**PROBLEMS FOR CHAPTER 9**

2. Given the following differential equation

$$\frac{dy}{dx} = 1 + x^2 y^2, \quad y(0) = 0$$

Solve by Euler's method in the range of 0 to 0.5 and h = 0.1

**Solution:**

$g(x, y) = \frac{dy}{dx} = 1 + x^2y^2$ and $y(0) = 0$; where $h = 0.1$

Euler Eq$^n$ => $y(x_1) = y(x_0) + hg(x_0, y(x_0))$

$y(0.1) = y(0) + 0.1\ [1 + x_0^2 * y(0)^2]$
$y(0.1) = 0 + 0.1\ (1 + 0^2 * 0^2) = 0.1$
$y(0.2) = 0.1 + 0.1\ (1 + 0.1^2 * 0.1^2) = 0.2$
$y(0.3) = 0.2 + 0.1\ (1 + 0.2^2 * 0.2^2) = 0.3002$
$y(0.4) = 0.3002 + 0.1\ (1 + 0.3^2 * 0.3002^2) = 0.4010$
$y(0.5) = 0.4010 + 0.1\ (1 + 0.4^2 * 0.4010^2) = 0.50355$

6. Use Euler's method to solve the following initial value problem,

$$\frac{dy}{dx} = y^2, y(0) = 0.2 \quad h = 0.5 \quad 0 \le x \le I$$

**Solution:**

$g(x, y) = \frac{dy}{dx} = y^2$ ; $h = 0.5$, $y(0) = 0.2$

$y(x_1) = y(x_0) + hg(x_0, y(x_0))$
$y(0.5) = y(0) + 0.5\ (y(0)^2)$
$= 0.2 + 0.5\ (0.2^2)$
$= 0.22$
$y(1) = 0.22 + 0.5\ (0.22^2)$
$= 0.2442$

10. Solve the differential equation

$$\ddot{y} + \dot{y} + y = \sin t$$

with $y(0) = 1$ and $\dot{y}(0) = 0$, and obtain $y(0.3)$ using Euler's method.

**Solution:**

$y'' + y' + y = \sin t$ ;  $y'(0) = 1$, $y(0) = 0$

we convert this second order ODE in 2 first order ODEs

$g(v, y)$ ;  $y' = v$    $v(0) = y'(0) = 1$

$v' = \sin t - y - v$    $y(0) = 0$

we take $h = 0.1$

$y(0.1) = y(0) + h(v(0))$

$v(0.1) = v(0) + h(\sin t_0 - y(0) - v(0))$

$y(0.1) = 0 + 0.1(1) = 0.1$

$v(0.1) = 1 + 0.1(\sin 0 - 0 - 1) = 0.9$

$y(0.2) = 0.1 + 0.1(0.9) = 0.19$

$v(0.2) = 0.9 + 0.1(\sin 0.1 - 0.1 - 0.9) = 0.81$

$y(0.3) = 0.19 + 0.1(0.81) = 0.271$

$v(0.3) = 0.81 + 0.1(\sin 0.2 - 0.2 - 0.81) = 0.7289$

14. An R-L circuit is described by the following second order differential equation

$$\frac{d^2i}{dt^2} + 8\frac{di}{dt} + 10\,i = 10, \text{ where } i(0) = 1,\ i'(0) = 1$$

Solve at t = 0.1 using

(a) Midpoint method

(b) Runge –Kutta method of order 4.

**Solution:**

$$\frac{d^2i}{dt} + 8\,\frac{di}{dt} + 10i = 10$$

We convert it into 2 first order ODEs

$\frac{di}{dt} = i' = v$

$v' = 10 – 8v – 10\,i$

a) Using Range-Kutta order 2 or mid-point

$i(t_1) = i(t_0) + hk_2$

$v(t_1) = v(t_0) + hl_2$

$k_2 = g\,[(t_0 + \frac{h}{2}), (i_0 + k_1\frac{h}{2}), (v_0 + l_1\frac{h}{2})]$

$l_2 = f\,[(t_0 + \frac{h}{2}), (i_0 + k_1\frac{h}{2}), (v_0 + l_1\frac{h}{2})]$

$k_1 = g(t_0, i_0, v_0)$

$l_1 = f(t_0, i_0, v_0)$

$k_1 = v_o = i'(0) = 1$

$l_1 = 10 – 8v_0 – 10\,i_0 = 10 - 8\,i'(0) – 10\,i(0)$

$= 10 – 8 * 1 – 10 * 1 = -8$

$k_2 = (v_0 + l_1\frac{h}{2}) = 1 + \frac{0.1*(-8)}{2} = 0.6$

$l_2 = 10 – 8\,(v_0 + l_1\frac{h}{2}) – 10(i_0 + k_1\frac{h}{2})$

$= 10 – 8\,(1 + \frac{0.1(-8)}{2}) – 10(1 + \frac{0.1*1}{2}) = -5.3$

$i(0.1) = i(0) + hk_2 = 1 + 0.1 * 0.6 = 1.06$

$v(0.1) = v(0) + hl_2 = 1 + 0.1 * 5.3 = 0.47$

therefore,

$i'(0.1) = 0.47$

b) Range-Kutta order 4

$k_1 = g(t_0, i_0, v_0)$

$l_1 = f(t_0, i_0, v_0)$

$k_2 = g\,[(t_0 + \frac{h}{2}), (i_0 + k_1 \frac{h}{2}), (v_0 + l_1 \frac{h}{2})]$

$l_2 = f\,[(t_0 + \frac{h}{2}), (i_0 + k_1 \frac{h}{2}), (v_0 + l_1 \frac{h}{2})]$

$k_3 = g\,[(t_0 + \frac{h}{2}), (i_0 + k_2 \frac{h}{2}), (v_0 + l_2 \frac{h}{2})]$

$l_3 = f\,[(t_0 + \frac{h}{2}), (i_0 + k_2 \frac{h}{2}), (v_0 + l_2 \frac{h}{2})]$

$k_4 = g\,[(t_0 + h), (i_0 + hk_3), (v_0 + h\,l_3)]$

$l_4 = f\,[(t_0 + h), (i_0 + hk_3), (v_0 + h\,l_3)]$

$k_1$, $k_2$, $l_1$, and $l_2$are the same as calculated in the previous part of this question

$k_3 = (v_0 + l_2 \frac{h}{2}) = 1 + \frac{0.1*(-5.3)}{2} = 0.735$

$l_3 = 10 - 8\,(v_0 + l_2 \frac{h}{2}) - 10(i_0 + k_2 \frac{h}{2})$

$= 10 - 8\,(1 + \frac{0.1(-5.3)}{2}) - 10(1 + \frac{0.1*0.6}{2}) = -6.18$

$k_4 = (v_0 + h\,l_3) = 1 + 0.1(-6.18) = 0.382$

$l_4 = 10 - 8\,(v_0 + hl_3) - 10(i_0 + h\,k_3)$

$= 10 - 8\,(1 + 0.1(-6.18)) - 10(1 + 0.735 * 0.1) = -3.791$

Therefore,

$i\,(0.1) = i\,(0) + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 1 + \frac{0.1}{6}(1 + 2(0.6) + 2(0.735) + 0.382)$

$= 1.06753$

$v\,(0.1) = v\,(0) + \frac{h}{6}(l_1 + 2l_2 + 2l_3 + l_4)$

$= 1 + \frac{0.1}{6}(-8 - 2(05.3) - 2(6.18) - 3.791)$

$i'(0.1) = 0.4208$