CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	205	All
Examination	**Date**	**Pages**
Final	December 2019	2
Instructors:	L. Dube, E. Mazzeo, U. Mgbemena	**Course Examiners**
	I. Pelczer, R. Sasani	A. Atoyan & H. Proppe
Special Instructions:	Only approved calculators are allowed. **Show all your work for full marks.**	

MARKS

[11] **1.** **(a)** Sketch a graph of the function

$$f(x) = \begin{cases} -\sqrt{9-x^2} - 1 & -3 \le x \le 0\,, \\ -2|x-3| + 2 & 0 < x \quad , \end{cases}$$

on the interval $-3 \le x \le 4$ and calculate the definite integral $\int_{-3}^{4} f(x)\,\mathrm{d}x$ in terms of area (*do not antidifferentiate*).

(b) Calculate the derivative of the function $F(x) = \sec(x) + \int_{\tan(3x)}^{1} e^{-t^2}\,\mathrm{d}t$

(Hint: use the Fundamental Theorem of Calculus and differentiation rules.)

[10] **2.** Calculate the following indefinite integrals:

(a) $\displaystyle\int \sin^5(x)\,\cos^2(x)\,dx$ **(b)** $\displaystyle\int \frac{4}{x^3+4\,x}\,dx$

[6] **3.** Find $F(t)$ such that $F'(t) = \sec^6(t)$ and $F\left(\frac{\pi}{4}\right) = 0$.

[12] **4.** Evaluate the following definite integrals (give the **exact** values):

(a) $\displaystyle\int_0^1 \frac{2^{\,x}}{4^x+4}\,dx$ **(b)** $\displaystyle\int_1^4 \frac{\ln^2 x}{\sqrt{x}}\,dx$

[8] **5.** Evaluate the given improper integral or show that it diverges:

(a) $\displaystyle\int_0^1 \frac{e^{-1/x}}{x^2}\,dx$ **(b)** $\displaystyle\int_0^\infty \frac{x}{x^2+9}\,dx$

[17] **6.** **(a)** Sketch the curves $y = 3x - x^3$ and $y = -x$, and find the area enclosed by these curves.

(b) Sketch the region enclosed by $y = \cos(x)$ and the $x-$axis on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$, and find the volume of the solid of revolution of this region about the line $y = 2$.

(c) Find the average value of the function $f(x) = x\sqrt{1+2x}$ on the interval $[0, 4]$.

[6] **7.** Find the limit of the sequence $\{a_n\}$ or prove that the limit does not exist:

$$\textbf{(a)} \quad a_n = \frac{\ln(n^3)}{\sqrt{n+1}} \qquad \textbf{(b)} \quad a_n = \frac{(-1)^n 2\,n}{\sqrt{1+100n^2}}$$

[12] **8.** Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

$$\textbf{(a)} \quad \sum_{n=2}^{\infty}(-1)^n \frac{1}{n\,\ln n^2} \qquad \textbf{(b)} \sum_{n=1}^{\infty} \frac{\sin n}{n^{3/2}+1} \qquad \textbf{(c)} \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2^n-10)^2}{4^n+2^n+100}$$

[10] **9.** Find the radius and the interval of convergence of the following series

$$\textbf{(a)} \quad \sum_{1}^{\infty} \frac{(3x)^n}{n!} \qquad \textbf{(b)} \quad \sum_{n=1}^{\infty} \frac{(x+1)^{3n}}{n\,8^n}$$

[8] **10.** **(a)** Derive the Maclaurin series of $f(x) = x^3\,\ln(1+2x^2)$ (HINT: start with the series for $\ln(1+z)$ where $z = 2x^2$).

(b) Use differentiability of power series to find the sum $F(x) = \sum_{1}^{\infty} \frac{(x-1)^n}{n}$ within its radius of convergence.

[5] **Bonus Question.** Calculate the definite integral

$$\int_0^{\pi} \sin t \cdot \sin^{11}(\cos t)\;\mathrm{d}t$$