PROBLEMS FOR CHAPTER 5

1. Compute the value of definite integrals using Trapezoidal rule.

(a) $\int_{3}^{6} \frac{x\,dx}{4+x^2}$ using 6 sub intervals

(b) $\int_{1}^{9} \frac{dx}{x}$ using 8 sub intervals

(c) $\int_{0}^{\pi}$ sinx dx using 6 sub intervals

(d) $\int_{0}^{1} e^x dx$ in steps of 0.5

Solutions:

(a) $\int_{3}^{6} \frac{x\,dx}{4+x^2}$ using 6 sub intervals

Therefore f(x) = $\frac{x\,dx}{4+x^2}$, n= 6 and h=0.5

n	0	1	2	3	4	5	6
x	3	3,5	4	4,5	5	5,5	6
F(x)	0,230769	0,215385	0,2	0,185567	0,172414	0,160584	0,15

$$I = \int_a^b f(x)dx \approx \frac{h}{2}\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)\right]$$

$$I \approx \frac{0.25}{2}\left[0.23077 + 2(0.2154 + 0.2 + 0.1856 + 0.1724 + 0.1606) + 0.15\right] = 0.562167$$

(b) $\int_{1}^{9} \frac{dx}{x}$ using 8 sub intervals

Therefore f(x) =, $\frac{1}{x}$ n= 8 and h= 1

n	0	1	2	3	4	5	6	7	8
x	1	2	3	4	5	6	7	8	9
F(x)	1	0,5	0,333333	0,25	0,2	0,166667	0,142857	0,125	0,111111

$$I = \int_a^b f(x)dx \approx \frac{h}{2}\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b) \right]$$

$$I \approx \frac{1}{2}\left[1 + 2(0.5 + 0.3333 + 0.25 + 0.2 + 0.16667 + 0.14286 + 0.125) + 0.1111\right] = 2.2734$$

(c) $\int_0^\pi$ sinx dx using 6 sub intervals

Therefore f(x) =sinx n= 6 and h= π/6 or 0.5236

n	0	1	2	3	4	5	6
x	0	0,523599	1,047198	1,570796	2,094395	2,617994	3,141593
F(x)	0	0,5	0,866025	1	0,866025	0,5	0

$$I = \int_a^b f(x)dx \approx \frac{h}{2}\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b) \right]$$

$$I \approx \frac{\pi}{2 \times 6}\left[0 + 2(0.5 + 0.866025 + 1 + 0.866025 + 0.5) + 0\right] = 1.19541$$

(d) $\int_0^1 e^x dx$ in steps of 0.5

Therefore f(x) =e^x n= 2 and h= 0.5

n	0	1	2
x	0	0,5	1
F(x)	1	1,648721	2,718282

$$I = \int_a^b f(x)dx \approx \frac{h}{2}\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b) \right]$$

$$I \approx \frac{0.5}{2}\left[1 + 2(1.6487) + 2.71823\right] = 1.753931$$

6. Apply Trapezoidal rule to integrate with 4 intervals.

$$I = \frac{1}{2}\int_0^4 \sqrt{x\sqrt{x}}\,dx$$

Solution:

$I = \dfrac{1}{2}\int_0^4 \sqrt{x\sqrt{x}}\,dx$

$f(x) = \frac{1}{2}\sqrt{x\sqrt{x}}$

n	0	1	2	3	4
x	0	1	2	3	4
$f(x)$	0	0.5	0.840896	1.13975	1.414214

I=$\frac{1}{2}[0 + 2(0.5 + 0.840896 + 1.1397) + 1.414214$

=3.187757

11. Compute $\int_1^4 (2x^3 - 11x^2 + 24x)dx$

using Simpson's rule with two intervals.

Solution:

$\int_1^4 (2x^3 - 11x^2 + 24x)\,dx$ with 2 intervals

$f(x) = (2x^3 - 11x^2 + 24x)$

n	0	1	2
x	1	2.5	4
fx	15	22.5	48

I = $\frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)]$

= $\frac{1.5}{3}[15 + 4(22.5) + 48] = 76.5$

21. Integrate I = $\int_0^1 \exp(x^2)dx$ using Gauss-Legendre quadrature. Use n =3 points.

Note: $\int_a^b f(x)dx = \int_{-1}^1 f\left[\frac{(b-a)t+b+a}{2}\right]\left(\frac{b-a}{2}\right)dt$

Roots	Coefficients
0.7746	0.5556
0.0000	0.8889
-0.7746	0.5556

Solution:

I = $\int_0^1 e^{x^2} dx$

The first step is to transform the function so that it will yield the same result in the interval [-1, 1].

Therefore t = $\frac{1}{(b-a)}(2x-a-b)$

In this case substitute a = 0 and b = 1

$$t=2x-1 \text{ or } x = \frac{t+1}{2} \text{ and } dx = \frac{1}{2}dt$$

Substituting these parameters for x and dx and with n = 3, we get

$$\int_0^1 e^{x^2} dx = \int_{-1}^1 \frac{1}{2} e^{(\frac{t+1}{2})^2} dt \approx \sum_{i=1}^3 C_i f(t_i)$$

Corresponding t_i and Ci from the table are:

t_i	C_i
0.7746	0.5556
0.0000	0.8889
-0.7746	0.5556

$$I \approx 0.5556*\left(\frac{1}{2}e^{(\frac{0.7746+1}{2})^2}\right) + 0.8889*\left(\frac{1}{2}e^{(\frac{0+1}{2})^2}\right) + 0.5556*\left(\frac{1}{2}e^{(\frac{-0.7746+1}{2})^2}\right)$$

$$I \approx 1.46241$$

23. Evaluate y = $\frac{1}{\pi}\int_0^{\pi}\cos(x\sin\theta)d\theta$ by Gauss quadrature with n=3 for x = 0 to 5π in steps of 1/4π. From your estimate find the smallest positive value of x for which y = 0.

Solution:

I = $\frac{1}{\pi}\int_0^{\pi}\cos(x\sin\theta)d\theta$

The first step is to transform the function so that it will yield the same result in the interval [-1, 1].

Therefore

$$t = \frac{1}{(b-a)}(2\theta - a - b)$$

In this case substitute a = 0 and b = π

$$t = \frac{1}{\pi}(2\theta-\pi) \text{ or } \theta = \frac{\pi(t+1)}{2} \text{ and } d\theta = \frac{\pi}{2}dt$$

Substituting these parameters for θ and dθ and with n = 3, we get

$$I = \frac{1}{\pi}\int_0^{\pi}\cos(x\sin\theta)d\theta = \frac{1}{\pi}\int_{-1}^{1}\cos\left(x\sin\left(\frac{\pi(t+1)}{2}\right)\right)\frac{\pi}{2}dt \approx \sum_{i=1}^{3}C_i f(t_i)$$

For x=0

Corresponding t_i and Ci from the table are:

t_i	C_i
0.7746	0.5556
0.0000	0.8889
-0.7746	0.5556

$$I \approx 0.5556^*\cos\left(0\times\sin\left(\frac{\pi(0.7746+1)}{2}\right)\right)\frac{1}{2} + 0.8889^*\cos\left(0\times\sin\left(\frac{\pi(0+1)}{2}\right)\right)\frac{1}{2} + 0.5556^*$$

$$\cos\left(0\times\sin\left(\frac{\pi(-0.7746+1)}{2}\right)\right)\frac{1}{2}$$

$$I \approx 1.00005$$

i	x	I(x)
0	0	1,00005
1	π/4	0,849402
2	π/2	0,475222
3	3π/4	0,066023
4	π	-0,1871
5	5π/4	-0,19884
6	3π/2	-0,03499
7	7π/4	0,13144
8	2π	0,127247
9	9π/4	-0,11392
10	5π/2	-0,50764
11	11π/4	-0,86394
12	3π	-0,99564
13	13π/4	-0,82637
14	7π/2	-0,43527
15	15π/4	-0,01209
16	4π	0,251044
17	17π/4	0,268076
18	9π/2	0,104416
19	19π/4	-0,06694
20	5π	-0,07242

The lowest value of x for y=0 is between $3\pi/4$ (2.3562) and π (3.14), therefore we use secant method discussed previously.

i	xi	fxi	xi-1	fx-1	f'xi	xi+1
0	2,356194	0,066023	1,570796	0,475222	-0,52101	2,482915
1	2,482915	0,010667	2,356194	0,066023	-0,43683	2,507335
2	2,507335	0,000549	2,482915	0,010667	-0,41434	2,50866
3	2,50866	5,34E-06	2,507335	0,000549	-0,41031	2,508673

Therefore for x=2.508673, y=0