Department of Mathematics & Statistics

Course	**Number**	**Section(s)**
Mathematics	203	All
Examination	**Date**	**Pages**
Final	April 2007	3
Instructors		**Course Examiner**
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Special Instructions

▷ Calculators are **not** allowed.

MARKS

[8] **1.** (a) Let $f(x) = x^2 + 1$ and $g(x) = \sin x$. Find $f \circ g$ and $g \circ f$.

(b) Find the inverse of the function $f(x) = e^{2x} - 1$. Determine the domain and range of f and f^{-1}.

[10] **2.** Evaluate the limits:

(a) $\lim\limits_{x \to 7} \dfrac{x^2 - 6x - 7}{\sqrt{x+2} - 3}$ (b) $\lim\limits_{x \to -\infty} \dfrac{4x^2\sqrt{9x^6 + 2x^2}}{x^5 + 3}$

Do not use l'Hopital's rule.

[12] **3.** (a) Consider the function $f(x) = \dfrac{|x+1|}{x^2 - 1}$.

Calculate both one-sided limits at the point(s) where the function is undefined.

(b) Find parameters a and b such that the function

$$f(x) = \begin{cases} 1 - \dfrac{x^2 - x}{x^2 - 1}, & \text{if} \quad 0 \le x < 1 \\ a, & \text{if} \quad x = 1 \\ x + b, & \text{if} \quad x > 1 \end{cases}$$

will be continuous at every point. Sketch the graph of this function.

[12] **4.** Find derivatives of the functions (do not simplify the answer):

(a) $f(x) = e^{-x^2}\left(\cos x + \tan x\right)^2$;

(b) $f(x) = \dfrac{\ln(1+x^2)}{1+\arctan(2x)}$;

(c) $f(x) = \sec^3(\sqrt{1+x^2})$;

(d) $f(x) = (\sin(5x))^{x^3}$ (use logarithmic differentiation).

[12] **5.** Given the function $f(x) = \sqrt{3x+1}$,

(a) Use appropriate differentiation rules to find the derivative of the function.

(b) Use the definition of derivative to verify (a).

(c) Find the differential of the function at $x = 1$.

(d) Use the differential above with the appropriate choice of $dx = \Delta x$ to estimate $\sqrt{5}$.

[10] **6.** (a) A curve called a lemniscate is defined implicitly by the equation $2(x^2+y^2)^2 = 25(x^2-y^2)$. Verify that the point $(3,1)$ belongs to the curve. Find an equation of the tangent line to the curve at this point.

(b) Use l'Hopital's rule to evaluate $\lim\limits_{x\to 0} \dfrac{1-\cos x}{(\ln(x+1))^2}$.

[8] **7.** (a) Let $f(x) = \arcsin(x^2)$. Find $f''(x)$.

(b) Let $f(x) = 3x^2 + 2x + 5$. Find a number c that satisfies the Mean Value Theorem for the function $f(x)$ on $[-1, 1]$.

[12] **8.** (a) A particle is moving along the curve $y = 4\sqrt{x}$. As the particle passes the point $(9, 12)$ its x-coordinate is increasing at a rate of 3 cm/sec. How fast is the distance from the particle to the origin changing at this instant?

(b) Find the area of the largest rectangle that has its base on the x - axis and its other two vertices above the x - axis and lying on the parabola $y = 12 - x^2$.

[16] **9.** Given the function $f(x) = \dfrac{e^x}{x}$,

(a) Find the domain and check for symmetry. Find asymptotes (if any).

(b) Calculate $f'(x)$ and use it to determine interval(s) where the function is increasing, interval(s) where the function is decreasing, and local extrema (if any).

(c) Calculate $f''(x)$ and use it to determine interval(s) where the function is concave upward, interval(s) where the function is concave downward and inflection point(s) (if any).

(d) Sketch the graph of the function.

[5] **Bonus Question**

Let $f(x) = x|x|$. Use the definition of the derivative to show that $f'(0)$ exists, then find $f'(x)$ for any x.