PHYS 205-Section 03
Electricity and Magnetism - Winter 2018
Assignment 3 – Solutions

Problems + Solutions

1. The current passing through a conductor is described by $I(t) = I_0\, e^{-t/\tau}$. If $I_0 = 2A$ and $\tau = 0.5\ s$, how many electrons flow through the conductor during the first 3 seconds? Take $e = 1.6 \times 10^{-19}\ C$. **(5 marks)**

Solution:
The current is defined as $I = \frac{dq}{dt}$, hence we have:

$$\Delta Q = \int_0^3 I(t)dt = \int_0^3 I_0\, e^{-t/\tau} dt = -(2)(0.5)\left(e^{-\frac{3}{0.5}} - 1\right) = 0.998\ C$$

Knowing the charge, we can calculate the number of electrons using:

$$\Delta Q = Ne \rightarrow N = \frac{\Delta Q}{e} = \frac{0.998\ C}{1.6 \times 10^{-19}\ C} = 6.2375 \times 10^{18}$$

2. During certain cellular activities, an ionic channel opens up for 1 ms (millisecond), during which it allows about 5000 Na^+ ions to pass through. Calculate the current passing through this channel. Take $e = 1.6 \times 10^{-19}\ C$. **(5 marks)**

Solution:
We should use the definition of current:

$$I = \frac{\Delta Q}{\Delta t} = \frac{Ne}{\Delta t} = \frac{(5000)(1.6 \times 10^{-19})}{10^{-3}} = 8 \times 10^{-13}\ A$$

3. The resistance of a copper wire with circular cross section of radius a and length L is R. If we melt this wire and reshape it into a wire with square cross section of side a, what would be the resistance of the new wire? **(5 marks)**

Solution:
When the wire is melted and reshaped, its volume remains constant (Since the quantity of the material remains the same). We can find the length of the new wire:

$$V = V' \rightarrow \pi a^2 L = a^2 L' \rightarrow L' = \pi L$$

$$R = \rho\epsilon_0 \frac{L}{A} = \epsilon_0 \frac{L}{\pi a^2}$$

$$R' = \rho\epsilon_0 \frac{L'}{A'} = \rho\epsilon_0 \frac{\pi L}{a^2} \rightarrow R' = \pi^2 R$$

4. The electrical resistance of human body is about 50 $k\Omega$ on average. If a person inadvertently touches the terminals of a 20 kV power supply, which has an internal resistance of 2 kV, determine:
   a) What would be the current passing through him? Given that a 0.1 mA (milliamp) current can kill a person, does he survive? **(2 marks)**
   b) The electrical resistance of a wet or damaged skin can be as low as1 $k\Omega$. What should be the internal resistance of such power supply to be considered safe for human being? **(3 marks)**

Solution:

The internal resistance of the battery is connected in series with the resistance of the human body.

a) To find the current:

$$I = \frac{\mathcal{E}}{r+R} = \frac{20\ kV}{(2+50)\ k\Omega} = 0.38\ A$$

Since this current is larger than 0.1 mA, it can kill the person.

b) Taking the threshold value of the safe current ($I = 0.1\ mA$), we solve for the internal resistance of the battery:

$$r = \frac{\mathcal{E} - IR}{I} = \frac{(2\times 10^4\ V) - (10^{-4}\ A)(10^3\ \Omega)}{10^{-4}\ A} \approx 2\times 10^8\ \Omega$$

5. The maximum current that can be provided to two resistors of same resistance R that are connected in series, is I. A larger current will melt the resistors! What is the maximum current that can be provided to the same resistors, but connected in parallel? **(5 marks)**

Solution:

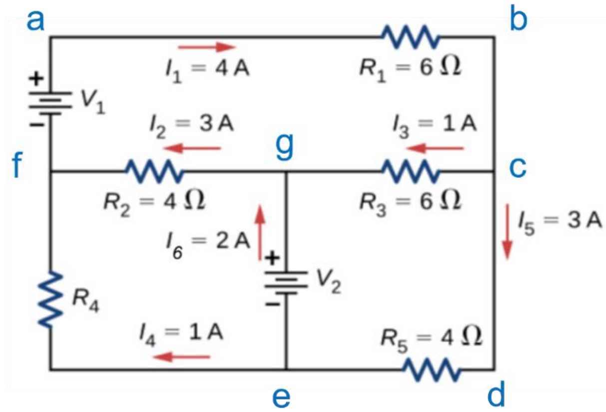
The max current that the two resistors connected in series can take is I. Since in series configuration, the same current passes through each resistor, then the max current a resistor can tolerate is I:

$$(I_{max})_{Series} = (I_{max})_R = I$$

When connected in parallel, the total current is the sum of the currents of each resistor. Hence the max current that the system of resistors in parallel configuration can tolerate is:

$$(I_{max})_{parallel} = 2I$$

6. In the circuit below, find V_1, V_2, and R_4. **(5 marks)**



Solution:

Using Kirchhoff's rule:

Loop abcgfa:

$$-I_1R_1 - I_3R_3 - I_2R_2 + V_1 = 0$$

$$V_1 = (4\ A)(6\ \Omega) + (1\ A)(6\ \Omega) + (3\ A)(4\ \Omega) = 42\ V$$

Loop egcde:

$$V_2 + I_3R_3 - I_5R_5 = 0$$

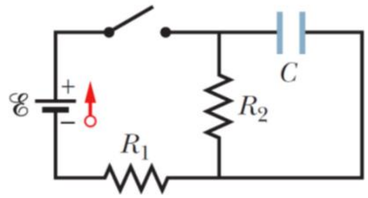
$$V_2 = (3\ A)(4\ \Omega) - (1\ A)(6\ \Omega) = 6\ V$$

Loop egfe:

$$V_2 - I_2R_2 + I_4R_4 = 0$$

$$R_4 = \frac{I_2R_2 - V_2}{I_4} = \frac{(3\ A)(4\ \Omega) - 6\ V}{1} = 6\ \Omega$$

7. In the figure below, the switch S is closed for a long time such that the system reaches a steady state (capacitor is fully charged). Find the current passing through R_2, 4 ms after the switch is opened. R_1 = 10 $k\Omega$, R_2 = 15 $k\Omega$, $\mathcal{E}$ = 20 V, C = 0.4 μF. **(5 marks)**



Solution:

When the switch is closed for a long time the capacitor gets fully charged. The potential difference across the capacitor will be the potential difference across R_2 (since the terminals are connected with ideal wires). This potential difference is IR_2. When the capacitor is fully charged there will be no currents passing through it. Hence to find I:

$$I = \frac{\mathcal{E}}{R_1 + R_2} = \frac{20\ V}{(10 + 15)\ k\Omega} = 0.8\ mA$$

$$\Delta V = IR_2 = (0.8\ mA)(15\ k\Omega) = 12\ V$$

When we open the switch, the capacitor discharges through R_2:

$$I(t) = \frac{Q_{max}}{C} e^{-\frac{t}{\tau}}$$

Where $\tau = RC = (15\ k\Omega)(0.4\ \mu F) = 6 \times 10^{-3}\ s$

We should find Q_{max}:

$$Q_{max} = C\Delta V = (0.4\ \mu F)(12\ V) = 4.8\ \mu C$$

$$I(t) = \frac{Q_{max}}{C} e^{-\frac{t}{RC}}$$

$$I(0.4\ ms) = \frac{4.8\ \mu F}{0.4\ \mu C} e^{-\frac{4\times10^{-3}}{6\times10^{-3}}} = 6.16\ A$$