CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number		Section(s)
Mathematics	208/4		All
Examination	**Date**	**Time**	**Pages**
Midterm	March 2019	1 Hour 30 minutes	2
Instructors			**Course Examiner**
F. Romanelli, M. Ahed Hindawi, T. Hughes, U. Tiwari			D. Sen

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\frac{(1+i)^n - 1}{i}, \quad PV = PMT\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- ▷ Answer all questions.
- ▷ **Only approved calculators are allowed.**

MARKS

[4+3+3] **1.** A firm producing poultry feed finds that the total cost $C(x)$ in dollars of producing and selling x units is given by

$$C(x) = 20x + 100.$$

Management plans to charge \$24 per unit for the feed.

(A) How many units must be sold for the firm to break even?

(B) What is the profit if 100 units of feed are sold?

(C) How many units must be sold to produce a profit of \$900?

[$2\frac{1}{2} \times 4$] **2.** Solve for x in the following equations:

(A) $\left(\frac{5}{3}\right)^{x^2-13} = \left(\frac{81}{625}\right)^{x-2}$

(B) $3^{\log_2 x} = 3^5$

(C) $(\sqrt{e^3})^{8x^2-6} = (e)^{3x-4}(e)^{5x+10}$

(D) $\log_3\left(\frac{x}{5}\right) + \log_3 725 + 2\log_3\sqrt{5} = \frac{1}{3}\log_3\sqrt[3]{7} + 5\log_3 1$

PLEASE TURN OVER

[6+4] **3.** (A) Given sequence

$$-60, -56, -52, -48, -44, \ldots\ldots\ldots$$

Find the 200th term and the sum of the first 200th terms of the sequence by only using a proper formula.

(B) Given sequence

$$100, 100(1.08), 100(1.08)^2, \ldots\ldots\ldots$$

Find the sum of the first twelve terms of the sequence by only using a proper formula.

[10] **4.** A person borrows \$6,000 and agrees to repay the loan in monthly installments over a period of 5 years. The agreement is to pay 1% of the unpaid balance each month for using the money and \$100 each month to reduce the loan. What is the total cost of the loan over the 5 years?

[4+6] **5.** At the end of each quarter, a 50–year old woman puts \$1200 in a retirement account that pays 7% interest compounded quarterly.

(A) How much will be in the account when she is at age 60?

(B) When she reaches 60, she withdraws the entire amount and places it in a mutual fund that pays 9% interest compounded monthly. From then on, she deposits \$300 in the mutual fund at the end of each month. How much is in the account when she reaches age 65?

[5+5] **6.** Lincoln Benefit Life offered an ordinary annuity that earned 6.5% compounded annually. A person plans to make equal annual deposits into this account for 25 years in order to then make 20 equal annual withdrawals of \$25,000, reducing the balance in the account to zero.

(A) How much must be deposited annually to accumulate sufficient funds to provide for these payments?

(B) How much total interest is earned during this entire 45 year process?

MATH 208 : Midterm Test : March 10, 2019
Solution

#1 $C(x) = 20x + 100$, Revenue: $R(x) = 24x$.
Profit: $P(x) = R(x) - C(x) = 4x - 100$

(A) Break even: $P(x) = 0 \Rightarrow 4x - 100 = 0 \Rightarrow x = 25$.

(B) For $x = 100$, $P(100) = 400 - 100 = \$300$.

(C) For $P(x) = \$900$, $900 = 4x - 100 \Rightarrow x = 250$ units.

#2 (A) $\left(\frac{5}{3}\right)^{x^2-13} = \left[\left(\frac{5}{3}\right)^{-4}\right]^{x-2} \Rightarrow x^2 - 13 = -4x + 8 \Rightarrow$

$x^2 + 4x - 21 = 0 \Longrightarrow (x+7)(x-3) = 0 \Rightarrow x = 3, -7$.

(B) $\log_3 x = 5 \Longrightarrow x = 3^5 \Rightarrow x = 243$

(C) $\left[e^{\frac{3}{2}}\right]^{8x^2-6} = e^{8x+6} \Rightarrow 12x^2 - 9 = 8x + 6 \Longrightarrow$

$12x^2 - 8x - 15 = 0 \Longrightarrow (2x-3)(6x+5) = 0 \Rightarrow$

$x = \frac{3}{2}, -\frac{5}{6}$.

(D): $\log_3\left(\frac{x}{5}\right) = \frac{1}{3}\log_3(7)^{\frac{1}{3}} + 0 - \log_3 725 - \log_3 5$

$\log_3\left(\frac{x}{5}\right) = \log\left[\frac{7^{1/9}}{725 \times 5}\right] \Rightarrow \frac{x}{5} = \frac{\sqrt[9]{7}}{725 \times 5}$

$x = \frac{\sqrt[9]{7}}{725}$ Ans.

#3. (A) Arithmetic sequence $a = -60$, $d = 4$, $n = 200$

n^{th} term: $T_n = a + (n-1)d \Rightarrow T_{200} = -60 + 796 = 736$

(B) G. Sequence: $a = 100$, $r = 1.08$

Formula: $S_n = \frac{a(r^n - 1)}{r - 1} \Rightarrow S_{12} = \frac{100[(1.08)^{12} - 1]}{0.08}$

$S_{12} = 1897.7126$

#4: Paid \$100 Loan/month. Total loan \$6000.
No of payments 60, \$100 each payment
Interest after 1st payment = 6000×0.01
" " 2nd " $= 5900 \times 0.01 = 59\$$
" " 60th " $= 100 \times 0.01 = 1\$$
Total Interest paid $= (60 + 59 + \cdots + 1)\$ \; (n = 60)$
$S_n = \frac{n}{2}[a_1 + a_n] \Rightarrow \text{Sum} = \frac{60}{2}[60+1] = 30 \times 61 = \$1830$

#5 Time $t = 10$ yrs, $m = 4$, $n = 10 \times 4 = 40$, PMT $= \$1200$
$i = \frac{r}{m} = 0.0175$

(A) $FV = \frac{PMT[(1+i)^n - 1]}{i} = \frac{1200[(1.0175)^{40} - 1]}{0.0175} = \$68680.97$

(B) Invest at 9% $= r$, $m = 12$, $i = \frac{r}{m} = 0.0075$ for $t = 5$ years
$P = \$68680.97$, $n = 12 \times 5 = 60$, P grows at comp rate
$\therefore A = P(1+i)^n = 58680.97(1.0075)^{60} = \$107,532.49$
Also \$300/month deposited at 9% comp $m = 12$ for $t = 5$ yrs
$FV = \frac{PMT[(1+i)^n - 1]}{i} = \frac{300[(1.0075)^{60} - 1]}{0.0075} = \$22,627.24$
Total amount $= \$107,532.49 + 22,627.24 = \$130,159.73$.

#6 (A) PV of the amount needed for \$25000 yearly withdrawals for 20 yrs: $r = 0.065$, $m = 1$, $n = 20$, $i = 0.065$
$PV = \frac{25000[1 - (1+i)^{-n}]}{i} = \frac{25000[1 - (1.065)^{-20}]}{0.065} = \$275,462.68$
is needed as F.V of all yearly deposits for 25 yrs
$FV = \frac{PMT[(1+i)^n - 1]}{i} \Rightarrow PMT = \frac{275,462.68 \times 0.065}{(1.065)^{25} - 1} = \$4677.76$/yr

(B) Total Interest = Total withdrawals − Total deposits
$= 25000 \times 20 - 4677.76 \times 25 = \$383,056.$