Department of Computer Science & Software Engineering

Comp232 Mathematics for Computer Science

Fall 2020

Assignment 1. Due date: Friday October 2

1. For each of the following statements use a truth table to determine whether it is a tautology, a contradiction, or a contingency.

   (a) $((p \vee r) \wedge (q \vee r)) \leftrightarrow ((p \wedge q) \vee r)$

   (b) $(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q)$

   (c) $(p \wedge (\neg q \rightarrow \neg p)) \rightarrow q$

   (d) $((p \rightarrow r) \vee (q \rightarrow r)) \rightarrow ((p \vee q) \rightarrow r)$

2. For each of the following logical equivalences state whether it is valid or invalid. If invalid then give a counterexample (*e.g.*, based on a truth table). If valid then give an algebraic proof using logical equivalences from Tables 6, 7, and 8 from Section 1.3 of textbook.

   (a) $p \rightarrow (q \rightarrow r) \equiv q \rightarrow (\neg p \vee r)$

   (b) $(p \rightarrow r) \wedge (q \rightarrow r) \equiv ((p \wedge q) \rightarrow r)$

   (c) $(p \rightarrow q) \wedge (p \rightarrow r) \equiv (p \rightarrow (q \wedge r))$

   (d) $((p \vee q) \wedge (\neg p \vee r)) \equiv (q \vee r)$

3. Write down the negations of each of the following statements *in their simplest form* (*i.e.*, do not simply state "It is not the case that..."). Below, x denotes a real number, $x \in \mathbb{R}$.

   (a) The plane is early or my watch is slow.

   (b) Doing the assignments is a sufficient condition for John to pass the cours e.

   (c) If x is positive, then x is not negative and x is not 0.

   (d) $(0 < x \leq 1) \vee (-1 < x < 0)$

4. Write the following statements in predicate form, using logical operators $\wedge$, $\vee$, $\neg$, and quantifiers $\forall$, $\exists$. Below $\mathbb{Z}^+$ denotes all positive integers $\{1, 2, 3, \ldots\}$.

   (a) The square of a positive integer is always bigger than the integer.

   (b) There is no integer solution to the equation x = x + 1.

   (c) The absolute value of an integer is not necessarily positive.

   (d) The absolute value of the sum of two integers does not exceed the sum of the absolute values of those integers.

5. Let P and Q be predicates on the set S, where S has two elements, say, $S = \{a, b\}$. Then the statement $\forall x P(x)$ can also be written in full detail as $P(a) \wedge P(b)$. Rewrite each of the statements below in a similar fashion, using P, Q, and logical operators, but without using quantifiers.

   (a) $\forall x \forall y \left(P(x) \vee Q(y)\right)$

   (b) $\exists x P(x) \vee \exists x Q(x)$

   (c) $\exists x P(x) \wedge \exists x Q(x)$

   (d) $\exists x \exists y \left(P(x) \wedge Q(y)\right)$

   (e) $\forall x \exists y \left(P(x) \wedge Q(y)\right)$

6. Let the domain for x be the set of all students in this class and the domain for y be the set of all countries in the world. Let $P(x, y)$ student x has visited country and $Q(x, y)$ student x has a friend in country Express each of the following using logical operations and quantifiers, and the propositional functions $P(x, y)$ and $Q(x, y)$.

   (a) Carlos has visited Bulgaria.

   (b) Every student in this class has visited the United States.

   (c) Every student in this class has visited some country in the world.

   (d) There is no country that every student in this class has visited.

   (e) There are two students in this class, who between them, have a friend in every country in the world.

7. For each part in the previous question, form the negation of the statement so that all negation symbols occur immediately in front of predicates. For example:

$$\neg\Big(\forall x\big(P(x) \wedge Q(x)\big)\Big) \equiv \exists x\Big(\neg\big((P(x) \wedge Q(x)\big)\Big) \equiv \exists x\Big(\big(\neg P(x)\big) \vee \big(\neg Q(x)\big)\Big)$$

8. Negate the following statements and transform the negation so that negation symbols immediately precede predicates. (See example in Question 7.)

   (a) $\exists x \exists y \left(P(x, y)\right) \vee \forall x \forall y \left(Q(x, y)\right)$

   (b) $\forall x \forall y \Big(Q(x, y) \leftrightarrow Q(y, x)\Big)$

   (c) $\forall y \exists x \exists z \Big(T(x, y, z) \wedge Q(x, y)\Big)$