Course	Number	Section(s)
Mathematics	204	All

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Problem N1

(a). Solve the following equation for matrix X.

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} X = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} X \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

[**Marks= 8**]

(b). Find all the 2×2 diagonal matrices D that satisfy

$$D^2 - D - 2I = 0$$

[**Marks= 2**]

Problem N2

(a). Compute the determinant

$$\begin{vmatrix} 4 & 0 & 0 & 2 & 0 \\ 3 & 4 & 5 & 0 & 1 \\ 1 & 2 & 3 & 2 & 1 \\ 3 & 4 & 3 & 2 & 1 \\ 1 & 1 & 2 & 2 & 1 \end{vmatrix}$$

[**Marks= 8**]

(b). Give an example of square matrices A and B, each with nonzero determinant, where

$$\det(A+B) = \det(A) + \det(B)$$

Size 2×2 for the matrices will suffice the purpose.

[**Marks= 2**]

Problem N3

(a). The planes α and β in $\mathbb{R}^3$ are given via equations

$$\alpha : \quad x + 2y - z - 2 = 0$$

and

$$\beta : \quad 2x - 2y + z - 1 = 0\,.$$

Their intersection $\alpha \cap \beta$ is the line l. Write down the parametric equation of the line l in the form

$$\vec{x} = \vec{x}_0 + t\vec{v}$$

where t is a real parameter and $\vec{x_0}$, $\vec{v}$ are given vectors.

[**Marks= 4**]

(b). Find the distance from the origin $\mathcal{O} = (0, 0, 0)$ to the line l.

[**Marks= 6**]

Problem N4

(a). Find a negative number x for which all three vectors $\vec{A} = (1, 3, 5)$, $\vec{B} = (1, 1 + x, 11)$ and $\vec{C} = (1, 5, x + 7)$ are parallel to the same plane.

[Marks= 5]

(b). Write down the coordinates of a unit normal vector to this plane (i. e. a vector of norm 1 which is orthogonal to the plane).

[Marks= 5]

Problem N5 Let four vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ and $\vec{v}_4$ be given by

$$\vec{v}_1 = (1, 0, 2), \quad \vec{v}_2 = (0, 1, -1)\,,$$

$$\vec{v}_3 = (1, 8, -6), \quad \vec{v}_4 = (1, 2, 0);$$

Let the vector space V be the linear span of $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ and $\vec{v}_4$.

(a). Find all values of k such that the vector $\vec{u} = (2, 1, k)$ belongs to V. Explain.

[Marks= 2]

(b). Are the vectors $\{\vec{v}_1, \vec{v}_2\}$ linearly independent? Explain.

[Marks= 1]

(c). Find the dimension and a basis of the space V.

[Marks= 2]

(d). Let $\mathbb{A}$ be the matrix whose columns are the vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ and $\vec{v}_4$ (written as columns). Find a basis and the dimension of the solution set of the homogeneous linear system $\mathbb{A}X = 0$.

[Marks= 5]

Problem N6

(a). Find a 2×2 matrix A such that $(-4, 0)$ and $(-2, -4)$ are eigenvectors of A corresponding eigenvalues 9 and -7.

[**Marks= 4**]

(b). Let B be the matrix

$$\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$$

Find the eigenvalues and corresponding eigenvectors of B.

[**Marks= 4**]

(c). Find the eigenvalue of C corresponding to the eigenvector $(-1, 1, 0)^t$ (t stands for transposition) given that C is the matrix

$$\begin{pmatrix} 10 & 5 & -9 \\ -9 & -4 & 9 \\ 5 & 5 & -4 \end{pmatrix}$$

[**Marks= 2**]