Concordia University

ENGR 233 - Tut UB - 1st Midterm Exam - February 16, 2009 Instructor: Galia Dafni
Total time: 75 minutes Total marks: 50
Allowable materials: Pencils, pens. You may **NOT** use notes, books, calculators or any other materials. A formula sheet can be found on the back of the exam.

Write your answers in the examination booklet. Write clearly and neatly and show all your work in order to receive full marks. You do not need to simplify or approximate numerical answers but you should evaluate any trigonometric functions if the question requires it.

Problem 1. (15 marks)

(a) Find a normal vector $\vec{N}$ to the plane $x + y + z = 0$, and give parametric equations for the normal line through the point $(0, 0, 0)$.

(b) Find the component $\text{comp}_{\vec{N}}\vec{F}$ of the vector $\vec{F} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$ in the direction given by the vector $\vec{N}$ from part (a). The sign of your answer will depend on the direction of the vector $\vec{N}$ you have chosen.

(c) Find the volume of the parallelepiped formed by the vectors $\langle 0, 1, -1\rangle$, $\langle 1, -1, 0\rangle$ and $\langle 3, 2, 1\rangle$.

Problem 2. (20 marks) The following questions refer to the curve in the xy-plane described by the vector function $\vec{r}(t) = t^2\hat{\imath} + t\hat{\jmath}$.

(a) Sketch the curve (hint: express x in terms of y).

(b) Find a tangent vector to the curve at the point $(1, 1)$.

(c) Find the curvature κ of the curve at the points $(0, 0)$ and at $(1, 1)$. Which is larger? What does this tell you about the difference in the shape of the curve at these points?

(d) Compute the gradient $\vec{\nabla} f$ of the function $f(x, y) = x - y^2$ and show it is perpendicular to this curve at the point $(1, 1)$. Explain why this will be true at all other points along the curve.

Problem 3 (15 marks). The following questions refer to the function

$$w = f(x, y, z) = \cos(x + y + z).$$

(a) Describe the level surfaces corresponding to $f(x, y, z) = C$ for different values of the constant C $(-1 \le C \le 1)$.

(b) Give the directional derivative of f at the point $(\pi/2, 0, 0)$ in the direction of the vector $3\hat{\imath} + 4\hat{\jmath}$.

(c) If $x = u^2$, $y = 2uv$, $z = v^2$, find $\frac{\partial w}{\partial u}$ when $u = v = \sqrt{\frac{\pi}{2}}$.