

Part A [18 Points] There are 6 **multiple-choice** questions in this part.

A correct choice for one question will get you 3 points. To choose an answer, simply draw a circle around the bullet (). An incorrect answer, marking several answers or selecting nothing for a question will get you -1. If you believe that more than one answer is correct, select the best answer. The minimum total mark for this part is 0.

- ✓ 1. Here is an incorrect kind of pseudo code a student provided for the algorithm which is supposed to determine whether a sequence of parentheses is balanced:

```

declare a character stack
while ( more input is available)
{
    read a character
    if ( the character is a '(' )
        push it on the stack
    else if ( the character is a ')' and the stack is not empty )
        pop a character off the stack
    else
        print "unbalanced" and exit
}
print "balanced"

```

Which of these unbalanced sequences does the above code think is balanced?

- ☒ a) ((()) (((())
☐ b)))((()
☐ c) (())) (((()
☐ d) ((()))

- ✓ 2. Suppose we have a circular array implementation of the queue class, with ten items in the queue stored at data [2] through data [11]. The current capacity is 42. Where does the insert method place the new entry in the array?

2 3 4 5 6 7 8 9 10 11

- ☐ a) data[1]
☐ b) data[2]
☐ c) data[11]
☒ d) data[12]

- ✓ 3. In the linked list implementation of the queue class, where does the insert method place the new entry on the linked list?

- ☐ a) At the head
☒ b) At the tail
☐ c) After all other entries that are greater than the new entry.
☐ d) After all other entries that are smaller than the new entry.

4. You have implemented the queue with a linked list, keeping track of a front node and a rear node with two reference variables. Which of these reference variables will change during an insertion into a NONEMPTY queue?

- a) Neither changes
- b) Only front changes.
- ☒ c) Only rear changes.
- d) Both change.

5. How many recursive calls will be made if the following method is called with 6?

```
void greeting(int n)
{
    if (n > 0)
    {
        System.out.println("Hello!");
        greeting(n+1);
    }
}
```

- a) 5
- b) 6
- ☒ c) infinitely
- d) 7

6. Consider the following List ADT operations (where the p_i 's represent positions):
 $p_1 = \text{insertFirst}(\text{Ann})$, $p_2 = \text{insertAfter}(p_1, \text{Alex})$,
 $p_3 = \text{insertBefore}(p_2, \text{Lee})$, $p_4 = \text{insertFirst}(\text{Jim})$, $\text{remove}(p_4)$,
 $\text{swapElement}(p_1, p_2)$, $\text{replaceElement}(p_3, \text{Kim})$,
 $p_5 = \text{insertAfter}(\text{first}(), \text{Bob})$.

Which of the following options describes the final list in a **correct** way?

- a) (Ann, Alex, Lee, Jim)
- b) (Alex, Kim, Lee, Bob)
- ☒ c) (Alex, Bob, Kim, Ann)
- d) (Lee, Kim, Alex, Jim)

Handwritten list evolution:

- [ann]
- [ann, alex]
p1 p2
- [ann, lee, alex]
p1 p3 p2
- [jim, ann, lee, alex]
p4 p1 p3 p2
- [ann, lee, alex]
p1 p3 p2
- [alex, lee, ann]
p2 p3 p1
- [alex, kim, ann]
p2 p3 p1
- [alex, bob, kim, ann]

Part B [57 Points]

B.1 [33 Points] A palindrome is a string that reads the same forward and backward, capitalization and space are ignored. For example *deed*, *go dog*, *level*.... are palindromes.

a) [7 Points] Write an iterative algorithm in pseudo code that tests whether a string is a palindrome.

Algorithm *palindrome*(*palindromeString*)

```

length ← length of palindromeString
for i ← 0 to floor of (length/2) - 1 do
    if charAt(i) ≠ charAt(length - 1 - i) of palindromeString then
        return false
return true
end of algorithm

```

b) [10 Points] Write a recursive algorithm in pseudocode that tests whether a string is a palindrome.

Algorithm *palindrome*(*testString*, *index*, *length*)

```

if index > floor of (length/2) - 1 then
    return true
else if charAt(i) = charAt(length - 1 - i) in testString then
    palindrome(testString, index + 1, length)
else then
    return false

```

c) [10 Points] Describe how you could use a *Stack* or *Queue* to check whether a string is a palindrome, and write the pseudo code for that.

Algorithm *palindrome*(*stringPal*)

```

for i ← 0 to floor of (length/2) - 1 do
    push(charAt(i))
for k ← ceiling of (length/2) to length - 1 do
    if pop() ≠ charAt(k) then
        return false
return true
end

```

d) [6 Points] What are the worst-case runtimes of above algorithms (a), (b), (c) (use the big-Oh notation). Justify your answers.

algorithm a is $O(n)$ since one for loop is used
 algorithm b is $O(n)$ because we do $n/2$ recursive calls
 algorithm c is $O(n)$ because we use 2 separate for loops

B.2 [24 Points] A group of students candidates went for a summer job interview with an IT company, here are some of the programming/algorithmic solutions they wrote. Describe the worst case running time of the following solutions in Big-Oh notation in terms of the variable n . Provide the answer in the box beside each algorithm. A correct choice for one question will get you 4 points.

Showing your work is not required (– don't spend a lot of time showing your work)

I.

```
void silly(int n, int x, int y) {
    if (x < y) {
        for (int i = 0; i < n; ++i)  $n$ 
            for (int j = 0; j < n * i; ++j)  $O(n^2)$ 
                System.out.println("y = " + y);
    } else {
        System.out.println("x = " + x);
    }
}
```

$O(n^3)$

II.

```
void silly(int n) {
    for (int i = 0; i < n * n; ++i) {  $n^2$  }  $n^3$ 
        for (int j = 0; j < n; ++j) {  $n$  }
            for (int k = 0; k < i; ++k)  $n^2$ 
                System.out.println("k = " + k);
            for (int m = 0; m < 100; ++m) 100
                System.out.println("m = " + m);
        }
}
```

$O(n^5)$

III.

```
void silly(int n) {
    for (int i = 0; i < n; ++i) {  $n$ 
        for (int j = 0; j < n; ++j)  $n$ 
            System.out.println("j = " + j);  $n^2$ 
        for (int k = 0; k < i; ++k) {  $n$ 
            System.out.println("k = " + k);  $n(n+1)/2$ 
            for (int m = 0; m < 100; ++m)
                System.out.println("m = " + m);
        }
    }
}
```

$O(n^2)$

IV.

```
void nero(int n) {
    for(int i=0; i < n; i++) {
        System.out.println("!");
    }
    for(int k=0; k < n*n*n; k++) {
        System.out.println("!");
    }
}
```

$$O(n^3)$$

V.

```
int vespasian(int n, int m) {
    for(int i=m; i > 0; i--) { m
        for(int j=0; j < 200; j++) { 200
            for(int k=0; k < n; k += 3) { n/3
                System.out.println("$");
            } } }
}
```

$$200(m \cdot n)/3$$

$$O(mn)$$

VI.

```
void claudius(int n) {
    for(int i=0; i < n; i++) { n
        int j = 1;
        while(j < n) {
            System.out.println("j = " + j);
            j = j * 2;
        } }
}
```

$$O(n \log n)$$

Part C [25 Points]

For each of the 5 questions in this part, mark T if the given statement is **ALWAYS** true. Otherwise mark F and **justify** your answer. If you do not justify the FALSE case you will lose $\frac{4}{5}$ of the mark. There is **no penalty** for selecting a wrong answer. **Hint:** a correct counter example and/or correct specification will give you better marks. A correct answer will get you 5 points.

1. If $f(n) = 5n^2$ then $f(n) \in \Omega(2^n)$

☐ T

☒ F

for $f(n)$ to be in $\Omega(g(n))$
 $f(n)$ must be bigger or equal to $c g(n)$
 in that case, $g(n)$ is 2^n ; therefore, there is no way
 $f(n) \in \Omega(2^n)$. It would work if it was big-O

2. The worst-case asymptotic running time for the best algorithm for finding something in a sorted array? is $O(n)$

☒ T

☒ F

the best algorithm for finding something is the binary search.
 using this method, we halve the data to analyze every
 iteration. the worst case of asymptotic runtime is then
 $O(\log n)$ not $O(n)$

3. The worst-case asymptotic running time of finding and removing all values greater than 12 from a stack implemented with a linked-list (leaving the rest of the stack in its original order) is $O(1)$

☐ T

☒ F

this is false since we would have to
 pop every element. Remove all the values greater
 than 12 and push the remaining element in reverse
 order. this would have a complexity of at least
 $O(n)$.

4. Removing at the tail of a singly linked list is very efficient: performed in a constant time $O(1)$.

☐ T

☒ F

Depending how one might implement a linked list,
 he might not include a pointer to the tail.
 To remove the tail, he would have to traverse the
 whole list of n elements and then remove the tail.
 this would be $O(n)$. Because of this exception, it is not always true,
 therefore I answered false

5. Having $f(n) \in O(g(n))$ implies $g(n) \in \Omega(f(n))$

☐ T

☒ F

big-O implies that $f(n) \leq C g(n)$
 for $n \geq n_0$
 big-Omega implies that $g(n) \geq C f(n)$

We can move the C from the $n \geq n_0$
 big-Omega to the left handed side
 by using a division. $\frac{1}{C}$ is a constant so we can rewrite it to C_2
 $f(n) \leq C g(n) \Leftrightarrow C_2 g(n) \geq f(n) \quad C_1 = C_2$

Same equation

End of Exam!