CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section
Mathematics	203	CA
Examination	**Date**	**Pages**
Final	August 2014	3
Instructor:		**Course Examiners**
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Special Instructions:	Only calculators approved by the Department are allowed	

MARKS

[12] **1.** **(a)** Sketch the graph of the function $f(x) = |(x+1)^2 - 3|$ starting from the graph $f(x) = x^2$ and using appropriate transformations.

(b) Given the functions $f(x) = x^2 - 2x$ and $g(x) = \sin\sqrt{\tan(x+1)}$. Find $f \circ g$, $g \circ f$, and $g \circ g$.

(c) Given the function $f = \dfrac{x+3}{2x-5}$ find the inverse function f^{-1} and determine domain and range of f and f^{-1}. $\ln(1+e^{2x}) = y \Rightarrow$

[10] **2.** Evaluate the limits. **Do not use l'Hôpital rule:**

(a) $\displaystyle\lim_{x\to-\infty} \frac{(x^2+x)^3(2x-4)^2}{x^7\sqrt{4x^2+2}}$ **(b)** $\displaystyle\lim_{x\to-2} \frac{x+2}{\sqrt{x+6}-2}$

[12] **3.** **(a)** Consider the function $f(x) = \dfrac{|x+4|}{x^2+3x-4}$. Calculate both one-side limits at the point(s) where the function is undefined.

(b) Find parameters a and b such that the function

$$f(x) = \begin{cases} x+1 & \text{if } x < 2 \\ ax^2 - 1 & \text{if } 2 \le x < 3 \\ 2x - a + bx & \text{if } x \ge 3 \end{cases}$$

$\dfrac{-(x+4)}{(x+4)(x-1)}$ $\dfrac{(x+4)}{(x+4)\cdot(x-1)}$

[13] **4.** Find the derivatives of the following functions (**You do not have to simplify!**):

(a) $f(x) = \dfrac{3\sqrt[9]{x^5} - 7x^3\sqrt{x} + x^{\frac{9}{11}}}{x^{\frac{4}{5}}}$

(b) $f(x) = \dfrac{\cos(3x)}{(1+x^2)^3}$

(c) $f(x) = \tan\left(\sqrt{2x^{-3} + 4e^{2x}}\right)$

✓ (d) $f(x) = (2x^3 - 4)^{\sin x}$ (use logarithmic differentiation)

$e^{\sin x \cdot \ln(2x^3-4)} = \exp[\sin x \cdot \ln(2x^3 - 4)]$ and then ...

[18] **5.** (a) A curve called a "Limacon of Pascal" is defined implicitly by the equation $(x^2 + y^2 - 4x)^2 = x^2 + y^2$. Verify that the point $(0, -1)$ belongs to the curve. Find an equation of the tangent line to the curve at this point.

(b) Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest from the point $(1, 0)$. $M_1(-1/3, \pm \frac{4}{3}\sqrt{2})$

(c) A particle is moving along the curve $\dfrac{xy^3}{1+y^2} = \dfrac{8}{5}$. Assume that the x-coordinate is increasing at the rate of 6 units per second when the particle is at the point $(1, 2)$. At what rate is the y-coordinate of the point changing at that instant?

[7] **6.** Strontium–90 has a half–life of 28 days.

(a) A sample has a mass of 50 mg initially. Find a formula for the mass remaining after t–days.

(b) Find the mass remaining after 40 days.

(c) How long does it take the sample to decay to a mass of 2 mg?

[12] 7. Consider the function $f(x) = \sqrt{x+2}$.

(a) Use the **definition of the derivative** to find the formula for $f'(x)$.

(b) Use appropriate differentiation rules to verify (a).

(c) Write the linearization formula for f at $a = 2$.

(d) Use this linearization to approximate the value of $f(3)$.

[16] 8. Given the function $f(x) = 3x^5 - 20x^3$.

(a) Find the domain of f and check for symmetry. Find asymptotes of f (if any).

(b) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).

(c) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

(d) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question.** Calculate the limit $\lim_{x\to 0^+} (1+ax)^{\frac{b}{x}}$. Hint: use the l'Hôpital rule.

$e^{b/x \cdot \ln(1+ax)}$ $= e^{\lim_{x\to 0^+} \frac{b\ln(1+ax)}{x}}$

$= e^{\lim_{x\to 0} \left(\frac{b \cdot a}{1+ax}\right)}$

$= e^{ba}$