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**MIDTERM** **ENGR251/2 Fall 12**

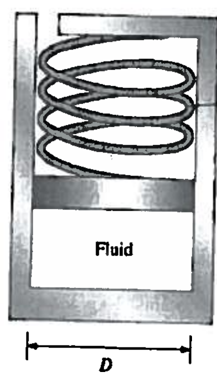
**CONCORDIA UNIVERSITY**
**FACULTY OF ENGINEERING AND COMPUTER SCIENCE**
**DEPARTMENT OF MECHANICAL ENGINEERING**

## PROBLEM I [20 pts]

A mass of 10 f of nitrogen is contained in the spring -loaded piston-cylinder device shown in the figure below. The spring constant is 1 kN/m and the piston diameter is 10 cm. When the spring exerts no force against the piston, the nitrogen is at 120 kPa and 27°C. The device is now heated until its volume is 10 percent greater than the original volume. Knowing that that there is linear relation between the variation in pressure and the variation in volume during the process,

1. Determine the final pressure.
2. Determine the change in internal energy (in kJ/kg) of the nitrogen.
3. Determine the change in enthalpy (in kJ/kg) of the nitrogen.

**<u>Hint:</u>** You have to determine the constant for the linear relation between variation in pressure and volume. For this you have to consider that the force applied by the spring at each state is equal to the product of the spring constant and the position of the piston (k*x)




for nitrogen
$R = 0.2968$ kJ/kg . K
$C_v = 0.743$ kJ/kg . K

## PROBLEM II [10]

- Explain physically why $C_p$ is higher than $C_v$ for an ideal gas?
- Show mathematically that $C_p - C_v = R$
- Show mathematically the variation of pressure with depth for an ideal gas.

**4-57** A spring-loaded piston-cylinder device is filled with nitrogen. Nitrogen is now heated until its volume increases by 10%. The changes in the internal energy and enthalpy of the nitrogen are to be determined.

***Properties*** The gas constant of nitrogen is $R = 0.2968$ kPa·m³/kg·K. The specific heats of nitrogen at room temperature are $c_v = 0.743$ kJ/kg·K and $c_p = 1.039$ kJ/kg·K (Table A-2a).

***Analysis*** The initial volume of nitrogen is

$$V_1 = \frac{mRT_1}{P_1} = \frac{(0.010\,\text{kg})(0.2968\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(27+273\,\text{K})}{120\,\text{kPa}} = 0.00742\,\text{m}^3$$ 2

The process experienced by this system is a linear $P$-$v$ process. The equation for this line is

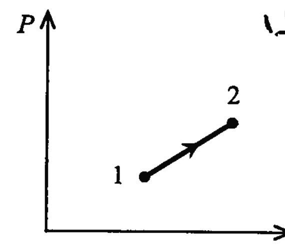
$$P - P_1 = c(V - V_1)$$ 2

where $P_1$ is the system pressure when its specific volume is $v_1$. The spring equation may be written as

$$P - P_1 = \frac{F_s - F_{s,1}}{A} = k\frac{x - x_1}{A} = \frac{kA}{A^2}(x - x_1) = \frac{k}{A^2}(V - V_1)$$ 3

Constant $c$ is hence

$$c = \frac{k}{A^2} = \frac{4^2 k}{\pi^2 D^4} = \frac{(16)(1\,\text{kN/m})}{\pi^2 (0.1\,\text{m})^4} = 16{,}211\,\text{kN/m}^5$$ 3




If the student can not get the formulation for c he/she can pick a value and continue the solution. In this case 6 pts have to be removed

The final pressure is then

$$P_2 = P_1 + c(V_2 - V_1) = P_1 + c(1.1V_1 - V_1) = P_1 + 0.1cV_1$$
$$= 120\,\text{kPa} + 0.1(16{,}211\,\text{kN/m}^5)(0.00742\,\text{m}^3)$$
$$= 132.0\,\text{kPa}$$ 3

The final temperature is

$$T_2 = \frac{P_2 V_2}{mR} = \frac{(132.0\,\text{kPa})(1.1\times 0.00742\,\text{m}^3)}{(0.010\,\text{kg})(0.2968\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})} = 363\,\text{K}$$ 3

Using the specific heats,

$$\Delta u = c_v \Delta T = (0.743\,\text{kJ/kg}\cdot\text{K})(363-300)\text{K} = \mathbf{46.8\,kJ/kg}$$ 2
$$\Delta h = c_p \Delta T = (1.039\,\text{kJ/kg}\cdot\text{K})(363-300)\text{K} = \mathbf{65.5\,kJ/kg}$$ 2

Problem II

1. because for the same amount of heat in, the variation in T° will be higher at constant volume than constant pressure. (2)

$C = \frac{Q}{\Delta T} \quad \Delta T|_{P=ct} < \Delta T|_{v=ct} \rightarrow C_p > C_v$

2.

$h = u + Pv$

$h = u + RT$ we have to assure ideal gas (3)

$dh = du + R\,dT$

$\frac{dh}{dT} = \frac{du}{dT} + R \rightarrow C_p - C_v = R$

3. $\frac{dP}{dz} = -\rho g$ for an ideal gas $P/\rho = RT$

$\rho = \frac{P}{RT}$ (5)

$\frac{dP}{dz} = -\frac{P}{RT} g$

$\frac{dP}{P} = \frac{-g}{RT} dz \rightarrow P_2 = P_1 \, e^{-g/RT (z_2 - z_1)}$