Concordia University

Dept. of Computer Science and Software Engineering

Introduction to Theoretical Computer Science

COMP335 – Introduction to Theoretical Computer Science

Fall 2017

Assignment 3

Submission through Moodle due on Thursday November 2nd at 23:55

Total mark is 50.

1. [15 Points] For each of the following languages, prove or disprove that it is regular.

   (a). $L_a = \{uav : uv \in L\}$, where L is any regular language over $\{a, b\}$.

   (b). $L_b = \{w : w \in \{a, b\}^* \text{ and } w = w^R\}$.

   (c). $L_c = \{b^k a^n b^n : n, k \geq 0\}$.

2. [5 Points] Show that context-free languages are closed under reversal, that is if L is CF, then its reverse L^R is CF as well.

3. [10 Points] Consider the following CFG G in which S is the start variable:

   $S \to A \,|\, B$
   $A \to aaA \,|\, \lambda$
   $B \to bB \,|\, bbC$
   $C \to B$

   (a). Remove λ-productions, unit-productions, and useless productions from G,

   (b). Convert G into an equivalent grammar in Chomsky normal form (CNF).

4. [5 Points] Show that every CF grammar $G = (V, T, S, P)$ can be converted into an equivalent CFG in which every production is of the form $A \to xBC$ or $A \to \lambda$, where $x \in T \cup \{\lambda\}$ and A, B, and C are variables.

5. [15 Points] For each of the following languages, if it is context-free, give a CFG. If it is not CF, simply state that it is not CF; no proof is required to provide.

   (a). $L_a = \{uvwv^R : u, v, w \in \{a, b\}^+, |u| = |w| = 2\}$.

   (b). $L_b = \{w \in \{a, b\}^* : w = w^R\}$.

   (c). $L_c = \{w \in \{a, b, c\}^* : n_a(w) > n_b(w) > n_c(w)\}$.

   (d). $L_d = \{a^i b^j a^i b^j : i \geq 0, j \geq 0\}$.

   (e). $L_e = \{w_1 c w_2 : w_1, w_2 \in \{a, b\}^*, w_1 \neq w_2\}$.