Concordia University

PHYS 204 – Mechanics
Practice Midterm – Solutions
75 minutes

Multiple Choice

1. **(5 marks)** Alice, who weighs 500 N is standing on a scale inside an elevator that is going up with constant speed of 5 m/s. The reading of the scale in the elevator is (take $g = 10\frac{m}{s^2}$):
   a) 750 N
   b) 500 N
   c) 250 N
   d) 1000 N

Elevator moves with constant speed → $a = 0$

$\Sigma F = ma = 0$

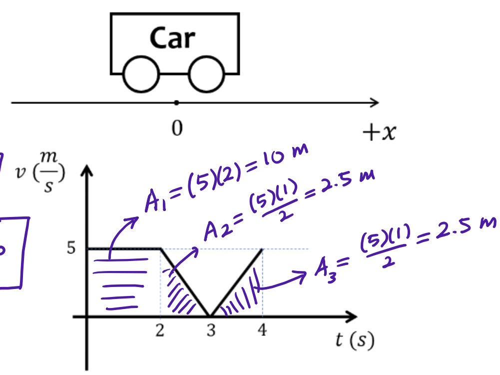
$\rightarrow N = \boxed{W = 500\text{ N}}$

2. **(5 marks)** The figure describes the velocity of a car moving in one dimension as a function of time. What is the displacement and average acceleration of the car during 4 seconds? The dotted lines are to guide the eye.
   a) $\Delta\vec{x} = 10\ m$ and $\vec{a} = 0$
   b) $\Delta\vec{x} = 0$ and $\vec{a} = 1.25\frac{m}{s^2}$
   c) $\Delta\vec{x} = -15\ m$ and $\vec{a} = 1.25\frac{m}{s^2}$
   d) $\Delta\vec{x} = 15\ m$ and $\vec{a} = 0$

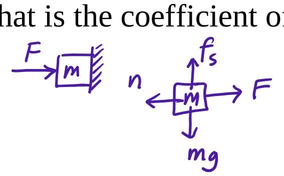
$\Delta\vec{x}$ = Area under graph

$= 10 + 2.5 + 2.5 = \boxed{15\text{ m}}$

$\vec{a} = \frac{\vec{V}_f - \vec{V}_i}{\Delta t} = \frac{5-5}{4} = \boxed{0}$



3. **(5 marks)** To hold a 1-kg block against a vertical wall, a minimum horizontal force of 20 N is required. What is the coefficient of static friction? (take $g = 10\frac{m}{s^2}$)
   a) $\mu_s = 0.1$
   b) $\mu_s = 0.5$
   c) $\mu_s = 0.3$
   d) The block will slide down due to the gravitational force



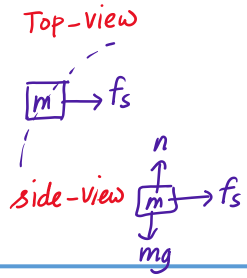
$\Sigma F_x = 0$
$F - n = 0$
$F = n$

$\Sigma F_y = 0$
$f_s - mg = 0$
$f_s = mg$

$\mu_s n = mg$ ($n = F$)
$\mu_s F = mg$
$\rightarrow \mu_s = \frac{mg}{F} = \frac{(1)(10)}{20} = \boxed{0.5}$

4. **(5 marks)** Bob is driving with constant speed in a curved path with radius $r = 50\ m$. If the coefficient of static friction between his tires and the road is 0.5, with what approximate maximum speed can he drive without skidding? Round your answer. Take $g = 10\frac{m}{s^2}$.
   a) 12 m/s

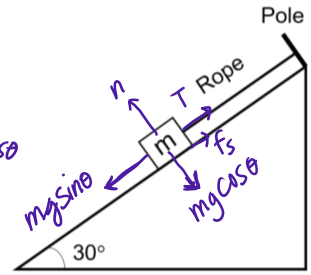
b) 16 m/s
c) 14 m/s
d) 20 m/s



$\sum F_y = 0$
$n - mg = 0$
$n = mg$

$\sum F_r = ma_r$
$f_s = m\frac{v^2}{r}$
$\mu_s n = m\frac{v^2}{r}$
$\mu_s \cancel{m} g = \cancel{m}\frac{v^2}{r} \rightarrow V = \sqrt{\mu_s r g} \approx \boxed{16\ m/s}$

5. **(5 marks)** A 2-kg block is connected to a rope that is tied to a pole on an inclined surface, as shown below. If the coefficient of static friction is 0.3, find the tension in the rope. Take $g = 10\frac{m}{s^2}$.



a) 6.4 N
b) 10.2 N
c) 0
d) 4.8 N

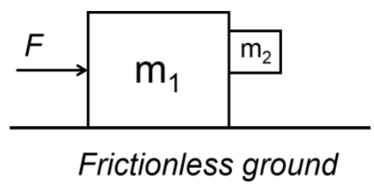
$\sum F_y = 0$
$\rightarrow n = mg\cos\theta$
$\sum F_x = 0$
$mg\sin\theta - T - f_s = 0$

$T = mg\sin\theta - f_s = mg\sin\theta - \mu_s mg\cos\theta$
$T = mg(\sin\theta - \mu_s\cos\theta) = \boxed{4.8\ N}$

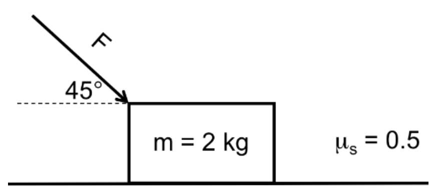
Long Answer

Instruction:
You MUST submit your detailed solution written on a white sheet of paper, through COLE.

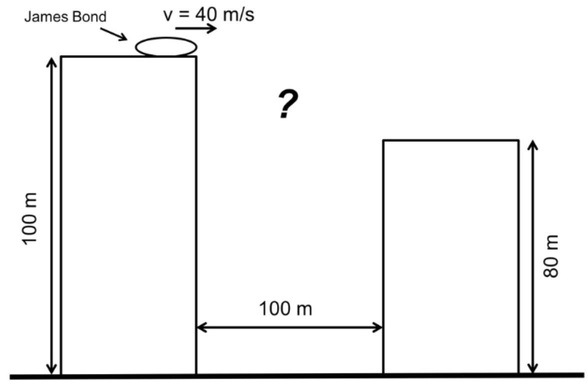
6. **(15 marks)** In the system shown in the figure, $m_1 = 1\,kg$ and $m_2 = 3\,kg$ (regardless of the dimension of the boxes in the figure). What should be the minimum force F, such that mass m_2 does not slide down (remains stationary with respect to mass m_1)? The coefficient of static friction between the boxes is $\mu_s = 0.5$ and there is no friction between m_1 and the ground. Draw the free body diagram. Show your detailed work.



7. **(15 marks)** As shown in the figure, a force F is applied to a box with mass m = 2kg at angle of 45° with the horizon. If the coefficient of static friction is $\mu_s = 0.5$, find the range of force F (F_{min}, F_{max}), for which the object remains stationary. Draw the free body diagram. Show your detailed work.



8. James Bond is driving his Aston Martin with speed of 40 m/s as he flies off a 100 m tall building, hoping to land on an 80 m tall building that is 100 m away (see the figure below). <u>Show your detailed work</u>. Take $g = 10\,\frac{m}{s^2}$.
    a) **(7 marks)** Determine the coordinates (x and y) of his landing point (or crashing point on the building!).
    b) **(8 marks)** What is his velocity when he lands (or crashes)?



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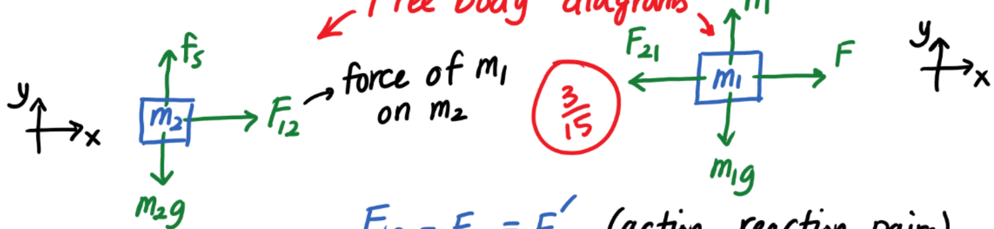
Frictionless ground

* Find min F such that m_2 does not slide down.

Free body diagrams



force of m_1 on m_2

$\frac{3}{15}$

$$F_{12} = F_{21} = F' \quad \text{(action – reaction pairs)}$$

Equations of motion

m_2

$$\sum F_x = m_2 a_x$$

$$F_{12} = m_2 a \rightarrow \boxed{F' = m_2 a} \; (1)$$

$\frac{7}{15}$

$$\sum F_y = 0$$

$$f_s = m_2 g \rightarrow \mu_s F_{12} = m_2 g$$

$$\mu_s F' = m_2 g$$

$$\rightarrow F' = \frac{m_2 g}{\mu_s} = \frac{(3)(10)}{0.5}$$

$$\rightarrow \boxed{F' = 60\text{ N}} \rightarrow \text{plug into (1)}:$$

$$F' = m_2 a \rightarrow a = \frac{F'}{m_2} = \frac{60}{3}$$

$$\boxed{a = 20 \ \frac{m}{s}} \rightarrow \text{same for both } m_1, m_2$$

m_1

$$\sum F_x = m_1 a_x$$

$$F - F_{21} = m_1 a$$

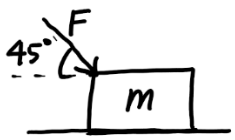
$$\boxed{F - F' = m_1 a} \; (2)$$

$$(1), (2) \rightarrow F = (m_1 + m_2) a$$

$$\boxed{F = (1+3)(20) = 80\text{ N}}$$

Correlating equations to find F

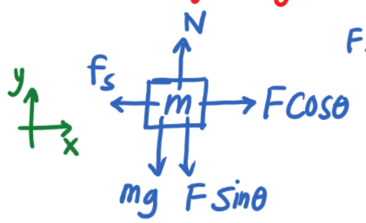
$\frac{5}{15}$

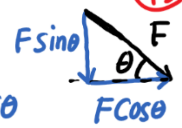
⑦ 

$\mu_s = 0.5$

* Find range of F for which $\underline{m}$ remains stationary

Free body diagram $\frac{3}{15}$





* F_{min} for which $\underline{m}$ remains stationary is $\underline{0}$: $F_{min} = 0$

In this condition, $f_s = 0$ (since $F_x = 0$) and $N = mg$ (since $F_y = 0$). $\frac{2}{15}$

equations of motion to find F_{max} $\frac{5}{15}$

stationary $\begin{cases} a_x = 0 \\ a_y = 0 \end{cases}$

$$\Sigma F_x = 0 \qquad\qquad \Sigma F_y = 0$$

$$F\cos\theta - f_s = 0 \qquad\qquad N - mg - F\sin\theta = 0$$

$$① \left(F\cos\theta = \underbrace{f_s = \mu_s N}_{\text{When F is max}}\right) \qquad\qquad \left(N = mg + F\sin\theta\right) ②$$

solving Eqs. ①&② to find F_{max} $\frac{5}{15}$

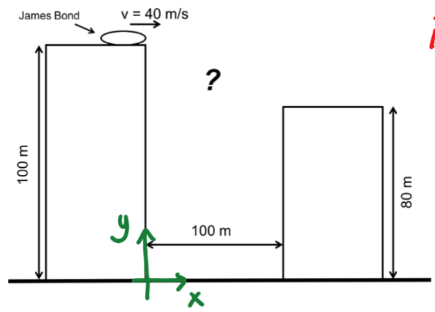
$$F\cos\theta = \mu_s N = \mu_s mg + \mu_s F\sin\theta$$

$$\rightarrow F\cos\theta - \mu_s F\sin\theta = \mu_s mg \qquad \boxed{0 \leq F \leq 28.28}$$

$$\rightarrow F(\cos\theta - \mu_s \sin\theta) = \mu_s mg$$

$$\rightarrow F = \frac{\mu_s mg}{\cos\theta - \mu_s \sin\theta} = \frac{(0.5)(2)(10)}{\sqrt{2}/2 - (0.5)\sqrt{2}/2} = \boxed{28.28 \text{ N}}$$

(8)



initial conditions

$$\begin{cases} x_i = 0 \\ y_i = 100\ m \\ V_{ix} = 40\ m/s \\ V_{iy} = 0 \\ a_x = 0\ ,\ a_y = -g \end{cases}$$

Equations of motion 3/7

$$\Delta x = V_{ix}\,\Delta t \qquad \Delta y = \cancel{V_{iy}}\,\Delta t - \tfrac{1}{2} g\,\Delta t^2$$

0

$$\Delta x = 40\,\Delta t \qquad \Delta y = -5\,\Delta t^2$$

The time it takes to travel $\Delta x = 100\,m$ Horizontally:

$$\Delta t = \frac{\Delta x}{40} = \frac{100}{40} = \boxed{2.5\ s}$$

Finding Δt to solve for landing coordinate

Within this $\Delta t = 2.5\,s$ he goes down: 4/7

$$\Delta y = -5\,\Delta t^2 = -5(2.5)^2 = \boxed{-31.25\ m}$$

To land on the 80m tall building safely, he could only go down 20m, so he crashes into the builing at:

$$\begin{cases} x = 100\ m \\ y = 100 - 31.25 = 68.75\,m \end{cases}$$ From ground

$$\begin{cases} x = 100\ m \\ y = -31.25\ m \end{cases}$$ → Also acceptable

b) $\vec{V}_f = \vec{V}_{fx}\,\hat{i} + V_{fy}\,\hat{j}$ Finding components of $\vec{V}$

$V_{fx} = V_{ix} = 40\ m/s \quad (a_x = 0)$ 3/8

$V_{fy} = \cancel{V_{iy}}^{\,0} - g\Delta t = -10(2.5) = -25\ m/s$ 3/8

$\rightarrow V_f = \boxed{(40\,\hat{i} - 25\,\hat{j})\ \frac{m}{s}}$ vector presentation 2/8

If presented with magnitude and angle, also accepted.

$\vec{V}_f = (47.17\ \frac{m}{s}, -32^\circ)$ OR

$\vec{V}_f = (47.17\ \frac{m}{s}, 328^\circ)$