Concordia University
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COMM 225 FALL 2012: SOLUTION TO REVIEW QUESTIONS

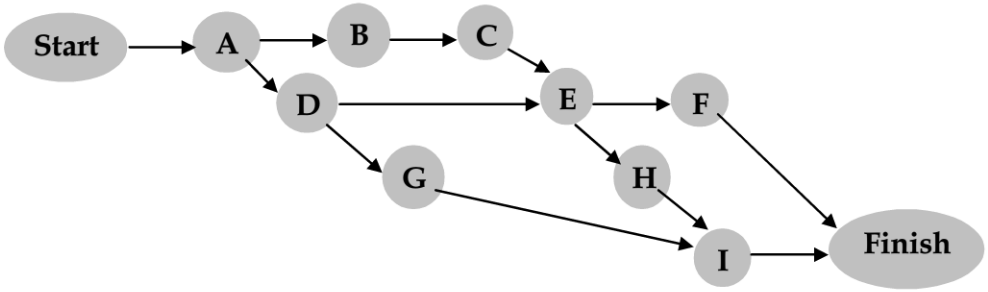
TOPIC: PROJECT MANAGEMENT

Q1.1: *An important client of RoboTech has asked for a redesign and redevelopment of one of RoboTech's industrial robots. The client wants to see a demonstration, a technical proposal and a cost proposal as soon as possible. The project team at RoboTech has identified the tasks involved as indicated in the table below. The activities and their predecessors are listed as follows:*

Activity	*Description of the activity*	*Predecessor*	*Duration (days)*
A	*designing the robot*	*-*	*20*
B	*Building the prototype*	*A*	*10*
C	*testing the prototype*	*B*	*8*
D	*estimating the material costs*	*A*	*11*
E	*refining the design*	*C,D*	*7*
F	*demonstration*	*E*	*6*
G	*estimating labor*	*D*	*12*
H	*preparing the technical proposal*	*E*	*13*
I	*submission of the proposal*	*G,H*	*5*

(a) *List all the paths and their durations.*
(b) *Identify the critical path(s) (by inspection).*
(c) *Determine the critical path(s) using early start, early finish, late start and late finish times. Also identify all the critical and non-critical activities.*

Solution:



(a) *The paths and their durations:*

Start- A –B – C – E –H- I – Finish,	**Duration = 20+10+8+ 7+ 13+5 = 63 (Longest)**
Start- A –B – C – E –F – Finish,	Duration = 20+10+8+ 7+ 6 =51
Start- A –D – G – I – Finish,	Duration = 20+11+12+ 5 = 48
Start- A –D – E – H – I – Finish,	Duration = 20+11+ 7+ 13+5 = 56
Start- A –D – E – F – Finish,	Duration = 20+11+6+ 7 = 44

Critical path(s): The path with longest duration is Start- A –B – C – E –H- I – Finish with a duration of 20+10+8+ 7+ 13+5 = 63 days. Hence, this is the critical path.

(b) *Critical path(s) (using ES, EF, LS, and LF).*

Activity	*Predecessor*	*Duration (days)*	*Early Start*	*Early Finish*	*Late Start*	*Late Finish*	*Slack*
A	-	20	0	20	20-10 = 0	20	0
B	A	10	20	20+10=30	30-10= 20	30	0
C	B	8	30	30+8=38	38-8 = 30	38	0
D	A	11	20	20+11=31	38-11= 27	min(38, 46) =	7

						38	
E	C,D	7	max(38,31) = 38	38+7=45	45-7 = 38	min(57,45) =45	0
F	E	6	45	45+6=51	63-6 = 57	63	12
G	D	12	31	31+12=43	58-12=46	58	15
H	E	13	45	45+13=58	58-13=45	58	0
I	G,H	5	max(43,58) = 58	58+5=63	63-5= 58	63	0

For the table above, we know that activity A, B, C, E, H and I are critical activities (because the slack is zero). This implies that these activities must be completed in time so that the project is completed by 63rd day. And the path containing these activities is the critical path (i.e. Start- A –B – C – E –H- I – Finish). The other activities such as D, F, and G are noncritical as they have positive slacks.

Q1.2: *Refer to the following table which lists the activities of a project with completion time estimate for each activity.*

Activity	*Predecessor*	*Time Estimates (Weeks)*		
		a	*m*	*b*
1	*-*	*8*	*8*	*8*
2	*1*	*11*	*12*	*13*
3	*2*	*9*	*10*	*12*
4	*3*	*5*	*7*	*10*
5	*1*	*5*	*6*	*7*
6	*5*	*14*	*18*	*26*
7	*6*	*8*	*10*	*14*
8	*5*	*13*	*13*	*13*
9	*8*	*10.5*	*13*	*15.5*
10	*1*	*10*	*12*	*14*
11	*10*	*7*	*10*	*12*
12	*11*	*10*	*11*	*12*
13	*12*	*6*	*6*	*6*

(a) Calculate the expected duration of each activity. Also, calculate the expected duration of each path.

(b) Calculate the variance of each activity and variance of each path. Also, calculate the standard deviation of each activity and the standard deviation of each path.

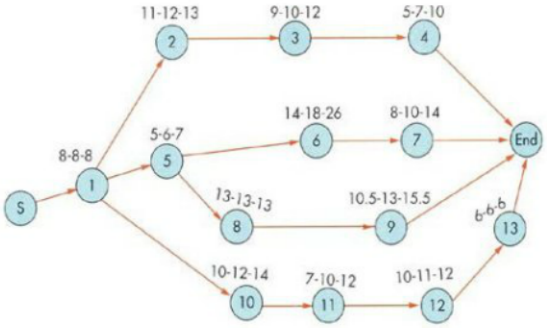
(c) Identify the critical path(s) (based on the expected durations).

(d) What is the probability that the project can be completed within 49 weeks? What is the probability that the project will NOT be completed within 49 weeks?

(e) What is the probability that the project will require 47 weeks or more? What is the probability that the project will NOT require more than 47 weeks?

(f) What is the probability that the project will be completed between 45 and 50 weeks?

Solution:



(a) and (b)

Activity	Mean (Expected Duration)	Variance	Standard Deviation
1	8	0	0
2	12	4/36	0.3334
5	6	4/36	0.3334
10	12	16/36	0.6667
3	10.17	9/36	0.5
6	18.67	144/36	2
8	13	0	0
11	9.83	25/36	0.8333
4	7.17	25/36	0.8333
7	10.33	36/36	1
9	13	25/36	0.8333
12	11	4/36	0.3334
13	6	0	0

Path	Mean (μ)	Var.	Std. dev. (σ)
1-2-3-4	37.34	38/36	1.027
1-5-6-7	43	184 /36	2.26
1-5-8-9	40	29/36	0.898
1-10-11-12-13	46.83	45/36	1.118

(c) *Identify the critical path(s) (based on the expected durations).*

Critical path is 1-10-11-12-13 with an expected duration/mean of 46.83 weeks.

(d) *What is the probability that the project can be completed within 49 weeks? What is the probability that the project will not be completed within 49 weeks?*

Caution: In order to calculate the probability of project-completion time, we will use the critical path(s) only. Note that this procedure on only approximate. Because there can be non-critical paths with high activity variances that might have the lower probability of completion time as compared to the critical path.

Path	Mean (μ)	Var.	Std. dev. (σ)	$z_{49} = \frac{49 - \mu}{\sigma}$	Probability
1-2-3-4	37.34	38/36	1.027	11.35	1
1-5-6-7	43	184 /36	2.26	2.65	0.9960
1-5-8-9	40	29/36	0.898	10.02	1
1-10-11-12-13	**46.83**	**45/36**	**1.118**	**1.94**	**0.9738**

- Probability that the project can be completed within 49 weeks =0.9738.
- Probability that the project will not be completed within 49 weeks = 1- 0.9738 = 0.0262

(e) *What is the probability that the project will require 47 weeks or more? What is the probability that the project will not require more than 47 weeks?*

Path	Mean (μ)	Var.	Std. dev. (σ)	$z_{47} = \frac{47 - \mu}{\sigma}$	Probability
1-2-3-4	37.34	38/36	1.027	9.406	1
1-5-6-7	43	184 /36	2.26	1.769	0.9608
1-5-8-9	40	29/36	0.898	7.795	1
1-10-11-12-13	**46.83**	**45/36**	**1.118**	**0.152**	**0.5596**

Probability that the project will NOT require more than 47 weeks = Probability that the project can be completed within 47 weeks=1- 0.5596 = 0.4404.

Probability that the project will require 47 weeks or more = Probability that the project will not be completed within 47 weeks = 1- 0.4404= 0.5596.

(f) *What is the probability that the project will be completed between 45 and 50 weeks?*

Path	Mean	Var.	Std. dev. (σ)	Z_{45}	P<=45	Z_{50}	P<=50
1-10-11-12-13	46.83	45/36	1.118	-1.64	1-0.9495= 0.0505	2.84	0.9977

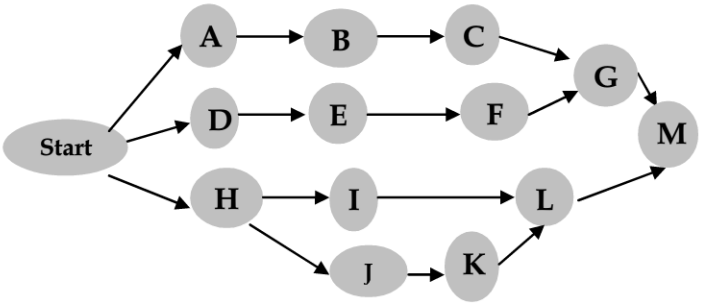
- Probability that the project can be completed within 45 weeks=0.0505
- Probability that the project can be completed within 50 weeks=0.9977
- Probability that the project will be completed between 45 and 50 weeks = Probability that the project can be completed within 50 weeks - Probability that the project can be completed within 45 weeks = 0.9977- 0.0505 = 0.9472.

Q1.3: *Fantasy products marketing manager has recently learned that its competitor is also in the process of developing a similar (new) product, which it intends to bring out to the market at exactly the same time and at lower price. The manager needs to decide whether to introduce the new product to the market 18 weeks from now. As a project management specialists, answer the following questions to help the marketing manager make a decision:*

(a) When would the project be completed using normal durations?
(b) Is it possible to complete the project in 18 weeks? What would the additional costs be? Which activities would need to the completed on a crash basis?
(c) Crash the project as much as possible. What is the optimum number of weeks to crash?
(d) Furthermore, market research has shown that if the early introduction is accomplished, it will bring a profit of $10,000 per week. Would you recommend some time frame shorter than 18 weeks? Justify your answer with appropriate calculation.

Activity	Predecessors	Normal Duration (Weeks)	Crash Duration (Weeks)	Normal Costs $	Crash Costs $
A	-	3	2	2,000	4,000
B	A	8	6	9,000	12,000
C	B	4	2	2,000	7,000
D	-	2	1	1,000	2,000
E	D	2	1	2,000	3,000
F	E	5	5	0	0
G	C, F	6	3	12,000	24,000
H	-	4	2	3,500	8,000
I	H	4	3	5,000	8,000
J	H	3	2	8,000	15,000
K	J	4	3	50,000	70,000
L	I, K	6	6	10,000	10,000
M	G, L	1	1	5,000	5,000

Solution:



Activity	Predecessors	Normal Duration (Weeks)	Crash Duration (Weeks)	Weeks can be Crashed	Normal Costs $	Crash Costs $	Crash Costs/ Weeks
A	-	3	2	1	2,000	4,000	2,000
B	A	8	6	2	9,000	12,000	1,500
C	B	4	2	2	2,000	7,000	2,500
D	-	2	1	1	1,000	2,000	1,000
E	D	2	1	1	2,000	3,000	1,000
F	E	5	5	-	0	0	-
G	C, F	6	3	3	12,000	24,000	4,000
H	-	4	2	2	3,500	8,000	2,250
I	H	4	3	1	5,000	8,000	3,000
J	H	3	2	1	8,000	15,000	7,000
K	J	4	3	1	50,000	70,000	20,000
L	I, K	6	6	-	10,000	10,000	-
M	G, L	1	1	-	5,000	5,000	-

(a) The normal duration of the project is **22 weeks** and the normal total cost is **$109,500.**

Path#	Path	Duration (Weeks)
1	Start-A-B-C-G-M-End	**3+8+4+6+1=22 weeks - Critical path**
2	Start-D-E-F-G-M-End	2+2+5+6+1=16
3	Start-H-I-L-M-End	4+4+6+1=15
4	Start-H-J-K-L-M-End	4+3+4+6+1=18

(b) Yes, it is possible to finish within 18 weeks. Some activities need to be crashed and additional crash cost will be incurred. The calculations are shown below:

Step 1:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	22	A	1	2,000
		B	2	1,500
		C	2	2,500
		G	3	4,000

- Crash activity B for 2 weeks at a crashing cost of $1500 x 2 =$3,000.
- Now, the duration of the A-B-C-G-M is 20 days.
- The resulting duration of the paths are as follows:

Path	Duration (Weeks)
Start-A-B-C-G-M-End	3+6+4+6+1=20*
Start-D-E-F-G-M-End	2+2+5+6+1=16
Start-H-I-L-M-End	4+4+6+1=15
Start-H-J-K-L-M-End	4+3+4+6+1=18

Step 2:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	20	A	1	2,000
		C	2	2,500
		G	3	4,000

- Crash activity A for one day at a crashing cost of $2,000.
- The resulting duration of the paths are as follows:

Path	Duration (Weeks)
Start-A-B-C-G-M-End	2+6+3+6+1=19*
Start-D-E-F-G-M-End	2+2+5+6+1=16
Start-H-I-L-M-End	4+4+6+1=15
Start-H-J-K-L-M-End	4+3+4+6+1=18

Step 3:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	20	C	2	2,500
		G	3	4,000

- Crash activity C for one week at a crashing cost of $2,500.
- Now, the duration of path A-B-C-G-M is 18 weeks. There are two critical paths:

Path	Duration (Weeks)
Start-A-B-C-G-M-End	2+6+3+6+1=18*
Start-D-E-F-G-M-End	2+2+5+6+1=16
Start-H-I-L-M-End	4+4+6+1=15
Start-H-J-K-L-M-End	4+3+4+6+1=18*

Project Length	Cumulative weeks shortened	Crashing costs per week	Cumulative crashing costs	Cumulative total costs
22	0	0	0	**$109,500**
21	1	$1,500	$1,500	**$111,000**
20	2	1,500	3,000	**$112,500**
19	3	2,000	5,000	**$114,500**
18	4	2,500	7,500	**$117,000**

(c)If we continue to further crash the project, we have:

Step 4:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	18	C	1	2,500
		G	3	4,000
H-J-K-L-M	18	H	2	2,250
		J	1	7,000
		K	1	20,000

- Crash activity C and H for one week at a crashing cost $2,500+$2,250 = $4,750
- When crashing H, path H-I-L-M is also crashed by one week to 14 weeks.

Path	Duration (Weeks)
Start-A-B-C-G-M-End	17*
Start-D-E-F-G-M-End	16
Start-H-I-L-M-End	14 (due to H)
Start-H-J-K-L-M-End	17*

Step 5:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	17	G	3	4,000
H-J-K-L-M	17	H	1	2,250
		J	1	7,000
		K	1	20,000

- Crash activity G and H for one week at a crashing cost of $4,000+$2,250 = $6,250
- When crashing G, path D-E-F-G-M is also crashed by one week to 15 weeks.
- When crashing H, path H-I-L-M is also crashed by one week to 13 weeks.

Path	Duration (Weeks)
Start-A-B-C-G-M-End	16*
Start-D-E-F-G-M-End	15 (due to G)
Start-H-I-L-M-End	13 (due to H)
Start-H-J-K-L-M-End	16*

Step 6:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	16	G	2	4,000
H-J-K-L-M	16	J	1	7,000
		K	1	20,000

- Crash activity G and J for one week at a crashing cost of $11,000.
- When crashing G, path D-E-F-G-M is also crashed by one week to 14 weeks.

Path	Duration (Weeks)
Start-A-B-C-G-M-End	15*
Start-D-E-F-G-M-End	14 (due to G)
Start-H-I-L-M-End	13
Start-H-J-K-L-M-End	15*

Step 7:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	15	G	1	4,000
H-J-K-L-M	15	K	1	20,000

- Crash activity G and K for one week at a crashing cost of 24,000 to reduce to 14 weeks.

Path	Duration (Weeks)
Start-A-B-C-G-M-End	14*
Start-D-E-F-G-M-End	13 (due to G)
Start-H-I-L-M-End	13
Start-H-J-K-L-M-End	14*

- Now, path A-B-C-G-M cannot be crashed any more.

Project Length	Cumulative weeks shortened	Crashing costs per week	Cumulative crashing costs	Cumulative total costs
22	0	0	0	**$109,500**
21	1	1,500	$1,500	**$111,000**
20	2	1,500	3,000	**$112,500**
19	3	2,000	5,000	**$114,500**
18	4	2,500	7,500	**$117,000**
17	5	4,750	12,250	**$121,750**
16	6	6,250	18,500	**$128,000**
15	7	**11,000**	29,500	**$139,000**
14	8	24,000	53, 500	**$163,000**

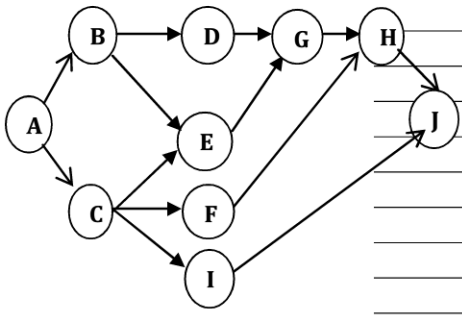
(d) Profit / week = $10,000. Because crash cost per week only jumps over $10,000 when the project is crashed from 16 down to 15 weeks, hence the project should be crashed to 16 weeks (see the table above).

Q 1.4: Given the following network and time & cost estimates, answer the following questions:

(a) What is the project completion time?

(b) What is the total cost required for completing this project on normal time?

(c) Crash the project by three weeks and calculate the new project completion cost.



Activity	Predecessor	Activity duration (weeks)		Activity cost ($)		Crash Cost / Week	# Days
		Normal	Crash	Normal	Crash		
A	-	6	4	10000	16000	3000	2
B	A	28	22	5000	9200	700	6
C	A	29	27	20000	20700	350	2
D	B	10	5	4000	6000	400	5
E	B,C	10	9	2500	3000	500	1
F	C	10	9	1000	7000	6000	1
G	D,E	15	14	1500	7500	6000	1
H	G,F	10	8	600	10600	5000	2
I	C	2	1	1000	2000	1000	1
J	H,I	10	8	900	8800	3950	2
TOTAL =				**46,500**			

Solution: The project has the following paths and their durations:

- A-B-D-G-H-J, Duration = 6 + 28 + 10 + 15 + 10 + 10 = 79
- A-B-E-G-H-J, Duration = 6 + 28 + 10 + 15 + 10 + 10 = 79
- **A-C-E-G-H-J, Duration = 6 + 29 + 10 + 15 + 10 + 10 = 80**
- A-C-F-H-J, Duration = 6 + 29 + 10 + 10 + 10 = 65
- A-C-I-J, Duration = 6 + 29 + 2 + 10 = 47

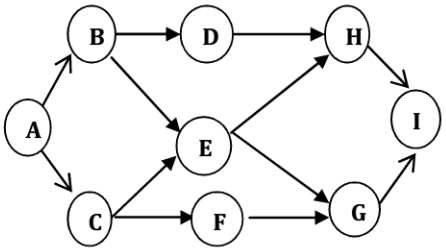
The project completion time is 80 weeks and the normal total cost (sum of all the activities) is **$46,500.**

Critical Path	Duration	It#1	It#2	It#3
A-B-D-G-H-J	79	79*	78*	77*
A-B-E-G-H-J	79	79*	78*	77*
A-C-E-G-H-J	80*	79*	78*	77*
A-C-F-H-J	65	64	64	63
A-C-I-J	47	46	46	45

It#	CP Before Crashing	Activity Crashed	CP After Crashing	Cumulative cost
1	A-C-E-G-H-J (80)	C by 1	A-B-D-G-H-J (79) A-B-E-G-H-J (79) A-C-E-G-H-J (79)	$ 350
2	A-B-D-G-H-J (79) A-B-E-G-H-J (79) A-C-E-G-H-J (79)	D by 1 E by 1	A-B-D-G-H-J (78) A-B-E-G-H-J (78) A-C-E-G-H-J (78)	$ 1250
3	A-B-D-G-H-J (78) A-B-E-G-H-J (78) A-C-E-G-H-J (78)	B by 1 C by 1	A-B-D-G-H-J (77) A-B-E-G-H-J (77) A-C-E-G-H-J (77)	$ 2300

After crashing by 3 weeks, the duration of the project is **77 weeks** and the total cost is **$46,500+ $2300 = $48,800.**

Q 1.5: The following table provides the necessary information for crashing a project. Project manager would like to crash the network by three weeks in the most economical way. Which activities should be crashed and by how many weeks?



Activity	Activity duration (weeks)		Activity cost ($)		Crash Cost/ Week
	Normal	Crash	Normal	Crash	
A	4	3	4000	6000	2000
B	3	2	5000	6000	1000
C	2	1	2000	2800	800
D	5	3	4000	6000	1000
E	6	5	2500	3000	500
F	3	2	1000	2000	1000
G	4	3	2000	2900	900
H	4	3	1500	2600	1100
I	6	5	5000	12000	7000

Solution:

The project has the following paths and their durations:

- A-B-D-H-I, Duration = 22
- **A-B-E-H-I, Duration = 23**
- **A-B-E-G-I, Duration = 23**
- A-C-E-H-I, Duration = 22
- A-C-E-G-I, Duration = 22
- A-C-F-G-I, Duration = 19

The normal duration of the project is 23 weeks and the normal total cost (sum of all the activities) is **$27,000.**

Critical Path	Duration	It#1	It#2	It#3
A-B-D-H-I	22	**22***	**21***	**20***
A-B-E-H-I	**23***	**22***	**21***	**20***
A-B-E-G-I	**23***	**22***	**21***	**20***
A-C-E-H-I	22	21	**21***	**20***
A-C-E-G-I	22	21	**21***	**20***
A-C-F-G-I	19	19	19	18

It#	CP Before Crashing	Activity Crashed	CP After Crashing	Cumulative cost
1	A-B-E-H-I (23) A-B-E-G-I (23)	E by 1	A-B-E-H-I (22) A-B-E-G-I (22) A-B-D-H-I (22)	$ 500
2	A-B-E-H-I (22) A-B-E-G-I (22) A-B-D-H-I (22)	B by 1	A-B-E-H-I (21) A-B-E-G-I (21) A-B-D-H-I (21)	$ 1500
3	A-B-E-H-I (21) A-B-E-G-I (21) A-B-D-H-I (21) A-C-E-H-I (21) A-C-E-G-I (21)	A by 1	A-B-E-H-I (20) A-B-E-G-I (20) A-B-D-H-I (20) A-C-E-H-I (20) A-C-E-G-I (20)	$ 3500

After crashing by 3 weeks, the duration of the project is **20 weeks** and the total cost is **$27,000+ $3500 = $30,500.**

Q 1.6: Kozar International, Inc. begun marketing a new instant-developing film project. The estimates of R&D activity time (weeks) for Kozar's project are given in the table below. The project has two paths: A-C-E-F and A-B-D-F. Assume the activity times are independent.

a) The company wants to be 95% confident that it can deliver the project without incurring any penalty, what time frame should it specify in the bid for project completion time?
b) If the time to complete the path A-B-D-F is normally distributed, what is the probability that this path will take at least 38 weeks to be completed?

Activity	Time(weeks)				Mean	Variance
	Predecessors	Optimistic time	Probable time	Pessimistic time		
A	-	9	9	9	9	0.00
B	A	8	10	12	10	0.44
C	A	9	12	18	12.5	2.25
D	B	5	8	11	8	1.00
E	C	5	7	10	7.166	0.69
F	D, E	10	12	14	12	0.44

Solution:

The project has two paths:

A-C-E-F:

- Expected Duration = 9 + 12.5 + 7.166 + 12 = 40.66 weeks (Critical Path).
- Variance $\sigma^2 = 0 + 2.25 + 0.69 + 0.44 = 3.38$, Standard Deviation = $\sigma = 1.838$

A-B-D-F

- Expected Duration = 9+ 10 + 8 + 12 = 39 weeks.
- Variance = 0+0.44+1.00+0.44 = 1.88, Standard Deviation = 1.371

Assumption: The probability of completing the project within a given timeframe depends on the critical path only. Note that this will provide us with approximate probability because there can be non-critical paths with high activity variances that might have the lower probability of completion time as compared to the critical path.

- The critical path A-C-E-F has an expected duration of 40.66 weeks and standard deviation of 1.838.
- The z value that corresponds to 95% is 1.645.
- This implies $z = 1.645 = \frac{\text{T}-\text{Expected Duration}}{\sigma} = \frac{T-40.66}{1.838}$ or T = 43.68.
- Hence, if the company wants to be 95% confident that it can deliver the project without incurring any penalty, then the time frame it should specify in the bid for project completion is **43.68 weeks.**

If the time to complete the path A-B-D-F is normally distributed, what is the probability that this path will take at least 38 weeks to be completed?

- The non-critical path A-B-D-F has an expected duration of 39 weeks and standard deviation of 1.371.
- This implies $z = \frac{\text{T}-\text{Expected Duration}}{\sigma} = \frac{38-39}{1.371}$ =-0.73.
- This z value corresponds to a probability of 0.2327.
- Probability that this path will take less than 38 weeks to be completed is 23.27%.
- Hence, the probability that this path will at least 38 weeks to be completed is 76.73%.