Concordia University

Faculty of Engineering and Computer Science
Department of Mechanical Engineering
Final Examination

Course:	**Advanced Calculus**
Number:	**ENGR 233 Section: CC**
Date:	**August 9, 2007**
Time and Place:	**11:00am → 2:00pm**
Number of Page:	**2 (including this one)**
Instructors:	**Dr. Pierre Q. Gauthier**
Material allowed:	**No**
Calculators Allowed:	**No**
Special instructions:	**None**

Answer the following six questions.
Please show each intermediate step to the answers.
Only non-programmable calculators are permitted.
Return the questions' paper with the answers book.

Name: ____________________________ ID:________________________

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1. Verify the Divergence Theorem for the volume bounded by the paraboloïd $z = x^2 + y^2$ and the plane $z = 1$, and the vector field: $\bar{F}(x, y, z) = (xz, \quad yz, \quad 3z^2)$.

2. Verify Stoke's Theorem for the vector field $\bar{F}(x, y, z) = (2x - y)\hat{i} - yz^2\hat{j} - zy^2\hat{k}$ and the hemisphere $x^2 + y^2 + z^2 = 1, \; z \geq 0$.

3. Verify Green's Theorem for the vector field $\bar{F}(x, y) = (y^2 - x^2)\hat{i} + (x^2 + y^2)\hat{j}$ and the curve C: the triangle bounded by y = 0, x = 3 and y = x.

4. Find the work done by the force field $\bar{F}(x, y, z) = (6x^2e^{yz}, \quad 2x^3ze^{yz}, \quad 2x^3ye^{yz})$ along the curve C, consisting of the series of straight lines connecting the following points in order: $(0, \; 0, \; 0,)$; $(1, \; -7, \; 5,)$; $(1, \; 1, \; 1,)$; $(2, \; 3, \; 4,)$; $(-1, \; 0, \; -7,)$; $(8, \; 8, \; 8,)$ and, finally, $(1, \; 2, \; 0,)$.

5. Given the function $f(x, y, z) = \ln(x^2 + y^2 + z^2) + y + 6z$,

   a) Find the directions in which the function increase and decrease most rapidly at: $\bar{x}_0 = (1, \; 1, \; 0)$.

   b) Find the derivative of the function at $\bar{x}_0$, in the direction: $\bar{v} = (1, \; -3, \; 2)$.

6. For the level surface: $\cos(\pi x) - x^2y + e^{xz} + yz = 4$ at the point $\bar{x}_0 = (0, \; 1, \; 2)$, find the equations for the

   a) Tangent plane.

   b) Normal line.

7. Consider the curve with the following parameterization: $\bar{r}(t) = (2t^2, \quad t, \quad 3), \; t \geq 0$.

   a) Find the Normal and Tangential components of acceleration at t = 1.

   b) Find the curvature at t = 1.

   c) Sketch the curve, showing the vectors found in part a).