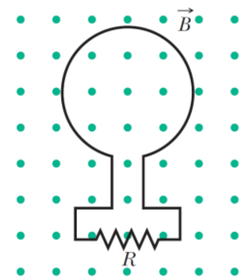
PHYS 205-Section 03
Electricity and Magnetism - Winter 2018
Assignment 5 – Solutions

Problems +Solutions

1. In the figure, the magnetic flux through the loop increases according to the relation $\Phi_B = 6t^2 + 7t$, where Φ_B is in milliwebers and t is in seconds.
(a) What is the magnitude of the emf induced in the loop when $t = 2\ s$? **(3 points)**
(b) Is the direction of the current through R to the right or left? **(2 points)**



Solution:

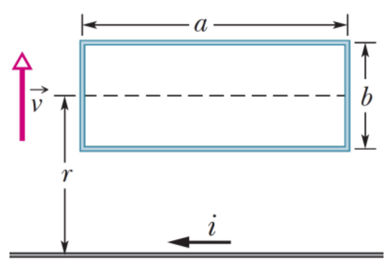
a) To find the magnitude of emf, we should find the rate of change of flux:

$$|\mathcal{E}| = \left|\frac{d\phi_B}{dt}\right| = 12t + 7 = 12(2) + 7 = 31\ mV$$

b) Using Lenz's law, considering the fact that the flux is increasing, the induced magnetic field should be in the opposite direction of the applied field, which means it should be pointing into the page. Using right-hand-rule:

The induced current should be **clockwise** in the loop

2. In the figure, a rectangular loop of wire with length $a = 2.2\ cm$, width $b = 0.8\ cm$, and resistance $R = 0.40\ m\Omega$ is placed near an infinitely long wire carrying current $i = 4.7\ A$. The loop is then moved away from the wire at constant speed $v = 3.2 \frac{mm}{s}$. When the center of the loop is at distance $r = 1.5b$, what are
(a) the magnitude of the magnetic flux through the loop? **(3 points)**
(b) the current induced in the loop? **(2 points)**



Solution:

a) Assuming that the flux in the loop is due to the magnetic field of the straight wire, and considering the fact that different elements of the loop's surface area see a different magnetic field (since they are at a different distance from the wire), we should calculate the flux passing through each element of the surface area dA and add them all up (integrate):

$$|\phi_B| = \int_{r-\frac{b}{2}}^{r+\frac{b}{2}} \vec{B}.d\vec{A} = \int_{r-\frac{b}{2}}^{r+\frac{b}{2}} BdA = \int_{r-\frac{b}{2}}^{r+\frac{b}{2}} \mu_0 \frac{i}{2\pi r}(adr) = \mu_0 \frac{ia}{2\pi} \ln\left(\frac{r+\frac{b}{2}}{r-\frac{b}{2}}\right)$$

For $r = \frac{b}{2}$:

$$\boxed{\phi_B = 1.4 \times 10^{-8}\, Wb}$$

b) To find the induced current, we should calculate the rate of change of this flux, considering $\frac{dr}{dt} = v$:

$$\boxed{I_{ind} = \left|\frac{\varepsilon}{R}\right| = \mu_0 \frac{ia}{2\pi R} \cdot \frac{d}{dt}\left[\ln\left(\frac{r+\frac{b}{2}}{r-\frac{b}{2}}\right)\right] = \mu_0 \frac{ia}{2\pi R} \frac{bv}{\left[r^2 - \left(\frac{b}{2}\right)^2\right]} = 10^{-5}\, A}$$

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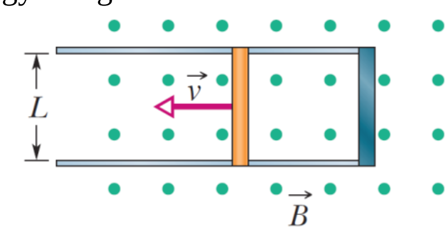
3. In the figure, a metal rod is forced to move with constant velocity $\vec{v}$ along two parallel metal rails, connected with a strip of metal at one end. A magnetic field of magnitude $B = 0.350\, T$ points out of the page.
   a) If the rails are separated by $L = 25\, cm$ and the speed of the rod is $55\ \frac{cm}{s}$, what emf is generated? **(2 points)**
   b) If the rod has a resistance of $18\ \Omega$ and the rails and connector have negligible resistance, what is the current in the rod? **(2 points)**
   c) At what rate is energy being transferred to thermal energy? **(2 points)**



Solution:

a) We can find the induced emf using:

$$\boxed{\mathcal{E} = BLv = 4.81 \times 10^{-2}\ V}$$

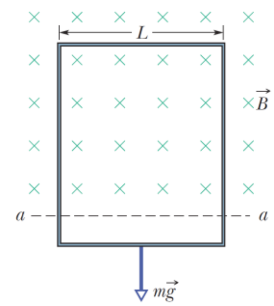
b) The induced current, which is clockwise, can be found from Ohm's law:

$$\boxed{I = \frac{\mathcal{E}}{R} = 2.67\ \times 10^{-3} A}$$

c) For power:

$$\boxed{P = I^2 R = 1.29 \times 10^{-4}\ W}$$

4. In the figure, a long rectangular conducting loop, of width L, resistance R, and mass m, is hung in a horizontal, uniform magnetic field $\vec{B}$ that is directed into the page and that exists only above line aa. The loop is then dropped; during its fall, it accelerates until it reaches a certain terminal speed v_t. Ignoring air drag, find an expression for v_t.
(4 points)



Solution:

When the loop reaches the terminal velocity, the net force applied on it is zero:

$$\sum F = F_B - mg = 0$$

The wire has 4 segments. The magnetic force on the side ones (vertical ones) cancel each other. There is no magnetic force on the bottom segment, as it experiences no magnetic field. The magnetic force countering the weight of the loop is on the top segment:

$$F = ILB$$

The current in the loop is the induced current, which can be found from:

$$I = \left|\frac{\mathcal{E}}{R}\right| = \frac{1}{R}\left|\frac{d\phi_B}{dt}\right| = \frac{B}{R}\left|\frac{dA}{dt}\right| = \frac{Bv_tL}{R}$$

$$ILB = mg \rightarrow I = \frac{mg}{LB} = \frac{Bv_tL}{R}$$

$$\boxed{v_t = \frac{mgR}{B^2L^2}}$$