FACULTY OF ENGINEERING AND COMPUTER SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING

ENGR-391 NUMERICAL METHODS FOR ENGINEERS

Student's Name: _____

I.D.: _____

Duration 3 hours

PROBLEM 1 [Solving Nonlinear Equations] [20 marks]

Solve the following nonlinear equation using the secant method with $x_0=0$ and $x_1=2$:

$$f(x) = x^2 \ \exp(-\frac{x}{2}) - 1 = 0$$

- Is there a root in the interval [0, 2]?
- Compute 5 iterations (i.e compute x_4). Check your result.

PROBLEM 2 [Solving Systems of Linear Equations] [40 marks]

Solve the following system of linear equations:

$$\begin{cases} 10x + 2y - z = 27 \\ -3x - 6y + 2z = -61.5 \\ x + y + 5z = -21.5 \end{cases}$$

a- Using LU decomposition
b- Using Gauss-Seidel method (3 iterations), choose as initial guesses: y=z=0
c- Check your results
d- Compare the two methods

PROBLEM 3 [Numerical Integration] [20 marks]

- Show that the following integral:

$$I = \int_0^2 e^{-x^2} dx$$

Is equivalent to: $I = \int_{-1}^{1} e^{-(x+1)^2} dx$

- Compute this integral using three-point Gauss quadrature rule.

C_1= 0.5555	x_1=-0.7746
C_2= 0.8888	x_2=0.0000
C_3= 0.5555	x_3=0.7746

PROBLEM 4 [Solving Ordinary Differential Equations] [20 marks]

1. Practice

Solve the following ODE:

$$\frac{dy}{dx} - y - x = 0 \qquad y(0) = 2$$

Using Runge-Kutta second order method (with Heun's method), for $0 \le x \le 1$ and a step of 0.2.
- Sketch your solution.

PROBLEM 5 [Theory questions] [20 marks]

1. You are given two methods to compute the integral of a function, the errors related to each one of them are:

Method 1: $\frac{(b-a)^3}{12n^2} \frac{\sum_{i=1}^{n} f''(x_i)}{n}$

Method 2: $\frac{(b-a)^5}{90n^4} \frac{\sum_{i=1}^{n/2} f''''(x_i)}{n}$

Discuss which one are you expected to converge faster.

2. What is the error on the determination of the kinetic energy (E) of a moving object, if its mass (M) is 5 kg and its velocity (V) 2 m/s. The error on the velocity measurement is 0.05 m/s and there is no error on the determination of the mass of the object.

$$E = \frac{1}{2} MV^2$$

3. Derive a forward finite difference formulation for the second derivative. What is the order of the error?

4. If the LU decomposition of matrix [A], using doolitle method is given by:

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \text{ and } [U] = \begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 5 \\ 0 & 0 & 8 \end{bmatrix}$$

Compute the det(A), without computing [A].

5. Comment following statement: *As the Gauss elimination algorithm is an exact algorithm (no approximations) it is insensitive to ill-conditioned problems.*

6. You are using Simpson 1/3 rule in its multiple version to compute an integral. You did get following results (*n* is the number of subdivions you used):

n=10 $I \cong 4.198021$
n=20 $I \cong 4.198039$

Based on these two results estimate the error and give an appropriate estimation of the integral (with the correct number of significant digits).