Emat 233 Midterm Exam
November 17, 2005

No caclulators allowed. Write all solutions in full. Do not assume we can read your mind!

[10 points] Problem 1. Compute the curl and the divergence of the vector field

$$\vec{F}(x, y, z) = yze^x\mathbf{i} + (2x - 3yz)\mathbf{j} + xy^2z^3\mathbf{k}.$$

[10 points] Problem 2. Evaluate the following integral by reversing the order of integration:

$$\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3 + 1}\,\mathrm{d}x\,\mathrm{d}y.$$

[10 points] Problem 3. Define a force field $\vec{F}$ by

$$\vec{F}(x, y) = (2xy)\mathbf{i} + (x^2)\mathbf{j}.$$

a. Show that this vector field is conservative, i.e. the line integrals $\int_{\mathcal{C}} \vec{F} \cdot \mathrm{d}\vec{r}$ are independent of path, and find a function $\phi(x, y)$ such that $\vec{F} = \nabla\phi$.

b. Let $\mathcal{C}$ be the upper half of the circle $x^2 + y^2 = 1$ in the xy plane, going from the point $(1, 0)$ to $(-1, 0)$. Compute the work done by the force $\vec{F}$ along the curve $\mathcal{C}$ in two different ways:
(i) Using the function ϕ found in part (a).
(ii) Computing the line integral $\int_{\mathcal{C}} \vec{F} \cdot \mathrm{d}\vec{r}$ directly along any convenient path between the endpoints.

[10 points] Problem 4. By using the appropriate theorem (which must be named) compute the circulation $\oint_{\mathcal{C}} \vec{F} \cdot \mathrm{d}\vec{r}$ of the vector field

$$\vec{F}(x, y) = (3xy - \mathrm{e}^{-x^4})\mathbf{i} + (5xy + \cos^2(y^{45}))\mathbf{j}$$

around the curve $\mathcal{C}$ given by the boundary of the rectangle of vertices $(1, 1)$, $(3, 1)$, $(3, 2)$, $(1, 2)$ oriented counterclockwise.

[10 points] Problem 5. Compute the double integral

$$\iint_{\mathcal{R}} \frac{y}{\sqrt{x^2 + y^2}} \sin(x^2 + y^2)\,\mathrm{d}A,$$

where $\mathcal{R}$ is the region above the x-axis, bounded by the x-axis and the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.