CONCORDIA UNIVERSITY
FACULTY OF ENGINEEING AND COMPUTER SCIENCE

ENGR 233: Applied Advanced Calculus - Winter 2016
Tuesday, March 1, 2016 Midterm Test (A) Time: 90 Minutes

DIRECTIONS:

- Books, and Notes are NOT allowed. Formula sheet provided. No Calculators.
- All seven questions are equally valued at 10 point each. Maximum mark is 60.
- **Solve any six questions or try all seven questions**. The best six marks will be recorded.
- Submit all sheets and papers at the end of the test.

1. Find the equation of the line l which is parallel to the plane P_1: $x + y + z = 2$, and the plane P_2: $x - 2y - z = 3$ and contains the point p (1,2,3).

2. Given two curves $C_1 : (x-1)^2 + y^2 = 2$, and $C_2 : (x+1)^2 + y^2 = 2$.
   (a) Find the point p (x, y) of the intersection of C_1 and C_2 such that $y > 0$.
   (b) Find the two normal vectors $\vec{n}_1$ to C_1 and $\vec{n}_2$ to C_2 at the point p.
   (c) Find the angle between the two vectors $\vec{n}_1$ and $\vec{n}_2$.

3. Calculate $\int_C 2x^2 y\, dx + (3x + y)\, dy$ where C is given by $x = y^2$ from (1, -1) to (1, 1).

4. Consider the curve C described by $\vec{r}(t) = t\,\hat{i} + t^2\,\hat{j} + t^3\,\hat{k}$.
   (a) Find the curvature κ at $t = 1$.
   (b) Find the work done by the force field $\vec{F}(x, y, z) = e^x\,\hat{i} + xe^{xy}\,\hat{j} + xye^{xyz}\,\hat{k}$ along the curve C from $t = 0$ to $t = 1$.

5. Let $\vec{F}(x,y,z) = (\frac{2x}{y^2} + 1)\hat{i} - (\frac{2x^2}{y^3})\hat{j} - 2z\hat{k}$. (a) Find $\nabla \times \vec{F}$.
   (b) Does there exist a scalar function $f(x,y,z)$ such that $\vec{F} = \nabla f$? Explain your answer. If the answer is "yes", then calculate $f(x,y,z)$.

6. A vector filed $\vec{F}$ is said to be solenoidal if $\nabla \bullet \vec{F} = 0$. Given the vector field $\vec{F}$:
   $\vec{F}(x,y,z) = < ax^2 + bx^2y + cxz^2,\ (b+c)xy - (3a+b)xy^2 + 4yz^2\ ,\ abcz^3 - (c-a)xz + 5xyz >$
   Determine the values of constants a, b and c such that $\vec{F}$ is solenoidal.

7. A particle moves along the curve of intersection of the elliptic cylinder surface S: $x^2 + 4y^2 = 4$ and the plane: $x + 2y + 4z = 4$. **Hint**: express $x = a\cos(t)$ & $y = b\sin(t)$ in terms of a time parameter t; then find a, b & $z(t)$, and solve the following:
   (a) Find the position vector of the particle at any time t.
   (b) Calculate at $t=\pi/2$, the particle (i) coordinates, (ii) velocity, (iii) speed, and (iv) acceleration.
   (c) At what time (if any) does the particle pass the xy-plane?

$\cos\theta = \vec{a} \bullet \vec{b} / (\|\mathbf{a}\| \|\mathbf{b}\|)$ | $\mathbf{comp}_{b}\vec{a} = \|\vec{a}\| \cos\theta = \vec{a}\cdot\hat{b}$ | $\mathrm{proj}_{b}\vec{a} = (\vec{a}\cdot\hat{b})\,\hat{b}$

Area of a parallelogram = $\|\vec{a}\times\vec{b}\|$ | Volume of a parallelepiped = $\|\vec{a}.(\vec{b}\times\vec{c})\|$

Equation of a line: $\vec{r} = \vec{r}_2 + t(\vec{r}_2 - \vec{r}_1) = \vec{r}_2 + t\vec{a}$

Equation of a plane: $a\,\mathbf{x} + b\,\mathbf{y} + c\mathbf{z} + d = 0$

also: $[(\vec{r}_2 - \vec{r}_1)\times(\vec{r}_3 - \vec{r}_1)] \bullet (\vec{r} - \vec{r}_1) = 0$

$$\frac{d\vec{r}(s)}{dt} = \frac{d\vec{r}}{ds}\frac{ds}{dt}$$

Length of a curve: $s = \int_{t_1}^{t_2} \|\vec{r}\,'(t)\|\, dt$

Curvature of a smooth curve: $\kappa = \left\|\frac{d\vec{T}}{ds}\right\| = \left\|\frac{d^2\vec{r}}{ds^2}\right\| = \frac{\|\vec{T}'\|}{\|\vec{r}'\|} = \frac{\|r'(t)\times r''(t)\|}{\|r'(t)\|^3}$

Acceleration: $\vec{\mathbf{a}}(t) = \kappa v^2\hat{\mathbf{N}} + \frac{dv}{dt}\hat{\mathbf{T}} = a_N\hat{\mathbf{N}} + a_T\hat{\mathbf{T}}$ | $\hat{\mathbf{N}} = \frac{d\mathbf{T}/dt}{\|d\mathbf{T}/dt\|}$ | $\hat{\mathbf{T}} = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

$a_T = \frac{dv}{dt} = \frac{\vec{v}\bullet\vec{a}}{\|\vec{v}\|}$ **&** $a_N = kv^2 = \frac{\|\vec{v}\times\vec{a}\|}{\|\vec{v}\|}$

The Binormal $\hat{\mathbf{B}} = \hat{\mathbf{T}}\times\hat{\mathbf{N}}$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} \quad \& \quad \frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y}$$

$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

$D_u(F) = \nabla F \bullet \hat{u},\ \hat{u} = \text{unit vector}$

Equation of Tangent Plane: $\vec{n}_o \bullet (\vec{r} - \vec{r}_o) = 0,\ \vec{n}_o = \nabla F$ at $P(\vec{r}_o)$

equation of normal line to a surface: $\vec{n}_o \times (\vec{r} - \vec{r}_o) = 0,\ \vec{n}_o = \nabla F$ at P

Also: $x = x_o + t\,F_x(x_o, y_o, z_o),\ y = y_o + t\,F_y(x_o, y_o, z_o),$

$z = z_o + t\,F_z(x_o, y_o, z_o)$

$$W = \int_C \vec{F}\bullet d\vec{r} = \int_c F_1(x,y,z)dx + \int_c F_2(x,y,z)dy + \int_c F_3(x,y,z)dz$$

$$\int_C F(x,y)ds = \int_a^b F(f(t), g(t))\sqrt{[f']^2 + [g']^2}\, dt = \int_a^b F(x, f(x))\sqrt{1 + [f']^2}\, dx$$