Robot Dynamics & Control – A.A. 2021/2022

Assignment 2: Recursive Inverse Dynamics

This assignment is focused on the **recursive Newton-Euler algorithm** for inverse dynamics.

Evaluation rules

You have to upload on Teams a '.zip' file named as **'SurnameName_Assignment_2.zip'** which contains:

- A **'.pdf'** report (**'SurnameName_Assignment_2_report.pdf'**) with the motivated answers for each exercise and eventually drawings/diagrams to better motivate the solution. (**Note:** Any answer provided without a full motivation will not be considered for evaluation. All the steps towards the solution must be justified.).
- The MATLAB file (**'SurnameName_Assignment_2_solution.m'**) with the numerical implementations of the exercises. This should be the main code, of course you can define helper functions in other files which also have to be included.

The assignment is individual, so each student must deliver his/her .zip archive.

Exercise 1 – Recursive Newton-Euler Implementation

Implement in MATLAB the recursive Newton-Euler algorithm for inverse dynamics, by keeping into account the following criteria.

- The code should be clean (no re-definition of variables) and readable (clear comments). Explain what each step does.
- The algorithm must be implemented as a general function, meaning that any parameter related to the specific robot or the input trajectory should be defined from the outside.
- The robot data should be represented by a unique data structure/object.
- The algorithm should support both 2D and 3D structures, regardless of the joint type

NB This exercise is mandatory for the next three, since are intended to evaluate the implemented algorithm. Be careful and implement the equations correctly.

Exercise 2 – Inverse Dynamics of 2R robot

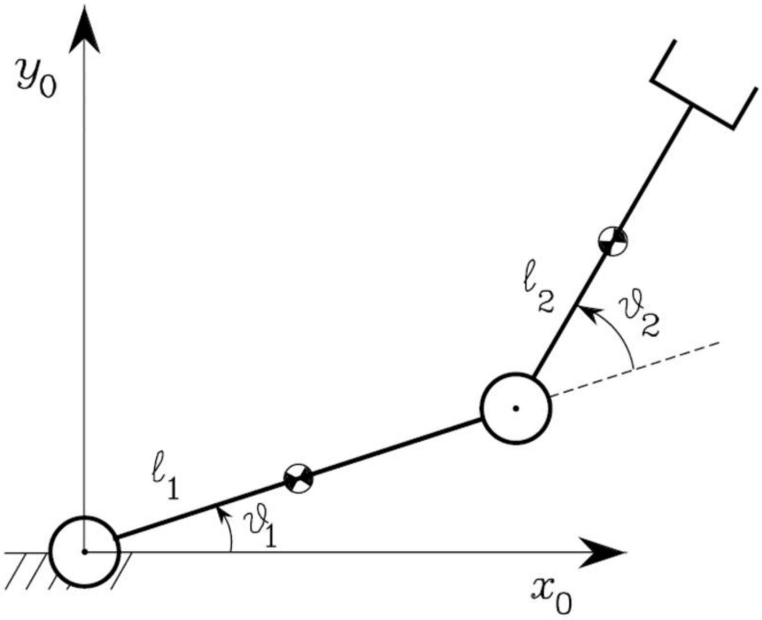

Figure 1: 2R Planar manipulator

$m_1 = 22Kg, m_2 = 19Kg$, $l_1 = 1m, l_2 = 0.8m$, $I_{zz1} = 0.4Kgm^2, I_{zz2} = 0.3Kgm^2$

Assumed that the links are prisms with a uniform mass distribution, the COM of each one is located at the geometric centre.

Find the inverse dynamics joint torques $\boldsymbol{\tau_1}$, $\boldsymbol{\tau_2}$ for the following motion snapshots, first without gravity, then with gravity acting along $\boldsymbol{y_0}$.

1. $\vartheta_1 = 20^{\circ}, \vartheta_2 = 40^{\circ}$;
   $\dot{\vartheta}_1 = 0.2rad/s, \dot{\vartheta}_2 = 0.15\ rad/s$;
   $\ddot{\vartheta}_1 = 0.1rad/s^2, \ddot{\vartheta}_2 = 0.085rad/s^2$

2. $\vartheta_1 = 90^{\circ}, \vartheta_2 = 45^{\circ}$;
   $\dot{\vartheta}_1 = -0.8rad/s, \dot{\vartheta}_2 = 0.35\ rad/s$;
   $\ddot{\vartheta}_1 = -0.4rad/s^2, \ddot{\vartheta}_2 = 0.1rad/s^2$

Exercise 3 – Inverse Dynamics of RP robot

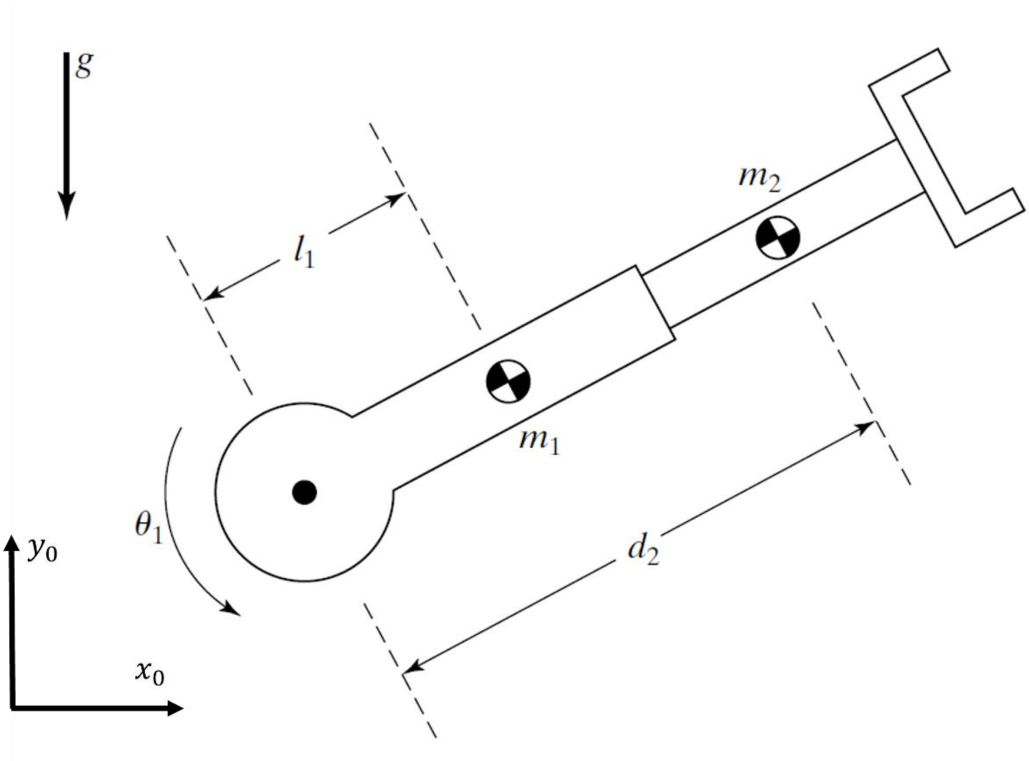

Figure 2: RP Planar manipulator

$m_1 = 10Kg, m_2 = 6Kg\ , l_1 = 1m, I_{zz1} = 0.4Kgm^2, I_{zz2} = 0.3Kgm^2$

The COM of each link is located as shown in figure.

Find the inverse dynamics joint torques $\boldsymbol{\tau_1}$, $\boldsymbol{\tau_2}$ for the following motion snapshots, first without gravity, then with gravity acting along $\boldsymbol{y_0}$.

1. $\vartheta_1 = 20^{\circ}, d_2 = 0.2m;$
   $\dot{\vartheta}_1 = 0.08 rad/s, v_2 = 0.03\ m/s;$
   $\ddot{\vartheta}_1 = 0.1 rad/s^2, a_2 = 0.01 m/s^2$

2. $\vartheta_1 = 120^{\circ}, d_2 = 0.6m;$
   $\dot{\vartheta}_1 = -0.4 rad/s, v_2 = -0.08\ m/s;$
   $\ddot{\vartheta}_1 = -0.1 rad/s^2, a_2 = -0.01 m/s^2$

Exercise 4 – Inverse Dynamics of 3R robot

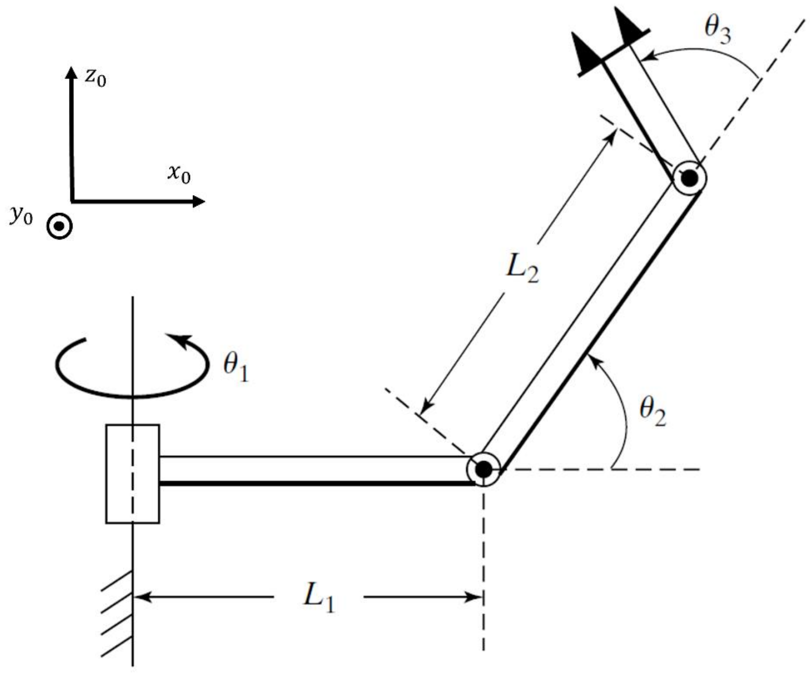

Figure 2: 3R Non-planar manipulator

$m_1 = 20Kg, m_2 = 20Kg, m_3 = 6Kg$

$l_1 = 1m, l_2 = 0.8m, l_3 = 0.35m$

$I_1 = diag(0.2Kgm^2, 0.2Kgm^2, 0.8Kgm^2)$

$I_2 = diag(0.2Kgm^2, 0.2Kgm^2, 0.8Kgm^2)$

$I_3 = diag(0.08Kgm^2, 0.08Kgm^2, 0.1Kgm^2)$

Assumed that the links are prisms with a uniform mass distribution, the COM of each one is located at the geometric centre.

Find the inverse dynamics joint torques $\boldsymbol{\tau_1}$, $\boldsymbol{\tau_2}$ for the following motion snapshot, first without gravity, then with gravity acting along $\boldsymbol{z_0}$.

1. $\vartheta_1 = 20^\circ, \vartheta_2 = 40^\circ, \vartheta_3 = 10^\circ;$
   $\dot{\vartheta}_1 = 0.2rad/s, \dot{\vartheta}_2 = 0.15\, rad/s, \dot{\vartheta}_3 = -0.2\, rad/s;$
   $\ddot{\vartheta}_1 = 0.1rad/s^2, \ddot{\vartheta}_2 = 0.085rad/s^2, \ddot{\vartheta}_3 = 0\, rad/s^2$