Ask Merlin Tomorrow if solved ok.
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CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	203	All
Examination	**Date**	**Duration**
Midterm Test	2 March, 2014	1 h 30 min
Special Instructions:	Only approved calculators are allowed **Show your work for full marks**	

1. (11 marks): **(a)** An open rectangular box (no top) with volume 3 cubic meters has a square base. If the length of each side of the base is x and the height is h, express the surface area S of the box as a function of x only (**not** x and h).

(b) Let $f(x) = \sqrt{2x-8}$ and $g(x) = x^2 - 5$ Find the composite functions $f \circ g$ and $g \circ f$, and determine their domains.

(c) Find the inverse function $f^{-1}(x)$ of $f(x) = \log_5(2x-1)$ and determine the domain and the range of $f^{-1}(x)$.

2. (8 marks) Find the limit or explain why the limit does not exist:

(a) $\lim_{t\to 0}\left(\frac{1}{t} - \frac{1}{t\sqrt{1+t}}\right)$

(b) $\lim_{x\to 3}\frac{x\,|x-3|}{x^2-9}$

3. (4 marks) Find all horizontal and vertical asymptotes of the graph

$$y = \frac{x^2 - x}{x^2 - 6x + 5}$$

4. (5 marks) Find the second derivative of

$$f(x) = x\,e^x(1 + e^{-x}).$$

(continued on the other side)

5. (16 marks) Find the derivatives of the following functions. (You don't need to simplify the final answer, but you must show how you calculate it):

(**a**) $f(x) = x\sqrt{x}\left(x - \dfrac{1}{x}\right)^2$

(**b**) $f(x) = (1 + x^3)\, e^{3x}$

(**c**) $f(x) = \dfrac{\cos^2 x}{1 + \tan x}$

(**d**) $f(x) = \sqrt{x^2 + \cos(e^{x^3 \sin x})}$

6. (6 marks) Given the function $f(x) = \dfrac{2x}{2 + x}$,

(a) Calculate $f'(x)$ using its definition as a limit of difference quotient.

(b) Write equation of the tangent line to the curve $y = f(x)$ at the point $(2, f(2))$.

Bonus Question (3 marks). Give an example of a function $f(x)$ for which $f'(0)$ exists but $f''(0)$ does not, or explain why this is impossible.