Boolean Algebra Theorems

Operations with 0 and 1

$X + 0 = X \qquad X \cdot 1 = X \qquad X + 1 = 1 \qquad X \cdot 0 = 0$

Idempotent laws

$X + X = X \qquad X \cdot X = X$

Involution law

$(X')' = X$

Complement Laws

$X + X' = 1 \qquad X \cdot X' = 0$

Commutative Laws

$XY = YX \qquad X + Y = Y + X$

Associative Laws

$(XY)Z = X(YZ) = XYZ \qquad (X + Y) + Z = X + (Y + Z) = X + Y + Z$

Distributive Laws

$X(Y + Z) = XY + XZ \qquad X + YZ = (X + Y)(X + Z)$

DeMorgan's Laws

$(X + Y)' = X'Y' \qquad (XY)' = X' + Y'$

simplification theorems

Uniting Theorems

$XY + XY' = X \qquad (X + Y)(X + Y') = X$

Absorption Theorems

$X + XY = X \qquad X(X + Y) = X$

Elimination Theorems

$X + X'Y = X + Y \qquad X(X' + Y) = XY$

Consensus Theorems

$XY + X'Z + YZ = XY + X'Z \qquad (X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z)$

Factoring Theorems

$(X + Y)(X' + Z) = XZ + X'Y' \qquad XY + X'Z = (X + Z)(X' + Y)$

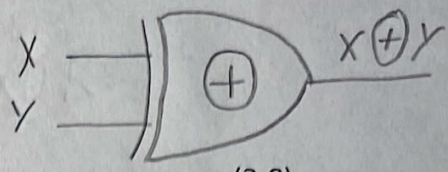
Duality

$(X + Y + Z + \cdots)^D = XYZ \cdots \qquad (XYZ \cdots)^D = X + Y + Z + \cdots$

$X = Y \rightarrow XY + X'Y'$

$X \oplus Y \rightarrow X'Y + XY'$

XOR Gate:



Theorems for exclusive-OR

$X \oplus 0 = X$		(3-8)
$X \oplus 1 = X'$		(3-9)
$X \oplus X = 0$		(3-10)
$X \oplus X' = 1$		(3-11)
$X \oplus Y = Y \oplus X$	(commutative law)	(3-12)
$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z$	(associative law)	(3-13)
$X(Y \oplus Z) = XY \oplus XZ$	(distributive law)	(3-14)
$(X \oplus Y)' = X \oplus Y' = X' \oplus Y = XY + X'Y'$		(3-15)

$(X \oplus Y)' = (X \equiv Y)$

Theorems for exclusive-NOR

$X \odot 0 = X'$	
$X \odot 1 = X$	
$X \odot X = 1$	
$X \odot X' = 0$	
$X \odot Y = Y \odot X$	(commutative law)
$(X \odot Y) \odot Z = X \odot (Y \odot Z) = X \odot Y \odot Z$	(associative law)
$(X \odot Y)' = X \oplus Y = X'Y + XY'$	

* Equivalence operation ($\equiv$):

$0 \equiv 0 = 1$

$0 \equiv 1 = 0$

$1 \equiv 0 = 0$

$1 \equiv 1 = 1$

(*) $X \equiv Y = 1$ iff $X = Y$

(*) Equivalence gate is also called NOR gate (exclusive NOR Gate)

NOR Gate: