CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Section(s)**
Mathematics	203	All
Examination	**Date**	**Pages**
Final	April 2008	3
Instructors		**Course Examiner**
A. Djerrahian, C. Grabowski, H. Greenspan, J. Li, H. Proppe		H. Proppe

Special Instructions

▷ **Only Sharp EL 531 or Casio FX 300 MS calculators are allowed.**

MARKS

[9] **1.** (a) Sketch the graph of the function $f(x) = |(x+2)^2 - 4|$ starting from the graph of the standard parabola and using appropriate transformations.

(b) Suppose $f(x) = \sqrt[3]{1+e^x}$, and $g(x) = \ln(x^3 - 1)$. Find $f \circ g$ and $g \circ f$. Determine the domain and range of $f \circ g$ and $g \circ f$.

(c) Evaluate $\cos(\sin^{-1}(t))$.

[8] **2.** Evaluate the limits:

(a) $\displaystyle\lim_{x\to 4} \frac{\sqrt{2x+1}-3}{x^3-64}$ (b) $\displaystyle\lim_{x\to -\infty} \frac{(x^4+1)(3-2x)^3}{(x+1)^5(2-x^2)}$

Do not use l'Hopital's rule.

[10] **3.** (a) Consider the function $f(x) = \dfrac{|x+1|}{x^2-1}$.

Calculate both one-sided limits at the point(s) where the function is undefined.

(b) Find parameters a and b such that the function

$$f(x) = \begin{cases} x+5 & \text{if} \quad x < 0 \\ (x-a)^2 + b, & \text{if} \quad 0 \le x < 2 \\ 1, & \text{if} \quad x \ge 2 \end{cases}$$

will be continuous at every point. Sketch the graph of this function.

[15] **4.** Find derivatives of the functions (do not simplify the answer):

(a) $f(x) = \dfrac{\sqrt[3]{x} - 2\sqrt[5]{x^3} + x^4}{\sqrt{x}}$;

(b) $f(x) = e^{-2x^3}(\sin x + \tan x)^2$;

(c) $f(x) = \sec(\arcsin 2x)$;

(d) $f(x) = \dfrac{\ln^2(\sqrt{x})}{1 + \sqrt{e^{2x}}}$;

(e) $f(x) = (\arctan x)^{1+x^2}$ (use logarithmic differentiation).

[6] **5.** Given the function $f(x) = x^2 + \dfrac{1}{x}$,

(a) Use the definition of derivative to find the derivative of the function.

(b) Use the appropriate differentiation rule(s) to verify (a).

[6] **6.** (a) Find the differential of the function $f(x) = \tan x$ at $a = \pi/4$.

(b) Use the differential above to estimate $\tan 0.8$ (to calculate dx, you may use 0.785 as an approximation of $\pi/4$) .

[10] **7.** (a) A curve called a "devil's curve" is defined implicitly by the equation $y^2(y^2 - 4) = x^2(x^2 - 5)$. Verify that the point $(0, -2)$ belongs to the curve. Find an equation of the tangent line to the curve at this point.

(b) Use l'Hopital's rule to evaluate $\displaystyle\lim_{x \to 0} \frac{\sin x - x}{x \cos x - x}$.

[10] **8.** (a) Let $f(x) = e^{\ln(\arctan(2x))}$. Find $f''(x)$.

(b) Let $f(x) = x^3 + x - 1$. Find a number c that satisfies the conclusion of the Mean Value Theorem for the function $f(x)$ on $[0, 2]$.

[10] **9.** (a) At what rate is the area of an equilateral triangle increasing if its base is 10 cm long and increasing at 0.5 cm/s?

(b) If 1200 cm^2 of material is available to make a box with a square base and no top, find the largest possible volume of the box.

[16] **10.** Given the function $f(x) = \dfrac{2x^2}{x^2 - 1}$,

(a) Find the domain and check for symmetry. Find all asymptotes.

(b) Calculate $f'(x)$ and use it to determine interval(s) where the function is increasing, interval(s) where the function is decreasing, and local extrema (if any).

(c) Calculate $f''(x)$ and use it to determine interval(s) where the function is concave upward, interval(s) where the function is concave downward and inflection point(s) (if any).

(d) Sketch the graph of the function, and label the local extrema and inflection point(s) you have found (if any) in parts (b) and (c).

[5] **Bonus Question**

Two runners start a race at the same time and finish in a tie. Show that at some time during the race they have exactly the same speed. [*Hint:* Let $g(t)$ be the distance the first runner covers in time t and let $h(t)$ be the distance the second runner covers in time t. Now put $f(t) = g(t) - h(t)$ and explain how to apply Rolle's Theorem to this situation].