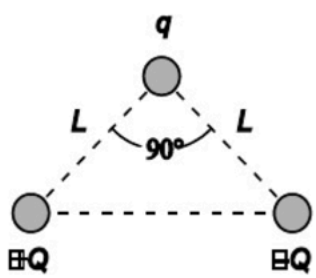
First name: Last name: ID #:

Notes:

1) In the entire exam, you need to start your solutions from the basic principles and show all details related to them on the booklet provided. Giving simple answers won't grant you any credit.
2) Transfer all your answers and details of your calculations to the booklet provided. The work on the exam sheets won't be evaluated; only the work on the booklet will be.
3) Please **answer in the order** of the given questions; reserve a space for a problem in the case you want to come back to tackle it (one page per problem should be sufficient)
4) Draw adequate, big enough, and clear pictures to support your proofs, where applicable. They can be given credit. A picture must take, at least, a quarter of the surface of a page, after dividing the page into four quarters.
5) The total of marks is 100.
6) There are 3 pages in total, including the formulae sheet on the back page

Q1 (20 pts) The exam is printed on both sides of papers.

(16 pts)If Q = 25 μC, q = 10 μC, the angle between the equal sides is 90°, and L = 40 cm in the figure, what is the magnitude of the electrostatic force on q? (4 pts)Draw a picture illustrating all the vectors and angles involved in your calculation.

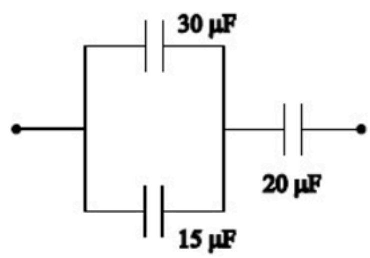


Q2 (18 pts) Theory question: (14 pts) Prove that, on an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature is the smallest. (4 pts) Draw an adequate picture.

Q3 (**14 pts**) (10 pts) Charge of uniform surface density (4.0 nC/m^2) is distributed on a spherical surface (radius = 2.0 cm). What is the total electric flux through a concentric spherical surface with a radius of 4.0 cm? (4 pts) Draw an adequate picture.

Q4 (14 pts) (10 pts) A proton (mass = 1.67×10^{-27} kg, charge = 1.60×10^{-19} C) moves from point A to point B under the influence of an electrostatic force only. At point A the proton moves with a speed of 50 km/s. At point B the speed of the proton is 80 km/s. Neglect the force of gravity on the proton and consider and only consider electric forces acting on the proton during this motion. Determine the potential difference $V_B - V_A$. (4 pts) Draw an adequate picture.

Q5 (20 pts) (16 pts) The picture shows three capacitors, 30-μF and 15-μF are connected in parallel, and the third capacitor has a 20-μF capacitance. What is the total energy stored in the group of capacitors shown if the charge on the 30-μF capacitor is 0.90 mC? (4 pts) Draw an adequate picture.



Q6 (14 pts) The temperature coefficient of resistivity of iron is 5.0×10^{-3} /°C; that of carbon is -0.50×10^{-3} /°C. When an iron wire and a carbon rod, each having the same 10-Ω resistance at 20 °C, are cooled from that temperature to −80 °C, the new ratio of the resistance of the carbon rod to the resistance of the iron wire at the lower temperature is

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

$$\vec{E} = \frac{\vec{F}_e}{q_0} = k_e \frac{q}{r^2}\hat{r}$$

$$k_e = \frac{1}{4\pi\epsilon_0} \qquad \vec{a} = \frac{q\vec{E}}{m}$$

$$\vec{E} = k_e \sum_i \frac{q_i}{r_i^2}\hat{r}_i$$

$$\vec{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2}\hat{r}_i = k_e \int \frac{dq}{r^2}\hat{r}$$

$$\Phi_E = \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A}$$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0} \qquad E = \frac{\sigma}{\epsilon_0}$$

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\vec{s}$$

$$V = \frac{U}{q_0} \qquad W = q\Delta V \qquad dV = k_e \frac{dq}{r}$$

$$\Delta V = -\int_A^B \mathbf{E} \cdot d\vec{\mathbf{s}} = -\mathbf{E} \cdot \int_A^B d\vec{\mathbf{s}} = -\mathbf{E} \cdot \vec{\mathbf{s}}$$

$$\Delta V = -E \int_A^B dx = -Ed$$

$$V = k_e \frac{q}{r} \qquad U = k_e \frac{q_1 q_2}{r_{12}}$$

$$E_x = -\frac{dV}{dx} \qquad E_r = -\frac{dV}{dr}$$

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

$$\wp = I\Delta V = I\varepsilon \qquad \tau = RC$$

$$\sum_{\text{junction}} I = 0 \qquad \sum_{\text{closed loop}} \Delta V = 0$$

$$q(t) = C\varepsilon(1 - e^{-t/RC}) = Q(1 - e^{-t/RC})$$

$$U = ½\, Q\varepsilon = ½\, C\varepsilon^2 \qquad q(t) = Qe^{-t/RC}$$

$$\vec{F}_B = q\vec{v} \times \vec{B} \qquad \vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$F_B = |q|\ v\ B \sin\theta$$

$$K = \frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m} \qquad \vec{F}_B = I\vec{L} \times \vec{B}$$

$$d\vec{F}_B = I\, d\vec{s} \times \vec{B} \qquad \vec{\tau} = I\vec{A} \times \vec{B}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\Delta V_H = E_H d = v_d B d$$

$$\Delta V_H = \frac{IB}{nqt} = \frac{R_H IB}{t}$$

1 u = 931.502 MeV/c^2
1 eV = 1.602×10^{-19} J
$m_p = 1.672 \times 10^{-27}$ kg
= 1.0073 u = 938.280 MeV/c^2
$m_n = 1.674 \times 10^{-27}$ kg
= 1.0087 u = 939.57 MeV/c^2
$m_e = 9.11 \times 10^{-31}$ kg
= 5.486×10^{-4} u = 0.511 MeV/c^2

$$C = \frac{Q}{\Delta V} \qquad \sigma_r = \sigma\left(1 - \frac{1}{\kappa}\right) \qquad C = \frac{ab}{k_e(b-a)}$$

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

$$C = \frac{\ell}{2k_e \ln(b/a)}$$

$$C = \frac{\epsilon_0 A}{d}$$

$$C_{eq} = C_1 + C_2 + C_3 + \dots \quad \text{(parallel combination)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \quad \text{(series combination)}$$

$$I = \frac{dQ}{dt} \qquad I_{avg} = \frac{\Delta Q}{\Delta t} = nqv_d A$$

$$J = \frac{I}{A} = nqv_d \qquad J = \sigma E \qquad R = \frac{\Delta V}{I}$$

$$R = \rho\frac{\ell}{A} \qquad J = nqv_d = \frac{nq^2E}{m_e}\tau$$

$$\rho = \rho_0[1 + \alpha(T - T_0)] \qquad \wp = I\Delta V$$

$$\wp = I^2R = \frac{(\Delta V)^2}{R} \qquad \rho = \frac{1}{\sigma}$$

$$K = \frac{1}{2}mV^2 \qquad K_R = \frac{1}{2}I\omega^2$$

$$a_c = \frac{v^2}{r}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\mathbf{s} \times \hat{r}}{r^2} \qquad B = \frac{\mu_0 I}{2\pi a}$$

$$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

$$B = \frac{\mu_0 I}{4\pi a}\theta \qquad B_x = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$$

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a} \qquad \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$$

$$B = \left(\frac{\mu_0 I}{2\pi R^2}\right)r \qquad (\text{for } r < R)$$

$$B = \frac{\mu_0 NI}{2\pi r} \qquad B = \mu_0 \frac{N}{\ell}I = \mu_0 nI$$

$$\mu = \left(\frac{e}{2m_e}\right)L$$

$$\mu_{spin} = \frac{e\hbar}{2m_e}$$

$$\mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24}\ \text{J/T}$$

$m_\alpha = 6.644656 \times 10^{-27}$ kg
= 4.00151 u
= 3727.409 MeV/c^2
e = 1.6×10^{-19} C
N = 6.02×10^{23} particles/mol
$c = 3.00\ 10^8$ m/s
$k_e = 8.9876 \times 10^9$ N.m^2/C^2
$\varepsilon_0 = 8.8542 \times 10^{-12}$ C^2/N.m^2
$\mu_0 = 4\pi \times 10^{-7}$ T.m/A
1T = 10^4 G

$$U_E = (\varepsilon_o E^2)/2; \quad C = \kappa C_o$$

$$\tau = p \times E;\ U = p \cdot E;\ E = E_o/\kappa$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \qquad \Delta V = E\ell = B\ell v$$

$$\mathcal{E} = -B\ell v \qquad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E}_L = -L\frac{dI}{dt} \qquad L = \frac{N\Phi_B}{I} \qquad L = -\frac{\mathcal{E}_L}{dI/dt}$$

$$L = \frac{N\Phi_B}{I} = \mu_0\frac{N^2}{\ell}A; \quad L = \mu\ l\ \ln(b/a)/2\pi$$

$$L = \mu_0\frac{(n\ell)^2}{\ell}A = \mu_0 n^2 A\ell = \mu_0 n^2 V$$

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}); \qquad \tau = \frac{L}{R}$$

$$I = \frac{\mathcal{E}}{R}e^{-t/\tau} = I_i e^{-t/\tau}$$

$$U = \frac{1}{2}LI^2 \qquad u_B = \frac{U}{V} = \frac{B^2}{2\mu_0} \qquad M_{12} = \frac{N_2\Phi_{12}}{I_1}$$

$$\mathcal{E}_1 = -M_{21}\frac{dI_2}{dt}$$

$$U = U_C + U_L = \frac{Q^2}{2C} + \frac{1}{2}LI^2$$

$$\omega = \frac{1}{\sqrt{LC}} \qquad Q = Q_{max}e^{-Rt/2L}\cos\omega_d t$$

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2} \qquad X_L = \omega L \qquad I_{max} = \frac{\Delta V_{max}}{X_L}$$

$$q = C\Delta V_{max}\sin\omega t$$

$$i_C = \omega C\Delta V_{max}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$X_C = \frac{1}{\omega C} \quad I_{max} = \frac{\Delta V_{max}}{X_C} \quad I_{max} = \frac{\Delta V_{max}}{Z}$$

$$\Delta v_R = I_{max}R\sin\omega t = \Delta V_R \sin\omega t$$

$$\Delta v_L = I_{max}X_L\sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L\cos\omega t$$

$$\Delta v_C = I_{max}X_C\sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C\cos\omega t$$

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} \qquad R_{eq} = \left(\frac{N_1}{N_2}\right)^2 R_L$$

$$\wp_{avg} = \frac{1}{2}I_{max}\Delta V_{max}\cos\phi \qquad \Delta v_2 = \frac{N_2}{N_1}\Delta v_1 \qquad I_1\Delta V_1 = I_2\Delta V_2$$

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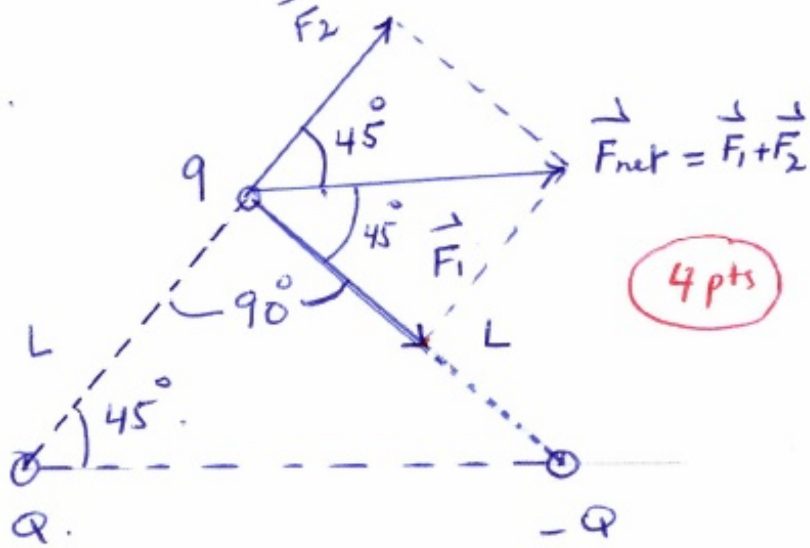
PHYS 205/01 section 01.

June 26, 2015.

solutions.



(4 pts)

Question 1 (20 pts)

$Q = 25 \mu C$

$q = 10 \mu C$.

$\theta = 90°$.

$L = 0.40$ m.

$$F_1 = F_2 = k_e \frac{|qQ|}{L^2}. \quad (4)$$

$$\vec{F_1} = k_e \frac{|qQ|}{L^2} (\cos 45° \hat{i} - \sin 45° \hat{j}) \quad (4)$$

$$\vec{F_2} = k_e \frac{|qQ|}{L^2} (\cos 45° \hat{i} + \sin 45° \hat{j}) \quad (4)$$

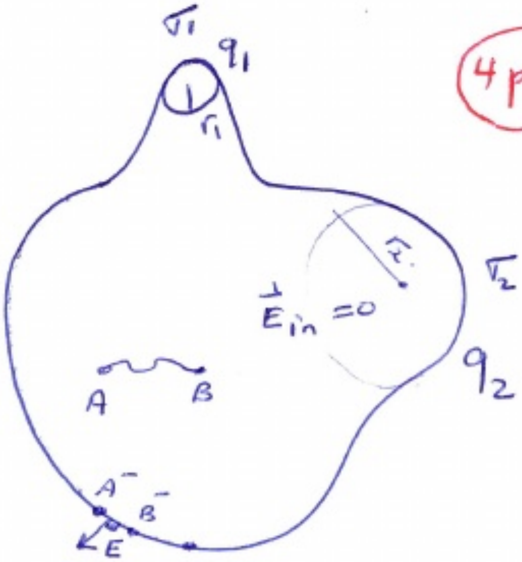
$$\vec{F}_{net} = \vec{F_1} + \vec{F_2} = k_e \frac{qQ}{L^2} (2 \cos 45°) \hat{i} \quad (2).$$

$$F_{net} = k_e \frac{qQ}{L^2} 2 \cos 45°.$$

$$= 9 \times 10^9 \frac{(10 \mu C)(25 \mu C)}{(0.40)^2 m^2} \times 2 \frac{\sqrt{2}}{2}.$$

$$= \frac{9 \times 10 \times 25 \times \sqrt{2}}{(0.40)^2} \times 10^9 \times 10^{-12} \text{ N}.$$

Question 2 (18 pts)



(4pts)

A conductor in equilibrium is an equipotential.

- $\Delta V = \int_A^B \vec{E}_{inside} \cdot \vec{ds} = 0$

  because $\underline{\overrightarrow{E_{inside}}} = 0$

- $\Delta V = 0 \Rightarrow V_B = V_A$
  for all A, B in the conductor.

Also
$$\Delta V = \int_{A^-}^{B^-} \vec{E}_\perp \cdot \vec{ds} = 0.$$

because $\vec{E}_\perp \perp \vec{ds}$
$\vec{ds}$ is on the surface.

$\Rightarrow V_{B^-} = V_{A^-} = V_B' = V_A$. (3)

Now:
$V_1 = k_e \frac{q_1}{r_1}$; $V_2 = k_e \frac{q_2}{r_2}$ (spheres of radii r_1 and r_2).

equipotential $V_1 = V_2$

$\Rightarrow k_e \frac{q_1}{r_1} = k_e \frac{q_2}{r_2}$ (3)

$\Rightarrow \frac{q_1}{q_2} = \frac{r_1}{r_2}$ (4)

but $q_1 = 4\pi r_1^2 \sigma_1$
and $q_2 = 4\pi r_2^2 \sigma_2$

$\Rightarrow \frac{q_1}{q_2} = \frac{4\pi r_1^2 \sigma_1}{4\pi r_2^2 \sigma_2} = \frac{r_1}{r_2}$

(4)

$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$

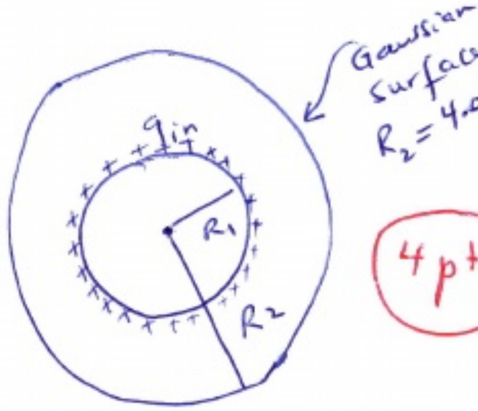
if $r_1 < r_2 \Rightarrow \sigma_1 > \sigma_2$ ∴

Question 3 (10 pts)

$\sigma = 4.0 \, nC/m^2$.

$R_1 = 2.0$ cm.

$R_2 = 4.0$ cm.



4 pts

$$\phi = \frac{q_{in}}{\epsilon_0}. \quad (6)$$

$$= \frac{\sigma \times 4\pi R_1^2}{\epsilon_0}.$$

$$= \frac{4.0 \times 10^{-9} \times 4\pi \, (2 \times 10^{-2})^2}{8.85 \times 10^{-12}} = 2.3 \, \frac{N}{C} m^2.$$

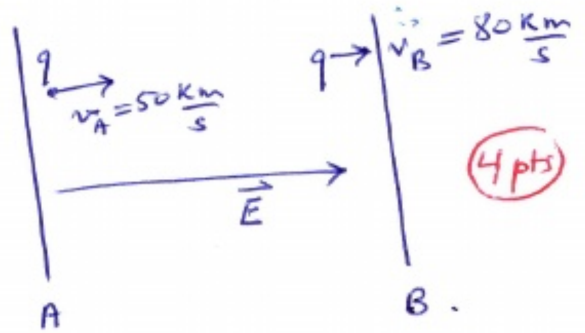
Question 4 (14 pts)

$m_p = 1.67 \times 10^{-27}$ kg.

$q = 1.6 \times 10^{-19}$ C.

$V_A = 50$ km/s

$V_B = 80$ km/s



4 pts

$V_B - V_A = ?$

Isolated system. (2)

$\Delta K + \Delta U = 0$

$\Delta K = -\Delta U = -q \Delta V$. (2)

$\frac{\Delta K}{-q} = \Delta V = V_B - V_A$.

$$\Rightarrow \Delta V = \frac{\frac{1}{2} m_p (v_B^2 - v_A^2)}{-q} \quad (4)$$

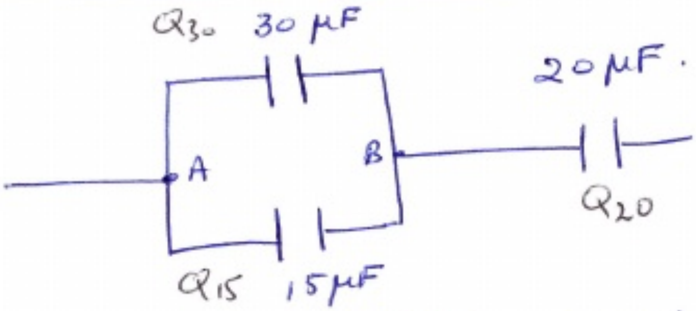
$$\Delta V = \frac{1}{2} \frac{1.67 \times 10^{-27} ((80)^2 - (50)^2) 10^6}{-1.6 \times 10^{-19}}$$

$\Delta V = -20.35$ V

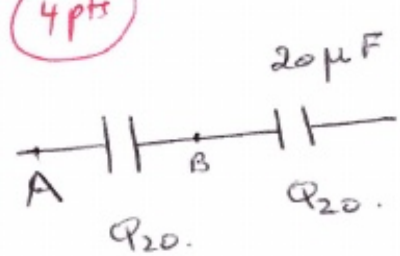
(2)

$\Delta V \simeq -20$ V.

Question 5 (20 pts)



(4 pts)



$$\Delta V = V_{ab} = \frac{Q}{C} = \frac{0.9\,mC}{30\,\mu F} = 30\,V. \quad (4)$$

$$Q_{15} = C_{15} V_{ab} = (15\,\mu F)(30\,V). \quad (4)$$

$$Q_{15} = 0.450\ mC.$$

$$Q_{20} = Q_{15} + Q_{30} = 0.450\,mC + 0.90\,mC. \quad (6)$$

$$Q_{20} = 1.35\ mC.$$

$$U_{total} = U_{30} + U_{15} + U_{20} = \frac{Q_{30}^2}{2C_{30}} + \frac{Q_{15}^2}{2C_{15}} + \frac{Q_{20}^2}{2C_{20}} \quad (2)$$

$$= \frac{(0.9\ 10^{-3})^2}{2\times 30\times 10^{-6}} + \frac{(0.450\times 10^{-3})^2}{2\times 15\times 10^{-6}} + \frac{(1.35\times 10^{-3})^2}{2\times 20\times 10^{-6}}$$

$$= 0.0659\,J. \quad (4)$$

Question 6 (14 pts).

$$\alpha_{Fe} = 5.0\times 10^{-3}/^\circ C.$$

$$\alpha_{C} = -0.50\times 10^{-3}/^\circ C.$$

$$R_{Fe} = R_C = 10\,\Omega. \text{ at } 20^\circ C.$$

$$T_0 = 20^\circ C \ ; \ T = -80^\circ C. \quad (10)$$

$$\Delta T = T - T_0 = -80^\circ C - 20^\circ C = -100^\circ C.$$

$$\left.\frac{R_C}{R_{Fe}}\right|_{T=-80^\circ C.} = \left(\frac{10(1-0.5\times 10^{-3}(-100))}{10(1+5\times 10^{-3}(-100))}\right) = \frac{1.05}{0.5} = 2.1.$$