X^2)$ *is positive* when $X \in \left]-\frac{1}{2}, 1\right[$

increasing on $\boxed{\left]-\frac{1}{2}, 1\right[}$

Q(53) The opposite figure represent the graph of $F(X)$
Which of the flowwing represent $F^{/}(X)$



①


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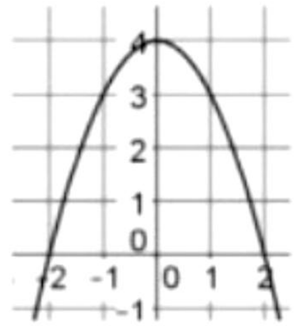
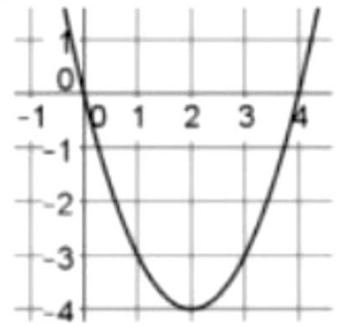
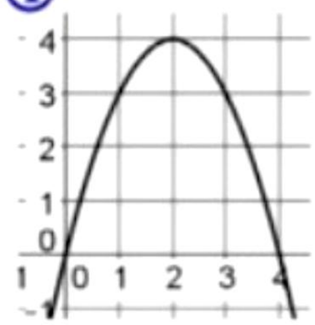


③


④


Q(54) The sum of two positive integer numbers is 5 and the sum of the cube of the smaller number and double the square of the other is as minimum as possible, find the two numbers.

$X + Y = 5 \quad \therefore Y = 5 - X$
$F(X) = X^3 + 2Y^2 = X^3 + 2(5 - X)^2 = X^3 + 2(25 - 10X + X^2)$
$F(X) = X^3 + 50 - 20X + 2X^2$
$F^{/}(X) = 3X^2 - 20 + 4X = 0 \quad \therefore X = 2$

Q(55) If f and h are two differentiable functions and f ' (x) < h ' (x) for each x ∈ R , function z where z (x) = f (x) - h(x) is .....

①is decreasing for each x ∈R

②has one local maximum value

③ is increasing for each x ∈R

④ has one local minimum value

$Z^{/}(X) = F^{/}(X) - h^{/}(X) < 0$
is decreasing for each x ∈R

Q(56) $F(X)$ is a function such that $when\ X > 3$, $\begin{cases} F^{/}(X) < 0 \\ F^{//}(X) > 0 \end{cases}$

, $when\ X < 3$, $\begin{cases} F^{/}(X) > 0 \\ F^{//}(X) > 0 \end{cases}$ which of the following figures represent F(X)

① ② ③ ④

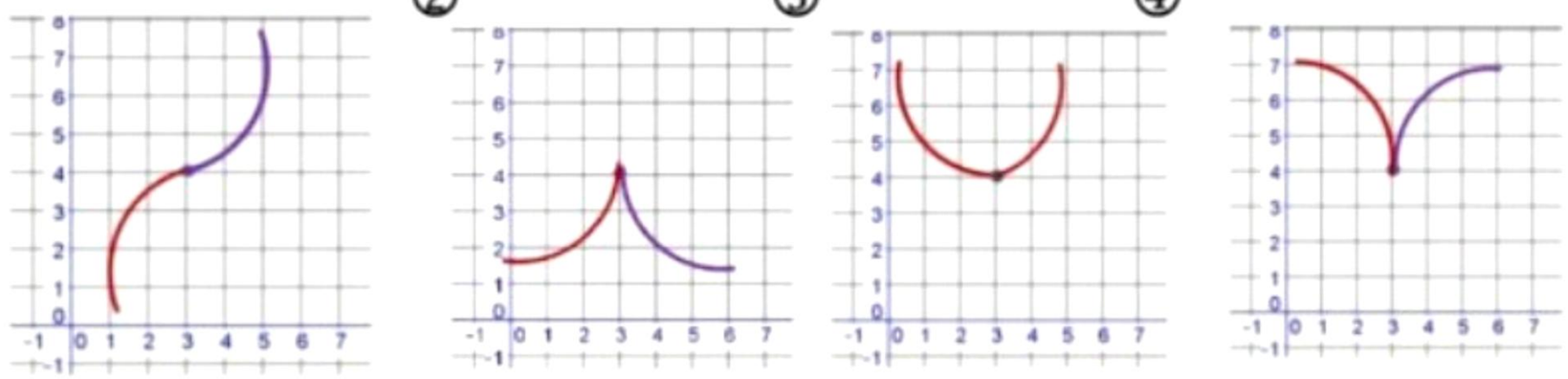
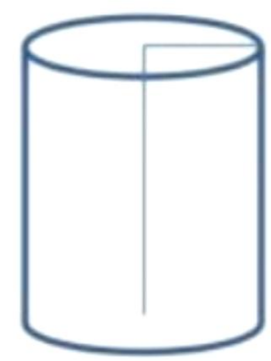


Figure ②

(58) The volume of a right circular cylinder is 16π find its minimum total surface area



$\pi r^2 h = 16\pi \quad \therefore h = 16r^{-2}$

$A = 2\pi r h + 2\pi r^2 \quad \therefore A = 2\pi r \times 16r^{-2} + 2\pi r^2$

$A = 32\pi r^{-1} + 2\pi r^2 \quad \therefore \dfrac{dA}{dr} = -32\pi r^{-2} + 4\pi r$

$\therefore r^3 = 8 \quad \therefore r = 2 \quad \therefore h = 4$

Q(59) A cuboid -like box whose base is in the form of square. If the sum of all its edges equals 240cm, find the dimensions of the box that will maximize its volume.

$8X + 4Y = 240 \quad \therefore 4Y = 240 - 8X \quad \therefore Y = 60 - 2X$

$V = X^2 Y = X^2(60 - 2X) = 60X^2 - 2X^3$

$\dfrac{dV}{dX} = 120X - 6X^2 \quad \therefore X = 20$

Q(60) The absolute maximum value of the function $F(X) = X^2 e^{-X}$ on $[0, 4]$ is

① 0 ② $\frac{4}{e^2}$ ③ e ④ $\frac{16}{e^4}$

$F^{/}(X) = 2Xe^{-X} - X^2 e^{-X} = 0$

$\therefore Xe^{-X}(2 - X) = 0 \therefore X = 0 \ or \ X = 2$

$F(0) = 0 \qquad F(4) = \frac{16}{e^4} \qquad F(2) = \frac{4}{e^2}$

Q(61) In the opposite figure Fin d the maximum area of the opposite rectangle

$Y_1 = -3X + 6 \ , Y_2 = 3X + 6$

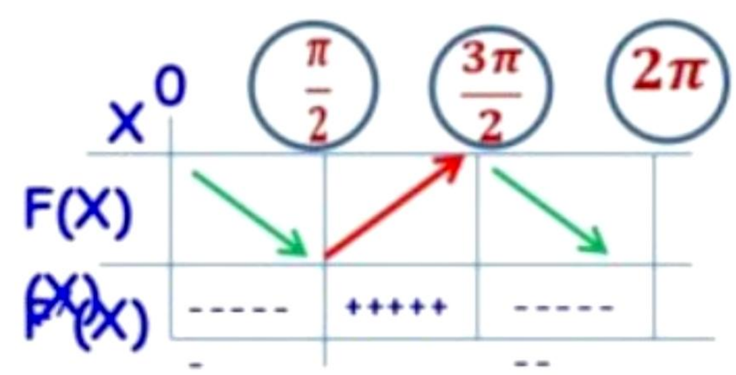
$A = 2\left(\frac{Y - 6}{3}\right) \times Y = \frac{2}{3}(Y^2 - 6Y)$

$\frac{dA}{dY} = \frac{2}{3}(2Y - 6) = 0 \ \therefore Y = 3$

Q(62) The function $F(X) = 1 - \sin X$, $X \in]0, 2\pi[$ increasing on .....

① $]0, \frac{\pi}{2}[$ ② $]\frac{\pi}{2}, \frac{3\pi}{2}[$ ③ $]\frac{2\pi}{2}, 2\pi[$ ④ $]\frac{\pi}{2}, 2\pi[$

$F^{/}(X) = -\cos X = 0 \ \therefore X = \frac{\pi}{2} \ or \ \frac{3\pi}{2}$



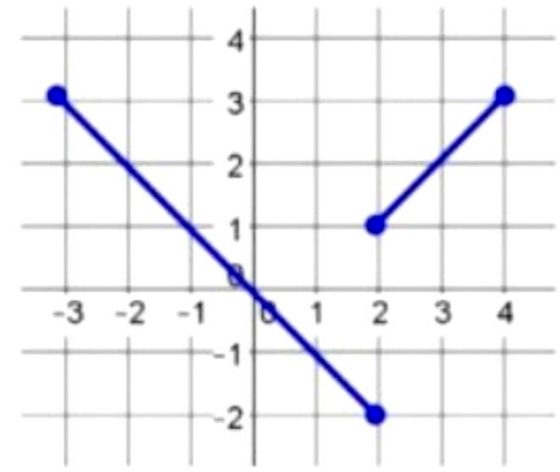
Q(63) The opposite graph represent $F^{/}(X)$

The curve is convex down on ....

① $]2,4[$ ② $]0,2[$

③ $]-3,0[$ ④ $]-3,4[$

$F^{//}(X) > 0$ *slope positive when* $X \in\ ]2,4[$



Q(64) In the opposite figure ABC is right angle triangle at A AC=6cm , AB=8cmfind the greatest area of rectangle ZDFE

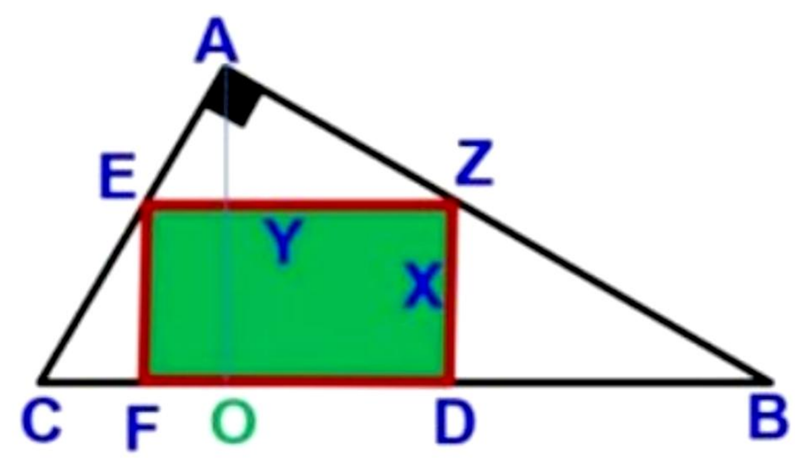
$BC = \sqrt{6^2 + 8^2} = 10 \ \therefore AO = \frac{6\times 8}{10} = 4.8$

$\frac{Y}{10} = \frac{4.8 - X}{4.8} \quad \therefore 4.8Y = 48 - 10X$

$Y = 10 - \frac{25}{12}X$

$A = XY = X\left(10 - \frac{25}{12}X\right) = 10X - \frac{25}{12}X^2$

$\therefore \frac{dA}{dX} = 10 - \frac{25}{6}X = 0 \quad \therefore X = 2.4 \quad \therefore = 5$



Q(65) Find the nearest point to point (0, 5) and lies on the curve

$Y = \frac{1}{2}X^2 - 4$

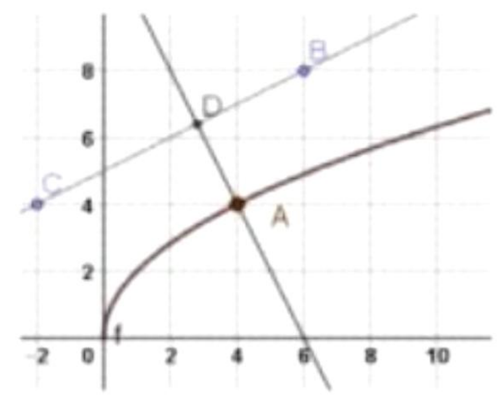
$D = \sqrt{(X_1 - X_2)^2 + (y_1 - y_2)^2}$

$D^2 = (X - 0)^2 + \left(\frac{1}{2}X^2 - 4 - 5\right)^2 = X^2 + \left(\frac{1}{2}X^2 - 9\right)^2$

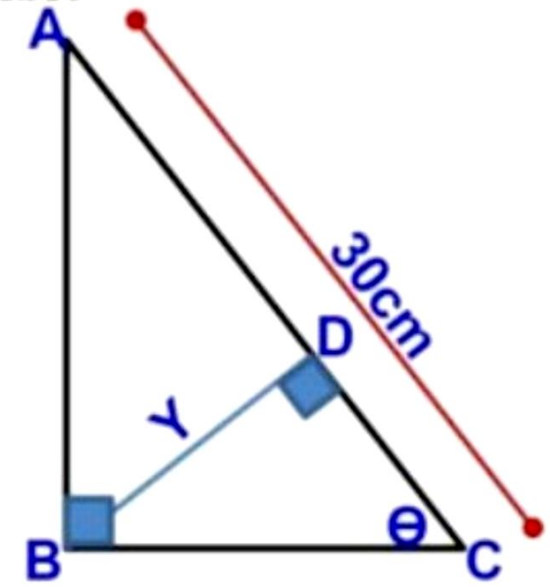
$D^2 = X^2 + \left(\frac{1}{2}X^2 - 9\right)^2 \quad \therefore 2D\frac{dD}{dX} = 2X + 2X\left(\frac{1}{2}X^2 - 9\right)$

$2X + 2X\left(\frac{1}{2}X^2 - 9\right) = 0 \quad \therefore 2X\left(\frac{1}{2}X^2 - 9 + 1\right) = 0$

$\frac{1}{2}X^2 - 8 = 0 \quad \therefore X^2 = 16 \quad \therefore X = \pm 4 \quad \therefore Y = \frac{1}{2}4^2 - = \pm 4$



Q(66) A right -angled triangle whose hypotenuse length is 30cm. Find the length of the two legs of the right angle if the length of the altitude drawn from the right angle on the hypotenuse is as maximum as possible



$BC = 30cos\theta \quad , BC = 30cos\theta$

$Y = \dfrac{30cos\theta \times 30cos\theta}{30} = 30sin\theta cos\theta$

$\therefore Y = 15sin2\theta \quad \therefore \dfrac{dY}{d\theta} = 30cos2\theta = 0$

$\therefore 2\theta = 90^o \quad \therefore \theta = 45^o$

$\dfrac{d^2Y}{dX^2} = -60sin2\theta < 0$

$\therefore Y = 15sin2\theta = 15sin2 \times 45^o = 15$

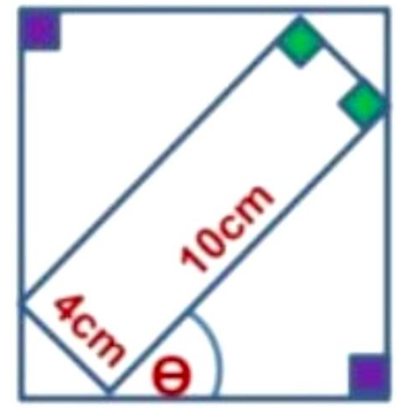
Q(67) If the function F: $F(X) = X^2 + \frac{b}{X}$ has a critical point at X=2, Then the value of b=

① -2 ② -3 ③ -8 ④ -9

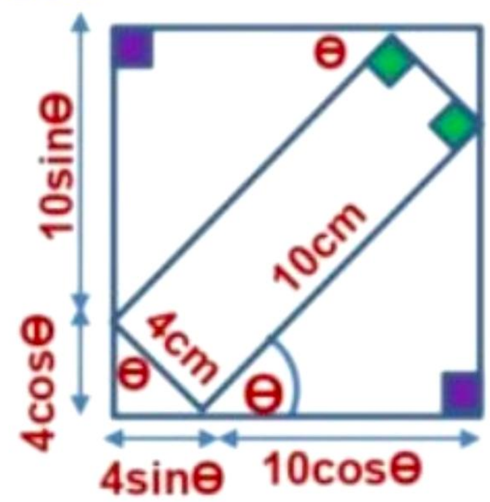
$F^{/}(X) = 2X - bX^{-2} \quad \therefore F^{//}(X) = 2 + 2bX^{-3} = 0$ when X=2

$\therefore \dfrac{2b}{2^3} = -2 \quad \therefore 2b = -16 \quad \therefore b = -8$

Q(68) In the opposite figure
Find the maximum area of the outside rectangle



$$A = (4\sin\theta + 10\cos\theta)(4\cos\theta + 10\sin\theta)$$
$$A = 16\sin\theta\cos\theta + 40\sin^2\theta + 40\cos^2\theta + 100\sin\theta\cos\theta$$
$$A = 116\sin\theta\cos\theta + 40 = 58\sin 2\theta + 40$$
$$\frac{dA}{d\theta} = 116\cos 2\theta = 0 \quad \therefore 2\theta = 90^o \quad \therefore \theta = 45^o$$
$$A = 58 + 40 = 98cm^2$$



(1) If the curve of the function $F(X) = aX^2 + 12X + 1$

has a critical point at X=2 then a=.....

(a)-1 (b)-3 (c)3 (d) 6

$F^{/}(X) = 2aX + 12 = 0$ *when* $X = 2$ $\therefore 4a + 12 = 0$ $\therefore a = -3$ answer (b)

(2) The function $F(X) = X^3 - 3X + 2$ decrease on

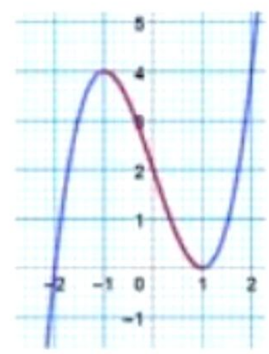
(a)$]-1, 1[$ (b)$]0, 1[$ (c)$]0, \infty[$ (d) $]1, \infty[$

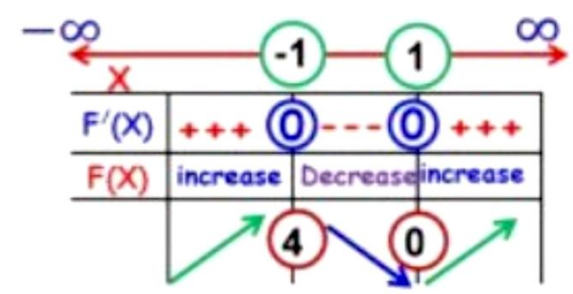
First derivative : $F^{/}(X) = 3X^2 - 3$

$\therefore 3X^2 - 3 = 3(X^2 - 1) = 0$

$\therefore (X - 1)(X + 1) = 0$

decrease in $]-1, 1[$ answer (a)





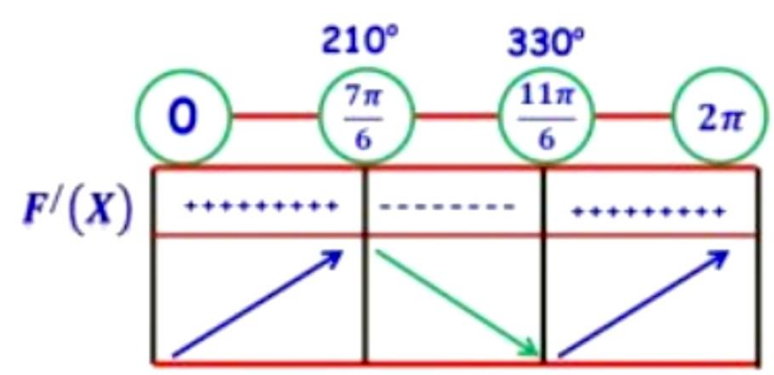
(3) The function $F(X) = X - 2\cos X$ such that $0 < X < 2\pi$

is decrease in the interval.....

(a)$]0, \frac{7\pi}{6}[$ (b)$]0, \frac{11\pi}{6}[$ (c)$]\frac{7\pi}{6}, \frac{11\pi}{6}[$ (d)$]\frac{7\pi}{6}, 2\pi[$

$F^{/}(X) = 1 + 2\sin X$ $\therefore \sin X = -\frac{1}{2}$

$\therefore X = \frac{7\pi}{6}$ *or* $X = \frac{11\pi}{6}$ Decreasing in $]\frac{7\pi}{6}, \frac{11\pi}{6}[$



answer (c)

(4) The function $F(X) = \frac{1}{3}X^3 + 3X^2 - 7X + 3$

has an inflection point at X=....

(a)-3 (b)3 (c)-1 (d)1

$F^{/}(X) = X^2 + 6X - 7$ $\therefore F^{//}(X) = 2X + 6$

The inflection point when $f''(X) = 0$ $\therefore 2X + 6 = 0$ $\therefore X = -3$ answer (a)

(5) If the function $F(X) = X^3 + aX^2 + 5X + 1$

has an inflection point At $X = -1$ then $a = \cdots$

(a)-1 (b)1 (c)3 (d)2

$F^{/}(X) = 3X^2 + 2aX + 5 \quad \therefore F^{//}(X) = 6X + 2a$

$6X + 2a = 0 \ when\ X = -1 \quad \therefore -6 + 2a = 0 \quad \therefore a = 3$ answer (c)

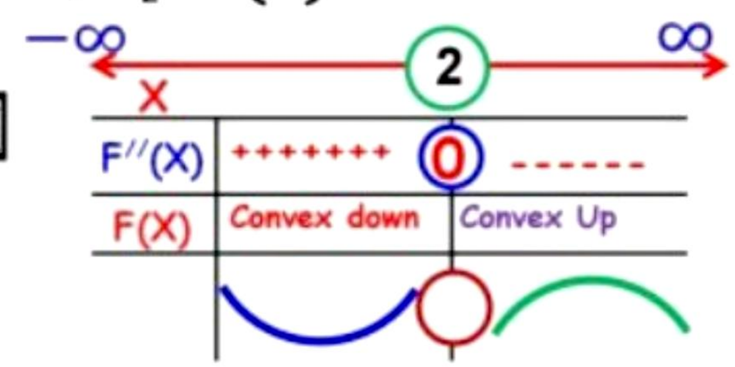
(6) The function $F(X) = 6X^2 - X^3$ convex upwards in

(a) $]2, \infty[$ (b) $]-\infty, 2[$ (c)$]-2, 2[$ (d)R

$F^{/}(X) = 12X - 3X^2, F^{//}(X) = 12 - 6X = 0 \therefore X = 2$

$\boxed{Convex\ upwards\ in\ ]2, \infty[}$, $\boxed{Convex\ downwards\ in\ ]-\infty, 2[}$

answer (a)



(7) If $X - 2Y = 4$ then the minimum value of XY

(a)-2 (b)2 (c)0 (d)-3

$X = 4 + 2Y \therefore F(Y) = XY = Y(4 + 2Y) = 4Y + 2Y^2$

$F^{/}(Y) = 4 + 4Y = 0 \quad \therefore Y = -1 \quad \therefore F^{//}(Y) = 4 > 0$

The minimum value$= 4 \times -1 + 2 \times 1^2 = -2$ answer (a)

(8) A cuboid -like box whose base is in the form of square. If the sum of all its edges equals 240cm, then the maximum volume of the box

(a)4000 (b)8000 (c)1600 (d)32000

$8X + 4Y = 240 \quad \therefore 4Y = 240 - 8X \quad \therefore Y = 60 - 2X$

$V = X^2Y = X^2(60 - 2X) = 60X^2 - 2X^3$

$\frac{dV}{dX} = 120X - 6X^2 \quad \therefore X = 20 \quad \therefore Y = 20 \quad \therefore V = 8000$ answer (b)

(9) The function $F(X) = 2\ln X - X^2$ decrease on .....

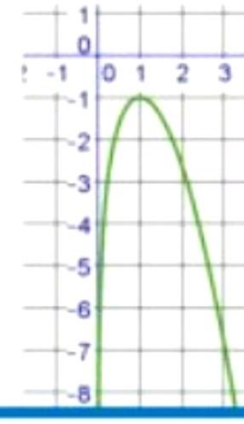
(a) $]-1, 1[$ (b) $]0, 1[$ (c) $]0, \infty[$ (d) $]1, \infty[$

Domain $]0, \infty[$

$$F^{/}(X) = \frac{2}{X} - 2X = \frac{2 - 2X^2}{X} = 0 \therefore X = 1$$

Increase $]0, 1[$, decrease $]1, \infty[$

answer (d)



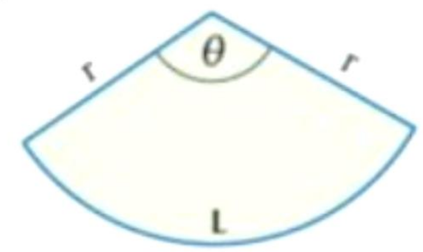
X	0		1	
F(X)		↗		↘
$F^{/}(X)$		+++++		-----

(10) If the perimeter of a circular sector is40 cm and its area is as maximum as possible , then the radius length of its circle.

(a)7 (b)8 (c)9 (d)10

$$2r + L = 40 \therefore L = 40 - 2r \therefore A = \frac{1}{2}Lr$$

$$\therefore A = \frac{1}{2} \times r(40 - 2r) = 20r - r^2 \therefore \frac{dA}{dr} = 20 - 2r = 0 \therefore r = 10$$



answer (b)

(11) The function $F(X) = aX + \frac{b}{X}, a, b, X > 0$ has least value when $X =$..

(a) $\frac{1}{2b}$ (b) $\sqrt{\frac{a}{b}}$ (c) $\sqrt{\frac{b}{a}}$ (d) $\sqrt{a}$

$$F^{/}(X) = a - \frac{b}{X^2} = 0 \therefore \frac{b}{X^2} = a \therefore X^2 = \frac{b}{a} \therefore X = \sqrt{\frac{b}{a}}$$

(12) If $F(X) = -X^3 + 6X^2 + 2X + 1$ find the greatest value of tangent to this curve

(a)12 (b)13 (c)14 (d)15

$m = -3X^2 + 12X + 2$ this is he function of the slope

$$\frac{dm}{dX} = -6X + 12 = 0 \therefore X = 2$$

Maximum slope if $-3(2)^2 + 12 \times 2 + 2 = 14$

answer (c)

(13) If the curve of curve $Y = X^3 + aX^2 + bX$ has an inflection point at $(2, 2)$ then $a + b =$

(a)15 (b)3 (c)6 (d)5

$\frac{dY}{dX} = 3X^2 + 2aX + b \quad \therefore \frac{d^2Y}{dX^2} = 6X + 2a = 0 \text{ when } X = 2$

$\therefore 2a = -12 \quad \therefore a = -6$

$\because (2, 2) \in$curve $\quad 2 = 2^3 - 6 \times 2^2 + 2b \quad \therefore b = 9$ answer (b)

(14) If$F'(X) = aX^2 + 8X$ has and inflection point when $x = 1$, then k equals:

(a)1 (b)2 (c)−4 (d)4

$F''(X) = 2aX + 8 \quad \therefore F''(X) = 0 \quad \therefore 2a + 8 = 0 \quad \therefore a = -4$ answer (c)

(15) A factory sells (X) printers monthly with a price $(840 - 10X)$ L.E fore each printer if the cost price for this printers Is $(4X^2 + 140X + 500)L.E$ then the number of printers should be buy to achieve a maximum profit is .....printers

(a)20 (b)22 (c)25 (d)30

$P = (840 - 10X)X - (4X^2 + 140X + 500) = -14X^2 + 700X - 500$

$\frac{dP}{dX} = -28X + 700 = 0 \quad \therefore X = 25$ answer (C)

(16) The minimum value of $\left(X^2 + \frac{250}{X}\right) = \cdots$

(a)45 (b)50 (c)25 (d)75

$2X - \frac{250}{X^2} = 0 \quad \therefore 2X - \frac{250}{X^2} = 0 \quad \therefore 2X = \frac{250}{X^2} \quad \therefore 2X^3 = 250$

$\therefore X^3 = 125 \quad \therefore X = 5 \quad \therefore$minimum value $= 25 + \frac{250}{5} = 75$ answer (d)

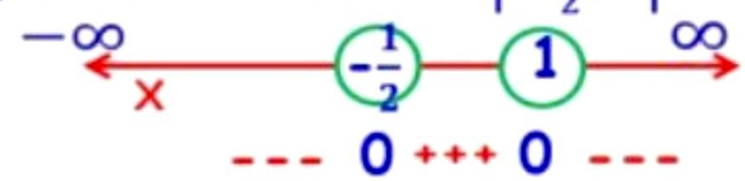
(17) The function $F(X) = Xe^{X(1-X)}$ is .....

(a)increasing on R (b)Decreasing on R

(c)increasing on $\left]-\frac{1}{2}, 1\right[$ (d)Decreasing on $\left]-\frac{1}{2}, 1\right[$

$F^{/}(X) = e^{X(1-X)} + X(1-2X)e^{X(1-X)} = e^{X(1-X)}(1 + X - 2X^2)$

$e^{X(1-X)}$ is always positive $(1 + X - 2X^2)$ *is positive* when $X \in \left]-\frac{1}{2}, 1\right[$



increasing on $\boxed{\left]-\frac{1}{2}, 1\right[}$

(18) $F(X) = X^2 - \frac{8}{X}$ convex upwards in the inteval

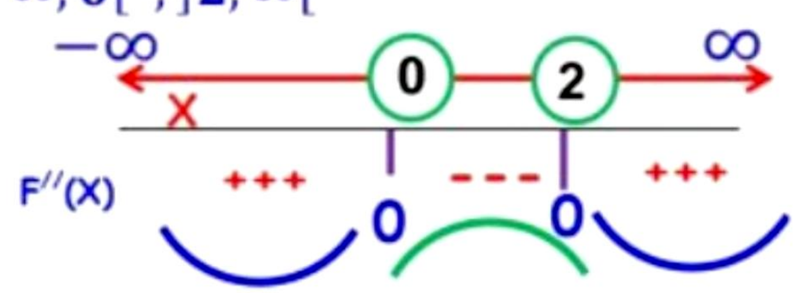
(a)$]0,2[$ (b) $]-\infty, 0[$ (c)$]0, 1[$ (d)$]-\infty, 1[$

$F^{/}(X) = 2X + \frac{8}{X^2} \Rightarrow F^{//}(X) = 2 - \frac{16}{X^3} = 0 \quad \therefore \frac{16}{X^3} = 2 \quad \therefore X^3 = 8 \quad \therefore X = 2$

Convex upward $]0,2[$, convex downward $]-\infty, 0[$, $]2, \infty[$

inflection point (2,0)

answer (a)



(19) A right circular cylinder the sum of of its base perimeter and its height 66cm find the radius to maximize the volume

(a)7cm (b)$11cm$ (c)$33cm$ (d)$66cm$

$2\pi r + h = 66 \quad \therefore h = 66 - 2\pi r$

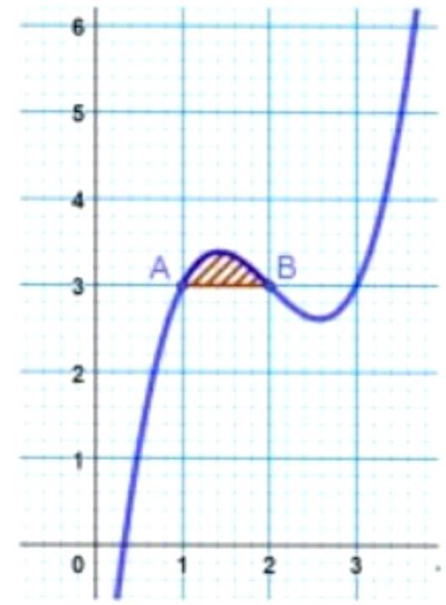
$V = \pi r^2 \times h = \pi r^2 \times (66 - 2\pi r) = 66\pi r^2 - 2\pi^2 r^3$

$\frac{dV}{dr} = 132\pi r - 6\pi r^2 = 0 \quad \therefore 6\pi r(22 - r\pi) = 0 \quad \therefore r = 7$ answer (a)

(20) The opposite figure represent

The curve of the $F(X) = X^3 - 6X^2 + 11X - 3$

The area of the shaded part equals



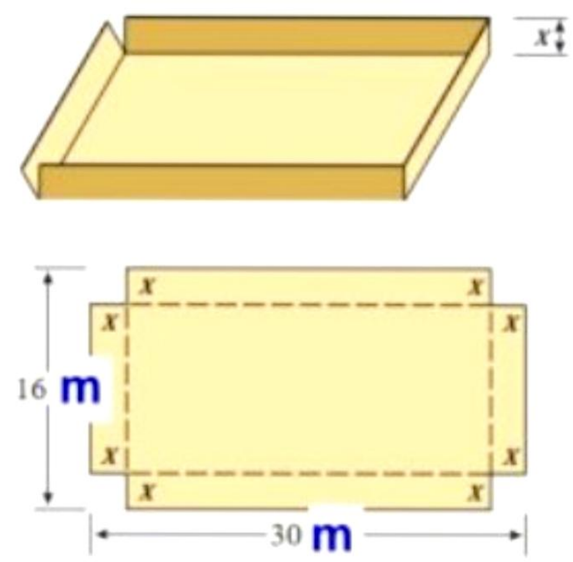
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{5}$

$\int_1^2 (X^3 - 6X^2 + 11X - 3)dX - 3 \times 1 = \frac{13}{4} - 4 = \frac{1}{4}$

answer (b)

(21) The value of x which makes the volume of the box is maximum is



(a) 12 (b) 8

(c) $\frac{10}{3}$ (d) $\frac{5}{3}$

$V = (16 - 2X)(30 - 2X)X$

$V = X(480 + 4X^2 - 92X) = 4X^3 - 92X^2 + 480X$

$\frac{dV}{dX} = 12X^2 - 184X + 480 \quad \therefore X = \begin{cases} 12 \\ \frac{10}{3} \end{cases} \quad \frac{d^2V}{dX^2} = 24X - 184$ At $X = \frac{10}{3}$ $\frac{d^2V}{dX^2} < 0$

answer (c)

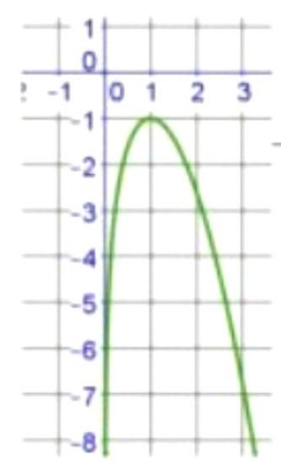
(22) The function $F(X) = 2\ln X - X^2$ decrease on .....

(a) $]-1, 1[$ (b) $]0, 1[$ (c) $]0, \infty[$ (d) $]1, \infty[$

Domain $]0, \infty[$

$F'(X) = \frac{2}{X} - 2X = \frac{2 - 2X^2}{X} = \therefore X = 1$

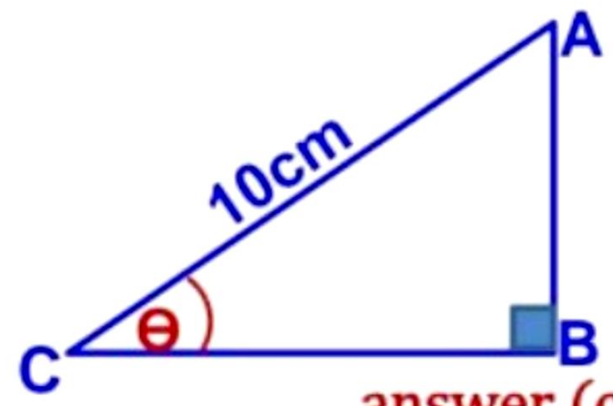
Increase $]0, 1[$, decrease $]1, \infty[$



X	0	1	
F(X)	↗		↘
F′(X)	+++++		-----

(23) Right angle triangle of hypotenuse length 10cm then its maximum area =

(a)$20cm^2$ (b)$50cm^2$ (c)$25cm^2$ (d)$75cm^2$



$A = \frac{1}{2} \times 10\sin\theta \times 10\cos\theta = 50\sin\theta\cos\theta$

$= 25\sin 2\theta \therefore \frac{dA}{d\theta} = 50\cos 2\theta = 0 \quad \therefore 2\theta = 90^o$

$\therefore \theta = 45^o \quad \therefore A = 25\sin 90^o = 25$

answer (c)

(24) If The function f: $f(X) = X + \frac{a}{X^3}$ has a critical point at $X = 3$ then the value of a equals

(a)3 (b)9 (c)27 (d)81

$f'(X) = 1 - \frac{3a}{X^4} = 0$ when X=3 $\therefore 3a = 3^4 \quad \therefore a = 27$

answer (c)

(25) The function $F(X) = 32\cos X + aX^2$ has an inflection point at $X = \frac{\pi}{3}$ then a=...

(a)2 (b)4 (c)8 (d)16

$F'(X) = -32\sin X + 2aX \quad \therefore F''(X) = -32\cos X + 2a = 0$

$2a = 16 \quad \therefore a = 8$

answer (c)

(26) If The function f: $f(X) = X\ln(KX)$ has a critical point at $X = 1$ then the value of (K) equals ....

(a)e (b)e^{-1} (c) e^{-2} (d)e^2

$f'(X) = \ln(KX) + \frac{K}{KX} \times X = 0 \quad \therefore \ln K + 1 = 0$

answer (b)

(27) If The function f: $f(X) = X\ln(KX)$ has a critical point at $X = 1$ then the value of (K) equals ....

(a) e (b) e^{-1} (c) e^{-2} (d) e^{2}

$f'(X) = \ln(KX) + \frac{K}{KX} \times X = 0 \quad \therefore \ln K + 1 = 0$

answer (b)

(28) In the opposite figure Find the maximum area of the rectangle

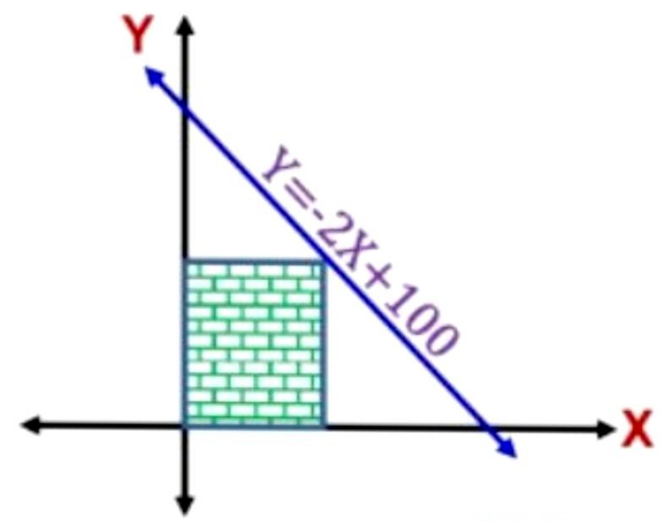
(a) 125 (b) 1200 (c) 1250 (d) 1500

$A = XY = X(-2X + 100) = -2X^2 + 100X$

$\frac{dY}{dX} = -4X + 100 = 0 \quad \therefore X = \frac{100}{4} = 25$

$\therefore Y = -2 \times 25 + 100 = 50$

$\frac{d^2Y}{dX^2} = -4X < 0 \quad \therefore A = XY = 25 \times 50 = 1250$



answer (c)

(29) The perimeter of a sector is (P) then the area of the sector is maximum when its radius is

(a) $\sqrt{P}$ (b) $\frac{1}{\sqrt{P}}$ (c) $\frac{P}{2}$ (d) $\frac{P}{4}$

$2r + L = P \quad \therefore L = P - 2r \quad A = \frac{1}{2}Lr \quad \therefore A = \frac{1}{2}r(P - 2r) = \frac{1}{2}rP - r^2$

$\frac{dA}{dr} = \frac{1}{2}P - 2r \quad \therefore 2r = \frac{1}{2}P \quad \therefore r = \frac{1}{4}P$

answer (d)

(30) The local maximum value for the function

$F(X) = 2X^3 - 3X^2 - 12X + 12$ is .....

(a) 2 (b) -8 (c) -1 (d) 19

$F'(X) = 6X^2 - 6X - 12 = 0 \quad \therefore \begin{cases} X = -1 \\ X = 2 \end{cases}$

$F''(X) = 12X - 6 \quad \therefore \begin{cases} F''(-1) = - \\ F''(2) = + \end{cases}$

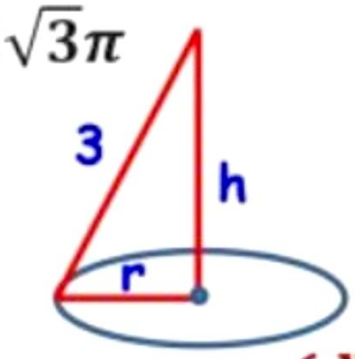
the function has local maximum value at X=-1

$F(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 12 = 19$

answer (d)

(31) The maximum volume in m^3 of a right circular cone of slant height 3meter is

(a) 6π (b) $3\sqrt{3}\pi$ (c) $\frac{4}{3}\pi$ (d) $2\sqrt{3}\pi$



$V = \frac{1}{3}\pi r^2 \times h \;\therefore V = \frac{1}{3}\pi h(9 - h^2) = \frac{1}{3}\pi(9h - h^3)$

$\therefore \frac{dh}{dt} = \frac{1}{3}\pi(9 - 3h^2) = 0 \;\; \therefore h = \sqrt{3} \therefore V = 2\sqrt{3}\pi$

answer (d)

(32) $F(X) = ln(sinX + cosX)$ where $X \in \left]\frac{-\pi}{4}, \frac{3\pi}{4}\right[$ is increasing on .....

(a) $\left]-\frac{\pi}{4}, \frac{\pi}{4}\right[$ (b) $\left]\frac{\pi}{4}, \frac{\pi}{2}\right[$ (c) $\left]0, \frac{3\pi}{8}\right[$ (d) $\left]\frac{\pi}{2}, \frac{3\pi}{4}\right[$

$F'(X) = \frac{cosX - sinX}{sinX + cosX} = \frac{1 - tanX}{1 + tanX} = tan\left(\frac{\pi}{4} - X\right) = 0 \;\; \therefore X = \frac{\pi}{4}$

increasing on $\left]-\frac{\pi}{4}, \frac{\pi}{4}\right[$

answer (a)

(33) 20 meter of wire is available for fencing off a flower bed in the form of a circular sector then the maximum area of the flower bed equals ....m^2

(a) 10 (b) 12.5 (c) 25 (d) 30

$P = 2r + L \;\; , A = \frac{1}{2}Lr \quad 2r + L = 20 \;\; \therefore L = 20 - 2r$

$A = \frac{1}{2}r(20 - 2r) = 10r - r^2 \;, \;\; \frac{dA}{dr} = 10 - 2r \;\; \therefore r = 5 \quad \therefore A = 25m^2$

(34) If the point $C(a, b) \in$ the curve of the function $F(X) = 17X - X^2$ then the maximum value of $a + b$ equals .....

(a) 27 (b) 81 (c) 90 (d) 93

$(X + 18X - X^2) = 18X - X^2 \quad \therefore 18 - 2X = 0 \;\; \therefore X = 9$

$\therefore a = 9$, b=$17 \times 9 - 9^2 = 72$

$\therefore a + b = 9 + 72 = 81$

(35) in the opposite figure

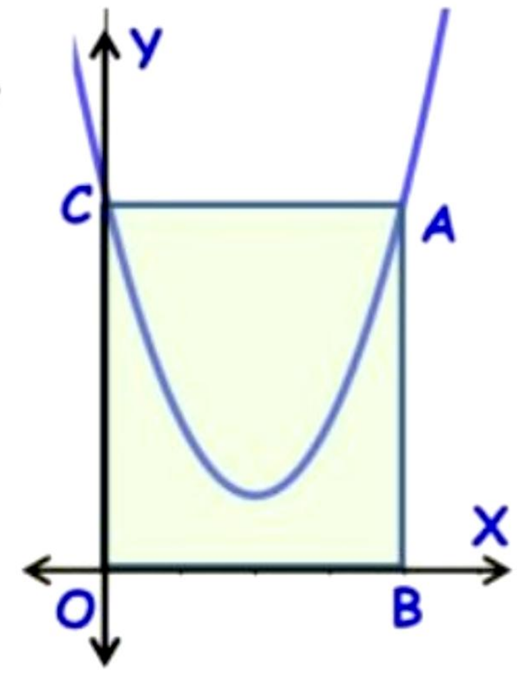
$A \in$ the curve of the function $F(X) = X^2 - 5X + 14$

Then the least value of the perimeter of

The rectangle OBAC equals .....length unit

(a)10 (b)12

(c)16 (d)20



$P = 2(X + Y) = 2(X^2 - 4X + 14)$

$\frac{dP}{dX} = 4X - 8 = 0 \quad \therefore X = 2 \quad \therefore P = 20$

answer (d)

(36) A cuboid its base is a square if the sum of its edges 240cm

Then its maximum a volume equals ....cm^3

(a)1000 (b)4000 (c)8000 (d)9000

$8X + 4Y = 240 \quad \therefore Y = 60 - 2X$

$V = X^2Y \quad \therefore \quad V = X^2(60 - 2X) = 60X^2 - 2X^3 \quad \therefore \frac{dV}{dX} = 1\,20X - 6X^2 = 0$

$\therefore X = 20cm \quad \therefore V = 20 \times 20 \times 20 = 8000cm^3$

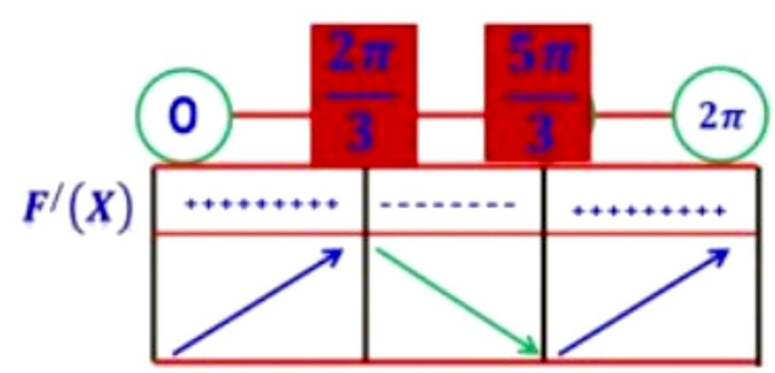
answer (c)

(37) The function $F(X) = \sqrt{3}\sin X - \cos X$ such that $0 < X < 2\pi$

is decrease in the interval.....

(a)$]0, \frac{\pi}{3}[$ (b)$]\frac{\pi}{3}, \frac{2\pi}{3}[$ (c)$]\frac{2\pi}{3}, \frac{5\pi}{3}[$ (d)$]\frac{\pi}{2}, \frac{5\pi}{3}[$

$F^{/}(X) = \sqrt{3}\cos X + \sin X \quad \therefore \frac{\sin X}{\cos X} = -\sqrt{3}$



$\therefore X = \frac{2\pi}{3}$ or $X = \frac{5\pi}{3}$ Decreasing in $]\frac{2\pi}{3}, \frac{5\pi}{3}[$

answer (c)

(38) A cuboid -like box whose base is in the form of square. If the sum of all its edges equals 240cm, then the maximum volume of the box

(a)4000 (b)8000 (c)1600 (d)32000

$8X + 4Y = 240 \quad \therefore 4Y = 240 - 8X \quad \therefore Y = 60 - 2X$

$V = X^2Y = X^2(60 - 2X) = 60X^2 - 2X^3$

$\frac{dV}{dX} = 120X - 6X^2 \quad \therefore X = 20 \quad \therefore Y = 20 \quad \therefore V = 8000$ answer (b)

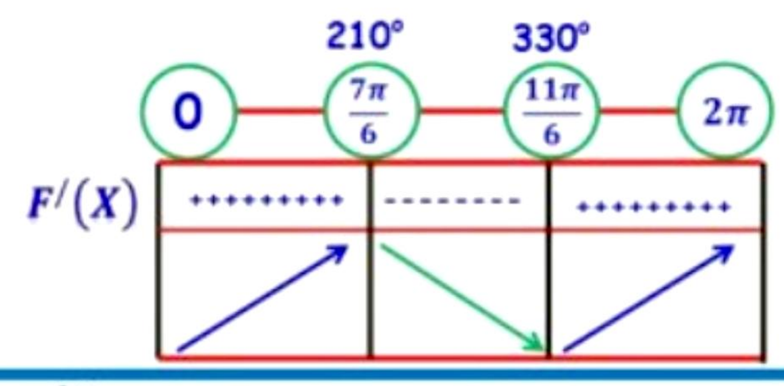
(39) The function $F(X) = X - 2\cos X$ such that $0 < X < 2\pi$

is decrease in the interval.....

(a)$]0, \frac{7\pi}{6}[$ (b)$]0, \frac{11\pi}{6}[$ (c)$]\frac{7\pi}{6}, \frac{11\pi}{6}[$ (d)$]\frac{7\pi}{6}, 2\pi[$

$F^{/}(X) = 1 + 2\sin X \quad \therefore \sin X = -\frac{1}{2}$

$\therefore X = \frac{7\pi}{6}$ or $X = \frac{11\pi}{6}$ Decreasing in $]\frac{7\pi}{6}, \frac{11\pi}{6}[$



(40) For the given function $Y = 5X - 2X^3$, $\frac{dX}{dt} = 2units/se$

Then the rate of increase of the slope tangent of the curve At X=3 is

(a)-36 (b)-72 (c)-108 (d)-144

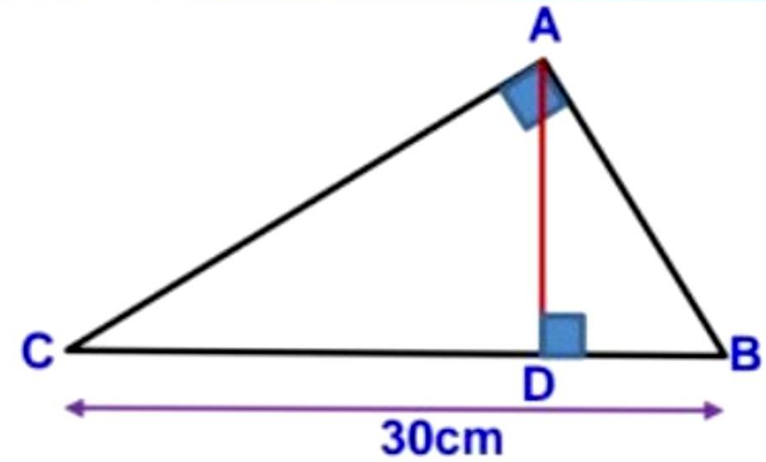
$m = 5 - 6X^2 \quad \therefore \frac{dm}{dt} = -12X\frac{dX}{dt} = -12 \times 3 \times 2 = -72$ answer (b)

(41) The opposite figure :

ABC is a right angle triangle at B

Find the length of $\overline{AB}$, $\overline{AC}$ such that

The length of $\overline{AD}$ Greatest as possible



(a) 10 (b)15 (c)25 (d)30

Let $BD = X \therefore DC = 30 - X$

$(AD)^2 = BD \times CD$

$(AD)^2 = X(30 - X) = 30X - X^2 \quad \therefore F(X) = \sqrt{30X - X^2}$

$$\frac{dM}{dX} = \frac{30 - 2X}{2\sqrt{30X - X^2}} = 0 \quad \therefore 2X = 30 \quad \therefore X = 15$$

Q(42) F(X) is a continuous on $[-2, 3]$ such that $F^{/}(0)$ does not exist

$F^{/}(2) = 0$ and $F^{//}(X) < 0$ for all values of X except X=0 which of the following is the graph of F(X)

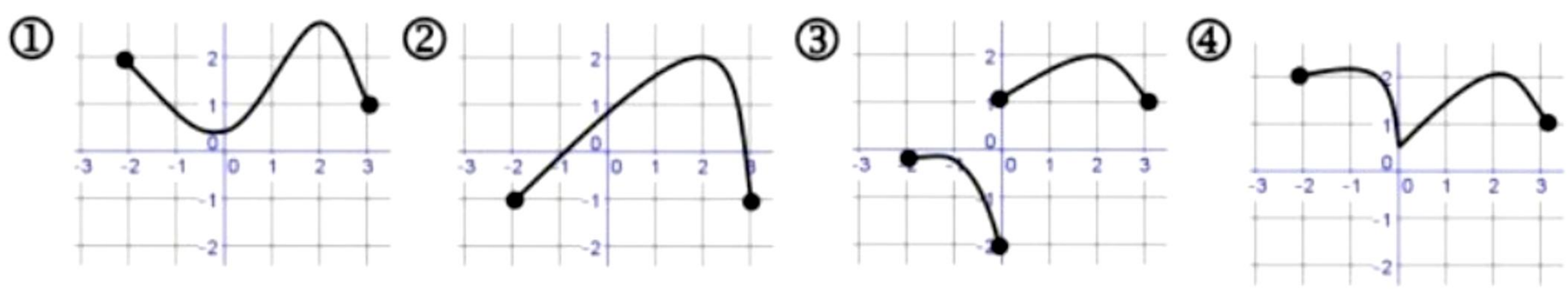


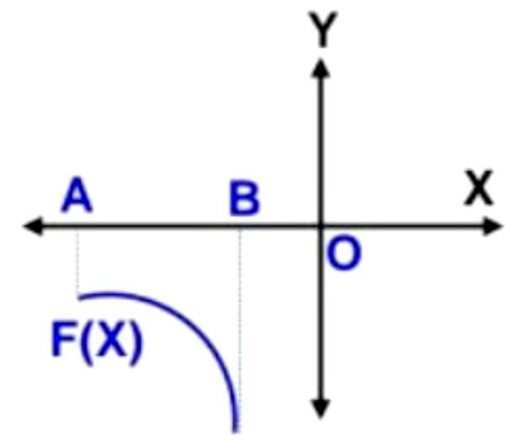
Figure (4)

Q(43) The function $F(X)$ in the third quadrant is defined in the intervel $[A, B]$

If $K(X) = e^X \times F(X)$ then K(X) is ....

(a) Increase in $[A, B]$ (b)decrease in $[A, B]$

(c)convex up in$[A, B]$ (d) convex down in$[A, B]$



$$K^{/}(X) = e^X \times F(X) + e^X \times F^{/}(X) = \underbrace{e^X}_{+}\left(\underbrace{F(X)}_{-} + \underbrace{F^{/}(X)}_{-}\right) = + \times - = -$$

$\therefore K(X)$ is decrease in $[A, B]$

Q(44) If the function $F(X)$ has a local maximum value at the point (2,3) and if $K(X) = \left(1 - F(X)\right)^3$ then K(X) has .....

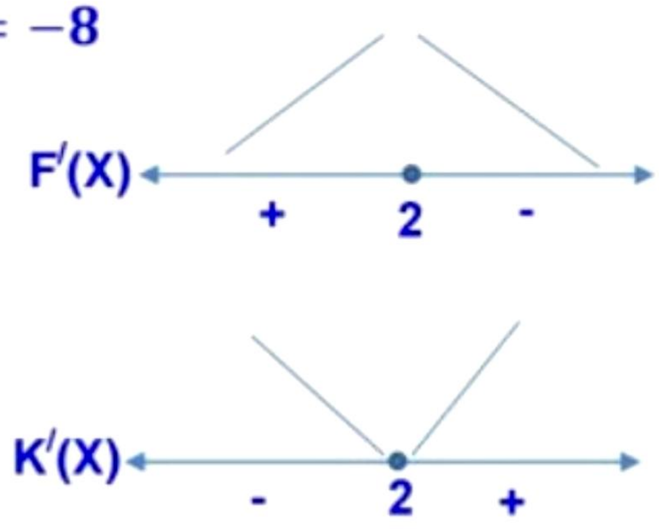
(a) local maximum value at(2,8) (b) local minimum value at(2,8)

(c) local maximum value at(2,-8) (d) local minimum value at(2,-8)

$\because F(2) = 3 \quad \therefore K(2) = \left(1 - F(2)\right)^3 = (1 - 3)^3 = -8$

$\because K(X) = \left(1 - F(X)\right)^3 \quad \therefore 1 - F(X) = \sqrt[3]{K(X)}$

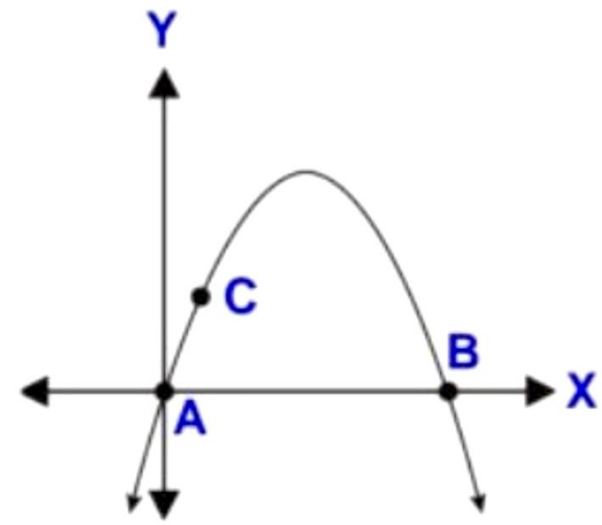
$\therefore -F^{/}(X) = \underbrace{\frac{1}{3}K(X)^{-\frac{2}{3}}}_{+} \times K^{/}(X)$



$\therefore K(X)$ has

local minimum value at(2,-8)

(45) For the given function $Y = 4X - X^2$, the maximum area of Δ ABC is when the X-coordinate of the point C is .....



(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) 2 (d) 3

$$A = \frac{1}{2} \times 4 \times (4X - X^2) = 8X - 2X^2$$

$$\frac{dA}{dX} = 8 - 4X = 0 \therefore X = 2$$

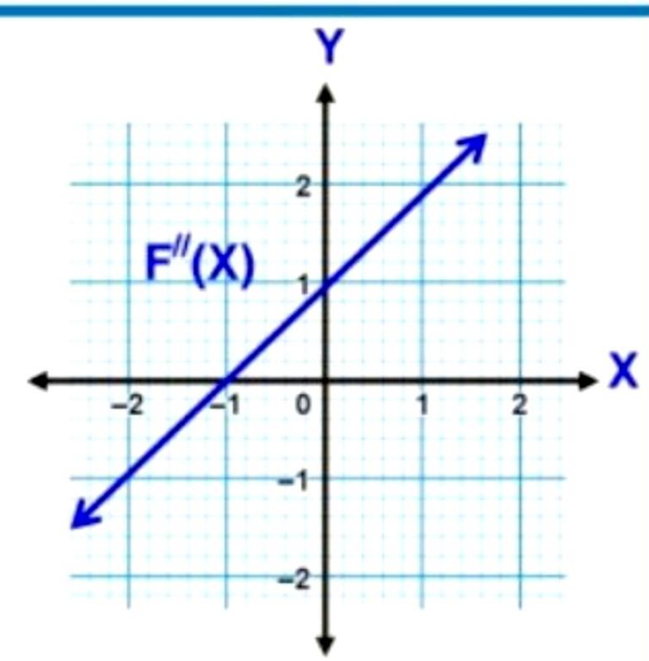
(46) $F(X) = \frac{1}{e^{sinX}}$ where $X \in [0, \pi]$ increase in ....

(a) $\left]0, \frac{\pi}{3}\right[$ (b) $\left]0, \frac{\pi}{2}\right[$ (c) $\left]\frac{\pi}{2}, \pi\right[$ (d) $]0, \pi[$

$$F^{/}(X) = -(cosX)e^{-sinX} = 0 \therefore cosX = 0 \therefore X = \frac{\pi}{2}$$

$\therefore F(X)$ increase in $\left]\frac{\pi}{2}, \pi\right[$

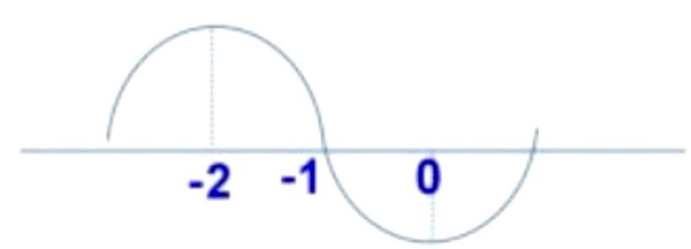
(47) The opposite graph represent the graph of $F''(X)$ of the function F(X) , $f'(-2) = f'(0) = 0$ then $F(X)$ is decrease in ....



(a) $]-\infty, -2[$ (b) $]-2, 0[$

(c) $]0, \infty[$ (d) $]0, 2[$

decrease in $]-2, 0[$



(48)The polynomial function F(X) defined on $[a, b]$ if $X_1, X_2 \in [a, b]$ such that $F'(X_1) - F'(X_2) < 0 \quad \forall X_1 > X_2$ then .....

(a) F(X) is increasing on $[a, b]$

(b) F(X) is decreasing on $[a, b]$

(c) F(X) is convex up on $[a, b]$

(d) F(X) is convex down on $[a, b]$

$X_1 > X_2 \quad , F'(X_1) < F'(X_2)$

F(X) is convex up on $[a, b]$