ENGR-233 Winter 2021

Sample final exam.

Problem 1. Using the Green's formula, find $\oint_C (x^2-y^2)dx+(x^2+y^2)dy$ if the contour C consists of the segment $x=1, 0\le y\le 1$, the segment $y=1, 0\le x\le 1$, and the portion of the circle $x^2+y^2=1$ in the first quadrant.

Problem 2. Let the vector field $\boldsymbol{F}=(2x-2z)\boldsymbol{i}+z\,\boldsymbol{j}+(y-2x)\boldsymbol{k}$. Find $\int_{(1,2,3)}^{(3,2,1)} \boldsymbol{F}\cdot d\boldsymbol{r}$ along any curve C connecting the points $(1,2,3)$ and $(3,2,1)$.

Problem 3. Find $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}$ if $\boldsymbol{F}(x,y,z)=(z^2, xy, 2y)$, and the contour C consists of the piece of the circle $x^2+y^2=1$, $z=0$ in the first quadrant, and the segment of the line connecting the points $(0,1,0)$ and $(0,0,1)$.

Problem 4. Using the Stokes Theorem, find $\oint_C \boldsymbol{F}\cdot d\boldsymbol{r}$ if $\boldsymbol{F}(x,y,z)=(y,-2x+z,x)$, and the contour C is the intersection of the surface $z=x^2-2y^2$ and the cylinder $x^2+y^2=4$.

Problem 5. Using the Divergence Theorem find the flux of the field $\boldsymbol{F}(x,y,z)=-xy\,\boldsymbol{i}+2yz\,\boldsymbol{j}+3xz\,\boldsymbol{k}$ through the surface of the body B: $x\ge 0$, $y\ge 0$, $z\ge 0$, $x+y+z\le 1$.

Problem 6. Find the moment of inertia I_z relative to the z - axis of the ball of radius R with a cylindrical hole of radius a whose axis is the z - axis if the density od the ball is δ.

Problem 7. Using the change of variables, find $\iint_D x\,y\,dA$ if D is the domain bounded by the curves $y=a_1x^2$, $y=a_2x^2$, $y=b_1x$, $y=b_2x$ ($a_1<a_2$, $b_1<b_2$).

Problem 8. The *cycloid* is the curve traced by a point on the wheel rolling along the x - axis. Its parametric equations are

$$x=t-\sin(t/R)\ ,\quad y=R(1-\cos(t/R))\ .$$

(a) Find the curvature of the cycloid as a function of t .

(b) What happens with the curvature as $t\to 0$?

Problem 9. Find the volume of the domain B defined by the inequalities $B:\ x^2+y^2+z^2\le 1,\ x^2+y^2-z^2\ge 0$.