Midterm Examination (Oct. 29, 10:15-11:30 AM)

If doubt exists as to the interpretation of any question, the student is urged to make a clear statement of any assumptions made.
Show all the steps of your calculations and justify your answers (except Question 1)
Submit your solutions as a PDF file (maximum 5-6 pages) on Moodle, under Midterm Exam
Every page of your solution file must contain your name and student ID

1. (10 points) Short answer questions. Just give your answer, no explanation is required.
   a) Give the number of binary strings of length 10 with at least two 1's.
   b) Give the number of binary strings of length 10 with odd number of 0's.
   c) In how many ways you can select (with repetitions allowed) k of n distinct objects?
   d) Give the number of 7 digit decimal numbers containing a 0 and a 1.
   e) What is the value of A, where $A = \sum_{k=0}^{39} \binom{80}{2k} - \sum_{k=0}^{39} \binom{80}{2k+1}$
   f) Evaluate: $\sum_{k=0}^{80} (-1)^{k+8} \binom{80}{80-k} 2^{84-k}$
   g) What is the Ordinaray generating function for the sequence 0, -1, 0, -1, 0, -1, ...
   h) What is the Exponential generating function for the sequence 0, -1, 0, -1, 0, -1, ...
   i) g) What is the Ordinaray generating function for the sequence $P(0), P(1), P(2), ...$, where $P(0) = 1$ and $P(n)$ is the number of partitions (without regarding the order) of positive integer n into summands that do not exceed 3.
   j) Solve the recurrence relation $a_n = a_{n-1} + (n-1)^2$ for all $n > 0$, and $a_0 = 0$.

2. (5 points) Find the number of all permutations of the word A N T A N A N A R I V O
   a) with non-distinguishable A's, non-distinguishable N's and no consecutive A's (2 points)
   b) with non-distinguishable A's, distinguishable N's and at least 3 consecutive A's (3 points)
   Show the details of your solution.
   (For no solution you will get 1 point)

3. (5 points) Define the integer sequence $a_0, a_1, a_2, ...$ recursively by
   1) $a_0 = a_1 = a_2 = 1$; and
   2) For $n \geq 3$, $a_n = a_{n-1} + a_{n-3}$
   Prove by induction that $a_{n+2} \geq (\sqrt{2})^n$ for all $n \geq 0$.
   (For no solution you will get 1 point)

4. (5 points) Show that $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$ using a combinatorial proof. Show the details of your solution. (For no solution you will get 1 point)

5. (5 points) Use the method of Generating function to find the number of 5 element subsets of the set $A = \{1, 2, 3, ..., 16\}$ with no consecutive integers. Show the details of your solution. For a complete solution using a different method you will get 2 points. (For no solution you will get 1 point)