ENGR-233 MOCK FINAL EXAM

Problem 1. Find the equation of the plane passing through the points $A(1,3,-2),\ B(3,-4,1),\ C(-1,2,1)$.

Problem 2. A planet of mass m moves around a star of mass M. The planet orbit is assumed to be a circle, the star being at its center.

(a) Suppose the orbit radius is R. Find the period of the planet (the duration of its "year").

(b) Suppose the speed of the planet is v. Find the radius R of the orbit.

(Hint: The force acting on the planet $\mathbf{F} = -GmM\frac{\mathbf{r}}{||\mathbf{r}||^3}$ where $\mathbf{r}$ is the vector connecting the star and the planet, and G is the gravity constant; the planet acceleration is defined by the 2-d Newton's Law $\mathbf{F} = m\mathbf{r}''$.)

Problem 3. (a) Find the divergence of the field $\mathbf{F} = (x^2 - y^2)\mathbf{i} + xyz\mathbf{j} + (z^2 - x^2)\mathbf{k}$ at the point $(1,2,3)$.

(b) Find the curvature of the curve defined by the parametric equations $x = e^t \cos t,\ y = e^t \sin t,\ z = e^t$ at the point $(1,0,1)$.

Problem 4. Consider the plane velocity field

$$\mathbf{u} = \left(\frac{y}{(x-1)^2 + y^2} + \frac{y}{(x+1)^2 + y^2}\right)\mathbf{i} - \left(\frac{x-1}{(x-1)^2 + y^2} + \frac{x+1}{(x+1)^2 + y^2}\right)\mathbf{j}.$$

(a) Find div $\mathbf{u}$;

(b) Find curl $\mathbf{u}$.

Problem 5. Find $\int\limits_C \sin y\,dx + \cos x\,dy$ where C is a union of the line segments from $(0,0)$ to $(0,\pi/2)$ and from $(0,\pi/2)$ to $(\pi/2,\pi/2)$.

Problem 6. (a) Find $\int\limits_C x^2y^2ds$ where C is the line $x = 2\cos t, y = 2\sin t,\ 0 \le t \le \pi/3$.

(b) Find the flux of the field $\mathbf{F} = (x^2 - y^2)\mathbf{i} + (y^2 - z^2)\mathbf{j} + (z^2 - x^2)\mathbf{k}$ through the surface of the sphere $x^2 + y^2 + z^2 = 4$ (use the Divergence Theorem).

Problem 7. (a) Find $\oint\limits_C \mathbf{F} \cdot ds$ if $\mathbf{F} = e^x \cos y\mathbf{i} - e^x \sin y\mathbf{j}$, and C is the circle $x^2 + (y - \pi)^2 = \pi^2$.

(b) Find the work done by the force $\mathbf{F} = y\mathbf{i} - x\mathbf{j}$ along the circle $(x - 1)^2 + y^2 = 1$ (use the Green's Theorem).

Problem 8. Evaluate $\iiint\limits_R xyzdV$ where R is a polyhedron bounded by the planes $x = 0,\ y = 0,\ z = 0,\ x + y + z = 1$.

Problem 9. (a) For the vector field $\mathbf{u} = \frac{x\mathbf{i}+y\mathbf{j}+z\mathbf{k}}{(x^2+y^2+z^2)^{3/2}}$, find div $\mathbf{u}$.

(b) For the same field, find $\iint\limits_S \mathbf{u} \cdot \mathbf{n}ds$ where S is the sphere $x^2 + y^2 + z^2 = 1$, and $\mathbf{n}$ is the unit outer normal vector to S.

(c) Explain why the results of (a) and (b) don't contradict the Divergence Theorem.

Problem 10. Using cylindrical coordinates, find the volume of the body of revolution formed by rotation of the disk $(x - 1)^2 + z^2 < 1$ around the z-axis (draw a picture).