CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Section(s)**
Mathematics	204	All except EC
Examination	**Date**	**Pages**
Final	April 2018	2
Instructors		**Course Examiner**
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Special Instructions:

- ▷ Only approved calculators are allowed.
- ▷ Justify all your answers.
- ▷ All questions have equal value.

MARKS

1. Use the Gauss-Jordan method to find all the solutions of the system:

$$\begin{array}{rcrcrcrcr} -3x_1 & + & 2x_2 & - & x_3 & + & 6x_4 & = & -7 \\ 7x_1 & - & 3x_2 & + & 2x_3 & - & 11x_4 & = & 14 \\ x_1 & & & & & - & x_4 & = & 1 \end{array}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1+s \\ -1-2s \\ 2-s \\ s \end{pmatrix}$$

2. Find the inverse of the matrix $A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$, if it exists.

$$A^{-1} = A^T = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3. Using Cramer's Rule, find the value of x_2 in the system:

$$\begin{array}{rcrcrcrcr} & & 3x_2 & & & - & 3x_4 & = & 1 \\ 2x_1 & - & x_2 & + & 3x_3 & - & 3x_4 & = & -2 \\ -2x_1 & + & 3x_2 & + & 2x_3 & + & 2x_4 & = & 0 \\ 2x_1 & & & + & 2x_3 & + & x_4 & = & -1 \end{array}$$

$$x_2 = \frac{-56}{-156} = \frac{14}{39}$$

4. Find the determinant of $A = \begin{pmatrix} 1 & 5 & 3 & -4 \\ -1 & 2 & -3 & 4 \\ 0 & 4 & 2 & -3 \\ 2 & 4 & 2 & -2 \end{pmatrix}$

$\det A = 0$

5. a) Let $u = (-3, 1, 2)$, $v = (1, 1, 1)$. Find the orthogonal projection of v on u. $\mathrm{pr}_{\bar{u}}\, \bar{v} = (0,0,0) = \bar{0}$

   b) Let $u_1 = (1, 0, 0)$, $u_2 = (1, 1, 0)$, $u_3 = (1, 1, 1)$. Find c_1, c_2, c_3 such that $c_1\, u_1 + c_2\, u_2 + c_3\, u_3 = (1, 0, 1)$. $(c_1, c_2, c_3) = (1, -1, 1)$

6. a) Find the area of a triangle with vertices $(1, 2, 1)$, $(0, 5, 2)$, $(6, 7, 3)$. Find a vector orthogonal to the plane of the triangle. $S_\Delta = 15/\sqrt{2}$, orth. vect. $(1, 7, -20)$

   b) Find the distance between the point $(2, 4)$ and the line $3x = 2y - 6$. dist $= 4/\sqrt{13}$

7. a) Let $u = (4, 0, -2)$, $v = (1, 3, 7)$, $w = (3, 3, 5)$. Are the vectors linearly dependent or independent? independent

   b) Find the parametric equations for the line in $\mathbb{R}^3$ passing through the point $(1, 2, -7)$ and perpendicular to the plane $2x - 3y + 5z = 4$.
   $\begin{cases} x = 2t+1 \\ y = -3t+2 \\ z = 5t-7 \end{cases}$

8. Let $A = \begin{bmatrix} 1 & 0 & 2 & 0 & 1 & 4 \\ 0 & 1 & 3 & 0 & 2 & 7 \\ 0 & 0 & 0 & 1 & 8 & 9 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \\ t \\ u \\ v \end{bmatrix}$. Basis: $(-2, -3, 1, 0, 0, 0)$, $(-1, -2, 0, -8, 1, 0)$, $(-4, -7, 0, -9, 0, 1)$

   Find a basis for the solution space of the homogeneous system $AX = 0$.

9. Find the standard matrices for following operators on $\mathbb{R}^2$: $A_a = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$

   a) a rotation counterclockwise of 60°.

   b) a reflection about line $y = -x$. $A_B = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

10. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -2 & -2 & -1 \\ 0 & 0 & -1 \end{bmatrix}$. Find a matrix P such that $P^{-1}AP = D$ a diagonal matrix.

Eigenvalues $0, -1$ (mult. 2)

$P = \begin{bmatrix} -1 & -1 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ for $D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$