Probability and Statistics in Engineering

QUESTIONS

1. A small printing company claims that a certain refilled laser printer cartridge can print minimum of 5000 pages. In order to verify the company's claim, Concordia University conducted a study among 50 refilled printer cartridges purchased form this company and found that cartridges can print average 4870 pages with a population standard deviation of 90 pages. Based on the given information
   a. Compute a 95% confidence interval for the population mean. (**6 points**)
   b. In order to reduce the error in estimating (half-width of) the population mean (*e*) by one half, how many more sample should be tested? (**4 points**)

2. During a Rock concert, failure of an amplifier is not an option. For the concert total 5 similar kinds of amplifiers are used. Based on the previous observations, it is known that an amplifier can perform without a failure with an average of 2 hours. Failures occasionally happen during the concert and the time that an amplifier works before it fails is found to be exponentially distributed. If the organizer likes to achieve 95 percent reliability during the upcoming concert which last about 2 hours, how many amplifier they should carry during the concert? (**10 points**)

3. Let Y denotes the diameter of a metal-pipe and X is the diameter of a plastic inserted in the joint to make a weatherproof system in an airplane. After scaling both X and Y, we obtain the following joint probability distribution for the system

$$f(x,y)=\begin{cases}\dfrac{1}{y} & 0<x<y<1\\ 0 & else\end{cases}$$

   a. If the designer wants the scaled tolerance (Y - X) between diameters not to be less than 0.05 and more than 0.10, find the probability of the system to comply with the given tolerance limits. In another word find
      $P(0.05 < Y\text{-}X < 0.10)$ (**7 points**)
   b. If we select 2 systems randomly, what is the probability that none of them violates the given tolerance specifications? (**3 points**)

4. From a group of 5 mechanical engineering (ME) and 3 industrial engineering (IE) students, how many committees of size 4 are possible?
   a. With no restrictions (**3 points**)
   b. Committee should include 2 ME and 2 IE (**2 points**)

5. Consider continuous random variables X and Y with the following joint probability density function:

$$f(x,y)=\begin{cases}\dfrac{1}{2} & x<y<x+2;\ -x<y<-x+2\\ 0 & else\end{cases}$$

   a. For

$$u = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}$$
$$v = \frac{-x}{\sqrt{2}} + \frac{y}{\sqrt{2}}$$

Determine the joint distribution of U and V. Call it g(u,v). (**7 marks**)

b. Determine region of support of g(u,v). That is determine the region in the (u,v) plane where g(u,v) is not zero. (**3 marks**).

6. We wish to study the flexural strength of a certain type of concrete beam. A random sample of size 30 is taken with the following strengths in Mega Pascals (MPa):

5.9	7.2	7.3	6.3	8.1	6.8	7.0	7.6	6.8	7.1
6.5	7.0	6.3	7.9	9.0	8.2	8.7	7.8	9.7	8.1
7.4	7.7	9.7	7.8	7.7	11.6	11.8	11.3	10.7	9.1

Assume that the population distribution is Gaussian.

a. Calculate the Sample Mean. (**2 marks**)
b. Calculate the Sample Median. (**1 mark**)
c. Calculate the Sample Standard Deviation. (**2 marks**)

If a beam is stronger than 8 MPa it has passed its strength test and can be considered a success.

d. Calculate the probability that the population mean is greater than 8 MPa. (**3 marks**)
e. Provide an estimate of what portion of the entire population of this type of beam would be successful. Justify your response. (**2 marks**).

7. Automobiles arrive at a vehicle equipment inspection station according to a Poisson Process with rate $\lambda = 10$ vehicles per hour. Each vehicle that arrives has a 50% chance of having no equipment violations. Assume violations on one vehicle are independent of violations on another.

a. What is the probability of exactly 10 vehicles arriving in one hour? (**2 marks**)
b. If 10 vehicles arrive what is the probability that all 10 have equipment violations? (**2 marks**)
c. What is the probability of exactly 10 vehicles arriving in one hour and all 10 having equipment violations. (**2 marks**)
d. For any fixed $y \geq 10$ what is the probability that y vehicles arrive in one hour and exactly 10 of them have equipment violations? (**2 marks**)
e. What is the probability that exactly 10 cars with equipment violations are found in one hour. (**2 marks**)