

COURSE

EXAMINATION

#### FACULTY OF ENGINEERING AND COMPUTER SCIENCE

DATE

**Numerical Methods in Engineering** 

Final Exam	Dec. 9, 2013	14:00-17:00	18		
PROFESSORS					
Dr.D. Davis, Dr. P. Gauthier and Dr. A. Kaushal					
MATERIALS ALLOWED - YES (C					
<ul> <li>Closed Book Exa</li> <li>You must show a</li> <li>Give the answer</li> <li>Please do not wr</li> </ul>	Il the questions 00; Time 180 Minutes 1m; Single sided 8 ½ x 11 fo all your steps leading to the 1s in the area provided. 1rite in red (colour used for colour adable will NOT be correct	e solution(s) prrection)			
	Good L				
Name:Surname, Signature:	Given names	I.D.:	MARKS		

NUMBER

TIME

**ENGR 391** 

SECTION

/4 (all)

# of pages (including title page)

## **Question #1 [Solving Nonlinear Equations] [10 marks]**

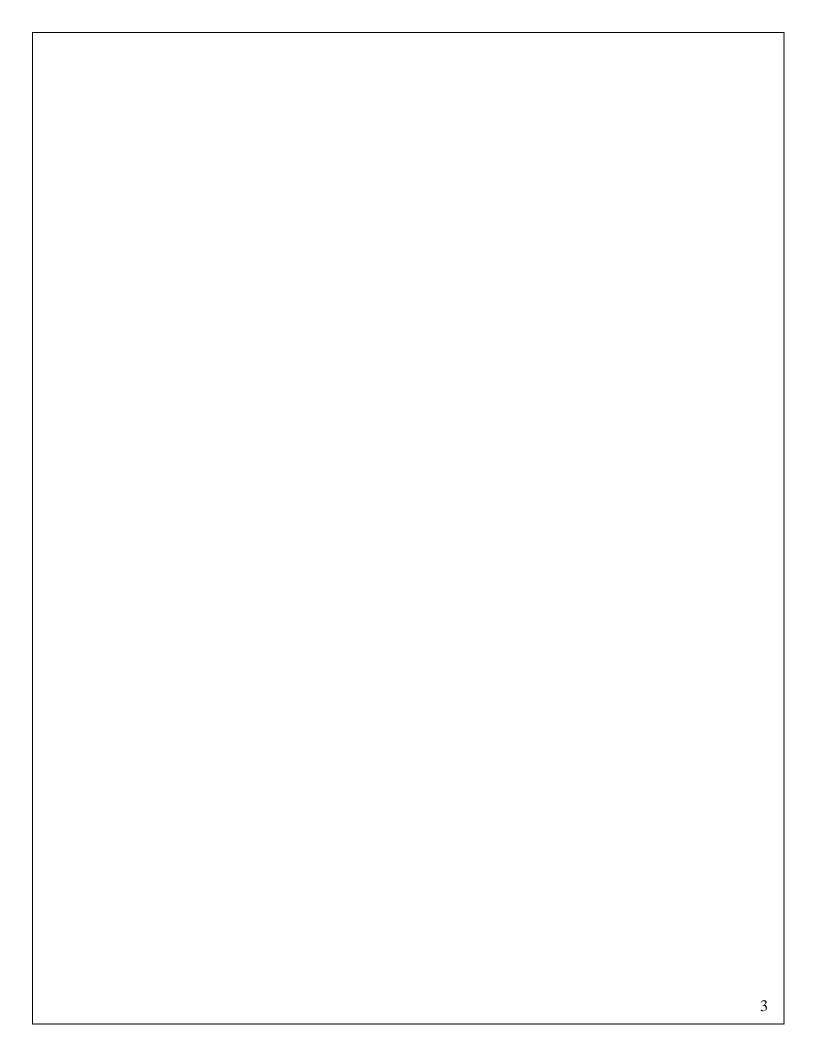
Obtain the first root above x = 0 for the following equation with accuracy of 4 digits (Hint: use incremental search to locate the region of the root)

$$e^{x} - 2x^{2} = 0$$

- a. Use the method of False Position
- b. Use Newton Raphson method

(5 Marks)

(5 Marks)



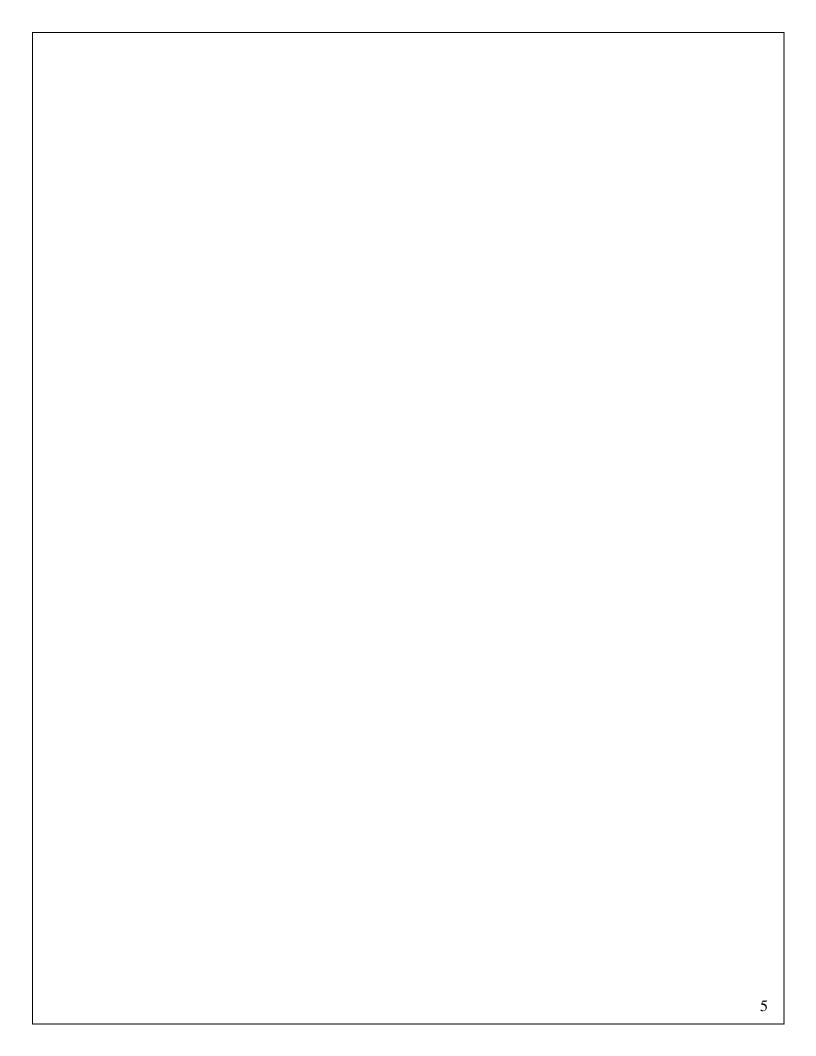
### Question #2 [Systems of linear and Nonlinear equations] [25 marks]

a) Consider the following system of linear equations  $[A]{X} = {B}$ 

$$\begin{bmatrix} 1 & 4 & 2 \\ 5 & 2 & 1 \\ 2 & 2 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

If using decimals instead of fraction number, keep 3 decimals in your calculations

- 1. Find the solution of this system of linear equations **using the LU decomposition** with **partial pivoting** (i.e. PA = LU). (10 Marks)
- 2. Find the first column of [A]<sup>-1</sup> (5 Marks)

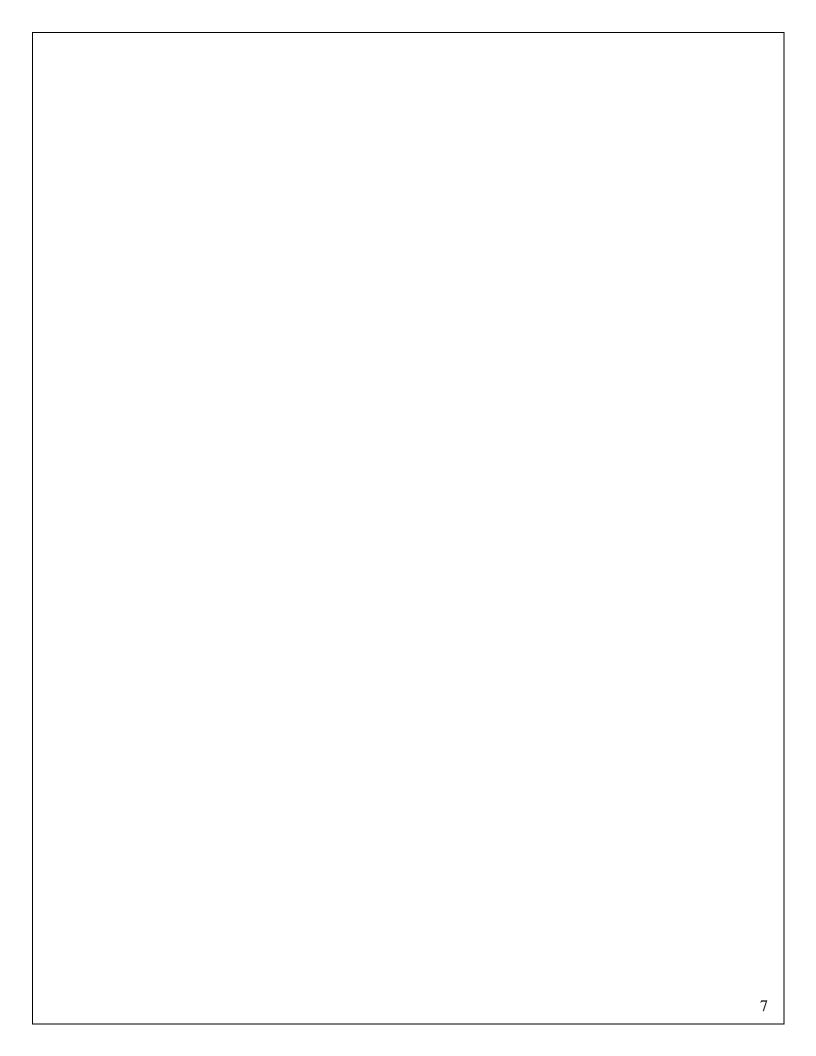


b) Obtain the solution to the following nonlinear equations using Newton's Method of the form  $\{X_{n+1}\}=\{X_n\}-\big[J(X_n)\big]^{-1}\{F(X_n)\}$ 

$$-x_1 + 2x_1^2 - 2x_1x_2 + x_2^2 = 1$$

$$x_1^2 - 2x_1x_2 - x_2 + x_2^2 = 0$$

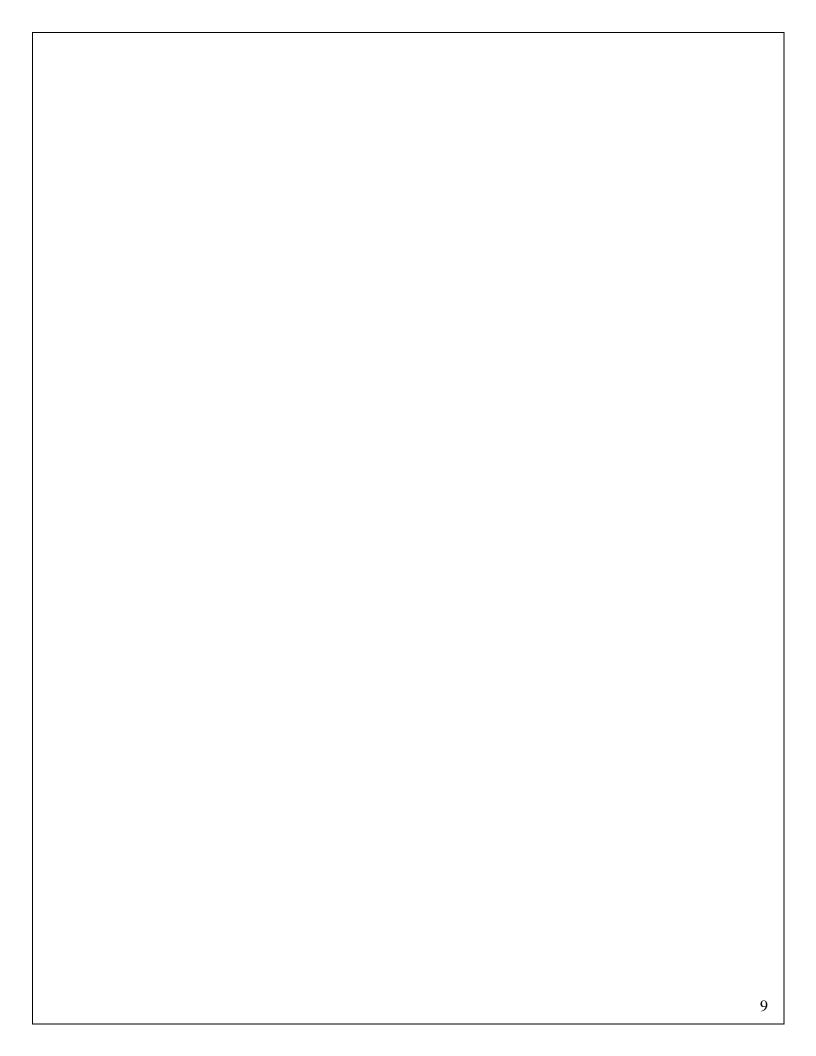
Assume the starting vector  $\{x_0\} = \begin{cases} 1 \\ 1 \end{cases}$ . Do **two iterations**. Compute the error **at each step** of the iteration using the  $||x||_2$  norm; Use **4 decimals** in your calculations (10 Marks)



## Question #3 [Curve Fitting ] [20 marks]

a) If the following points are related by a formula of the type,  $P(x) = Ae^{Mx}$ Find the best value of A and M; Keep **4 decimals** in your calculations. (10 Marks) (Hint: Change the form to a linear equation and use least squares regression)

Xi	1	2	3	4
P <sub>i</sub>	7	11	17	27



b)	<ul> <li>Use the Lagrange Interpolating Polynomial to approximate Cos (0.750) using the followalues; (Note the given values are in radians)</li> </ul>				following (10 Marks)
	Cos(0.698) = 0.7	661 Cos(0.	733) = 0.7432	Cos(0.768	) = 0.7193
					10

#### **Question #4 [Numerical Differentiation & Integration] [25 marks]**

a) Evaluate the Integral:

$$I = \int_{0}^{2.4} \frac{2x}{x^2 + 1} dx$$

Gauss Quadrature

$$\begin{array}{ll} \underline{n=4}:\\ c_1=0.3478548; & x_1=-0.86113631;\\ c_2=0.6521452; & x_2=-0.33998104;\\ c_3=c_2; & x_3=-x_2;\\ c_4=c_1; & x_4=-x_1 \end{array}$$

1. Analytically

(2.5 Marks)

2. Using Simpson's 1/3 method, using 6 sub-intervals.

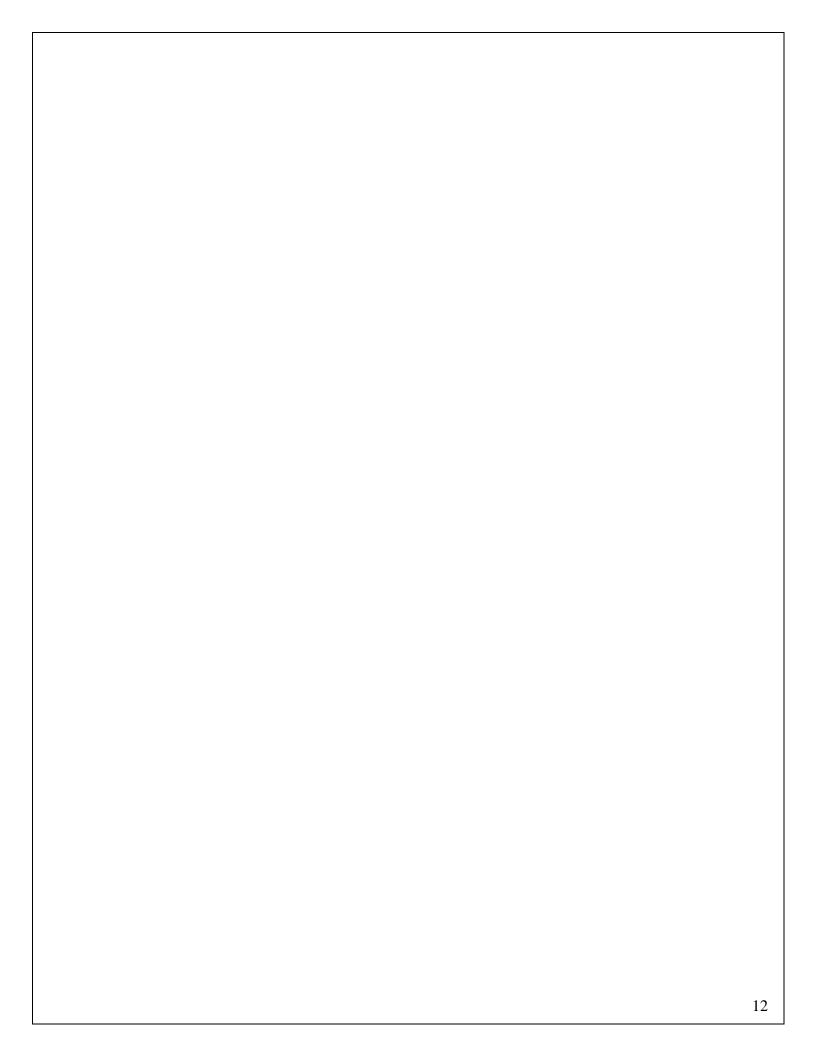
(5 Marks)

3. Using four-point Gauss Quadrature

(5 Marks)

4. Using the exact solution found in part a) evaluate the percent relative error associated with each of the approximations found in parts 2) and 3) (2.5 Marks)

Keep 3 decimals in your calculations.



b. The following data is given for the stopping distance of a truck on a wet road versus the speed at which it begins breaking:

v (km/h)	20.0	40.5	62.5	80	100.5	125
d(m)	6	19	38	65	99	135

- 1. Calculate the rate of change of the stopping distance at a speed of 100.5 km/h using a two-point backward difference formula. (5 Marks)
- 2. Estimate the stopping distance at 125 km/h using the result from part 1) and a two-point central difference formula applied at the speed of 100.5 km/h. (5 Marks)

# **Question #5 [Ordinary Differential Equations] [20 marks]**

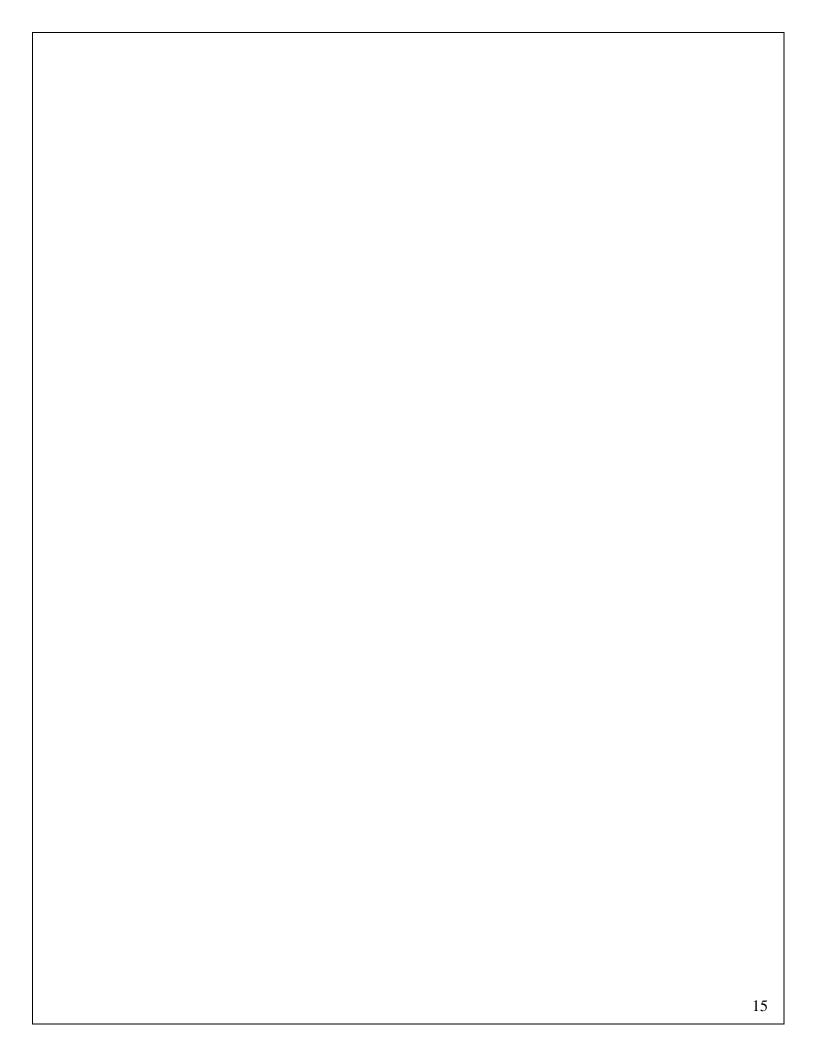
a. Given the differential equation

$$\frac{dy}{dx} = \frac{4x}{y} - xy$$

Fill out the following table using the classical fourth- order Runge Kutta method; Keep **6 decimals** in your calculations.

(10 Marks)

Xi	Yi
0.00000	3.000000
0.10000	
0.20000	



b)	Solve numerically using Euler's method the following second order ordinary	differential
	equation with a step size, $h = 0.5$ , from $t=1$ to $t=2$ , for given initial condition	ıs:

(10 Marks)

$$\frac{d^2y}{dt^2} = 4y^3$$

y(1)=1/4 and dy/dt(1)=0

Keep 5 decimal places in your calculations

