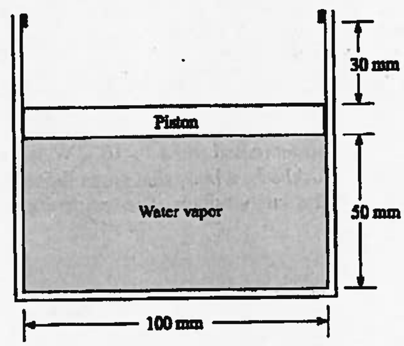
CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING

PROBLEM I [12 pts]
A frictionless piston shown in the figure below has a mass of 16 kg. Heat is added until the temperature reaches 400°C. If the initial quality is 20%, find:

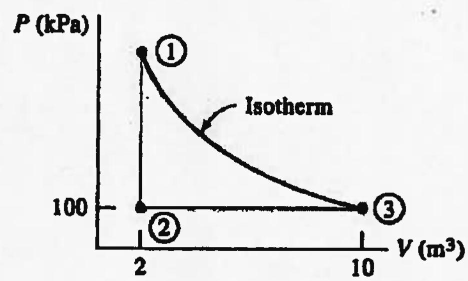
a) the initial pressure,
b) the mass of the water,
c) the quality when the piston hits the stops,
d) the work done.



P_1	120 kPa
m	0.001373 kg
x_2	0.32
W	28.3 J//

PROBLEM II [12 pts]
Two kilograms of air experiences the three-process cycle in the figure below. Calculate the net work.

W	-809 kJ



PROBLEM III [6 pts]
1. Express mathematically the variation of pressure with depth for an ideal gas.

2. Demonstrate that the compressive/expensive work (like in piston-cylinder assembly), can be computed as: $\int P\, dV$

3. What is thermodynamic equilibrium?

CONSTANTS FOR ALL PROBLEMS: P_{atm}= 100 kPa For air: R=0.2870 kJ/ kg K

TABLE A–5

Saturated water—Pressure table

		Specific volume, m^3/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg · K		
Press., P kPa	Sat. temp., T_{sat} °C	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
1.0	6.97	0.001000	129.19	29.302	2355.2	2384.5	29.303	2484.4	2513.7	0.1059	8.8690	8.9749
1.5	13.02	0.001001	87.964	54.686	2338.1	2392.8	54.688	2470.1	2524.7	0.1956	8.6314	8.8270
2.0	17.50	0.001001	66.990	73.431	2325.5	2398.9	73.433	2459.5	2532.9	0.2606	8.4621	8.7227
2.5	21.08	0.001002	54.242	88.422	2315.4	2403.8	88.424	2451.0	2539.4	0.3118	8.3302	8.6421
3.0	24.08	0.001003	45.654	100.98	2306.9	2407.9	100.98	2443.9	2544.8	0.3543	8.2222	8.5765
4.0	28.96	0.001004	34.791	121.39	2293.1	2414.5	121.39	2432.3	2553.7	0.4224	8.0510	8.4734
5.0	32.87	0.001005	28.185	137.75	2282.1	2419.8	137.75	2423.0	2560.7	0.4762	7.9176	8.3938
7.5	40.29	0.001008	19.233	168.74	2261.1	2429.8	168.75	2405.3	2574.0	0.5763	7.6738	8.2501
10	45.81	0.001010	14.670	191.79	2245.4	2437.2	191.81	2392.1	2583.9	0.6492	7.4996	8.1488
15	53.97	0.001014	10.020	225.93	2222.1	2448.0	225.94	2372.3	2598.3	0.7549	7.2522	8.0071
20	60.06	0.001017	7.6481	251.40	2204.6	2456.0	251.42	2357.5	2608.9	0.8320	7.0752	7.9073
25	64.96	0.001020	6.2034	271.93	2190.4	2462.4	271.96	2345.5	2617.5	0.8932	6.9370	7.8302
30	69.09	0.001022	5.2287	289.24	2178.5	2467.7	289.27	2335.3	2624.6	0.9441	6.8234	7.7675
40	75.86	0.001026	3.9933	317.58	2158.8	2476.3	317.62	2318.4	2636.1	1.0261	6.6430	7.6691
50	81.32	0.001030	3.2403	340.49	2142.7	2483.2	340.54	2304.7	2645.2	1.0912	6.5019	7.5931
75	91.76	0.001037	2.2172	384.36	2111.8	2496.1	384.44	2278.0	2662.4	1.2132	6.2426	7.4558
100	99.61	0.001043	1.6941	417.40	2088.2	2505.6	417.51	2257.5	2675.0	1.3028	6.0562	7.3589
101.325	99.97	0.001043	1.6734	418.95	2087.0	2506.0	419.06	2256.5	2675.6	1.3069	6.0476	7.3545
125	105.97	0.001048	1.3750	444.23	2068.8	2513.0	444.36	2240.6	2684.9	1.3741	5.9100	7.2841
150	111.35	0.001053	1.1594	466.97	2052.3	2519.2	467.13	2226.0	2693.1	1.4337	5.7894	7.2231
175	116.04	0.001057	1.0037	486.82	2037.7	2524.5	487.01	2213.1	2700.2	1.4850	5.6865	7.1716
200	120.21	0.001061	0.88578	504.50	2024.6	2529.1	504.71	2201.6	2706.3	1.5302	5.5968	7.1270
225	123.97	0.001064	0.79329	520.47	2012.7	2533.2	520.71	2191.0	2711.7	1.5706	5.5171	7.0877
250	127.41	0.001067	0.71873	535.08	2001.8	2536.8	535.35	2181.2	2716.5	1.6072	5.4453	7.0525
275	130.58	0.001070	0.65732	548.57	1991.6	2540.1	548.86	2172.0	2720.9	1.6408	5.3800	7.0207
300	133.52	0.001073	0.60582	561.11	1982.1	2543.2	561.43	2163.5	2724.9	1.6717	5.3200	6.9917
325	136.27	0.001076	0.56199	572.84	1973.1	2545.9	573.19	2155.4	2728.6	1.7005	5.2645	6.9650
350	138.86	0.001079	0.52422	583.89	1964.6	2548.5	584.26	2147.7	2732.0	1.7274	5.2128	6.9402
375	141.30	0.001081	0.49133	594.32	1956.6	2550.9	594.73	2140.4	2735.1	1.7526	5.1645	6.9171
400	143.61	0.001084	0.46242	604.22	1948.9	2553.1	604.66	2133.4	2738.1	1.7765	5.1191	6.8955
450	147.90	0.001088	0.41392	622.65	1934.5	2557.1	623.14	2120.3	2743.4	1.8205	5.0356	6.8561
500	151.83	0.001093	0.37483	639.54	1921.2	2560.7	640.09	2108.0	2748.1	1.8604	4.9603	6.8207
550	155.46	0.001097	0.34261	655.16	1908.8	2563.9	655.77	2096.6	2752.4	1.8970	4.8916	6.7886
600	158.83	0.001101	0.31560	669.72	1897.1	2566.8	670.38	2085.8	2756.2	1.9308	4.8285	6.7593
650	161.98	0.001104	0.29260	683.37	1886.1	2569.4	684.08	2075.5	2759.6	1.9623	4.7699	6.7322
700	164.95	0.001108	0.27278	696.23	1875.6	2571.8	697.00	2065.8	2762.8	1.9918	4.7153	6.7071
750	167.75	0.001111	0.25552	708.40	1865.6	2574.0	709.24	2056.4	2765.7	2.0195	4.6642	6.6837

TABLE

Saturat

Press., P kPa
800
850
900
950
1000
1100
1200
1300
1400
1500
1750
2000
2250
2500
3000
3500
4000
5000
6000
7000
8000
9000
10,000
11,000
12,000
13,000
14,000
15,000
16,000
17,000
18,000
19,000
20,000
21,000
22,000
22,064

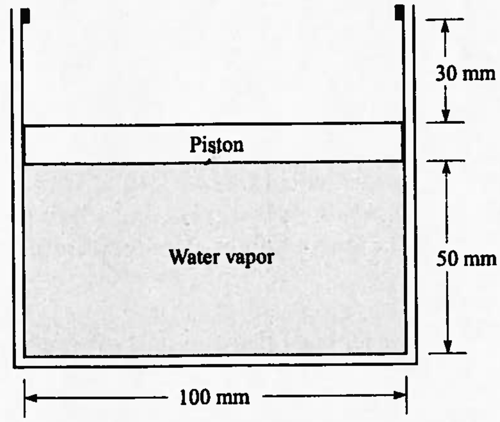


Fig. 3-12

3.2 The frictionless piston shown in Fig. 3-12 has a mass of 16 kg. Heat is added until the temperature reaches 400 °C. If the initial quality is 20 percent, find (*a*) the initial pressure, (*b*) the mass of water, (*c*) the quality when the piston hits the stops, (*d*) the final pressure, and (*e*) the work done on the piston.

3

(*a*) A force balance on the piston allows us to calculate the initial pressure. Including the atmospheric pressure, which is assumed to be 100 kPa, we have

$$P_1 A = W + P_{\text{atm}} A \qquad P_1 \frac{\pi(0.1)^2}{4} = (16)(9.81) + (100\,000)\frac{\pi(0.1)^2}{4}$$

$$\therefore P_1 = 120\,000 \text{ Pa} \quad \text{or } 120 \text{ kPa}$$

3

(*b*) To find the mass, we need the specific volume. Using entries from Table C-2, we find

$$v_1 = v_f + x(v_g - v_f) = 0.001 + (0.2)(1.428 - 0.001) = 0.286 \text{ m}^3/\text{kg}$$

The mass is then

$$m = V_1/v_1 = \frac{\pi(0.1)^2}{4}\left(\frac{0.05}{0.286}\right) = 0.001373 \text{ kg}$$

3

(*c*) When the piston just hits the stops, the pressure is still 120 kPa. The specific volume increases to

$$v_2 = V_2/m = \frac{\pi(0.1)^2}{4}\left(\frac{0.08}{0.001373}\right) = 0.458 \text{ m}^3/\text{kg}$$

The quality is then found as follows, using the entries at 120 °C:

$$0.458 = 0.001 + x_2(1.428 - 0.001) \qquad \therefore x_2 = 0.320 \quad \text{or } 32.0\%$$

(*d*) ~~After the piston hits the stops, the specific volume ceases to change since the volume remains constant. Using $T_3 = 400$ °C and $v_3 = 0.458$, we can interpolate in Table C-3, between pressure 0.6 MPa and 0.8 MPa at 400 °C, to find~~

$$\cancel{P_3 = \left(\frac{0.5137 - 0.458}{0.5137 - 0.3843}\right)(0.8 - 0.6) + 0.6 = 0.686 \text{ MPa}}$$

3

(*e*) There is zero work done on the piston after it hits the stops. From the initial state until the piston hits the stops, the pressure is constant at 120 kPa; the work is then

$$W = P(v_2 - v_1)m = (120)(0.458 - 0.286)(0.001373) = 0.0283 \text{ kJ} \quad \text{or } 28.3 \text{ J}$$

PII

1

The work for the constant-volume process from state 1 to state 2 is zero since $dV = 0$. For the constant-pressure process the work is

$$W_{2-3} = \int P\,dV = P(V_3 - V_2) = (100)(10-2) = 800 \text{ kJ} \quad 2$$

The work needed for the isothermal process is

$$W_{3-1} = \int P\,dV = \int \frac{mRT}{V}\,dV = mRT \int_{V_3}^{V_1} \frac{dV}{V} = mRT \ln\frac{V_1}{V_3} \quad 2$$

To find W_{3-1} we need the temperature. It is found from state 3 to be

$$T_3 = \frac{P_3V_3}{mR} = \frac{(100)(10)}{(2)(0.287)} = 1742 \quad 3$$

Thus, the work for the constant-temperature process is

$$W_{3-1} = (2)(0.287)(1742) \ln\frac{2}{10} = -1609 \text{ kJ} \quad 1$$

Finally, the net work is

$$W_{\text{net}} = \cancel{W}^{0}_{1-2} + W_{2-3} + W_{3-1} = 800 - 1609 = -809 \text{ kJ} \quad 3$$

The negative sign means that there must be a net input of work to complete the cycle in the order shown above.

3.6 A paddle wheel (Fig. 3-15) requires a torque of 20 ft-lbf to rotate it at 100 rpm. If it rotates for 20 sec, calculate the net work done by the air if the frictionless piston raises 2 ft during this time.

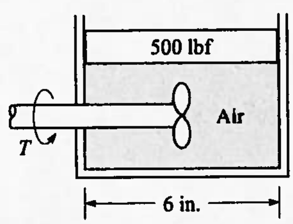


Fig. 3-15

The work input by the paddle wheel is

$$W = -T\omega\,\Delta t = (-20 \text{ ft-lbf})\left[\frac{(100)(2\pi)}{60} \text{ rad/sec}\right](20 \text{ sec}) = -4190 \text{ ft-lbf}$$

The negative sign accounts for work being done on the system, the air. The work needed to raise the piston requires that the pressure be known. It is found as follows:

$$PA = P_{\text{atm}}A + W \qquad P\frac{\pi(6)^2}{4} = (14.7)\frac{\pi(6)^2}{4} + 500 \qquad \therefore P = 32.4 \text{ psia}$$

The work done by the air to raise the piston is then

$$W = (F)(d) = (P)(A)(d) = (32.4)\frac{\pi(6)^2}{4}(2) = 1830 \text{ ft-lbf}$$

and the net work is $W_{\text{net}} = 1830 - 4190 = -2360$ ft-lbf.

Pr $\underline{III}$

1) $\frac{dP}{dz} = -\rho g \iff dP = -\rho g\, dz$

however for an ideal gas: $\frac{P}{\rho} = RT \iff \rho = \frac{P}{RT}$

$\frac{dP}{P} = \frac{-g}{\bar{R}T} dz \iff \ln\left(\frac{P_2}{P_1}\right) = \frac{-g}{\bar{R}T}(z_2 - z_1)$ 2

$\iff P_2 = P_1\, e^{-g/RT\, \Delta z}$

2) See next page 3

3) Thermodynamic eq. $\begin{cases} \cdot \text{ Thermal eq.} \\ + \text{ Mechanical eq.} \\ + \text{ Phase eq.} \\ + \text{ chemical eq.} \end{cases}$ 1

CHAPTER III

Energy Transfer by Heat and Work

This chapter is an important transition between the properties of pure substances and the most important chapter which is: the first law of thermodynamics
In this chapter, we will introduce the notions of heat, work and conservation of mass.

III.1. Work

Work is basically defined as any transfer of energy (except heat) into or out of the system. In the next part, we will define several forms of work. But, first we will focus our attention on a particular kind of work called: compressive/expansive work. Why is this important? Because it's the main form of work found in gases and it's vitally important to many useful thermodynamic applications such as engines, refrigerators, free expansions, liquefactions, etc.

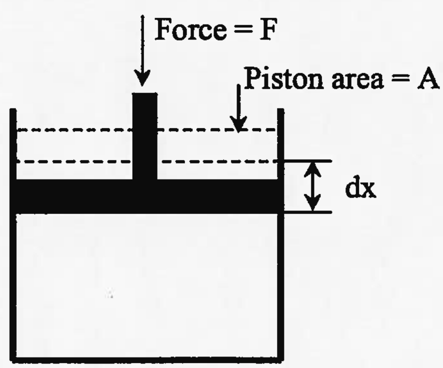
By definition, if an applied force F causes an infinitesimal displacement ds then, the work done dW is given by:

$$dW = F.ds$$

and as that force keep acting, those infinitesimal work contributions add up such that:

$$W = \sum dW = \int F.ds$$

This is the general definition of work, however, for a gas it is more convenient to write this expression under an other form. Consider first the piston-cylinder arrangement:



Here we can apply a force F to the piston and cause it to be displaced by some amount *dx*. But, in thermodynamics, it's better to talk about the pressure $P = F/A$ rather than the force because the pressure is size-independent. Making this shift gives a key result:

$$W = \int PA(dx) = \int PdV$$