This is a closed book exam. Solve all questions. Only faculty approved calculators are allowed.   Good luck!

PROBLEM No. 1. Solve the IVP:

$$y'' + 6y' - 7y = 0,$$ subject to the initial conditions: $y(0) = 5; \ y'(0) = -3.$

PROBLEM No. 2. Solve the following differential equation by the method of undetermined coefficients:

$$y'' + 6y' + 9y = 27x + 16e^{-2x}$$

PROBLEM No. 3. Solve the following differential equation by the method of variation of parameters:

$$y'' + y = \sin^2 x$$

PROBLEM No. 4. Solve the following Cauchy equation:

$$x^2 y'' + 5xy' + 12y = 0$$