# CONCORDIA UNIVERSITY
## Department of Mathematics and Statistics

| Course | Number | Section(s) |
|---|---|---|
| Mathematics | 204 | All - except EC |

| Examination | Date | Pages |
|---|---|---|
| Final | December 2016 | 3 |

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## Special Instructions:

- Only approved calculators are allowed

- Answer all questions

1. **[10]** Solve the system of equations

$$\begin{cases} 2x_1 + 2x_2 - x_3 + x_4 = 4 \\ 4x_1 + 3x_2 - x_3 + 2x_4 = 6 \\ 8x_1 + 5x_2 - 3x_3 + 4x_4 = 12 \\ 3x_1 + 3x_2 - 2x_3 + 2x_4 = 6 \end{cases}$$

2. **[10]** Solve the following equation for $(2 \times 2)$ matrix X:

$$\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} X + \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}^{-1} X \begin{pmatrix} 1 & 5 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

3. **[10]** For which $a$ the following matrix

$$\begin{pmatrix} a & 2a & 3a & 4a \\ 2 & 2a+2 & 16 & 10 \\ 3 & 6 & 3a^2+6 & 18 \\ 4 & 8 & 12 & 32-4a^2 \end{pmatrix}$$

is invertible?

4. [10]

(A) Find the distance from the point $A = (1, 2, 5)$ to the point of intersection of the plane $\alpha$:

$$x - y + 3z = 1$$

with line $(AB)$ passing through the point $A$ and the point $B = (2, 2, -3)$

(B) Find the distance from the point $B$ to the plane $\alpha$.

5. [10]

(A) Find the distance from the point $A = (1, 3, 4)$ to the line $\mathcal{L}$ of intersection of the planes

$$x - y + 3z = 2$$

and

$$2x - 3y + z = 1.$$

(B) Write down the equation of the plane passing through the point $A$ and the line $\mathcal{L}$.

6. [10] (A) Write down the equation of the plane $\alpha$ passing through the point $A = (1, 5, 7)$ and parallel to the lines

$$\mathcal{L} : x = 3 - 2t, \quad y = 1 - 5t, \quad z = 2 - t$$

and

$$\mathcal{M} : x = 2 - 2t, y = 3 - 2t, z = 1 + 4t$$

(B) Let $B = (1, -1, 3)$. Find the coordinates of the point $C$ of the plane $\alpha$ such that the distance from $B$ to $C$ is minimal.

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7. [10] Do the vectors $(1, 1, 1, 1)$,  $(-1, 2, 3, 4)$.  $(1, -1, 1, 1)$ and $(4, 1, 7, 7)$ form a basis of the space $\mathbf{R}^4$?

8. [10] Find the basis of the solution space of the following system of four equations with six unknowns

$$\begin{cases} x_1 + 2x_2 + x_4 + x_5 = 0 \\ x_1 + x_2 - x_4 - x_5 = 0 \\ 3x_1 + 4x_2 - x_4 - x_5 = 0 \\ x_1 + x_3 + x_6 = 0 \end{cases}$$

9. [10] Find the eigenvalues and eigenvectors of the matrix

$$D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 4 & 3 & -3 \\ 2 & 3 & -2 \\ 4 & 4 & -3 \end{pmatrix} \qquad P = \begin{pmatrix} 3 & 1 & -1 \\ 2 & 0 & 1 \\ 4 & 1 & 0 \end{pmatrix}$$

Is $A$ diagonalizable? If yes, find the matrix $P$ such that $P^{-1}AP$ is diagonal and compute the matrix $A^8$.

10. [10] Let

$$A = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}$$

Find the eigenvectors of $A^9$.