CONCORDIA UNIVERSITY
FACULTY OF ENGINEEING AND COMPUTER SCIENCE

ENGR 233: Applied Advanced Calculus - Winter 2016
Tuesday, March 1, 2016 Midterm Test (B) Time: 90 Minutes

DIRECTIONS:

- Books, and Notes are NOT allowed. Formula sheet provided. No Calculators.
- All seven questions are equally valued at 10 point each. Maximum mark is 60.
- Solve any six questions or try all seven questions. The best six marks will be recorded.
- Submit all sheets and papers at the end of the test.

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1. Find the equation of the plane P which is parallel to the line L_1: $\frac{x-1}{2} = \frac{y+2}{24} = \frac{z+3}{-2}$, and the line L_2: $x = 3 - 4t,\ y = -1 + t, z = 2 + 3t,$ and contains the point p (3,2,1).

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2. Given the surface S: $x^2 + y^2 + (z-1)^2 = 1$, and the line L: $x = y = z$.
   (a) Find the point p (x, y) of the intersection of surface S and line L other than the origin.
   (b) Find the normal vector $\vec{n}$ to S at the point p.
   (c) Find the angle between the vector $\vec{n}$ and the vector $\vec{a}$ parallel to the line L.

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3. Calculate $\int_C y\,dx + z\,dy + x\,dz$ where C is given by $x = 3t, y = t^3, z = \frac{5}{4}t^2$, $0 \le t \le 2$.

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4. Consider the curve C described by $\vec{r}(t) = \hat{i} + t^2\ \hat{j} + t^3\ \hat{k}$.
   (a) Find the curvature κ at t = 2.
   (b) Find the work done by the force field $\vec{F}(x,y,z) = ye^{z^2}\ \hat{i} + e^{xy}\ \hat{j} + ye^{yz}\ \hat{k}$ along the curve C from $t = 0$ to $t = 2$.

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5. Let $\vec{F}(x,y,z) = (\frac{y}{1+x^2y^2})\hat{i} + (\frac{x}{1+x^2y^2})\hat{j} + z\hat{k}$. (a) Find $\nabla \times \vec{F}$.
   (b) Does there exist a scalar function $f(x,y,z)$ such that $\vec{F} = \nabla f$? Explain your answer. If the answer is "yes", then calculate $f(x,y,z)$.

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6. A vector filed $\vec{F}$ is said to be solenoidal if $\nabla \bullet \vec{F} = 0$. Given the vector field $\vec{F}$:
   $\vec{F} =< axy^2z + cx^2y + (b+c)xz, (b+c)zy^3 - \left(\frac{a}{2} + c\right)xy^2 + 4yz, abcz^2 - (b-a)xyz + 5y^2z^2 >$.
   Determine the values of constants a, b and c such that $\vec{F}$ is solenoidal.

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7. A particle moves in space with the acceleration $\vec{a}(t) =< 2t, 4, 2 >$. At time t = 0, the initial position of the particle is p (1, -3, 3) and the initial velocity is $\vec{v}_o =< 0, 0, -4 >$.
   (a) Find the position vector of the particle at any time t.
   (b) Calculate the times t_1 and t_2 at which the particle pass the xy-plane.
   (c) Calculate at time t_1 and t_2, the particle (i) coordinates, (ii) velocity, (iii) speed, and (iv) acceleration.

$\cos\theta = \vec{a} \bullet \vec{b} / (\| \mathbf{a} \| \| \mathbf{b} \|)$

$\mathbf{comp}_{\mathrm{b}}\vec{\mathrm{a}} = \| \vec{\mathrm{a}} \| \cos\theta = \vec{\mathrm{a}} \cdot \hat{\mathrm{b}}$

$\mathrm{proj}_{\mathrm{b}}\vec{\mathrm{a}} = (\vec{\mathrm{a}} \cdot \hat{\mathrm{b}})\,\hat{\mathrm{b}}$

Area of a parallelogram = $\| \vec{a} \times \vec{b} \|$

Volume of a parallelepiped = $\| \vec{a}.(\vec{b} \times \vec{c}) \|$

Equation of a line : $\vec{r} = \vec{r}_2 + t(\vec{r}_2 - \vec{r}_1) = \vec{r}_2 + t\vec{a}$

Equation of a plane : $a\,\mathbf{x} + b\,\mathbf{y} + c\mathbf{z} + d = 0$

also: $[(\vec{r}_2 - \vec{r}_1) \times (\vec{r}_3 - \vec{r}_1)] \bullet (\vec{r} - \vec{r}_1) = 0$

$$\frac{d\vec{r}(s)}{dt} = \frac{d\vec{r}}{ds}\frac{ds}{dt}$$

Length of a curve : $s = \int_{t_1}^{t_2} \| \vec{r}\,'(t) \| \, dt$

Curvature of a smooth curve: $\kappa = \left\| \frac{d\vec{\mathrm{T}}}{ds} \right\| = \left\| \frac{d^2\vec{\mathrm{r}}}{ds^2} \right\| = \frac{\| \vec{\mathrm{T}}' \|}{\| \vec{\mathrm{r}}' \|} = \frac{\| \mathrm{r}'(t) \times \mathrm{r}''(t) \|}{\| \mathrm{r}'(t) \|^3}$

Acceleration: $\vec{\mathbf{a}}(t) = \kappa v^2 \hat{\mathbf{N}} + \frac{dv}{dt}\hat{\mathbf{T}} = a_N \hat{\mathbf{N}} + a_T \hat{\mathbf{T}}$

$\hat{\mathbf{N}} = \frac{d\mathbf{T}/dt}{\| d\mathbf{T}/dt \|}$

$\hat{\mathbf{T}} = \frac{\mathbf{r}'(t)}{\| \mathbf{r}'(t) \|}$

$a_T = \frac{dv}{dt} = \frac{\vec{\mathrm{v}} \bullet \vec{\mathrm{a}}}{\| \vec{\mathrm{v}} \|}$ & $a_N = kv^2 = \frac{\| \vec{\mathrm{v}} \times \vec{\mathrm{a}} \|}{\| \vec{\mathrm{v}} \|}$

The Binormal $\hat{\mathbf{B}} = \hat{\mathbf{T}} \times \hat{\mathbf{N}}$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} \quad \& \quad \frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y}$$

$$\nabla = \hat{\mathrm{i}}\frac{\partial}{\partial x} + \hat{\mathrm{j}}\frac{\partial}{\partial y} + \hat{\mathrm{k}}\frac{\partial}{\partial z}$$

$D_u(F) = \nabla F \bullet \hat{u}, \ \hat{u} = \text{unit vector}$

Equation of Tangent Plane: $\vec{n}_o \bullet (\vec{r} - \vec{r}_o) = 0, \ \vec{n}_o = \nabla F \text{ at } \mathrm{P}(\vec{r}_o)$

equation of normal line to a surface : $\vec{n}_o \times (\vec{r} - \vec{r}_o) = 0, \ \vec{n}_o = \nabla F$ at P

Also: $x = x_o + t\,F_x(x_o, y_o, z_o), \ y = y_o + t\,F_y(x_o, y_o, z_o),$

$\mathrm{z} = z_o + t\,F_z(x_o, y_o, z_o)$

$$W = \int_C \vec{F} \bullet d\vec{r} = \int_c F_1(x,y,z)dx + \int_c F_2(x,y,z)dy + \int_c F_3(x,y,z)dz$$

$$\int_C F(x,y)ds = \int_a^b F(f(t), g(t))\sqrt{[f']^2 + [g']^2}\, dt = \int_a^b F(x, f(x))\sqrt{1 + [f']^2}\, dx$$