Concordia University

Dept. of Computer Science and Software Engineering

COMP 335 – Introduction to Theoretical Computer Science

Fall 2018

Assignment 4

Submission through Moodle due on Thursday Nov. 29th at 23:55

Note: This is a theoretical course. That means, while the WHAT is important, the WHY is absolutely essential. Show steps of your solution for the full mark.

Total mark is 60.

1. [10 Points] Show that the family of context-free languages are closed under each of the following operations.

   (a). Reversal (Note: $L^R = \{w^R : w \in L\}$, for any language L.)

   (b). Homomorphism.
   If Σ and Γ are alphabets, a homomorphism h is a function from Σ to Γ^*, that is h replaces a single symbol in Σ to a string over Γ. This definition can be extended naturally to strings, that is, if $w = a_1 \cdots a_n$ is a string, then $h(w) = h(a_1) \cdots h(a_n)$.

2. [20 Points] Which of the following languages are CF and which ones are not? For each language, if it is CF, give a CFG that generates it; otherwise prove your claim.

   (a). $L_a = \{w : w \neq uu,\ u \in L((a+b)^*)\}$
   Hint 1: Note that L_a contains every string of length odd and certain strings of length even.
   Hint 2: Note that every even length string w in L_a can be written/viewed as xy, where $|x|$ is odd, $|y|$ is odd, AND the central symbol of x is different from the central symbol of y. You can use these hints to design a CFG or a PDA for L_a.

   (b). $L_b = \{w : w = uu,\ u \in L(1^*01^*)\}$

   (c). $L_c = \{w : w \in \{a,b,c\}^*,\ n_a(w) > n_b(w) > n_c(w)\}$

   (d). $L_d = \{a^{n^2} : n \geq 0\} \cup \{a^n : n \geq 0\}$

3. [20 Points] For each of the following languages, show the transition diagram of a Turing machine (TM) that accepts the language.

   (a). $L_e = \{w : w \in \{a,b,c\}^+, n_a(w) = n_b(w) = n_c(w)\}$

   (b). $L_f = \{ww : w \in \{a,b\}^+\}$

   (c). $L_g = \{ww^R : w \in \{a,b\}^+\}$

4. [10 Points] For each of the following functions over the positive integers, represented in*unary*, give the transition diagram of a TM that computes the function.

   (a). $f(x) = x \bmod 5$

   (b). $g(x) = \lfloor \frac{x}{2} \rfloor$