CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Section**
MATH 264/MAST 218	264/218	AA
Examination	**Date**	**Pages**
Final Exam Version B	April 2020	3
Instructor		
Dr. D. Dryanov		

Instructions: The value of each problem is 10 marks out of a possible total of 100 marks. Justify your answers by appropriate explanations.

Problem 1. Consider the plane curve defined by the parametric equations

$$x = \sin(t), \quad y = \cos(2t), \quad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

(a) Find

$$\frac{d^2y}{dx^2}$$

in terms of the parameter t.
(b) For which values of t in the interval $(-\pi/2, \pi/2)$ is the curve concave downward?

Problem 2. **(a)** Sketch the curves expressed here in polar coordinates

$$r_1 = 1 + \sin(\theta); \quad r_2 = 2 - \sin(\theta).$$

(b) Find the area of the plane region that lies inside both curves: $r_1 = 1 + \sin(\theta)$ and $r_2 = 2 - \sin(\theta)$.

Problem 3. **(a)** Find an equation of the plane that contains the line l_1 given with the vector equation

$$\mathbf{r}(t) = \langle 1 - 2t,\, 1 + 3t,\, 2t \rangle$$

and is parallel to the line l_2 given with parametric equations $x = 1+2s$, $y = 1-4s$, $z = 1 + 3s$.
(b) Determine if the planes given by the Cartesian equations $3x - 2y + 57 = 6$ and $2x - 2y - 2z = -17$ are orthogonal, parallel, or neither.

Problem 4. The acceleration of a moving object is given by

$$\mathbf{a}(t) = 3t\mathbf{i} - 4e^{-t}\mathbf{j} + 12t^2\mathbf{k}.$$

The initial velocity of the object is $\mathbf{v}(0) = \langle 0, 1, -3\rangle$ and the initial position of the object is $\mathbf{r}(0) = \langle -5, 2, -3\rangle$. Determine the velocity and the position function of the object at any time t.

Problem 5. Consider the plane curve C with Cartesian equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\,,$$

where $a > b$ are positive numbers.
(a) Find an one-parametric vector function $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$ representing the curve C.
(b) Use the vector function obtained in **(a)** to compute the curvature $\kappa(t)$ as a function of the parameter t.
(c) Find the maximum and the minimum values of the curvature κ of the curve C and all points on the curve C at which these values occur.

Problem 6. Fir each of the following limits find the limit or show that the limit does not exists

$$\textbf{(a)} \lim_{(x,y)\to(0,0)} \frac{x^4 + y^4}{x^2 + y^2}; \quad \textbf{(b)} \lim_{(x,y)\to(2,4)} \frac{\sqrt{x+2} - \sqrt{y}}{x - y + 2}.$$

Problem 7. Suppose that the temperature T at every point (x, y, z) in a ball of radius 10 centered at the origin is defined by

$$T(x, y, z) = 100e^{-2x^2-3y^2-4z^2}\,.$$

(a) Find the rate of change of T at the point $(2, -1, 2)$ in the direction of $\mathbf{v} = -\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$.
(b) Find the direction in which the temperature is increasing most rapidly at the point $(2, -1, 2)$. Find the maximum rate of increase.

Problem 8. Find and classify the critical points of the function

$$f(x, y) = y^3 - x^2 + 7x - 8y + 2xy - 25\,.$$

Problem 9. Find the absolute maximum and minimum of the function

$$f(x, y) = x^2 + y^2 - 4y + 100$$

on the closed triangular region with vertices $(2, 0)$, $(2, 2)$, and $(4, 2)$.

Problem 10. Find the maximum and minimum values of the function

$$f(x, y, z) = \ln(1 + xyz)$$

subject to the constraint $x^2 + y^2 + z^2 = 1$.

Bonus Question. Find a vector equation of the tangent line to the curve of intersection of the surfaces $x^2 + 2y^2 + 3z^2 = 6$ and $x^2yz^3 = 1$ at the point $(1, 1, 1)$.
