

SOEN 321

Prob. 1 Let $x=111$ and $y=19301$. Factor $n=21311$ using the fact that $x^2 \equiv y^2 \pmod n$.

Ans.

Note that

$$x^2 \equiv y^2 \pmod n \rightarrow x^2 - y^2 \equiv 0 \pmod n \rightarrow (x-y)(x+y) \equiv 0 \pmod n \rightarrow$$

$(x-y)(x+y) = Kn = Kpq$ for some integer K . Let $K = k_1 k_2$. Thus we have

$$(x-y) = k_1 p \quad \& \quad (x+y) = k_2 q$$

Then we can factor n as follows:

$$\gcd(x \pm y, n) = p \text{ or } q.$$

Prob. 2 Suppose Bob has an RSA Cryptosystem with a large modulus n for which the factorization cannot be found in a reasonable amount of time. Suppose Alice sends a message to Bob by representing each alphabetic character as an integer between 0 and 25 (i.e., $A \leftrightarrow 0$, $B \leftrightarrow 1$, etc.), and then encrypting each residue modulo n as a separate plaintext character. Describe how Eve can easily decrypt a message which is encrypted in this way.

Ans. Eve can construct a lookup table for all the valid 26 ciphertexts by encrypting the letters A to Z using Bob's public key. Then Eve can use this table (or more precisely the inverse of this table) to decrypt any ciphertext encrypted by Alice.

Prob. 3 . Determine the problems in the following protocol in which A wants to establish a shared session key with B using the help of a trusted authority S

$A \rightarrow S: A, B$

$S \rightarrow A: K_{AB}$

$A \rightarrow B: A, K_{AB}$

Ans. The key is sent in the clear.

Prob. 4 Consider the following authentication protocol

$A \rightarrow B: T_A, \text{Sig}_A(T_A, B)$

(i) What is the objective of the time stamp T_A ?

(ii) After this protocol is executed

(a) B is authenticated to A

(b) A is authenticated to B

(c) Both A and B are authenticated to each other

Ans. The time stamp ensures the freshness of the signature and prevents replay attacks. "A" is authenticated to B.