Mathematics 209
Midterm test, July 12, 2012 1h30min

Only authorized calculators permitted

[100 pts=100%]

1. [15pts] Find the limits

(a) $\lim_{x\to 4}(3x^2-5x-3)$, (b) $\lim_{x\to 3}\frac{x^2+4x+7}{x-3}$, (c) $\lim_{x\to 3^-}\frac{b^2}{x+3}$ where b is a real number.

2. [30pts](7+7+8+8) Without simplifying find the derivatives $f'(x)$ of the following

(a) $f(x)=\frac{1}{2}x^{-3}+6\sqrt{x}+5$, (b) $f(x)=3(x^2+7)^5(x^2-7)^6$,
(c) $f(x)=\frac{-3x}{x^5+1}$, (d) $f(x)=5\ln(3x^3+2)$.

3. [5pts] Prove from the definition of the derivative: if the functions $f(x)$ and $g(x)$ are differentiable at $x=5$ then the function $h(x)=f(x)+g(x)$ is also differentiable at $x=5$.

4. [10pts] If interest is compounded continuously at the rate $r=0.07$ (7% annually), how many whole years are needed for principal of 8,000 dollars to become the future value of 18,000 dollars?

5. [15pts] The function $t(x)$ is given implicitly by the equation $2e^t-t+x=3e-1$. Calculate the slope of the tangent line at the point $(e,1)$

6. [15pts] Market studies for a new camera show that the demand as a function of price p, is $x=600,000-500p$.

(a) Find the marginal revenue depending on p at $p=\$50$.
(b) For what p does the revenue reach its *maximum*?

7. [10pts] A point is moving along the graph of $y^3=x^2$. When the point is at $(x,y)=(-8,4)$ its y coordinate is decreasing by 2 units per second. How fast is the x coordinate changing at that moment?

MATH 209
Midterm class test
October 21 2011

1h15min
[100 pts=100%]

1. Find the limits [15pts]

(a) $\lim_{x\to 3} \frac{x+1}{x-2}$, (b) $\lim_{x\to -1} \frac{x^2+3x+2}{x+1}$, (c) $\lim_{x\to 2^-} \frac{x+1}{x-2}$.

2. Find the derivatives $f'(x)$ of the following [30pts]

(a) $f(x) = 2x^{0.5} + x^{20} + 2e$, (b) $f(x) = (x^2+10x+2)^8$,
(c) $f(x) = \frac{x-1}{x^2-1}$, (d) $f(x) = e^{(\sqrt{x^2+1})}$, (e) $f(x) = (x^2+2)^5(x^4-1)^{-3}$.

find derivative
add t'
solve for t'
plug in

3. [15pts] The function $t(x)$ is given implicitly by the equation $\ln(t+1) + t^2 + x^2 = 1$. Calculate the slope of the tangent line at $(x,t) = (1,0)$.

4. [15pts] The cost function of producing x TVs is $C(x) = 1,000 + 5x - 0.01x^2$.
(a) Find the average cost at $x = 100$.
(b) Use the marginal average cost to estimate how the average cost will change if we increase production by ~~10~~ 1 unit units at the production level $x = 100$.

5. [15pts] Market studies for a new icamera showed that the demand of it, depending on the price p, is $x = 500,000 - 2,000p$.

(a) Find the marginal *revenue* depending on x at $x = 100,000$.
(b) Find the approximate change of revenue if x is increased by 10% at $x = 100,000$.
(c) [bonus question: 5pts] At what x the *revenue* reaches its *maximum*? (Hint: the *marginal revenue* is equal to zero at this value of x.)

6. [10pt] The point is moving along the graph of $2y^2 - e^x = 1$. When the point is at $(x,y) = (0,1)$ its x coordinate is increasing at the rate of 0.4 units per second. How fast is the y coordinate changing at that moment?

2D) $f'(x) = \left(e^{(x^2+1)^{1/2}}\right)\left(\tfrac{1}{2}e^{(x^2+1)^{-1/2}}\right)(2x)$