**PROBLEMS FOR CHAPTER 2**

5. Determine the smallest positive root between (0,2) of the given equation using Bisection method with an error less than 0.02.

$$2x = e^x - 0.9$$

**Solution**:

$2x - e^x + 0.9 = 0$

using incremental search we get

| x | 0 | 1 | 2 |
|---|---|---|---|
| f(x) | -0.1 | 0.181718 | 2.48906 |

Therefore, root is in between 0 and 1

a = 0 and b =1

f(a)=-0.1 and f(b) = 0.181718

p = 0.5 and f(p)= 0.2513

Therefore p is the new b. continue using bisection we get

| n | A | b | p | fa | fb | fp | er |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0.5 | -0.1 | 0.181718 | 0.251279 | |
| 2 | 0 | 0.5 | 0.25 | -0.1 | 0.251279 | 0.115975 | 1 |
| 3 | 0 | 0.25 | 0.125 | -0.1 | 0.115975 | 0.016852 | 1 |
| 4 | 0 | 0.125 | 0.0625 | -0.1 | 0.016852 | -0.03949 | 1 |
| 5 | 0.0625 | 0.125 | 0.09375 | -0.03949 | 0.016852 | -0.01079 | 0.333333 |
| 6 | 0.09375 | 0.125 | 0.109375 | -0.01079 | 0.016852 | 0.003169 | 0.142857 |
| 7 | 0.09375 | 0.109375 | 0.101563 | -0.01079 | 0.003169 | -0.00377 | 0.076923 |
| 8 | 0.101563 | 0.109375 | 0.105469 | -0.00377 | 0.003169 | -0.00029 | 0.037039 |
| 9 | 0.105469 | 0.109375 | 0.107422 | -0.00029 | 0.003169 | 0.00144 | 0.018181 |

Therefore, x = 0.106446

10. Use the method of False Position to find the positive root of the equation correct to 4 decimal places.

$$2x + cosx - 2 = 0$$

Solution:

$2x + Cosx - 2 = 0$

Using incremental search we get

| x | 0 | 1 |
|---|---|---|
| f(x) | -1 | 0.5403 |

using false position method

$$p = \frac{a\,f(b) \,+\, bf(a)}{f(b) - f(a)}$$

$= \frac{0\,(0.5403)-1\,(-1)}{(0.5403)-(-1)}$ = 0.6492

f(p) = 0.095

p = b new

continuing in table form

| n | a | B | p | fa | fb | fp | er |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0.649223 | -1 | 0.540302 | 0.095 | |
| 2 | 0 | 0.649223 | 0.592898 | -1 | 0.095 | 0.015121 | 0.095 |
| 3 | 0 | 0.592898 | 0.584066 | -1 | 0.015121 | 0.00236 | 0.015121 |
| 4 | 0 | 0.584066 | 0.582691 | -1 | 0.002359 | 0.000367 | 0.00236 |
| 5 | 0 | 0.582691 | 0.582477 | -1 | 0.000367 | 5.71E-05 | 0.000367 |
| 6 | 0 | 0.582477 | 0.582444 | -1 | 5.66E-05 | 8.81E-06 | 5.71E-05 |
| 7 | 0 | 0.582444 | 0.582439 | -1 | 8.78E-06 | 1.37E-06 | 8.81E-06 |
| 8 | 0 | 0.582439 | 0.582438 | -1 | 1.53E-06 | 2.38E-07 | 1.33E-06 |

12. Evaluate the smallest positive root of the following equation by Newton-Raphson method and 4 significant digits.

$$3x^2 - tanx$$

**Solution:**

from incremental search we get

| $x_1$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 |
|---|---|---|---|---|---|
| $f(x_1)$ | 1 | -0.0703 | -0.0827 | -0.0393 | 0.0572 |

Therefore,

$x_0 = 0.3$

$f'(x) = 6x - \sec^2(x) \quad \Rightarrow \quad f'(0.3) = 0.7043$

therefore,

$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

$x_1 = 0.3 - \frac{-0.03934}{0.7043} = 0.35585$

Continuing in table form

| i | xi | fxi | f'xi | xi+1 | error |
|---|---|---|---|---|---|
| 0 | 0.3 | -0.03934 | 0.704311 | 0.355851 | 0.15695 |
| 1 | 0.355851 | 0.008216 | 0.996963 | 0.34761 | 0.023708 |
| 2 | 0.34761 | 0.000175 | 0.95438 | 0.347426 | 0.000529 |
| 3 | 0.347426 | 8.74E-08 | 0.953429 | 0.347426 | 2.64E-07 |

23. Apply Secant method to find the root of the following equation to 5 decimal places.

$$2x + 1 - e^x = 0$$

Solution:

$2x + 1 - e^x = 0$

From incremental search method

| x | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
|---|---|---|---|---|---|---|---|---|---|
| f(x) | 0 | -0.08517 | -0.1414 | -0.16986 | -0.17182 | -0.14872 | -0.10212 | -0.03375 | 0.054459 |

Therefore,

$x_n = 0.7 \quad x_{n-1} = 0.6$

$f'(x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} = \frac{(-0.03375) - (-0.10212)}{0.7 - 0.6} = 0.683661$

$x_{n+1} = x_n - \left(\frac{f(x_n)}{f'(x_n)}\right) = 0.7 - \left(\frac{-0.03375}{0.683661}\right) = 0.749371$

Continuing in table form

| i | $x_{n-1}$ | $x_n$ | $f(x_n)$ | $f(x_{n-1})$ | $f'(x_n)$ | $x_{n+1}$ | $\sum$ |
|---|---|---|---|---|---|---|---|
| 0 | 0.6 | 0.7 | -0.03375 | -0.10212 | 0.683661 | 0.749371 | 0.065883 |
| 1 | 0.7 | 0.749371 | -0.007445 | -0.03375 | 0.83445 | 0.740449 | 0.012049 |
| 2 | 0.749371 | 0.740449 | -0.00035 | -0.007445 | 0.873381 | 0.740847 | 0.000537 |
| 3 | 0.740449 | 0.740847 | $-3.2 * 10^{-6}$ | -0.00035 | 0.865298 | 0.74085 | $5 * 10^{-6}$ |

Therefore,

x = 0.74085