There are **TWENTY FIVE problems** in all, each carrying **4 marks**.

There are **THREE** types of problems:

1. For each of the problems 1 to 12, **indicate your choice** by mentioning **one of the letter (a) to (d) only**. There is no need to provide an explanation.

2. For each of the problems 13 to 22, **provide suitable text only for the blank space** so that the resulting statement is correct. There is no need to provide an explanation.

3. For each of the problems 23 to 25, provide a solution. You must show **all steps** of your solution.

1. (a). **5.** (d). **9.** (b).

2. (c). **6.** (c). **10.** (b).

3. (d). **7.** (d). **11.** (a).

4. (d). **8.** (b). **12.** (a).

13. [4 MARKS]

(a) 64.

(b) $3^2 \cdot 7$.

Marking Scheme: 2 marks for each part. There are no partial marks.

14. [4 MARKS]

Let the universe of discourse for x, y, and z be **Z**. Let $P(x, y, z)$ denote $xy^2 = z$. A **counterexample** to $\forall x \, \forall z \, \exists y \, P(x, y, z)$ is _____.

SOLUTION.

A counterexample is a negation of the given logical expression. There are many possible counterexamples. For example, a combination of x and z that makes z/x a negative integer or a non-integer rational number would work.

$x = 1$ and $z = -1$.
$x = 2$ and $z = 3$.
. . .

Marking Scheme: There are partial marks. Providing specific values for x, y, and z such that $xy^2 \neq z$ is not a counterexample because that would mean that $xy^2 \neq z$ for that specific y. There is deduction of 2 marks if the value of y is "hardwired".

15. [4 MARKS]

5.

Marking Scheme: There are no partial marks.

16. [4 MARKS]

64.

Marking Scheme: There are no partial marks.

17. [4 MARKS]

$n = 2$.

Marking Scheme: There are no partial marks.

18. [4 MARKS]

$S = \{x \mid -1 \leq x < 0 \vee 0 < x \leq 1\}$.

Marking Scheme: There are partial marks. 2 marks if any one of two correct intervals of S is given.

19. [4 MARKS]

3.

Marking Scheme: There are no partial marks.

20. [4 MARKS]

0 or 2.

Marking Scheme: There are partial marks. 2 marks for each correct answer.

21. [4 MARKS]

$\{\}$.

Marking Scheme: There are no partial marks.

22. [4 MARKS]

$\mathbf{Z} \times \mathbf{Z}$.

Marking Scheme: There are no partial marks.

23. [4 MARKS]

(a) The equivalence classes of R:

$[2]_R = \{2, 7, 12\}$.
$[3]_R = \{3, 8\}$.
$[5]_R = \{5, 10\}$.

(b) $\{\}$.

Marking Scheme: 1 marks for each correct equivalence class in (a). 1 mark for (b). There are no partial marks.

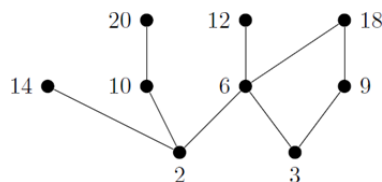**24. [4 MARKS]**

(a) For n = 0. $f_{3 \cdot 0} = f_0 = 0$.

(b) For n > 0. $f_{3n} = f_{3n-1} + f_{3n-2} = (f_{3n-2} + f_{3n-3}) + f_{3n-2} = 2f_{3n-2} + f_{3n-3} = 2f_{3n-2} + f_{3(n-1)}$.

Marking Scheme: 1 mark for (a) and 3 marks for (b) for each correct value or expression. There are no partial marks.

25. [4 MARKS]

(a) 20 12 18

or

(b) It does not exist.

(c) 2.

(d) It is not a lattice.

Marking Scheme: 1 marks for each part. There are no partial marks.