Concordia UNIVERSITY

FACULTY OF ENGINEERING AND COMPUTER SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING

ENGR-391 NUMERICAL METHODS FOR ENGINEERS

Student's Name: _____

I.D.: _____

Duration 60 minutes

PROBLEM 1 [Taylor series] (20 points)

Consider the following function:

$$f(x) = \frac{1}{x^2}$$

1- Compute the analytical derivative of f(x) for x=2.
2- Compute numerically the derivative of f(x) for x=2, using the forward, backward and centered formulations using h=0.01 and then for h=0.05. What is the most accurate formulation and what is the effect of increasing h?

NOTE: Keep 5 significant digits for the results.

PROBLEM 2 [Newton-Raphson Method] (40 points)

Consider the following function:

$$f(x) = x^6 - x - 1$$

1- Explain graphically Newton-Raphson method.

2- Solve for f(x)=0 using Newton-Raphson Method in the interval **[1 2]**.
- Use as an initial guess **x=1.5**.
- Compute **five iterations**, and compute the **relative error for each iteration**.

NOTE: Keep 5 significant digits for the results.

PROBLEM 3 [LU Decompostion] **(40 points)**

Consider the following system of linear equations:

$$\begin{cases} 4x_1 - x_2 + x_3 = 6 \\ 8x_1 + 3x_2 - x_3 = 10 \\ 3x_1 + x_2 + x_3 = 9 \end{cases}$$

1- Write the system under the form: $[A]\{X\} = \{B\}$
2- Is the system ill-conditioned?
3- Solve the system using LU decomposition.
4- Replace your solution in the system.

ENGR 391
2006/2007
Midterm.

Problem I: (Taylor series)

1/ Analytical derivatives:

$$f(x) = \frac{1}{x^2} \iff f'(x) = -\frac{2}{x^3}$$

for $x = 2$ $\quad f'(x) = -0.25$ (2)

2/ Numerical derivatives:

for $h = 0.01$

- forward: $f'(2) = \dfrac{f(2.01) - f(2)}{0.01} = -0.24814$ (2)
- backward: $f'(2) = \dfrac{f(2) - f(1.99)}{0.01} = -0.25189$ (2)
- Centered: $f'(2) = \dfrac{f(2.01) - f(1.99)}{0.02} = -0.25001$ (2)

for $h = 0.05$

forward : $f'(2) = \dfrac{f(2.05) - f(2)}{0.05} = -0.24093$ (2)

backward : $f'(2) = \dfrac{f(2) - f(1.95)}{0.05} = -0.25969$ (2)

Centered : $f'(2) = \dfrac{f(2.05) - f(1.95)}{2(0.05)} = -0.25031$ (2)

Errors (relative error)

	$h = 0.01$	$h = 0.05$
forward	0.74 %	3.63 %
backward	0.76 %	3.88 %
Centered	0 %	0.124 %

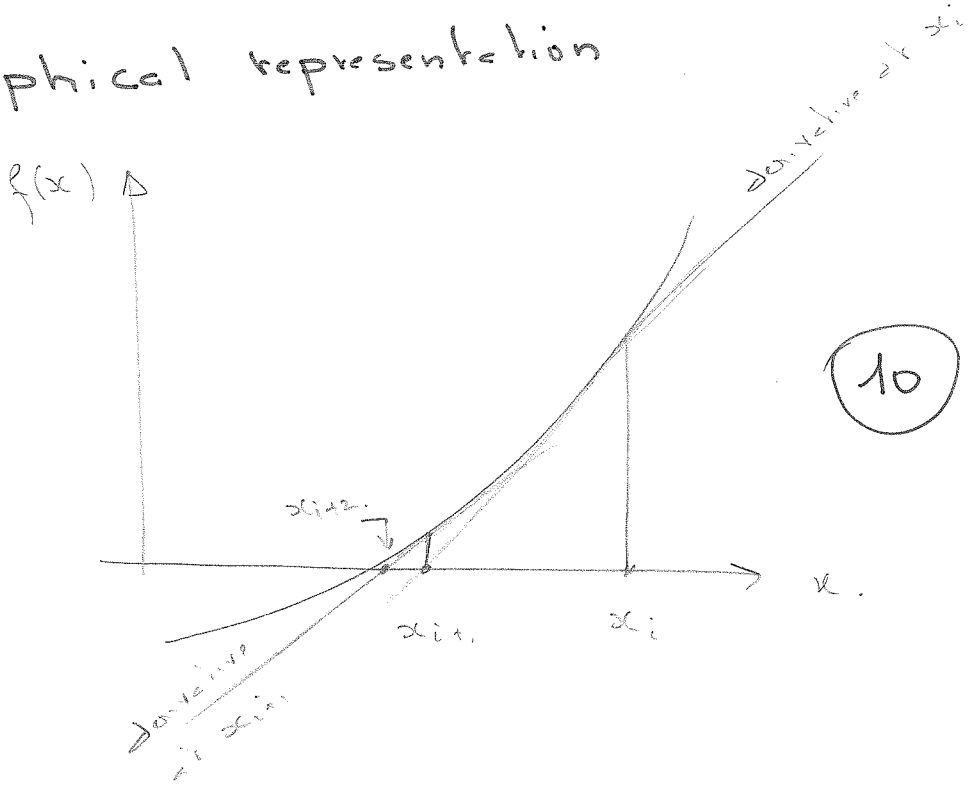
(2)

Comment:

- the centered formula is the more accurate (2)
- increasing (h) increase the error on the determination of the derivative. (2)

• Problem II (Newton-Raphson)

1- Graphical representation



(10)

2/ Solving in [1 2] with $x_0 = 1.5$ (5)

$f(x): x^6 - x - 1.$

$f'(x): 6x^5 - 1.$ (5)

$$\varepsilon_a = \left| \frac{\text{Previous} - \text{actual}}{\text{actual}} \right|$$

or

$$\varepsilon_a = \left| \frac{\text{Previous} - \text{actual}}{\text{Previous}} \right|$$

n.	x_n	ε_a
0	1.5 (3)	(3)
1	1,30049 (3)	13% (3)
2	1,18148 (3)	10,07% (3)
3	1,13946 (3)	3,69% (3)
4	1,13478 (3)	0,41% (3)
5	1,13472	0,00492%

The students may stop after n: 4.

- in case of a numerical error remove (6) points for the error and consider the rest as correct.

Problem III

1/ writing the system under matrix form (5)

$$[A]\{x\} = \{b\} \iff \begin{bmatrix} 4 & -1 & 1 \\ 8 & 3 & -1 \\ 3 & 1 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 10 \\ 9 \end{Bmatrix}$$

2/ is the system ill-conditionned? (5)

$$\det(A) = \det \begin{bmatrix} 4 & -1 & 1 \\ 8 & 3 & -1 \\ 3 & 1 & 1 \end{bmatrix} = 26 \neq 0$$

the system is not ill-conditionned. (5)

3/ (5) (5)

$$[L] = \begin{bmatrix} 4 & 0 & 0 \\ 8 & 5 & 0 \\ 3 & 1.75 & 1.3 \end{bmatrix} ; \quad [U] = \begin{bmatrix} 1 & -1/4 & 1/4 \\ 0 & 1 & -3/5 \\ 0 & 0 & 1 \end{bmatrix}$$

checking if $[L][U] = [A]$ (5)

Note that the students may put the ones in [L] instead of [U], which is correct.

- computation of $\{z\}$

$$[L][U]\{x\} = \{b\} \quad \Leftrightarrow \quad [L]\{z\} = \{b\}$$

$$\{z\} = [U]\{x\}$$

$$z = \begin{Bmatrix} 1.5 \\ -0.4 \\ 4 \end{Bmatrix}$$

(표) (1)

- computation of $\{x\}$

$$[U]\{x\} = \{z\}$$

(표) (1)

$$x = \begin{Bmatrix} 1 \\ 2 \\ 4 \end{Bmatrix}$$

(5)

→ replacing in the system.