**Cairo University**
**Faculty of Computers and Artificial Intelligence**

# Systems Modeling and Simulation (DS311) - Fall 2024

Third year OR&DS – FCAI

Instructor: Assoc. Prof. Ayman Ghoneim

Problem I [Petrol Station Multi-Channel Queue]

Problem II [Hospital Inventory System]

| Name | ID |
|---|---|
| Youssef Hanaie Hussien | 20220417 |
| Ammar Mohsen Abdulaziz | 20220214 |
| Mohamed Gomaa Mohamed | 20220449 |
| Salma Mohamed Mostafa | 20221074 |

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# Problem I [Petrol Station Multi-Channel Queue]

## Part 1:

### Problem Formulation:

The problem is about simulating a multi-channel queue petrol system with these features:

1. Car Categories:

- **Category A:** Uses 95-octane petrol (20% of cars).
- **Category B:** Can use both 90-octane and 95-octane petrol (35% of cars).
- **Category C:** Can use 90-octane petrol or gas (45% of cars)

2. Pumps: The petrol has three pumps one for 95-octane petrol and one for 90-octane and one for gas

3. Car arrival: Car arrive following these time-between arrival table 1

| Time between Arrivals (Minutes) | Probability |
|---|---|
| 0 | 0.17 |
| 1 | 0.23 |
| 2 | 0.25 |
| 3 | 0.35 |

4. Queues: Each pump has its own queue use (**FCFS**) behavior a car could switch form queue to another if there is these conditions

**Category B cars:**

- Prefer the **90-octane pump**.(cheaper drivers want to pay less)
- Switch to the **95-octane pump** with a probability of **0.6** if the queue length for the 90-octane pump exceeds three cars.

**Category C cars:**

- Prefer the **gas pump**. (cheaper drivers want to pay less)
- Switch to the **90-octane pump** with a probability of **0.4** if the queue length for the gas pump exceeds four cars.

5. Service Time: The time required to serve a car at a pump is random and depend on these Tables

Table 2 | Table 3

| Category A & B Service Time (Minutes | Probability | Category C Service Time (Minutes) | Probability |
|---|---|---|---|
| 1 | 0.20 | 3 | 0.20 |

| Category A & B Service Time (Minutes | Probability | Category C Service Time (Minutes) | Probability |
|---|---|---|---|
| 2 | 0.30 | 5 | 0.50 |
| 3 | 0.50 | 7 | 0.30 |

Objectives:

The main objectives of this simulation are:

1. Queue Analysis:
    - Track the length of queues at each pump.
    - Analyze the waiting times for cars in different categories.
2. Service efficiency:
    - Average waiting time for cars.
    - Number of cars served per pump.
3. Driver Behavior: Track how the driver switch from pump to another due to queue length
4. Statistical Reporting:
    - Average service time per pump.
    - Total number of cars served.
    - Queue length statistics (average, maximum.).
5. Cost Analysis:
    - Assess the trade-offs between driver behavior (time vs. cost preferences) and system efficiency

Part 2:

System Components:

| System | Entities | Attributes | Activities | Events | State Variables |
|---|---|---|---|---|---|
| Petrol Station Queue | Cars | Category (A, B, C); | Refueling, Switching Pumps | Arrival at station; Queue joining; Start of service; End of service; Pump switching decision | Number of cars in each queue; Number of cars being served; Pump utilization; |

System analysis:

| Car ID | Category | Arrival Time | Start Time | End Time | Wait Time | Service Time | Idle Time | Assigned Pump |
|---|---|---|---|---|---|---|---|---|
| 1 | B | 0 | 0 | 2 | 0 | 2 | 0 | 90 |
| 2 | B | 1 | 2 | 4 | 1 | 2 | 0 | 90 |
| 3 | A | 2 | 2 | 4 | 0 | 2 | 2 | 95 |
| 4 | B | 4 | 4 | 7 | 0 | 3 | 0 | 90 |
| 5 | C | 4 | 4 | 9 | 0 | 5 | 4 | Gas |
| 6 | A | 6 | 6 | 9 | 0 | 3 | 2 | 95 |
| 7 | B | 6 | 7 | 8 | 1 | 1 | 0 | 90 |
| 8 | B | 7 | 8 | 9 | 1 | 1 | 0 | 90 |
| 9 | B | 10 | 10 | 12 | 0 | 2 | 1 | 90 |
| 10 | B | 11 | 12 | 15 | 1 | 3 | 0 | 90 |
| 11 | C | 11 | 11 | 16 | 0 | 5 | 2 | Gas |
| 12 | C | 12 | 16 | 19 | 4 | 3 | 0 | Gas |
| 13 | B | 12 | 15 | 18 | 3 | 3 | 0 | 90 |
| 14 | B | 13 | 18 | 20 | 5 | 2 | 0 | 90 |
| 15 | A | 14 | 14 | 17 | 0 | 3 | 5 | 95 |
| 16 | B | 17 | 20 | 21 | 3 | 1 | 0 | 90 |
| 17 | B | 19 | 21 | 22 | 2 | 1 | 0 | 90 |
| 18 | B | 19 | 22 | 24 | 3 | 2 | 0 | 90 |
| 19 | A | 19 | 19 | 21 | 0 | 2 | 2 | 95 |
| 20 | B | 21 | 24 | 27 | 3 | 3 | 0 | 90 |

The system involves probabilistic arrivals of cars from three categories, each with specific fuel preferences and associated service times. Category A cars are limited to 95 octane petrol, while category B cars have flexibility and can switch to 95 octane petrol under certain conditions, and category C cars can choose between 90 octane petrol or gas based on queue lengths. The analysis must account for varying service times, customer behavior influenced by queue length, and the dynamics of fuel availability to minimize wait times and optimize station throughput

**Time between Arrivals (Table 1)**

| Time Between Arrivals (Minutes) | Probability | Cumulative Probability | Random-Digit Assignment |
|---|---|---|---|
| 0 | 0.17 | 0.17 | 01–17 |
| 1 | 0.23 | 0.40 | 18–40 |
| 2 | 0.25 | 0.65 | 41–65 |
| 3 | 0.35 | 1.00 | 66–00 |

**Car Category Table (2)**

| Category | Probability | Cumulative Probability | Random-Digit Assignment |
|---|---|---|---|

| | | | |
|---|---|---|---|
| A | 0.20 | 0.20 | 01–20 |
| B | 0.35 | 0.55 | 21–55 |
| C | 0.45 | 1.00 | 56–00 |

## Service Time for Categories A & B (Table 3)

| Service Time (Minutes) | Probability | Cumulative Probability | Random-Digit Assignment |
|---|---|---|---|
| 1 | 0.20 | 0.20 | 01–20 |
| 2 | 0.30 | 0.50 | 21–50 |
| 3 | 0.50 | 1.00 | 51–00 |

## Service Time for Category C (Table 4)

| Service Time (Minutes) | Probability | Cumulative Probability | Random-Digit Assignment |
|---|---|---|---|
| 3 | 0.20 | 0.20 | 01–20 |
| 5 | 0.50 | 0.70 | 21–70 |
| 7 | 0.30 | 1.00 | 71–00 |

Part 3:

Experimental Design Parameters

### 1. Simulation Time

- **Parameter**: Total time (or number of arrivals) simulated.
- **Value**: 20 arrivals.

### 2. Number of Runs

- **Parameter**: Total number of runs.
- **Value**: 30 runs

### 3. Queue Thresholds for Switching

- **Category B**: Switches to 95 octane if the 90-octane queue has more than 3 cars.
- **Category C**: Switches to 90 octane if the gas queue has more than 4 cars.

### 4. Probability of Switching

- **Category B**: 60% probability for 95 octane if threshold exceeded.
- **Category C**: 40% probability for 90 octane if threshold exceeded.

### 5. Pump Configurations

- **Number of Pumps**: One pump each for 95 octane, 90 octane, and gas.

### 6. Inter-arrival and Service Times

- **Source**: Table 1 for inter-arrival times and Tables 2 & 3 for service times.

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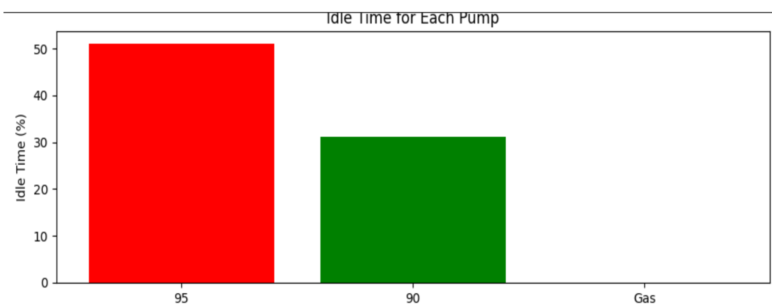
Justification of Parameter Values

- **Simulation Time**: Long duration ensures reliable statistics.
- **Number Of Runs**: the more you run the simulation the more It will approach the theoretical and that’s what we are looking for
- **Queue Thresholds and Switching Probabilities**: Reflect real-world trade-offs between time and cost.
- **Probability of Switching** Reflects typical petrol station setup, ensuring focus on queue dynamics and decision-making.
- **Pump Configurations**: Simulates resource constraints in typical petrol stations.
- **Inter-arrival and Service Times**: Capture variability to evaluate system efficiency under realistic conditions.

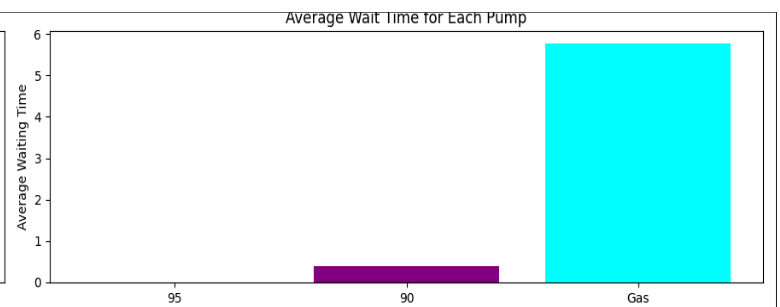
Part 4:

Results Analysis

Graph for idle time for each pump




Graph for Average wait time for each pump




1. The average service time of cars in the three categories:

   - Average Service Time (Experimental): {'A': 2.3307881773399015, 'B': 2.3941329966329965, 'C': 5.058469863469864}

2. The average waiting time in the queues for each pump, and the average waiting time for all cars

   - Average Waiting Time (Pump): {'95': 0.0, '90': 0.4, 'Gas': 5.777777777777778}

   - Average Waiting Time (All Cars): 2.7

3. The maximum queue length for each pump

- Max Queue Lengths: {'95': 1.0, '90': 1.0, 'Gas': 3.0}

4. The probability that a car wait for each pump.

- Probability (Car Waits): {'95': 0.0, '90': 0.2, 'Gas': 0.8888888888888888}

5. The portion of idle time of each pump

- Idle Times (Portion): {'95': 51.11111111111111, '90': 31.11111111111111, 'Gas': 0.0}

6. Does the theoretical average service time of the service time distribution match with the experimental one for the three categories?

- Theoretical Average Service Time: {'A, B': 2.3, 'C': 5.1}
- Average Service Time (Experimental): {'A': 2.3307881773399015, 'B': 2.3941329966329965, 'C': 5.058469863469864}

  it kind of matching in category A and B but in Category C there is a small difference but it converges more and more to it with more runs and by comparing both, we see they approach each other and there is not big difference between them

7. Does the theoretical average inter-arrival time of the inter-arrival time distribution match with the experimental one?

- Experimental Avg Inter-arrival Time: 1.7
- Theoretical Avg Inter-arrival Time: 1.78

There is a difference between experimental and theoretical but it is close to it.

8. If the petrol station is investigating the addition of one extra pump, what kind of pump (95 octane, 90 octane or gas) will result in the most decrease in the average waiting time for all cars?

Gas pump will cause a huge decrease in the average time waiting because as you can see in **table 2**

Most of cars will fall into category of C and as you can see at service time **table 4** that the service time of gas pump take a lot of time which will cause into more cars waiting in queue and also the car of category of **C** has a 0, 4 probability to change to 90-octane pump which make also a huge jump in average waiting time

## Conclusion

The petrol station system effectively models the behavior of diverse car categories under real-world conditions. By incorporating probabilities for car categories, switching behavior based on queue thresholds, and realistic inter-arrival and service times, the simulation provides insights into pump utilization, queue dynamics, and customer preferences. This analysis can guide optimization strategies to minimize delays and improve resource allocation.

# Problem 2

## Part 1:

The problem revolves around simulating the inventory management of a hospital that includes two levels of inventory: **First Floor Inventory** and **Basement Inventory**

## System Description:

1. **First Floor Inventory**:
    - Maximum capacity: 10 boxes.
    - Serves five patient rooms.
    - Each room consumes one box per day.
    - When boxes run out, it requests 10 boxes from the basement inventory (or less, based on basement availability).

2. **Basement Inventory**:
    - Maximum capacity: 30 boxes.
    - Supplies boxes to the first floor inventory.
    - Orders are placed to make the basement inventory to its maximum capacity during the **review period** (N = 6 days).

3. **Order Lead Time**:
    - The time taken to make the basement inventory varies and is random

**Table 1**

| Lead Time | Lead Time Probabilities |
|---|---|
| 1 | 0.4 |
| 2 | 0.35 |
| 3 | 0.25 |

4. **Demand for Rooms**:
    - Number of rooms occupied by patients follows a probability distribution

**Table 2**

| # of Rooms Occupied | Probability |
|---|---|
| 1 | 0.1 |

| | |
|---|---|
| 2 | \| 0.15 |
| 3 | \| 0.35 |
| 4 | \| 0.2 |
| 5 | \| 0.2 |

## Objectives:

1. **Minimize Shortages**:
   - Ensure the first floor inventory has sufficient supply to meet patient room demand each day.
   - Minimize instances of shortages and track their occurrence.
2. **Efficient Replenishment**:
   - Optimize the restocking process for the basement inventory to avoid overstocking or understocking.
   - Ensure lead times are factored into planning orders to maintain inventory levels.
3. **Generate Statistics:**
   - Provide insights into system performance, such as average daily inventory levels, frequency of shortages, and average lead-time.

## Part 2:

## System components:

| System | Entities | Attributes | Activities | Events | State Variables |
|---|---|---|---|---|---|
| **Inventory System** | Patients' Rooms | Demand of Boxes | Replenishing, Ordering | Daily demand, | Inventory levels in first floor & basement |
| | | | Consumption of inventory | Order placement | Number of demands |
| | | | Receiving orders | Inventory review | Number of shortages |

System analysis:

| Day | Beginning Inventory | Demand | Ending Inventory | Shortage_first_floor | Order Quantity | Lead Time | Basement Inventory |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 4 | 0 | 0 | 0 | 0 | 30 |
| 2 | 10 | 4 | 6 | 0 | 0 | 0 | 20 |
| 3 | 6 | 4 | 2 | 0 | 0 | 0 | 20 |
| 4 | 2 | 3 | 0 | 1 | 0 | 0 | 20 |
| 5 | 10 | 3 | 7 | 0 | 0 | 0 | 9 |
| 6 | 7 | 3 | 4 | 0 | 21 | 3 | 9 |
| 7 | 4 | 1 | 3 | 0 | 21 | 2 | 9 |
| 8 | 3 | 1 | 2 | 0 | 21 | 1 | 9 |
| 9 | 2 | 4 | 0 | 2 | 21 | 0 | 9 |
| 10 | 10 | 4 | 6 | 0 | 0 | 0 | 18 |
| 11 | 6 | 5 | 1 | 0 | 0 | 0 | 18 |
| 12 | 1 | 4 | 0 | 3 | 12 | 1 | 18 |
| 13 | 10 | 5 | 5 | 0 | 12 | 0 | 5 |
| 14 | 5 | 4 | 1 | 0 | 0 | 0 | 17 |
| 15 | 1 | 5 | 0 | 4 | 0 | 0 | 17 |
| 16 | 10 | 3 | 7 | 0 | 0 | 0 | 3 |
| 17 | 7 | 5 | 2 | 0 | 0 | 0 | 3 |
| 18 | 2 | 2 | 0 | 0 | 27 | 2 | 3 |
| 19 | 3 | 3 | 0 | 0 | 27 | 1 | 0 |
| 20 | 0 | 2 | 0 | 2 | 27 | 0 | 0 |

The hospital's inventory system involves two levels: a first floor inventory serving patient rooms with a capacity of 10 boxes and a basement inventory with a capacity of 30 boxes, which replenishes the first floor when needed. The first floor sends requests for boxes when it runs out, relying on the basement inventory's availability, while orders are placed to refill the basement inventory with a lead-time, which is a random variable. The system operates with a fixed review period (N = 6 days), where inventory is assessed and orders are placed to maintain adequate stock levels.

## Table 1: Number of Rooms Occupied

| # of Rooms Occupied | Probability | Cumulative Probability | Random Digit Assignment |
|---|---|---|---|
| 1 | 0.10 | 0.10 | 01–10 |
| 2 | 0.15 | 0.25 | 11–25 |
| 3 | 0.35 | 0.60 | 26–60 |
| 4 | 0.20 | 0.80 | 61–80 |
| 5 | 0.20 | 1.00 | 81–00 |

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## Table 2: Lead Time

| Lead Time | Probability | Cumulative Probability | Random Digit Assignment |
|---|---|---|---|
| 1 | 0.40 | 0.40 | 01–40 |

| Lead Time | Probability | Cumulative Probability | Random Digit Assignment |
|---|---|---|---|
| 2 | 0.35 | 0.75 | 41–75 |
| 3 | 0.25 | 1.00 | 76–00 |

Part 3:

Experimental Design Parameters

1. **Simulation Time**
   - Total time (or number of arrivals) simulated we will make it 30 Days
2. **Number of runs**
   - Total number of runs of simulation we will make it 30
3. **Review Period (N)**
   - Definition: The number of days after which the inventory is reviewed, and an order is placed to refill the basement inventory to its maximum capacity.it is reviewed at the end of business day and order comes early
   - Selected Value: N=6 days.
4. **Basement Inventory Capacity (M)**
   - Definition: The maximum number of medical supply boxes that the basement can hold.
   - Selected Value: M=30 boxes.
5. **First Floor Inventory Capacity**
   - Definition: The Number of boxes that we will start with
   - Selected Value: 4 boxes.
6. **Lead Time Distribution**
   - Definition: The number of days between placing an order and receiving it.
   - Values: Based on a random variable as shown in **Table 1.**
7. **Daily Demand Distribution**
   - Definition: Number of rooms occupied and corresponding daily consumption of medical supply boxes.
   - Values: Randomly determined as per **Table 2.**

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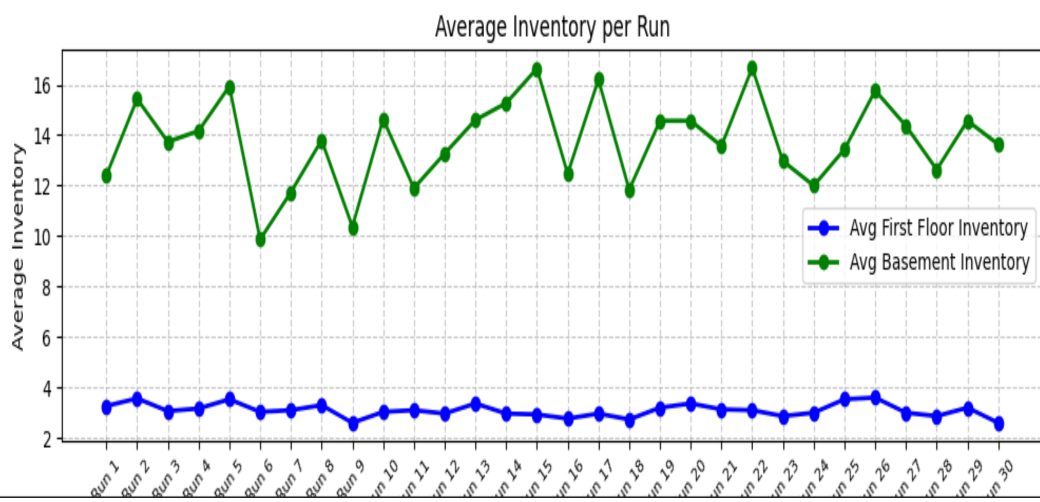
Justification of Experiment Parameter Values

1. **Simulation Time**
   - A 30-day duration provides sufficient time to observe variability in the system, including daily, weekly, and potential seasonal trends, and By the Central Limit Theorem (CLT), as the simulation generates data over multiple iterations or time intervals, the distribution of sample means approaches a normal distribution.
2. **Number of runs**
   - The more you run the more you approach the theoretical and that is what we are aiming for
3. **Review Period (N = 6 days)**
   - A period of six days was chosen to balance between minimizing stock outs and reducing ordering frequency. Increasing N would reduce ordering costs but could increase shortages, while decreasing N would increase ordering frequency unnecessarily.

4. **Basement Inventory Capacity (M = 30 boxes)**
   - A maximum capacity of 30 boxes provides a buffer to meet demand during lead times and prevents frequent orders while considering space constraints.
5. **First Floor Inventory Capacity (4 boxes)**
   - We Used This Capacity to see if we do not have full capacity in first what will happen and how to handle this case.
6. **Lead Time Distribution**
   - Using a probabilistic lead-time reflects realistic supply chain uncertainty, enabling a more robust simulation of inventory dynamics.
7. **Daily Demand Distribution**
   - The demand variability accounts for the dynamic nature of patient occupancy and ensures the model adapts to fluctuations in room utilization.

## Part 4:

## Graph for Inventory levels




1- The average ending units in first floor inventory and the basement inventory.

- Average Ending Inventory (First Floor): 3.10
- Average Basement Inventory:  1 3.77

2- The number of days when a shortage condition occurs.

- Days with Shortage: 0.50

3- Does the theoretical average demand of boxes match the experimental one?

- Experimental average Demand: 3.21
- Theoretical Demand: 3.25

The experimental is really close to the theoretical one but it doesn't match it, it approach to it the more runs

4- Does the theoretical average lead time of the lead time distribution match the

Experimental one?

- Experimental average Lead Time: 1.67
- Theoretical Lead Time: 1.85

No, it doesn't match perfectly but it's kind of close to theoretical that's also because the center limit theorem

5- Is there a better value for the review period variable N to minimize the

Shortages of medical supplies boxes?

No, because the maximum demand is five the basement has 30 boxes and the first floor has 4, which makes 34 boxes available the basement gives boxes instant to the first floor which makes it difficult to have days with shortage but it in worst case scenario 5*6=30 and the lead time is 3 then I will have 3 days of shortage so I can make review period 4 instead of 6 but that won't make a big difference

6- Is there a better value for the maximum capacity M of the basement inventory to minimize the shortages of medical supplies boxes?

The more the better of course but in this case it will cost more and 30 is enough to handle a maximum of 5 demand daily but it also In worst case I could make it 35 boxes but also that won't make a big difference

## Conclusion

The hospital inventory system ensures a reliable supply of medical boxes to meet patient demand while managing stock levels efficiently. By balancing the review period, basement capacity, and first-floor inventory limits, the system minimizes shortages and ordering costs. Incorporating realistic lead-time and demand variability enhances the model's ability to adapt to operational uncertainties.

Good Luck ☺