COMP 232, Winter 2019 **CONCORDIA UNIVERSITY**

MATHEMATICS FOR COMPUTER SCIENCE
Assignment 2.
Due: February 22, 2019.

IMPORTANT NOTE: Proofs must be presented with great clarity, using proper notation, so that an informed reader can easily understand it.

1. Consider the implication
$$p \wedge (\neg q \rightarrow \neg p) \;\Rightarrow q$$
   (a) Prove the implication by a direct proof, *i.e.* assume that the left hand side of the implication is True, and then show that the right hand side of the implication must be True also,
   (b) Prove the implication by contradiction.

2. For each of the statements below state whether it is *True* or *False*. If *True* then give a proof. If *False* then give a counterexample.
   (a) If $a, b \in \mathbb{Z}^+$ are odd then the product ab is odd.
   (b) If $a, b \in \mathbb{Z}^+$, where a is even and b is odd then $a + b$ is odd.
   (c) Let $a, b \in \mathbb{Z}^+$. If $a + b$ is even, then a^2 or b^2 is even.
   (d) For all $a \in \mathbb{Z}^+$, if $a > 3$ then $a^2 - 4$ is composite.

3. Let $n \in \mathbb{Z}^+$. Prove that the following statements are equivalent:
   (a) n^3 is odd
   (b) n^2 is odd
   (c) $1 - n$ is even
   (d) $n^2 + 1$ is even

4. (a) Use a proof by cases to show that 10 is not the square of a positive integer.
   (b) Prove by contraposition that if n is an integer and $3n + 2$ is even, then n is even.

5. For each of the statements below state whether it is *True* or *False*. If *True* then give a proof. If *False* then explain why, *e.g.*, by giving a counterexample.

   (a) The sum of the squares of any two rational numbers is a rational number.

   (b) For all positive $x \in \mathbb{R}$, if x is irrational then $\sqrt{x}$ is irrational.

   (c) For all $x, y \in \mathbb{R}$, if x and y are irrational then $x^2 + y$ is irrational.

   (d) $\log_{10}(2)$ is irrational.

6. Give a proof of each of the following:

   (a) If the integers $1,\ 2,\ 3,\ \cdots,\ 7,$ are placed around a circle, in any order, then there exist two adjacent integers that have a sum greater than or equal to 9 .

   (b) If the integers $1,\ 2,\ 3,\ \cdots,\ 16$ are placed around a circle, in any order, then there exist three integers in consecutive locations around the circle that have a sum greater than or equal to 27.

7. For each of the following, determine whether it is valid or invalid. If valid then give a proof. If invalid then give a counter example.

   (a) $(A \cap B) \cup (C \cap D) \ = \ (A \cap D) \cup (C \cap B)$

   (b) $A - (B \cup C) = (A - B) \cap (A - C)$

   (c) $B \cap C \subseteq A \quad \Rightarrow \quad (C - A) \cap (B - A)$ is empty

   (d) $(A \cup B) - (A \cap B) = A \quad \Rightarrow \quad B$ is empty