Theoretical Computer Science

Winter 2021

Assignment 2

Show all the steps of your solution for full mark.

Note that in questions below $n_a(w)$ and $n_b(w)$ indicate the number of a's and the number of b's in string w.

- 1. Find a regular expression for the following languages over $\Sigma = \{a, b\}^*$
 - (a) $\{w : w \text{ does not contain substring } bbb\}$
 - (b) $\{w : w \text{ contains exactly three } a \text{'s and it ends with } abb\}$
 - (c) $\{w : n_a(w) \mod 3 = 0 \text{ and } w \text{ begins with } ab\}$
 - (d) $\{w : (n_a(w) + n_b(w)) \mod 3 \ge 2\}$
 - (e) $\{w : |w| \mod 2 = 0\}$
 - (f) $L = \{a^m b^n : mn > 4\}$
- 2. Find an NFA that accepts the language defined by the following regular expression $ab(a+ab)^*(a+aa^*) + (abab)^* + (aaa^*+b)^*$
- 3. Find a regular expression for the following regular languages
 - (a) $\{w : 2n_a(w) + 3n_b(w) \text{ is even } \}$
 - (b) $\{w : (n_a(w) n_b(w)) \mod 3 = 1\}$
- 4. Find a regular grammar for the following languages
 - (a) $L(aa^*(ab+a)^*)$
 - (b) $L((aab^*ab)^*)$
 - (c) $L = \{w : (n_a(w) n_b(w)) \mod 3 = 1\}$
- 5. Prove that the following languages are not regular
 - (a) $L = \{ww : w \in \{a, b\}^*\}$
 - (b) $L = \{a^n b^k : n \ge k\}$
 - (c) $L = \{a^n b^m c^k : n + m > k > 0\}$
 - (d) $L = \{w \in \{a, b\}^* : 2n_a(w) = 3n_b(w)\}$
- 6. Find context free grammars for the following languages
 - (a) $L = \{a^n b^m : n, m \ge 0 \text{ and } n \ne m\}$
 - (b) $L = \{a^n b^m c^k : n \ge m \ge 0 \text{ or } m \ge k \ge 0\}$
 - (c) $L = \{a^n b^m c^k : n + m > k > 0\}$
 - (d) $L = \{w \in \{a, b, c\}^* : n_b(w) = n_a(w) + n_c(w)\}\$