1. (1 point) Suppose $f(x) = \frac{3}{x-2}$,
$f'(4) =$ ______
Use this to find the equation of the tangent line to the curve $y = \frac{3}{x-2}$ at the point $(4, \frac{3}{2})$. Write you answer in the form $y = mx+b$, where m is the slope and b is the y-intercept
The equation of the tangent line is ______.
Correct Answers:

- `-0.75`
- `0.75*x+y = 4.5`

2. (1 point) For what values of x is the tangent line of the graph of

$$f(x) = 4x^3 - 6x^2 - 48$$

parallel to the line $y = -0.5$?

$x =$ ______
Correct Answers:

- `0, 1`

3. (1 point) Find the y-intercept of the tangent line to

$$y = \frac{-0.7}{\sqrt{6+5x}}$$

at $(3.5, -0.144398974476231)$.
The y-intercept = ______
Correct Answers:

- `-0.0153616*3.5+-0.144399`

4. (1 point) Let $f(t) = \dfrac{\sqrt{6}}{t^3}$. Find $f'(t)$.
$f'(t) =$ ______
Find $f'(5)$.
$f'(5) =$ ______
Correct Answers:

- `-(2.44949*3*t^2/[(t^3)^2])`
- `-0.0117576`

5. (1 point) Let

$$f(x) = \frac{\sqrt{x}-5}{\sqrt{x}+5}$$

$f'(9) =$ ______
Correct Answers:

- `5/[sqrt(9)]/([sqrt(9)+5]^2)`

6. (1 point) If $f(x) = 6x\sqrt{x} + \dfrac{5}{x^2\sqrt{x}}$, then

$f'(9) =$ ______
Correct Answers:

- `118073/4374`

7. (1 point) If $f(x) = 5\sqrt{x}(x^3 - 8\sqrt{x} + 2)$, find $f'(x)$.
$f'(x) =$ ______

Find $f'(3)$.
$f'(3) =$ ______
Correct Answers:

- `3.5*5*x^2.5-5*8+0.5*5*2*x^-0.5`
- `3.5*5*3^2.5-5*8+0.5*5*2*3^-0.5`

8. (1 point) Evaluate the limit

$$\lim_{x\to 0} \frac{\sin 3x}{7x}$$

______

Correct Answers:

- `0.428571428571429`

9. (1 point) Let $f(x) = \dfrac{\tan(x) - 4}{\sec(x)}$. Find the following:

1. $f'(x)$ = ______
2. $f'(5)$ = ______

Correct Answers:

- `([sec(x)]^2*sec(x)-[tan(x)-4]*sec(x)*tan(x))/([sec(x)]^2)`
- `-3.55203`

10. (1 point) Let $f(x) = -2x(\sin(x) + \cos(x))$. Find the following:

1. $f'(x)$ = ______
2. $f'(\frac{\pi}{6})$ = ______

Correct Answers:

- `-(2*[sin(x)+cos(x)]+2*x*[cos(x)-sin(x)])`
- `-3.11535`

11. (1 point) If $f(x) = (4x+2)^{-4}$,

Find $f'(x)$.
Then $f'(x) =$ ______

Find $f'(5)$.

Then $f'(5) =$ ______

Correct Answers:

- `-4*(4*x+2)^(-4-1)*4`
- `-4*(4*5+2)^(-4-1)*4`

---

12. (1 point) Let $f(x) = \sin(x^4)$.

$f'(x) =$ ______________

$f'(3) =$ ______________

Correct Answers:

- `(cos(x^4))*(4*x^(4-1))`
- `(cos(3^4))*(4*3^(4-1))`

---

13. (1 point) If $f(x) = \sin^4(x)$,
find $f'(x) =$ ______________.
Find $f'(1) =$ ______________.

Correct Answers:

- `4*[sin(x)]^3*cos(x)`
- `1.2877`

---

14. (1 point) Let $f(x) = \cos(3x+6)$.

$f'(x) =$ ______________

$f'(4) =$ ______________

Correct Answers:

- `-[sin(3*x+6)]*3`
- `-[sin(3*4+6)]*3`

---

15. (1 point) Let $f(x) = -6\sin(\sin(x^2))$

$f'(x) =$ ______________

Correct Answers:

- `-6*2*x^(2-1)*cos(sin(x^2))*cos(x^2)`

---

16. (1 point) Let

$$f(x) = -9e^{x\sin x}$$

$f'(x) =$ ______________

Correct Answers:

- `-9*2.71828^[x*sin(x)]*[sin(x)+x*cos(x)]`

---

17. (1 point)

By using known trig identities, $\frac{\sin(2x)}{1+\cos(2x)}$ can be written as

- A. $\csc(2x)$
- B. $\sec(x)$
- C. $\tan(2x)$
- D. $\tan(x)$
- E. All of the above
- F. None of the above

Correct Answers:

- `D`

---

18. (1 point) Use a sum or difference formula or a half angle formula to determine the value of the trigonometric functions. Give exact answers. Do not use decimal numbers. The answer should be a fraction or an arithmetic expression. If the answer involves a square root it should be enter as sqrt; e.g. the square root of 2 should be written as sqrt(2);

$\sin(\frac{5\pi}{12})$ = ________

$\sin(\frac{11\pi}{8})$= ________

$\cos(\frac{11\pi}{8})$= ________

$\cos(\frac{\pi}{12})$= ________

Correct Answers:

- `0.965925826289068`
- `-0.923879532511287`
- `-0.38268343236509`
- `0.965925826289068`

---

19. (1 point) The expressions A,B,C,D, E are left hand sides of identities. The expressions 1,2,3,4,5 are right hand side of identities. Match each of the left hand sides below with the appropriate right hand side. Enter the appropriate letter (A,B,C,D, or E) in each blank.

A. $\tan(x)$
B. $\cos(x)$
C. $\sec(x)\csc(x)$
D. $\frac{1-(\cos(x))^2}{\cos(x)}$
E. $2\sec(x)$

___1. $\tan(x)+\cot(x)$
___2. $\sin(x)\sec(x)$
___3. $\sin(x)\tan(x)$
___4. $\frac{\cos(x)}{1-\sin(x)} + \frac{1-\sin(x)}{\cos(x)}$
___5. $\sec(x) - \sec(x)(\sin(x))^2$

Correct Answers:

- `C`
- `A`
- `D`
- `E`
- `B`

---

20. (1 point) Simplify and write the trigonometric expression in terms of sine and cosine:

$$\frac{2+\tan^2 x}{\sec^2 x} - 1 = (f(x))^2$$

$f(x) =$ _____.

Correct Answers:

- `cos(x)`

21. (1 point) Is the function below exponential?

$$G(t) = (5 \cdot t)^3$$

If so, write the function in the form $G(t) = ab^t$ and enter the values you find for a and b in the indicated blanks below. If the function is not exponential, enter **NONE** in both blanks below.
$a =$ _____
$b =$ _____

Solution:
SOLUTION
No, $G(t)$ is not exponential. If can be written as

$$G(t) = (5 \cdot t)^3 = 5^3 t^3$$

which is not of the form $G(t) = ab^t$. Do not get this confused with $G(t) = 5^3 \cdot (3)^t$ which is exponential. The variable t must be the power and 3 the base. In this example t is the base and 3 is the power.

Correct Answers:

- `NONE`
- `NONE`

---

22. (1 point) Is the function below exponential?

$$K(x) = \frac{7^x}{\sqrt{2} \cdot 4^x}$$

If so, write the function in the form $K(x) = ab^x$ and enter the values you find for a and b in the indicated blanks below. If the function is not exponential, enter **NONE** in both blanks below.
$a =$ _____
$b =$ _____

Solution:
SOLUTION
Yes, $K(x)$ is exponential. It can be rewritten in standard form as

$$K(x) = \frac{7^x}{\sqrt{2} \cdot 4^x} = \left(\frac{1}{\sqrt{2}}\right) \cdot \left(\frac{7^x}{4^x}\right) = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{7}{4}\right)^x.$$

So $a = \frac{1}{\sqrt{2}}$ and $b = \frac{7}{4}$.

Correct Answers:

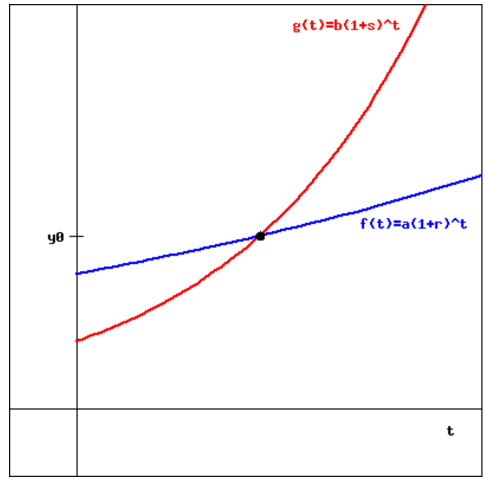
- `1/[sqrt(2)]`
- `7/4`

---

23. (1 point) Suppose y_0 is the y-coordinate of the point of intersection of the graphs below. Complete the statement below in order to correctly describe what happens to y_0 if the value of b (in the red graph of $g(t) = b(1+s)^t$ below) is increased, and all other quantities remain the same.

As b increases, the value of y_0

- A. increases
- B. decreases
- C. remains the same



(click on image to enlarge)

Solution:
SOLUTION
As b is increased, the y-intercept of the red graph increases. The entire red graph will be shifted up, and the point of intersection shifts to the left and down, so the y-coordinate decreases.

Correct Answers:

- `B`

---

24. (1 point)

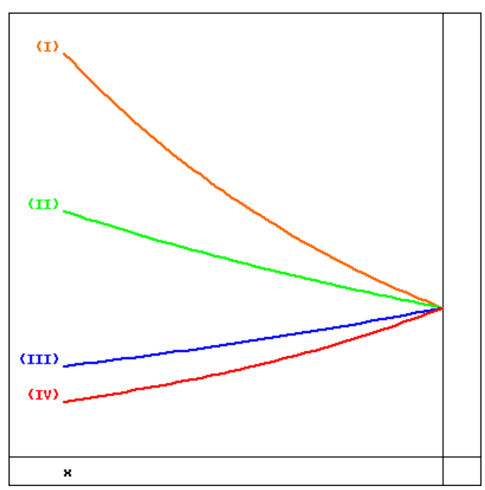
Without using a calculator, match each exponential function with its graph.

[?] e^x

[?] e^{-x}

[?] $e^{0.8x}$

[?] $e^{-0.8x}$



(Click on graph to enlarge)

Solution:
SOLUTION
The functions given in (b) and (d) represent exponential decay while the functions given in (c) and (a) represent exponential growth. Thus, (b) and (d) correspond to (I) and (II) (not necessarily in that order) while (c) and (a) correspond to (III) and (IV) (not necessarily in that order).

The function in (a) grows faster than the function in (c) since it grows continuously at a rate of 1% while the function in (a) grows continuously by 0.8%. Since (a) has a higher continuous growth rate, its graph must decrease the fastest to 0 as $x \to -\infty$, thus it has graph IV and (c) has graph III.

Graphs (I) and (II) correspond to the exponential decay formulas, with graph (I) decaying at a more rapid rate (since it increases to ∞ quicker as $x \to -\infty$ it similarly decreases to 0 faster as $x \to +\infty$). Thus formula I corresponds to graph (b) and formula (d) corresponds to graph II. We have:
a) IV
b) I
c) III
d) II

Correct Answers:

- `IV`
- `I`
- `III`
- `II`

---

25. (1 point) If $f(x) = 6^x$ and $g(x) = \dfrac{6x}{x+12}$, find a simplified formula for:
$g(f(x)) =$ ______________

Solution:
SOLUTION
We solve by substituting the expression $f(x) = 6^x$ in for x in $g(x) = \dfrac{6x}{x+12}$:

$$g(f(x)) = \frac{6f(x)}{f(x)+12} = \frac{6 \cdot 6^x}{6^x+12}.$$

Correct Answers:

- `6*6^x/(6^x+12)`

---

26. (1 point) If $f(x) = e^{x/5}$, $g(x) = 9x+2$, and $h(x) = \sqrt{x}$, Find a simplified formula for:
$f(g(x))h(x) =$ ______________

Solution:
SOLUTION
We start by computing $f(g(x)) = e^{5g(x)} = e^{5(9x+2)} = e^{45x+10}$. Next we compute $f(g(x))h(x)$ by multiplying the expression above by $h(x) = \sqrt{x}$:

$$f(g(x))h(x) = e^{45x+10} \cdot \sqrt{x}.$$

Correct Answers:

- `e^(9/5*x+2/5)*sqrt(x)`

---

27. (1 point) Let $f(x) = x^{4/3}$, $g(x) = \frac{(4x-3)^3}{64}$, and $h(x) = \tan(2x)$. Find values for the constants A and P which result in the simplified expression for $h(x)/f(g(x))$ equal to the combination of functions below:

$$\frac{h(x)}{f(g(x))} = \frac{A\tan(2x)}{(4x-3)^P}$$

$A =$ _____, and
$P =$ _____

Solution:
SOLUTION
We start by computing $f(g(x)) = (g(x))^{4/3} = \left(\dfrac{(4x-3)^3}{64}\right)^{4/3} = \dfrac{(4x-3)^4}{(4^3)^{4/3}} = \dfrac{(4x-3)^4}{256}$.
Thus $\dfrac{h(x)}{f(g(x))} = \dfrac{\tan(2x)}{(4x-3)^4/256} = \dfrac{256\tan(2x)}{(4x-3)^4}$.

Correct Answers:

- `256`
- `4`

---

28. (1 point) Let $f(x) = 6x^4\sqrt{x} + \dfrac{-4}{x^3\sqrt{x}}$.

$f'(x) =$ ______________

[NOTE: Your answer should be a function in terms of the variable 'x' and not a number!]

Correct Answers:

- `6*(4+1/2)*x^(4-1/2)--4*(3+1/2)/[x^(3+3/2)]`

---

29. (1 point) Let $f(x) = \dfrac{5\sin(x)}{2+\cos(x)}$. Find the following:

1. $f'(x)$ = ______________________
2. $f'(1)$ = ______________________

Correct Answers:

- `[5*cos(x)*[2+cos(x)]+5*sin(x)*sin(x)]/([2+cos(x)]^2)`
- `1.61209`

---

30. (1 point) A function $f(x)$ is said to have a **removable** discontinuity at $x = a$ if:
1. f is either not defined or not continuous at $x = a$.
2. $f(a)$ could either be defined or redefined so that the new function is continuous at $x = a$.

Let $f(x) = \begin{cases} \frac{5}{x} + \frac{-4x+25}{x(x-5)}, & \text{if } x \neq 0,5 \\ 4, & \text{if } x = 0 \end{cases}$

Show that $f(x)$ has a removable discontinuity at $x = 0$ and determine what value for $f(0)$ would make $f(x)$ continuous at $x = 0$.
Must redefine $f(0) =$_______.
Hint: Try combining the fractions and simplifying.
The discontinuity at $x = 5$ is not a removable discontinuity, just in case you were wondering.

Correct Answers:

- `-0.2`

31. (1 point) Let $f(x) = \frac{x-3}{x^2+14x+49}$. Use interval notation to indicate the domain of $f(x)$.

Note: You should enter your answer in **interval notation**. If the set is empty, enter ”” without the quotation marks.

Domain = ______

Correct Answers:

- `(-infinity,-7) U (-7,infinity)`

32. (1 point) Let $f(x) = 3-4x$ and $g(x) = 3x+4x^2$.

Evaluate each of the following:

$f(-6) =$ ______
$g(-10) =$ ______
$f(-10)+g(-10) =$ ______
$g(-6)-f(-10) =$ ______
$f(-10)\cdot g(-5) =$ ______
$\dfrac{f(-6)}{g(-5)} =$ ______

Correct Answers:

- `27`
- `370`
- `413`
- `83`
- `3655`
- `0.317647058823529`

33. (1 point) Suppose that

$$f(x) = \sqrt{x^2-3^2} \quad \text{and} \quad g(x) = \sqrt{3-x}.$$

For each function h given below, find a formula for $h(x)$ and the domain of h. Use **interval notation** for entering each domain.

(A) $h(x) = (f\circ g)(x)$.

$h(x) =$ ______
Domain = ______

(B) $h(x) = (g\circ f)(x)$.

$h(x) =$ ______
Domain = ______

(C) $h(x) = (f\circ f)(x)$.

$h(x) =$ ______
Domain = ______

(D) $h(x) = (g\circ g)(x)$.

$h(x) =$ ______
Domain = ______

Correct Answers:

- `sqrt(3-x-3^2)`
- `(-infinity,-6]`
- `sqrt(3-sqrt(x^2-3^2))`
- `[-4.24264068711929,-3] U [3,4.24264068711929]`
- `sqrt(x^2-3^2-3^2)`
- `(-infinity,-4.24264068711929] U [4.24264068711929,infinity`
- `sqrt(3-sqrt(3-x))`
- `[-6,3]`

34. (1 point) Suppose that

$$f(x) = \sqrt{3x}, \quad g(x) = \frac{x}{x-3}, \quad \text{and} \quad h(x) = \sqrt[3]{9x}.$$

Find $(f\circ g\circ h)(x)$.

$(f\circ g\circ h)(x) =$ ______

Correct Answers:

- `((3*(9*x)**(1/3))/((9*x)**(1/3) - 3))**(1/2)`

35. (1 point) Evaluate the following:

$$\lim_{x\to 2} \frac{-1+\sqrt{x}}{\sqrt{-1+x}}.$$

Enter **I** for ∞, **-I** for $-\infty$, and **DNE** if the limit does not exist.

Limit = ______

Correct Answers:

- `0.414213562373095`

36. (1 point) Use continuity to evaluate

$$\lim_{x\to\pi} \sin(2x+\sin 7x)$$

Enter **I** for ∞, **-I** for $-\infty$, and **DNE** if the limit does not exist.

Limit = ______

Correct Answers:

- `0`

37. (1 point) Evaluate the limit

$$\lim_{x\to -1} \sqrt{3-7x}.$$

(If the limit does not exist, enter “DNE”.)

Limit = ______

Correct Answers:

- `3.16228`

38. (1 point) Evaluate the limit :

$$\lim_{x\to 2^+} \frac{|x-2|}{x-2}.$$

If the limit does not exist, enter **DNE**.

Limit = ______

Correct Answers:

- `1`

39. (1 point) Evaluate the limit

$$\lim_{x\to -3} \frac{x^2+12x+27}{x+3}.$$

(If the limit does not exist, enter "DNE".)

Limit = ____________

Correct Answers:

- `6`

40. (1 point) Evaluate the limit

$$\lim_{y\to 1} \frac{y^3-y}{y^2-1}.$$

(If the limit does not exist, enter "DNE".)

Limit = ____________

Correct Answers:

- `1`

41. (1 point) Evaluate the limit

$$\lim_{t\to 8} \frac{\frac{1}{t}-\frac{1}{8}}{t-8}.$$

(If the limit does not exist, enter "DNE".)

Limit = ____________

Correct Answers:

- `-0.015625`

42. (1 point)

A function $f(x)$ is said to have a **jump** discontinuity at $x=a$ if:

1. $\lim\limits_{x\to a^-} f(x)$ exists.

2. $\lim\limits_{x\to a^+} f(x)$ exists.

3. The left and right limits are not equal.

Let $f(x) = \begin{cases} x^2+5x+4 & \text{if } x<7 \\ 18 & \text{if } x=7 \\ -7x+2 & \text{otherwise} \end{cases}$

Show that $f(x)$ has a jump discontinuity at $x=7$ by calculating the limits from the left and right at $x=7$.

$\lim\limits_{x\to 7^-} f(x) =$ ________

$\lim\limits_{x\to 7^+} f(x) =$ ________

Now for fun, try to graph $f(x)$.

Correct Answers:

- `88`
- `-47`

43. (1 point) Find the value of the constant b that makes the following function continuous on $(-\infty,\infty)$.

$$f(x) = \begin{cases} 8x-3 & \text{if } x\le 10 \\ -3x+b & \text{if } x>10 \end{cases}$$

$b =$ ____________

Now draw a graph of f.

Correct Answers:

- `107`

44. (1 point) For what value of the constant c is the function f continuous on $(-\infty,\infty)$ where

$$f(x) = \begin{cases} c & \text{if } x=0 \\ x\sin\frac{1}{x} & \text{otherwise} \end{cases}$$

$c =$ ____________

Hint: Compute $\lim\limits_{x\to 0} f(x)$?

Solution: Since $\lim\limits_{x\to 0} f(x) = 0$, the solution is $c=0$.

Correct Answers:

- `0`

45. (1 point) Let $f(x) = 11e^{-x/3}$.

$f^{(8)}(-1) =$ ______________

Correct Answers:

- `0.00233985`

46. (1 point)

Differentiate:

$$z = w^{3/2}(w+ce^w)$$

$z' =$ ____________________

Correct Answers:

- `w^(3/2) (1+ce^w)+ (3/2)w^(1/2) (w+ce^w)`

47. (1 point) Let $f(x) = \dfrac{6x+6}{3x+1}$. Find $f^{-1}(x)$.

$f^{-1}(x)$ = __________________

Correct Answers:

- `(6-x)/(3*x-6)`

48. (1 point) In each part, find the value of x in simplest form.

(a) $x = \log_3\left(\frac{1}{27}\right)$

$x =$ _____

(b) $x = \log_{10}\sqrt[6]{10}$

$x =$ _____

(c) $x = \log_{10} 0.1$

$x =$ _____

Correct Answers:

- `-3`
- `0.166666666666667`
- `-1`

49. (1 point) Use the laws of logarithms to rewrite the expression $\ln \sqrt[3]{xy}$ in terms of $\ln x$ and $\ln y$.

After rewriting

$$\ln \sqrt[3]{xy} = A \ln x + B \ln y$$

we find $A =$ ____________ and $B =$ ____________.

Correct Answers:

- `0.333333`
- `0.333333`

---

50. (1 point) Solve the following equation. If necessary, enter your answer as an expression involving natural logarithms or as a decimal approximation that is correct to at least four decimal places.

$$2^{2x+12} = 3^{x-14}$$

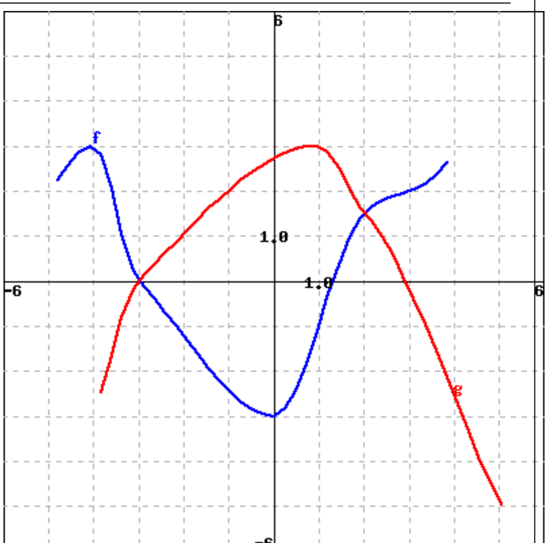
$x =$ __________

Correct Answers:

- `-82.3768`

---

51. (1 point)



(Click on the graph to see a larger version in a separate window.)

Use the graphs of f (in blue) and g (in red) to answer these questions:

_______ (A) What is the value of f at -4?

_______ (B) For what values of x is $f(x) = g(x)$? (Separate your answers with commas.)

_______ (C) Estimate the solution of the equation $g(x) = 3$.

_______ (D) On what interval is the function f decreasing? (Use **interval notation** for your answer.)

Correct Answers:

- `3`
- `-3, 2`
- `1`
- `(-4,0)`

---

52. (1 point) A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 26 feet, express the area A of the window as a function of the width x (across the base) of the window.

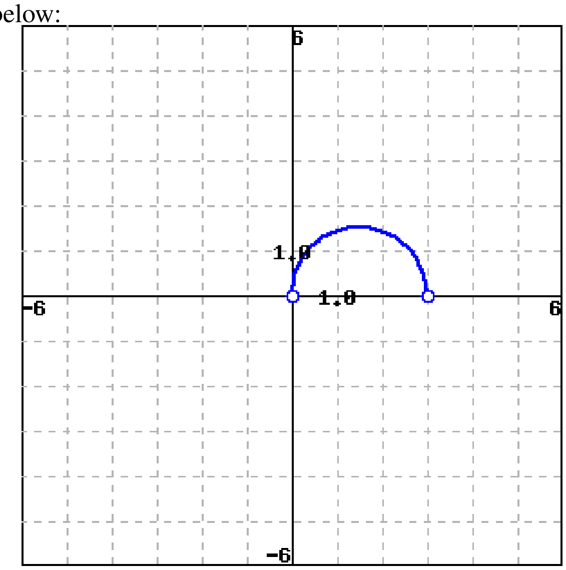
$A(x) =$ ________________

Correct Answers:

- `x*(26-x-pi*x/2)/2 + pi*x^2/8`

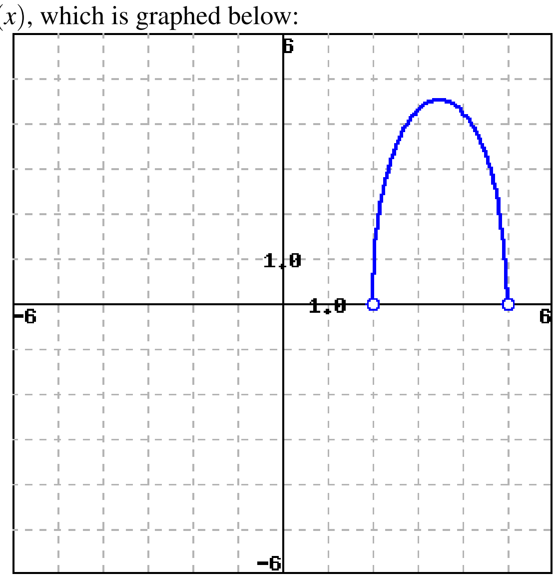
---

53. (1 point) The function $f(x) = \sqrt{3x - x^2}$ is given graphed below:



Note: Click on graph for larger version in new browser window.

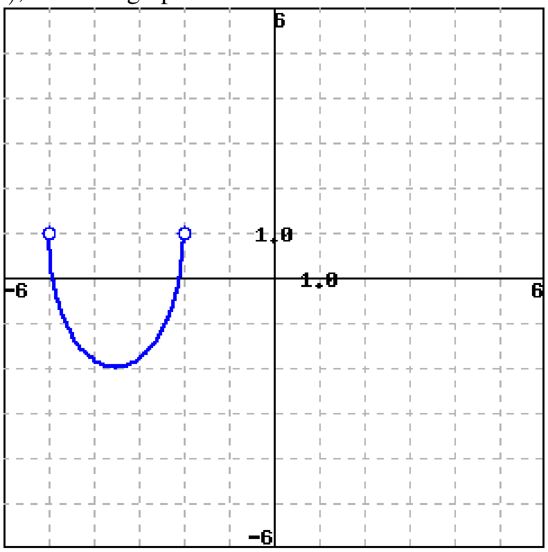
(A) Starting with the formula for $f(x)$, find a formula for $g(x)$, which is graphed below:



Note: Click on graph for larger version in new browser window.

$g(x) =$ ________________

(B) Starting with the formula for $f(x)$, find a formula for $h(x)$, which is graphed below:



Note: Click on graph for larger version in new browser window.

$h(x) =$ ______

Correct Answers:

- `3*(3*(x-2)-(x-2)**2)**(1/2)`
- `1+-2*(3*(x--5)-(x--5)**2)**(1/2)`

---

54. (1 point)

Find the inverse function of $g(x) = \frac{\sqrt{x}+4}{3-\sqrt{x}}$. If the function is not invertible, enter **NONE** .

$g^{-1}(x) =$ ______

(notice in this problem the independent variable in the inverse is x)

Solution:

SOLUTION

Start with our property of inverse functions $g\left(g^{-1}(x)\right) = x$, and substitute y for $g^{-1}(x)$ to get $g(y) = x$. Now, using the formula for g we get $g(y) = \dfrac{\sqrt{y}+4}{3-\sqrt{y}} = x$ and solving for y yields

$$x = g(y)$$
$$x = \frac{\sqrt{y}+4}{3-\sqrt{y}}$$
$$x(3-\sqrt{y}) = \sqrt{y}+4$$
$$3x - x\sqrt{y} = \sqrt{y}+4$$
$$3x - 4 = \sqrt{y} + x\sqrt{y}$$
$$3x - 4 = \sqrt{y}(1+x)$$
$$\sqrt{y} = \frac{3x-4}{1+x}$$
$$y = \left(\frac{3x-4}{1+x}\right)^2.$$

As the range of the original function $f(x)$ is $(-\infty,-1)\cup[2,\infty)$, the inverse function is

$$f^{-1}(x) = \left(\frac{3x-6}{1+x}\right)^2, x \in (-\infty,-1)\cup[2,\infty).$$

Correct Answers:

- `( (3 x - 4)/(1+x) )^2`

---

55. (1 point) If $\cos(t) = -\frac{9}{11}$ where $\pi < t < \frac{3\pi}{2}$, find the values of the following trigonometric functions.

Note: Give exact answers, do not use decimal numbers. The answer should be a fraction or an arithmetic expression. If the answer involves a square root it should be enter as sqrt; e.g. the square root of 2 should be written as sqrt(2).

$\cos(2t) =$ ______

$\sin(2t)=$ ______

$\cos(\frac{t}{2})=$ ______

$\sin(\frac{t}{2})=$ ______

Correct Answers:

- `41/121`
- `2*9*sqrt(40)/121`
- `-0.301511344577764`
- `0.953462589245592`

---

56. (1 point) If $\tan x = -1/3, \cos x > 0,$, then

$\sin 2x =$ ____;

$\cos 2x =$ ____;

$\tan 2x =$ ____.

Correct Answers:

- `-0.6`
- `0.8`
- `-0.75`