CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Section
Mathematics	204	AA
Examination	**Date**	**Pages**
Final	June 2018	2
Instructor		**Course Examiner**
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Special Instructions:

- ▷ Only approved calculators are allowed.
- ▷ Justify all your answers.
- ▷ All questions have equal value.

<u>ARKS</u>

1. Use the Gauss-Jordan method to find all the solutions of the system:

$$\begin{aligned} 2x_1 - 2x_2 - 6x_3 + x_4 &= 3 \\ -x_1 + x_2 + 3x_3 - x_4 &= -3 \\ x_1 - 2x_2 - x_3 + x_4 &= 2 \end{aligned}$$

$$\begin{cases} x_1 = 1+5s \\ x_2 = 1+2s \\ x_3 = s \\ x_4 = 3 \end{cases}$$

2. Using Cramer's Rule, find the value of x_3 in the system:

$$\begin{aligned} 3x_1 + x_2 &= 5 \\ -x_1 + 2x_2 + x_3 &= -2 \\ -x_2 + 2x_3 &= -1 \end{aligned}$$

$x_1 = \frac{28}{17}$

$x_2 = \frac{1}{17}$

$\boxed{x_3 = \frac{-8}{17}}$

3. a) Let $u = (2, 1, -3)$, $v = (1, 2, 6)$. Find the orthogonal projection of v on u. $\text{pr}_{\vec{u}} \vec{v} = (-2, -1, 3)$

   b) Let $u_1 = (1, 2, 1)$, $u_2 = (2, 1, 1)$, $u_3 = (1, 1, 2)$. Find scalar c_1, c_2, c_3 such that $c_1\, u_1 + c_2\, u_2 + c_3\, u_3 = (3, 4, 5)$.

   $c_1 = 1, \ c_2 = 0, \ c_3 = 2$

4. Find the inverse of $A = \begin{pmatrix} 1 & -2 & 1 \\ -3 & 7 & -6 \\ 2 & -3 & 0 \end{pmatrix}$, if it exists.

$$A^{-1} = \begin{pmatrix} -18 & -3 & 5 \\ -12 & -2 & 3 \\ -5 & -1 & 1 \end{pmatrix}$$

5. Find the determinant of A = $\begin{pmatrix} 3 & -4 & 0 & 5 \\ 2 & 1 & -7 & 1 \\ 0 & -3 & 2 & 2 \\ 5 & 8 & -2 & -1 \end{pmatrix}$ $\det A = -26$

6. a) Let $u = (3, -1, 1)$, $v = (9, 2, 0)$, $w = (0, -5, 6)$.
Are the vectors linearly dependent or independent? $\det[\bar{u}|\bar{v}|\bar{w}] = 45$
The vectors are independent

b) Find the parametric equations for the line in $\mathbb{R}^3$ passing through $(2, 5, 6)$ and perpendicular to the plane $3x - 4y + 7z = 2$.
$\begin{cases} x = 2 + 3t \\ y = 5 - 4t \\ z = 6 + 7t \end{cases}$

7. a) Find the area of a triangle with vertices $\overset{A}{(2, 1, 0)}$, $\overset{B}{(1, 5, 6)}$, $\overset{C}{(7, 4, 3)}$.
Find a vector orthogonal to the plane of the triangle.
$\overline{AB} = (-1, 4, 6)$
$\overline{AC} = (5, 3, 3)$
$\bar{n} = (-6, 33, -23)$
$S_\Delta = \frac{\sqrt{1654}}{2} = \sqrt{\frac{82}{2}}$

b) Find the distance between the point $(-1, 2)$ and the line $3x = 4y + 7$.
$\text{dist} = \frac{|-3-8-7|}{\sqrt{9+16}} = \frac{18}{5}$

8. Let $A = \begin{bmatrix} 1 & 2 & 0 & 4 & 0 & 6 \\ 0 & 0 & 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \\ t \\ u \\ v \end{bmatrix}$. $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 0 \\ -4 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$

Find a basis for the solution space of the homogeneous system $AX = 0$.

9. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -2 \\ 2 & 4 & 2 \end{bmatrix}$. Find a matrix P such that $P^{-1}AP = D$ $P = \begin{bmatrix} 1 & 1 & 1 \\ -2 & -1 & -2 \\ 2 & 2 & 3 \end{bmatrix}$;
a diagonal matrix. $D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

10. Find the standard matrices for following operators on $\mathbb{R}^2$:

a) a rotation counterclockwise of 90°. $R_{\frac{\pi}{2}} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

b) a reflection about the line $y = 0$. $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$