CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING

PROBLEM I [10 pts]

A piston-cylinder device contains 0.15 kg of the air initially at 2 MPa and 350°C. The air is first expanded isothermally to 500 kPa, then compressed polytropically with a polytropic exponent of 1.2 to the initial pressure, and the finally compressed at the constant pressure to the initial state.

- Determine the boundary work for each process and the net work of the cycle.

for air
R = 0.287 kJ/kg . K

PROBLEM II [10 pts]

A piston-cylinder device initially contains 0.07 m^3 of nitrogen gas at 130 kPa and 120°C. The nitrogen is now expanded polytropically to a state of 100 kPa and 100°C.

- Determine the boundary work done during this process.

for nitrogen
R = 0.2968 kJ/kg . K

PROBLEM III [10]

1. Under what conditions will none of the heat supplied to a piston-cylinder device containing air be converted into work?

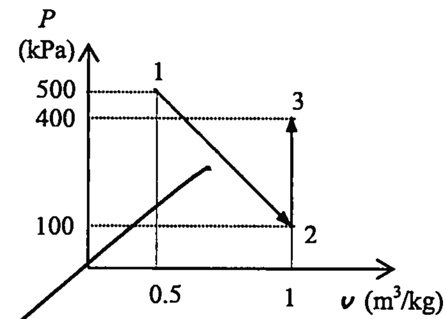
2. What does the area under a Cp *.vs* T graph represents?

3. Show <u>mathematically</u> the variation of pressure with depth for <u>an ideal gas</u>.

4-6 The boundary work done during the process shown in the figure is to be determined.

Assumptions The process is quasi-equilibrium.

Analysis No work is done during the process 2-3 since the area under process line is zero. Then the work done is equal to the area under the process line 1-2:

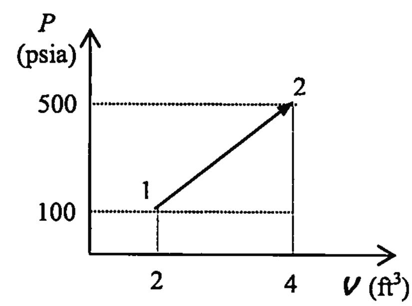


$$W_{b,\text{out}} = \text{Area} = \frac{P_1+P_2}{2}m(v_2-v_1)$$
$$= \frac{(100+500)\text{kPa}}{2}(2\text{ kg})(1.0-0.5)\text{m}^3/\text{kg}\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right)$$
$$= \mathbf{300\ kJ}$$

4-7E The boundary work done during the process shown in the figure is to be determined.

Assumptions The process is quasi-equilibrium.

Analysis The work done is equal to the area under the process line 1-2:



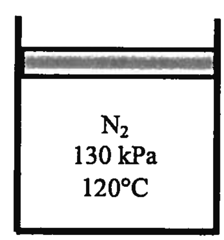
$$W_{b,\text{out}} = \text{Area} = \frac{P_1+P_2}{2}(V_2-V_1)$$
$$= \frac{(100+500)\text{psia}}{2}(4.0-2.0)\text{ft}^3\left(\frac{1\text{ Btu}}{5.404\text{ psia}\cdot\text{ft}^3}\right)$$
$$= \mathbf{111\ Btu}$$

problem II

4-8 A piston-cylinder device contains nitrogen gas at a specified state. The boundary work is to be determined for the polytropic expansion of nitrogen.

Properties The gas constant for nitrogen is 0.2968 kJ/kg.K (Table A-2).

Analysis The mass and volume of nitrogen at the initial state are



$$m = \frac{P_1V_1}{RT_1} = \frac{(130\text{ kPa})(0.07\text{ m}^3)}{(0.2968\text{ kJ/kg.K})(120+273\text{ K})} = 0.07802\text{ kg} \quad 2$$

2

$$V_2 = \frac{mRT_2}{P_2} = \frac{(0.07802\text{ kg})(0.2968\text{ kPa.m}^3/\text{kg.K})(100+273\text{ K})}{100\text{ kPa}} = 0.08637\text{ m}^3$$

The polytropic index is determined from

$$P_1V_1^n = P_2V_2^n \longrightarrow (130\text{ kPa})(0.07\text{ m}^3)^n = (100\text{ kPa})(0.08637\text{ m}^3)^n \longrightarrow n = 1.249 \quad 4$$

The boundary work is determined from

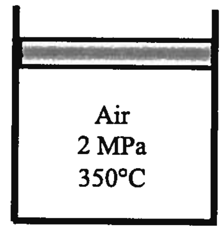
$$W_b = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{(100\text{ kPa})(0.08637\text{ m}^3)-(130\text{ kPa})(0.07\text{ m}^3)}{1-1.249} = \mathbf{1.86\ kJ} \quad 2$$

Problem (I)

4-25 A piston-cylinder device contains air gas at a specified state. The air undergoes a cycle with three processes. The boundary work for each process and the net work of the cycle are to be determined.

Properties The properties of air are $R = 0.287$ kJ/kg.K , $k = 1.4$ (Table A-2a).

Analysis For the isothermal expansion process:



623   0.0894 m³/kg

$$V_1 = \frac{mRT}{P_1} = \frac{(0.15\text{ kg})(0.287\text{ kJ/kg.K})\overbrace{(350+273\text{ K})}^{623}}{(2000\text{ kPa})} = \overbrace{0.01341}^{}\text{ m}^3 \quad (1)$$

0.3576 m³/kg

$$V_2 = \frac{mRT}{P_2} = \frac{(0.15\text{ kg})(0.287\text{ kJ/kg.K})(350+273\text{ K})}{(500\text{ kPa})} = \overbrace{0.05364}^{}\text{ m}^3 \quad (1)$$

$$W_{b,1-2} = \overbrace{P_1V_1}^{mRT_1}\ln\left(\overbrace{\frac{V_2}{V_1}}^{P_1/P_2}\right) = (2000\text{ kPa})(0.01341\text{ m}^3)\ln\left(\frac{0.05364\text{ m}^3}{0.01341\text{ m}^3}\right) = \mathbf{37.18\ kJ} \quad (2)$$

For the polytropic compression process:

$$P_2V_2^n = P_3V_3^n \longrightarrow (500\text{ kPa})(0.05364\text{ m}^3)^{1.2} = (2000\text{ kPa})V_3^{1.2} \longrightarrow V_3 = 0.01690\text{ m}^3 \quad (2)$$

$$W_{b,2-3} = \frac{P_3V_3 - P_2V_2}{1-n} = \frac{(2000\text{ kPa})(0.01690\text{ m}^3) - (500\text{ kPa})(0.05364\text{ m}^3)}{1-1.2} = \mathbf{-34.86\ kJ} \quad (2)$$

For the constant pressure compression process:

$$W_{b,3-1} = P_3(V_1 - V_3) = (2000\text{ kPa})(0.01341 - 0.01690)\text{m}^3 = \mathbf{-6.97\ kJ} \quad (1)$$

The net work for the cycle is the sum of the works for each process

$$W_{\text{net}} = W_{b,1-2} + W_{b,2-3} + W_{b,3-1} = 37.18 + (-34.86) + (-6.97) = \mathbf{-4.65\ kJ} \quad (1)$$

Compr. P=cte   Compr. Polyt

1 ← 3 ← 2

Expansión

1 → 2

isothermal

Problem III

1. if $\vartheta = c^t$ (2)

$\forall = cte$

2. it is h (specific enthalpy) (3)

3. $\frac{dP}{dz} = -\rho g$ for an ideal gas $P/\rho = RT$

$\rho = \frac{P}{RT}$ (5)

$$\frac{dP}{dz} = -\frac{P}{RT} g$$

$$\frac{dP}{P} = \frac{-g}{RT} dz \longrightarrow P_2 = P_1 e^{-g/RT (z_2 - z_1)}$$

ABou ~~[illegible]~~, Carlo Kitra

MIDTERM **ENGR251/2 Fall 12**

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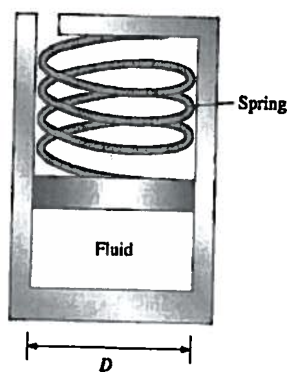
PROBLEM I [20 pts]

A mass of 10 f of nitrogen is contained in the spring -loaded piston-cylinder device shown in the figure below. The spring constant is 1 kN/m and the piston diameter is 10 cm. When the spring exerts no force against the piston, the nitrogen is at 120 kPa and 27°C. The device is now heated until its volume is 10 percent greater than the original volume. Knowing that that there is linear relation between the variation in pressure and the variation in volume during the process,

1. Determine the final pressure.
2. Determine the change in internal energy (in kJ/kg) of the nitrogen.
3. Determine the change in enthalpy (in kJ/kg) of the nitrogen.

Hint: You have to determine the constant for the linear relation between variation in pressure and volume. For this you have to consider that the force applied by the spring at each state is equal to the product of the spring constant and the position of the piston (k*x)



for nitrogen
$R = 0.2968$ kJ/kg . K
$C_v = 0.743$ kJ/kg . K

PROBLEM II [10]

- Explain physically why C_p is higher than C_v for an ideal gas?
- Show mathematically that $C_p - C_v = R$
- Show mathematically the variation of pressure with depth for an ideal gas.

4-57 A spring-loaded piston-cylinder device is filled with nitrogen. Nitrogen is now heated until its volume increases by 10%. The changes in the internal energy and enthalpy of the nitrogen are to be determined.

Properties The gas constant of nitrogen is $R = 0.2968$ kPa·m³/kg·K. The specific heats of nitrogen at room temperature are $c_v = 0.743$ kJ/kg·K and $c_p = 1.039$ kJ/kg·K (Table A-2a).

Analysis The initial volume of nitrogen is

$$V_1 = \frac{mRT_1}{P_1} = \frac{(0.010\text{ kg})(0.2968\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(27+273\text{ K})}{120\text{ kPa}} = 0.00742\text{ m}^3$$ 2

The process experienced by this system is a linear P-v process. The equation for this line is

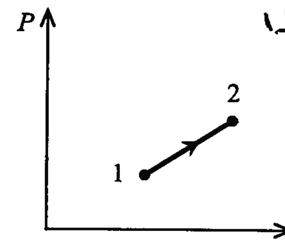
$$P - P_1 = c(V - V_1)$$ 2

where P_1 is the system pressure when its specific volume is v_1. The spring equation may be written as

$$P - P_1 = \frac{F_s - F_{s,1}}{A} = k\frac{x - x_1}{A} = \frac{kA}{A^2}(x - x_1) = \frac{k}{A^2}(V - V_1)$$ 3

Constant c is hence

$$c = \frac{k}{A^2} = \frac{4^2 k}{\pi^2 D^4} = \frac{(16)(1\text{ kN/m})}{\pi^2 (0.1\text{ m})^4} = 16{,}211\text{ kN/m}^5$$ 3



If the student can not get the formulation for C he/she can pick a value and continue the solution. In this case 6 pts have to be removed

The final pressure is then

$$P_2 = P_1 + c(V_2 - V_1) = P_1 + c(1.1V_1 - V_1) = P_1 + 0.1cV_1$$

$$= 120\text{ kPa} + 0.1(16{,}211\text{ kN/m}^5)(0.00742\text{ m}^3)$$

$$= 132.0\text{ kPa}$$ 3

The final temperature is

$$T_2 = \frac{P_2 V_2}{mR} = \frac{(132.0\text{ kPa})(1.1 \times 0.00742\text{ m}^3)}{(0.010\text{ kg})(0.2968\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})} = 363\text{ K}$$ 3

Using the specific heats,

$$\Delta u = c_v \Delta T = (0.743\text{ kJ/kg}\cdot\text{K})(363 - 300)\text{K} = \mathbf{46.8\text{ kJ/kg}}$$ 2

$$\Delta h = c_p \Delta T = (1.039\text{ kJ/kg}\cdot\text{K})(363 - 300)\text{K} = \mathbf{65.5\text{ kJ/kg}}$$ 2

Problem II

1. because for the same amount of heat in, the variation in T° will be higher at constant volume than constant pressure. (2)

$C = \frac{Q}{\Delta T}$ $\Delta T|_{P=ct} < \Delta T|_{v=ct} \rightarrow C_p > C_v$

2.

$h = u + Pv$

$h = u + RT$ we have to assure ideal gas (3)

$dh = du + R\,dT$

$\frac{dh}{dT} = \frac{du}{dT} + R \rightarrow C_p - C_v = R$

3. $\frac{dP}{dz} = -\rho g$ for an ideal gas $P/\rho = RT$

$\rho = \frac{P}{RT}$ (5)

$\frac{dP}{dz} = -\frac{P}{RT} g$

$\frac{dP}{P} = \frac{-g}{RT} dz \rightarrow P_2 = P_1 \, e^{-g/RT(z_2 - z_1)}$