

**MATH 209 - Section AC**

**Midterm test - May 22, 2013 (6:30 - 8:00 pm)**

**Only approved calculators are premitted.**

**Total marks: 80 (20% of the final grade)**

**P1.** Find the following limits

[6] (a)  $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x + 3}$

[6] (b)  $\lim_{x \rightarrow 0} \frac{\sqrt{49 + x} - 7}{x}$

[6] (c)  $\lim_{x \rightarrow \infty} \frac{-2x^2 + 3x + 5}{x^2 - 7x + 10}$

**P2.**

[6] (a) Given  $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 3x & \text{if } x > 2 \end{cases}$ . Check the continuity of  $f(x)$  at  $x = 2$  and draw the graph of  $f(x)$ .

[6] (b) Using the definition (four-step method), find the derivative of the function  $f(x) = 2x^2 - 5x + 6$ .

**P3.** Find the derivative of the following functions. (Do not simplify)

[5] (a)  $y = (2x^3 + x^2 - 2)(x^3 + 5)$

[5] (b)  $y = \frac{1}{5}x^{-4} - 3\sqrt{x} + 5$

[5] (c)  $y = e^{3x^2+7} + \ln(1 + x^3)$

[5] (d)  $y = 4(3x^4 + x^3 - 2)^6$

[5] (e)  $y = \frac{2x^2 - 3x + 5}{x^3 + 2}$

**P4.** The total cost (in dollars) of manufacturing  $x$  auto body frames is  $C(x) = 60,000 + 300x$

[3] (a) Find the average cost per unit if 500 frames are produced

[4] (b) Find the marginal average cost at a production level of 500 units and interpret the result.

[3] (c) Use the results from parts (a) and (b) to estimate the average cost per frame if 501 frames are produced.

[5] **P5.** How long will it take money to double if it is invested at 6.5% compounded continuously?

**P6.**

[5] (a) Using implicit differentiation, find  $y' \frac{dy}{dx}$  at  $(-1, 3)$  given  $2x^3y - x^3 + 5 = 0$

[5] (b) Find the equation of the tangent line to the graph of the above equation at the point  $(-1, 3)$ .