Concordia University

# CONCORDIA UNIVERSITY
## FACULTY OF ENGINEERING AND COMPUTER SCIENCE
## DEPARTMENT OF MECHANICAL ENGINEERING

***NUMERICAL METHODS IN ENGINEERING***

***Mid-term***

***Time: 75min***

- Read carefully all questions
- Give the answers in the boxes provided.
- You can use the back of the paper to do your auxiliary calculations (will not be corrected)
- Please do not write in red (colour used for correction)
- **No questions will be answered during the exam. Answer all questions to your best knowledge. Add any assumptions in case you made some**
- This exam has in total 3 problems
- Including this page there are 5 pages

**Name:** ______________________________

**Student ID:** ______________________________

✓

**Problem 1 (10 marks)**

We want to solve the following equation:

$$(1)\ x^3 - 3x + 1 = 0$$

using the bisection algorithm.

✓ a) Find an interval $[a, b]$ such that $|a - b| = 1$ containing at least one root of equation (1).

$[0, 1]$

✓ b) Compute five iterations of the bisection algorithm by using the following table

| $a_i$ | $b_i$ | $\frac{a_i + b_i}{2}$ | Signe of $f(a_i)$ | Signe of $f(b_i)$ | Signe of $f(c_i)$ |
|---|---|---|---|---|---|
| 0 | 1 | 0.5 | + | - | - |
| 0 | 0.5 | 0.25 | + | - | + |
| 0.25 | 0.5 | 0.375 | + | - | - |
| 0.25 | 0.375 | 0.3125 | + | - | + |
| 0.3125 | 0.375 | 0.34375 | + | - | + |
| 0.34375 | 0.375 | 0.359375 | + | - | |

✓ c) Based on your calculations from (b), give the solution of equation (1) with as many **correct** <u>significant digits as you can.</u>

0.3 because there is no other repeating digit in the iterations performed

7/10

**Problem 2 (10 marks)**

Consider the following system of linear equations:

$$\begin{cases} 3x - 2y + z = -10 \\ 2x + 6y - 4z = 44 \\ -x - 2y + 5z = -26 \end{cases}$$

3, 7, 11
7, 1, 3
4, 2, 3

a) Write the system in matrix form

$$\begin{bmatrix} 3 & -2 & 1 \\ 2 & 6 & -4 \\ -1 & -2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 44 \\ -26 \end{bmatrix}$$

b) Solve your system using the LU decomposition algorithm

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 2 & 6 & -4 \\ -1 & -2 & 5 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ ? & 1 & 0 \\ ? & ? & 1 \end{bmatrix}$$

$-\frac{2}{3}R_1 + R_2 \rightarrow R_2$ New

$$\begin{bmatrix} 3 & -2 & 1 \\ 0 & 22/3 & -\frac{14}{3} \\ -1 & -2 & 5 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ ? & ? & 1 \end{bmatrix}$$

$\frac{1}{3}R_1 + R_3 \rightarrow R_3$ New

$$\begin{bmatrix} 3 & -2 & 1 \\ 0 & 22/3 & -\frac{14}{3} \\ 0 & -8/3 & 16/3 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ -\frac{1}{3} & ? & 1 \end{bmatrix}$$

$\frac{8}{22}R_2 + R_3 \rightarrow R_3$ New

$$\begin{bmatrix} 3 & -2 & 1 \\ 0 & 22/3 & -\frac{14}{3} \\ 0 & 0 & 40/11 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ -\frac{1}{3} & 8/22 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 22/3 & -14/3 \\ 0 & 0 & 40/11 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ -1/3 & 8/22 & 1 \end{bmatrix} \quad b = \begin{bmatrix} -10 \\ 44 \\ -26 \end{bmatrix}$$

$$L \cdot Z = b$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -10 \\ 2/3 & 1 & 0 & 44 \\ -1/3 & 8/22 & 1 & -26 \end{array}\right]$$

$z_1 = -10$

$\frac{2}{3}(-10) + Z_2 = 44 \quad Z_2 = 50.\overline{6}$

$-\frac{1}{3}(-10) + \frac{8}{22}(50.\overline{6}) + Z_3 = -26$

$Z_3 = -47.75757576$

-3

$$Z = \begin{bmatrix} -10 \\ 50.66666667 \\ -47.75757576 \end{bmatrix}$$

-2

$$U \cdot X = Z$$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & -10 \\ 0 & 22/3 & -\frac{14}{3} & 50.66666667 \\ 0 & 0 & 40/11 & -47.75757576 \end{array}\right]$$

$\frac{40}{11} x_3 = -47.75757576$

$\boxed{x_3 = -13.13333333}$

$\frac{22}{3} x_2 - \frac{14}{3}(-13.13333333) = 50.66666667$

$\boxed{x_2 = -1.448484848}$

$3x_1 - 2(-1.448484848) + (-13.13333333) = -10$

$\boxed{x_1 = 0.078787878}$

$$\boxed{x = \begin{bmatrix} 0.078787878 \\ -1.448484848 \\ -13.13333333 \end{bmatrix}}$$

**Problem 3 (5 marks)**

Consider the function $f(x) = x\sin(x^2)$. The value $r = 0$ is a root of $f(x)$.

a) What is the multiplicity of the root $= 0$ ?

$f'(x) = \sin(x^2) + 2x^2\cos(x^2) \Rightarrow f'(0) = 0 + 0 = 0$

$f''(x) = \cos(x^2)2x + 4x\cos(x^2) - 4x^3\sin(x^2) \Rightarrow f''(0) = 0 + 0 - 0 = 0$

$f'''(x) = 2\cos(x^2) - 4x^2\sin(x^2) + 4\cos(x^2) - 8x^2\sin(x^2) - 12x^2\sin(x^2) - 8x^4\cos(x^2)$

$f'''(0) = 2 - 0 + 4 - 0 - 0 - 0 = 2 + 4 = 6 \neq 0$

The root $x=0$ has a multiplicity of 3

b) Find the forward and backward errors for the approximation $x_c = 0.01$ of the root 0 of the function $f(x) = x\sin(x^2)$.

backward $= |f(x_c)| = 0.000000999 = 0.0000999\ \%$

forward $= |r - x_c| = |0 - 0.01| = 0.01 = 1\%$

c) Discuss your answer in b) based on your finding from a) (give **maximum** 2 sentences).

The solution seems to have a large error multiplication factor. Finding roots with high precision becomes challenging when the function has multiple roots.