

FACULTY OF ENGINEERING AND COMPUTER SCIENCE
FINAL EXAMINATION FOR APPLIED DIFFERENTIAL EQUATIONS
ENGR 213 - FALL 2012

Date: December 11, 2012

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Special instructions: Do all problems
Only Faculty approved calculators are allowed
ALL PROBLEMS CARRY THE SAME WEIGHT

PROBLEM No. 1. Solve the following equation by the separation of variables method.

$$2tx \frac{dx}{dt} + (t^2 - x^2) = 0$$

(Hint: use the idea of homogeneous functions)

PROBLEM No. 2. Find the solution to the following equations by the exact differentials method:

$$(a) \quad \frac{dy}{dx} = -\frac{2xy^2 + 1}{2x^2y}$$

$$(b) \quad \frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2} y = \cos x \quad x > 0$$

PROBLEM No. 3. Solve the following linear differential equations using the integrating factor method:

$$(a) \quad \frac{dy}{dx} - \frac{y}{x} = 1$$

$$(b) \quad \frac{dy}{dx} - 2y = e^x$$

PROBLEM No. 4. Solve the following Bernoulli equation,

$$\frac{dy}{dx} - 5y = -\frac{5}{2}xy^3$$

PROBLEM No. 5. Give the general solutions of the following differential equations:

$$(a) \quad \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$$

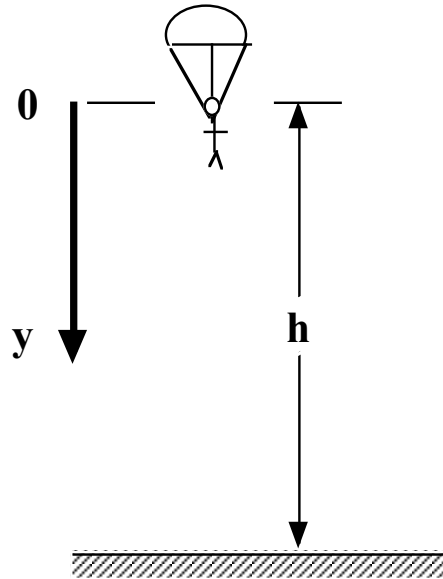
$$(b) \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

PROBLEM No. 6. Give the general solutions of the following differential equations:

(a) $y'' - y = \cos x$

(b) $y'' + y' - 2y = x^2$

Problem No. 7. A parachutist whose mass is 75 kg drops from a helicopter hovering 4000 m above the ground and falls towards the earth under the influence of gravity. Assume that the gravitational force remains constant during the flight, and that the drag due to air resistance is proportional to the velocity of the parachutist, with a proportionality constant $b = 150 \text{ N s/m}$ when the chute is open. The chute opens immediately after the parachutist leaves the helicopter.



The equation of motion (Newton's 2nd law) is:

$$m \frac{dV}{dt} = mg - bV$$

Obtain an expression for the velocity of the parachutist as a function of time. What is her/his terminal velocity? ($g = 9.81 \text{ m/s}^2$)

PROBLEM No. 8. Solve the differential equation by the method of variation of parameters:

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + y = e^{-t}$$

PROBLEM No. 9. Solve the following system of differential equations using the method of your preference:

$$\frac{dx}{dt} = x - y$$

$$\frac{dy}{dt} = y - 4x$$

Problem No. 10. Use the power series method to solve the differential Equation:

$$y'' + 2xy' + y = 0$$

(write the first 6 non-zero terms of the power series solution)