FACULTY OF ENGINEERING AND COMPUTER SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING

ENGR-391 NUMERICAL METHODS FOR ENGINEERS

Student's Name: _____

I.D.: _____

Duration 3 hours

PROBLEM 1 [Solving Nonlinear Equations] [20 marks]

1. Practice

Solve the following nonlinear equation using the secant method with $x_0=0$ and $x_1=1$:

$$f(x) = x^3 + x - 1$$

Use 5 iterations. Check your result.

2. Theory

- What is the main difference between Newton-Raphson method and secant method?
- Sketch the bisection and the Newton-Raphson method.

PROBLEM 2 [Solving Systems of Linear Equations] [40 marks]

1. Practice

Solve the following system of linear equations:

$$\begin{cases} 4x - y + z = 7 \\ 4x - 8y + z = -21 \\ -2x + y + 5z = 15 \end{cases}$$

a- Using LU decomposition
b- Using Gauss-Seidel method (3 iterations), choose as initial guesses: y=z=0
c- Check your results
d- Compare the two methods

2. Theory

- Ill conditioning of equations occurs when the determinant of [A] is close to zero?
  True / I really do not know / False
- What is an upper-diagonal matrix?
- When two rows are interchanged in [A], then the determinant of the new matrix is the same as det[A]?
  True / I really do not know / False

PROBLEM 3 [Numerical Integration] [20 marks]

1. Practice

Compute the following integral:

$$I = \int_0^1 e^{-x^2} dx$$

Using multi-segment Simpson's (1/3) rule and a step equal 0.25.

2. Theory

- In general, for the same number of segments, Simpson's rule is less accurate than the trapezoidal rule?
  True / I really do not know / False
- The Gauss quadrature formula is applicable to integrals having arbitrary, but finite, limits of integration?
  True / I really do not know / False
- Improper integrals (integrals with at least one limit equal to infinity) can not be computed with numerical algorithms.
  True / I really do not know / False
- To increase the accuracy of the computation of an integral using a quadrature formula, one has normally to reduce the step-size (increase the number of segments for a multi-segment version of the quadrature formula). What do you think about the following statement:
  The accuracy of my result is only limited by the computation time, i.e, with a computer; you will be able to increase the number of segments and to more accurately predict the value of the integral?
  True / I really do not know / False
  Explain why (even for I really do not know)?

PROBLEM 4 [Solving Ordinary Differential Equations] [20 marks]

1. Practice

Solve the following ODE:

$$\frac{dy}{dx} + y - x = 0 \qquad y(0) = 0$$

Using Runge-Kutta second order method (with Heun's method), for $0 \le x \le 1$ and a step of 0.25.

2. Theory

- Euler's method can be considered as the first order Runge-Kutta method?
  True / I really do not know / False
- What is the difference between a boundary-value problem and an initial- value problem?
- The order of the most popularly used Runge-Kutta methods is:
  4 / 3 / 2?