

**ENGR 213 –ORDINARY DIFFERENTIAL EQUATIONS**  
**FALL 2018                      Date: October 19, 2018**

**Midterm Test 1**

**Section XX**

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**Name and Surname:**

**ID:**

1. (4 marks) Solve the following initial value problem for the first order differential

equation  $\frac{dy}{dx} = \frac{3x^2y}{1+x^3}, \quad y(1)=2.$

**Solution**

$$\frac{dy}{y} - \frac{3x^2}{1+x^3} dx = 0 \rightarrow \int \frac{dy}{y} - \int \frac{3x^2}{1+x^3} dx = C \rightarrow \ln y - \ln(1+x^3) = C \rightarrow \ln y = \ln(1+x^3) + \ln C,$$

$$\rightarrow y = C(1+x^3).$$

$$\text{When } x=1, y=2 \rightarrow 2 = C(1+1) \rightarrow C=1$$

$$y = (1+x^3) \text{ Particular solution.}$$

2. (4 marks) Solve the following differential equation of the first order

$$\frac{dy}{dx} + 2xy = -xy^4.$$

**Solution**

$$\text{The transformation } u = y^{-3}, \quad \frac{du}{dx} = -3y^{-4} \frac{dy}{dx}, \quad \frac{dy}{dx} = -\frac{1}{3}y^4 \frac{du}{dx}$$

reduces the equation to

$$-\frac{1}{3}y^4 \frac{du}{dx} + 2xy = -xy^4, \quad -\frac{1}{3} \frac{du}{dx} + \frac{2x}{y^3} = -x, \quad \frac{du}{dx} - 6xu = 3x$$

Therefore  $\mu(x) = e^{\int P(x)dx} = e^{-\int 6x dx} = e^{-3x^2}$ , is the integrating factor.

Then,

$$\frac{d}{dx} \left[ e^{-3x^2} u \right] = 3xe^{-3x^2}.$$

$$\int \frac{d}{dx} \left[ e^{-3x^2} u \right] dx = \int 3xe^{-3x^2} dx$$

$$e^{-3x^2} u = -\frac{1}{2} e^{-3x^2} + C$$

$$\text{Solution } \rightarrow y^{-3} = -\frac{1}{2} + Ce^{3x^2}$$

3. (4 marks) Solve the differential equation  
 $(4x^3 + 3x^2 + 3y - 1)dx + (3x + 2y + 1)dy = 0$ .

**Solution**

$$\frac{\partial M}{\partial y} = 3 = \frac{\partial N}{\partial x} \text{ and the equation is exact.}$$

$$\mu(x,y) = \int_0^x (4x^3 + 3x^2 + 3y)dx = x^4 + x^3 + 3xy + g(y),$$

$$\frac{\partial \mu}{\partial y} = 3x + g'(y) = N(x,y) = 3x + 2y + 1 \rightarrow g'(y) = 2y + 1 \rightarrow g(y) = y^2 + y,$$

$$\text{Solution} \rightarrow x^4 + x^3 + 3xy + y^2 + y = C.$$

4. (4 marks) Solve  $(x^3 + y^3)dx - 3xy^2 dy = 0$ ,  $y(1)=1$ .

**Solution**

The equation is homogeneous of degree 3.

We use the substitution  $y=vx$ ,  $dy=vdx + xdv$ ,  $\rightarrow$  1 mark

$$x^3 \left[ (1 + v^3)dx - 3v^2(vdx + xdv) \right] = 0. \text{ The variables are separable.}$$

$$x^3 \left[ (1 + v^3) - 3v^3 \right] dx - 3v^2 x^4 dv = 0, \quad (1 - 2v^3)dx - 3v^2 x dv = 0,$$

$$\frac{dx}{x} = \frac{3v^2}{1 - 2v^3} dv$$

$$\ln x = \frac{1}{2} \ln(1 - 2v^3) + \ln C, \quad 2 \ln x = \ln(1 - 2v^3) + \ln C,$$

$$\ln \left[ \frac{x^2}{1 - 2v^3} \right] = \ln C, \quad \frac{x^2}{1 - 2v^3} = C, \quad \frac{x^2}{1 - 2\left(\frac{y}{x}\right)^3} = C.$$

$$\text{Solution: } x^3 - 2y^3 = Cx.$$

But  $y(1)=1$  thus  $C=-1$  then  $x^3 - 2y^3 = -x$  is the particular solution.

5. (4 marks) In a certain culture of bacteria the rate of increase is proportional to the number present. If it is found that the number triples in 4 hours, how many may be expected at the end of 12 hours?

*Hint: Assume that  $x = x_0$  at time  $t = 0$ .*

**Solution**

$$\frac{dx}{dt} = kx \rightarrow \frac{dx}{x} = kdt \rightarrow \ln x = kt + \ln C_1 \rightarrow x = Ce^{kt}.$$

Assuming that  $x = x_0$  at time  $t = 0 \rightarrow C = x_0 \rightarrow x = x_0 e^{kt}$ .

At time  $t = 4$ ,  $x = 3x_0 \rightarrow 3x_0 = x_0 e^{4k} \rightarrow e^{4k} = 3$ .

When  $t = 12$ ,  $x = x_0 e^{12k} = x_0 (e^{4k})^3 = x_0 (3)^3 = 27x_0$ .

that is, there are 27 times the original number.