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1. (1 point) Find dy/dx in terms of x and y if $\cos^2(7y)+\sin^2(7y)=y+3$.

$\frac{dy}{dx}=$ ____________________

Solution:

SOLUTION

Using the relation $\cos^2\theta+\sin^2\theta=1$, the equation becomes: $1=y+3$ or $y=-2$. Hence, $\frac{dy}{dx}=0$.

Correct Answers:

- `0`

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2. (1 point) Find the slope of the tangent line to the curve (a lemniscate)

$$2(x^2+y^2)^2=25(x^2-y^2)$$

at the point $(3,1)$.

The slope of the lemniscate at the given point is ___.

Solution: Implicit differentiation gives

$$4(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right)=25\left(2x-2y\frac{dy}{dx}\right),$$

or

$$y\big(25+4(x^2+y^2)\big)\frac{dy}{dx}=x\big(25-4(x^2+y^2)\big),$$

and so

$$\frac{dy}{dx}=\frac{x\big(25-4(x^2+y^2)\big)}{y\big(25+4(x^2+y^2)\big)}.$$

If $x=3$ and $y=1$, then $x^2+y^2=10$, and therefore

$$\left.\frac{dy}{dx}\right|_{(3,1)}=\frac{(3)(25-40)}{(1)(25+40)}=-\frac{9}{13}$$

is the slope of the tangent line to the lemniscate at the point $(3,1)$.

Correct Answers:

- `-9/13`

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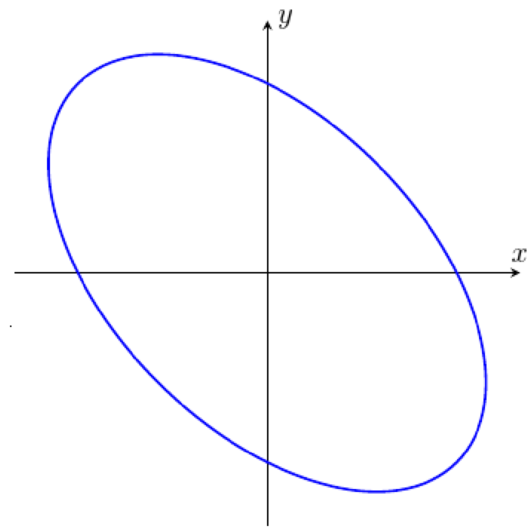
3. (1 point) The graph of the equation $x^2+xy+y^2=9$ is an ellipse lying obliquely in the plane, as illustrated in the figure below.



a. Compute $\frac{dy}{dx}$.

$\frac{dy}{dx}=$ __________.

b. The ellipse has two horizontal tangents. Find an equation of the lower one.

The lower horizontal tangent line is defined by the equation $y=$ _____.

c. The ellipse has two vertical tangents. Find an equation of the rightmost one.

The rightmost vertical tangent line is defined by the equation $x=$ _____.

d. Find the point at which the rightmost vertical tangent line touches the ellipse.

The rightmost vertical tangent line touches the ellipse at the <u>**point**</u> ____________.

Hint: The horizontal tangent is of course characterized by $\frac{dy}{dx}=0$. To find the vertical tangent use symmetry, or solve $\frac{dx}{dy}=0$.

Solution: Differentiating implicitly with respect to x gives

$$2x+y+x\frac{dy}{dx}+2y\frac{dy}{dx}=0,$$

and so

$$\frac{dy}{dx}=-\frac{2x+y}{x+2y}.$$

The tangent line to the ellipse is horizontal where $\frac{dy}{dx}=0$, i.e., where

$$2x+y=0, \qquad \text{or} \qquad x=-\tfrac{1}{2}y.$$

Combining this with the original equation gives

$$\tfrac{1}{4}y^2-\tfrac{1}{2}y^2+y^2=9, \qquad \text{i.e.,} \qquad \tfrac{3}{4}y^2=9, \qquad \text{or} \qquad y^2=12.$$

Hence, the lower horizontal asymptote is defined by the equation $y=-\sqrt{12}$, or $y=-2\sqrt{3}$.

By symmetry the vertical asymptotes occur where $x^2=12$, and the rightmost one is defined by the equation $x=2\sqrt{3}$. This vertical tangent line touches the ellipse at the point where $y=-\frac{1}{2}x=-\frac{1}{2}\left(2\sqrt{3}\right)=-\left(\sqrt{3}\right)$, i.e., at the point $\left(2\sqrt{3},-\left(\sqrt{3}\right)\right)$.

Correct Answers:

- `-[(2*x+y)/(x+2*y)]`
- `-2*sqrt(3)`
- `2*sqrt(3)`
- `(2*sqrt(3),-[sqrt(3)])`

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4. (1 point) For the equation given below, evaluate y' at the point $(1,2)$.

$$4e^{xy}-5x=y+22.6.$$

y' at $(1,2)$ = __________

Correct Answers:

- `-1.8949440949046`

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5. (1 point) Let

$$f(x)=(\ln x)^7$$

$f'(x)=$ ____________________

$f'(e^2)=$ ____________________

Correct Answers:

- `7 / x *(ln(x))^(7 - 1)`
- `60.6302069104801`

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6. (1 point) Let

$$f(x)=\ln[x^3(x+3)^7(x^2+3)^9]$$

$f'(x)=$ ____________________

Correct Answers:

- `3/x+7/(x+3)+2*x*9/(x^2+3)`

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7. (1 point) Evaluate $\dfrac{d}{dx}\sqrt[4]{\ln(10-x^2)}$ at $x=1$.

Answer: __________

Correct Answers:

- `2*[ln(9)]^(-1+1/4)/-36`

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8. (1 point) Find dy/dx in terms of x and y if $ax^3-by^2=c^3$. Assume that a, b and c are constants.

$\dfrac{dy}{dx}=$ ________

Solution: Differentiating both sides,

$$3ax^2-2by\frac{dy}{dx}=0,$$

so

$$\frac{dy}{dx}=\frac{3ax^2}{2by}.$$

Correct Answers:

- `3*a*x^2/(2*b*y)`

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9. (1 point) Find $\dfrac{dy}{dx}$ for each of the following functions

$$y=\ln\left(\frac{4x-14}{x\sqrt[4]{x^2+1}}\right)$$

$\dfrac{dy}{dx}=$ ____________________

$$y=x^{\cos(x)}$$

$\dfrac{dy}{dx}=$ ____________________

Correct Answers:

- `4/(4*x-14)-1/x-(2*x)/(4*(x^2+1))`
- `x^(cos(x))*(cos(x)/x - sin(x)*ln(x))`

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10. (1 point) Let $f(x)=\log_4(6x^2-4x-4)$. Find $f'(x)$.

$f'(x)=$ _________________________

Correct Answers:

- `0.721348/(6*x^2-4*x-4)*(12*x-4)`