Engr 233, Section QA; Class test, Monday, October 25, 2021

Instructor A. Kokotov

Time: 60 min

Answer all questions. Closed book

1. **(10 points)** Let l be the line of intersection of the planes

$$x + 2y - z - 1 = 0$$

and

$$x - y + 2z + 4 = 0\,.$$

Find the distance $\mathrm{dist}\,(l, m)$ from the line l to the line m passing through the origin and the point $(1, 1, 2)$.

2. **(10 points)** The curve γ is the intersection of the plane $x = -1$ and the cone $z^2 = x^2 + y^2$. Find the curvature of γ at the point $(-1, 2, \sqrt{5})$.

3. **(10 points)** Find the length of the part of the curve

$$\vec{R}(t) = t\vec{i} + \frac{\sqrt{2}}{2}t^2\vec{j} + \frac{1}{3}t^3\vec{k}$$

between the origin and the point $(1, \sqrt{2}/2, 1/3)$.

4. Find the distance from the point $(1, 2, 3)$ to the tangent plane to the surface $x^2 + y^3\sqrt{z} = 2$ at the point $(1, 1, 1)$.

5. **(10 points)** The scalar field f is given via $f(x, y, z) = y(x^2 + z^3)$. The vector field $\vec{A}(x, y, z)$ is given by

$$\vec{A}(x, y, z) = x^2\vec{i} + z\vec{j} + y\vec{k}$$

Find

$$\mathrm{div}(\vec{A} \times \mathrm{grad}\, f)$$

and

$$\mathrm{div}\,\mathrm{curl}(\vec{A} \times \mathrm{grad}\, f)$$