Youssef Mahmoud
905854027
PS3, Qs 1-4

HW5

① a) If $X_1 > X_2$, then $C(X_2) \geq C(X_1)$

$t=0$	$t=T$
$C(X_2)$	$Max(0, S_T - X_1)$
$-C(X_1)$	$-Max(0, S_T - X_2)$
≥ 0	≥ 0

→ Arbitrage exists

b) $X_2 > X_1$ → $P(X_1) \geq P(X_2)$

$t=0$	$t=T$
$P(X_1)$	$Max(0, X_2 - S_T)$
$-P(X_2)$	$Max(0, X_1 - S_T)$

→ Arbitrage exists

① c) $P > Xe^{-rT} - S_0$

Sys $P \leq Xe^{-rT} - S_0$

$P + S_0 \leq Xe^{-rT}$

→ Payoff is not -ve

→ Arbitrage exists

J) i) $D + Xe^{-rT} > S_0$

$t=0$	$t=T$
D	$X - S_T$
Xe^{-rT}	$-X$
$-S_0$	S_T
>0	0

Arbitrage exists in both cases.

ii) $D + Xe^{-rT} < S_0$

$t=0$	$t=T$
$-D$	$-X + S_T$
$-Xe^{-rT}$	X
S_0	$-S_T$
>0	0

(2) Short sell:

	$t=0$	$t=T$
	S_0	$-S_T$
	$-S_0$	$S_0 . e^{rT}$
		$S_0 . e^{rT} - S_T$

$$C + Xe^{-rT} = P + S_0$$

$t=0$	$t=T$ $S_T > X$	$t=T$ $S_T < X$
C	$-(S_T - X)$	0
Xe^{-rT}	$-X$	$-X$
$-P$	0	$X - S_T$
$-S_0$	$S_0 e^{rT}$	$S_0 e^{rT}$
Payoff	$S_0 e^{rT} - S_T$	$S_0 e^{rT} - S_T$

③ a)

$S_1^+ = 110$

$S_0 = 100$

$S_1^- = 90$

$P_1^+ = 0 = \Delta 110 - F$

P_0

$P_1^- = 10 = \Delta 90 - F$

$$0 - 10 = \Delta(110 - 90)$$

$$-10 = \Delta(20)$$

$$\Delta = -\frac{1}{2}$$

$$0 = -\frac{1}{2}(110) - F$$

$$F = -55 \qquad P = -\frac{1}{2}(100) - \frac{(-55)}{1.015} = 4.187$$

③ b) $10 = 110\Delta - F$

$0 = 90\Delta - F$

$\Delta = \frac{1}{2} \quad F = 45$

$C = \frac{1}{2} \times 100 - \frac{45}{1.015} = 5.665$

$5.665 + \frac{100}{1.015} = 4.187 + 100 = 104.187$

→ Put parity holds!

c) The replicating portfolio for the put option shorts the stock as the Δ is -ve.

④ a) sps $u \leq (1+r) \leq d$

for $u \leq (1+r) \rightarrow uS_0 \leq (1+r)S_0$

and $1+r > d$

$t=0$, Short-sell Stock and long zero coupon bond with $F = (1+r)S_0$

$t=1$, Receive $(1+r)S_0$ from the ZCB, buy the stock again at uS_0

if Price↑, Payoff $= (1+r)S_0 - uS_0 > 0$

if Price↓, Payoff $= (1+r)S_0 - dS_0 > 0$

for $(1+r) \leq d \rightarrow (1+r)S_0 \leq dS_0$

$t=0$, short zero coupon bond with $F = S_0(1+r)$, and long S_0.

$t=1$, return $= (1+r)S_0$ for the ZCB and sell stock at S_T

if Price↑, Payoff $= dS_0 - (1+r)S_0 > 0$

if Price↓, Payoff $= uS_0 - (1+r)S_0 > 0$

(4) b)

$$\begin{tikzcd}[row sep=small]
 & S_1^+ = uS_0 \\
S_0 \arrow[ur] \arrow[dr] & \\
 & S_1^- = dS_0
\end{tikzcd}
\qquad
\begin{tikzcd}[row sep=small]
 & A_d^+ = 0 \\
A_d \arrow[ur] \arrow[dr] & \\
 & A_d^- = 1
\end{tikzcd}$$

Payoffs $0 = \Delta u S_0 - F \Longrightarrow \Delta = -\frac{1}{(u-d)S_0}$

$1 = \Delta d S_0 - F \Longrightarrow F = \frac{u}{d-u}$

$$A_d = \Delta S_0 - \frac{F}{1+r} = \frac{S_0}{(d-u)S_0} - \frac{u/d-u}{(1+r)}$$

$$= \frac{u-(1+r)}{(1+r)(u-d)}$$