CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	204	All except EC
Examination	**Date**	**Pages**
Final	December 2010	2
Instructors		**Course Examiner**
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Special Instructions:

- ▷ Only approved calculators are allowed.
- ▷ Justify and explain all your answers.
- ▷ All questions have equal value.

MARKS

1. Use the Gauss-Jordan method to find all the solutions of the system

$$\begin{aligned} 2x_1 - 2x_2 + 2x_3 &= 0 \\ -2x_1 + 5x_2 + 2x_3 &= 1 \\ 8x_1 + x_2 + 4x_3 &= -1. \end{aligned}$$

2. Let $M = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 7 & 3 \\ -1 & 0 & 1 & 2 \\ 2 & 1 & 5 & 7 \end{bmatrix}$.

   a) Calculate M^{-1}.

   b) Find the matrix C such that $MC = B$.

3. a) Use Cramer's rule to solve the system of equations

$$\begin{aligned} x - 4y + z &= 6 \\ 4x - y + 2z &= -1 \\ 2x + 2y - 3z &= -20. \end{aligned}$$

   (No marks given if you don't use Cramer's rule.)

   b) Calculate the determinant of the matrix $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix}$.

4. a) Let $\underline{u} = (1,2,4)$, $\underline{v} = (-1,0,1)$. Find the orthogonal projection $\underline{u}_1$ of $\underline{u}$ on $\underline{v}$ and a vector $\underline{u}_2$ so that $\underline{u} = \underline{u}_1 + \underline{u}_2$.

   b) Find the distance from the point $(2,1)$ to the line $x - 7y = 6$.

5. a) Find vectors $\underline{v}$ and $\underline{w}$ which are the orthogonal to $\underline{u} = (1,-7,2)$ and so that $\underline{v}$ is orthogonal to $\underline{w}$.

   b) Find the area of a triangle with vectors $(1,2,1)$, $(7,1,3)$, $(-2,0,1)$.

6. a) Find the equation of the plane that contains the points $(1,0,1)$, $(1,1,0)$ and $(0,1,1)$.

   b) Find the parametric equations for the line in $\mathbb{R}^3$ passing through the point $(1,1,2)$ and perpendicular to the plane $-x + 3y - 2z = 7$.

7. a) Let $\underline{u}_1 = (1,0,-1)$, $\underline{u}_2 = (-2,7,2)$, $\underline{u}_3 = (3,-7,-3)$. Show that $\underline{u}_1, \underline{u}_2, \underline{u}_3$ are linear dependent.

   b) Find the basis for the subspace of $\mathbb{R}^3$ containing $\underline{u}_1, \underline{u}_2, \underline{u}_3$.

8. Let

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 7 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \\ u \\ v \\ w \end{bmatrix}.$$

   Find a basis for the solution space of the homogeneous system of linear equations $AX = 0$.

9. Find the standard matrix for the composition of the following 2 linear operations on $\mathbb{R}^2$: A rotation counterclockwise of $90°$ followed by a reflection about the y- axis.

10. Let $A = \begin{bmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & 2 \end{bmatrix}$. Find the invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.