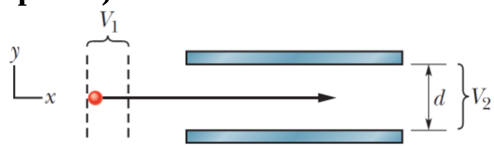
**PHYS 205-Section 03**
**Electricity and Magnetism - Winter 2018**
**Assignment 4 – Solutions**

**Problems + Solutions**

1. In the figure below, an electron accelerated from rest through potential difference $V_1 = 1\ kV$ enters the gap between two parallel plates having separation $d = 20\ mm$ and potential difference$V_2 = 100\ V$. The lower plate is at the lower potential. Assume that the electron's velocity vector is perpendicular to the electric field vector between the plates. In unit-vector notation, what uniform magnetic field allows the electron to travel in a straight line in the gap? **(5 points)**



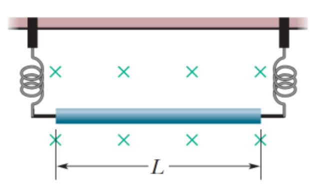

**Solution:**

Straight-line motion will result from zero net force acting on the system; we ignore gravity. Thus, $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = 0$. Note that $\vec{v} \perp \vec{B}$ so $\left|\vec{v} \times \vec{B}\right| = vB$. Thus, obtaining the speed from the formula for kinetic energy, we obtain

$$B = \frac{E}{v} = \frac{E}{\sqrt{2K/m_e}} = \frac{100\ \text{V}/(20\times10^{-3}\text{m})}{\sqrt{2\left(1.0\times10^{3}\text{V}\right)\left(1.60\times10^{-19}\text{C}\right)/\left(9.11\times10^{-31}\text{kg}\right)}} = 2.67\times10^{-4}\,\text{T}.$$

In unit-vector notation, $\vec{B} = -(2.67\times10^{-4}\ \text{T})\hat{\text{k}}$.

2. A 13 g wire of length L= 62 cm is suspended by a pair of flexible leads in a uniform magnetic field of magnitude 0.440 T (as shown in the figure).What are the (a) magnitude and (b) direction (left or right) of the current required to remove the tension in the supporting leads? **(5 points)**



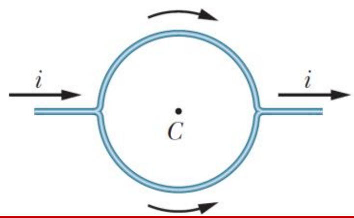
**Solution:**

(a) The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force $mg$ on the wire. Since the field and the current are perpendicular to each other the magnitude of the magnetic force is given by $F_B = iLB$, where $L$ is the length of the wire. Thus,

$$iLB = mg \quad \Rightarrow \quad i = \frac{mg}{LB} = \frac{(0.0130\,\text{kg})(9.8\,\text{m/s}^2)}{(0.620\,\text{m})(0.440\,\text{T})} = 0.467\,\text{A}.$$

(b) Applying the right-hand rule reveals that the current must be from left to right.

3. A straight conductor carrying current $i = 5\,A$ splits into identical semicircular arcs as shown in the figure. What is the magnetic field at the center C of the resulting circular loop? **(5 points)**

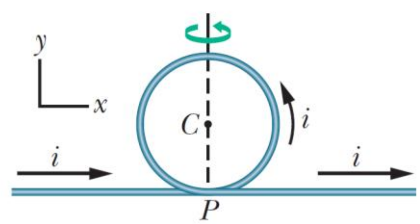



**Solution:**

The straight segments of the wire do not produce any magnetic field at the center, since $d\vec{s} \perp \hat{r}$ and hence $d\vec{s} \times \hat{r} = 0$. For the curved segments, the magnetic field produced by the upper semicircle has the same magnitude as the lower segment, but is in opposite direction (using right-hand-rule). Hence, for there is no magnetic field at the center:

$$\vec{B}_{tot} = 0$$

4. In the figure below, part of a long insulated wire carrying current $i = 5.78\,mA$ is bent into a circular section of radius $R = 1.89\,cm$. In unit-vector notation, what is the magnetic field at the center of curvature C if the circular section (a) lies in the plane of the page as shown and (b) is perpendicular to the plane of the page after being rotated 90° counterclockwise as indicated? **(5 points)**



**Solution:**

20. (a) The contribution to $B_C$ from the (infinite) straight segment of the wire is

$$B_{C1} = \frac{\mu_0 i}{2\pi R}.$$

The contribution from the circular loop is $B_{C2} = \frac{\mu_0 i}{2R}$. Thus,

$$B_C = B_{C1} + B_{C2} = \frac{\mu_0 i}{2R}\left(1+\frac{1}{\pi}\right) = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(5.78\times10^{-3}\,\mathrm{A})}{2(0.0189\,\mathrm{m})}\left(1+\frac{1}{\pi}\right) = 2.53\times10^{-7}\,\mathrm{T}.$$

$\vec{B}_C$ points out of the page, or in the $+z$ direction. In unit-vector notation, $\vec{B}_C = (2.53\times10^{-7}\,\mathrm{T})\hat{\mathrm{k}}$

(b) Now, $\vec{B}_{C1} \perp \vec{B}_{C2}$ so

$$B_C = \sqrt{B_{C1}^2 + B_{C2}^2} = \frac{\mu_0 i}{2R}\sqrt{1+\frac{1}{\pi^2}} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(5.78\times10^{-3}\,\mathrm{A})}{2(0.0189\,\mathrm{m})}\sqrt{1+\frac{1}{\pi^2}} = 2.02\times10^{-7}\,\mathrm{T}.$$

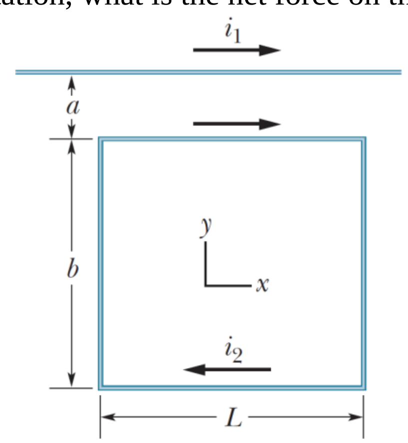
and $\vec{B}_C$ points at an angle (relative to the plane of the paper) equal to

$$\tan^{-1}\left(\frac{B_{C1}}{B_{C2}}\right) = \tan^{-1}\left(\frac{1}{\pi}\right) = 17.66°.$$

In unit-vector notation,

$$\vec{B}_C = 2.02\times10^{-7}\,\mathrm{T}\,(\cos 17.66°\,\hat{\mathrm{i}} + \sin 17.66°\,\hat{\mathrm{k}}) = (1.92\times10^{-7}\,\mathrm{T})\,\hat{\mathrm{i}} + (6.12\times10^{-8}\,\mathrm{T})\,\hat{\mathrm{k}}.$$

5. As shown in the figure, a long wire carries a current $i_1 = 30\ A$ and a rectangular loop carries current $i_2 = 20\ A$. Take the dimensions to be $a = 1\ cm$, $b = 8\ cm$, and $L = 30\ cm$. In unit vector notation, what is the net force on the loop due to $i_1$? **(5 points)**



**Solution:**

Let's break down the wire into 4 segments, two of which are parallel to the wire with $i_1$ and the other two perpendicular to it. The force on the parallel components can be found from:

$$F_{||} = \frac{\mu_0 i_1 i_2 L}{2\pi r}$$

where $r = a$ for the top segment and $r = a + d$ for the bottom one. But for the perpendicular segments, since the distance between every little segment of the wire ($d\vec{s}$) and the wire with $i_1$ varies, we have to calculate the force on each little segment and add them up:

$$F_{\perp \text{ sides}} = \int_a^{a+b} \frac{i_2 \mu_0 i_1}{2\pi y} dy.$$

Fortunately, these forces on the two perpendicular sides of length $b$ cancel out. For the remaining two (parallel) sides of length $L$, we obtain

$$F = \frac{\mu_0 i_1 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a+d}\right) = \frac{\mu_0 i_1 i_2 b}{2\pi a(a+b)}$$

$$= \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(30.0\mathrm{A})(20.0\mathrm{A})(8.00\mathrm{cm})(300\times10^{-2}\,\mathrm{m})}{2\pi(1.00\mathrm{cm}+8.00\mathrm{cm})} = 3.20\times10^{-3}\,\mathrm{N},$$

and $\vec{F}$ points toward the wire, or $+\hat{\mathrm{j}}$. That is, $\vec{F} = (3.20\times10^{-3}\,\mathrm{N})\hat{\mathrm{j}}$ in unit-vector notation.