

Question 1:

X is a discrete random variable with a given PMF:

[a]

X={0.2,0.4,0.5,0.8,1}

[b] Find

P(X≤0.5):

Add the probabilities where X≤0.5:

P(X≤0.5)=P(X=0.2)+P(X=0.4)+P(X=0.5)=0.1+ 0.2+ 0.2=0.5

[c] Find P(0.25<X<0.75):

P(0.25<X<0.75)=P(X<0.75)-P(X<0.25)=P(X=0.4)+P(X=0.5)  
0.2 +0.2=0.4

#####

##Question 2:

X is a discrete random variable with a given PMF:

#####

[a] Find E[X]E[X]E[X] (expected value):Use the formula:

E[X]= ∑ Xi⊗P(Xi)  
E[X]=[0.2][0.1]+[0.4][0.2]+[0.5][0.2]+[0.8][0.3]+[1][0.2]  
E[X]=0.02+0.08+0.1+0.24+0.2=0.64.

[b] Find Var[X]Var(X)Var(X) (variance):Use the formula:

Var(X)=E[X2]−[E[X]]2  
=[0.2\*0.2][0.1]+[0.4\*0.4][0.2]+[0.5\*0.5][0.2]+[0.8\*0.8][0.3]+[1][0.2].  
E[X2]=0.004+0.032+0.05+0.192+0.2=0.478.  
Var(X)=E[X2]−[E[X]]2  
=0.478−[0.64]2=0.478−0.4096=0.0684

[c] E[Y]

Y=[X−2]2  
E[Y]= ∑ P(Xi)⊗(Xi−2)2.  
E[Y]=0.324+0.512+0.45+0.432+0.2=1.918.

#####



##Question 3:

#####

Var(2X−Y)=4Var(X)+Var(Y)=6  
Var(X+2Y)=Var(X)+4Var(Y)=9.  
Let Var(X)=a and Var(Y)=b  
4a+b=6, a+4b=9.  
16a+4b=24,  
15a=15⊗a=1  
4(1)+b=6⊗b=2  
Var(X)=1,Var(Y)=2.

#####

##Question 4:

#####

P(X=k)={20k}⊗0.5k⊗[1−0.5]20−k  
P(X > 15) = P(X = 16) + P(X = 17) + P(X = 18) + P(X = 19) + P(X = 20)

#####

##Question 4:

#####

P(X=11)=0.0663  
P(X=12)=0.0829  
P(X=13)=0.0956  
P(X=14)=0.1024  
P(X=15)=0.1024  
P(10<X≤15)=P(X=11)+P(X=12)+...+P(X=15).  
P(X=k)=k!λke−λ.  
P(10<X≤15)=0.0663+0.0829+0.0956+0.1024+0.1024=0.4496.

#####

##Question 6:

#####

Given the CDF F(X):

- 1. P(X=5)=F(5)−F(4)=0.75-.05=0.25 .1
- 2. P(2<X<5)=F(4)−F(2)=0.5-0.25= 0.25
- 3. P(X≥3)=1−F(2)=1-0.25=7

#####

##Question 7:

#####

- 1. Mean:
  - 1. μ=n⊗p=20⊗0.5=10.
- 2. Variance:
  - 2. σ2=n⊗p⊗(1-p)=20⊗0.5⊗0.5=5.