2. (1 point) Consider the function $f(x) = x^2 e^{3x}$.

$f(x)$ has two inflection values at x = C and x = D with $C \leq D$
where C is ______
and D is ______
Finally for each of the following intervals, tell whether $f(x)$ is concave up (type in CU) or concave down (type in CD).
$(-\infty, C]$: ______
$[C, D]$: ______
$[D, \infty)$ ______

Correct Answers:

- `-1.1380711874577`
- `-0.195262145875635`
- `CU`
- `CD`
- `CU`

3. (1 point) Let $f(x) = -x^4 - 8x^3 + 8x + 6$. Find the open intervals on which f is concave up (down). Then determine the x-coordinates of all inflection points of f.

1. f is concave up on the intervals ______
2. f is concave down on the intervals ______
3. The inflection points occur at x = ______

Notes: In the first two, your answer should either be a single interval, such as (0,1), a comma separated list of intervals, such as (-inf, 2), (3,4), or the word "none".

In the last one, your answer should be a comma separated list of x values or the word "none".

Correct Answers:

- `(-4,0)`
- `(-infinity,-4), (0,infinity)`
- `0, -4`

4. (1 point) A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola $y = 10 - x^2$. What are the dimensions of such a rectangle with the greatest possible area?

Width = ______
Height = ______

Correct Answers:

- `3.65`
- `6.66667`

5. (1 point) A cylinder is inscribed in a right circular cone of height 7.5 and radius (at the base) equal to 6.5. What are the dimensions of such a cylinder which has maximum volume?

Radius = ______ Height = ______

Correct Answers:

- `4.33333`
- `2.5`

6. (1 point) If 1600 square centimeters of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

Volume = ______ (include **units**)

Solution:

Solution:

To solve this problem, we will need to write a formula for the volume of the box in terms of one of its dimensions, and then use derivatives to find the dimensions at which the box has a maximum volume. Let x be the length of the sides of the square base. Then, if h is the height of the box, the volume is given by $x^2 h$. We need to find an expression for the height h in terms of x.

This is where we use our information about the amount of material used in constructing the box. If the base of the box has sides of length x, then x^2 square centimeters of material are used to make the base. Therefore, we have only $1600 - x^2$ square centimeters of material left to make the sides, of which there are four. Each of the sides uses hx square centimeters of material. Therefore, we get the formula:

$$1600 - x^2 = 4(hx) \Rightarrow h = \frac{1600 - x^2}{4x}$$

Plugging this into our formula for volume, we can now write out $v(x)$ as:

$$v(x) = x^2 \left(\frac{1600 - x^2}{4x} \right) = \frac{1600x - x^3}{4}$$

Now, we take the derivative of this expression, using the rules for taking derivatives of polynomials, to get $v'(x) = \frac{1600}{4} - \frac{3}{4}x^2$. Setting this equal to 0 will give us the critical points. When solving, remember that this is a real world situation, so we can not have a negative value for x (which is a length).

$$\begin{array}{rcl} v'(x) & = & 0 \\ \frac{1600}{4} - \frac{3}{4}x^2 & = & 0 \\ \frac{3}{4}x^2 & = & \frac{1600}{4} \\ x^2 & = & \frac{1600}{3} \\ x & = & \sqrt{\frac{1600}{3}} \approx 23.09 \end{array}$$

Now, plugging this width into our formula for volume, $v(x)$, we get the maximal volume of $v(23.09) = !6158.4 \text{ cm}^3 : \%5.2f$.

Correct Answers:

- `6158.4 cm^3`

7. (1 point) A rancher wants to fence in an area of 1000000 square feet in a rectangular field and then divide it in half with a fence down the middle, parallel to one side.

What is the shortest length of fence that the rancher can use?

Length of fence = ________ feet.

Correct Answers:

- `4898.98`

8. (1 point) Find the point on the line $-4x+4y+5=0$ which is closest to the point $(3,-2)$.

Answer is ____________

Correct Answers:

- `(1.1,-0.12)`

9. (1 point) A fence 7 feet tall runs parallel to a tall building at a distance of 2 feet from the building.

What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?

Length of ladder = ________ feet.

Correct Answers:

- `12.0179`