PSFML Session 2 Homework

Problem 1: Expected Value of Soccer Days

Applying the Formula:

- Let's list out the values and their probabilities:
  - $x_1=0$ $P[X=0]=0.2$
  - $x_2=1$, $P[X=1]=0.5$
  - $x_3=2$, $P[X=2]=0.3$
- Now, compute each term:
  - $0\times0.2=0$
  - $1\times0.5=0.5$
  - $2\times0.3=0.6$
- Summing these up:
- $E[X]=0+0.5+0.6=1.1$

Problem 2: Mean and Standard Deviation of Even Numbers on Die Rolls

1. **Identifying Possible Values of X:**
   - X can be 0, 1, or 2.
     - X=0: Both rolls are odd.
     - X=1: One roll is even, and the other is odd.
     - X=2 Both rolls are even.

1. **Calculating Probabilities for Each Value of XX:**
   - determine the probability of rolling an even number on a single die:
     - There are 3 even numbers (2, 4, 6) out of 6 possible outcomes
     - So, **P(Even)=3/6 =0.5**
     - Similarly, **P(Odd)=0.5**
   - calculate the probabilities for each value of X:
     - $P(X=0)$: Both rolls are odd.
     - **P(X=0)=P(Odd)×P(Odd)=0.5×0.5=0.25**
     - $P(X=1)$: One even and one odd. There are two scenarios:
       - First roll even, second roll odd.
       - First roll odd, second roll even.
     - **P(X=1)=2×(P(Even)×P(Odd))=2×(0.5×0.5)=0.5**
     - $P(X=2)$: Both rolls are even.
     - **P(X=2)=P(Even)×P(Even)=0.5×0.5=0.25**

Calculating the Mean μμ:

E(X)= $\Sigma(x_i \cdot P(X=x_i))=0\times0.25+1\times0.5+2\times0.25=0+0.5+0.5=1$

Calculating the Variance σ2σ2:

$\sigma^2=E(X^2)-[E(X)]^2$

E(X2)= $\Sigma(x_i^2 \cdot P(X=x_i))=0^2\times0.25+1^2\times0.5+2^2\times0.25=0+0.5+1=1.5$

$\sigma^2=1.5-(1)^2=1.5-1=0.5$

$\sigma=0.5\approx0.7071$

Problem 3: Expected Values Involving Random Variable

1. **Part (a): E[2X−4]**
   - **Calculation:**
   - $E[2X-4]=2E[X]-4=2(1)-4=2-4=-2$
   - **Conclusion:**
   - $E[2X-4]=-2$

2. **Part (b): E[X2]**
   - We know $E[X]=1$ but we need $E[X^2]$
   - Recall that $var(X)=E[X^2]-(E[X])^2$
   - Given $var(X)=4$ and $E[X]=1$
   - **$4=E[X^2]-(1)^2 \Rightarrow E[X^2]=4+1=5$**
3. **Part (c): E[(2X−4)2]E[(2X−4)2]**
   - **$(2X-4)^2=4X^2-16X+16$**
   - **$E[4X^2-16X+16]$**
   - **$E[4X^2-16X+16]=4(5)-16(1)+16=20-16+16=20$**

Problem 4:

1. **Part 1**

   $$P(X=x)=\sum_y P(X=x,Y=y)$$

   - **For X=0**
   - **P(X=0)=P(X=0,Y=0)+P(X=0,Y=1)=.**
   - **For X=1**
   - **P(X=1)=P(X=1,Y=0)+P(X=1,Y=1)=0.10+0.40=0.50**

$$P(Y=y)=\sum_x P(X=x,Y=y)$$

- **For Y=0**
- **P(Y=0)=P(X=0,Y=0)+P(X=1,Y=0)=0.45+0.10=0.55**
- **For Y=1**
- **P(Y=1)=P(X=0,Y=1)+P(X=1,Y=1)=0.05+0.40=0.45**

2. **Part (ii): Are *X* and *Y* Independent?**
2. Two random variables X and Y are independent if:
2. **$P(X=x,Y=y)=P(X=x) \cdot P(Y=y)$ for all x,y**
2. **P(X=0,Y=0)=0.45**
2. **$P(X=0) \cdot P(Y=0)=0.50 \cdot 0.55=0.275$**
2. Since $0.45\neq0.2750$ X and Y are **not independent**.

3. **Part (iii): Find the Mean and Variance of X**
3. **$E[X]=\sum_x x \cdot P(X=x)$**
3. **$E[X]=0 \cdot 0.50+1 \cdot 0.50=0+0.50=0.50$**
3. **$Var(X)=E[X^2]-(E[X])^2$**
3. **$E[X^2]=\sum_x x^2 \cdot P(X=x)$**
3. **$E[X^2]=0^2 \cdot 0.50+1^2 \cdot 0.50=0+0.50=0.50$**
3. **$Var(X)=0.50-(0.50)^2=0.50-0.25=0.25$**
4. **Part (iv): Find the Mean and Variance of Y**
   - E[Y]=0.45$E[Y]=0.45$, Var(Y)=0.2475$Var(Y)=0.2475$