CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	**Date**	**Pages**
Final	December 2015	2
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Special	Only approved calculators are allowed.	
Instructions:	**Show all your work for full marks.**	

MARKS

[10] **1.** **a.** Sketch the graph of the function

$$f(x) = \begin{cases} 1 + \sqrt{4 - x^2} & -2 \le x \le 2 \\ 3 - x & 2 < x \end{cases}$$

on the interval $[-2, 5]$, and find the definite integral $\int\limits_{-2}^{5} f(x)\,\mathrm{d}x$ in terms of area (do **not** antidifferentiate).

b. Use the Fundamental Theorem of Calculus to calculate the derivative of $F(x) = \int\limits_{-x^2}^{x} e^{1-t^2}\,\mathrm{d}t$, and determine whether F is increasing or decreasing at $x = 1$.

[10] **2.** Find the following indefinite integrals:

(**a**) $\displaystyle\int \frac{\cos^3(x)}{\sin^2(x)}\,dx$ (**b**) $\displaystyle\int (e^x + \ln x)\,dx$

[6] **3.** Find $F(x)$ such that $F'(x) = \dfrac{x^2+4}{x^2-4}$ and $F(-1) = 0$.

[18] **4.** Evaluate the following definite integrals (give the **exact** answers, do not approximate):

(**a**) $\displaystyle\int_0^{\pi/2} \frac{\cos(x)}{4 + \sin^2(x)}\,dx$ (**b**) $\displaystyle\int_0^{\pi/4} \sec^4(x)\,dx$ (**c**) $\displaystyle\int_0^{3} x^2\sqrt{1+x}\,dx$

[8] **5.** Evaluate the given improper integral or show that it diverges:

(**a**) $\displaystyle\int_e^{\infty} \frac{dx}{x\ln^3(x)}$ (**b**) $\displaystyle\int_{-1}^{1} \frac{x}{x^2-1}\,dx$

[16] **6.** **a.** Sketch the curves $y = 6 - x^2$ and $y = 2 - 3x$, and find the area enclosed.

b. Find the volume of a solid obtained by rotating the region bounded by the curve $y = \sin(x)$ and the x-axis on the interval $0 \le x \le \pi$ about the axis $y = -1$.

c. Find the exact average value of $f(x) = \dfrac{x}{\sqrt{16 + x^2}}$ on the interval $[0, 3]$.

[6] **7.** Find the limit of the sequence $\{a_n\}$ at $n \to \infty$ or prove that it does not exist:

(**a**) $a_n = \dfrac{(3^n + 1)^2}{6^n}$ (**b**) $a_n = \ln(1 + 2n^2) - \ln(30 + 2n^2)$

[12] **8.** Determine whether the series is divergent or convergent, and if convergent, whether absolutely or conditionally :

(**a**) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{1 + n^3}}{n^2}$ (**b**) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{5 + e^n}$ (**c**) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln^2(n)}$

[6] **9.** Find **(a)** the radius of convergence, and **(b)** the interval convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^n}{(n+1)\, 2^n}$.

[8] **10.** **(a)** Derive the Maclaurin series of $f(x) = x^2 \ln(1 + 2x)$ (HINT: start with the series for $\ln(1 + z)$ where $z = 2x$).

(b) Use integrability of power series to find the sum $S(x) = \sum_{n=1}^{\infty} n\, x^{2n-1}$ in the form of an elementary function within the radius of convergence of $S(x)$. (HINT: first find the sum for the antiderivative of $S(x)$, then differentiate).

[5] **Bonus question.** If f is continuous, prove that

$$\int_0^{\pi/2} f(\cos x)dx = \int_0^{\pi/2} f(\sin x)dx$$