

CONCORDIA UNIVERSITY
Dept. of Computer Science and Software Engineering
COMP 335 – Introduction to Theoretical Computer Science
Fall 2018

Assignment 3

Submission through Moodle due on Thursday November 8 at 23:55

1. (15 pts) Give context-free grammars for the following languages:

- (a) $\{(ab)^i c^j d^k : j = |i - k|\}$
- (b) $\{w \in \{a, b\}^* : n_a(w) \neq n_b(w)\}$
- (c) $\{w \in \{a, b, c\}^* : |w| = 3n_a(w)\}$

2. (15 pts) Give push-down automata for the languages in Q 1 above. (Note: Do NOT use the construction to convert CFGs to PDAs. You must give PDAs obtained "directly.")
3. (15 pts) Convert the following CFGs to PDAs using the procedure given in class.

- (a) $S \rightarrow aSba \mid Ab$
 $A \rightarrow aA \mid b$
- (b) $S \rightarrow AB$
 $A \rightarrow aAcc \mid \lambda$
 $B \rightarrow ccBb \mid \lambda$

4. (15 pts) Consider the following context-free grammar

$$S \rightarrow Sb \mid aaSb \mid b$$

- (a) Show that G is an ambiguous grammar.
 - (b) Describe $L(G)$.
 - (c) Give an unambiguous grammar G' such that $L(G') = L(G)$.
5. (5 pts) Give a context-free grammar that generates the set of all regular expressions on the alphabet $\{a, b\}$. Clearly specify the set of variables, and the set of terminals in your grammar.
6. (10 pts) Convert the following grammars to Chomsky Normal Form.

- (a) $S \rightarrow aSb \mid bSa \mid a \mid b$
- (b) $S \rightarrow aPbQRaT \mid aS \mid Rb$
 $P \rightarrow PQR \mid aP \mid PbT \mid \lambda$
 $Q \rightarrow R \mid bQ \mid \lambda$
 $R \rightarrow aSb \mid S \mid \lambda$
 $T \rightarrow ab$

7. (5 pts) Suppose G is a grammar in CNF, let $w \in L(G)$, and let T be a derivation tree for w . Prove that if the longest path in T is of length n , then $|w| \leq 2^{n-1}$.
8. (10 pts) Given a context-free grammar G , show how to convert G into a context-free grammar G' so that G' generates prefixes of strings in $L(G)$, that is, $L(G') = \{u : uv \in L(G)\}$.