**CONCORDIA UNIVERSITY**

**Department of Mathematics & Statistics**

| Course | Number | Section(s) |
|---|---|---|
| MATH | 204 | All |

| Examination | Date | Pages |
|---|---|---|
| Alternate | April 2019 | 3 |

| Instructors | Course Examiner |
|---|---|
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**Special Instructions:**

- Only approved calculators are allowed
- Answer all questions

1. **[10]** Solve the system of equations

$$\begin{cases} 2x_1 + 3x_2 + 11x_3 + 5x_4 = 2 \\ x_1 + x_2 + 5x_3 + 2x_4 = 1 \\ 2x_1 + x_2 + 3x_3 + 2x_4 = -3 \\ x_1 + x_2 + 3x_3 + 4x_4 = -3 \end{cases}$$

2. **[10]** Solve the following equation for (3 x 3) matrix X:

$$\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} X = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}$$

3. **[10]** Find the determinant

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$$

4. **[10]**

(A) Find the distance from the point A = (1, 2, 5) to the line $\mathcal{L}$ of intersection of the plane

α:

$$x - y + 3z = 1$$

with the plane β:

$$2x - 3y + 4z = 2$$

(B) Write down the equation of the plane passing through the point A and the line $\mathcal{L}$.

5. **[10]**

(A) Find the distance from the point A = (1, 3, 4) to the plane α passing through the origin and orthogonal to the vector (1, 2, 3).

(B) Find the point of intersection of the plane α and the line $\mathcal{L}$ passing through the point A and the point (5, 4, 1).

6. **[10]** (A) Write down the equation of the plane α passing through the point A = (2, 3, 5) and parallel to the lines

$$\mathcal{L}: x = 3 - 2t, y = 1 - 4t, z = 1 - t$$

and

$$\mathcal{M}: x = -2 - 2t, y = 3 - 3t, z = -1 + 4t$$

(B) Let B = (1, 2, 6). Find the coordinates of the point C of the plane α such that the distance from B to C is minimal.

7. **[10]** For which $x$ the vectors (1, 1, 1, 1), (1, 2, 3, 4), (2, 5, 1, 1) and (-1, x, 3, -4) DO NOT form a basis of the space $R^4$?

8. **[10]** Find the bases of the solution space of the following system of four equations with six unknowns

$$\begin{cases} x_1 + x_3 + x_4 + x_5 = 0 \\ x_1 + 2x_3 + 2x_4 + x_5 = 0 \\ 3x_1 + 4x_3 + 4x_4 + 3x_5 = 0 \\ x_2 + x_6 = 0 \end{cases}$$

9. **[10]** Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 2 & 6 & -15 \\ 1 & 1 & -5 \\ 1 & 2 & -6 \end{pmatrix}$$

Is A diagonalizable?

10. [10] Let

$$A = \begin{pmatrix} -4 & 5 \\ -2 & 3 \end{pmatrix}$$

Find the eigenvectors of the matrix $A^7$.