Solving Non-linear equations

$$f(x) = 2\sin(x) - \frac{x^2}{10}$$

is there a root in [2 3] ?

$f(2) = 1.4186$ 5

$f(3) = -0,6177$

yes there is at least one root in [2 3].

Solving $f(x)=0$ using fixed point method

$$f(x)=0 \iff 2\sin(x) - \frac{x^2}{10} = 0$$

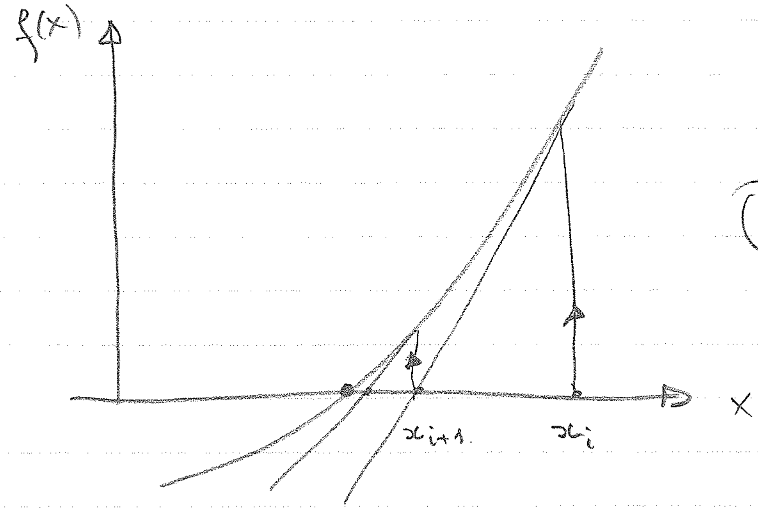
$$\iff 2\sin(x) = \frac{x^2}{10}.$$

so $x_{i+1} = \sqrt{20 \sin(x_i)}$ 5 Valid if $\sin(x_i) \geq$

Note: This formulation is not unique.

4 Solving $f(x)=0$ using Newton-Raphson

a)



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b) $x_0 = 2.5$

$$f(x) = 2\sin(x) - \frac{x^2}{10}$$

$$f'(x) = 2\cos(x) - \frac{x}{5}$$

and $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$

$x_0 = 2.5$

$x_1 = 2.772$ 10

$x_2 = 2.753$. 10

c) using the secant Method, we have to:

(2)
- → approximate the derivative of the function @ x_i
- → use Two initial guesses.

(3) we will not theoretically converge faster since in the secant method we have approximated the derivative @ x_i.

LU decomposition

$$[A] = \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix}$$

$$[L] = \begin{bmatrix} 3 & 0 & 0 \\ 0.1 & 7.033 & 0 \\ 0.3 & -0.190 & 10.012 \end{bmatrix}$$ 6

$$[U] = \begin{bmatrix} 1 & -0.033 & -0.067 \\ 0 & 1 & -0.042 \\ 0 & 0 & 1 \end{bmatrix}$$ 6

Solving the system using LU decomposition

2 - $[A]\{x\} = \{b\} \iff [L]\underbrace{[U]\{x\}}_{\{z\}} = \{b\}$

2 - So we solve for $\{z\}$ i.e $[L]\{z\} = \{b\}$ by forward substitution.

2 - The we solve for $\{x\}$; ie $[U]\{x\} = \{z\}$ by backward substitution.

Gauss Seidel: convergence Gauss seidel (2.5)

yes: the diagonal is dominent

$$\begin{cases} 3x_1 - 0.1x_2 - 0.2x_3 = 7.85 \\ 0.1x_1 + 7x_2 - 0.3x_3 = -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 = 71.4 \end{cases}$$

first iteration:

$$x_1 = \frac{7.85 + 0 + 0}{3} = 2.6167 \quad (3)$$

$$x_2 = \frac{-19.3 - 0.1(2.6167) + 0}{7} = -2.7945 \quad (3)$$

$$x_3 = \frac{71.4 - 0.3(2.6167) + 0.2(-2.7945)}{10} = 7.0056 \quad (3)$$

second iteration:

$x_1 = 2.9906$ (3) $\quad |\varepsilon_a| = 12.5\%$ (0.5)

$x_2 = -2.4996$ (3) $\quad |\varepsilon_a| = 11.8\%$ (0.5)

$x_3 = 7.0003$ (3) $\quad |\varepsilon_a| = 0.076\%$ (0.5)

III

A/ see Tutorial (on line) 10

B/ We have

(1) $\rightarrow$ $f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{1}{2} f''(x_i)h^2 + o(h^2)$

(2) $\rightarrow$ $f(x_{i+2}) = f(x_i) + f'(x_i)(2h) + \frac{1}{2} f''(x_i)(2h)^2 + o(h$

(1) $\times (-2)$ + (2) gives

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} + o(h^2)$$

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