CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	203	All
Examination	**Date**	**Duration**
Midterm Test	24 February, 2013	1 h 30 min
Special Instructions:	Only approved calculators are allowed **Show all your work**	

1. (6 marks): Solve for x (find the *exact* values, do not approximate)

(a) $\log_3(3\,x) - \log_3(x^5) = 3$

(b) $4^{\log_2 x} - 6\,x = 0$

2. (6 marks) **(a)** Let $f(x) = \sqrt{x^2 - 1}$ and $g(x) = \dfrac{1}{x}$. Find the composite functions $f \circ g$ and $g \circ g$, and determine their domains.

(b) Given the function $f(x) = \dfrac{1}{1 + 2\,e^x}$, find the inverse function f^{-1} and the domain and the range of f^{-1}.

3. (8 marks) Find the limit or explain why the limit does not exist:

(a) $\displaystyle\lim_{x \to 1} \frac{\sqrt{x+3} - 2}{x - 1}$

(b) $\displaystyle\lim_{x \to -1} \frac{10\,x + 10}{|\,x + 1|}$

4. (5 marks) Find all horizontal and vertical asymptotes, as well as $x-$ and $y-$intercepts of the graph $y = \dfrac{3x + 1}{\sqrt{4x^2 - 16}}$.

(continued on the other side)

5. (16 marks). Find the derivatives of the following functions (you don't need to simplify the final answer, but you must show how you calculate it):

(a) $f(x) = \dfrac{4 + 2\sqrt{x} + 3x^2}{x}$

(b) $f(x) = \dfrac{\cos x \, \sin x}{1 - \tan x}$

(c) $f(x) = (\pi + e^x)\, 2^x$

(d) $f(x) = \cos^3(\sin\sqrt{x^2 + 9})$

6. (9 marks) Given the function $f(x) = x^2 - 2x$:

(a) Calculate $f'(x)$ using its definition as a limit (of difference quotient). Check that your calculation is correct using standard rules for differentiation of power functions.

(b) Compute the average rate of change of $f(x)$ on the interval $[0, 3]$, call it m.

(c) Find whether there is a point a on that interval such that the instantaneous rate of change of f at that point is equal to the average rate of change, $f'(a) = m$. Calculate a if it exists.

Bonus Question (3 marks).

A cylinder is inscribed in a right circular cone with height $h = 8\,\text{m}$ and radius at the cone's base $r = 4\,\text{m}$. Express the volume of the cylinder V as a function of its radius x.