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Answer all 8 questions in 3 hours. Questions have the same weight.

Question 1:

In the computer market, the price of a computer normally depends on the speed that it can run. High speed computers normally have higher price. Assume that in the market 10% of computers have high price, 30% have medium price and 60% have low price. In the past, 90% of high price computers, 50% of medium price computers and 10% of low price computers have had high speed.

a) What is the probability that a randomly selected computer does not have high speed?

b) If a computer is high speed what is the probability that the computer does not have low price?

Question 2:

The receptionist of a company receives phone calls regularly. The time between phone calls in a typical day is known to be exponentially distributed with mean of 2 minutes.

a) What is the probability that the time between two received calls is between 2 and 5 minutes?

b) What is the probability that the receptionist receives four to six calls between 10:30AM up to 10:45 AM on a specific day?

Question 3:

The function $f_{XY}(x,y) = cxy^2$ is a joint probability density function over the range of $0 \le x \le 2,\ 0 \le y \le 5$.

a) Determine the constant "c". Are random variables X and Y independent? Justify your answer.

b) Determine the conditional density function of Y given $X = 1.5$. Calculate $E(Y)$ and $V(Y)$.

Question 4:

Transportation Safety Board of Canada (TSB) report shows that in Canada, the probability of avionic accident because of pilot error is 20%, because of icing is 25%, because of mechanical failure is 30% and the cause of rest of the accidents are unknown. Suppose there are 10 accidents and the type of each accident is independent from accident to accident. Let random variables X, Y, Z and W denote the number of accidents because of pilot error, icing, mechanical and unknown, respectively. Calculate the following:

a) $P(X = 3, Y = 3, Z = 2, W = 2)$ and $P(X = 8)$

b) $P(X \ge 7 | Y = 2)$

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Question 5:

Suppose 10 units of a certain type of processor are tested, and their failure times in hours are

300, 350, 370, 375, 380, 340, 400, 390, 385, and 410. Suppose the failure times are normally distributed.

a) Calculate the sample mean and the sample variance of the failure time of the processor.

b) Determine a 96% upper confident bound on the population mean.

Problem 6:

Suppose $\bar{X}_1$ and $\bar{S}_1^2$ are the sample mean and sample variance from a population with mean μ_1 and variance σ_1^2, and $\bar{X}_2$ and $\bar{S}_2^2$ are the sample mean and sample variance from a second independent population with mean μ_2 and variance σ_2^2. The sample sizes are n_1 and n_2, respectively.

a) $\bar{X}_1 + \bar{X}_2$ is an estimator of $\mu_1 + \mu_2$. Is it biased or unbiased? Justify your answer.

b) Find the standard error of $\bar{X}_1 + \bar{X}_2$.

Problem 7:

A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed, with standard deviation 0.25 volt. He/she selects a random sample of 8 units with sample mean of 4.95 volts.

a) Test the hypothesis that the average output voltage is 5 volts against that the average output voltage is not 5 volts. Use a 0.05 of significance and formulate appropriate hypothesis test.
b) Construct a 95% confidence interval on the mean output voltage. Discuss the results of parts (a) and (b).

Problem 8:

The weight of a certain product in mg is normally distributed, with standard deviation of $\sigma = 3$ mg. A random sample of 36 items is tested and the mean weight is obtained which is equal to 74 mg.

a) To test H0: $\mu = 75$ versus H1: $\mu > 75$, calculate P-value.
b) The probability of type I error is 6%. What is the probability of type II error if the true mean is 77?