# CONCORDIA UNIVERSITY
## Department of Mathematics & Statistics

| Course | Number | Section(s) |
|---|---|---|
| Mathematics | 208/2 | All |

| Examination | Date | Time | Pages |
|---|---|---|---|
| Midterm | October 2019 | 1 Hour 30 minutes | 2 |

| Instructors | Course Examiner |
|---|---|
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## FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\frac{(1+i)^n - 1}{i}, \quad PV = PMT\frac{1-(1+i)^{-n}}{i}$$

## Special Instructions:

▷ Answer all questions.
▷ Only approved calculators are allowed.

## *MARKS*

[5+5] **1.** A manufacturer has been selling 1200 television sets a week at \$480 each. A market survey indicates that for each \$30 rebate offered to a buyer, the number of sets sold will increase by 300 per week.

   (A) Find the demand function $p(x)$, where $x$ is the number of the television sets sold per week and $p(x)$ is the price of one set.

   (B) How large rebate should the company offer to a buyer, in order to maximize its revenue?

$[2\frac{1}{2} \times 4]$ **2.** Solve for $x$ in the following equations:

   (A) $\left(\dfrac{5}{4}\right)^{x^2-42} = \left(\dfrac{64}{125}\right)^{2x-10}$

   (B) $2\ln(2x) - \ln(3x-2) = 3\ln 4 - \dfrac{4}{5}\ln 32 + \ln 1$

   (C) $e^{-2x^2+x-5} = \left(\dfrac{1}{e}\right)^{x^2-3x+20}$

   (D) $\log(x+5) - \log(2x-1) = 2\log\sqrt{3}$

PLEASE TURN OVER

[5+5]  **3.**

    (A) If the first and 10th terms of an arithmetic sequence are 3 and 30 respectively, find the 48th term of the sequence.

    (B) If the first term of a geometric sequence is 300 and the third term is 3, find the sum of the first 40 terms.

[10]  **4.** A person deposits \$2000 to a savings account at the end of each year. The account pays 2% interest compounded annually. If the first deposit is made at the end of the year 2012 and the last deposit is made at the end of the year 2022 , what will be the balance in the account immediately after the last deposit.

[5+5]  **5.** A small business owner contributes \$3,000 at the end of each quarter to a retirement account that earns 8% compounded quarterly.

    (A) How long will it be until the account is worth \$150,000?

    (B) Suppose that when the account reaches \$150,000, the business owner increases the contributions to \$5,000 at the end of each quarter. What will the total value of the account be after 15 more years?

[4+3+3]  **6.** You are buying a \$20,000 boat with a down-payment of \$4,000 and you finance the remaining amount with a 3 year loan at 7.8% compounded weekly.

    (A) What are your weekly payments?

    (B) What is the remaining balance after 2 years?

    (C) How much are you saving by paying the remaining balance after 2 years?

Math 208, Midterm Test : October 2019
(Solution)

**#1.** Two point $A(1200, 480)$, $B(1500, 450)$ given

(A) $p(x) = mx + b$ where $m = \dfrac{450 - 480}{1500 - 1200} = -\dfrac{1}{10}$

$p(x) = -\dfrac{1}{10}x + b$. Also $450 = -\dfrac{1}{10}(1500) + b \Rightarrow$

$b = 600$ $\therefore$ $p(x) = -\dfrac{1}{10}x + 600$

(B) Revenue $R(x) = p(x) \cdot x = -\dfrac{1}{10}x^2 + 600x$

$R(x)$ is max at $x$ coordinate of vertex $= -\dfrac{b}{2a} = 3000$

$\Rightarrow p = -\dfrac{1}{10}(3000) + 600 = 300\$$ Rebate $= 480 = 300$

Rebate $= \$180$ for max revenue

**#2 A:** $\left(\dfrac{5}{4}\right)^{x^2 - 42} = \left[\left(\dfrac{5}{4}\right)^{-3}\right]^{2x - 10} \Rightarrow x^2 - 42 = -6x + 30 \Rightarrow$

$x^2 + 6x - 72 = 0 \Rightarrow (x + 12)(x - 6) = 0 \Rightarrow x = -12, 6.$

(B) $\ln(2x)^2 - \ln(3x - 2) = \ln 4^3 - \ln(32)^{4/5} + 0 \Rightarrow$

$\ln \dfrac{4x^2}{3x - 2} = \ln \dfrac{64}{16} \Rightarrow \dfrac{4x^2}{3x - 2} = 4 \Rightarrow x^2 - 3x + 2 = 0$

$x = 1, 2.$

(C) $e^{-2x^2 + x - 5} = (e^{-1})^{x^2 - 3x + 20} \Rightarrow -2x^2 + x - 5 = -x^2 + 3x - 20$

$\Rightarrow x^2 + 2x - 15 = 0 \Rightarrow (x + 5)(x - 3) = 0 \Rightarrow x = \begin{cases} -5 \\ 3 \end{cases}$

(d) $\log(x + 5) - \log(2x - 1) = 2 \log \sqrt{3} \Rightarrow$

$\log \dfrac{x + 5}{2x - 1} = \log 3 \Rightarrow \dfrac{x + 5}{2x - 1} = 3 \Rightarrow 5x = 8 \Rightarrow x = \dfrac{8}{5}$

**#3 (A):** Here $a_1 = 3$, $a_{10} = 30 \Rightarrow a_{10} = a_1 + (10 - 1)d$

$9d = 27 \Rightarrow d = 3$. Thus $a_{48} = a_1 + (48 - 1)d = 3 + 47 \cdot 3$

$a_{48} = 144$

**3 (B)** $b_1 = 300$ and $b_3 = 3 \Rightarrow 3 = 300r^2 \Rightarrow r = \pm \dfrac{1}{10}$

for $r = \dfrac{1}{10}$, $S_{40} = 300 \dfrac{1 - \left(\frac{1}{10}\right)^{40}}{1 - \frac{1}{10}} \approx \dfrac{3000}{9}$

Ans 2 : $S_{40} = 300 \dfrac{1 - \left(-\frac{1}{10}\right)^{40}}{1 + \frac{1}{10}} = \dfrac{3000}{11}$

## Midterm october 2019 (soln)

**#4**   $i = 0.02$    $n = 11$    $FV = PMT \dfrac{(1+i)^n - 1}{i}$,   $PMT = 2000$

$$FV = 2000 \frac{1.02^{11} - 1}{0.02} = \underline{24,337.43}$$

### # 5.A

$i = 0.02$   $PMT = 3000$,   $FV = PMT \dfrac{(1+i)^n - 1}{i} \Rightarrow 150,000 = 3000 \dfrac{1.02^n - 1}{0.02}$

$\Rightarrow 1 = 1.02^n - 1 \Rightarrow 1.02^n = 2 \Rightarrow \ln(1.02)^n = \ln(2) \Rightarrow n = \dfrac{\ln(2)}{\ln(1.02)} = 35$

Ans: $\underline{35 \text{ quarters}}$.

### # 5.B

$$FV_+ = 5{,}000 \frac{1.02^{60} - 1}{0.02} = \underline{\$570{,}257.70} \quad \text{and}$$

$150{,}000 \cdot (1.02)^{60} = \$492{,}154.62$    Total $= \underline{\$1{,}062{,}412.32}$

### # 6.A

$PV = 20{,}000 - 4{,}000 = 16{,}000$,   $n = 3 \cdot 52 = 156$,   $i = \dfrac{0.078}{52} = 0.0015$

$$PMT = \frac{PV \cdot i}{1 - (1+i)^{-n}} = \frac{16{,}000 \cdot 0.0015}{1 - 1.0015^{-156}} = \underline{\$115.11}$$

### #6.B

$$PV_2 = PMT \frac{1 - (1+i)^{-52}}{i} = 115.11 \frac{1 - 1.0015^{-52}}{0.0015} = \underline{\$5754.08}$$

### # 6.C

the total to pay in the last year is $PMT \cdot 52 = 115.11 \cdot 52 = \$5985.72$

Interest saved:   $\$5985.72 - \$5754.08 = \underline{\$231.64}$