Concordia University

Dept. of Computer Science and Software Engineering

COMP 335 – Introduction to Theoretical Computer Science

Sample Midterm Exam

Student ID: .....
First Name: (Please PRINT).....
Last Name: (Please PRINT).....

Important Notes:

* **Books, notes, calculators, extra sheets are NOT allowed.**

* Maximum time is **70 minutes** and the Maximum point is **24**.

* Write with **PEN** only. For drawing graphs, you may use a pencil as well.

* There are 3 types of questions as follows.
  - (Detailed): For Q1 and Q2 on pages 2 and 3 you need to write detailed answers.
  - (T/F): Mark True or False for questions on page 4. Briefly justify if false.
  - (Multiple-Choice Questions): Mark on the scan sheet for MCQ's on pages 5 to 9.

* For Detailed questions, answer in the area below them. Use other areas for rough work.

* Questions are not in specific order of difficulties. So browse through and do the easy ones first.

* Make sure you have **9 pages** including this cover page.

Q1. **[2 Points]** Let L be any language over $\Sigma = \{a, b\}$. Using L, we define a new language L' which includes every string w if both w and its reverse are in L. Show that L' is regular whenever L is regular.

Q2. Consider the regular expression $r = ab^*a^* + (ab)^*ba$.

(a). **[1 Point]** List all strings in $L(r)$ whose size/length is at most 3.

(b). **[2 Points]** Give a finite state automaton that accepts $L(r)$.

Q3. [**5 Points**] (T/F Questions) In each of the following questions, mark T if it is **always** true. In this case, no explanation is needed. Otherwise mark F and justify briefly, for instance, by giving a counter-example.

(a). Suppose R is a regular language and L is any subset of R. Then L is regular.

• T • F

(b). Let $L = \{a^n b^n : n \geq 0\}$. Then the string aba^4b^4 is in L^3 (note: $L^3 = LLL$).

• T • F

(c). The language $L(\emptyset\emptyset^* + \emptyset)$ has no strings.

• T • F

(d) If L_1 is finite and $L_1 \cup L_2$ is regular, then L_2 is regular.

• T • F

(e) Let L be any language. Then $R = L^* - L$ is the complement of L, that is, R includes every string that is not in L.

• T • F

[14 Points] (Multiple Choice Questions). Each question has exactly one answer. Each correct answer gets 1 point. Providing a wrong or multiple answers to each question gets 0. Mark your answer by drawing a circle around the answer. Use PENS and NOT pencils.

1. Let $L = L_3^*(L_1 \cup L_2)L_3$ be a language, where $L_1 = \{ab^n : n \geq 1\}$, $L_2 = L(b^*a^*)$, and $L_3 = \{aa, aaa\}$. The number of strings of length at most 3 in L is ...?

   (a) 3

   (b) 4

   (c) 5

   (d) 6

2. Let $L = \{ab, aab, aba\}$. Which of the following strings is NOT in $L^* - L^2$?

   (a) λ

   (b) ab

   (c) ba

   (d) aba

3. Let $L_1 = \emptyset$ (the empty set) and $L_2 = \{a^n : n \geq 0\}$. Which of the following is $L_1^*L_2$?

   (a) $\emptyset$

   (b) $\{\lambda\}$

   (c) L_1

   (d) L_2

4. Which of the following statements is NOT correct?

   (a) If L^R is regular, then L is regular.

   (b) If $\overline{L}$ is regular, then L is regular.

   (c) If L^* is regular, then L is regular.

   (d) If $\overline{L}$ is finite, then L is regular.

5. What is the minimum number of states required for a DFA that accepts the language: $L = \{w : w \in \{a, b\}^*, w \text{ does not end with } ab\}$?

   (a) 1

   (b) 2

   (c) 3

   (d) 4

6. What is the language $L(r)$ described by the following regular expression? $r = (b + c)^*(\lambda + a)(b + c)^*(\lambda + a)(b + c)^*(\lambda + a)(b + c)^*$?

   (a) every string in $L(r)$ has at least one a

   (b) every string in $L(r)$ has at least two a's

   (c) every string in $L(r)$ has at least three a's

   (d) none of the above

7. Consider the following FA M in which q_0 is both the intial and the final state.

State	λ	a	b
q_0	ϕ	$\{q_0\}$	$\{q_1, q_2\}$
q_1	$\{q_0\}$	$\{q_1\}$	$\{q_2\}$
q_2	$\{q_1\}$	$\{q_2\}$	$\{q_1\}$

Of the following, which one is the result of the function $\delta^*(q_0, babb)$?

(a) $\{q_0\}$

(b) $\{q_0, q_1\}$

(c) $\{q_1, q_2\}$

(d) $\{q_0, q_1, q_2\}$

8. Which of the following regular expressions corresponds to the language accepted by the FA M defined above?

(a) $(a + b)^*$

(b) $a^* + (ba)^*$

(c) $a^* + b(a + bb)^*$

(d) $a^* + b(a + bab)^*$

9. A regular expression for the set of strings that begin with ab and end with bba is:

(a) $ab(a + b)^*ba$

(b) $a(b + a)^*bba$

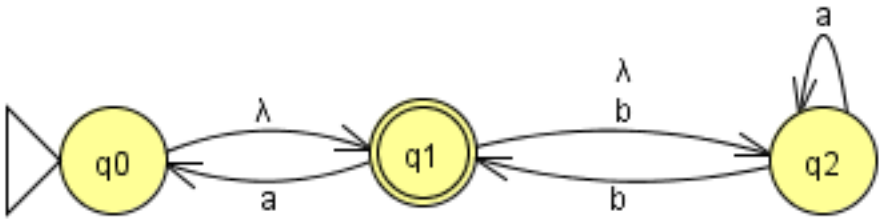
(c) $ab(a + b)^*bba$

(d) $ab(b + a)^*bba + abba$

10. Let $r = (a+b)^*b(c+cd)^*$ be a regular expression. How many strings of length at most 3 are there in the language denoted by r?

    (a) 9
    (b) 10
    (c) 11
    (d) 12

11. Of the following languages over $\Sigma = \{a, b\}$, which one is accepted by the FA below?



    (a) Σ^*
    (b) The set of strings that end with bb
    (c) The set of strings that end with a or b
    (d) In addition to λ, the set of strings that end with ba^*b

12. In the previous question, what is the minimum number of states of an equivalent DFA?

    (a) 1
    (b) 2
    (c) 3
    (d) 4

13. Which of the folloiwng statements is correct?

    (a) If L_1 is regular and $L_1 \cap L_2$ is regular, then L_2 is regular.
    (b) The union of infinitely many regular languages is regular.
    (c) If $L_1 L_2$ is a finite language, then L_1 and L_2 are both finite.
    (d) If L_1 and L_2 are regular languages, then their difference $(L_1 - L_2)$ is also regular.

14. For any alphabet Σ, the language $\emptyset^* \emptyset \Sigma^* \Sigma$ is equal to ...?

    (a) $= \emptyset$
    (b) $= \Sigma$
    (c) $= \{\lambda\}$
    (d) $= \Sigma^+$