Answers for the final of April 2018

1.

(a) $x = \frac{10}{3}$.

(b) $f^{-1}(x) = \log_3\left(\frac{-6x}{x-4}\right)$ with domain $(0, 4)$ and range $(-\infty, +\infty)$.

2.

(a) Does not exist:

$$\lim_{x \to 3^-} \frac{|x-3|}{x^2 - x - 6} = -\frac{1}{5} \neq \frac{1}{5} = \lim_{x \to 3^+} \frac{|x-3|}{x^2 - x - 6}.$$

(b) -3.

(c) 0.

3. $f''(x) = 2ae^{ax} + (x-1)a^2 e^{ax}$; $f''(0) = 2a - a^2$.

4.

(a) $2^{x+1} \ln(2)\left(x^3 + 3x^{1/3}\right) + 2^{x+1}\left(3x^2 + x^{-2/3}\right)$.

(b)
$$\frac{1}{x^4}4x^3 - \frac{1}{x+3}.$$

(c)
$$\frac{\frac{1}{1+x^2}\left(\tan(x) + x\right) - \arctan(x)\left(\frac{1}{\cos^2(x)} + 1\right)}{\left(\tan(x) + x\right)^2}.$$

(d)
$$\frac{1}{x^2 \sin(x) + x\cos(x^2)}\left[2x\sin(x) + x^2\cos(x) + \cos(x^2) + x\left[-\sin(x^2)\right]2x\right].$$

(e)
$$(1 + 2x)^{x^2}\left[2x\ln(1+2x) + x^2\frac{2}{1+2x}\right].$$

5.

(a) $y = \frac{11}{3}(x-2) + 1$.

(b) $V' = -\frac{81\pi}{20}$.

(c) $\frac{1}{2}$.

6.

  (a) $m = 1$.

  (b) $c = 2$.

7.

  (a)
  $$\lim_{h \to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} = \cdots = \frac{1}{\sqrt{2x+1}}.$$

  (b) $L(x) = \frac{1}{3}(x-4) + 3$.

  (c) $\sqrt{11} \approx L(5) = \frac{10}{3}$.

8.

  (a) Absolute minimum 0 (at $x = 0$); absolute maximum $\frac{2}{3}$ (at $x = 1$).

  (b) Dimensions: 4×8.

9.

  (a) Domain $(-\infty, \infty)$; f is odd; horizontal asymptote $y = 0$ ($\lim_{x \to \pm\infty} f(x) = 0$) and no vertical asymptotes.

  (b) $f'(x) = e^{-2x^2}(1 - 4x^2)$. f is increasing on $\left(-\frac{1}{2}, \frac{1}{2}\right)$ and decreasing on $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$. Local minimum $\left(-\frac{1}{2}, -\frac{1}{2\sqrt{e}}\right)$ and local maximum $\left(\frac{1}{2}, \frac{1}{2\sqrt{e}}\right)$.

  (c) $f''(x) = e^{-2x^2}(-12x + 16x^3)$. f is concave up on $\left(-\frac{\sqrt{3}}{2}, 0\right) \cup \left(\frac{\sqrt{3}}{2}, \infty\right)$ and concave down on $\left(-\infty, -\frac{\sqrt{3}}{2}\right) \cup \left(0, \frac{\sqrt{3}}{2}\right)$. Inflection points $\left(\pm\frac{\sqrt{3}}{2}, \pm\frac{\sqrt{3}}{2}e^{-3/2}\right)$ and $(0, 0)$.

(d) Local extrema, inflection points and asymptotes computed above.