COMP 232, Fall 2020 **CONCORDIA UNIVERSITY**

MATHEMATICS FOR COMPUTER SCIENCE
Assignment 2.
Due: October 23, 2020.

1. Let $P(x, y, z)$ denote the statement "$x + y \leq z$, " where $x, y, z \in \mathbb{Z}^+$. What is the truth value of each of the following? Explain your answers.
   (a) $\forall x \exists y \exists z P(x, y, z)$.
   (b) $\forall y \exists x \forall z P(x, y, z)$.
   (c) $\exists z \exists y \forall x P(x, y, z)$.

2. For each of the premise-conclusion pairs below, give a valid step-by-step argument (proof) along with the name of the inference rule used in each step. For examples, see pages 73 and 74 in textbook.
   (a) Premise: $\{\neg p \vee q \rightarrow r,\ s \vee \neg q,\ \neg t,\ p \rightarrow t,\ \neg p \wedge r \rightarrow \neg s\}$, conclusion: $\neg q$.
   (b) Premise: $\{\neg p \rightarrow r \wedge \neg s,\ t \rightarrow s,\ u \rightarrow \neg p,\ \neg w,\ u \vee w\}$, conclusion: $\neg t \vee w$.
   (c) Premise: $\{p \vee q,\ q \rightarrow r,\ p \wedge s \rightarrow t,\ \neg r,\ \neg q \rightarrow u \wedge s\}$, conclusion: t.

3. For each of the following, determine whether the argument is valid. You may use a counterexample or equivalence transformations to justify your answer.
   (a) $$\begin{array}{l} p \rightarrow q \\ \underline{\neg p} \\ \therefore\ \neg q \end{array}$$
   (b) $$\begin{array}{l} \underline{\neg p \rightarrow \neg q} \\ \therefore\ (\neg p \rightarrow q) \rightarrow p \end{array}$$
   (c) $$\begin{array}{l} p \rightarrow r \\ q \rightarrow r \\ \underline{\neg (p \vee q)} \\ \therefore\ \neg r \end{array}$$
   (d) $$\begin{array}{l} p \rightarrow q \\ \underline{p \rightarrow (q \rightarrow \neg p)} \\ \therefore\ \neg p \end{array}$$

4. For each of the arguments below, indicate whether it is valid or invalid.
   (a) A convertible car is fun to drive.
   Isaac's car is not a convertible.
   $\therefore$ Isaac's car is not fun to drive.
   (b) All healthy people eat an apple a day.
   Herbert is not a healthy person.
   $\therefore$ Herbert does not eat an apple a day.
   (c) If a product of two real numbers is 0, then at least one of the numbers is 0.
   For a particular real number x, neither $(x-1)$ nor $(x+1)$ equals 0.
   $\therefore$ The product $(x-1)(x+1)$ is not 0.
5. Use rules of inference to show that if $\forall x\big(P(x) \to Q(x)\big)$, $\forall x\big(Q(x) \to R(x)\big)$, and $\exists x\big(\neg R(x)\big)$ are true, then $\exists x\big(\neg P(x)\big)$ is true.
6. (a) Give a direct proof of: "If x is an odd integer and y is an even integer, then $x+y$ is odd."
   (b) Give a proof by contradiction of: "If n is an odd integer, then n^2 is odd."
   (c) Give an indirect proof of: "If x is an odd integer, then $x+2$ is odd."
   (d) Use a proof by cases to show that there are no solutions in positive integers to the equation $x^4 + y^4 = 100$.
   (e) Prove that given a nonnegative integer n, there is a unique nonnegative integer m, such that $m^2 \leq n < (m+1)^2$.
7. For each of the statements below state whether it is True or False. If True then give a proof. If False then explain why, e.g., by giving a counterexample.
   (a) The difference of any two odd integers is odd.
   (b) Let a and b be integers. If $a+b$ is even, then either a or b is even.
   (c) For all positive integers n, it holds that n is even if and only if $3n^2+8$ is even.
   (d) For all positive $x, y \in \mathbb{R}$, if x is irrational and y is irrational then $x+y$ is irrational.
   (e) $\forall x, y \in \mathbb{R}$, if x is irrational and y is rational then $x \cdot y$ is irrational.