**CONCORDIA UNIVERSITY**
**Department of Mathematics & Statistics**

| **Course** | **Number** | **Section** |
|---|---|---|
| Mathematics | 203 | AA |
| **Examination** | **Date** | **Pages** |
| Final | June 2013 | 3 |
| **Instructor:** | | **Course Examiners** |
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| **Special Instructions:** | Only calculators approved by the Department are allowed | |

*MARKS*

[10] **1.** **(a)** Let $f(x) = \dfrac{x}{1+x}$ and $g(x) = \sin(2x)$. Find $h = g \circ f$ and determine the domain of $h$.

**(b)** Given the function $f = e^{2\sqrt{x}}$ find the inverse function $f^{-1}$, and determine the domain of $f$ and the domain of $f^{-1}$.

[12] **2.** Evaluate the limits. **Do not use l'Hôpital rule**:

**(a)** $\displaystyle\lim_{x\to 5} \frac{x^2-9x+20}{x^2-25}$ **(b)** $\displaystyle\lim_{x\to 1} \frac{\sqrt{3x+1}-2}{x^2-1}$ **(c)** $\displaystyle\lim_{x\to\infty} \frac{(x^2+4)^2}{x^2(2x-3)^2}$

[5] **3.** Calculate both one-sided limits of $f(x) = \dfrac{|x-3|}{x^2-9}$ at the point(s) where the function $f$ is undefined.

[16] **4.** Find the derivatives of the following functions (**Do Not Simplify!**):

(**a**) $f(x) = \dfrac{\sqrt{x} + 3\sqrt[3]{x^2}}{2x\sqrt[3]{x}}$

(**b**) $f(x) = \left(\ln(1+\sin^2(4x))\right)^3$

(**c**) $f(x) = e^{\sin x}(x^5 + 2\ln x)$

(**d**) $f(x) = \sin(x + \cos(x + \tan x))$

(**e**) $f(x) = (3x^2+2)^{\arctan x}$ (use logarithmic differentiation)

[16] **5.** **(a)** Verify that the point $(3, 1)$ belongs to the curve defined by the equation $2(x^2 + y^2)^2 = 25(x^2 - y^2)$, and find the equation of the tangent line to the curve at that point.

**(b)** A spotlight on the ground shines on a wall 12 m. away. If a man 2 m. tall walks from the spotlight toward the building at a speed of 1.6 m/sec , how fast is the length of his shadow on the building decreasing when he is 4 m. from the building?

**(c)** Use l'Hôpital's rule to evaluate the $\lim_{x \to 0} \dfrac{x \ln(1 + 2x)}{\tan^2 x}$.

[6] **6.** Let $f(x) = x^3 + x - 1$.

**(a)** Find the slope $m$ of the secant line joining the points $(1, f(1))$ and $(3, f(3))$.

**(b)** Find all points $x = c$ (if any) on the interval [1, 3] such that $f'(c) = m$.

[9] **7.** Consider the function $f(x) = \sqrt{x + 9}$.

**(a)** Use the **definition of the derivative** to find the formula for $f'(x)$.

**(b)** Write the linearization formula for $f$ at $a = 7$

**(c)** Use this linearization to approximate the value of $f(8) = \sqrt{17}$

[12] **8.** **(a)** Find the absolute extrema of $f(x) = x - \ln x$ on the interval $[1/2, 2]$.

**(b)** The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of printed material on the poster is fixed at 384 $\text{cm}^2$, find the dimensions of the poster with the smallest area.

[14] **9.** Given the function $f(x) = \dfrac{x^2}{x^2+3}$.

**(a)** Find the domain of $f$ and check for symmetry. Find asymptotes of $f$ (if any).

**(b)** Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).

**(c)** Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

**(d)** Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question** Given the equation $x^5 + 4x - 3 = 0$,

**(a)** Use the Intermediate Value Theorem to show that there is a root between 0 and 1.

**(b)** Use the Mean Value Theorem to show that the equation has exactly one root in this interval.