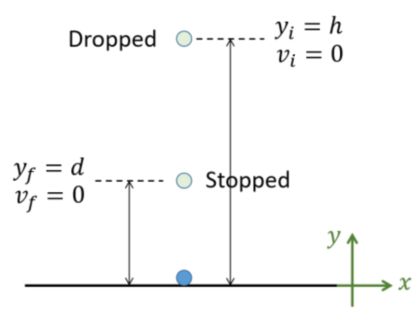
PHYS 205-Section 03
Electricity and Magnetism - Winter 2018
Assignment 2 – Solutions

Problems + Solutions

1. A particle of mass m and charge q is dropped from the height of h towards a point charge Q, which lies on the ground. What is the minimum distance between the charges? Your should be in terms of m, q, Q, h, g) **(5 marks)**
Hint: Using conservation of energy will make your life easier!

Solution:
The forms of energy involved in this problem are gravitations potential energy and electric potential energy. Since the charged particle is initially dropped and finally stops, the kinetic energy has no contribution to this problem. Considering the system to be the two charged particles and the Earth and applying the conservation of energy:

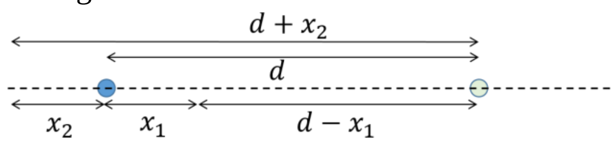


$$\Delta E = 0$$
$$\Delta U_g + \Delta U_e = 0$$
$$mg(d-h) + k_e qQ\left(\frac{1}{d} - \frac{1}{h}\right) = 0$$
$$mg(d-h) + k_e qQ\left(\frac{h-d}{dh}\right) = 0$$
$$(d-h)\left(mg - \frac{k_e qQ}{h}\frac{1}{d}\right) = 0$$
$$mg - \frac{k_e qQ}{h}\frac{1}{d} = 0$$
$$\boxed{d = k_e \frac{qQ}{mgh}}$$

2. Point charges Q and $-2Q$ are at a distance d. At what distances from Q is the electric potential zero? Only consider the points on the line that connects Q_1 and Q_2. **(5 marks)**

Solution:

Considering that electric potential is a scalar quantity, and considering that it is inversely proportional to the distance from the charge, we expect the electric potential to become zero at points (x_1 and x_2) closer to the smaller charge. Taking these points to be along the line that connects the two charges:



For x_1:

$$\frac{k_e Q}{x_1} - \frac{2k_e Q}{d - x_1} = 0$$

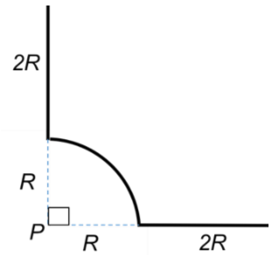
$$\frac{1}{x_1} = \frac{2}{d - x_1} \rightarrow \boxed{x_1 = \frac{d}{3}}$$

For x_2:

$$\frac{k_e Q}{x_2} - \frac{2k_e Q}{d + x_2} = 0$$

$$\frac{1}{x_2} = \frac{2}{d + x_2} \rightarrow \boxed{x_2 = d}$$

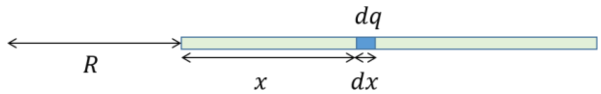
3. In the figure below, the rod is uniformly charged (λ). Find the electric potential at point P. **(5 marks)**



Solution:

We consider the rod as two straight segments and one curved (quarter of a circle). Then we calculate the electric potential of each segment and add them together.

Considering a uniform charge distribution, for the straight segments:



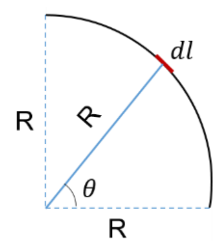
$$dV_1 = k_e \frac{dq}{r} = k_e \frac{dq}{R+x} = k_e \frac{\lambda dx}{R+x}$$

$$V_1 = \int_0^{2R} k_e \frac{\lambda dx}{R+x} = k_e \lambda \ln(3)$$

Since the electric potential is a scalar quantity and considering the symmetry, for the other straight segment we have:

$$V_2 = k_e \lambda \ln(3)$$

And for the curved segment:



$$dV_3 = k_e \frac{dq}{r} = k_e \frac{dq}{R}$$

$$V_1 = \frac{k_e}{R} \int dq = \frac{k_e Q_3}{R} = \frac{k_e}{R} \lambda \left(\frac{2\pi R}{4}\right) = \frac{k_e \lambda \pi}{2}$$

where Q_3 is the charge on the curved segment (quarter of a circle).

For the net electric potential we have:

$$\boxed{V_{net} = V_1 + V_2 + V_3 = k_e \lambda \left(2\ln(3) + \frac{\pi}{2}\right)}$$

4. We charge a 6 μF capacitor with potential difference of 100 V. We then disconnect it from the battery and connect it to an empty of charge capacitor of 4 μF. Find the charge on each capacitor. **(5 marks)**

Solution:

After the capacitors are connected, two concepts need to be considered here: conservation of charge and conservation of energy.
Since one capacitor is initially empty of charge, the net charge of the system before connection is the charge on the 6 μF capacitor:

$$Q_i = CV = (6 \times 10^{-6})(100) = 600\ \mu C$$

After connecting the capacitors, the charges will redistribute until the potential difference across the two capacitors becomes equal. Note that the net charge after connection should remain unchanged:

$$Q_f = Q_1 + Q_2 = Q_i = 600\ \mu C$$

$$V_1 = V_2 \rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \rightarrow Q_1 = \frac{C_1}{C_2}Q_2 = \frac{4}{6}Q_2$$

$$Q_1 + Q_2 = \frac{2}{3}Q_2 + Q_2 = \frac{5}{3}Q_2 = 600\ \mu C$$

$$\boxed{Q_2 = 360\ \mu C \text{ and } Q_1 = 240\ \mu C}$$

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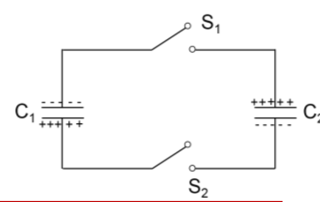
5. The capacitors of $C_1 = 2\ \mu F$ and $C_2 = 3\ \mu F$ are each fully charged with potential difference of 30 V. What is the charge on each capacitor if we close switches S_1 and S_2 simultaneously? **(5 marks)**



Solution:

This is very similar to the previous problem, except that both capacitors are charged and are connected with opposite polarity:

$$Q_i = Q_2 - Q_1 = C_2V - C_1V = (C_2 - C_1)V = 30\ \mu C$$

After connection:

$$Q_f = Q_1 + Q_2 = Q_i = 30\ \mu C$$

$$V_1 = V_2 \rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \rightarrow Q_1 = \frac{C_1}{C_2}Q_2 = \frac{2}{3}Q_2$$

$$Q_1 + Q_2 = \frac{2}{3}Q_2 + Q_2 = \frac{5}{3}Q_2 = 30\ \mu C$$

$$\boxed{Q_2 = 18\ \mu C \text{ and } Q_1 = 12\ \mu C}$$

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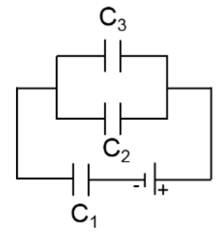
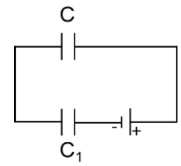
6. In the configuration below, the battery is 50 V. Find the charge and energy stored on capacitor C_2.
$C_1 = 2\ \mu F$, $C_2 = 3\ \mu F$, $C_3 = 4\ \mu F$ **(5 marks)**



Solution:

The first step is to find the total charge stored in the system, by finding the equivalent capacitance. We'll do it in two steps:

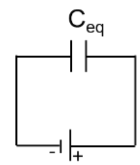
1. Capacitors C_1 and C_2 are connected in parallel:



$$C = C_2 + C_3 = 7\ \mu F$$

2. Capacitors C_1 and C are connected in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C} \rightarrow C_{eq} = \frac{(2)(7)}{(2+7)} = \frac{14}{9}\ \mu F$$



Total charge:

$$Q_{tot} = C_{eq}V = \left(\frac{14}{9}\right)(50) = \frac{700}{9}\ \mu C$$

Now we go backwards, step by step, towards the original circuit. Since C_1 and C are connected in series, then:

$$Q_1 = Q = Q_{tot} = \frac{700}{9}\mu C$$

Knowing Q, and the fact that C_2 and C_3 are connected in parallel:

$$V_2 = V_3 \rightarrow \frac{Q_2}{C_2} = \frac{Q_3}{C_3} \rightarrow Q_3 = \frac{C_3}{C_2} Q_2$$

$$Q_2 + Q_3 = Q$$

$$\left(\frac{C_3}{C_2} + 1\right) Q_2 = Q \rightarrow \boxed{Q_2 = \left(\frac{700}{9}\right)\left(\frac{3}{7}\right) = \frac{100}{3}\ \mu C}$$

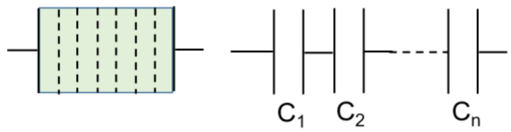
To find the energy stored in C_2:

$$\boxed{U = \frac{Q^2}{2C_2} = 185.2\ \mu J}$$

Bonus: In a parallel plate capacitor, the dielectric constant linearly changes from 1.5 ϵ_0 on one plate to 3 ϵ_0 on the other plate. If the surface area of the plates is A and the distance between the plates is d, what is the capacitance in terms of $\epsilon_0 \frac{A}{d}$? **(0 marks)** 😉

Solution:

Since the dielectric constant continuously changes along the length of the dielectric, we should break it into many pieces such that the dielectric constant of each piece can be considered to remain constant. This can be done by inserting many thin conducting sheets in between the plates of the parallel plate capacitor, without changing the total capacitance (see Example 26.7 of the textbook). Doing so, we'll have many capacitors that are connected in series:



Each capacitor will have a dielectric with a different constant, depending on its position:

$$C = \frac{A\epsilon(x)}{dx}$$

Where dx is separation between the plates of each capacitor. Since these capacitors are connected in series:

$$\frac{1}{c_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{C_i} = \frac{1}{A}\int_0^d \frac{dx}{\epsilon(x)}$$

Since $\epsilon(x)$ changes linearly from $1.5\epsilon_0$ to $3\epsilon_0$ along d, hence:

$$\epsilon(x) = \frac{1.5\epsilon_0}{d}x + 1.5\epsilon_0$$

Hence we'll have:

$$\frac{1}{c_{eq}} = \frac{1}{A}\int_0^d \frac{dx}{\frac{1.5\epsilon_0}{d}x + 1.5\epsilon_0} = \left(\frac{1}{A}\right)\left(\frac{d}{1.5\epsilon_0}\right)\ln(2)$$

$$\boxed{C_{eq} = 1.5\left(\frac{\epsilon_0 A}{d}\right)\left(\frac{1}{\ln(2)}\right)}$$