1. (1 point) Solve the following equation. If necessary, enter your answer as an expression involving natural logarithms or as a decimal approximation that is correct to at least four decimal places.

$$2^{2x+3} = 3^{x-4}$$

$x =$ ________

Correct Answers:

- `-22.5036`

2. (1 point) The population of a colony of rabbits grows exponentially. The colony begins with 10 rabbits; 5 years later there are 380 rabbits.

(a) Express the population of the colony of rabbits, P, as a function of time, t, in years.

$P(t) =$ ____________

(b) Use the graph to estimate how long it takes for the population of rabbits to reach 1000 rabbits.

It will take ________ years. (round to nearest 0.01 year)

Solution:

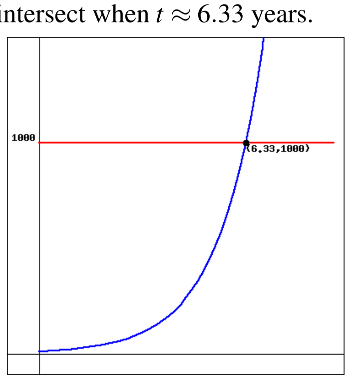
SOLUTION

a) Since the population grows exponentially, it can be described by $P = ab^t$, where P is the number of rabbits and t is the number of years which have passed. We know that a represents the initial number of rabbits, so $a = 10$ and $P = 10(b)^t$. After 5 years, there are 380 rabbits so

$$380 = 10(b)^5$$
$$38 = b^5$$
$$(b^5)^{1/5} = 38^{1/5}$$
$$b \approx 2.07$$

From this, we know that $P(t) = 10(2.07)^t$.

b) We want to find t when $P = 1000$. Using a graph of $P = 10(2.07)^t$, we see (figure below) that the line $P = 1000$ and $P = 10(2.07)^t$ intersect when $t \approx 6.33$ years.



Correct Answers:

- `10*2.07^t`
- `6.33`

3. (1 point)

If $f(x) = e^{9x}$, $g(x) = 4x+5$, and $h(x) = \sqrt{x}$. Find a simplified formula for the function below:

$f\big(g(x)\big)h(x) =$ ____________

Solution:

SOLUTION

We start by computing $f\big(g(x)\big) = e^{9g(x)} = e^{9(4x+5)} = e^{36x+45}$.

Next we compute $f\big(g(x)\big)h(x)$ by multiplying the expression above with $h(x) = \sqrt{x}$:

$$f\big(g(x)\big)h(x) = e^{36x+45} \cdot \sqrt{x}.$$

Correct Answers:

- `e^(36 x + 45)*sqrt(x)`

4. (1 point)

What is the domain of $y = \ln(x^2 - 3x - 4)$?

____

click here for help using interval notation

Solution:

SOLUTION

The quadratic $y = x^2 - 3x - 4 = (x-4)(x+1)$ has zeros at $x = 4, \ -1$.

It is positive outside of this interval and negative within this interval. Therefore, the function $y = \ln(x^2 - x - 6)$ is undefined on the interval $-1 \le x \le 4$, and it is defined when $x < -1$ or $x > 4$. In interval notation we can express the domain as

$$(-\infty, -1)(4, \infty)$$

Correct Answers:

- `(-infinity,-1) U (4,infinity)`

5. (1 point) Using the properties of logarithms, decide whether each equation is true or not.

$\boxed{?}$ 1. $p \cdot \ln(A) = \ln(A^p)$

$\boxed{?}$ 2. $\sqrt{\ln(A)} = \ln(A^{(1/2)})$

$\boxed{?}$ 3. $\ln(A)\ln(B) = \ln(A) + \ln(B)$

$\boxed{?}$ 4. $\log(\sqrt{A}) = \frac{1}{2}\log(A)$

$\boxed{?}$ 5. $\dfrac{\log(A)}{\log(B)} = \log(A) - \log(B)$

$\boxed{?}$ 6. $\log(AB) = \log(A) + \log(B)$

Solution: SOLUTION(EV3P(¡¡'END$_{S}OLUTION'$));
SOLUTION

1. The first question is True.

2. The second question False. The property $\ln(\sqrt{A}) = \frac{1}{2}\ln(A)$ is true; therefore, the given equation $\sqrt{\ln(A)} = \ln(A^{1/2})$ is not correct.

3. The third question is False. Notice the difference between the given incorrect equation $\ln(A)\ln(B) = \ln(A) + \ln(B)$, and the property with which you may be confusing, $\ln(AB) = \ln(A) + \ln(B)$. You may only split multiplication INSIDE the log into the sum of two logs.

4. The fourth question is True.

5. The fifth question is False. Notice the difference between the given incorrect equation $\frac{\log(A)}{\log(B)} = \log(A) - \log(B)$, and the property with which you may be confusing, $\log(A/B) = \log(A) - \log(B)$. You may only split division INSIDE the log into the difference of two logs.

6. The sixth question is True.

Correct Answers:

- `T`
- `F`
- `F`
- `T`
- `F`
- `T`

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6. (1 point)

Find the inverse function (if it exists) of $h(x) = \frac{x}{2x+7}$. If the function is not invertible, enter **NONE** .

$h^{-1}(x) =$ ______________

(notice in this problem the independent variable in the inverse is x)

Solution:

SOLUTION

Start with our property of inverse functions $h\left(h^{-1}(x)\right) = x$, and substitute y for $h^{-1}(x)$ to get $h(y) = x$. Now, using the formula for h we get $h(y) = \dfrac{y}{2y+7} = x$ and solving for y yields

$\frac{y}{2y+7} = x$
$y = x(2y+7)$
$y = 2yx + 7x$
$y - 2yx = 7x$
$y(1-2x) = 7x$
$y = \frac{7x}{1-2x}$.

Now replacing y by $h^{-1}(x)$, we have our formula, $h^{-1}(x) = \dfrac{7x}{1-2x}$.

Correct Answers:

- `(7 x)/(1-2 x)`

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7. (1 point)

Find the inverse function (if it exists) of $f(x) = \ln(3-4x)$. If the function is not invertible, enter **NONE** .

$f^{-1}(x) =$ ______________

(notice in this problem the independent variable in the inverse is x)

Solution:

SOLUTION

Start with our property of inverse functions $f\left(f^{-1}(x)\right) = x$, and substitute y for $f^{-1}(x)$ to get $f(y) = x$. Now, using the formula for f we get $f(y) = \ln(3-4y) = x$ and solving for y yields

$x = f(y)$
$x = \ln(3-4y)$
$e^x = e^{(\ln(3-4y)}$
$e^x = 3-4y$
$4y = 3-e^x$
$y = \frac{3-e^x}{4}$.

Thus $y = f^{-1}(x) = \dfrac{3-e^x}{4}$.

Correct Answers:

- `(3 - e^x)/4`

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8. (1 point)

Find a formula for the inverse of the function.

$f(x) = \frac{1+e^x}{1-e^x}$.
$f^{-1}(x) =$ ______________________

Correct Answers:

- `ln((x-1)/(x+1))`

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9. (1 point)

Find the EXACT solution to the equation below *(do not give a decimal approximation)* .

$$\frac{\log(x^3) + \log(x^5)}{\log(6x)} = 5$$

$x =$ ________________

Note: log is the natural logarithm with base e, i.e. ln.

Solution:

SOLUTION

First we can multiply both sides by $\log(6x)$ and get:
$\log(x^3) + \log(x^5) = 5\ \log(6x)$
$\log(x^{3+5}) = 5\ \log(6x)$
$\log(x^8) = \log\left((6x)^5\right)$

Now we can exponentiate both sides and we get the following equation which we solve for x :
$x^8 = (6x)^5$
$x^8 = (6)^5 x^5$
$\frac{x^8}{x^5} = (6)^5$
$x^{(8-5)} = (6)^5$
$x^3 = (6)^5$
$x = (6)^{5/3}$

Correct Answers:

- `(6)^(5/3)`

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10. (1 point) Evaluate the following expressions. Your answer must be an angle $-\pi/2 \leq \theta \leq \pi$ in radians.
$\sin^{-1}(-\frac{\sqrt{2}}{2})$ _____
$\sin^{-1}(-\frac{1}{2})$ _____
$\cos^{-1}(\frac{\sqrt{3}}{2})$ _____
$\cos^{-1}(-1)$ _____

Correct Answers:

- `-0.785398163397448`
- `-0.523598775598299`
- `0.523598775598299`
- `3.14159265358979`