CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING

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Subsection: UA UB UC

PROBLEM I: Solving non-linear equations [40 pts]

- Consider the following function: $f(x) = 2\ sin(x) - \frac{x^2}{10}$

- Is there a root in the interval [2 3]? Why?

Note: x should be in radian

- Write a formulation to solve for $f(x) = 0$ using the fixed point method.
- We will try to solve the problem using Newton-Raphson method:

a- Sketch the Newton-Raphson method.

b- Solve $f(x) = 0$ with as initial guess $x_0 = 2.5$, compute 2 iterations and always keep **4** significant digits (example: **0.001**)

Iteration	**X**
0	2.5
1	
2	

c- If we want to solve the same problem but with the secant method, what do you have to change is your procedure. Will you theoretically converge faster? Why?

Newton-Raphson Method: $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

PROBLEM II: System of linear equations [40 pts]

Consider the following system of linear equations:

$$\begin{cases} 3x_1 - 0.1x_2 - 0.2x_3 = 7.85 \\ 0.1x_1 + 7x_2 - 0.3x_3 = -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 = 71.4 \end{cases}$$

A\ LU Decomposition:

- Decompose the system into L and U using the **Crout's method**. [keep **4** significant digits (example: **0.001**)]. **You HAVE to use the formulations that are given below.**

[L]	[U]

- Explain, briefly, the steps that you will have to follow to solve this system of linear algebraic equations using LU decomposition method (you do not have to solve the system).

LU decomposition:

$$l_{ij} = \left\{ a_{ij} - \sum_{k=1}^{j-1} l_{ik} u_{kj} \right\}$$; i≥j; i=**1**,2,…,**n**

$$u_{ij} = \left\{ \frac{a_{ij} - \sum_{k=1}^{i-1} l_{ik} u_{kj}}{l_{ii}} \right\}$$; i<j; j=**2**,3,…,**n**

and $u_{ii} = 1$; i=1,2,…,n

B\ Gauss Seidel:
We want to solve the above system using Gauss-Seidel method.

- Will the Gauss-Seidel method converge? Why?
- Solve the system using as initial guesses $x_2 = x_3 = 0$. Use 2 iterations and compute the relative error. Always keep **5** significant digits (example: **0.0001**).

Iteration	X_1	X_2	X_3	Errors
0		0	0	
1				
2				

Gauss-Seidel:

$$x_i^{k+1} = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1}\underbrace{a_{ij}x_j^{k+1}}_{NEW} - \sum_{j=i+1}^{n}\underbrace{a_{ij}x_i^{k}}_{OLD}\right]; \text{ i=1,2,...,n; k=1,2,3, ...}$$

PROBLEM III: Theory [20 pts]

A\ The tolerance, *tol*, of the solution in the bisection method is given by: $tol = \frac{1}{2}(b_n - a_n)$, where a_n and b_n are the endpoints of the interval after the n^{th} iteration. The number of iterations *n* that are required for obtaining a solution with a tolerance that is equal to or smaller that a specified tolerance can be determined before the solution is calculated. Show that *n* is given by:

$$n \geq \frac{\log(b-a) - \log(tol)}{\log 2}$$

B\ Show that the expression of the second derivative of any continuous function can be approximated using the following formulation:

$$f''(x_i) \cong \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}$$

What is the order of the truncation error?

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)}{2!}(x_{i+1} - x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i+1} - x_i)^3 + \ldots + \frac{f^n(x_i)}{n!}(x_{i+1} - x_i)^n + R_n$$

With $h=x_{i+1}-x_i$

Good LucK