PHYS 205-Section 03
Electricity and Magnetism - Winter 2018
Assignment 1 - Solution

Problems and Solutions

1. Assume a classical hydrogen atom, in which the electron orbits the proton in a circular path due to the Coulomb's force. Taking the mass of electron to be $9\times10^{-31}kg$ and the radius of the orbit to be $5.2\times10^{-11}\ m$, calculate the speed of the electron in its orbit. **(5 points)**
Hint: You need to know circular motion from PHYS 204.

Solution:
To calculate the speed of the electron, we use Newton's 2nd law in circular motion. The force applied on the electron is the Coulomb force, which is responsible for the centripetal acceleration:

$$\sum F = ma_c = m\frac{v^2}{r}$$

$$k_e\frac{Qq}{r^2} = m\frac{v^2}{r}$$

$$v = \sqrt{k_e\frac{Qq}{mr}} = \sqrt{9\times10^9\frac{(1.6\times10^{-19})^2}{9\times10^{-31}(5.2\times10^{-11})}} = 2.2\times10^6\ \left(\frac{m}{s}\right)$$

2. The gravitational force close to the surface of the Earth is described by $F_g = mg$, where $g = 9.8\ m/s^2$. If we want an electron to float very close to the surface of the Earth, what should be the sign and magnitude of the charge we should place at the center of the Earth? Take the mass of electron $9\times10^{-31}kg$ and the radius of the Earth to be 6370 km. **(5 points)**

Solution:
For the electron to float, there should be no net force applied on it. The forces applied on the electron are the gravitational force of the Earth and the electric force due to the charge placed at the center of the Earth (Q). Since the electric force is repulsive, Q should be negative. Here, we calculate the magnitude of Q.

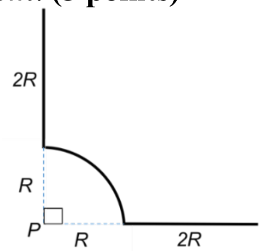
$$\sum F = 0$$

$$k_e\frac{Qq}{r^2} - mg = 0$$

$$Q = \frac{mgr^2}{k_eq} = \frac{(9\times10^{-31})(9.8)(6.37\times10^6)^2}{(9\times10^9)(1.6\times10^{-19})} = 2.5\times10^{-7}\ \ (C)$$

Pay attention to the magnitude of this charge (source of electric force) as compared to the mass of the Earth $\approx 6\times10^{24}\ kg$ (source of gravitational force), which applies the same (magnitude) force on the electron!

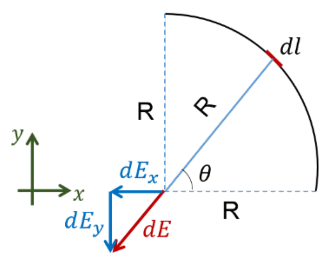
3. In the figure below, the rod is uniformly charged with $\lambda = +3\times10^{-3}\,\frac{C}{m}$. Find the net electric field at point P. $R = 20\ cm$. **(5 points)**



Solution:
Breaking the rod into 3 parts (two straight segments of length 2R and a curved segment of length $\frac{\pi R}{2}$ (one-quarter of a circle), we calculate the electric field of each segment at point P and add them (vector sum) to find the net field.
The electric field of a straight rod has been calculated in your textbook (Example 23.7). So for the straight segments we'll have:

$$\vec{E}_1 = k_e\lambda\left(\frac{1}{R} - \frac{1}{3R}\right)(-\hat{\imath}) = \frac{2k_e\lambda}{3R}\ (-\hat{\imath})$$
$$\vec{E}_2 = k_e\lambda\left(\frac{1}{R} - \frac{1}{3R}\right)(-\hat{\jmath}) = \frac{2k_e\lambda}{3R}\ (-\hat{\jmath})$$

Now for the curved segment, we take an element of length dl, with charge dq and find the *x* and *y*-components of its electric field $d\vec{E}$. Note that all the elements are at the same distance of R from point P. Also we take $dq = \lambda dl = \lambda R d\theta$ where I used the arc length equation for dl.



$$dE_3 = k_e\frac{dq}{r^2} = k_e\frac{\lambda dl}{R^2} = k_e\frac{\lambda R d\theta}{R^2}$$
$$d\vec{E}_{3x} = dE\cos\theta\ \ (-\hat{\imath})$$
$$d\vec{E}_{3y} = dE\sin\theta\ \ (-\hat{\jmath})$$
$$E_x = \int_0^{90} dE\cos\theta = k_e\frac{\lambda}{R}\int_0^{90}\cos\theta d\theta = k_e\frac{\lambda}{R}$$

By symmetry, the *y*-component of the electric field has the same magnitude: $E_y = k_e\frac{\lambda}{R}$.
So we'll have:

$$\vec{E}_x = \vec{E}_{1x} + \vec{E}_{2x} + \vec{E}_{3x} = k_e\lambda\left(\frac{2}{3R} + \frac{1}{R}\right)(-\hat{\imath}) = \frac{5}{3}k_e\frac{\lambda}{R}\ (-\hat{\imath})$$
$$\vec{E}_y = \vec{E}_{1y} + \vec{E}_{2y} + \vec{E}_{3y} = k_e\lambda\left(\frac{2}{3R} + \frac{1}{R}\right)(-\hat{\jmath}) = \frac{5}{3}k_e\frac{\lambda}{R}\ (-\hat{\jmath})$$
$$\vec{E} = \vec{E}_x + \vec{E}_y = -2.25\times10^{8}\ (\hat{\imath} + \hat{\jmath})\ \left(\frac{N}{C}\right)$$

4. A point charge $q = 2\times10^{-3}\ C$ is placed at the center of a cube with length $a = 25\ cm$. What is the flux passing through one side? **(5 points)**

Solution:
We use Gauss's law to calculate the flux passing through the cube:

$$\phi_E = \frac{q}{\epsilon_0}$$

Since the charge is at the center of the cube, by symmetry, same flux passes through the 6 faces of the cube. Hence, the flux passing through one face is:

$$\phi_E = \frac{q}{6\epsilon_0} = \frac{2\times10^{-3}}{(6)(8.85\times10^{-12})} = 3.8\times10^{7}\left(\frac{Nm^2}{C}\right)$$

Note that this does not depend on the length of the side of the cube (a).

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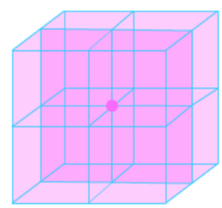
5. A point charge $q = 2\times10^{-3}\ C$ is placed at the corner of a cube with length $a = 25\ cm$. What is the flux passing through one side? **(5 points)**

Solution:
Since the charge is at the corner of the cube, all the electric field lines coming out of the point charge won't pass through the cube. But if we had 7 more cubes (creating a large cube with 8 cubes, as shown in the figure) and then place the charge at the center of this big cube, the electric flux passing through it would be: $\phi_E = \frac{q}{\epsilon_0}$



So the flux passing through 1 cube will be:

$$\phi_E = \frac{q}{8\epsilon_0}$$

Now this cube has 6 faces, but 3 of these faces are adjacent to the point charge (the point charge is placed where 3 faces of the cube meet). There will be no electric flux passing through these 3 faces as the electric field lines will be parallel to these 3 faces. So to find the flux passing through one side, we divide the total flux passing through the cube by 3:

$$\phi_E = \frac{q}{24\epsilon_0} = \frac{2\times10^{-3}}{(24)(8.85\times10^{-12})} = 9.4\times10^{6}\left(\frac{Nm^2}{C}\right)$$

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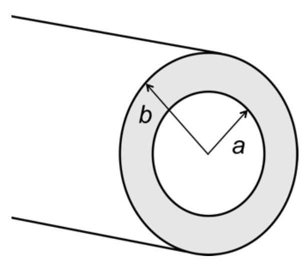
6. The volume charge density of the hollow cylinder of length 25 cm, shown in the figure below, is $\rho = 4\times10^{-5}\ \frac{C}{m^3}$. If $a =$ 10 cm and $b = 15\ cm$. Find the electric field at:
   a) $r = 5\ cm$ **(2 points)**
   b) $r = 12\ cm$ **(2 points)**
   c) $r = 25\ cm$ **(2 points)**



Solution:

We use Gauss's law to do this problem.

a) We choose a Gaussian cylinder with radius of $r = 5\ cm$ and arbitrary length of l.

$$\phi_E = \int \vec{E}.d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

Since $q_{in} = 0$ inside this hypothetical cylinder, then $\vec{E} = 0$.

b) We choose a Gaussian cylinder with radius of $r = 12\ cm$ and arbitrary length of l. We then apply Gauss's law:

$$\phi_E = \int \vec{E}.d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

Ignoring the edge effects of the cylinder, the electric field $\vec{E}$, which goes radially outward, will be parallel to the surface area vector $d\vec{A}$ for every element of the surface. Hence:

$$\vec{E}.d\vec{A} = EdA$$

The electric field will be constant on the surface of the cylinder, so:

$$\int EdA = EA = E(2\pi rl)$$

Now all we need to do is to calculate q_{in}. To do so, we use the volume charge density:

$$q_{in} = \rho V_G = \rho\pi(r^2 - a^2)l = l\pi(1.76\times10^{-7})(C)$$

$$E(2\pi rl) = \frac{q_{in}}{\epsilon_0} \to E = \frac{q_{in}}{2\pi\epsilon_0 rl} = 8.3\times10^4 \left(\frac{N}{C}\right)$$

c) We choose a Gaussian cylinder with radius of $r = 15\ cm$ and arbitrary length of l. In this case, $q_{in} = Q$, which is the total charge of the hollow cylinder:

$$Q = \rho V = \rho\pi(b^2 - a^2)l = l\pi(5\times10^{-7})(C)$$

$$E(2\pi rl) = \frac{Q}{\epsilon_0} \to E = \frac{Q}{2\pi\epsilon_0 rl} = 1.9\times10^5 \left(\frac{N}{C}\right)$$