Parallelism and Concurrency

Parallelism : In parallel computing, multiple parts of a program execute at the same time , on separate processors for example, with the goal of speeding up computations.

Concurrency : concurrent computing consists of process *lifetimes* overlapping, but execution need not happen at the same instant.(process 1 then process 2 then process

Parallelism

Week 1 : Basics of parallel computing and parallel program analysis

We will try to unsure the following *important* property :

> **No Race Conditions :** If one parallel thread writes to a variable (or array entry), no other thread may read or write this variable at the same time.

Our first parallel program

Suppose we cant to compute the norm of a vector $\left(\sum_{i=1}^{n} |a_i|^p\right)^{1/p}$

The following program can do it from us when we call `sumSegment(a,0,n-1)`

```
def sumSegment(a: Array[Int], p: Double, s: Int, t: Int): Int = {
    var i= s; var sum: Int = 0
        while (i < t) {
            sum= sum + power(a(i), p)
            i= i + 1
        }
    sum
}
```

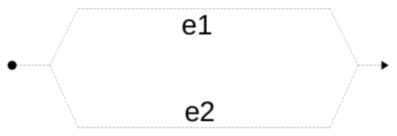
We can also split the to sum to two sums ($\sqrt{\sum_{i=0}^{m-1} |a_i|^2 + \sum_{i=m}^{N-1} |a_i|^2}$

```
def pNormTwoPart(a: Array[Int], p: Double): Int = {
    val m = a.length / 2
    val (sum1, sum2) = (sumSegment(a,p, 0, m), sumSegment(a,p, m, a.length))
power(sum1 + sum2, 1/p)
```

Non can we run the two parts in *parallel* :

suppose we have access to `parallel(e1,e2)` that computes `e1` and `e2` in parallel and return the pair of results.

```
val (sum1, sum2) =
    parallel(sumSegment(a, p , 0, m), sumSegment(a, m, a.length))
```



Splitting the task in 4 is easy, now what if we have an unlimited number of threads : we use recursion.

```
def segmentRec(a: Array[Int], p: Double, s: Int, t: Int) = {
if (t - s < threshold)
    sumSegment(a, p, s, t)
// small segment: do it sequentially
else {
    val m = s + (t - s)/2
    val (sum1, sum2) = parallel(segmentRec(a, p, s, m),
                                segmentRec(a, p, m, t))
sum1 + sum2 } }
```

The signature of parallel would be as follows :

```
def parallel[A, B](taskA: => A, taskB: => B): (A, B) = { ... }
```

Notice that it takes arguments *by name* `task1: => A` , because we do not want to evaluate them yet . If it was not the case , execution would be *sequential*.

Parallelism is not always the solution :

```
def sum1(a: Array[Int], p: Double, s: Int, t: Int): Int = {
    var i= s; var sum: Int = 0
    while (i < t) {
        sum= sum + a(i) // no exponentiation!
        i= i + 1
    }
sum }

val ((sum1, sum2),(sum3,sum4)) = parallel(
                    parallel(sum1(a, p, 0, m1), sum1(a, p, m1, m2)),
                    parallel(sum1(a, p, m2, m3), sum1(a, p, m3, a.length)))
```

`sum1` doesn't get faster when parallelized , contrarily to `sumSegment` .

WHY ? Because when we do not use exponentiation (which is very time consuming) the code is so fast that the bottleneck becomes the RAM.

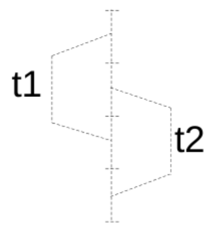
Tasks :

Instead of :

```
val (v1, v2) = parallel(e1, e2)
```

we can write :

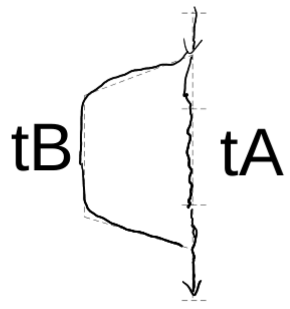
```
val t1 = task(e1)
val t2 = task(e2)
val v1 = t1.join
val v2 = t2.join
```



- `t.join` blocks and waits until the result is computed
- subsequent `t.join` calls quickly return the same result

We can now implement `parallel` ourselves :

```
def parallel[A, B](cA: => A, cB: => B): (A, B) = {
    val tB: Task[B] = task { cB }
    val tA: A = cA
    (tA, tB.join)
}
```



be careful , this is not parallel

```
def parallelWrong[A, B](cA: => A, cB: => B): (A, B) = {
    val tB: B = (task { cB }).join
    val tA: A = cA
    (tA, tB.join)
}
```

Asymptotic analysis of parallel algorithms :

```
e1
e2
```

Total running time = running time(e1) + running time(e2)

```
parallel(e1,e2)
```

Total running time is the maximum of the two running times.

```
def segmentRec(a: Array[Int], p: Double, s: Int, t: Int) = {
    if (t - s < threshold)
        sumSegment(a, p, s, t)
    else {
        val m= s + (t - s)/2
        val (sum1, sum2)= (segmentRec(a, p, s, m),
                           segmentRec(a, p, m, t))
sum1 + sum2 } }
```

we have the following recurrence:

$$W(s,t) = \begin{cases} c_1(t-s)+c_2, & \text{if } t-s < \text{threshold} \\ W(s,m)+W(m,t)+c_3 & \text{otherwise, for } m = \lfloor (s+t)/2 \rfloor \end{cases}$$

the solution is $O(t-s)$

if we parallel it :

```
val (sum1, sum2)= parallel(segmentRec(a, p, s, m), segmentRec(a, p, m, t))
```

$$D(s,t) = \begin{cases} c_1(t-s)+c_2, & \text{if } t-s < \text{threshold} \\ \max(D(s,m), D(m,t))+c_3 & \text{otherwise, for } m = \lfloor (s+t)/2 \rfloor \end{cases}$$

the solution is $O(log(t-s))$

Work and depth :

Work W(e): number of steps e would take if there was no parallelism

- this is simply the sequential execution time
- treat all parallel `(e1,e2)` as `(e1,e2)`

Depth D(e): number of steps if we had unbounded parallelism

Key rules are:

- $W(parallel(e_1, e_2)) = W(e_1) + W(e_2) + c_2$
- $D(parallel(e_1, e_2)) = max(D(e_1), D(e_2)) + c_1$

we also have

- $W(f(e_1, \ldots, e_n)) = W(e_1) + \ldots + W(e_n) + W(f)(v_1, \ldots, v_n)$
- $D(f(e_1, \ldots, e_n)) = D(e_1) + \ldots + D(e_n) + D(f)(v_1, \ldots, v_n)$

Here v_i denotes values of e_i. If f is primitive operation on integers, then $W(f)$ and $D(f)$ are constant functions, regardless of v_i.

Time Estimate of parallel algorithm :

- $D(e)$ assumes an unlimited number of thread or CPUs so $D(e)$ is our lower bound.
- Regardless of $D(e)$, cannot finish sooner than $W(e)/P$: every piece of work needs to be done

  So it is reasonable to use this estimate for running time: $D(e) + \frac{W(e)}{P}$

so for `segmentRec` the time is $b1 \times log(t-s) + b2 + \frac{b3(t-s)+b4}{P}$

The speedup is $1/\left(f+\dfrac{1-f}{P}\right)$ by Amdahl's law

Benchmarking parallel programs :

Measuring performance is difficult – there multiples ways to enhance it's precision

- multiple repetitions
- statistical treatment – computing mean and variance
- eliminating outliers
- ensuring steady state (warm-up)
- preventing anomalies (GC, JIT compilation, aggressive optimizations)

ScalaMeter is a library that helps with that , to use it :

- add as dependency :

```
libraryDependencies += "com.storm-enroute" %% "scalameter-core" % "0.6"
```

- use :

```
val time = measure {
    (0 until 1000000).toArray
    }
println(s"Array initialization time: $time ms")
```

This is a naïve testing method. We will get very different result when running it multiple times.

WHY ? When a JVM program starts, it undergoes a period of *warmup*, after which it achieves its maximum performance (at the *steady state*)

So we should test *after* warmup :

```
import org.scalameter._
val time = withWarmer(new Warmer.Default) measure {
    (0 until 1000000).toArray
}
```

Week2 : Parallel algorithms and operations :

Parallel merge sort :

We will implement parallel merge :

1. recursively split in two halves treated in parallel.
2. Sequentially merge the two halves by copying into a temporary array.
3. copy the temporary array back into the original array.

```
def parMergeSort (xs: Array[Int], maxDepth: Int): Unit = {
    val ys = new Array[Int] (xs.length)

    def sort(from: Int, until: Int, depth: Int): Unit = {
    if (depth == maxDepth) {
        quickSort(xs, from, until - from)
    }
    else {
        val mid = (from + until) / 2
        parallel (sort (mid, until, depth + 1), sort (from, mid, depth + 1))
        val flip = (maxDepth depth) % 2 == 0
        val src= if (flip) ys else xs
        val dst = if (flip) xs else ys
        merge(src, dst, from, mid, until)
        }
    }

    sort(0, xs.length, 0)
}
```

```
    def copy(src: Array[Int], target: Array[Int],
      from: Int, until: Int, depth: Int): Unit = {
        if (depth == maxDepth) {
        Array.copy(src, from, target, from, until - from)
        } else {
        val mid = (from + until) / 2
          val right = parallel(
            copy(src, target, mid, until, depth + 1),
          copy(src, target, from, mid, depth + 1)

    if (maxDepth % 2 == 0) copy(ys, xs, 0, xs.length, 0)
```

Operations on collections

we will study the following operation :

- `map` : `List(1,3,8).map(x => x*x) == List(1, 9, 64)`
- `fold` : `List(1,3,8).fold(100)((s,x) => s + x) == 112`
- `scan` : `List(1,3,8).scan(100)((s,x) => s + x) == List(100, 101, 104, 112)`

Note that `List` are not good for parallel use because we cannot efficiently :

- split them in half
- combine them

We will mostly use : **Arrays** and **Trees**

Map on Lists

Main properties :

- `list.map(x => x) == list`
- `list.map(f.compose(g)) == list.map(g).map(f)`

Sequential maps :

```
    // ON LIST
    def mapSeq[A,B](lst: List[A], f : A => B): List[B] = lst match {
        case Nil => Nil
        case h :: t => f(h) :: mapSeq(t,f)
    } // NOT PARALLIZABLE
```

```
// ON ARRAY
def mapASegSeq[A,B](inp: Array[A], left: Int, right: Int, f : A => B,
out: Array[B]) = {

    var i= left
    while (i < right) {
        out(i)= f(inp(i))
        i= i+1
    }
}
```

Parallel map :

```
def mapASegPar[A,B](inp: Array[A], left: Int, right: Int, f : A => B,
out: Array[B]): Unit = {
    // Writes to out(i) for left ≤ i ≤ right-1
    if (right - left < threshold)
        mapASegSeq(inp, left, right, f, out)
    else {
        val mid = left + (right - left)/2
        parallel(mapASegPar(inp, left, mid, f, out),
        mapASegPar(inp, mid, right, f, out))
    }
}
```

- we need to write to **disjoint memory addresses** (nondeterministic behavior otherwise)
- threshold needs to be large (loose of efficiency otherwise)

Performance measure :

We have 4 functions , we want to compute $\text{Array}(a_1, a_2, \ldots, a_n) \longrightarrow \text{Array}(|a_1|^p, |a_2|^p, \ldots, |a_n|^p)$:

- `mapASegSeq` : uses map but sequentially
- `mapASegPar` : uses map but parallel
- `normOfSeq` : normal sequential function with loop
- `normOfPar` : computes in parallel without map

```
def normsOfPar(inp: Array[Int], p: Double, left: Int, right: Int,
    out: Array[Double]): Unit = {
    if (right - left < threshold) {
        // compute sequentially
        normsOfSeq()
    } else {
    val mid = left + (right - left)/2
    parallel(normsOfPar(inp, p, left, mid, out),
    normsOfPar(inp, p, mid, right, out))
    }
}
```

```
mapASegSeq(inp, 0, inp.length, f, out) // sequential
mapASegPar(inp, 0, inp.length, f, out) // parallel
```

We get :

- inp.length = 2000000
- threshold = 10000
- Intel(R) Core(TM) i7-3770K CPU @ 3.50GHz (4-core, 8 HW threads), 16GB RAM

expression	*time*(*ms*)
mapASegSeq(*inp*, 0, *inp.length*, *f*, *out*)	174.17
mapASegPar(*inp*, 0, *inp.length*, *f*, *out*)	28.93
normsOfSeq(*inp*, *p*, 0, *inp.length*, *out*)	166.84
normsOfPar(*inp*, *p*, 0, *inp.length*, *out*)	28.17

- Parallel `map` this way is efficient.

Maps on Trees

Let's consider the following implementation of trees :

```scala
sealed abstract class Tree[A] { val size: Int }
    case class Leaf[A](a: Array[A]) extends Tree[A] {
    override val size = a.size
}
case class Node[A](l: Tree[A], r: Tree[A]) extends Tree[A] {
    override val size = l.size + r.size
}
```

we can implement map (parralel) this way :

```scala
def mapTreePar[A:Manifest,B:Manifest](t: Tree[A], f: A ⇒ B) : Tree[B] =
t match {
    case Leaf(a) ⇒ {
        val len = a.length; val b = new Array[B](len)
        var i= 0
        while (i < len) { b(i)= f(a(i)); i= i + 1 }
        Leaf(b) }
    case Node(l,r) ⇒ {
        val (lb,rb) = parallel(mapTreePar(l,f), mapTreePar(r,f))
        Node(lb, rb) }
}
```

Note that the time complexity is $O(h)$, h being the height of the tree.

List vs immutable tree :

Arrays:

- (+) random access to elements, on shared memory can share array
- (+) good memory locality
- (-) imperative: must ensure parallel tasks write to disjoint parts
- (-) expensive to concatenate

Immutable trees:

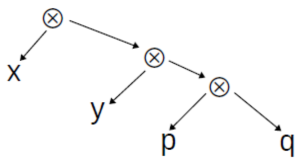
- (+) purely functional, produce new trees, keep old ones
- (+) no need to worry about disjointness of writes by parallel tasks
- (+) efficient to combine two trees
- (-) high memory allocation overhead
- (-) bad locality

Fold operations :

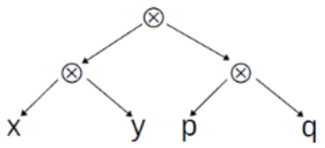
```
List(1,3,8).fold(100)((s,x) => s + x) == 112
// the difference between fold and foldLeft/foldRight is that the order of the
operations is non deterministic for fold , therefore f must be associative
List(1,3,8).foldLeft(100)((s,x) => s - x) == ((100 - 1) - 3) - 8 == 88
List(1,3,8).foldRight(100)((s,x) => s - x) == 1 - (3 - (8-100)) == -94
List(1,3,8).reduceLeft((s,x) => s - x) == (1 - 3) - 8 == -10
List(1,3,8).reduceRight((s,x) => s - x) == 1 - (3 - 8) == 6
```

When we are working in parallel we want to be able to *choose* the order of our operations .

The reason for that is that instead of doing the usual `foldRight\Left` operations that look like this :



We want to apply divide and conquer to be able to parallelize. Thus our execution of the operators will be like this one :



These two orders yield the same result only for **associative** operations $\otimes$ st $(x \otimes (y \otimes z)) = ((x \otimes y) \otimes z)$

Reduce on Trees:

```
def reduce[A](t: Tree[A], f : (A,A) => A): A = t match {
    case Leaf(v) => v
    case Node(l, r) => {
        val (lV, rV) = parallel(reduce[A](l, f), reduce[A](r, f))
        f(lV, rV)
    }
}
```

Reduce on Array :

reduce on arrays follows naturally by divide and conquer :

```
def reduceSeg[A](inp: Array[A], left: Int, right: Int, f: (A,A) => A): A = {
if (right - left < threshold) {
    var res= inp(left); var i= left+1
    while (i < right) { res= f(res, inp(i)); i= i+1 }
    res
} else {
        val mid = left + (right - left)/2
        val (a1,a2) = parallel(reduceSeg(inp, left, mid, f),
        reduceSeg(inp, mid, right, f))
        f(a1,a2)
    }
}

def reduce[A](inp: Array[A], f: (A,A) => A): A =reduceSeg(inp, 0, inp.length, f)
```

example of use : Compute with map / reduce $\sum_{i=s}^{t-1} \lfloor |a_i|^p \rfloor$

Answer : `reduce( map(a , pow(abs(_),p))  , _ + _ )`

⚠: parallel reduce works only for **associative** operators.

Associativity and Commutativity

Associative : $f(x, f(y,z)) = f(f(x,y),z)$ e.g : addition , multiplication of **integers**

⚠ floating points + not associative : `(1 + 1e20) + (-1e20) = 0` , `1 + (1e20 + (-1e20))= 1`

Commutative : $f(x,y) = f(y,x)$ addition , concatenation

concatenation commutative but not associative. many are the same.

Making an operation commutative : Easy

`def f(x: A, y: A) = if (less(y,x)) g(y,x) else g(x,y)` , even if g is not commutative f will be.

No such trick for associativity.

Associative operations on tuple : associativity extends to tuples.

If f_1 associative and f_2 associative then $f(x_1, x_2, y_1, y_2) = (f_1(x_1, y_1), f_2(x_2, y_2))$ associative.

`times((x1,y1), (x2, y2)) = (x1*x2, y1*y2)` associative because multiplication is .

Another way to prove associativity

If `f` *commutative* and `f(f(x,y), z) = f(f(y,z), x)` then f is also *associative*.

How to do prefix sum in parallel : the example of scanLeft

`List(1,3,8,4).scanLeft(100)(_ + _) == List(100, 101, 104, 112,116)`

We will only work on associative operations.

The problem is that we try to do the usual divide and conquer , we will need values that are to be calculated by other treads. e.g we divide problem into

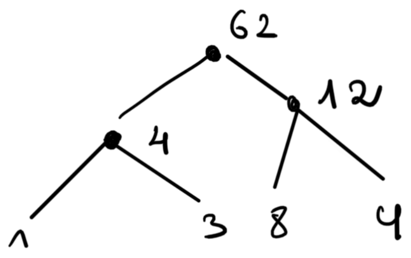
`[1,3]` and `[8,4]` . The sequence `[8,4]` will need the value returned by `[1,3]` to compute 112 = 104 + 8 .

What to do ? What to do ?

We will solve the problem in two steps :

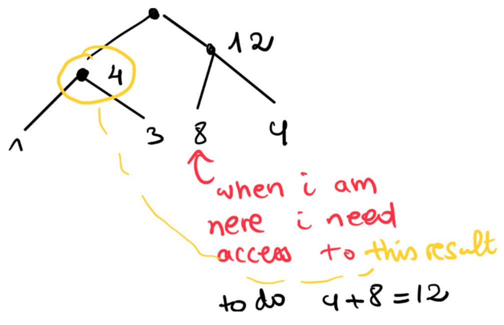
1. `upsweep` :

We will solve the problem for each interval indenpendently and store it in a tree.



Here we will neglect the initial accumulator (100 in the example), we can add it later.

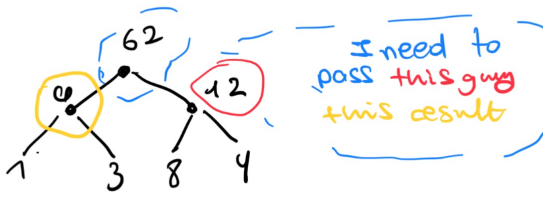
This idea resides in this observation :



And that's exactly what we will do in 2. `backsweep`

2. `backsweep`

Now we well traverse the tree that we saved :



that is expressed in code in this way :

`parallel(downsweep[A](l, a0, f), downsweep[A](r, f(a0, l.res), f))`

when we are in the root (blue) and we go to right (red) we pass to it `l.res` (which is 4 above = yellow).

Now the full code :

```scala
def upsweep[A](t: Tree[A], f: (A,A) => A): TreeRes[A] = t match {
    case Leaf(v) => LeafRes(v)
    case Node(l, r) => {
        val (tL, tR) = parallel(upsweep(l, f), upsweep(r, f))
        NodeRes(tL, f(tL.res, tR.res), tR)
    }
}

// āa0ā is reduce of all elements left of the tree ātā
def downsweep[A](t: TreeRes[A], a0: A, f : (A,A) => A): Tree[A] = t match {
    case LeafRes(a) => Leaf(f(a0, a))
    case NodeRes(l, _, r) => {
        val (tL, tR) = parallel(downsweep[A](l, a0, f),
        downsweep[A](r, f(a0, l.res), f))
        Node(tL, tR) }
}
```

Week3 : Data-Parallelism

Task-parallelism : we have multiple processors , we give each a task.

Data-parallelism: we have multiple process , we give all same task on different data(we distribute the *data*).

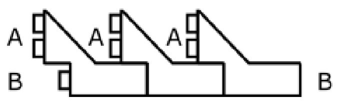
Example : for loop

```scala
def initializeArray(xs: Array[Int])(v: Int): Unit = {
    for (i <- (0 until xs.length).par) { // will create a parallel array
        xs(i) = v
    }
}
```

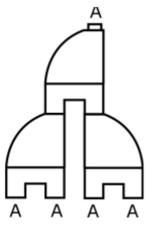
Let's consider `def foldLeft[B](z: B)(f: (B, A) => B): B` , we know it is not parallelizable without extra assuming on the operator `f` (associativity).

That is because if `f` is not associative there's only one possible order to execute it.

It is the same for `foldRight` , `reduceLeft` , `reduceRight` and `scanRight`



Let us see it's consider `def fold(z: A)(f: (A, A) => A): A` , we can parallelize it :



`fold` is useful :

```scala
def sum(xs: Array[Int]): Int = {
    xs.par.fold(0)(_ + _)
}
def max(xs: Array[Int]): Int = {
    xs.par.fold(Int.MinValue)(math.max)
}
```

for the fold operation to work , it must hold that: `f` is associative and `z` is neutral (`f(z,x)=x`)

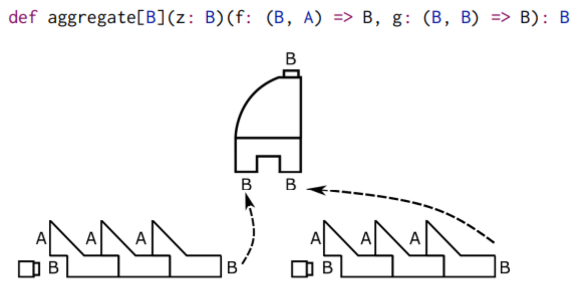
What if we want to do this ?

```scala
Array('E','P', 'F', 'L').par
.fold(0)((count, c) => if (isVowel(c)) count + 1 else count)
// DOES NOT COMPILE
Array('E','P', 'F', 'L').par.aggregate(0)(
(count, c) => if (isVowel(c)) count + 1 else count,
_ + _
)n// WORKS
```

`aggregate` is a combination of `fold` and

```scala
def aggregate[B](z: B)(f: (B, A) => B, g: (B, B) => B): B
```

Do the `foldingLeft` with `f` and then combine them with `g`



```scala
def aggregate[B](z: B)(f: (B, A) => B, g: (B, B) => B): B
```

It is important to note that `aggregate` splits the input arbitrarily :

`xs.aggregate(z)(f, g)` might result in `g(f(z, x1), f(f(z, x2), x3))` or

`g(f(f(z, x1), x2), f(z, x3))` .

When does `aggregate` always give the same results ?

- **split invariance :**

  `g( xs.foldLeft(z)(f) ,ys.foldLeft(z)(f) ) == (xs ++ ys).foldLeft(z)(f) for all xs,ys`

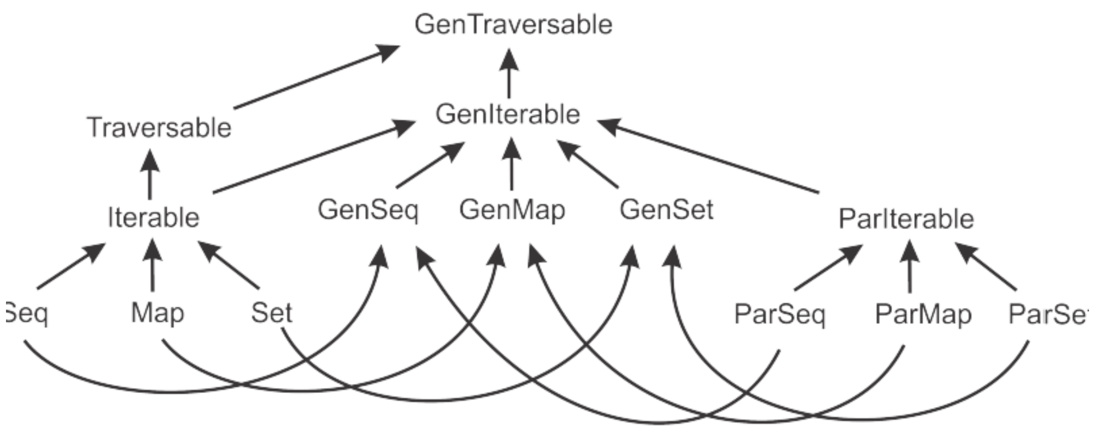
  it implies correctness of agregate.

- The following two properties imply split invariance :
  1. `g(u,f(v,x)) == f(g(u,v),x) for all x,y,u                (g-f-associative)`
  2. `g(z,x ) = x                                             (g-right-unit)`

Scala parallel collections

There exists traits `ParIterable` , `ParSet` , `ParSeq` and `ParMap[K,V]` the parallel counterparts of `Iterable` , `Sequence` , `Set` and `Map` .

`Interable[T]` : collection of elements operations implemented using `interator` .



Collections prefixed with `Gen` are super classes of normal collections , it helps to write code that is unware of parallelization.

```
def largestPalindrome(xs: GenSeq[Int]): Int = {
xs.aggregate(Int.MinValue)(
(largest, n) =>
if (n > largest && n.toString == n.toString.reverse) n else     largest,
    math.max)
}
val array = (0 until 1000000).toArray
largestPalindrome(array) // works
largestPalindrome(array.par) // works too and is parallized
```

add `.par` to make the collection parallel eg : `a.par`

- ParArray[T] – parallel array of objects, counterpart of Array and ArrayBuffer
- ParRange – parallel range of integers, counterpart of Range
- ParVector[T] – parallel vector, counterpart of Vector

but `list.par` returns `ParVector[]` => converts to closes parallel collection

Side effecting operations

```
def intersection(a: GenSet[Int], b: GenSet[Int]): Set[Int] = {
    val result = mutable.Set[Int]()
    for (x ← a) if (b contains x) result += x
    result
}
intersection((0 until 1000).toSet, (0 until 1000 by 4).toSet)

intersection((0 until 1000).par.toSet, (0 until 1000 by 4).par.toSet)
// DOES NOT WORK , DIFFERENT PROCESS MODIFIE RESULT
```

> **RULE :** Avoid mutations to the same memory location without proper synchronization

Solutions :

```
// 1. OBSCURE CONCURRENCY LIBRARY
import java.util.concurrent._
def intersection(a: GenSet[Int], b: GenSet[Int]) = {
    val result = new ConcurrentSkipListSet[Int]()
    for (x ← a) if (b contains x) result += x
    result
}

// 2. NO SIDE EFFECTS : FUNCTIONAL
def intersection(a: GenSet[Int], b: GenSet[Int]): GenSet[Int] = {
    if (a.size < b.size) a.filter(b(_))
    else b.filter(a(_))
}
```

Modification during traversal

```
val graph = mutable.Map[Int, Int]() ++= (0 until 100000).map(i ⇒ (i, i + 1))
graph(graph.size - 1) = 0
for ((k, v) ← graph.par) graph(k) = graph(v)
val violation = graph.find({ case (i, v) ⇒ v ≠ (i + 2) % graph.size })
// DOES NOT WORK
// 1. WE CHANGE graph while traversing it
// 2. WE READ SOME VALUES FROM graph THAT are currently being modified by other
process
```

This is a solution

```
val graph =
concurrent.TrieMap[Int, Int]() ++= (0 until 100000).map(i ⇒ (i, i + 1))
graph(graph.size - 1) = 0
val previous = graph.snapshot()
for ((k, v) ← graph.par) graph(k) = previous(v)
val violation = graph.find({ case (i, v) ⇒ v ≠ (i + 2) % graph.size })
println(sŏviolation: $violationŏ)
```

a snapshot saves that specific version of the data structure.

It is done is $O(1)$ time !

Splitters

```
trait Splitter[A] extends Iterator[A] {
    def split: Seq[Splitter[A]]
    def remaining: Int
}
def splitter: Splitter[A] // on every parallel collection
```

The *splitter contract*:

- after calling `split`, the original splitter is left in an undefined state
- the resulting splitters traverse disjoint subsets of the original splitter
- `remaining` is an estimate on the number of remaining elements
- `split` is an efficient method – $O(\log n)$ or better

`fold` on `Splitter`:

```
def fold(z:A)(f:(A,A) ⇒ A): A = {
    if (remaining < threshhold ) foldLeft(z)(f)
    else{
        val children = for (child ← split) yield task {child.fold(z)(f) }
        children.map(_.join()).foldLeft(z)(f)
    }
}
```

Builder

```
trait Builder[A, Repr] {
    def +=(elem: A): Builder[A, Repr]
    def result: Repr
}
def newBuilder: Builder[A, Repr] // on every collection
```

The *builder contract*:

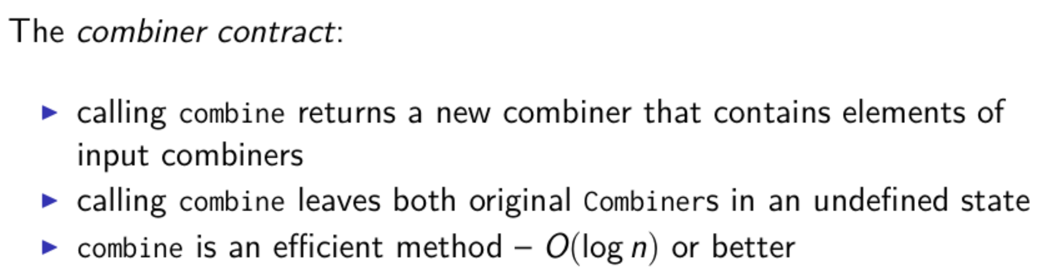
- calling `result` returns a collection of type `Repr`, containing the elements that were previously added with `+=`
- calling `result` leaves the `Builder` in an undefined state

```
def filter(p: T ⇒ Boolean): Repr = {
    val b = newBuilder
    for (x ← this) if (p(x)) b += x
    b.result
}
```

Combiner :

Like builder but in arbitrary order and in parallel

```scala
trait Combiner[A, Repr] extends Builder[A, Repr] {
    def combine(that: Combiner[A, Repr]): Combiner[A, Repr]
}
def newCombiner: Combiner[T, Repr] // on every parallel collection
```



The *combiner contract*:

- ▶ calling `combine` returns a new combiner that contains elements of input combiners
- ▶ calling `combine` leaves both original `Combiners` in an undefined state
- ▶ `combine` is an efficient method – $O(\log n)$ or better

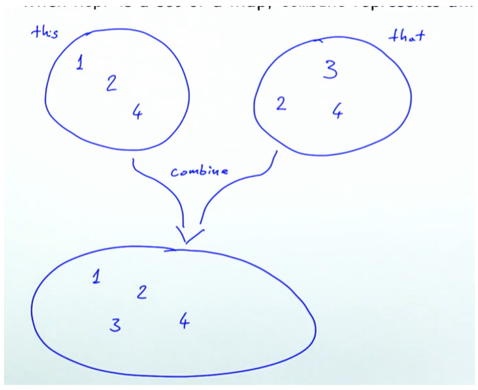
- `result` **must be parallelizable**

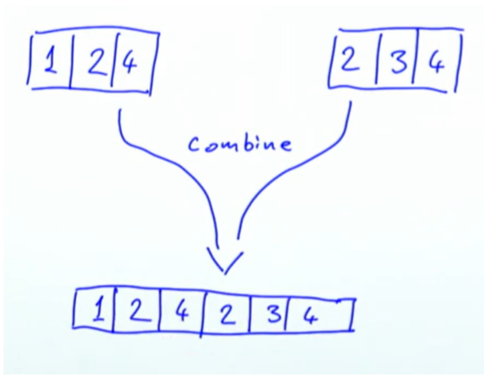
Week4 : Data-structures for efficient combining

Let's remember the `combiner` trait that extends the `Builder` trait :

```scala
trait Builder[T, Repr] {
    def +=(elem: T): this.type
    def result: Repr
}

trait Combiner[T, Repr] extends Builder[T, Repr] {
    def combine(that: Combiner[T, Repr]): Combiner[T, Repr]
}
```

The meaning of the combine depends on the type of `Repr` :





case of `Set` or `Map` : union   case of `Array` : concatenation

Why is it useful ? efficient data parallel computing : example

```
def filter(start : Int , end : Int , A : Array[Int] , threshold :Int ) : Array[Int]=
    def filter_com((start : Int , end : Int ) : Combiner[Int,Array] =
        if( end - start < threshold ) then // solve sequentially
        else
            val mid = (start+end)/2
            val left = filter_com(start,mid)
            val right = filter_com(mid,end)
            left.combine(right)

    filter_com(start,end).result
```

How to implement a combiner ?

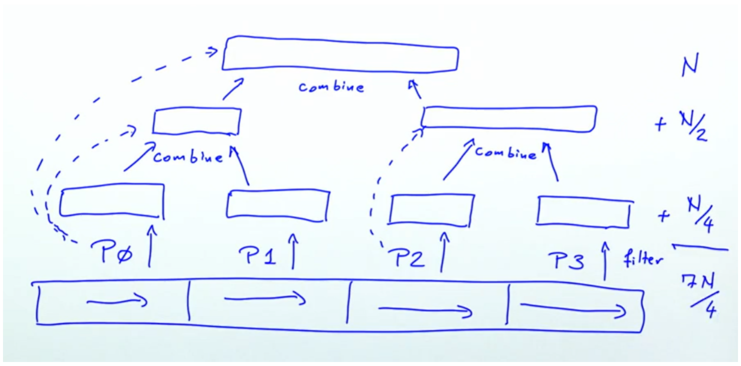
Naïve way : Very bad $O(n+m)$ where n, m are the sizes of `n` and `m` respectively

Example on an array :

```
def combine(xs: Array[Int], ys: Array[Int]): Array[Int] = {
    val r = new Array[Int](xs.length + ys.length)
    Array.copy(xs, 0, r, 0, xs.length)
    Array.copy(ys, 0, r, xs.length, ys.length)
    r
}
```

Why is that bad ?

Imagine we implement parallel `filter` using a $O(n+m)$ time `combine` we would get worse running times than the simple iteration linear `filter` :



We want better running times i.e $O(log(n) + log(m))$ running times for `combine`

Two phase construction

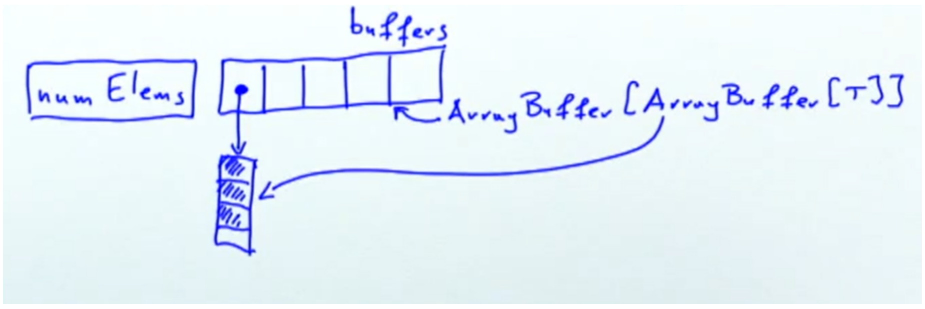
We will use an intermediate data structure that we will convert to our final (wanted) data structure at the end.

The intermediate data structure must have :

- $O(log(n) + log(m))$ `combine` or better
- efficient `+=` method
- can be converted to our desired data structe in $O(n/p)$ where p is the number of processors.

Example on arrays

```scala
class ArrayCombiner[T <: AnyRef: ClassTag](val parallelism: Int) {
    private var numElems = 0
    private val buffers = new ArrayBuffer[ArrayBuffer[T]]
    buffers += new ArrayBuffer[T]
```



```scala
def +=(x: T) = {
    buffers.last += x
    numElems += 1
    this
}
```

We add a new element to the last `ArrayBuffer` in our array of `ArrayBuffers` , this is done in $O(1)$ time.

```scala
def combine(that: ArrayCombiner[T]) = {
    buffers ++= that.buffers
    numElems += that.numElems
    this
}
```

Combining is just appending (the reference of) buffers . (concatenation will happen on the buffer level) . Given that we will have one `Combiner` working per processor that size of `buffers` will never have more than p `ArrayBuffer` s.

So concatenation will take $O(p)$ time.

```scala
def result: Array[T] = {
    val array = new Array[T](numElems)
    // PARTITION INDICES IN
    val step = math.max(1, numElems / parallelism)
    val starts = (0 until numElems by step) :+ numElems
    val chunks = starts.zip(starts.tail)

    val tasks = for ((from, end) <- chunks) yield task {
    copyTo(array, from, end)  // COPIES FROM THE BUFFERS
    }
    tasks.foreach(_.join())
    ar
```

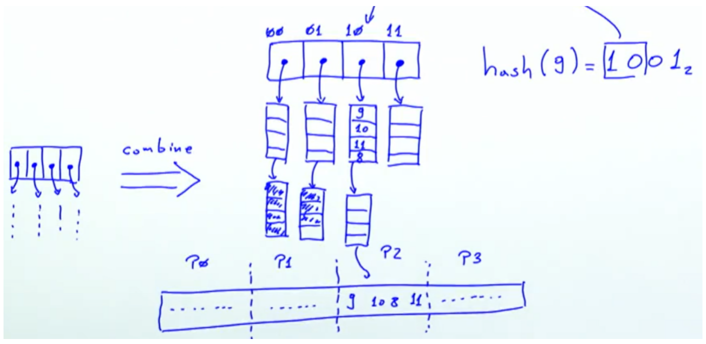
Two steps :

1. partition indices
2. copy the combiners elements into the array in parallel

For `maps` :

1. Partition the hash codes into buckets

2. Element from different buckets will be in different region of the final table, therefore we can fill the table in parallel.



ConcTree

ConcTree is a data structure that accepts efficient concatenation

> Trees are only good for parallelism when they are balanced. Otherwise we cannot balance the workload equally between processors.

So ConcTree will be balanced :

```scala
sealed trait Conc[+T] {
    def level: Int
    def size: Int
    def left: Conc[T]
    def right: Conc[T]
}

case object Empty extends Conc[Nothing] {
    def level = 0
    def size = 0
}
class Single[T](val x: T) extends Conc[T] {
    def level = 0
    def size = 1
}
// <> is a confusing name for node
case class <>[T](left: Conc[T], right: Conc[T]) extends Conc[T] {
    val level = 1 + math.max(left.level, right.level)
    val size = left.size + right.size
}
```

Properties of ConcTree

1. A node `<>` cannot contain an empty subtree.
2. The `level` difference between the left and right subtree of a node `<>` is at most 1.

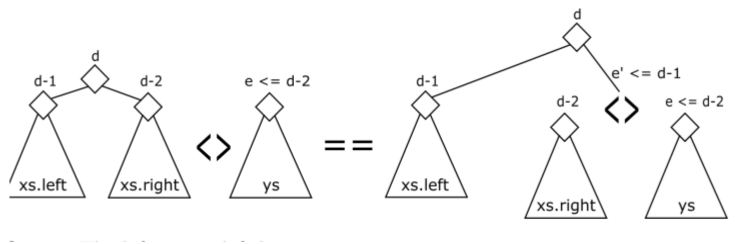
```scala
def <>(that: Conc[T]): Conc[T] = { // CONSTRUCTOR OF <>
    if (this == Empty) that
    else if (that == Empty) this
    else concat(this, that)         // "merge" them and return root
}
```

The challenge : Concatenation

1. if they have same level (± 1), merge

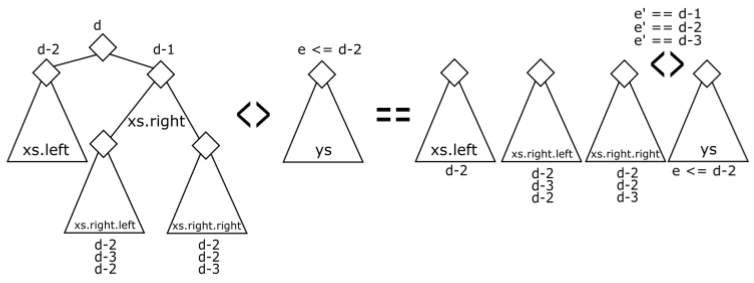
2. Let's assume (WLOG because we can swap them) that the left is bigger :
   a. Case left is itself left leaning

```
  if (xs.left.level >= xs.right.level) {
      val nr = concat(xs.right, ys)
      new <>(xs.left, nr)
  }
```



   b. Case the right is right leaning : then there 4 subtrees at play

   link the two smallest first (like the Huffman code)



```
  } else {
      val nrr = concat(xs.right.right, ys)
      if (nrr.level == xs.level - 3) {
          val nl = xs.left
          val nr = new <>(xs.right.left, nrr)
          new <>(nl, nr)
      } else {
          val nl = new <>(xs.left, xs.right.left)
          val nr = nrr
          new <>(nl, nr)
      }
  }
```

It takes $O(h_1 - h_2)$ time where h_1, h_2 are the heights

Append in amortized constant time

very simple append :

```
  var xs: Conc[T] = Empty
      def +=(elem: T) {
      xs = xs <> Single(elem)
  }
```

this works in $O(log(n))$ time.

Can we do better ? Yes.

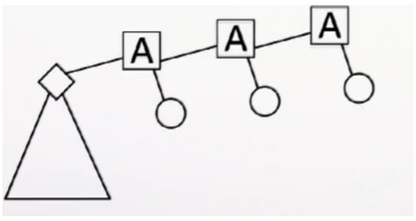
The idea is to store our append requests in an intelligent way in our tree.

```
case class Append[T](left: Conc[T], right: Conc[T]) extends Conc[T] {
    val level = 1 + math.max(left.level, right.level)
    val size = left.size + right.size
}
```

suppose we do this

```
def appendLeaf[T](xs: Conc[T], y: T): Conc[T] = Append(xs, new Single(y))
```

It will make the tree unbalanced



We will do not do that. Our technique will ensure that the number of `Append` nodes does not exceed $log(n)$.

the `Append` nodes are not balanced as `Conc` nodes but they satisfy these invariants :

- the right subtree of an `Append` node is never another `Append` node. (we only append to the left)
- if an Append node a has another Append node b as the left child, then a.right.level < b.right.level.

```
def appendLeaf[T](xs: Conc[T], ys: Single[T]): Conc[T] = xs match {
case Empty => ys
case xs: Single[T] => new <>(xs, ys)
case _ <> _ => new Append(xs, ys)
case xs: Append[T] => append(xs, ys)
}


@tailrec private def append[T](xs: Append[T], ys: Conc[T]): Conc[T] = {
    if (xs.right.level > ys.level) new Append(xs, ys) // verify append invariant
    else {
    val zs = new <>(xs.right, ys)
    //       xs                  ys
    //    /         \           big subtree
    //   xs.left   xs.right

    //      <>
    //   /      \               xs.left = ws
    // xs.right     ys
     //           big subtree
    xs.left match { // these violate append invariant
        case ws @ Append(_, _) => append(ws, zs) // assure invariant
        case ws if ws.level <= zs.level => ws <> zs // violate append invariant
        // making right side bigger , trying to balance .
        case ws => new Append(ws, zs)  // ws.level >= zs.level
        // invariant verified
        }
    }
}
```

Chuck nodes

Like `Single` nodes but hold multiple elements. `ConcBuffer` adds all added elements to a buffer and pushes them once they exceed a certain threshold.

```
class ConcBuffer[T: ClassTag](val k: Int, private var conc: Conc[T]) {
private var chunk: Array[T] = new Array(k) // buffer to group elements and push them
in one time
private var chunkSize: Int = 0
```

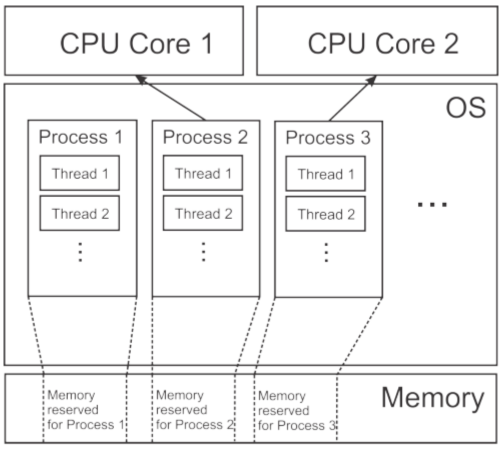
When we add

```
final def +=(elem: T): Unit = {
    if (chunkSize ≥ k) expand() // if full add it to tree
    chunk(chunkSize) = elem
    chunkSize += 1
}
private def expand() {
    conc = appendLeaf(conc, new Chunk(chunk, chunkSize))
    chunk = new Array(k)
    chunkSize = 0
}

final def combine(that: ConcBuffer[T]): ConcBuffer[T] = {
    val combinedConc = this.result <> that.result
    new ConcBuffer(k, combinedConc)
}

def result: Conc[T] = {
conc = appendLeaf(conc, new Chunk(chunk, chunkSize))
conc
}
```

Concurrency

Concurrency : concurrent computing consists of process *lifetimes* overlapping, but execution need not happen at the same instant.(process 1 then process 2 then process 1 again ...)

Week 5:



The OS schedules **threads** to run on **cores**.

```
def thread(b: => Unit) = {
    val t = new Thread {
        override def run() = b
    }
    t.start()
    t
}
```

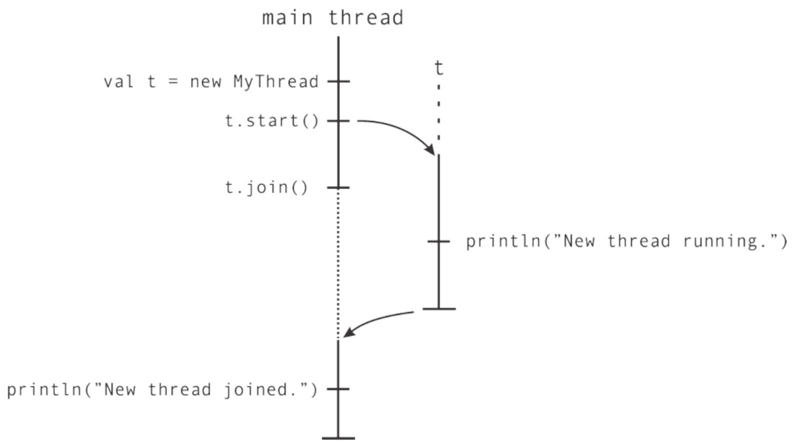
To start a thread on scala:

1. inherit from `java.lang.Thread` and redefine the `run` method
2. create an instance of the class
3. run it using `.start`

The call `t.join()` lets the calling thread wait until thread `t` has terminated.

First example :

```
val t = thread { println(s"New thread running") }
t.join()
println(s"New thread joined")
```



Non-deterministic behavior :

```
val t = thread {
println("New thread running")
}
println(" … ")
println(" … ")
t.join()
println("New thread joined")
```

Sometimes "New threadrunning , ... ,.... " is printed , other times "....,New threadrunning,...." is printed.

Instructions are *interleaved* this makes a lot of valid sequential programs invalid with concurrency.

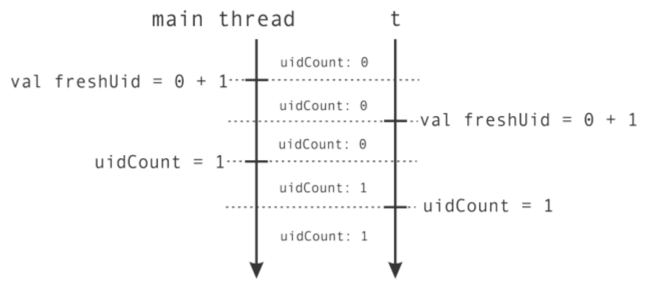
Consider the following piece of code that returns a unique id.

```
object ThreadsGetUID extends App {
    var uidCount = 0
    def getUniqueId() = {
        val freshUID = uidCount + 1
        uidCount = freshUID
        freshUID
    }
}
```

we test it *concurrently* with :

```
// test it with :
def printUniqueIds(n: Int): Unit = {
    val uids = for (i ← 0 until n) yield getUniqueId()
    println(s"Generated uids: $uids")
}

val t = thread { printUniqueIds(5) } // one call on seperate thread
printUniqueIds(5)                   // one call on the main thread
t.join()

//> ThreadsGetUID // first run of the code
//[53:thread] Generated uids: Vector(2, 5, 7, 9, 10)
//[1:main] Generated uids: Vector(1, 3, 4, 6, 8)

//> ThreadsGetUID // second run of the code
//[1:main] Generated uids: Vector(1, 2, 3, 4, 5)
//[55:thread] Generated uids: Vector(5, 6, 7, 8, 9) // 5 is repeated
```

1. non-deterministic behavior
2. Ids are not unique.



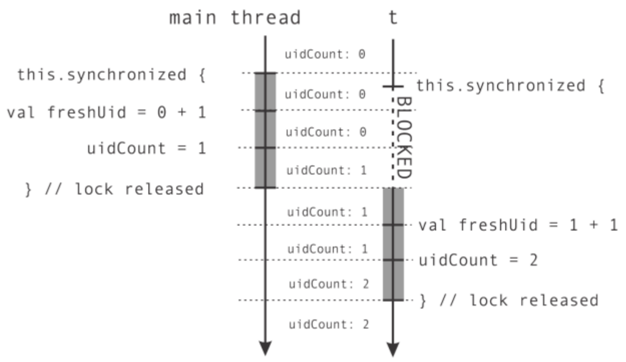
Synchronization and Atomic execution

The problem above is that we use a shared variable `uidCount` that is *not synchronized* properly.

We want to the instructions of `getUniqueId` to be run sequentially without interleaving with another thread. That's what we call **atomic execution**. To do that :

```
object GetUID:
    var uidCount = 0
    def getUniqueId() = synchronized {
        val freshUID = uidCount + 1
        uidCount = freshUID
        freshUID
    }
```

It means that only one thread can run the block inside `synchronized` at at time.



Two possible syntax :

- `synchronized{block}`
- `obj.synchronized{block}` where `obj` is a an instance of `anyRef`
  - this one puts a lock on `obj` : any thread that wants to use it should wait until the thread that has the lock on it is done.

Ledger example

```
object Ledger:
import scala.collection._
private val transfers = mutable.ArrayBuffer[String]()
    def logTransfer(name: String, n: Int) = transfers.synchronized {
        transfers += s"transfer to account $name = $n"
    }// notice that synchronized here is necessary
    def getlog = transfers

class Account(val name: String, var initialBalance: Int):
    private var myBalance = initialBalance
    private var uid = getUID
    def balance: Int = this.synchronized { myBalance } // synchronized here is
optional

    def add(n: Int): Unit = this.synchronized {
        myBalance += n
        // Log only if more than 10 CHF is transferred
        if n > 10 then logTransfer(name, n)
    }
```

Deadlock

Let's make a function to transfer money

```
def transfer(from: Account, to: Account, n: Int) =
    from.synchronized {
        to.synchronized {
            from.add(-n)
            to.add(n)
        }
    }
```

suppose we launch the following program :

```
val jane = new Account("Jane", 1000)
val john = new Account("John", 2000)
log("started ... ")
val t1 = thread { for i <- 0 until 100 do transfer(jane, john, 1) }
val t2 = thread { for i <- 0 until 100 do transfer(john, jane, 1) }
```

1. `t1` locks `jane` and `t2` locks `john`
2. `t1` tries to lock `john` but cannot because `t2` has it so it waits
3. `t2` tries to lock `jane` but cannot because `t1` has it so it waits

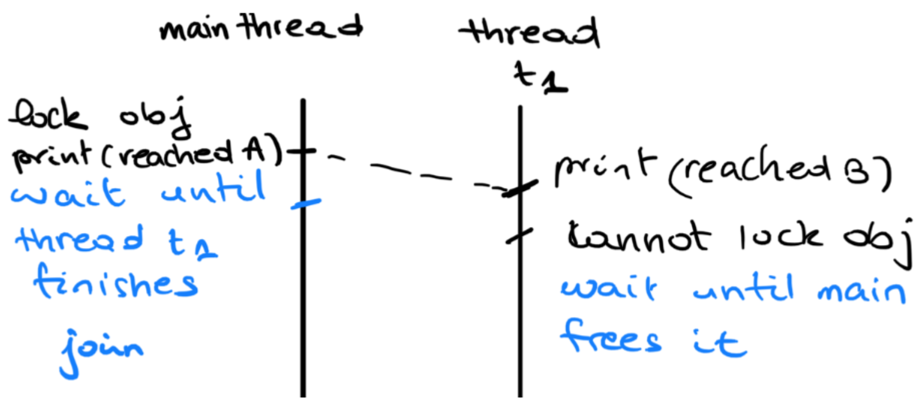
Solution : One approach is to always acquire resources in the same order

```
def transfer(from: Account, to: Account, n: Int) =
    def adjust() { to.add(n); from.add(-n) }
    if from.getUID < to.getUID then
        from.synchronized { to.synchronized { adjust() } }
    else
        to.synchronized { from.synchronized { adjust() } }
```

Another deadlock example :

```
val obj = AnyRef
obj.synchronized {
    println("Reached A")
    thread {
        println("Reached B")
        obj.synchronized {
            println("Reached C")
        }
    }.join
    println("Reached D")
}
```

This will not halt.



```
// one solution
val lock = AnyRef
lock.synchronized {
println("Reached A")
}
thread {
    println("Reached B")
    lock.synchronized {
        println("Reached C")
    }
}.join
println("Reached D")
```

Classical example

When using a one place buffer. We distinguish two thread roles :

1. consumers : take element from buffer

   if thread is empty consumers must wait
2. producers : put elements in buffer

   if thread is full producers have to wait
3. at most one element can be in the buffer at any one time

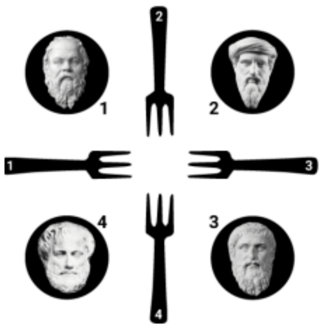
```
def put(e: Elem) = synchronized {
    while bufferIsFull do {}
        putElementInTheBuffer(e)
        bufferIsFull = true
    }
def get(): Elem = synchronized {
    while !bufferIsFull do {}
        elem = getElementFromTheBuffer()
        bufferIsFull = false
        elem
    }
// DEAD LOCK SITUATION
```

Solution: Hold the lock for a short duration and release it after checking the buffer is full (for producers) empty (for consumers). Repeat the operation without always holding the lock.

```
// SOLUTION
class TempObj[Elem]:
    var e:Elem = uninitialized
    class OnePlaceBuffer[Elem]:
    private var elem: Elem = uninitialized
    private var bufferIsFull: Boolean = false

    def put(e: Elem) =
        while !tryToPut(e) do {}
        def tryToPut(e: Elem): Boolean = this.synchronized {
            if bufferIsFull then false
            else { elem = e; bufferIsFull = true; true }
        }
    def get(): Elem =
        var temp = new TempObj[Elem]
        var bufferIsEmpty: Boolean = true
        while bufferIsEmpty do
            this.synchronized {
                if bufferIsFull then
                bufferIsFull = false; temp.e = elem; bufferIsEmpty = false
            }
        return temp.e
```

the example of the dining philosophers

There are N philosphers sitting around a circular table eating spaghetti and discussing philosphy. The problem is that each philosopher needs 2 forks to eat, and there are only N forks, one between each 2 philosophers



```
def philosophersDining(n: Int) =
    val forks = new Array[Fork](n)
    val philosophers = new Array[Thread](n)
    val waiter = new Waiter
    for p <- 0 to n - 1 do
        forks(p) = new Fork()
    for p <- 0 to n-1 do
        philosophers(p) = thread{
            while (!philosopherTurn(w, forks(p%n), forks((p+1)%n))) {}
        }
    for p <- 0 to n - 1 do
    philosophers(p).join()

def philosopherTurn(w: Waiter, left: Fork, right: Fork): Boolean =
    Thread.sleep(100) // wait for some time
    w.synchronized {
        if !left.inUse && !right.inUse then
            left.inUse = true
            right.inUse = true
        }
        else
            false
    Thread.sleep(1000) // eating
    w.synchronized {
        left.inUse = false
        right.inUse = false
    }

    true
```