CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	204	All Except EC
Examination	**Date**	**Pages**
Final	April 2015	2
Instructors		**Course Examiners**
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Special Instructions

- ▷ Only approved calculators are allowed.
- ▷ Justify all your answers.
- ▷ All questions have equal value.

1. Using Gauss-Jordan method, find all solutions of the following system of equations:

$$\begin{array}{rcrcrcrcr} x_1 & + & x_2 & - & 2x_3 & + & 4x_4 & = & 5 \\ 2x_1 & + & 2x_2 & - & 3x_3 & + & x_4 & = & 3 \\ 3x_1 & + & 3x_2 & - & 4x_3 & + & 2x_4 & = & 1 \end{array}$$

2. Let $M = \begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix}$.

   (a) Find M^{-1}.

   (b) Calculate the matrix C so that $MC = B$, where $B = \begin{pmatrix} 1 & 2 \\ 1 & 4 \\ -1 & 0 \end{pmatrix}$.

3. (a) Use Cramer's rule to solve the following system of equations:

$$\begin{array}{rcrcrcr} 2x & + & 3y & - & z & = & 1 \\ 3x & + & 5y & + & 2z & = & 8 \\ x & - & 2y & - & 3z & = & -1 \end{array}$$

   (b) Evaluate the determinant of the matrix $A = \begin{pmatrix} 2 & -1 & 3 & -4 \\ 2 & 1 & -2 & 1 \\ 3 & 3 & -5 & 4 \\ 5 & 2 & -1 & 4 \end{pmatrix}$

4. Let $\mathcal{L}$ be the line with parametric equations $x = 2 - t$, $y = 1 + t$, $z = 1 - 3t$ and let $v = (4, -2, 3)$. Find vectors w_1, w_2 such that $v = w_1 + w_2$, and such that w_1 is parallel to $\mathcal{L}$ and w_2 is perpendicular to $\mathcal{L}$.

5. Let $P_1(2,3,6)$, $P_2(1,-1,2)$, and $P_3(1,4,-2)$ be 3 points
   (a) Find the area of a triangle with vertices $P_1, , P_2, P_3$.
   (b) Find the equation of the plane containing $P_1, , P_2, P_3$.

6. Let $\mathcal{L}$ be the line with parametric equations $x = 1 - 5t$, $y = 3 - t$, $z = 2 - t$ and let $\mathcal{P}$ be the plane $-x + y + 4z = 6$
   (a) Prove that $\mathcal{L}$ and $\mathcal{P}$ are parallel.
   (b) Find the distance between $\mathcal{L}$ and $\mathcal{P}$.

7. Let $v_1 = (3,1,4)$ and $v_2 = (1,-2,5)$
   (a) Find scalars x and y such that $xv_1 + yv_2 = (5,4,3)$.
   (b) Find a vector v_3 such that v_1, v_2, v_3 is a basis of $\mathbb{R}^3$.

8. Let $A = \begin{pmatrix} 1 & 0 & 2 & 0 & 3 & 7 \\ 0 & 1 & 4 & 0 & 6 & 8 \\ 0 & 0 & 0 & 1 & 2 & 3 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \\ z \\ u \\ v \\ w \end{pmatrix}$. Find a basis for the solution space of the homogeneous system $AX = 0$.

9. Let $A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$. FInd all eigenvalues of A.
   Is A diagonalizable? If yes, find P so that $P^{-1}AP = D$ diagonal.

10. Let $A = \begin{pmatrix} 5 & 6 \\ -2 & -2 \end{pmatrix}$. Find A^{100}.