ENGR 371 August 1992 Final

1. (11 Marks)

A random sample of measured values of diameter of the pipe consists of:

$$X_i = \{1.01; 1.03; 1.04; 0.97; 0.98; 0.99; 0.99; 1.01; 1.03; 1.01\}cm$$

1) Find the following statistics: a. Sample Mean; b. Sample Median; c. Sample Mode; d. Sample Variance.
2) Using Chebyshev's Theorem, calculate value of x if

$$P(\overline{X} - x \leq X_i \leq \overline{X} + x) \geq 75\ \%$$

3) Find 95 % confidence interval of the standard deviation, assuming a normal distribution of X_i
4) Find 95 % confidence interval of the mean sample and the maximum value of the error.

2.(11 Marks)

Suppose that X has probability density function (pdf):

$$f_X(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0 & elsewhere \end{cases}$$

1) Find Moment Generating Function $M_X(t)$
2) Using $M_X(t)$, find mean value and variance of X

Let random variable X be transformed to a new variable Y:

$$Y = \frac{1}{(1+X)^2}$$

3) Find and sketch the probability density function $f_Y(y)$

3. (10 Marks)

20 % of the delegates on a meeting are electronic engineers. Of these 75 % gave talks. Of those who are not electronic engineers, 20 % gave talks.
What is the probability that randomly selected lecturer is an electronic engineer?

4. (11 Marks)

Suppose that a random variable X can take only 4 values: -1; 0; 1; 2 with the same probability. Let Y be defined by the relation: $Y = X^2$.

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1) Find and sketch probability distributions $p(x_i)$ and $p(y_i)$
2) Find mean values and variances of X and Y
3) Find correlation coefficient r_{XY}.

5. (11 Marks)

Consider the following probability density function:

$$f_X(x) = \begin{cases} kx & 0 \le x < a \\ k(2a - x) & a \le x < 2a \\ 0 & elsewhere \end{cases}$$

1) Find value of k
2) Find mean and variance of X
3)Find and sketch the commulative distribution function $F_X(x)$

6. (11 Marks)

Suppose that A, B, ϕ are statistically independent random variables;

A has one-sided exponential pdf with the unit mean,

B has a normal pdf with mean equals 2 and variance equals 4,

ϕ is uniformly distributed over the range $0 \le \phi \le \pi$.

The considered random variables are parameters of a random process:

$$Y(t) = A + B\cos(\omega t - \phi)$$

where ω is a constant value.

1) Find mean value of $Y(t)$
2) Find autocorrelation function $R_{XX}(t, \tau)$
3) Is $Y(t)$ a stationary process? Justify your answer.

Concordia UNIVERSITY

Course	Number	Section
Probability and Statistics in Engineering	ENGR 371	CA

Examination	Date	Time	# of pages
Final Examination	August 2000	3 Hours	8

Instructor(s)
Dr. M. Kahrizi

Materials allowed: [X] No [ ] Yes (Please specify)

Calculators allowed: [ ] No [X] Yes

No materials allowed.

Special Instructions:

Attempt all questions. Please number and begin each question on a new page.
Show all steps clearly in neat and legible handwriting (preferably in ink).
Students are required to return question paper together with exam booklet(s).

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Department of Electrical and Computer Engineering
1455 de Maisonneuve Blvd West, Montréal, Québec, Canada H3G 1M8 Fax: (514) 848-2802

1. A particular type of tennis racket comes in a midsize version and an oversize version. Sixty percent (60%) of all customers at a certain store want the oversize version.

   (a) Among ten randomly selected customers, what is the probability that the number who want the oversize version is within one standard deviation of the mean value?

   (b) Among ten randomly selected customers who want this type of racket, what is the probability that at least six want the oversize version?

   (c) The store currently has seven rackets of each version. What is the probability that all of the next ten customers who want this racket can get the version they want from current stock?

2. The Faculty of Engineering of a large university is designing a placement test to be given to the incoming freshman classes. Members of the faculty feel the average grade for this test will vary from one freshman class to another. This variation of the average class grade is expressed subjectively by a normal distribution with mean $\mu = 72$ and variance $\sigma^2 = 5.76$.

   (a) What prior probability does the faculty assign to the actual average grade being somewhere between 71.8 and 73.4 for next year's freshmen class?

   (b) If the test is tried on a random sample of 100 freshman students from the next incoming freshman class resulting in an average grade of 70 with a variance of 64, construct a 95% Bayesian interval for μ.

   (c) What posterior probability should the faculty assign to the event of part (a)?

3. If S_1^2 and S_2^2 represent the variances of independent random samples of size $n_1 = 8$ and $n_2 = 12$ taken from normal populations with equal variances, find the $P(S_1^2/S_2^2 < 4.89)$.

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4. Measured values of diameters of 10- pipes manufactured by a plant are:

$$S_i = \{1.01;\ 1.03;\ 1.04;\ 0.97;\ 0.98;\ 0.99;\ 0.99;\ 1.01;\ 1.03;\ 1.01\}$$ done

   (a) Find the following statistics: sample mean; sample median; sample mode; sample variance.

   (b) Assuming the population of pipes is normal, find 95% confidence intervals of:
      (i) The standard deviation.
      (ii) The mean sample and the maximum error in the estimate of mean sample.

5. The lifetime in weeks of a certain type of transistor is known to follow a normal distribution with mean 10 weeks and standard deviation $\sqrt{50}$ weeks.

   (a) What is the probability that the transistor will last at most 50 weeks?
   (b) What is the probability that the transistor will not survive the first 10 weeks?

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Concordia
UNIVERSITY

Course	Number	Section	
Probability and Statistics	ENGR 371		
Examination	**Date**	**Time**	**# of pages**
FINAL	April 2000	3	

Instructor(s)

Dr. E. Plotkin

Materials allowed: [X] No [ ] Yes (Please specify)

Calculators allowed: [ ] No [Y] Yes

Special Instructions:

-

All questions carry equal weight.

Q1.

Measured values of diameters of 10 pipes manufactured by the plant are:

$$X_i = \{1.01; 1.03; 1.04; 0.97; 0.98; 0.99; 0.99; 1.01; 1.03; 1.01\} cm$$

1) Find the following statistics: a. Sample Mean; b. Sample Median; c. Sample Mode; d. Sample Variance.

2) Assuming that the population of pipes is normal, find 95 % confidence intervals of a) the standard deviation σ_X and b) the mean sample μ_X, and the maximum error in the estimate of μ_X.

3) Using Chebyshev's Theorem, calculate value of x if

$$P(\bar{X} - x \leq X_i \leq \bar{X} + x) \geq 75\%$$

Hint: Use the point estimate of σ_X^2.

Q2.

20 % of the delegates on a meeting are electronic engineers. Of these 75 % gave talks. Of those who are not electronic engineers, 20 % gave talks.

What is the probability that randomly selected lecturer is an electronic engineer?

Hint: Use the Bayes' Theorem.

P.S.

NAME [redacted] ID [redacted]

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CONCORDIA UNIVERSITY

COURSE	**Probability and Statistics in Engineering**	NUMBER	**ENGR 371**	SECTION	S, U, V		
EXAMINATION	**FINAL EXAM**	DATE	Dec. 19, 2001	TIME	3 hours	# OF PAGES	8
INSTRUCTOR	Dr. A.K. Athienitis (Coord.), Dr. P. Saathoff, Dr. I. Stateikina, Dr. I. Diouf						
MATERIALS ALLOWED:	Calculator						
SPECIAL INSTRUCTIONS:	Answer **all** problems. Each problem is worth 10 marks. State all assumptions. Return exam paper.						

1. The concentration of a pollutant in a lake was measured at 30 locations. The sample mean concentration was 2.52 mg/l and the sample standard deviation was 2.05 mg/l.

   (a) Find the 99% confidence interval for the population mean.
   (b) Find the 95% confidence interval for the population variance.

2. We want to destroy a target and we have 5 missiles. The target will be destroyed only if it is hit at least 3 times.

   (a) Find the probability that the target will be destroyed, given that the probability of a successful hit is 0.3.
   (b) Solve this problem using the Poisson distribution.
   (c) Discuss the validity of using the Poisson distribution for this problem.

3. The burning rates of two different solid-fuel rocket propellents are being studied. It is known that both propellants have the same standard deviations of burning rate : $\sigma_1 = \sigma_2 = 3$ cm/s. Two random samples of $n_1 = 20$ and $n_2 = 20$ specimens are tested and the sample mean burning rates are $\overline{x}_1 = 24$ cm/s and $\overline{x}_2 = 18$ cm/s.

(a) Construct a 99% confidence interval for the mean difference in burning rate

(b) Suppose that the sample means $\overline{X}_1$, $\overline{X}_2$ are unknown random variables while population means are:

$$\mu_1 = \mu_{\overline{X}_1} = 20, \qquad \mu_2 = \mu_{\overline{X}_2} = 19.5.$$

Find $P[(\overline{X}_1 - \overline{X}_2) < 1]$

4. The voltage output of a device is a random variable (X) and is uniformly distributed over the interval (-10, 10). This voltage is applied to a nonlinear device with the transfer characteristic:

$$Y = 10 + 5e^{-x/10}$$

(a) Find the mean value of Y

(b) Find the probability density function f(y)

(c) Sketch the probability density functions f(x) and f(y)

5. We have two coins; the first is fair and the second has two heads. We pick one of the coins at random. toss it twice and heads are obtained both times. Find the probability that the coin picked is fair.

6. Given the following joint pdf of X and Y:

$$f(x,y) = \begin{cases} x^2 + xy/3 & 0<x<1,\ 0<y<2 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Determine $P(x > 0, y > 1)$.

(b) find the marginal distribution of y.

(c) Determine $P(x > 0.5 \mid y=2)$.

Table A.3 (*continued*) Areas Under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TRIGONOMETRIC IDENTITIES

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y) \quad \text{(C-1)}$$

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y) \quad \text{(C-2)}$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp\sin(x) \quad \text{(C-3)}$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm\cos(x) \quad \text{(C-4)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) \quad \text{(C-5)}$$

$$\sin(2x) = 2\sin(x)\cos(x) \quad \text{(C-6)}$$

$$2\cos(x) = e^{jx} + e^{-jx} \quad \text{(C-7)}$$

$$2j\sin(x) = e^{jx} - e^{-jx} \quad \text{(C-8)}$$

$$2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y) \quad \text{(C-9)}$$

$$2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y) \quad \text{(C-10)}$$

$$2\sin(x)\cos(y) = \sin(x-y) + \sin(x+y) \quad \text{(C-11)}$$

$$2\cos^2(x) = 1 + \cos(2x) \quad \text{(C-12)}$$

$$2\sin^2(x) = 1 - \cos(2x) \quad \text{(C-13)}$$

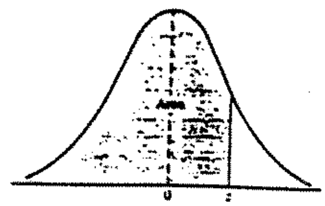

Table A.3 Areas Under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

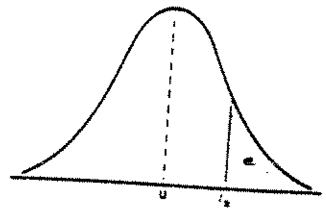

Table A.4* Critical Values of the *t*-Distribution

ν	α				
	0.10	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
inf.	1.282	1.645	1.960	2.326	2.576

* From Table IV of R. A. Fisher, *Statistical Methods for Research Workers*, published by Oliver & Boyd, Edinburgh, by permission of the author and publishers.

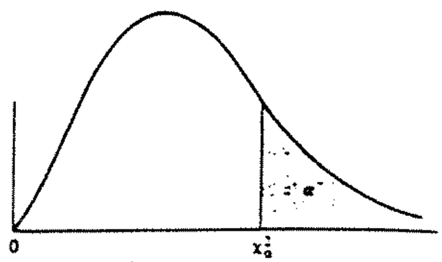

Table A.5 Critical Values of the Chi-Squared Distribution

v	α = .995	.99	.98	.975	.95	.90	.80	.75	.70	.50
1	$.0^4393$	$.0^3157$	$.0^3628$	$.0^3982$	.00393	.0158	.0642	.102	.148	.455
2	.0100	.0201	.0404	.0506	.103	.211	.446	.575	.713	1.386
3	.0717	.115	.185	.216	.352	.584	1.005	1.213	1.424	2.366
4	.207	.297	.429	.484	.711	1.064	1.649	1.923	2.195	3.357
5	.412	.554	.752	.831	1.145	1.610	2.343	2.675	3.000	4.351
6	.676	.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.036	11.721	14.339

v	α = .30	.25	.20	.10	.05	.025	.02	.01	.005	.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465
5	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697

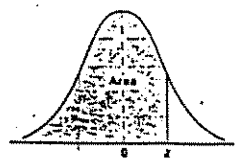

Table A.3 ***Areas Under the Normal Curve***

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A.3 (continued) ***Areas Under the Normal Curve***

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

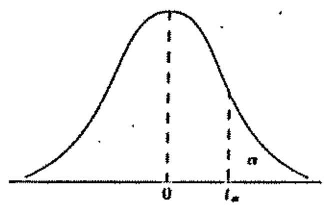

Table A.4 Critical Values of the t-Distribution

	α						
v	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

l Tables

Table A.4 (continued) Critical Values of the t-Distribution

	α						
v	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.125	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291

13

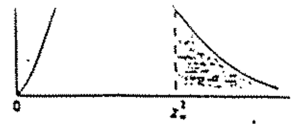

Table A.5 Critical Values of the Chi-Squared Distribution

v	α									
	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.75	0.50
1	0.0^4393	0.0^3157	0.0^3628	0.0^3982	0.00393	0.0158	0.0642	0.102	0.148	0.455
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.575	0.713	1.386
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.036	11.721	14.339
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337
23	9.260	10.196	11.293	11.688	13.091	14.848	17.187	18.137	19.021	22.337
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336

Table A.5 (continued) Critical Values of the Chi-Squared Distribution

v	α 0.025 √									
	0.30	0.25	0.20	0.10	0.05	~~0.25~~	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465
5	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620
26	29.246	30.434	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052
27	30.319	31.528	32.912	36.741	40.113	43.194	44.140	46.963	49.645	55.476
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.893
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.302
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703

Probability and statistics

Week 6.

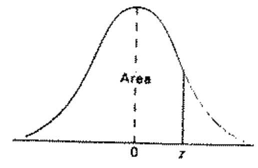

Table A.3 Areas Under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A.3 (continued) Areas Under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

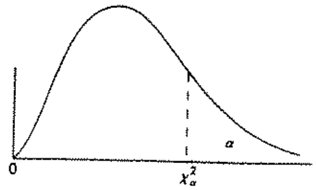

Table A.5 Critical Values of the Chi-Squared Distribution

v	α 0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.75	0.50
1	0.0^4393	0.0^3157	0.0^3628	0.0^3982	0.00393	0.0158	0.0642	0.102	0.148	0.455
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.575	0.713	1.386
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.036	11.721	14.339
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337
23	9.260	10.196	11.293	11.688	13.091	14.848	17.187	18.137	19.021	22.337
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336

Table A.5 (continued) Critical Values of the Chi-Squared Distribution

0.025

v	α 0.30	0.25	0.20	0.10	0.05	0.25	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465
5	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620
26	29.246	30.434	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052
27	30.319	31.528	32.912	36.741	40.113	43.194	44.140	46.963	49.645	55.476
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.893
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.302
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703

$$J = \begin{vmatrix} \partial x_1/\partial y_1 & \partial x_1/\partial y_2 \\ \partial x_2/\partial y_1 & \partial x_2/\partial y_2 \end{vmatrix}$$

Moment generating functions $M_X(t) = E(e^{Xt})$

$\frac{\partial^r M_X(t)}{dt^r}\Big|_{t=0} = E(X^r)$. If $Y = X_1 + X_2$ (X_1 and X_2 are independent).

$M_X(t) = M_{X_1}(t)M_{X_2}(t)$. Shows sums of normal are normal and sums of Poisson are Poisson.

Sampling $X_1, X_2, \dots X_n$ independent samples from a population with mean μ and variance σ^2.

Sample mean $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, sample variance $= \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$.

$E(\bar{X}) = \mu$ (unbiased estimator). $\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} \to 0$ for $n \to \infty$

$\bar{X}$ is normal if population normal or if n large > 30.

$z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ — standard normal.

$\chi = \frac{(n-1)s^2}{\sigma^2}$ — Chi-square with $\nu = n-1$ degrees of freedom.

$T = \frac{\bar{X}-\mu}{s/\sqrt{n}}$ — t-distribution $\nu = n-1$ degrees of freedom.

Confidence intervals for mean of population,

$P(\bar{X} - z_{\alpha/2}\sigma/\sqrt{n} \le \mu \le \bar{X} + z_{\alpha/2}\sigma/\sqrt{n}) = 1-\alpha$, σ known.

$P(\bar{X} - t_{\alpha/2}s/\sqrt{n} \le \mu \le \bar{X} + t_{\alpha/2}s/\sqrt{n}) = 1-\alpha$, σ unknown.

Estimation error: $e \le z_{\alpha/2}\sigma/\sqrt{n}$.

Difference of the means

$$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

(σ_1^2 and σ_2^2 known)

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\ s_p$$

$\nu = n_1 + n_2 - 2$ ($\sigma_1^2 = \sigma_2^2$ but unknown)

Confidence interval for variance of population

$$P\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right) = 1-\alpha$$

Confidence interval for the ratio of two variances of two populations

$$P\left(\frac{s_1^2}{s_2^2}\frac{1}{f_{\alpha/2}(\nu_1,\nu_2)} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2} f_{\alpha/2}(\nu_2,\nu_1)\right) = 1-\alpha$$

Bayesian Methods of Estimation

mean & variance of posterior distribution

$$\mu^* = \frac{n\bar{x}\sigma_0^2 + \mu_0\sigma^2}{n\sigma_0^2 + \sigma^2}, \quad \sigma^* = \sqrt{\frac{\sigma_0^2\sigma^2}{n\sigma_0^2 + \sigma^2}}$$

$$\mu^* - z_{\alpha/2}\sigma^* < \mu < \mu^* + z_{\alpha/2}\sigma^*$$

Name: Class No.:

Concordia University
Department of Electrical and Computer Engineering

Final Examination: April 19, 2001 Probability ENGR 371/4 Time Allowed : 3 Hours

Examiners : Dr. Mumtaz B. Gawargy Section Y
Dr. Mary Danial Section W
Dr. Irina Stateikina Section T

Important: 1. *Write your class number, ID, and section followed by your last name on each book used*
2. *Start each question or part of a question on a new page*
3. *Use a pen. If you use a pencil, push hard on the pencil*
4. *Highlight your answers*

Question 1 (12 marks)

a) We have two coins; the first is fair and the second has two heads. We pick one of the coins at random, we toss it twice and heads shows both times. Find the probability that the coin picked is fair.

b) Two random variables X and Y have $E(X) = 2$, $E(Y) = 0$, $\sigma_x = 1$ and $\sigma_y = 2$, correlation coefficient between x and y, $\rho_{xy} = 0.5$.
If $Z = Y - X - b$. Find the minimun value of $E(Z^2)$.
Hint: Find the coefficient b to minimize $E(Z^2)$.

Question 2 (10 marks)

a) Given the probability density function $f(x) = (1/900)\, e^{(-x/900)}$, $x>0$ Demonstrate that Chebyshev's theorem holds for $k = 2$.

b) From 4 red, 5 green, and 6 yellow apples, how many selections of 9 apples are possible if 3 of each colour to be selected.

Question 3 (10 marks)

a) From a lot of ten missiles, four are selected at random and fired. If the lot contains three defective missiles that will not fire, what is the probability, that all four will fire? At most two will fire ?

b) A company generally purchase large lots of certain kind of electronic devices, a method is used that rejects a lot if two or more defective units are found in a random sample of hundred units. What is the probability of accepting a lot that is 5% defective?

Question 4 (12 marks)

a) Statistics released by the national highway traffic safety administration and national safety council show that on average weekend night, one out of every ten drivers on the road is drunk. If 400 drivers are randomly checked next Saturday night, what is the probability that the number of drunk drivers will be at least 35 but less than 47? Hint: Use normal approximation to the binomial.

b) During a laboratory experiment the average number of radioactive particles passing through a counter in one millisecond is four. What is the probability that six particles enter the counter in a given millisecond. (Don't use Poissons Tables).

4. Given $X_i = \{1.01, 1.03, 1.04, .97, .98, .99, .99, 1.01, 1.03, 1.01\}$

   a) Find 95% confidence interval of the standard deviation assuming a normal distributions of X_i.

   b) Find 95% confidence interval of the mean sample and the maximum value of the error.

   c) Use Chebyshev's theorem to find x if

   $P(\bar{x} - x \leq x_i \leq \bar{x} + x) \geq \frac{8}{9}$

5. a) An electrical firm manufactures light bulbs that have a length of life that is normally distributed with mean equal to 1000 hours and a standard deviation of 50 hours.

   Find the probability that a bulb burns between 900 and 1200 hours.

   b) If observations X are normally distributed, what percent of these differ from the mean by more than 1.5σ.

6. For the given process $Y(t) = A+B \cos(wt + \theta)$, where w is a constant.

   A, B and θ are statistically independent random variables.

   A is uniformly distributed over the range $-10 < A < 10$

   B has a normal pdf with mean $\mu_B = 5$ and variance $\sigma_B^2 = 25$

   θ is uniformly distributed between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$

   Find
   a) mean value of Y(t)
   b) Variance of Y(t)
   c) is Y(t) a stationary process? Justify your answer.

Question 5 (12 marks)

a) Let X_1 and X_2 be two random variables with joint probability function that only exist between $x_1 = 0$ and $x_1 = 1$ for X_1 and between $x_2 = 0$ and $x_2 = 1$ for X_2 and $f(x_1,x_2) = 4x_1x_2$ in these intervals. Find the joint probability distribution of $Y_1 = X_1^2$ and $Y_2 = X_1 X_2$, showing the region of Y_1, Y_2.

b) Find the moment generating function of random variable X having a Chi-Squared distribution with degrees of freedom = 2.

Question 6 (10 marks)

a) Find the probability that a random sample of 11 observations from a normal population with variance $\sigma^2 = 5$ will have a variance s^2 between 2.789 and 11.309. Assume the sample variance to be continuos measurements.

b) Evaluate $\Gamma(0.5)$ from first principal, and use it to find $\Gamma(2.5)$.

hyper geometri

3 Sample Final Exams. For ENGR 371 1.20

Concordia UNIVERSITY

Course	Number	Section
Probability and Introduction to Random Processes in Engineering	ENGR 371	CB

Examination	Date	Time	# of pages
Final	July 1993	3 hours	6

Instructor(s)

Dr. Mumtaz B. Gawargy

Materials allowed: [ ] No [X] Yes (Please specify) -ONE $8\frac{1}{2}$" x 11" size sheet with formulae and other notes (two sides)

Calculators allowed: [ ] No [X] Yes

Special Instructions:

1. 30% of the delegates on a meeting are electronic engineers. Of these, 80% gave talks. Of these who are not electronic engineers, 15% gave talks. What is the probability that a randomly selected lecturer is an electronic engineer?

2. Two random variables have the joint probability density function given by

$$f(x,y) = \begin{cases} K(x^2+y^2) & 0<x<2,\ 1<y<4 \\ 0 & \text{else where} \end{cases}$$

   a) Find K
   b) Find $P(1<x<2\ ,\ 2<y<3)$
   c) $P(1<x<2)$
   d) $P(x+y>4)$

3. a) Given $f(x) = \begin{cases} e^{-x} & x>0 \\ 0 & \text{elsewhere} \end{cases}$

   Find the probability density function of Y

   where $Y = \dfrac{3}{(1+x)^2}$

   b) Check your answer by finding $\int f(y)\,dy$

$P(E) = 0.3$ $P(E') = 0.7$

$P(T) = 0.8$

$P(T \cap E') = 0.15$

$P(E|T) =$

Summary of formulae

S-space of experimental outcomes. Events are subsets of S, and $P(A)$ measures of A. $P(A') = 1 - P(A)$. If $A \cap B = \emptyset$ (disjoint events), $P(A \cup B) = P(A) + P(B)$. In general $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Calculation of probability by counting $P(A) = n_A/n$; n = total number of outcomes, n_A number of ways for A, if all outcomes equiprobable. Choose k from n with order $\frac{n!}{(n-k)!}$ without order $\binom{n}{k}$.

Conditional probability $= P(A/B) = \frac{P(A \cap B)}{P(B)}$; If A and B independent $P(A \cap B) = P(A)P(B)$. Sets $B_1, B_2 \ldots, B_n$ partition the space if $B_i \cap B_j = \emptyset$, $i \neq j, \bigcup_{i=1}^{n} B_i = S$.

Law of total probability $P(A) = \sum_{i=1}^{n} P(A \cap B_i)$

Bayes' formula $P(B_r/A) = P(B_r)P(A/B_r)/\sum_{i=1}^{n} P(B_i)P(A/B_i)$

Random Variables discrete case — $f(x) = P(X = x)$, continuous case — $f(x)\Delta x = P(x < X \leq x + \Delta)$ or $P(a < X \leq b) = \int_a^b f(x)dx$

Joint RV's discrete $f(x,y) = P(X = x, Y = y)$, continuous $f(x,y)\Delta x \Delta y = P(x < X \leq x + \Delta x, y < Y \leq y + \Delta y), P(X, Y \in A) = \int_A dx \int dy f(x,y)$

Marginal distribution $g(x) \begin{cases} = \sum_y f(x,y) & \text{discrete} \\ = \int_{-\infty}^{\infty} f(x,y) & \text{continuous} \end{cases}$

conditional distribution $f(y/x) = f(x;y)/g(x)$

Expectation

$$\mu = E(X) = \begin{cases} \sum_x f(x)x \\ \int_{-\infty}^{\infty} x f(x)dx \end{cases} ; Y = g(X), E(Y) = \begin{cases} \sum g(x)f(x) \\ \int_{-\infty}^{\infty} g(x)f(x)dx \end{cases} ;$$

Variance $\sigma_x^2 = E((X-\mu)^2) = E(X^2) - \mu^2$,

Covariance $\sigma_{XY} = [E(X-\mu)(Y-\mu)] = E(XY) - \mu_X\mu_Y$ If X and Y independent $\sigma_{XY} = 0$

$Z = aX + \beta Y + c, E(Z) = aE(X) + \beta E(Y) + c, \sigma_Z^2 = a^2\sigma_X^2 + \beta^2\sigma_Y^2$

Chebychev Theorem $[P[\mu - k\sigma < x \leq \mu + k\sigma] \geq 1 - 1/k^2$.

Binomial distribution $b(x;n,p) = \binom{n}{x} p^x(1-p)^{n-x}; x = 0, 1, \ldots, n$

$E(X) = np, \sigma_X^2 = np(1-p)$. Let $n \to \infty, p \to 0, np \to \mu$.

Poisson distribution $P(x;\mu) = \mu^x e^{-\mu}/x!;\quad x = 0, 1, \ldots$

$E(X) = \mu, \sigma_X^2 = \mu$ "average is what counts"

normal distribution; $f_X(x) = \exp(-(x-\mu)^2/2\sigma^2)/\sqrt{2\pi}\sigma; -\infty < x < \infty$

$E(X) = \mu, \sigma_X^2 = \sigma^2$

Central limit theorem — sums of RVs tend toward normal distribution.

Exponential distribution $f(x) = \frac{1}{\beta}e^{-x/\beta}; x > 0,\quad E(X) = \beta,\quad \sigma_X^2 = \beta^2$

One-to-one transformations $Y = g(X), X = w(Y)$

discrete $f_Y(y) = f(Y = y) = P(X = w(y)) = f_X(w(y))$

continuous $f_Y(y) = f_X(w(y)) \left|\frac{dw(y)}{dy}\right|$

Transformation of two variables