

PHYS 204 - Practice Final - Solutions

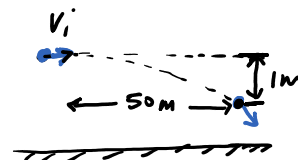
1. (c) Regardless of their motion along the x-axis (horizontal), the ball and the bullet go through free fall motion with $(v_y)_i = 0$ along the y-axis (vertical). Hence, they both hit the ground at the same time (but at different locations).

2. (c)

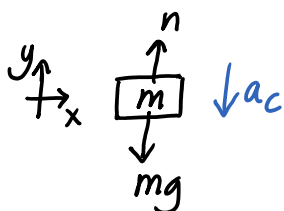
$$\begin{cases} \Delta x = 50 \text{ m} \\ \Delta y = -1 \text{ m} \end{cases} \quad \begin{cases} v_{ix} = v_i \\ v_{iy} = 0 \end{cases}$$

$$\Delta x = v_i \Delta t \rightarrow \Delta t = \frac{\Delta x}{v_i} = \frac{50}{v_i}$$

$$\Delta y = -\frac{1}{2} g \Delta t^2 \Rightarrow -1 = -\frac{1}{2} (10) \left(\frac{50}{v_i} \right)^2 \rightarrow \boxed{v_i \approx 112 \frac{\text{m}}{\text{s}}}$$



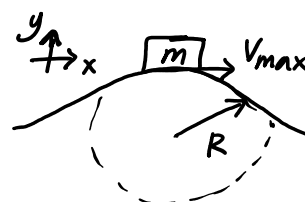
3. (b) At max speed, the normal force is zero and the car is at the verge of leaving the surface.



$$\sum F_y = ma_c$$

$$\rightarrow n - mg = -m \frac{v^2}{R}$$

$$mg = m \frac{v^2}{R} \rightarrow v = \sqrt{Rg} = \boxed{10 \text{ m/s}}$$



4. (b) This is a completely inelastic collision, in which the momentum of the system conserves but the mechanical energy does not:

$$p_i = p_f$$

$$mv = (m + m)v' \rightarrow v' = \frac{v}{2}$$

$$\frac{K_f}{K_i} = \frac{\frac{1}{2} (2m) v'^2}{\frac{1}{2} m v^2} = \frac{1}{2} = 50\%$$

The rest of the kinetic energy of the first block is spent to fuse the two blocks together, and also comes out as heat/sound.

5. (b) Airbags increase the time during which passengers' speed changes from v (speed of the car before accident) to 0. Considering Impulse momentum principle, passengers experience a smaller force.

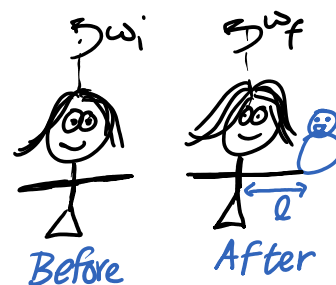
6. (b) Taking the system as the girl and the bird together, there is no external torque and hence the system is isolated. Using conservation of angular momentum:

$$L_i = L_f \rightarrow I_i \omega_i = I_f \omega_f$$

$$\rightarrow I_{\text{girl}} \omega_i = (I_{\text{girl}} + M_{\text{bird}} l^2) \omega_f$$

$$\rightarrow \frac{\omega_f}{\omega_i} = \left[\frac{I_{\text{girl}}}{(I_{\text{girl}} + M_{\text{bird}} l^2)} \right] < 1$$

Her angular velocity decreases.



7. (b) The ball and the box both have the same energy initially (gravitational potential energy). As the box slides down, this energy transforms completely into translational kinetic energy at the bottom of the inclined surface. Since the ball rolls, the potential energy converts into translational and rotational kinetic energy. Hence the **box** will have a larger velocity of center of mass:

Box:

$$mgh = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2gh}$$

Ball:

$$mgh = \frac{1}{2}mv'^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv'^2 + \frac{1}{2}I\left(\frac{v'}{R}\right)^2 = \frac{1}{2}\left(m + \frac{I}{R^2}\right)v'^2$$

$$v' = \sqrt{\left(\frac{m}{m + \frac{I}{R^2}}\right) 2gh} < \sqrt{2gh}$$

8. (c) They will both reach the same height. Regardless of where the bullet enters, the linear momentum of the system conserves. Hence, both boxes with the bullets embedded inside them will have the same velocity after this completely inelastic collision. Using conservation of energy, they will both reach the same height.

You can watch the interesting demonstration of this problem here:

http://demo.webassign.net/sercp11/html/bullet_block/index.html?_cf_chl_jschl_tk_=bcdcb00b7efaefbd3d2469c2dc9e4042ba22d261-1606926230-0-ATUVyN8pcZwAOcD57spMwBfSIB1Gu16Axhhw3TPfQyV9wdfypTIHhG0u9EPmxvXqaWBhj94KWd hfMmSFcPd4aM79SGLGROxJWUJcad37pfdcv3IBqctJZ3Pw6dfFiTvoAxfB-EdybgBwt50KJxqsTtOq8J0rVuUcsXznw0IHcvttUNuGxV77qKEfjJPireirLpy7IIvzpUGRKPJQtPAFWFr ddh9NIPKdsMbWts8vsJjUOaMN1Z9uwhKcFs-RRWmmyEtW51oxLq7JAah-45xbWu6I9yYI1Qkv6ibOadg-MofGPT9wyIFei5o214ZIZE6RxMbl-oAmG7GM7YeolhdpagkJ2Fuglb4qNhueOg0l8bzIrvgzY8yKLHX3Ed8v-9hT9zT2rPWBFIKYGL67QhP3 5tEhFFvo_GbQV6bxQVo0CGs

9. (c) This is a completely inelastic collision in rotational motion.

As the angular momentum of this system conserves:

$$L_i = L_f$$

$$I_i \omega = I_f \omega' \rightarrow \frac{\omega'}{\omega} = \frac{I_i}{I_f} = \frac{\frac{1}{12} mL^2}{\frac{1}{12} (2m)(2L)^2} = \frac{1}{8}$$

10. (c) The velocity is given by:

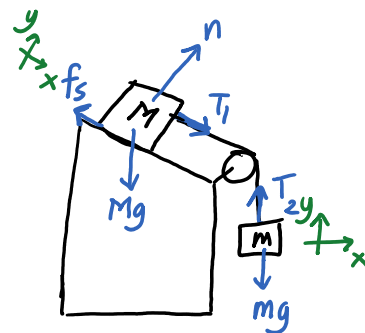
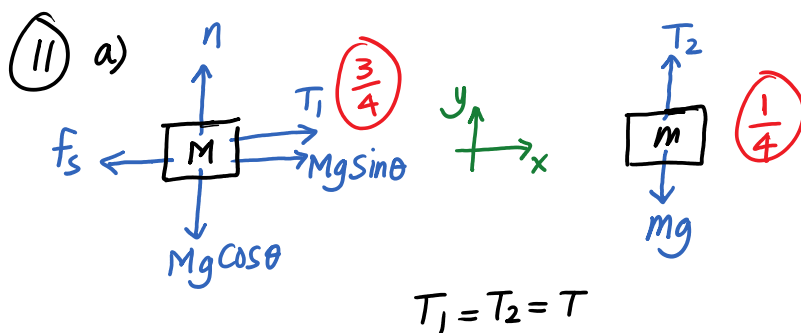
$$v(t) = -\omega A \sin(\omega t)$$

From the $x(t)$ equation: $A = 3$ and $\omega = 2\pi$

$$\omega = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{\omega} = 1 \text{ s}$$

$$v(t) = -6\pi \sin(2\pi t)$$

$$v\left(\frac{T}{4}\right) = v\left(\frac{1}{4}\right) = -6\pi \sin\left(\frac{2\pi}{4}\right) = -6\pi$$



b)

Block M:

$$\sum F_y = 0 \quad \left(\frac{2}{8}\right)$$

$$n - Mg \cos \theta = 0 \quad \left(\frac{3}{8}\right)$$

$$n = Mg \cos \theta$$

$$\sum F_x = 0 \quad \left(\frac{3}{8}\right)$$

$$T + Mg \sin \theta - f_s = 0$$

$$\mu_s n = T + Mg \sin \theta$$

$$\rightarrow \mu_s Mg \cos \theta = mg + Mg \sin \theta$$

$$\rightarrow \mu_s = \frac{m + M \sin \theta}{M \cos \theta} \quad \left(\frac{1}{8}\right)$$

Block m:

$$\sum F_y = 0 \quad \left(\frac{2}{8}\right)$$

$$T - mg = 0$$

$$T = mg$$

12. It is best to use the work-energy principle to solve this problem. Since the cylinder rolls without slipping, we should consider the kinetic energy of rolling motion:

System = Cylinder + Earth
Environment = Inclined surface

$\left(\frac{4}{12}\right) W_{f_k} = \Delta E = E_f - E_i$

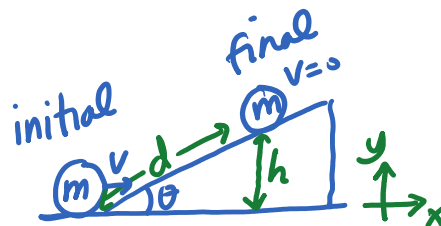
$$W_{f_k} = (K_f + U_{gf}) - (K_i + U_{gi})$$

$$-f_k d = mgh - \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)$$

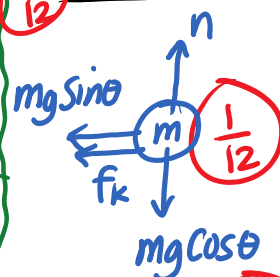
$$\omega = \frac{v}{R}, \quad I = \frac{1}{2}mR^2, \quad f_k = \mu_k mg \cos \theta, \quad h = d \sin \theta$$

$\left(\frac{3}{12}\right) - \mu_k mg d \cos \theta = mgh - \left(\frac{1}{2}mv^2 + \frac{1}{4}mR^2 \frac{v^2}{R^2}\right)$

$$d = \frac{\frac{3}{4}v^2}{g(\sin \theta + \mu_k \cos \theta)} = 2.01 \text{ m} \quad \left(\frac{1}{12}\right)$$



$\left(\frac{1}{12}\right) h = d \sin \theta$



$$\sum F_y = 0 \quad \left(\frac{2}{12}\right)$$

$$n = mg \cos \theta$$

$$f_k = \mu_k n = \mu_k mg \cos \theta$$

13. To do this problem, we should:

- 1) Find the speed of Tarzan before reaching Jane from conservation of energy
- 2) Find the speed of Tarzan and Jane after their inelastic collision
- 3) Find the maximum angle through conservation of energy

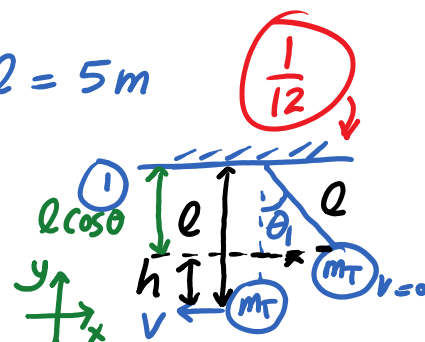
$$m_T = 70 \text{ kg}, \quad m_J = 50 \text{ kg}, \quad \theta_1 = 60^\circ, \quad \ell = 5 \text{ m}$$

① Conservation of energy:

$$h = \ell - \ell \cos \theta = \ell (1 - \cos \theta)$$

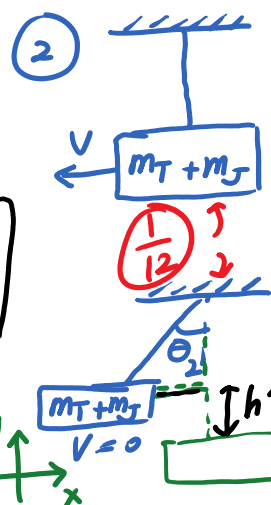
$$\Delta E = 0 \rightarrow \Delta K + \Delta U_g = 0$$

$$K_f - K_i + U_{gf} - U_{gi} = 0 \rightarrow \frac{1}{2} m_T v^2 = m_T g h \rightarrow v = \sqrt{2gh}$$



② conservation of momentum:

$$P_i = P_f \rightarrow m_T v = (m_T + m_J) v' \rightarrow v' = \left(\frac{m_T}{m_T + m_J} \right) v$$



③ Conservation of energy:

$$\Delta E = 0 \rightarrow \Delta K + \Delta U_g = 0 \rightarrow K_f - K_i + U_{gf} - U_{gi} = 0$$

$$(m_T + m_J) g h' = \frac{1}{2} (m_T + m_J) v'^2 = \frac{1}{2} (m_T + m_J) \cdot \left(\frac{m_T}{m_T + m_J} \right)^2 v^2 \rightarrow 2gh$$

$$h' = \frac{1}{2} \left(\frac{m_T}{m_T + m_J} \right)^2 \frac{2gh}{g} = \left(\frac{m_T}{m_T + m_J} \right)^2 h$$

$$\ell (1 - \cos \theta') = \left(\frac{m_T}{m_T + m_J} \right)^2 \ell (1 - \cos \theta)$$

$$\cos \theta' = 1 - \left(\frac{m_T}{m_T + m_J} \right)^2 (1 - \cos \theta) = 0.83$$

$$\theta' = \cos^{-1}(0.83) = 33.9^\circ$$

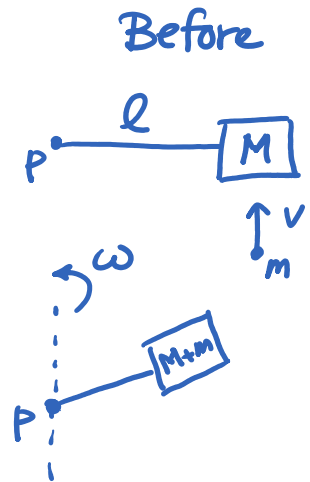
$$\begin{cases} h' = \ell (1 - \cos \theta') \\ h = \ell (1 - \cos \theta) \end{cases}$$

14. This problem involves collision and rotation of the system after collision. We should use conservation of angular momentum, since there is no external torque on the system (Bullet + Block).

$$\left(\frac{2}{12} \right) L_i = L_f, \quad \begin{cases} L_i = \vec{r} \times \vec{p} = m v \ell \\ L_f = I \omega = (m+M) \ell^2 \omega \end{cases}$$

$$\rightarrow m v \ell = (m+M) \ell^2 \omega$$

$$\rightarrow \omega = \left(\frac{m}{m+M} \right) \frac{v}{\ell} = 4.5 \text{ rad/s}$$

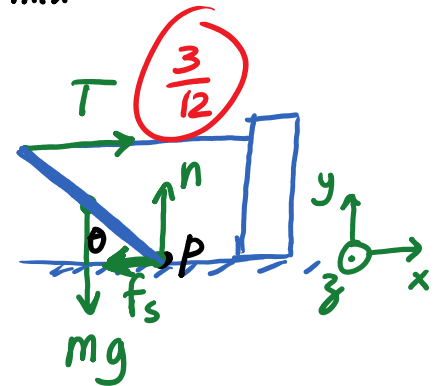


15. This is a static equilibrium problem, as Bob is not moving or rotating. We should set up Newton's laws of motion for linear and rotational motion:

(a) $\Sigma F_x = 0 \rightarrow f_s = T \rightarrow \mu_s n = T$ (1) f_s is max, since we're looking for θ_{min}

$$\Sigma F_y = 0 \rightarrow n = mg$$

$$(1), (2) \rightarrow T = \mu_s mg = 250 \text{ N}$$



(b) $\Sigma \vec{\tau} = 0$ about P

$$\vec{\tau}_T + \vec{\tau}_{mg} + \vec{\tau}_n + \vec{\tau}_{f_s} = 0$$

$$\vec{\tau}_T = \vec{r}_T \times \vec{T} = \ell T \sin \phi (-\hat{k})$$

$\phi = 180^\circ - \theta \rightarrow \sin \phi = \sin \theta \rightarrow \vec{\tau}_T = \ell T \sin \theta (-\hat{k})$ (3)

right-hand-rule (or turning clockwise)

$$\vec{\tau}_{mg} = \vec{r}_{mg} \times m\vec{g} = \frac{\ell}{2} mg \sin \phi = \frac{\ell}{2} mg \sin(90^\circ + \theta) (\hat{k}) \rightarrow \vec{\tau}_{mg} = \frac{\ell}{2} mg \cos \theta (\hat{k})$$

$$(3), (4) \rightarrow \ell T \sin \theta = \frac{\ell}{2} mg \cos \theta \rightarrow \tan \theta = \frac{mg}{2T}$$

$$\rightarrow \theta = \tan^{-1} \left(\frac{mg}{2T} \right) = 45^\circ$$