COMP 352/4 **Concordia University** Winter 2001

# Data Structures and Algorithms

## Mid-term Exam, Sec. Z, Solutions

1. 3 pts. Assume that function $f = 5n^2 + 20n^2 \log n + 15\sqrt{n^5}$ represents the run-time of a program $P$.

   (a) Consider the 3 terms in $f$ and state which is the most important term, the second most important term, the least important term:
   (1) $15\sqrt{n^5}$, (2) $20n^2 \log n$, (3) $5n^2$.

   (b) Indicate using the big $O$ notation the time complexity of $P$:
   $O(\sqrt{n^5})$

2. 4 pts. Consider the following function in which only the loops are indicated. All other statements in the function need time $O(1)$.

```
void EX(int n)
 {
   int i = 2*n;
   while (i >= 1){
     ...
     for (int j= 3; j <= n-2; j+=2) {
       ...
      }
     i = i/ 3;
   }
 }
```

   Determine the run-time complexity of EX as a function of $n$, using the big O notation:

   The while loop is repeated $log_3 2n$ times and the for loop is repeated $(n-4)/2+1$ times. Loops are nested, thus the run-time complexity is $O(nlogn)$.

3. 5 pts.

   (a) Give an informal definition of a queue:
   It is a restricted *list* in which any item is inserted at one end of the list and any deletion is done at the other end of the list.

   (b) Assume we have a circular array implementation of a queue. Draw the picture of the queue and the position of $front$ and $rear$ indices resulting from the following sequence of operations (assume we have a queue of integers):

```
Queue v(6);
v.enqueue(4);
v.enqueue(2);
int i=v.dequeue();
v.enqueue(3);
v.enqueue(i);
```

queue is array[0]...array[6].
queue contains 2 in location 2, 3 in location 3, 4 in location 4.
front=1, rear = 4.

4. 10 pts. Assume that we have a class *list*

```
class list {       // a linked list
private:
 link* head;       // pointer to the list header node
 link* tail;       // pointer to the tail
 link* curr;       // pointer to the current element
public:
  list(const int = LIST_SIZE); //constructor
  ~list();        // destructor
                   //here we have the usual operations
                   //on lists
};
```

This list is implemented using a singly linked list, so the class link is defined as

```
class link {
  public:
     ELEM element;   // ELEM value for this node
     link* next;     // pointer to the next node
     link(const ELEM& elemval, link* nextp=NULL);   //constructor
         { element = elemval; next = nextp}
  ~link() { };       // destructor
}
```

We want to add to the class *list* a function

```
void remEvery(const ELEM & item);
```

which removes, in the linked list pointed to by *head*, every node containing *item*.

```
void remEvery(const ELEM & item){
 if (head==tail) return;          // list is empty
 link * ltemp1 = head;            // start from the head nodde
 link * ltemp2 = ltemp1->next;    // ltemp2 is the node following ltemp1
                                  // in the list
 while (ltemp2 != NULL) {
      if (ltemp2->elem == item)       // delete ltemp2 node
         { ltemp1 ->next = ltemp2->next;
           if (ltemp2 == curr) curr = ltemp1; // node pointed to by
                     // current pointer is deleted, adjust curr pointer
           delete(ltemp2);
          }
      else ltemp1 = ltemp2;    // shift ltemp1 to the next node in the list
      ltemp2 = ltemp1->next    // shift ltemp2 to the next node in the list
    }
 tail = ltemp1;                  // adjust tail pointer
}
```

5. 5 pts.

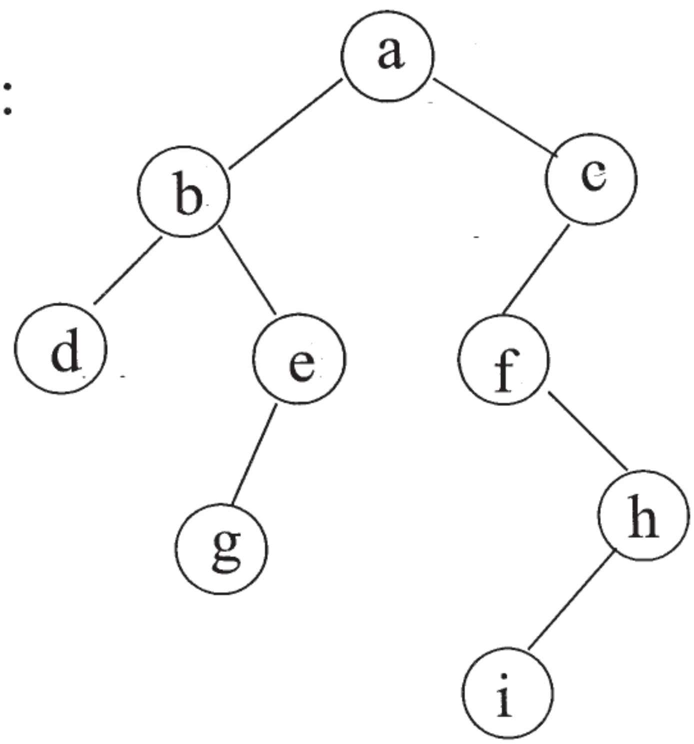
(a) How many internal nodes are there in $T$?
6

(b) Give the preorder and postorder traversals of the $T$.
preorder: $a, b, d, e, g, c, f, h, i$
postorder: $d, g, e, b, i, h, f, c, a$

(c) At least how many nodes do you have to add to $T$ to make it a <u>full</u> binary tree?
4

(d) At least how many nodes do you have to add to $T$ to make it a <u>complete</u> binary tree?
17

T:



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CONCORDIA UNIVERSITY
COMP 352 : Data Structures and Algorithms
Fall 2000
Section: V

**Midterm Examination**

October 17th, 2000
Total Marks: 20 (20% of the final grade)

Name: [redacted]

Student ID: [redacted]

-0.5

***Question1*** (3 marks)

What is the asymptotic time complexity in the average case for the following operations

(a) Create an array-based stack $\Theta(1)$

(b) Create a linked stack $\Theta(1)$

(c) Find an element in a linked list $\Theta(n)$

(d) Insert an element into a linked queue (enqueue operation) $\Theta(1)$

(e) Insert an element into an array-based queue (enqueue operation) $\cancel{\Theta(n)}$ $\Theta(1)$

(f) Clear an array-based list $\Theta(1)$

***Question 2*** (3 marks)

(a) For the following code fragment give

T($n$) and $\Theta(n)$ in the best case $T(n) = c_1$ ; $\Theta(1)$

T($n$) and $\Theta(n)$ in the average case $T(n) = \frac{n(n+1)}{2}$ ; $\Theta(n)$

T($n$) and $\Theta(n)$ in the worst case $T(n) = n$ ; $\Theta(n)$

```
int SeqSearch (int* array, int k) {          // Find element k
        for (int i=1; i<n; i++)                  // For each element
                if (array[i] = k)                // If found
                        return i;                // Return its position
        return NOT_FOUND;                        // Return const - flag
}
```

(b) For the following code fragment give

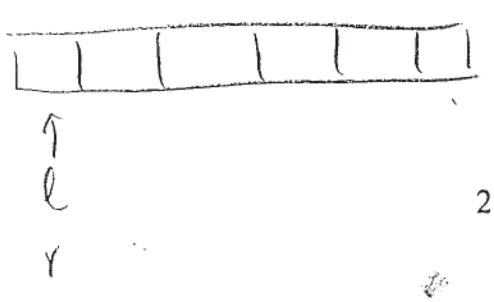
T($n$) and $\Theta(n)$ in the best case $T(n) = c_1$ ; $\Theta(1)$ // $c_1$ is constant

T($n$) and $\Theta(n)$ in the average case $T(n) = \log n$ ; $\Theta(\log n)$

T($n$) and $\Theta(n)$ in the worst case ~~$T(n) = n/2$ ; $\Theta(n)$~~

−0.5

```
int binarySearch (int* array, int k, int left, int right) {
        // find element k
        int l = left + 1;              //l and r beyond the bounds
        int r = right + 1;
        while (l + 1 != r){
                int i = (l + r)/2
                if ( k == array[i]) return i; // return its position
                else if ( k < array[i]) r = i;      // in left half
                else ( k > array[i]) l = i;   //in right half
        }
        return NOT_FOUND;              // return const - flag
}
```



***Question 3***

(a) (1 mark) Convert the following infix expression into a postfix form

(5 + 1) * (3 - 2) / 3 - 4 * 2

5 1 + 3 2 - * 3 / 4 2 * -

(b) (1 mark) Evaluate the following postfix expression

5 2 + 6 * 9 3 / -

5 + 2, 6 * 9 3 / -

(5 + 2) * 6 (9 / 3) -

(5 + 2) * 6 - (9 / 3) = 39

(5 + 2) * 6 (9 / 3) -

(5 + 2) * 6 + (9 / 3)

***Question 4*** (2 marks)

Each data element is 16 bytes. Size of a pointer is 8 bytes. Maximum number of elements is 100. Consider a queue that contains *n* elements. Calculate break-even point beyond which the array based implementation is more space efficient.

P = 8 bytes

E = ~~100~~ 16

D = 16 ~~bytes~~ 100

P+E

DE

-0.5

Array = DE

List = n(P+E)

$n > \frac{DE}{P+E}$ (P+E = 24) $\Rightarrow n > \frac{1600}{24}$ ($\cancel{108}$)

16/24

n > 14.815

or $n \geq 15$

Thus, by the break-even analysis, it requires at least 15 elements ($n \geq 15$) in order for the array-based implementation to be more efficient.

***Question 5*** (3 marks)

Fill in the code to remove an element from the top of the stack, and return it to the calling routine, in a linked stack implementation with the following definitions:

```
class Node { // Node class
public:
     char aValue;
     Node* next;
};

class Stack { // Linked stack class
private:
     Node* topOfStack;          // Pointer to top element
public:
     Stack() {topOfStack = NULL;}  // Constructor
     Node* Remove ();
     ...
     ...
     ...
};
```

```
Node* Stack::Remove()
{
Node* temp = topOfStack;   // make "temp" point to sam
topOfStack = topOfStack->next; // memory location as "topOfSt
                           // make topOfStack point t
                           // next node in the list

return temp;               // return pointer "temp" since
}                          // the return argument is
           Empty Stack.    // expecting a pointer
```

1

***Question 6*** (7 marks)

Using the public functions for the class `List` given below, write a C++ function,

```
List* Alternate (List*& L1, List*& L2)
```

that creates a list from elements in L1 and L2 by alternating elements in the two lists and then appending the remaining nodes of the longer of two lists.

For example, if L1 = (10,20, 30) and L2 = (15, 25, 35, 45, 55), the call to `Alternate(L1, L2)` produces the list (10, 15, 20, 25, 30, 35, 45, 55), and L1and L2 become empty lists. Assume that list elements are of type `Elem`.

```
class List { // Linked list class
private:
     Link* head;          // Pointer to list header
     Link* tail;          // Pointer to last Elem
     Link* curr;          // Pos of "current" Elem
public:
     List();                           // Constructor
     ~List();                          // Destructor
     void clear();                     // Remove all Elems
     void insert(const Elem);          // Insert at current pos
     void append(const Elem);          // Insert at tail
     Elem remove();                    // Remove/return Elem
     void setFirst();                  // Set curr to first pos
     void prev();                      // Move curr to prev pos
     void next();                      // Move curr to next pos
     int length() const;               // Return length
     void setPos(int);                 // Set current pos
     void setValue(const Elem);        // Set current value
     Elem currValue() const;           // Return current value
     bool isEmpty() const;             // TRUE if list is empty
     bool isInList() const;            // TRUE if now in list
     bool find(Elem);                  // Find value
};
```

```
while ( L2. isInList( ))
{
  alter. insert ( L1. currValue( ));
  alter. insert(L2. currValue( ));
  L1. remove( );  // remove current pointer of L1
  L2. remove( );  //     "       "       "    " L2
  L1. next( );    // go to next node in L1
  L2. next( );    //  "  "   "    "   " L2
} //end of while
while (L1. isInList( ))
{
  alter. append(L1. currValue( ));
  L1. remove( );
  L1. next( );
} //end of while
} // end of else

} //end of Alternate
```

```
void Alternate (List *& L1, List *& L2) // given
{
 List alter ; // create object of type "List" to make new
                                                  // list
 L1->setFirst(); // set pointer to first node in L1
 L2->setFirst(); //  ''    ''    ''   ''   ''  L2

if (L1->length() <= L2->length())
{
   while (L1->isInList())
   {
    alter.insert(L1->currValue());
    alter.insert(L2->currValue());
    L1->remove();    // remove current pointer in L1
    L2->remove();    //   ''     ''      ''    '' L2

    L1.next();     // go to next node
    L2.next();     // go to  ''  node
   } //end of while
  while(L2.isInList())
  {
   alter.append(L2.currValue());
   L2.remove();
   L2.next();
  } // end of while
} // end of if
else
 {
  // See next page
```

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CONCORDIA UNIVERSITY
COMP 352 : Data Structures and Algorithms
Summer 2001

**Midterm Examination**

May 30th, 20001
Total Marks: 50 (25% of the final grade)

Name:________________________________________

Student ID:____________________________________

***Question 1*** (9 marks)

Give the asymptotic time complexity in the average case for the following operations:

(a) Clear a linked list ______________________________

(b) Delete an element in an array based list____________________

(c) Find an element in a linked list ________________________

(d) Create a linked stack ____________________________

(e) Insert an element into a linked stack (push operation) ______________

(f) Insert an element into an array-based stack (push operation) ___________

(g) Create a linked queue ____________________________

(h) Insert an element into a linked queue (enqueue operation) ___________

(i) Insert an element into an array-based queue (enqueue operation) ________

**Question 2** (8 marks)

Give T($n$) and $\Theta$($n$) for the following code fragments

(a)

```
sum = 0;
for (j=1; j<=n; j++)
  for (i=1; i<=n; i++)
    sum++;
for (k=0; k<n; k++)
  A[k] = k;
```

(b)

```
sum2 = 0;
for (i=1; i<=n; i++)
  for (j=1; j<=i; j++)
    sum2++;
```

(c)

```
sum1 = 0;
for (k=1; k<=n; k*=2)   log2n
  for (j=1; j<=k; j++)
    sum1++;
```

k j
1 1
2 2
4 4
n

(e)

```
sum = 0;
k = 5;
for (i=0; i<k; i++)
  for (j=0; j<n; j++)
    sum++;
```

$2^{\log n + 1} - 1$

***Question 3*** (3 marks)

Each data element is 16 bytes. Size of a pointer is 8 bytes. Maximum number of elements is 100.

(a) Determine the space required for a linked list implementation (Number of elements in the list is *n* )

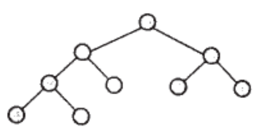
(b) Determine the space required for an array based implementation (Number of elements in the list is *n* )

(c) Calculate break-even point (Break-even point is the number of elements at which both the linked list and array-based list implementations are of equal space efficiency.)

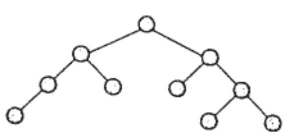
***Question 4*** (5 marks)

For each of the following trees please state whether it is full, complete, neither, or both

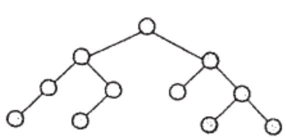
(a)



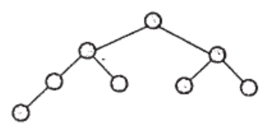
(b)



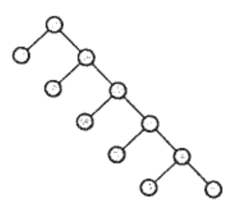
(c)



(d)



(e)



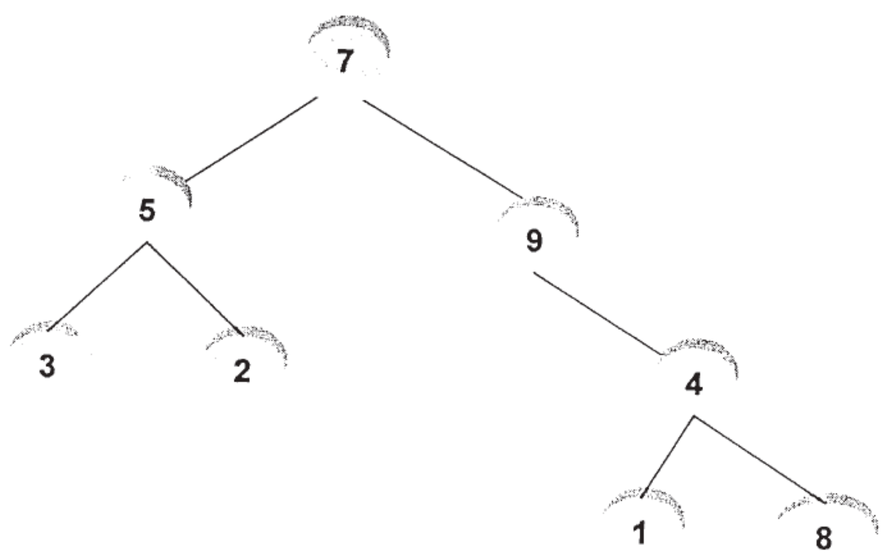
***Question 5*** (4 marks)

(a) Traverse the tree below using inorder traversal (Write the numbers corresponding to the nodes).

______________________________________________

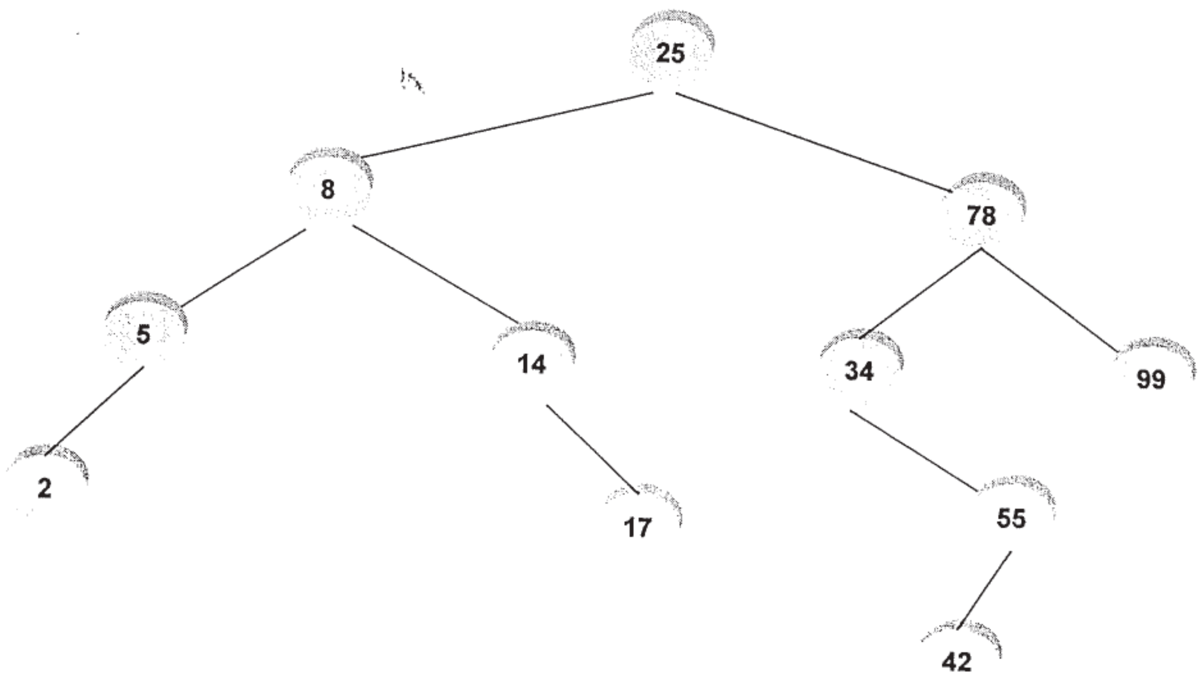
(b) Traverse the tree below using preorder traversal (Write the numbers corresponding to the nodes).

______________________________________________



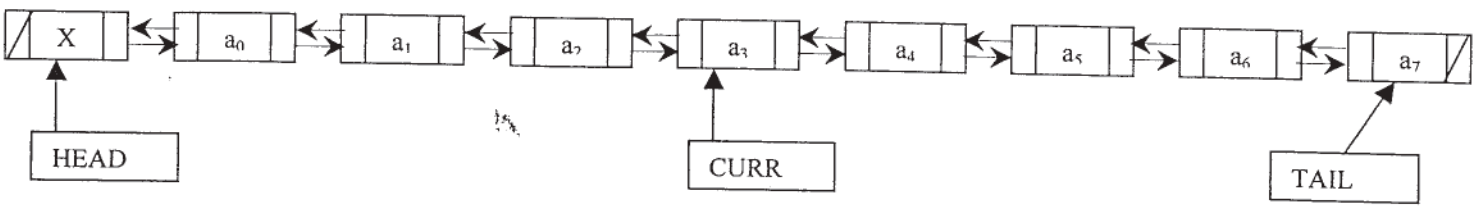
***Question 6*** (5 marks)

Draw the BST that results from deleting value 25 from the BST bellow

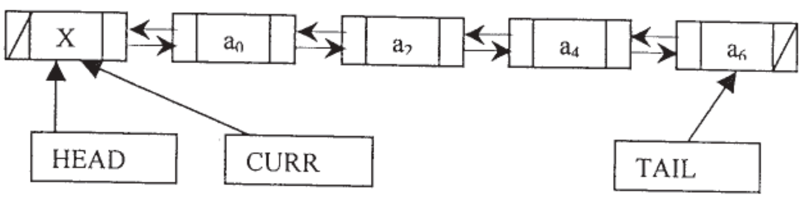


***Question 7*** (16 marks)

Given a doubly linked list A




the *trim* operation applied to list A gives




that is, all elements $a_i$ with odd subscript *i* are removed.
Add to the doubly linked list class implementation a member function

```
void trim();
```

which trims the list. To keep this question simple, `trim()` should just reset the `curr` pointer to the head of the list.

**Try to make your function as efficient as possible.**

You should assume the doubly – linked list node and doubly – linked list classes as given in your textbook. For your reference, the declarations for the list node class and for the linked list class are reproduced below:

```
class Link {                    // Doubly - linked node
public:
     Elem element;              // Elem value for node
     Link *next;                // Pointer to next node
     Link* prev;                // Pointer to prev node
     Link(const Elem elemval, Link* nextval =NULL,
                                       Link* prevp =NULL)
     { element = elemval; next = nextval; prev = prevp;}
     Link(Link* nextval =NULL, Link* prevp = NULL)
     { next = nextval; prev = prevp;}
};

class List { // Linked list class
private:
     Link* head;                // Pointer to list header
     Link* tail;                // Pointer to last Elem
     Link* curr;                // Pos of "current" Elem
public:
     List();                              // Constructor
```

```cpp
    ~List();                           // Destructor
    void clear();                      // Remove all Elems
    void insert(const Elem);           // Insert at current pos
    void append(const Elem);           // Insert at tail
    Elem remove();                     // Remove/return Elem
    void setFirst();                   // Set curr to first pos
    void prev();                       // Move curr to prev pos
    void next();                       // Move curr to next pos
    int length() const;                // Return length
    void setPos(int);                  // Set current pos
    void setValue(const Elem);         // Set current value
    Elem currValue() const;            // Return current value
    bool isEmpty() const;              // TRUE if list is empty
    bool isInList() const;             // TRUE if now in list
    bool find(Elem);                   // Find value
};
```