## CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

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Course	Number	Section
Mathematics	203	0.4
Examination		CA
Final	Date	Pages
	August 2014	3
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Instructions:	Only calculators approved by the	
mseructions:	Department are allowed	

## MARKS

- (a) Sketch the graph of the function  $f(x) = |(x+1)^2 3|$  starting from the [12] 1. graph  $f(x) = x^2$  and using appropriate transformations.
  - (b) Given the functions  $f(x) = x^2 2x$  and  $g(x) = \sin \sqrt{\tan(x+1)}$ . Find  $f \circ g$ ,  $g \circ f$ , and  $g \circ g$ .
  - (c) Given the function  $f = \frac{x+3}{2x-5}$  find the inverse function  $f^{-1}$  and determine domain and range of f and  $f^{-1}$ .  $\ln \left( 1 e^{2x} \right) = y \Rightarrow \exp \left( 1 e^{2x} \right) = y \Rightarrow \exp \left( 1 e^{2x} \right)$
- [10] 2. Evaluate the limits. Do not use l'Hôpital rule:

(a) 
$$\lim_{x \to -\infty} \frac{(x^2 + x)^3 (2x - 4)^2}{x^7 \sqrt{4x^2 + 2}}$$

(b) 
$$\lim_{x \to -2} \frac{x+2}{\sqrt{x+6}-2}$$

- [12] 3. (a) Consider the function  $f(x) = \frac{|x+4|}{x^2 + 3x 4}$ . Calculate both one-side limits at the point(s) where the function is undefined.

$$f(x) = \begin{cases} x+1 & \text{if } x < 2\\ ax^2 - 1 & \text{if } 2 \le x < 3\\ 2x - a + bx & \text{if } x \ge 3 \end{cases}$$

$$\frac{-(x+a)}{(x+a)(x-1)} \frac{(x+a)}{(x+a)(x-1)}$$

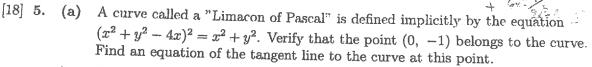
Find the derivatives of the following functions (You do not have to simplify!):

(a) 
$$f(x) = \frac{3\sqrt[9]{x^5} - 7x^3\sqrt{x} + x^{\frac{9}{11}}}{x^{\frac{4}{5}}}$$

(b) 
$$f(x) = \frac{\cos(3x)}{(1+x^2)^3}$$

(c) 
$$f(x) = \tan\left(\sqrt{2x^{-3} + 4e^{2x}}\right)$$

(d)  $f(x) = (2x^3 - 4)^{\sin x}$  (use logarithmic differentiation)  $= e^{\sin x - \ln \left(\frac{x^2 - 4}{x^2}\right)} = \exp\left[\frac{\sin x - \ln \left(\frac{x^2 - 4}{x^2}\right)}{2x^2 + \ln \left(\frac{x^2 - 4}{x^2}\right)}\right] = \exp\left[\frac{\sin x - \ln \left(\frac{x^2 - 4}{x^2}\right)}{2x^2 + \ln \left(\frac{x^2 - 4}{x^2}\right)}\right]$ 



- (b) Find the points on the ellipse  $4x^2 + y^2 = 4$  that are farthest from the point M. (-3, + 3. V2) (1, 0).
- (c) A particle is moving along the curve  $\frac{xy^3}{1+y^2} = \frac{8}{5}$ . Assume that the x-coordinate is increasing at the rate of 6 units per second when the particle is at the point (1, 2). At what rate is the y-coordinate of the point changing at that instant?
- Strontium-90 has a half-life of 28 days.
  - (a) A sample has a mass of 50 mg initially. Find a formula for the mass remaining after t-days.
  - (b) Find the mass remaining after 40 days.
  - (c) How long does it take the sample to decay to a mass of 2 mg?

- Consider the function  $f(x) = \sqrt{x+2}$ .
- Use the definition of the derivative to find the formula for f'(x).

- (d) Use this linearization to approximate the value of f(3).
- 5 00 Given the function  $f(x) = 3x^5 - 20x^3$ .
- (a) Find the domain of f and check for symmetry. Find asymptotes of f (if any).
- (b) Calculate f'(x) and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).
- <u></u> Calculate f''(x) and use it to determine intervals where the function is inflection points (if any). concave upward, intervals where the function is concave downward, and the
- Sketch the graph of the function f(x) using the information obtained above.
- Bonus Question. Calculate the limit  $\lim_{x\to 0^+} (1+ax)^{\frac{b}{x}}$ . Hint: use the l'Hôpital rule. TOWN TO THE STATE OF THE STATE