CONCORDIA UNIVERSITY
Dept. of Computer Science and Software Engineering

COMP 335 – Introduction to Theoretical Computer Science
Fall 2018

Assignment 1
Submission deadline: Thursday September 27th at 23:55

Note: This is a theoretical course. That means, while the WHAT is important, the WHY is absolutely essential. Show the steps of your solution for the full mark.

Total mark is 70.

1. [10 Points] Let R and L be any languages over an alphabet Σ. Show the following statements are true.

   (a) $L^2 \subseteq L$ if and only if $L = L^+$.

   (b) If $L \subseteq R$, then $min\{|x| :\ x \in L\} \geq min\{|y| :\ y \in R\}$.

2. [10 Points] Suppose L_1 and L_2 are languages over an alphabet Σ. In each of the following cases, find at least three instances of the languages for which the equality holds.

   (a) $L_1 L_2 = L_2$.

   (b) $L_1 \Sigma^* = L_1$.

3. [20 Points] For each of the following languages over $\Sigma = \{a, b\}$, design a DFA.

   (a) $L_1 = \{w :\ w \in \Sigma^*$, where w does not have bbb as a substring$\}$.

   (b) $L_2 = \{w :\ w \in \Sigma^*$, where $n_a(w)$ mod $3 = 0$ and w begins with $ab\}$.

   (c) $L_3 = \{vwv :\ v, w \in \Sigma^*$, and $|v| = 2\}$.

   (d) $L_4 = \{a^m b^n:\ \mathrm{mn} > 4\}$.

4. [5 Points] Give a regular expression for $(L_1 \cup L_2)L_3$, where L_1, L_2, and L_3 are the languages defined in question 3 above.

5. [5 Points] Give a simple, verbal description of the complement of $L = \{a^n b^{n+1} : n \geq 0\}$.

6. [10 Points] Let L be any regular language over an alpgabet Σ. Using L, we define $chop(L) = \{w :\ \exists\ x, y, z \in \Sigma^*,\ xyz \in L,\ w = xz\}$.

   Show that $chop(L)$ is regular or give a counter-example.

7. [10 Points] Give a FA for $L = L_1 \cup \bar{L_3}$, where L_1 and L_3 are languages defined in question 3 above.

   Extra homework, NOT marked: Give a DFA for L.