Concordia University

EMAT 233 - Second Midterm Exam - March 11, 2005 **Instructor: Galia Dafni**
Total time: 75 minutes **Total marks: 100**
Allowable materials: Pencils, pens. You may NOT use notes, books, calculators or any other materials.

Write your answers in the examination booklet. Write clearly and neatly and show all your work in order to receive full marks. You do not need to simplify or approximate numerical answers.

Problem 1 (33 marks). The following questions refer to the function

$$z = f(x, y) = \ln \sqrt{x^2 + y^2}, \quad (x, y) \neq (0, 0).$$

(i) Find the directional derivative of the function at the point $(1, 1)$ in the direction of the vector $2i + 3j$.
(ii) In which direction is the function increasing most rapidly at the point $(1, 1)$? Find the maximum value of the directional derivative at this point.
(iii) Find an equation of the tangent plane to the graph of the function at the point $(1, 1)$.

Problem 2 (33 marks). Consider a force given by the vector field

$$\mathbf{F}(x, y, z) = (10xe^z - y \sin x)\mathbf{i} + \cos x\mathbf{j} + 5x^2e^z\mathbf{k}$$

in the whole of 3-dimensional space.
(i) Compute the divergence and the curl of the vector field $\mathbf{F}$.
(ii) Explain why $\mathbf{F}$ is a conservative vector field and find a potential function f so that $\vec{\nabla} f = \mathbf{F}$.
(iii) Find the work done by the force in moving a particle along the line from $(\pi, 0, 0)$ to $(\pi, 2, 1)$.

Problem 3 (34 marks).
(i) Let C be the closed curve in the xy-plane made up of the line segment from $(0, 0)$ to $(1, 0)$, the arc of the circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, and the line segment from $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ to $(0, 0)$, traversed in the counterclockwise direction. Compute the line integral

$$\int_C x^2 dy - dx.$$

(ii) Let R be the region bounded by the curve C, namely a $\frac{1}{8}$-th sector of the disk of radius 1 centered at the origin. Denote by $(\overline{x}, \overline{y})$ the coordinates of the center of mass (centroid) of the lamina that corresponds to the region R, assuming constant density ρ. SET UP THE DOUBLE INTEGRALS corresponding to $\overline{x}$, $\overline{y}$ in RECTANGULAR (x and y) coordinates (with whichever order of integration you choose) and POLAR coordinates. The limits of integration and all other ingredients in the integrals must be given explicitly. You may use the fact that the area of the sector is $\frac{1}{8}$-th the area of the disk of radius 1, i.e. $A = \frac{\pi}{8}$.
(iii) Find the center of mass $(\overline{x}, \overline{y})$ in part (ii) by evaluating whichever integrals you choose.