Taylor Series:

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1}-x_i) + \frac{f''(x_i)}{2!}(x_{i+1}-x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i+1}-x_i)^3 + \cdots + \frac{f^{(n+1)}(\xi)}{(n+1)!}(x_{i+1}-x_i)^{n+1}$$

Root finding: False Position:

$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

Root finding: Newton-Raphson:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Secant:

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1}-x_i)}{f(x_{i-1}) - f(x_i)}$$

Modified Newton-Raphson:

$$x_{i+1} = x_i - \frac{f(x_i)f'(x_i)}{[f'(x_i)]^2 - f(x_i)f''(x_i)}$$

System of linear algebraic equations: [A] {X} = {B}. Set [L] [U] = [A] with [L] and [U] as follows:

LU (Doolittle) decomposition:

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}; \quad [U] = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Crout decomposition:

$$[L] = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}; \quad [U] = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Gauss-Seidel: sufficient condition for convergence: $|a_{ii}| > \sum_{j=1;j\neq i}^{n} |a_{ij}|$

Polynomial least squares regression analysis:

$y = a_0 + a_1 x + a_2 x^2 + ..$

2x2 matrix (only a_0, a_1): linear,

3x3 matrix: quadratic, etc.

Quadratic Regression:

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{Bmatrix};$$

where $\sum = \sum_{i=1}^{n}$

Error Analysis for linear regression:

Standard error of the estimate: $s_{y/x} = \sqrt{\frac{S_r}{n-2}}$

$$S_r = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)^2$$

Non linear regression … (fill in)

Lagrange interpolating polynomials: $f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$; where $L_i(x) = \prod_{j=0, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}$

$$f_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

Newton interpolating polynomials: ….(fill in)

Numerical integration: $\int_a^b f(x)dx$: trapezoidal: $I = \frac{h}{2}\left(f(x_0) + \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right)$

Simpson's 3/8

…..(fill in)

Simpson's 1/3

$$I = \frac{h}{3}\left(f(x_0) + 4\sum_{i=1,3,5..}^{n-1} f(x_i) + 2\sum_{j=2,4,6,..}^{n-2} f(x_j) + f(x_n) \right)$$

Gauss quadrature (Gauss-Legendre polynomials): $\int_a^b g(x)dx = \int_{-1}^{1} f(x)dx = c_0 f(x_0) + c_1 f(x_1) + ... + c_{n-1} f(x_{n-1})$

Romberg: ….(fill in)

Numerical differentiation using finite divided difference formulas:

Forward: $f'(x_i) = \left(f(x_{i+1}) - f(x_i)\right)/h + O(h)$; $f'(x_i) = \left(-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)\right)/(2h) + O(h^2)$

$f''(x_i) = \left(f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)\right)/h^2 + O(h)$; $f''(x_i) = \left(-f(x_{i+3}) + 4f(x_{i+2}) - 5f(x_{i+1}) + 2f(x_i)\right)/h^2 + O(h^2)$

Backward: $f'(x_i) = \big(f(x_i) - f(x_{i-1})\big)/h + O(h)$; $f'(x_i) = \big(3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})\big)/(2h) + O(h^2)$ $f''(x_i) = \big(f(x_i) - 2f(x_{i-1}) + f(x_{i-2})\big)/h^2 + O(h)$; $f''(x_i) = \big(2f(x_i) - 5f(x_{i-1}) + 4f(x_{i-2}) - f(x_{i-3})\big)/h^2 + O(h^2)$
Centered: $f'(x_i) = \big(f(x_{i+1}) - f(x_{i-1})\big)/(2h) + O(h^2)$ $f'(x_i) = \big(-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})\big)/(12h) + O(h^4)$ $f''(x_i) = \big(f(x_{i+1}) - 2f(x_i) + f(x_{i-1})\big)/h^2 + O(h^2)$ $f''(x_i) = \big(-f(x_{i+2}) + 16f(x_{i+1}) - 30f(x_i) + 16f(x_{i-1}) - f(x_{i-2})\big)/(12h^2) + O(h^4)$ $f'''(x_i) = \big(f(x_{i+2}) - 2f(x_{i+1}) + 2f(x_{i-1}) - f(x_{i-2})\big)/(2h^3) + O(h^2)$
System of nonlinear equations … (fill in)
Solving ODEs (initial value problems): $y' = f(x, y)$; y_0 given: **Euler:** $y_{i+1} = y_i + f(x_i, y_i)h$ RK2 methods: **Heun's**: …(fill in) **Midpoint**: $y_{i+1} = y_i + k_2 h$, where $k_1 = f(x_i, y_i)$ and $k_2 = f(x_i + h/2, y_i + k_1 h/2)$ **Ralston**: …(fill in) **Classical RK4**: $y_{i+1} = y_i + (k_1 + 2k_2 + 2k_3 + k_4)h/6$, where $k_1 = f(x_i, y_i)$; $k_2 = f(x_i + h/2, y_i + k_1 h/2)$; $k_3 = f(x_i + h/2, y_i + k_2 h/2)$ and $k_4 = f(x_i + h, y_i + k_3 h)$