DEPARTMENT OF COMPUTER SCIENCE & SOFTWARE ENGINEERING
COMP232 MATHEMATICS FOR COMPUTER SCIENCE
FALL 2018

Assignment 1. Due date: Thursday September 27

1. For each of the following statements use a truth table to determine whether it is a tautology, a contradiction, or a contingency.

   (a) $\big((p \vee r) \wedge (q \vee r)\big) \leftrightarrow \big((p \wedge q) \vee r\big)$

   (b) $(p \oplus q) \wedge (p \oplus \neg q)$

   (c) $\big(p \rightarrow (q \rightarrow r)\big) \leftrightarrow \big(p \rightarrow (q \wedge r)\big)$

   (d) $\big(p \wedge (\neg q \rightarrow \neg p)\big) \rightarrow q$

2. For each of the following logical equivalences state whether it is valid or invalid. If invalid then give a counterexample (*e.g.*, based on a truth assignment). If valid then give an algebraic proof using logical equivalences from Tables 6, 7, and 8 from Section 1.3 of textbook.

   (a) $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

   (b) $(p \rightarrow q) \vee (p \rightarrow r) \equiv (p \vee q) \rightarrow r$

   (c) $\Big(\big((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)\big) \rightarrow r\Big) \equiv T$

   (d) $\Big(\big((p \rightarrow q) \wedge (q \rightarrow r)\big) \rightarrow (p \rightarrow r)\Big) \equiv T$

3. Which of the following conditions are *necessary*, and which conditions are *sufficient* for the natural number n to be divisible by 6. The natural numbers are $\mathbb{N} = \{0, 1, 2, \ldots, \}$.

   (a) n is divisible by 3.

   (b) n is divisible by 9.

   (c) n is divisible by 12.

   (d) $n = 24$

   (e) n^2 is divisible by 3.

   (f) n is even and divisible by 3.

4. A set of propositions is *consistent* if there is an assignment of truth values to each of the variables in the propositions that makes each proposition true. Is the following set of propositions consistent?

   > *If the file system is not locked, then new messages will be queued.*
   > *If the file system is not locked, then the system is functioning normally, and conversely.*
   > *If new messages are not queued, then they will be sent to the message buffer.*
   > *If the file system is not locked, then new messages will be sent to the message buffer.*
   > *New messages will not be sent to the message buffer.*

5. Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y, where $x = 1, 2,$ or 3, and $y = 1, 2,$ or 3. Write out the propositions below using disjunctions and conjunctions only.

   (a) $\exists x\, P(x, 3)$

   (b) $\forall y\, \neg P(2, y)$

   (c) $\forall x\, \exists y\, P(x, y)$

   (d) $\exists x\, \forall y\, \neg P(x, y)$

6. Let the domain for x be the set of all students in this class and the domain for y be the set of all countries in the world. Let $P(x, y)$ denote "*student x has visited country y*" and $Q(x, y)$ denote "*student x has a friend in country y.*" Express each of the following using logical operations and quantifiers, and the propositional functions $P(x, y)$ and $Q(x, y)$.

   (a) *Carlos has visited Bulgaria.*

   (b) *Every student in this class has visited the United States.*

   (c) *Every student in this class has visited some country in the world.*

   (d) *There is no country that every student in this class has visited.*

   (e) *There are two students in this class, who between them, have a friend in every country in the world.*

   (f) *Nobody in this class has visited a country in which they did not have a friend.*

7. For each part in the previous question, form the negation of the statement so that all negation symbols occur immediately in front of predicates. For example:

$$\neg\Big(\forall x\big(P(x) \wedge Q(x)\big)\Big) \equiv \exists x\Big(\neg\big((P(x) \wedge Q(x))\big)\Big) \equiv \exists x\Big(\big(\neg P(x)\big) \vee \big(\neg Q(x)\big)\Big)$$

8. Negate the following statements and transform the negation so that negation symbols immediately precede predicates. (See example in Question 7.)

   (a) $\Big(\exists x\, \exists y\, P(x, y)\Big) \vee \Big(\forall x\, \forall y\, Q(x, y)\Big)$

   (b) $\forall x\, \forall y\, \Big(Q(x, y) \leftrightarrow Q(y, x)\Big)$

   (c) $\forall y\, \exists x\, \exists z\, \Big(T(x, y, z) \wedge Q(x, y)\Big)$