CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	**Number**	**Section(s)**
MATH	204	All except EC
Examination	**Date**	**Pages**
Final	December 2018	2
Instructors		**Course Examiner**
All		E. Cohen

Special Instructions:

- Only approved calculators are allowed
- Justify all your answers.
- All questions have equal value.

1. Use Cramer's Rule to compute the solution of the system:

$$\begin{aligned} x_1 \quad + \quad x_2 \qquad\qquad &= 3 \\ -3x_1 \qquad\qquad + \quad 2x_3 &= 0 \end{aligned}$$

2. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}$, if it exists.

3. Find all solutions of the system:

$$x_1 + 6x_2 + 2x_3 - 5x_4 - 2x_5 = -4$$
$$2x_3 - 8x_4 - x_5 = 3$$
$$x_5 = 7$$

4. Find the determinant of $A = \begin{pmatrix} 1 & 3 & 0 & 5 \\ 2 & 7 & 1 & 3 \\ 1 & 2 & 1 & 6 \\ 2 & 3 & 4 & 5 \end{pmatrix}$

5. a,) Let $u = (1,5,-2)$, $v = (3,-1,5)$. Find the orthogonal projection of v on u.

   b.) Let $u_1 = (2, 3, 1)$, $u_2 = (3, 1, 2)$, $u_3 = (1, 2, 3)$. Find c_1, c_2, c_3 such that

   $c_1u_1 + c_2u_2 + c_3u_3 = (1, 0, 1)$.

6. a.) Find the area of a triangle with vertices (1, 1, 2), (0, 1, 4), (1, 2, 5).

   Find a vector orthogonal to the plane of the triangle.

   b.) Find the distance between the point (2, -3) and the line $2x = 3y + 4$.

7. a.) Are the vectors (2, -2, 1), (1, -3, 2), (-7, 5, 4) linearly dependent or independent?

   b.) Find the parametric equations for the line in $\mathbb{R}^3$ passing through (1, 4, 5) and perpendicular to the plane $2x - 4y + 3x = 1$.

8. Let $A = \begin{bmatrix} 1 & 3 & 4 & 0 & 2 & 0 & 7 \\ 0 & 0 & 0 & 1 & 6 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 \end{bmatrix}$ $and\ X = \begin{bmatrix} x \\ y \\ z \\ t \\ u \\ v \\ w \end{bmatrix}$.

   Find a bases for the solution space of the homogenous system $AX = 0$.

9. Find the standard matrices for the following equation on $\mathbb{R}^2$:

   a) a rotation clockwise of $45°$

   b) a reflection about line $y = -x$

10. Let $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$. Find a matrix P such that $P^{-1}AP = D$, a diagonal matrix.