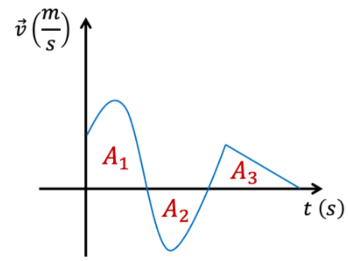
PHYS 204 – Mechanics
Sections EC
Midterm Examination – Summer 2021

Multiple choice

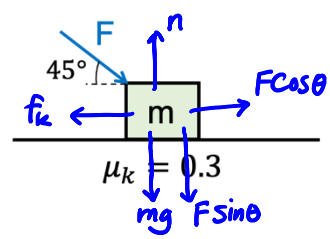
1. **(5 marks)** The motion of an object, moving along a straight line, is shown in the $v-t$ graph. If the areas $A_1 = 5$, $A_2 = 3$, and $A_3 = 2$, what is the total <u>displacement</u> of the object?



   a) 10 m
   b) -10 m
   c) 4 m
   d) -4 m
   e) 12 m

Displacement $= A_1 - A_2 + A_3$
$= 5 - 3 + 2 = 4\ m$

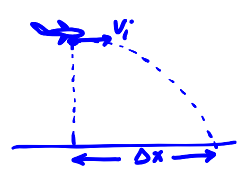
2. **(5 marks)** A 20 N force is applied on a 2-kg block, initially at rest on a horizontal surface with coefficient of kinetic friction $\mu_k = 0.3$, as shown in the figure. What is the speed of the block after it travels 5 m? Take $g = 10\,\frac{m}{s^2}$.



   a) 3.2 m/s
   b) 1.95 m/s
   c) 2.3 m/s
   d) 4.4 m/s

$\Sigma F_y = 0$
$n = mg + F\sin\theta$
$\Sigma F_x = ma$
$F\cos\theta - f_k = ma$
$F\cos\theta - \mu_k n = ma$
$F\cos\theta - \mu_k(mg + F\sin\theta) = ma$
$\rightarrow \boxed{a = 1.95\ m/s^2}$
$v_f^2 - \cancel{v_i^2}^{\,0} = 2a\Delta x$
$v_f = \sqrt{2a\Delta x} = \boxed{4.4\ m/s}$

3. **(5 marks)** An airplane flying horizontally with constant speed of 100 m/s, drops a box from height of 500 m above ground. How far from the release point will the box hit the ground? Ignore air resistance. Take $g = 10\,\frac{m}{s^2}$.



   a) 0
   b) 500 m
   c) 5 km
   d) 1 km
   e) 2 km

$\Delta y = \cancel{v_{iy}}^{\,0}\Delta t - \frac{1}{2}g\Delta t^2$
$-500 = -\frac{1}{2}(10)\Delta t^2 \rightarrow \boxed{\Delta t = 10\ s}$
$\Delta x = v_{ix}\Delta t = 100\,(10) = 1000\ m$
$= \boxed{1\ km}$

4. **(5 marks)** A block, initially at $\vec{r}_1 = (3\hat{\imath} + \hat{\jmath})\,m$ moves to $\vec{r}_2 = (6\hat{\imath} + 5\hat{\jmath})\,m$ in 5 seconds. What is the magnitude of its average velocity?

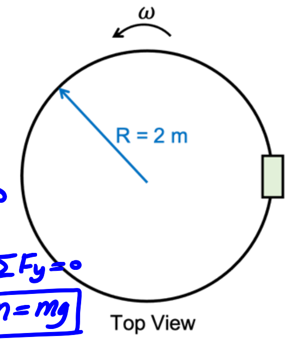
   a) 5 m/s

b) 1 m/s
c) 1.5 m/s
d) 2 m/s

$\vec{V}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2-\vec{r}_1}{\Delta t} = \frac{3\hat{i}+4\hat{j}}{5} = (0.6\hat{i}+0.8\hat{j})\frac{m}{s}$

$V_{avg.} = \sqrt{0.6^2+0.8^2} = \boxed{1\ \frac{m}{s}}$

5. **(5 marks)** A book is placed on the edge of a circular turntable with radius of $2\ m$, which rotates with constant angular speed ω. If the coefficient of static friction between the book and the table is $\mu_s = 0.5$, with what maximum angular speed can the table turn such that the book does not slip? Take $g = 10\frac{m}{s^2}$.



a) 1.58 rad/s
b) 3.16 rad/s
c) 10 rad/s
d) 2.56 rad/s

$f_s = ma_c = m\frac{v^2}{r} \rightarrow v = \sqrt{\mu_s r g}$

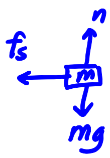
$\mu_s n = m\frac{v^2}{r}$

$\mu_s \cancel{m} g = \cancel{m}\frac{v^2}{r}$

$\omega = \frac{v}{r} = \frac{\sqrt{\mu_s r g}}{r}$

$= \boxed{1.58\ \frac{rad}{s}}$

side view

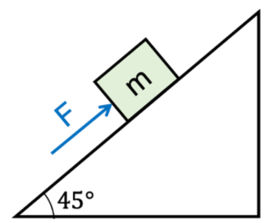


$\Sigma F_y = 0$

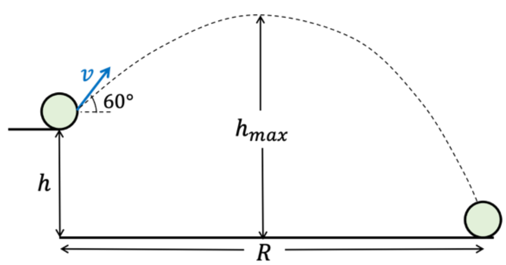
$\boxed{n = mg}$

Long Answer

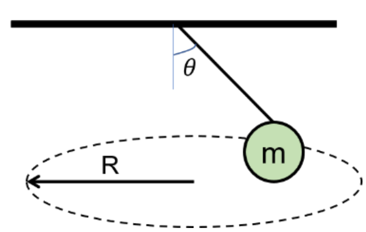
6. **(30 marks)** If $m = 4\ kg$ and $\mu_s = 0.5$, what is the rage of Force F (maximum and minimum force) for which the block does not move? Force F is applied parallel to the surface. Draw the free-body diagram. Take $g = 10\frac{m}{s^2}$. <u>Show your detailed work</u>.



7. A projectile is launched from height $10\ m$ above ground, with speed of $10\frac{m}{s}$ at angle 60° above horizon. Determine:
a) **(10 marks)** How far from its launching point (R) will the projectile land?
b) **(10 marks)** What is the maximum height h_{max} it reaches (from the ground)?
<u>Show your detailed work</u>.



8. **(25 marks)** In a conical pendulum, the pendulum bob of mass m is revolving in a horizontal circle of radius R with constant speed v. If the rope can tolerate maximum tension of T, find the maximum speed the pendulum bob can have without breaking the rope. Draw the free-body diagram. Provide your answer <u>symbolically</u>. <u>Show your detailed work</u>.



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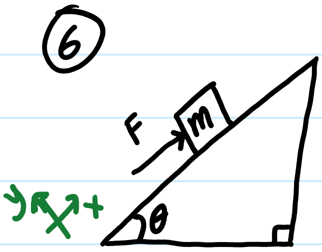
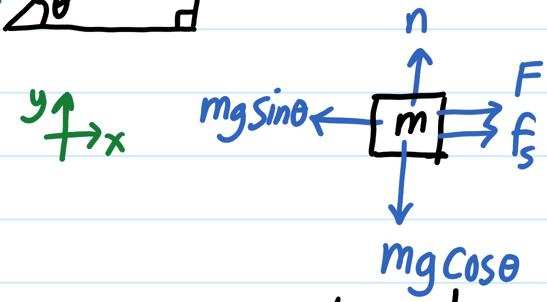


To find min Force, f_s should be in the same direction as F:



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Applying Newton's 2nd law (static equilibrium):

$\sum F_y = 0$

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$n - mg\cos\theta = 0$

$\boxed{n = mg\cos\theta}$

$\sum F_x = 0$

$F + f_s - mg\sin\theta = 0$

7/30

$F + \mu_s n - mg\sin\theta = 0$

$F + \mu_s mg\cos\theta - mg\sin\theta = 0$

$\rightarrow F = mg(\sin\theta - \mu_s\cos\theta)$

$F = (4)(10)\left[\frac{\sqrt{2}}{2} - (0.5)\left(\frac{\sqrt{2}}{2}\right)\right] \rightarrow \boxed{F_{min} = 14.14\ N}$

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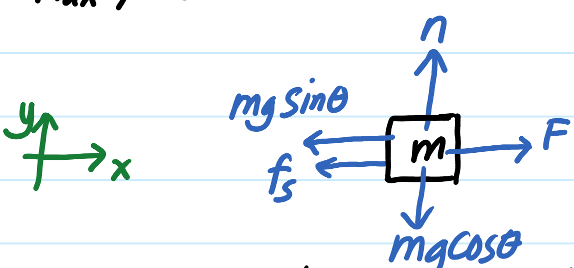
To find F_{max}, f_s should be in the opposite direction of F:



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We don't need to re-write the equation along y-axis:

$\sum F_x = 0$

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$F - f_s - mg\sin\theta = 0$

$F - \mu_s mg\cos\theta - mg\sin\theta = 0 \rightarrow F = mg(\sin\theta + \mu_s\cos\theta)$

$F = (4)(10)\left[\frac{\sqrt{2}}{2} + (0.5)\frac{\sqrt{2}}{2}\right]$

$\boxed{F_{max} = 42.43\ N}$

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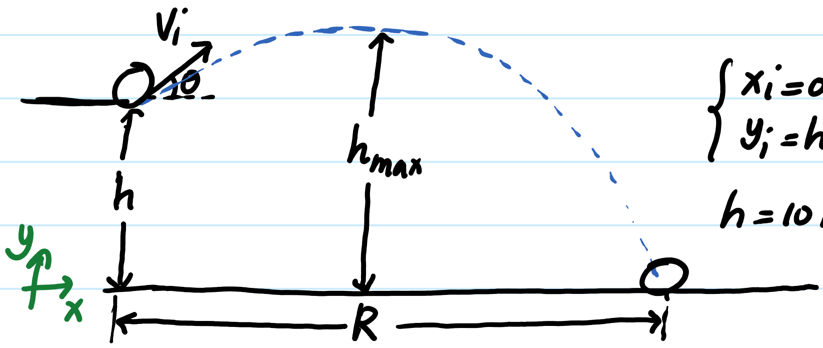


$$\begin{cases} x_i = 0 \\ y_i = h \end{cases} \qquad \begin{cases} V_{ix} = V_i \cos\theta \\ V_{iy} = V_i \sin\theta \end{cases}$$

$h = 10\,m$, $V_i = 10\,\frac{m}{s}$, $\theta = 60°$

$$\begin{cases} x_f = R \\ y_f = 0 \end{cases}$$

a) $\Delta x = V_{ix}\,\Delta t$ $\frac{3}{20}$

$R = V_i \cos\theta\,\Delta t$

$\frac{3}{20}$ $\Delta y = V_{iy}\,\Delta t - \frac{1}{2} g \Delta t^2$

$\Delta y = V_i \sin\theta\,\Delta t - \frac{1}{2} g \Delta t^2$

We should find Δt from the equation along y-axis and then plug into the one for x-axis:

$$\Delta y = V_i \sin\theta\,\Delta t - \frac{1}{2} g \Delta t^2$$

$$-10 = 10\left(\frac{\sqrt{3}}{2}\right)\Delta t - \frac{1}{2}(10)\Delta t^2 \rightarrow 5\Delta t^2 - 5\sqrt{3}\,\Delta t - 10 = 0$$

$\frac{3}{20}$

$$\Delta t^2 - \sqrt{3}\,\Delta t - 2 = 0$$

$$\begin{cases} \boxed{\Delta t = 2.52} \; ✓ \\ \Delta t = -0.79 \; \times \end{cases}$$

$$R = V_i \cos\theta\,\Delta t = 10\left(\frac{1}{2}\right)(2.52) \rightarrow \boxed{R = 12.6\,m}$$ $\frac{1}{20}$

b) At max height $V_y = 0$ $\frac{2}{20}$

$$\cancel{V_y^2}^{\,0} - V_{yi}^2 = -2g\,\Delta y \rightarrow \Delta y = \frac{V_{yi}^2}{2g} = \frac{V_i^2 \sin^2\theta}{2g}$$ $\frac{5}{20}$

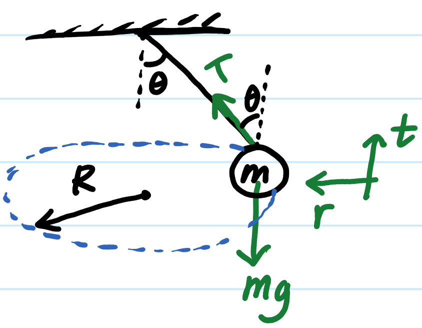
$$\Delta y = \frac{(10)^{\cancel{2}}(\sqrt{3}/2)^2}{2(\cancel{10})} = \frac{30}{8} = \boxed{3.75\,m}$$

from ground level:

$\frac{2}{20}$ $h_{max} = h + \Delta y = 10 + 3.75$

$$\boxed{h_{max} = 13.75\,m}$$ $\frac{1}{20}$

⑧



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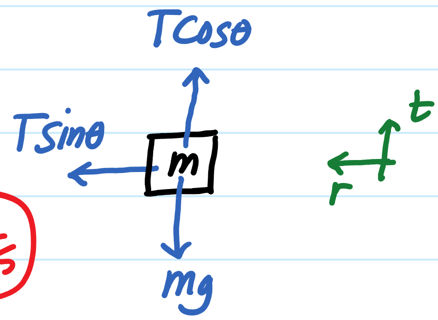

$$\sum F_r = m a_r = m \frac{v^2}{R}$$

$$T\sin\theta = m\frac{v^2}{R} \quad (1)$$

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$$\sum F_t = 0$$

$$T\cos\theta - mg = 0$$

5/25

$$T\cos\theta = mg$$

$$T = \frac{mg}{\cos\theta} \quad (2)$$

plugging (2) into (1):

$$T\sin\theta = m\frac{v^2}{R}$$

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$$\underbrace{\frac{mg}{\cos\theta}\cdot\sin\theta}_{\tan\theta} = m\frac{v^2}{R} \rightarrow \cancel{m}g\tan\theta = \cancel{m}\frac{v^2}{R}$$

$$v^2 = Rg\tan\theta$$

$$\boxed{V_{max} = \sqrt{Rg\tan\theta}}$$

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