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QUESTIONS

1. The life in hours of a battery is known to be normally distributed, with standard deviation of 1.5 hours. A random sample of 12 batteries has a sample mean life time of 50 hours.
 - a. Test the hypothesis that the mean battery life is 50.5 hours? (Use $\alpha = 0.05$) (5 marks)
 - b. What is the P-value for the test in part (a)? (5 marks)
 - c. Find the β -error for the test in part (a), if the true mean life is 52 hours? (5 marks)
 - d. What sample size is required to ensure that β does not exceed 0.10, if the true mean life is 52 hours? (5 marks)
2. A random sample of 36 observations has been drawn from a normal distribution with mean of 50 and standard deviation of 12.
 - a. Find the probability that the sample mean is in the interval of $47 \leq \bar{X} \leq 53$. (7 marks)
 - b. Is the assumption of normality important? Why? (3 marks)
3. The tar content in 9 samples of cigar tobacco follows 5.40, 5.67, 5.79, 6.85, 6.92, 5.70, 6.08, 5.48, and 5.44. Assume the tar content has a normal distribution.
 - a. Calculate the sample mean and variance (4 marks)
 - b. Estimate 95% confidence upper bound of the population mean (6 marks)
4. For a certain virus, if a person is exposed to the virus there is a 10% chance that the person will be infected. Assume that persons who were infected are independent of one another.
 - a. If we examine exposed people one by one, find the probability that we must examine 5 or more people before we find two who are infected. (5 marks)
 - b. What is the mean number or the expected number of people we must examine before we find two people who are infected? (5 marks)
5. A hydraulic system contains two pumps and two valves. Let P_i be the event that pump i works (here $i=1,2$). Similarly let V_j be the event that valve j works (here $j=1,2$). The entire system will work if at least one pump and at least one valve work. Let F be the event that the whole system works. Write down F in terms of the P_i and V_j . (5 marks)
6. Four friends are given tickets to a football game. The tickets give seats all in the same row of 12 seats. However the tickets are for random seats in that row. What is the probability that the four friends will be seated in consecutive seats (i.e. all four seated one beside the other)? (5 marks)