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CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	203	All
Examination	**Date**	**Pages**
Final	April 2013	3
Instructors:		**Course Examiners**
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Special Instructions:	Only calculators approved by the Department are allowed	

MARKS

[10] **1.** (a) Let $f(x) = \sqrt{4-x^2}$. Find $h = f \circ f$ and determine the domain and the range of f and the domain of h.

(b) Given the function $f = \log_3(2^x - 4)$ find the inverse function f^{-1}, and determine the domain of f and the domain of f^{-1}.

[12] **2.** Evaluate the limits **Do not use l'Hôpital rule**:

(a) $\displaystyle\lim_{x\to-3} \frac{x^2+5x+6}{x^2-9}$ (b) $\displaystyle\lim_{x\to 1} \frac{\sqrt{x+8}-3}{x-1}$ (c) $\displaystyle\lim_{x\to\infty} \frac{x\sqrt{2+16x^2+x^4}}{1-3x^3}$

[5] **3.** Calculate both one-sided limits of $f(x) = \dfrac{|x^2-5x|}{x-5}$ at the point(s) where the function f is discontinuous.

[16] **4.** Find the derivatives of the following functions:

(a) $f(x) = \dfrac{\sqrt{x^5} - x^2 + e\sqrt{x}}{x^{3/2}}$

(b) $f(x) = \ln \dfrac{x^3}{(x+3)^4}$

(c) $f(x) = e^{-x}(x^2 \arctan x)$

(d) $f(x) = \sin(x + \sin(x + \sin x))$

(e) $f(x) = (1+2x)^{\ln x}$ (use logarithmic differentiation)

[16] 5. (a) Verify that the point (1,-2) belongs to the curve defined by the equation $xy - x\sqrt{5+y^2} + 6 = x^2$, and find the equation of the tangent line to the curve at that point.

(b) A particle is moving on the trajectory $(x(t), y(t))$ described by the equation $x^2 + 4y^2 = 8$ in the (x, y) plane. At an instant t when the (x, y) coordinates are $(2, 1)$ the x-coordinate is changing at the rate $\frac{\mathrm{d}x}{\mathrm{d}t} = 16\,\frac{\mathrm{m}}{\mathrm{sec}}$. How fast is the y-coordinate changing at that instant?

(c) Use l'Hôpital's rule to evaluate the $\lim\limits_{x\to 0} \frac{\sin(3x) - 3x}{3x^4 - x^3}$.

[6] 6. Let $f(x) = x^3 - 3x + 2$.

(a) Find the slope m of the secant line joining the points $(0, f(0))$ and $(2, f(2))$.

(b) Find all points $x = c$ (if any) on the interval [0,2] such that $f'(c) = m$.

[9] 7. Consider the function $f(x) = \sqrt{1+3x}$.

(a) Use the **definition of the derivative** to find the formula for $f'(x)$.

(b) Write the linearization formula for f at $a = 5$

(c) Use this linearization to approximate the value of $f(6) = \cancel{\sqrt{18}}$ $\sqrt{19}$

[12] 8. (a) Find the absolute extrema of $f(x) = \frac{x-1}{3+2x}$ on the interval $[-1, 2]$.

(b) Find the radius r and the height h of the cylinder that has a given volume V, but has the smallest possible surface area including both its bottom and the top (i.e. express r and h as functions of V).

[14] **9.** Given the function $f(x) = x^4 - 6x^2$.

(a) Find the domain of f and check for symmetry. Find asymptotes of f (if any).

(b) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).

(c) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

(d) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question**

We know that a function f is differentiable on the interval [0,2] and has values $f(0) = 0$, $f(1) = 1$ and $f(2) = -1$. Is this information sufficient to claim, using the Mean Value theorem, that the tangent line to the graph of $f(x)$ must be horizontal at least at one point x in the interval (0,2)? Explain why yes or why not.