CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	203	All
Examination	**Date**	**Pages**
Final	December 2017	3
Instructors:	J. Brody, X. Gao, U. Mgbemena B. Rhodes, N. Rossokhata, S. Vikram	**Course Examiners** A. Atoyan & H. Proppe
Special Instructions:	Only approved calculators are allowed. **Show all your work for full marks.**	

MARKS

[9] **1.** **(a)** Solve for x : $\log_2(x-4) + \log_2(x+2) = 2\,\log_2 4$

(b) Given the function $f = 1 + \ln(x^3 - 8)$, find the inverse function f^{-1}, the domain and the range of f and the domain and the range of f^{-1}.

[6] **2.** **(a)** Calculate both one-sided limits at $x = 2$ where the function

$$f(x) = \frac{x^2 + x - 6}{|\,x - 2|} \text{ is undefined.}$$

(b) Is it possible to define f at $x = 2$ to make $f(x)$ continuous everywhere? Explain.

[5] **3.** Find all horizontal and all vertical asymptotes of the function

$$f(x) = \frac{x\sqrt{4x^2 + 2x + 1} + 2x^2}{x^2 + 25}$$

.

[15] **4.** Find the derivatives of the following functions (**show your work for full marks**):

(**a**) $f(x) = (x^{1/2} + x^{-1/2})\,\sqrt{x}\,\ln(x)$

(**b**) $f(x) = \dfrac{\arcsin(x^2)}{\sqrt{1-x^2}} + e^2x^2$

(**c**) $f(x) = \ln[(x^2 - 2x)\,\ln(x^3 + 3x)]$

(**d**) $f(x) = \dfrac{e^x + e^{-x}}{\cos(x) + \sin(x)}$

(**e**) $f(x) = (1 + x^2)^{x^2+1}$ (use logarithmic differentiation)

[10] **5.** Consider the function $y = \sqrt{1+2x}$.

(a) Use the **definition of derivative** to find the formula for dy/dx.

(b) Find the linearization $L(x)$ of the function $y(x)$ at $a = 4$

(c) Use this linearization to approximate the exact value of $\sqrt{10}$.

[7] **6.** Let $f(x) = x^3 - 2x + 3$.

(a) Find the slope m of the secant line joining the points $(-2, f(-2))$ and $(0, f(0))$.

(b) Find all points $x = c$ (if any) on the interval [-2,0] such that $f'(c) = m$.

[8] **7.** **(a)** Verify that the point (2,1) belongs to the curve defined by the equation $y^2 + x\sqrt{3+y} = 1 + x^2$, and find the implicit derivative $\dfrac{dy}{dx}$ at this point.

(b) Write the equation of the tangent line to the curve at the point (2,1).

[15] **8.** **(a)** Find the absolute extrema of $f(x) = \dfrac{x}{x^2 - x + 1}$ on the interval $[0, 3]$.

(b) The length of a rectangle is increasing at the rate of 8 cm/s and its width is increasing at the rate of 5 cm/s. When the length is 20 cm and the width is 12 cm, how fast is the area of the rectangle increasing at that instant?

(c) Use l'Hôpital's rule to evaluate the $\lim\limits_{x\to 0} \dfrac{\ln(1+x^2)}{1-\cos(2x)}$.

[11] **9.** **(a)** Find the point (x_0, y_0) on the line $y = 2 - 2x$ that is closest to the point (5,2).

(b) If $A = 1200\,\text{cm}^2$ of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

---

[14] **10.** Given the function $f(x) = x^4 - 6x^2$.

(a) Find the domain of $f(x)$, check for symmetry, and also find asymptotes (if any).

(b) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).

(c) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

(d) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question.** Find the value of the 10-th derivative $f^{(10)}(0)$ of the function $f(x) = (e^{2x} + e^{-2x})$ at the point $x = 0$.
(HINT: study the pattern of higher-order derivatives of $f(x)$.)