MATH 209/2 all sections except EC: - Fundamental Mathematics II

Midterm - Sunday, October 20, 2013, 2pm (1h30min)
Only approved calculators are permitted.

MARKS

[7] . 1. (a) Find $\lim\limits_{x \to -2} \dfrac{x^2 - x - 6}{x^2 + x - 2}$.

[7]
(b) Give examples of functions $g(x)$ and $h(x)$ with the following properties:

(i) $\lim\limits_{x \to 5} g(x) = 0$ (ii) $\lim\limits_{x \to 5} h(x) = 0$ (iii) $\lim\limits_{x \to 5} \dfrac{h(x)}{g(x)} = 0$

[7] 2. Let $h(x) = 7 - x^3$. Work out the following in detail:

$$\lim_{t \to 0} \frac{h(x+t) - h(x)}{t}$$

[12] 3. (a) If $f(x) = 3\sqrt[4]{x^3} - \frac{1}{x^6}$, find $f'(1)$. You need not simplify.

(b) If $g(x) = [2 + ln(x^2)][3x^3 - 5]$, find $g'(2)$. You need not simplify.

(c) Find $h'(x)$ if $h(x) = \frac{x^3 - 3}{x^2 + 7e^x}$. You need not simplify.

(d) Find the value of dy if $y = x^3 - 2$, $x = 3$, and the change in x is 0.1.

[7] 4. A stock portfolio grows from ten thousand dollars to forty thousand dollars in eight years. Find the associated annual rate of growth assuming that it is compounded continuously.

[10] 5. Consider the cost function for the production of headphone sets $C(x) = 7,000 + 2x$.

(a) Find $\overline{C}(x)$ and $\overline{C}'(x)$.

(b) Find $\overline{C}(100)$ and $\overline{C}'(100)$, and interpret these quantities.

(c) Use the results in part (B) to estimate the average cost per headphone set at a production level of 101 headphone sets.

[10] . 6. Find x' for the function $x(t)$ defined implicitly below. Evaluate x' at the indicated point.

$$x^2 - tx^2 - 16 = 0; \quad (-3, -2).$$

[10] 7. Helium is pumped into a spherical balloon at a constant rate of 4 cubic feet per second. How fast is the radius increasing after 1 minute?