Concordia University

Department of Mathematics and Statistics

Course	Number	Section	
MATH	204/3	CA	
Examination	**Date**	**Time**	**Pages**
Mid-term	July 2020	1 hour 15 mins	2
Instructor	**Course Examiner**		**Marks**
U. Tiwari	A. Kokotov		20
Special Instructions:	**Online proctored Closed Book Exam**		

Q 1. [**Marks = 5**] Use Gauss-Jordan elimination method to solve the system of equations:

$$\left.\begin{aligned} x + y + 4z &= 2 \\ 2x + 5y + 20z &= 10 \\ -x + 2y + 8z &= 4 \end{aligned}\right\} \Rightarrow \begin{aligned} x &= 0 \\ y &= 2 - 4t \\ z &= t \quad , \quad -\infty < t < \infty \end{aligned}$$

t is a parameter

Q 2. [**Marks = 3.5+1.5=5**] For matrix

$$A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & -3 & -1 \\ 3 & 2 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -1 \\ -3 & 2 & 6 \end{bmatrix}$$

Find

(a) A^{-1} if exists

(b) Use A^{-1} to solve the system

$$\begin{aligned} 4x + 2y + z &= -3 \\ -3x - 3y - z &= -9 \\ 3x + 2y + z &= 2 \end{aligned} \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \\ 3 \end{pmatrix}$$

Q 3. **[Marks = 5]** Evaluate the determinant of the matrix

$$A = \begin{bmatrix} 3 & 0 & 1 & 5 \\ 1 & -1 & 2 & 1 \\ 0 & 1 & 0 & -2 \\ 4 & 0 & 3 & 4 \end{bmatrix}$$

$\det A = 0$

Q 4. **[Marks = 1+2+2]** For square matrices A and B, prove or disprove each of the following:

(a) $det(A^T) = det\ A$

See texbook.

(b) $det(A + B) = det\ A + det\ B$

(c) $(A^{-1})^{-1} = A$

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Solution

#1 Augmented matrix of the system

$$\begin{bmatrix} 1 & 1 & 4 & 2 \\ 2 & 5 & 20 & 10 \\ -1 & 2 & 8 & 4 \end{bmatrix} \xrightarrow[R_3+R_1]{R_2-2R_1} \begin{bmatrix} 1 & 1 & 4 & 2 \\ 0 & 3 & 12 & 6 \\ 0 & 3 & 12 & 6 \end{bmatrix} \xrightarrow[\frac{1}{3}R_2]{R_3-R_2}$$

$$\begin{bmatrix} 1 & 1 & 4 & 2 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1-R_2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Longrightarrow \begin{aligned} x &= 0 \\ y &= 2-4t \\ z &\equiv t \end{aligned}$$

where $-\infty < t < \infty$.

#2 (a) $[A \vdots I] = \begin{bmatrix} 4 & 2 & 1 & \vdots & 1 & 0 & 0 \\ -3 & -3 & -1 & \vdots & 0 & 1 & 0 \\ 3 & 2 & 1 & \vdots & 0 & 0 & 1 \end{bmatrix} \xrightarrow[R_3+R_2]{R_1+R_2}$

$$\begin{bmatrix} 1 & -1 & 0 & \vdots & 1 & 1 & 0 \\ -3 & -3 & -1 & \vdots & 0 & 1 & 0 \\ 0 & -1 & 0 & \vdots & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_2+3R_1} \begin{bmatrix} 1 & -1 & 0 & \vdots & 1 & 1 & 0 \\ 0 & -6 & -1 & \vdots & 3 & 4 & 0 \\ 0 & -1 & 0 & \vdots & 0 & 1 & 1 \end{bmatrix} \xrightarrow[R_2 \leftrightarrow R_3]{-R_3}$$

$$\begin{bmatrix} 1 & -1 & 0 & \vdots & 1 & 1 & 0 \\ 0 & 1 & 0 & \vdots & 0 & -1 & -1 \\ 0 & -6 & -1 & \vdots & 3 & 4 & 0 \end{bmatrix} \xrightarrow[R_3+6R_2]{R_1+R_2} \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 & 0 & -1 \\ 0 & 1 & 0 & \vdots & 0 & -1 & -1 \\ 0 & 0 & -1 & \vdots & 3 & -2 & -6 \end{bmatrix} \xrightarrow{-R_3}$$

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 1 & 0 & -1 \\ 0 & 1 & 0 & \vdots & 0 & -1 & -1 \\ 0 & 0 & 1 & \vdots & -3 & 2 & 6 \end{bmatrix} \Longrightarrow A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -1 \\ -3 & 2 & 6 \end{bmatrix}$$

(b) $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -1 \\ -3 & 2 & 6 \end{bmatrix} \begin{bmatrix} -3 \\ -9 \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ 7 \\ 3 \end{bmatrix}$$ ✓

#3 $\begin{vmatrix} 3 & 0 & 1 & 5 \\ 1 & -1 & 2 & 1 \\ 0 & 1 & 0 & -2 \\ 4 & 0 & 3 & 4 \end{vmatrix} \xrightarrow{R_1 \leftrightarrow R_2} = - \begin{vmatrix} 1 & -1 & 2 & 1 \\ 3 & 0 & 1 & 5 \\ 0 & 1 & 0 & -2 \\ 4 & 0 & 3 & 4 \end{vmatrix} \xrightarrow[R_4 - 4R_1]{R_2 - 3R}$

$= - \begin{vmatrix} 1 & -1 & 2 & 1 \\ 0 & 3 & -5 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 4 & -5 & 0 \end{vmatrix} \xrightarrow{R_2 \leftrightarrow R_3} = + \begin{vmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 3 & -5 & 2 \\ 0 & 4 & -5 & 0 \end{vmatrix}$

$\xrightarrow[R_4 - 4R_2]{R_3 - 3R_2} \begin{vmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -5 & 8 \\ 0 & 0 & -5 & 8 \end{vmatrix} \xrightarrow{R_4 - R_3} \begin{vmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -5 & 8 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0$

#4. (a) $\det(A) = \det(A^T)$:

True, since transposing a matrix changes its rows to columns and its columns to rows, the cofactor expansion of A along any row is the same as cofactor expansion of A^T along the corresponding column.

(b) $\det(A+B) \neq \det A + \det B$ in general.

Example $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$\det A = 1$ and $\det B = 0$, $\det(A+B) = 0$

(c) See Textbook