# Concordia University

## Department of Computer Science & Software Engineering

## COMP 232/4 Introduction to Discrete Mathematics Winter 2019

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### Assignment 4

Due date: Monday, April 1st, 2019

1. Find a formula for
$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{n(n+1)}$$
by examining the values of this expression for small values of $n$. Use mathematical induction to prove your result.

2. Show that
$$1^3+2^3+\ldots+n^3=\left(\frac{n(n+1)}{2}\right)^2$$
whenever $n$ is a positive integer.

3. Use mathematical induction to show that 3 divides $n^3+2n$ whenever $n$ is a nonnegative integer.

4. Show that $n$ lines separate the plane into $(n^2+n+2)/2$ regions if no two of these lines are parallel and no three pass through a common point.

5. Show by strong induction that any integer $n \geq 6$ can be written as $3a+4b$ for some non-negative integers $a$ and $b$.

6. Use strong induction to show that every positive integer can be written as a sum of distinct powers of two.

   *Hint: In the inductive step, consider separately the cases when $n+1$ is even and $n+1$ is odd.*

7. The Fibonacci numbers are defined as follows: $f_1=1$, $f_2=1$, and $f_{n+2}=f_n+f_{n+1}$ whenever $n \geq 1$.

   (a) Characterize the set of integers $n$ for which $f_n$ is even and prove your answer using induction.

   (b) Use induction to prove that $\sum_{i=1}^{n} i f_i = n f_{n+2} - f_{n+3} + 2$ for all $n \geq 1$.