Note: There are no official solutions for the past finals I uploaded. This file contains only final answers without any explanations. I hope it is still useful in case you want to check your own solutions, but remember that in the exam you have to show your work and justify some steps. If you get stuck or do not know how to solve some exercise, you can ask me (Francesc) directly. It is important that you try to solve all the exercises of these past finals, as your exam is going to have similar questions.

Answers for the final of December 2017

1.

(a) $x = 6$

(b) $f^{-1}(x) = \sqrt[3]{e^{x-1} + 8}$ with domain $(-\infty, +\infty)$ and range $(2, +\infty)$.

2.

(a)

$$\lim_{x\to 2^-} f(x) = -5 \qquad \text{and} \qquad \lim_{x\to 2^+} f(x) = 5\ .$$

(b) No (no limit).

3. Horizontal asymptotes: $y = 0$ and $y = 4$. No vertical asymptotes.

4.

(a) $\ln(x) + 1 + \frac{1}{x}$.

(b)

$$\frac{\frac{2x}{\sqrt{1-(x^2)^2}}\sqrt{1-x^2} - \arcsin\left(x^2\right)\frac{-2x}{2\sqrt{1-x^2}}}{1-x^2} + 2e^2x\ .$$

(c)

$$\frac{1}{\left(x^2-2x\right)\ln\left(x^3+3x\right)}\left[(2x-2)\ln\left(x^3+3x\right) + \left(x^2-2x\right)\frac{1}{x^3+3x}\left(3x^2+3\right)\right].$$

(d)

$$\frac{\left(e^x - e^{-x}\right)\left(\cos(x)+\sin(x)\right) - \left(e^x + e^{-x}\right)\left(-\sin(x)+\cos(x)\right)}{\left(\cos(x)+\sin(x)\right)^2}\ .$$

(e)

$$\left(1+x^2\right)^{x^2+1}\left[2x\ln\left(1+x^2\right)+\left(x^2+1\right)\frac{1}{1+x^2}2x\right].$$

5.

(a)

$$\lim_{h\to 0}\frac{\sqrt{1+2(x+h)}-\sqrt{1+2x}}{h}=\cdots=\frac{1}{\sqrt{1+2x}}.$$

(b) $L(x)=\frac{1}{3}(x-4)+3.$

(c) $\sqrt{10}\approx L\left(\frac{9}{2}\right)=\frac{19}{6}.$

6.

(a) $m=2.$

(b) $c=-\sqrt{\frac{4}{3}}.$

7.

(a)

$$\left.\frac{dy}{dx}\right|_{(x,y)=(2,1)}=\frac{4}{5}.$$

(b) $y=\frac{4}{5}(x-2)+1.$

8.

(a) Absolute mininum 0 (at $x=0$); absolute maximum 1 (at $x=1$).

(b) $196\,\mathrm{cm}^2/\mathrm{s}.$

(c) $\frac{1}{2}.$

9.

(a) $(x_0,y_0)=(1,0).$

(b) $V_{\max}=4000\,\mathrm{cm}^3.$

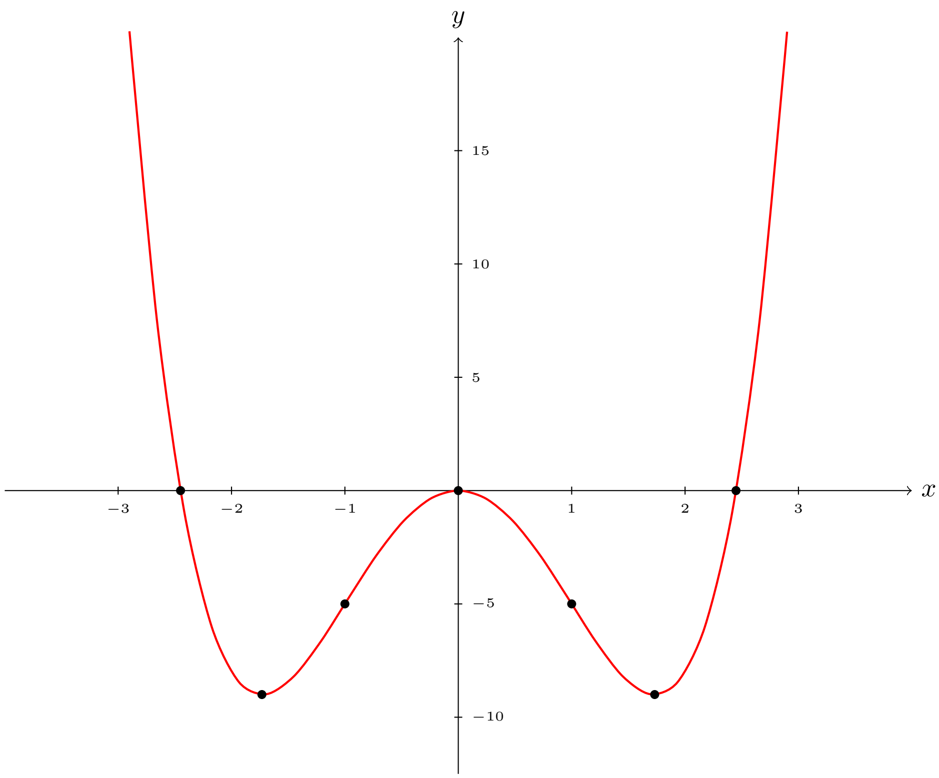
10.

(a) Domain $(-\infty, \infty)$; f is even; no asymptotes (but $\lim_{x \to \pm\infty} f(x) = +\infty$).

(b) $f'(x) = 4x^3 - 12x$. f is increasing on $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$ and decreasing on $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$. Local minima $(\pm\sqrt{3}, -9)$ and local maximum $(0, 0)$.

(c) $f''(x) = 12x^2 - 12$. f is concave up on $(-\infty, -1) \cup (1, \infty)$ and concave down on $(-1, 1)$. Inflection points $(\pm 1, -5)$.

(d) Local extrema, inflection points and asymptotes computed above. We can also compute some more points: $(\pm\sqrt{6}, 0)$, ...



Bonus. $f^{(n)}(x) = 2^n e^{2x} + (-2)^n e^{2x}$; $f^{(10)}(0) = 2^{11} = 2048$.