Concordia University

Department of Computer Science & Software Engineering

COMP 232/4 Introduction to Discrete Mathematics Winter 2019

Assignment 1

Due date: Monday, 28.1.2019

1. Show that $((p \leftrightarrow q) \leftrightarrow T)$ and $(p \leftrightarrow (q \leftrightarrow T))$ are logically equivalent
   (a) using truth tables
   (b) using transformations
2. Suppose the universe of discourse of the propositional function $P(x, y)$ consists of pairs x and y, where x is 1,2, or 3 and y is 1,2, or 3. Write out the following propositions using disjunctions and conjunctions.
   (a) $\exists x P(x, 3)$
   (b) $\forall y P(1, y)$
   (c) $\exists y \neg P(2, y)$
   (d) $\forall x \neg P(x, 2)$
3. Express the negation of the following propositions using quantifiers and then express this negation in English.
   (a) Some drivers do not obey the speed limit.
   (b) All Swedish movies are serious.
   (c) No one can keep a secret.
   (d) There is someone in this class who does not have a good attitude.
4. Express each of the following statements using only quantifiers, logical connectives, and as predicates use only the mathematical operators $\times$ (multiplication), $>, <, \geq, \leq, /$ (division), $-$ (negative numbers), $-$ (difference), $+$ or any of the ten digits.
   (a) The product of two negative integers is positive.
   (b) The average of two positive integers is positive.
   (c) The difference of two negative integers is not necessarily negative.
5. Determine the truth value of each of the following statements if the universe of discourse of each variable consists of all real numbers.
   (a) $\forall x \exists y (x + y = 1)$
   (b) $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$
   (c) $\forall x \exists y (x + y = 2 \wedge 2x - y = 1)$
   (d) $\forall x \forall y \exists z (z = (x + y)/2)$

6. Negate the following statements and transform the negation so that negation symbols immediately precede predicates.
   (a) $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$
   (b) $\exists x \exists y (Q(x, y) \leftrightarrow Q(y, x))$
   (c) $\forall y \exists x \exists z (T(x, y, z) \vee Q(x, y))$

7. Express the negations of these propositions using (i) quantifiers and (ii) in English.
   (a) There is a student in this class who has never seen a computer.
   (b) There is a student in this class who has taken every mathematics course offered by Concordia.

8. $pNANDq$, also written $p|q$ and called the Sheffer stroke, is false only when both p and q are true.
   (a) Show that $p|q \equiv \neg(p \wedge q)$.
   (b) Express $p \wedge q$ using only the Sheffer stroke and the propositional symbols p and q.
   (c) Express $\neg p$ using only the Sheffer stroke and the propositional symbol p.
   (d) Construct a valid argument that all of propositional logic can be expressed using only the Sheffer stroke and propositional symbols.