July 2014 Practice Midterm

**EX2-1**

In a certain class there are Electrical, Civil and Mechanical Engineering students only. At this class the students must give a presentation. Students feel that the order of presentations is very important..

a) If there are 9 students in the class and all give their presentation on one day. How many different ways can you arrange the order of the presentation?

$9! = 362880$

b) If there are 18 students in the class but only 9 can give their presentation on the first day, how many different orders of presentation can you have for first day?

$\binom{18}{9} 9!$ or ${}_{18}P_9$

c) If the class contains 12 Mechanical, 18 Civil and 6 Electrical students. The teacher requires that the 8 individual who give their presentations on the first day must include 3 Mechanical, 3 Civil and 2 Electrical students. How many different orders of presentation are there for the first day?

$\binom{12}{3}\binom{18}{3}\binom{6}{2} \cdot 8!$

d) Suppose that all of the presentations that you counted in part c) are equally likely. Suppose that you are one of the 6 electrical students. What is the probability that you will be the first one to give a presentation on the first day?

$$\frac{\binom{12}{3}\binom{18}{3}\binom{5}{1} 7!}{\binom{12}{3}\binom{18}{3}\binom{6}{2} 8!} = .04167$$

**EX2-2**

An inspector working for a manufacturing company has a 99% chance of correctly identifying defective items and a 0.5% chance of incorrectly classifying a good item as defective. The company has evidence that its line produces 0.9% of nonconforming items.

a) What is the probability that an item selected for inspection is classified as defective?(4 marks) $\mathbb{P}(S) = \mathbb{P}(S|A)\mathbb{P}(A) + \mathbb{P}(S|\bar{A})\mathbb{P}(\bar{A})$

$= (.99)(.009) + (.005)(.991) = .013865$

b) If an item selected at random is classified as non-defective, what is the probability that it is indeed good?(4 marks)

S = says defective

A = Actually is defective

$\mathbb{P}(S|A) = .99 \rightarrow \mathbb{P}(\bar{S}|A) = .01$

$\mathbb{P}(S|\bar{A}) = .005 \rightarrow \mathbb{P}(\bar{S}|\bar{A}) = .995$

$\mathbb{P}(A) = .009$

$\mathbb{P}(\bar{A}) = .991$

(b) $\mathbb{P}(\bar{A}|\bar{S}) = \dfrac{\mathbb{P}(\bar{S}|\bar{A})\mathbb{P}(\bar{A})}{\mathbb{P}(\bar{S}|\bar{A})\mathbb{P}(\bar{A}) + \mathbb{P}(\bar{S}|A)\mathbb{P}(A)}$

$= \dfrac{(.995)(.991)}{(.995)(.991) + (.01)(.009)} = .9999$

## EX3-3

Let X be a random variable with the following probability distribution, where "A" is a constant that you must determine.

| X | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| *f(x)* | 0.080 | 0.125 | 0.300 | 0.225 | 0.200 | A |

a) Compute the value of "A".

b) Find and plot the cumulative distribution function (CDF), F(x).

c) Evaluate the mean and the standard deviation of the random variable X.

d) Find P(X=2) using the CDF in (b)

e) If another random variable $Y=X^2+1$ is formed, find the mean E[Y].

## EX3-4

Because all airline passengers do not show up for their reserved seat, the airline company sells 125 tickets for a flight that holds only 120 passengers. The probability that a passenger does not show up is 0.10, and all passengers behave independently.

a) What is the probability that every passenger who shows up can take the flight? (4 marks)

b) What is the probability that the flight departs with empty seats?(5 marks)

<u>Binomial</u>

$X = \#$ of show ups

$n=125$

$p=.9$

a) $\mathbb{P}(X \leq 120) = 1-\mathbb{P}(X>120) = .9961$

b) $\mathbb{P}(X \leq 119) = 1-\mathbb{P}(X>119) = .9886$