ENGR 233/4 MIDTERM I-A FEBRUARY, 2009 Prof. R. Stern

The five questions have equal value. 75 minutes. Calculators allowed. No other materials.

1. Consider the surface given by the equation $z = x^2y + y^3$.
   (a) Give the equation of the tangent plane to the surface going through the point $(1, 1, 2)$.
   (b) Give the parametric equations of the normal line through that point.
   (c) Find the directional derivative of f at the point $(1, 1)$ in a direction parallel to $(3, 4)$.
   (d) In what direction is the directional derivative minimized at the point $(1, 1)$? What is that minimum value?
2. Let $v = (1, 2, 3)$ and $w = (0, -1, 4)$.
   (a) Find the cosine of the angle between the vectors v and w.
   (b) Find the component of v on w and the projection of of v on w.
3. (a) Find the equation of the plane containing the three points $(1, 1, 1)$, $(-2, 0, 3)$ and $(4, 5, 1)$.
   (b) Find the parametric equations of the line normal to the plane you found, and passing through $(-2, 0, 3)$.
4. Consider the space-curve $r(t) = (3\cos(4t), 3\sin(4t), 6t)$.
   (a) Find the arc-length between $t = 0$ and $t = 2$.
   (b) Find the curvature when $t = \pi$.
5. Consider the function $f(x, y) = e^{x^2-y^2}\cos(2xy)$.
   (a) Verify that $f_{xx} = -f_{yy}$.
   (b) Suppose $x(t) = t^2$ and $y(t) = t$. Use the appropriate multivariate chainrule in order to find $\frac{d}{dt}f(x(t), y(t))$ when $t = 1$.