1. (a) Demonstrate that for an ideal gas: $C_p - C_v = R$. [2 marks]

(b) For an ideal gas undergoing a process, what does the area under a C_v vs. T graph represent? [2 marks]

(c) For an ideal gas undergoing a process, what does the area under a C_p vs. T graph represent? [2 marks]

(a) for ideal gas

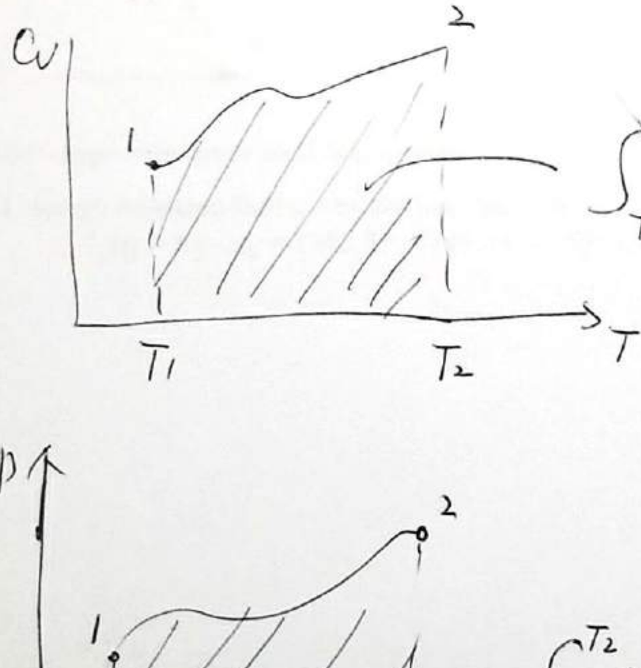
$$\Delta u = \int_{T_1}^{T_2} c_v\, dT \qquad \Delta h = \int_{T_1}^{T_2} c_p\, dT$$

$$\Delta h = \Delta u + \Delta(p v) = \Delta u + R\Delta T = \Delta u + \int_{T_1}^{T_2} R\, dT$$

$$= \int_{T_1}^{T_2} (c_v + R)\, dT = \int_{T_1}^{T_2} c_p\, dT$$

$$c_p = c_v + R$$

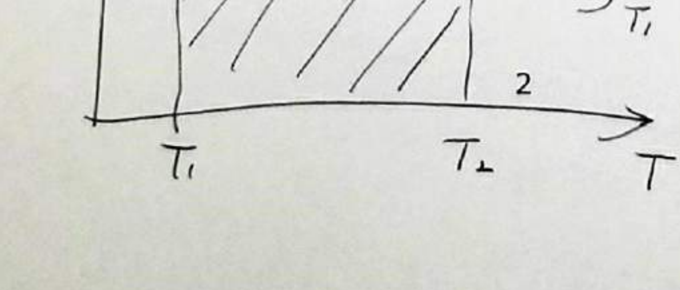
(b)



$$\int_{T_1}^{T_2} c_v\, dT = \underline{\Delta u}$$

Change in specific internal energy

(c)



$$\int_{T_1}^{T_2} c_p\, dT = \Delta h$$

Change in specific enthalpy.

Problem 2 Solution

Ammonia, NH_3, is contained in a sealed rigid tank at 0°C, $x = 50\%$ and is then heated to 100°C. Find the final state P_2, u_2 and the specific work and heat transfer.

Solution:

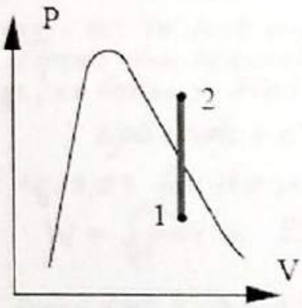
Continuity Eq.: $m_2 = m_1$;

Energy Eq.5.11: $E_2 - E_1 = {}_1Q_2$; $({}_1W_2 = 0)$

Process: $V_2 = V_1 \Rightarrow v_2 = v_1 = 0.001566 + 0.5 \times 0.28763 = 0.14538 \text{ m}^3/\text{kg}$

Table B.2.2: v_2 & $T_2 \Rightarrow$ between 1000 kPa and 1200 kPa

$$P_2 = 1000 + 200 \frac{0.14538 - 0.17389}{0.14347 - 0.17389} = \mathbf{1187\ kPa}$$



$$u_2 = 1490.5 + (1485.8 - 1490.5) \times 0.935 = 1485.83 \text{ kJ/kg}$$

$$u_1 = 179.69 + 0.5 \times 1138.3 = 748.84 \text{ kJ/kg}$$

Process equation gives no displacement: ${}_1w_2 = 0$;

The energy equation then gives the heat transfer as

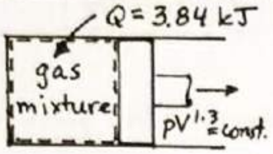
$${}_1q_2 = u_2 - u_1 = 1485.83 - 748.84 = \mathbf{737\ kJ/kg}$$

Problem 3 solution

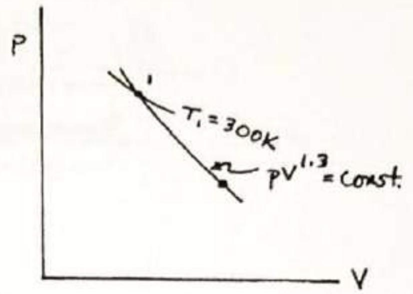
KNOWN: A gas mixture expands with a known pressure-volume relation. The initial state is fixed and the heat transfer for the process is known.

FIND: Determine (a) the final temperature, (b) the final pressure, (c) the final volume, and (d) the work.

SCHEMATIC & GIVEN DATA:



$M = 33$ kg/kmol
$p_1 = 3$ bar
$T_1 = 300$K
$V_1 = 0.1\ m^3$



For the mixture: $c_v = 0.6$ c_v in kJ/kg·K)

ASSUMPTIONS: (1) The gas mixture is the closed system. (2) The gas mixture behaves as an ideal gas. (3) The process is polytropic with n = 1.3. (4) Kinetic and potential energy effects are neglected.

ANALYSIS: (a) To find the final temperature, start with the energy balance and express the internal energy change and work in terms of temperature change, as follows. First

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - W$$

From Eq. 3.57 for the polytropic process

$$W = \int_{V_1}^{V_2} p\,dV = \frac{mR(T_2 - T_1)}{1-n} \qquad (*)$$

Also, from the given c_v relation

$$\Delta U = m\int_{T_1}^{T_2} c_v(T)\,dT = m\,[(0.6)(T_2 - T_1)] \text{ [redacted]} \qquad (**)$$

Incorporating (*) and (**) into the energy balance

$$m\,[(0.6)(T_2 - T_1)] \text{ [redacted]} = Q - \frac{mR(T_2 - T_1)}{1-n} \qquad (***)$$

The mass is found using the ideal gas equation at state 1

$$m = \frac{p_1 V_1}{RT_1} = \frac{(3\text{ bar})(0.1\text{ m}^3)}{\left(\frac{8.314}{33}\right)\frac{\text{kJ}}{\text{kg·K}}(300\text{K})}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N·m}}\right| = 0.397\text{ kg}$$

[redacted] with m = 0.397 kg, T_1 = 300K, Q = 3.84 kJ, and R = 8.314/33 kJ/kg·K to solve for T_2:

[redacted]

Result

T_2 = [redacted] 259.5 K ______ T_2

(b) To find p_2, use Eq. 3.56 for the polytropic process

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{n-1/n} \Rightarrow p_2 = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} p_1 = \left(\frac{\overset{259.5}{\text{[redacted]}}\ K}{300\ K}\right)^{\frac{1.3}{.3}} (3\ bar)$$

$$= \overset{1.6}{\text{[redacted]}}\ bar \longleftarrow p_2$$

(c) Again using Eq. 3.56

$$\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1} \Rightarrow V_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} V_1 = \left(\frac{\overset{3\ bar}{3}\ bar}{\underset{1.6}{\text{[redacted]}}\ bar}\right)^{\frac{1}{1.3}} (0.1\ m^3)$$

$$= \underset{0.162\ m^3}{\text{[redacted]}} \longleftarrow V_2$$

(d) Now, using (*) to evaluate the work

$$W = \frac{mR(T_2 - T_1)}{(1-n)}$$

$$= \frac{(0.397\ kg)\left(\frac{8.314}{33}\right)\frac{kJ}{kg \cdot K}\left(\overset{259.5}{\text{[redacted]}} - 300\right)}{(1-1.3)}$$

$$= \overset{13.5}{\text{[redacted]}}\ kJ \longleftarrow W$$