Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	204/2	All, except EC
Examination	**Date**	**Pages**
Final	December 2012	2

Instructors	Course Examiner
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Special Instructions: ▷ Only approved calculators are allowed.

Answer 10 questions. All questions have equal value.

1. Using the Gauss-Jordan method (i.e. reduced row echelon form method), find all the solutions of the following system of equations

$$\begin{aligned} 2x - 2y \quad\quad + 2u + 3v &= 1 \\ 3x - 3y - z + 5u + 2v &= 3 \\ 2x - 2y - 2z + 6u \quad\quad &= -2. \end{aligned}$$

2. Let $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 2 & 0 \end{bmatrix}$.

a) Calculate M^{-1}.

b) Find the matrix C such that $MC = B$.

3. a) Use Cramer's rule to solve the following system of equations

$$\begin{aligned} 2x + 3y \quad\quad &= -2 \\ x \quad\quad + 3z &= -1 \\ 2y + z &= 2. \end{aligned}$$

(No marks given if you don't use Cramer's rule.)

b) Calculate the determinant of the matrix $\begin{bmatrix} 1 & 2 & 0 & 2 \\ 1 & 0 & 2 & 3 \\ 0 & 3 & 1 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix}$.

4. Let $\mathcal{L}$ be the line with parametric equations $x = 2 + t$, $y = 1 - t$, $z = 1 + 3t$. and et $\mathbf{v} = (1, 2, 0)$. Find vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ such that $\mathbf{v} = \mathbf{w}_1 + \mathbf{w}_2$, and such that $\mathbf{w}_1$ is parallel to $\mathcal{L}$ and $\mathbf{w}_2$ is perpendicular to $\mathcal{L}$.

5. Let $P_1 = (1, -1, 1)$, $P_2 = (2, 1, -1)$ and $P_3 = (1, -2, -1)$.

   a) Find the area of the triangle with vertices P_1, P_2 and P_3.

   b) Find an equation of the plane containing P_1, P_2 and P_3.

6. Let $\mathcal{L}$ be the line with parametric equations $x = 1 + 2t$, $y = 2 - 3t$, $z = 3 - t$, and let $\mathcal{P}$ be the plane $x + y + z - 10 = 0$.

   a) Prove that $\mathcal{L}$ and $\mathcal{P}$ are parallel.

   b) Find the distance between $\mathcal{L}$ and $\mathcal{P}$.

7. Let $\mathbf{v}_1 = (2, -1, -2)$ and $\mathbf{v}_2 = (1, 2, -2)$.

   a) Find scalars x and y such that $x\mathbf{v}_1 + y\mathbf{v}_2 = (5, -10, -2)$.

   b) Find a vector $\mathbf{v}_3$ such that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis of $\mathbb{R}^3$. Justify your answer.

8. Let

$$A = \begin{bmatrix} 1 & 0 & -2 & 0 & -1 & 3 \\ 0 & 1 & 3 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 & -2 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \\ z \\ u \\ v \\ w \end{bmatrix}.$$

   Find a basis for the solution space of the homogeneous system of linear equations $AX = 0$.

9. Find the standard matrix for the composition of the following two linear operators on $\mathbb{R}^2$: A rotation counterclockwise of 30^o followed by a reflection about the y-axis.

10. Let $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

11. Let $A = \begin{bmatrix} -1 & 1 \\ -3 & 5/2 \end{bmatrix}$. Calculate A^{1000}.