QUESTION 1: (30 pts)

$f(x) = 25x^3 - 6x^2 + 7x - 88$ $\quad f(1) = -62$

$f'(x) = 75x^2 - 12x + 7$ $\quad f'(1) = 70$

$f''(x) = 150x - 12$ $\quad f''(1) = 138$

$f'''(x) = 150$ $\quad f'''(1) = 150$

$f^{IV}(x) = 0$ $\quad f^{IV}(1) = 0$

a)

1st order Taylor

$f(x) = f(1) + f'(1)\frac{h}{1!}$ where $h = x - 1$

$f(x) = -62 + 70(x-1)$

2nd order Taylor

$f(x) = f(1) + f'(1)\frac{h}{1!} + f''(1)\frac{h^2}{2!}$

$= -62 + 70(x-1) + 69(x-1)^2$

3rd order Taylor

$f(x) = f(1) + f'(1)\frac{h}{1!} + f''(1)\frac{h^2}{2!} + f'''(1)\frac{h^3}{3!}$

$= -62 + 70(x-1) + 69(x-1)^2 + 25(x-1)^3$

b)

1st order $f(3) \approx -62 + 70(3-1) = 78$

2nd order $f(3) \approx -62 + 70(2) + 69(2)^2 = 354$

3rd order $f(3) \approx -62 + 70(2) + 69(2)^2 + 25(2)^3 = 554$

c)

True solution $f(3) = 35(3)^3 - 6(3)^2 + 7(3) - 88 = 554$

$$\varepsilon_{t_1} = \left|\frac{554 - 78}{554}\right| \times 100 = 85.9\%$$

$$\varepsilon_{t_2} = \left|\frac{554 - 354}{554}\right| \times 100 = 36.1\%$$

$$\varepsilon_{t_3} = \left|\frac{554 - 554}{554}\right| \times 100 = 0\%$$

QUESTION 2: (40 pts)

a)

$$\begin{bmatrix} 10 & 2 & -1 \\ -3 & -6 & 2 \\ 1 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 27 \\ -61.5 \\ -21.5 \end{bmatrix}$$

b)

$$Cond(A) = \|A\|_\infty \cdot \|A^{-1}\|_\infty$$

$\|A\|_\infty$ = max row sum = $\max\{13, 11, 7\} = 13$.

$\|A^{-1}\|_\infty = \max\{0.155709, 0.294119, 0.224914\} = 0.294119$.

$Cond(A) = (13)(0.294119) = 3.823547 \longrightarrow$ Well Conditioned

c)

$$\begin{bmatrix} 10 & 2 & -1 \\ -3 & -6 & 2 \\ 1 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & 2 & -1 \\ -3 & -6 & 2 \\ 1 & 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -0.3 & 1 & 0 \\ 0.1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & 2 & -1 \\ 0 & -5.4 & 1.7 \\ 0 & 0.8 & 5.1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -0.3 & 1 & 0 \\ 0.1 & -0.148148 & 1 \end{bmatrix} \begin{bmatrix} 10 & 2 & -1 \\ 0 & -5.4 & 1.7 \\ 0 & 0 & 5.351852 \end{bmatrix}$$

d)

Define $\{z\}$ such that $[U]\{z\} = \{z\}$ & $[L]\{z\} = \{b\}$

$$\begin{bmatrix} 1 & 0 & 0 \\ -0.3 & 1 & 0 \\ 0.1 & -0.148148 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 27 \\ -61.5 \\ -21.5 \end{bmatrix}$$

$z_1 = 27$

$-0.3(27) + z_2 = -61.5 \rightarrow z_2 = -53.4$

$0.1(27) - 0.148148(-53.4) + z_3 = -21.5$

$\hookrightarrow z_3 = -32.111$

$$\begin{bmatrix} 10 & 2 & -1 \\ 0 & -5.4 & 1.7 \\ 0 & 0 & 5.351852 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 27 \\ -53.4 \\ -32.111 \end{bmatrix}$$

$$x_3 = \frac{-32.111}{5.351852} = -5.999979$$

$-5.4\, x_2 + 1.7(-5.999979) = -53.4 \rightarrow x_2 = 8.0000066$

$10\, x_1 + 2(8.0000066) - 1(-5.999979) = 27 \rightarrow x_1 = 0.4999889$

e) $[A]\{x\} = \begin{Bmatrix} 26.9998812 \\ -61.4999643 \\ -21.4998995 \end{Bmatrix}$

QUESTION 3: (30 pts)

a)

$f'(x) = 5.5 - 8x + 1.5x^2$

Newton Raphson $x_{i+1} = x_i - \dfrac{-1 + 5.5x_i - 4x_i^2 + 0.5x_i^3}{5.5 - 8x_i + 1.5x_i^3}$

$x_0 = 6$

it # 1

$$x_1 = 6 - \frac{(-1) + 5.5(6) - 4(6)^2 + 0.5(6)^3}{5.5 - 8(6) + 1.5(6)^3} = 6.347826$$

$$\varepsilon_a = \left|\frac{x_1 - x_0}{x_1}\right| \times 100$$

$\varepsilon_{a_1} = 5.47945\%$

it # 2

$x_2 = 6.306535$ $\varepsilon_{a2} = 0.654728\%$

it # 3

$x_3 = 6.305898$ $\varepsilon_{a3} = 0.010114\%$

b)

$f'(4.52) < -0.0144 \simeq 0$ function is flat-

hence first iteration will shoot top value very far from root