ENGR 233 X Midterm 1

Friday, Feb 16, 2018, 90 min, 5 problems

1. [10 Marks] For $\vec{r}(t) = \langle \sin(4t), 10 + 3t, \cos(4t) \rangle$ find the unit tangent, normal, and binormal vectors as functions of $t \in \mathbb{R}$. Then parametrize the curve given by $\vec{r}(t)$ by its arc length s.

2. [10 Marks] The position of a moving object is given by
$$\vec{r}(t) = \left\langle t^2, \cos t + t \sin t, \sin t - t \cos t \right\rangle,$$
where $t > 0$ is the time. Find the acceleration $\vec{r}''(t)$ and its tangent $a_T(t)$ and normal $a_N(t)$ scalar components.

3. [10 Marks] Let $f(x, y) = \sqrt{x^2 - 16xy - y^2}$ and $P = (1, -1)$. Find the gradient of f as a function of x and y. Then find the directional derivative of f at the point P in the direction of the vector $\vec{v} = \langle 3, -4 \rangle$. Find $\vec{u}$ that minimizes the value of $D_{\vec{u}} f$ at P. What is the maximum value of $D_{\vec{u}} f$ at P?

4. [10 Marks] Let $f(x, y) = 4x^2 + 4xy + y^2$.
   - A. Write an equation of the plane tangent to the graph of scalar function f at the point $P = (-2, 3, 1)$.
   - B. Write symmetric equations of the line normal to the graph of f at the point $P = (1, -1, 1)$.

5. [10 Marks] Find the divergence and curl of the vector field
$$\vec{F}(x, y, z) = \left\langle y e^{zy}, x \sin(zx^2), z^3 \cos(yx) \right\rangle.$$

Formula Sheet:

$$\vec{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|}, \quad \vec{N}(t) = \frac{\vec{T}'(t)}{\|\vec{T}'(t)\|}, \quad \vec{B}(t) = \vec{T}(t) \times \vec{N}(t),$$

$$s(t) = \int_0^t \|\vec{r}'(\tilde{t})\| \, d\tilde{t}, \quad \kappa(t) = \left\| \frac{d\vec{T}}{ds}(t) \right\| = \left\| \frac{\vec{T}'(t)}{\vec{r}'(t)} \right\| = \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3},$$

$$\vec{a}(t) = \left(\frac{d}{dt} \|\vec{v}(t)\| \right) \vec{T}(t) + \kappa(t) \|\vec{v}(t)\|^2 \vec{N}(t),$$

$$\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right\rangle \text{ in } \mathbb{R}^2, \quad \nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \text{ in } \mathbb{R}^3,$$

$$D_{\vec{u}} f(x, y) = \nabla f(x, y) \cdot \vec{u}, \quad \operatorname{grad} f = \nabla f, \quad \operatorname{curl} \vec{F} = \nabla \times \vec{F}, \quad \operatorname{div} \vec{F} = \nabla \cdot \vec{F}$$