

## ENGR 233 MIDTERM OCT. 20, 2016

INSTRUCTIONS: Answer all six questions. The six questions are equally valued. Formula sheet is attached. Legal calculators are permitted.

1. Consider the 3-dimensional vector field  $\vec{F} = \langle y^2, 2xy + e^{3z}, 3ye^{3z} \rangle$ 
  - (a) Show that this field is conservative.
  - (b) Find a potential function.
  - (c) Evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where  $C$  is the space-curve given by

$$\vec{r}(t) = \langle t^2 + t^3, e^{5t}, 7 \rangle, \quad 0 \leq t \leq 3$$

2. A mountain is described by the equation  $z = f(x, y) = 7 - x^2 - 2y^2$  where  $z$  represents altitude, and a skier is standing at the point  $(1, 1, 4)$ .
  - (a) Find the directional derivative in the unit direction  $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$ .
  - (b) In which unit direction should the skier face if he or she wants to go downhill the fastest? What is the directional derivative in that direction?
  - (c) Sketch the level curve  $f(x, y) = 4$  and draw the gradient of  $f$  at the point  $(1, 1)$
3. A cannon is on the edge of a 100 ft. cliff, and a cannonball is shot at a 45 degree upward angle with a speed of  $\sqrt{18}$  ft/sec.
  - (a) Find the time at which the cannonball hits the ground.
  - (b) What is the horizontal range of the cannonball?
4. Given the space-curve  $\vec{r}(t) = \langle \cos 2t, \sin 2t, 6t \rangle$ :
  - (a) Find the arc-length as  $t$  goes from 0 to 1.
  - (b) Find the curvature at  $t = 0$ .
5. Consider the surface given by  $z = 2x^3 - e^{xy}$ .
  - (a) Find the tangent plane at the point  $(1, 0, 1)$ .
  - (b) Find the normal line at the point  $(1, 0, 1)$ .
6. (a) Find the area of the triangle with vertices  $(1, 1, 1)$ ,  $(3, 4, 5)$  and  $(8, 1, 4)$ .
  - (b) Show that the following three vectors are coplanar:  
 $\vec{a} = \langle 1, 4, -7 \rangle$ ,  $\vec{b} = \langle 2, -1, 4 \rangle$ ,  $\vec{c} = \langle 0, -9, 18 \rangle$ .