ENGR371 Probability and Statistics for Engineering

Note: start a new page for each question

1. Cement and concrete research reports the investigation of the compressive strength of concrete when mixed with fly ash. It was found that the compressive strength of concrete has a Normal distribution. For this test, nine samples are selected randomly, and the compressive strength for the nine samples are 40.2 30.4 28.9 30.5, 22.4 25.8 18.4 14.2 15.3.
   (a) Find the sample mean and standard deviation (3 marks)
   (b) Find a 98% lower one-sided confidence interval on mean compressive strength (5 marks)
   (c) Find a 95% two-sided confidence interval on mean compressive strength. Explain why the lower end-point of the interval is or is not the same as in (a) (5 marks)
   (d) Find a 95% two-sided confidence interval on standard deviation of compressive strength. (5 marks)

2. The white blood cell (WBC) recovery is investigated for patients after a new treatment. Now 19 patients are randomly selected and the WBC recovery times are 18, 16, 13, 16, 15, 12, 9, 14, 12, 8, 16, 12, 10, 8, 14, 9, 5, 18 and 12 days. The recovery time is assumed a Normal distribution.
   (a) Is there sufficient evidence to support a claim that the mean WBC recovery time exceeds 12 days? Use a level of significance 0.05 (8 marks)
   (b) Find a 95% prediction interval for a new patient's recovery time. (8 marks)

3. Saguaro cacti are large cacti indigenous to the southwestern United States and Mexico. Assume that the number of saguaro cacti in a region follows a Poisson distribution with a mean of 280 per square kilometer. Find: (6 marks for each)
   (a) Mean number of cacti per 10,000 square meters.
   (b) Probability of no cacti in 10,000 square meters.
   (c) Area of a region such that the probability of at least two cacti in the region is 0.9.

4. (a) A bearing assembly contains 10 bearings. The bearing diameters are assumed to be independently and normally distributed with a mean of 1.5 millimeters and a standard deviation of 0.025 millimeter. What is the probability that the maximum diameter bearing in the assembly exceeds 1.6 millimeters? (7 marks)
   (b) Let Y=2X+1, where X is the diameter of a bearing in (a). Calculate E(Y) and V(Y). (5 marks)

5. Consider the bivariate density function

$$f(x,y) = \begin{cases} \frac{1}{4}x(3y^2+1) & 0 \le x \le 2, 0 \le y \le 1 \\ 0 & Otherwise \end{cases}$$

   a) Find the marginal probability density functions of *x and y*. (5 marks)
   b) Are the random variables *x and y* statistically independent? Justify your answer. (5 marks)
   c) Find the conditional probability $P(\frac{1}{4} < x < \frac{1}{2} \mid y = \frac{1}{3})$ (8 marks)

6. The sick-leave time of employees in a firm in a month is normally distributed with a mean of 100 hours and a standard deviation of 20 hours. (6 marks for each)
   a) What is the probability that the sick-leave time for next month will be between 50 and 80 hours?
   b) How much time should be budget for sick leave if the budgeted amount should be exceeded with a probability of only 10%?
   c) What do you think the value of the mean will be if the number of sick-leave between 97.7 and 101.3 hours?