PSFML Session 2 Homework

Problem 1: Expected Value of Soccer Days

Applying the Formula:

- Let's list out the values and their probabilities:
  - $x1=0$ $P[X=0]=0.2$
  - $x2=1$, $P[X=1]=0.5$
  - $x3=2$, $P[X=2]=0.3$
- Now, compute each term:
  - $0\times0.2=0$
  - $1\times0.5=0.5$
  - $2\times0.3=0.6$
- Summing these up:
- $E[X]=0+0.5+0.6=1.1$

Problem 2: Mean and Standard Deviation of Even Numbers on Die Rolls

1. **Identifying Possible Values of X:**
   - X can be 0, 1, or 2.
     - $X=0$: Both rolls are odd.
     - $X=1$: One roll is even, and the other is odd.
     - $X=2$ Both rolls are even.

1. **Calculating Probabilities for Each Value of X*X*:**
   - determine the probability of rolling an even number on a single die:
     - There are 3 even numbers (2, 4, 6) out of 6 possible outcomes
     - So, **P[Even]=3/6 =0.5**
     - Similarly, **P[Odd]=0.5**
   - calculate the probabilities for each value of X:
     - $P(X=0)$: Both rolls are odd.
     - **P[X=0]=P[Odd]×P[Odd]=0.5×0.5=0.25**
     - $P(X=1)$: One even and one odd. There are two scenarios:
       - First roll even, second roll odd.
       - First roll odd, second roll even.
     - **P[X=1]=2×(P[Even]×P[Odd])=2×(0.5×0.5)=0.5**
     - $P(X=2)$: Both rolls are even.
     - **P[X=2]=P[Even]×P[Even]=0.5×0.5=0.25**

Calculating the Mean μμ:

E(X)= Σ (xi⋅P(X=xi))=0×0.25+1×0.5+2×0.25=0+0.5+0.5=1

Calculating the Variance σ2σ2:

σ2=E(X2)−[E(X)]2

E(X2)= Σ (xi2⋅P(X=xi))=02×0.25+12×0.5+22×0.25=0+0.5+1=1.5

σ2=1.5−(1)2=1.5−1=0.5

σ=0.5≈0.7071

Problem 3: Expected Values Involving Random Variable

1. **Part (a): E[2X−4]**
   - **Calculation:**
   - $E[2X-4]=2E[X]-4=2(1)-4=2-4=-2$
   - **Conclusion:**
   - $E[2X-4]=-2$

2. **Part (b): E[X2]**
   - We know E[X]=1 but we need E[X2]
   - Recall that var[X]=E[X2]−(E[X])2
   - Given var[X]=4 and E[X]=1
   - **4=E[X2]−(1)2⋅E[X2]=4+1=5**
3. **Part (c): E[(2X−4)2]*E*[(2*X*−4)2]**
   - **(2X−4)2=4X2−16X+16**
   - **E[4X2−16X+16]**
   - **E[4X2−16X+16]=4(5)−16(1)+16=20−16+16=20**

Problem 4:

1. **Part 1**

   P(X=x)=y Σ P(X=x,Y=y)
   - **For X=0**
   - **P[X=0]=P[X=0,Y=0]+P[X=0,Y=1]=.**
   - **For X=1**
   - **P[X=1]=P[X=1,Y=0]+P[X=1,Y=1]=0.10+0.40=0.50**

P(Y=y)=x Σ P(X=x,Y=y)

- **For Y=0**
- **P[Y=0]=P[X=0,Y=0]+P[X=1,Y=0]=0.45+0.10=0.55**
- **For Y=1**
- **P[Y=1]=P[X=0,Y=1]+P[X=1,Y=1]=0.05+0.40=0.45**

2. **Part (ii): Are *X* and *Y* Independent?**
2. Two random variables X and Y are independent if:
2. **P[X=x,Y=y]=P[X=x]⋅P[Y=y]for all x,y**
2. **P[X=0,Y=0]=0.45**
2. **P[X=0]⋅P[Y=0]=0.50⋅0.55=0.275**
2. Since 0.45≠0.2750 X and Y are **not independent**.

3. **Part (iii): Find the Mean and Variance of X**

3. **E[X]=x** Σ x⋅P(X=x)
3. **E[X]=0⋅0.50+1⋅0.50=0+0.50=0.50**
3. **Var(X)=E[X2]−(E[X])2**
3. **E[X2]=x** Σ x2⋅P(X=x)
3. **E[X2]=02⋅0.50+12⋅0.50=0+0.50=0.50**
3. **Var(X)=0.50−(0.50)2=0.50−0.25=0.25**

4. **Part (iv): Find the Mean and Variance of Y**
   - E[Y]=0.45*E*[*Y*]=0.45, Var[Y]=0.2475Var[*Y*]=0.2475