

Concordia University

ENGR 233 - Tut UA - 1st Midterm Exam - February 16, 2009 Instructor: Galia Dafni

Total time: 75 minutes

Total marks: 50

Allowable materials: Pencils, pens. You may **NOT** use notes, books, calculators or any other materials. A formula sheet can be found on the back of the exam.

Write your answers in the examination booklet. Write clearly and neatly and show all your work in order to receive full marks. You do not need to simplify or approximate numerical answers but you should evaluate any trigonometric functions if the question requires it.

Problem 1. (10 marks)

(a) Determine the work done by the constant force $\vec{F} = 2\hat{i} + 3\hat{j}$ in moving a particle from the point $(-1, 1)$ to the point $(4, 6)$ along a straight line.

(b) Find the area of the triangle with vertices at the points $(0, 1, 2)$, $(1, 2, 3)$ and $(3, 2, 1)$.

Problem 2. (20 marks) Let $\vec{r}(t) = 2\hat{i} + \cos t\hat{j} + \sin t\hat{k}$ be the position vector of a moving particle.

(a) Describe the shape of the curve traced by the trajectory of the particle (hint: find an equation relating y and z).

(b) Find the velocity vector $\vec{v}(t)$, the acceleration vector $\vec{a}(t)$, and the speed at any t .

(c) Find the tangential component of the acceleration at any t . What does this tell you about the motion?

(d) Give parametric equations for the tangent line to the curve at time $t = \pi/2$.

Problem 3 (15 marks). The following questions refer to the function

$$z = f(x, y) = e^{1-x^2-y^2}.$$

(a) In the xy -plane, sketch the level curves corresponding to $f(x, y) = C$ for different values of the constant C ($0 < C \leq e$).

(b) If $x = \cos t$ and $y = \sin t$, find $\frac{dz}{dt}$ at $t = \pi$.

(c) Give the direction along which the function f increases most rapidly at the point $(0, 1)$, and find the maximum rate of increase.

Problem 4 (5 marks). Give the equation of the tangent plane to the graph of

$$z = 25 - x^2 - y^2$$

at the point $(3, -4, 0)$.