CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	203	All
Examination	Date	Duration
Midterm	18 February, 2012	1 h 30 min
Special Instructions:	Only approved calculators are allowed	
mstructions:	Show all your work	

MARKS

[6] 1. Solve for x (Note: guessing the answer and checking by substitution is not an acceptable solution.)

(a)
$$\log_2(3x) - \log_2(1-x) = 2$$

(b)
$$9^x - 4 \cdot (3^x) = 12$$

[7] 2. (a) Let $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x^2 - 1}$. Find the following functions and their domains:

(i)
$$f \circ g$$

(ii)
$$g \circ f$$

(b) Given the function $f(x) = \frac{1}{3} \ln(2 + e^x)$, find the inverse function f^{-1} and the domain of f^{-1} .

[6] 3. Evaluate the limits:

(a)
$$\lim_{x\to 0} \frac{\sqrt{3x^2+4}-2}{x^2}$$

(b)
$$\lim_{x \to -\infty} \frac{\sqrt[3]{1 + 6x^4 - 8x^9}}{3x^3 - 4x^2 + 2}$$

[3] 4. Let
$$f(x) = \frac{x-4}{x^2+2x-24} + 2$$
.

Find the equations of all horizontal and vertical asymptotes.

(continued on the other side)

[12] 5. Find the derivatives of the following functions (you don't need to simplify the final answer, but you must show how you calculate it):

(a)
$$f(x) = \frac{3x^{2/3}}{1+x^3}$$

(b)
$$f(x) = e^x(\sin x - \cos x)$$

(c)
$$f(x) = \sqrt{x + \sqrt{x^2 + 1}}$$

(d)
$$f(x) = \ln(1 + \arctan x)$$

- [6] 6. Let $f(x) = \frac{1}{x}$.
 - (a) Use the definition of derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
 to find $f'(2)$.

- (b) Find the equation of the tangent line T(x) to the graph of f(x) at the point (2, f(2)).
- [3] Bonus Question.

Use the definition $|x| = \sqrt{x^2}$ and the Chain Rule to show that if $f(x) = |\sin x|$ then $f'(x) = \frac{\sin x}{|\sin x|} \cos x$ whenever $\sin x \neq 0$.

What can you say about f'(x) when $\sin x = 0$?