Concordia University
ENGR 371 - Probability and Statistics

Midterm Exam

October 23, 2011

General Instructions

- Answer all five questions.
- All questions have equal weight.
- Show all work. Final answers will not be accepted.
- Make reasonable assumptions if need be.
- You can only use the provided space for your answers.
- Do not detach the sheets.
- ENCS approved calculators may be used only.
- The exam is closed-book, closes-notes.
- Write your name and ID number on all pages.
- Exam time is two hours.

Name: ....
ID: ....

1) (*10 Marks*) In the layout of a printed circuit board for an electronic product, there are 12 different locations that can accommodate chips.
   a) If there are five chips that are different, which are to be placed on the board. How many different layouts are possible? Justify your answer.

$$P^{12}_{5} = \frac{12!}{7!} = 12 \times 11 \times \cdots \times 8 = 95,040$$

reason: they are 5 different chips, and there are 12 locations, so order is important

   b) If the five chips that are placed on the board are of the same type, how many different layouts are possible. Justify your answer.

$$C^{12}_{5} = \frac{12!}{7!\,5!} = 792$$

because the 5 chips are the same, so order is not important.

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2. (*10 Marks*) In an HIV blood test, the sensitivity of the test which is defined by the following probability: $P(\text{test is positive}|\text{disease is present})$ is equal to 95%. There are two possible errors in the test called "false positive" and "false negative" defined as follows: *false positive*: $P(\text{test is positive}|\text{disease is absent})$, *false negative*: $P(\text{test is negative}|\text{disease is present})$. Assuming that in this test the false positive and false negative are equal and that approximately 10% of the population is infected with HIV; find the probability that a tested person is infected given the test result is positive.

Define the following event

A = person has disease (ie, infected)

B = test result is positive

$P(B \mid A) = 0.95$

$P(A) = 0.10 \longrightarrow P(\bar{A}) = 0.90$

$P(B \mid \bar{A}) \Rightarrow$ false positive

$P(\bar{B} \mid A) \Rightarrow$ false negative

Since $P(B \mid A) = 0.95$, $P(\bar{B} \mid A) = 1 - 0.95 = 0.05$

Also, $P(B \mid \bar{A}) = P(\bar{B} \mid A) = 0.05$

$$P(A \mid B) = \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|\bar{A})\,P(\bar{A})} = \frac{0.095}{0.14} = 0.678$$

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3. (*10 Marks*) Grandma bakes chocolate chip cookies in batches of 100. She puts 300 chips into the dough. When the cookies are done, she gives you one.
   a) What is the probability that your cookie contains no chocolate chips?

Assume Poisson Distribution

$$\lambda = \frac{300}{100} = 3$$

Let X be the number of chips on a cookie

$$P(X=0) = \frac{e^{-3}(3)^0}{0!} = 0.0498$$

   b) What is the probability that your cookie has at least two chocolate chips?

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$

$$= 1 - \frac{e^{-3}(3)^0}{0!} - \frac{e^{-3}(3)^1}{1!}$$

$$= 1 - 0.0498 - 0.1494$$

$$= 0.8008$$

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4. (*10 Marks*) Assume that each of your calls to a popular radio station has a probability of 0.02 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent.
   a) What is the probability that your first call that connects is your tenth call?

Geometric distribution

$$P(X=10) = 0.02(0.98)^9$$
$$= 0.01667$$

   b) What is the probability that your second call (assuming you had a follow up comment) that connects is your 20th call?

Negative binomial distribution

$X=20$
$r=2$

$$P(X=20) = \binom{19}{1}(0.02)^2(0.98)^{18}$$
$$= 5.28\times10^{-3}$$

   c) What is the average number of calls needed to connect the first time?

Geometric distribution

$$\mu = \frac{1}{p} = \frac{1}{0.02} = 50$$

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5. (*10 Marks*) The thickness of wood paneling, in millimeters, is a random variable with the following cumulative distribution function (measurements are rounded to the nearest millimeter):

x	1	2	3	4	5
$F(x)$	0	0.25	0.55	0.80	1

$\Rightarrow$

x	1	2	3	~~0.25~~ 4	5
$f(x)$	0	0.25	0.3	0.25	0.2

Determine:

a) $P(X > 3.3)$ $= 1 - P(X \le 3.3)$

$= 1 - F(3) = 1 - 0.55 = 0.45$

b) $P(2 \le X \le 3)$ $= f(2) + f(3)$

$= 0.25 + 0.3 = 0.55$

c) $P(X = 4.0)$ $= f(4) = F(4) - F(3)$

$= 0.80 - 0.55 = 0.25$

d) The expected value and standard deviation of the paneling thickness.

$\mu = \sum_x x f(x) = 1 \times 0 + 2 \times 0.25 + 3 \times 0.3 + 4 \times 0.25 + 5 \times 0.2$

$= 3.4$

$\sigma^2 = \sum_x x^2 f(x) - \mu^2$

$\sum_x x^2 f(x) = 1 \times 0 + 4 \times 0.25 + 9 \times 0.3 + 16 \times 0.25 + 25 \times 0.2 = 12.7$

$\sigma = \sqrt{12.7 - 3.4^2} = \sqrt{1.14} = 1.0677$