Concordia University

Department of Computer Science & Software Engineering

COMP 232/4 Introduction to Discrete Mathematics Winter 2019

Assignment 5

Due date: Thursday, April 11st, 2019

1. For each of the following relations on the set $\mathbb{Z}$ of integers, determine if the relation is reflexive, symmetric, anti-symmetric, or transitive. On the basis of these properties, state whether or not it is an equivalence relation or a partial order.
   (a) $R = \{(a, b) \mid a^2 = b^2\}$
   (b) $S = \{(a, b) \mid |a - b| \leq 1\}$.

2. Prove that $\{(x, y) \mid x - y \in \mathbb{Q}\}$ is an equivalence relation on the set of real numbers, where $\mathbb{Q}$ denotes the set of rational numbers.

3. Definition: a relation on A is called *irreflexive*, if no element of the domain A is related to itself.

   Prove or disprove the following statements:
   (a) Let R be a relation on the set $\mathbb{Z}$ of integers such that xRy if and only if $xy \geq 1$. Then, R is irreflexive.
   (b) Let R be a relation on the set $\mathbb{Z}$ of integers such that xRy if and only if $x = y + 1$ or $x = y - 1$. Then, R is irreflexive.
   (c) Let R and S be reflexive relations on a set A. Then, $R - S$ is irreflexive.

4. Let R be the relation on $\mathbb{Z}^+$ defined by xRy if and only if $x < y$. Then, in Set Builder Notation, $R = \{(x, y) \mid y - x > 0\}$.
   (a) Use Set Builder Notation to express the transitive closure of R.
   (b) Use Set Builder Notation to express the composite relation R^n, where n is a positive integer.

5. Give the transitive closure of the relation $R = \{(a, c), (b, d), (c, a), (d, b), (e, d)\}$ on domain $A = \{a, b, c, d, e\}$.

6. Give an example to show that when the symmetric closure of the reflexive closure of the transitive closure of a relation is formed, the result is not necessarily an equivalence relation.

7. Show that the symmetric closure of the union of two relations is the union of their symmetric closures.

8. Let $S = \{1, 2, 3, 4\}$. With respect to the lexicographic order based on the usual less than relation,
   (a) find all pairs in $S \times S$ less than $(2, 3)$
   (b) find all pairs in $S \times S$ greater than $(3, 1)$
   (c) draw the Hasse diagram of the poset $(S \times S, \preccurlyeq)$.