Department of Computer Science & Software Engineering

Comp232 Mathematics for Computer Science

Fall 2020

Assignment 4. Due date: Friday December 4

1. Use mathematical induction to solve the following:
   (a) Find a formula for $\frac{1}{1.2}+\frac{1}{2.3}+\cdots+\frac{1}{n(n+1)}$ by examining the values of this expression for small values of n.
   (b) Show that $7^n - 1$ is a multiple of 6 for all $n \in \mathbb{N}$

2. Suppose that a bank machine can dispense money in either 3$ or 10$ bills. Show that any amount over 17$ could be dispensed with combinations of only the 3$ or the 10$ bills

3. Use mathematical induction to show that n lines in the plane passing through the same point divide the plane to $2n$ parts.

4. Let $a_1 = 2$, $a_2 = 9$, and $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 3$. Use strong induction to show that $a_n \leq 3^n$ for all positive integer n.

5. Give an example of the following relations:
   (a) A relation on $\{a, b, c\}$ that is reflexive and transitive, but not antisymmetric
   (b) A relation on $\{1, 2\}$ that is symmetric and transitive, but not reflexive.
   (c) A relation on $\{1, 2, 3\}$ that is reflexive and transitive, but not symmetric.

6. Give a recursive definition of the sequence $\{a_n\}$. where $n = 1, 2, 3, \ldots$ if
   a) $a_n = 4n - 2$
   b) $a_n = 1 + (-1)^n$
   c) $a_n = n(n+1)$

   In all parts of this question you must proof and verify your answer.

7. Consider the following relations on the set of positive integers.
   $R_1 = \{(x, y) \mid x + y > 10\}$
   $R_2 = \{(x, y) \mid y \text{ divides } x\}$
   $R_3 = \{(x, y) \mid gcd(x, y) = 1)\}$
   $R_4 = \{(x, y) \mid x \text{ and } y \text{ have the same prime divisors }\}$

   Which of these relations are reflexive, symmetric, antisymmetric or transitive? Justify your answer.

8. Suppose A is the set composed of all ordered pairs of positive integers. Let R be the the relation defined on A where $(a, b)R(c, d)$ means that $ad = bc$. Show that R is an equivalence relation.