Department of Electrical and Computer Engineering

ENGR 371 Probability and Statistics in Engineering
Midterm Exam – July 30, 2014
Time: 2 hours

37/40

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SECTION: GD

Special Instructions:

~~CELL PHONES OR ANY ELECTRONIC DEVICES ARE NOT PERMITTED.~~

- Attempt all questions. If you have any difficulty you may try to make REASONABLE assumptions. State the assumptions and how those assumptions limit your answers. Show all your work in detail and justify your answers.
- Marks are given for how an answer is arrived at, not just the answer itself.
- All answers are to be written into the question papers. Use the back pages and extra blank papers for your rough work.
- Show your work and put a box around your final answer. **Providing ONLY the Final answer (without any calculations) will earn ONLY a Zero in that problem.**
  - **Write Big, clear and legible. Provide a neat and professional presentation.**

Problem 1- A hospital has three models, (Model A, Model B and Model C) of infusion pumps. The hospital has 300 "Model A" infusion pumps, 500 "Model B" infusion pumps and 350 "Model C" infusion pumps. According to the manufacturer; Model A has a 1% chance of malfunctioning, Model B has a 0.7 % chance of malfunctioning and model C has 1.3% chance of malfunctioning.

a) What is the probability that if a pump is randomly selected it is defective? (4 Marks)

D = defective

	D
A = 300	.01
B = 500	.007
C = 350	.013

$P(D|A) = .01$ $P(A) = .261$

$P(D|B) = .007$ $P(B) = .434$

$P(D|C) = .013$ $P(C) = .304$

$P(D) = P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C)$

$= (.01)(.261) + (.007)(.434) + (.013)(.304)$

$= \boxed{.0096}$

b) **If the randomly selected pump is defective compute the probability that it is a Model C pump. (3 Marks)**

$$P(C|D) = \frac{P(D|C)\,P(C)}{P(D)} = \frac{(.013)(.304)}{.0096} = \boxed{.4116}$$

$P(D)$ = calculated in (a)

c) What is the probability that if Model A pump is selected it is defective? **(3 Marks)**

$$P(A|D) = \frac{P(D|A)\,P(A)}{P(D)} = \frac{(.01)(.261)}{.0096} = \boxed{.2718}$$

Problem 2- A sample of 6 people out of group of 50 people are tested for drug use. If 3 of the 50 people are drug users.

a) How many ways of selecting 6 people will end up with 2 drug users? **(4 Marks)**

n = sample = 6
T = Total = 50
D = Drug user = 3
D' = Non Drug = 47

$$\binom{47}{4}\binom{3}{2} = \boxed{535095}$$

b) **Compute the probability of selecting 6 people and having two drug users in the group. (6 Marks)**

$$\frac{\binom{47}{4}\binom{3}{2}}{\binom{50}{6}} = \boxed{.0336}$$

Problem 3- Let X be a discrete random variable with the following CDF

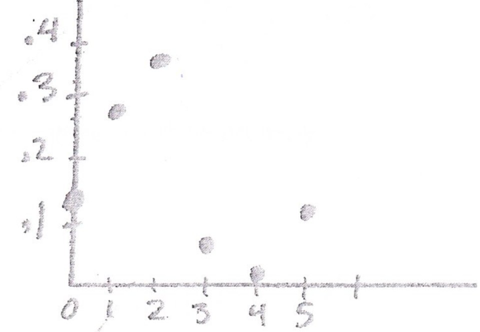
X	0	1	2	3	4	5
F(X)	0.1245	0.423	0.7697	0.8608	0.87504	?

$\boxed{? = 1}$

a) Find and plot the probability mass function (pmf). (3 marks)

$$F(x) = \begin{cases} 0 & 0 > x \\ .1245 & 0 \le x < 1 \\ .423 & 1 \le x < 2 \\ .7697 & 2 \le x < 3 \\ .8608 & 3 \le x < 4 \\ .87504 & 4 \le x < 5 \\ 1 & 5 \le x \end{cases}$$

x	0	1	2	3	4	5
f(x)	.1245	.2985	.3467	.0911	.0142	.1249



b) Find the mean and the variance of the random variable X. (2marks)

$E(x) = \mu = x\, f(x) = \boxed{1.9465}$

$E(x^2) = 5.8549$

$\sigma^2 = E(x^2) - [E(x)]^2 = 5.8549 - 3.788$

$\boxed{\sigma^2 = 2.0668}$ $\boxed{\sigma = 1.437}$

c) Find P(2<X<4) using the CDF in (a) (2 marks)

$P(2 < x < 4) = P(x = 3)$

$P(2 < x < 4) = F(3) - F(2) = .8608 - .7697 = \boxed{.0911}$

d) If another random variable $Y=X^2+2$ is formed, find the mean E[Y]. (3 marks)

$Y = x^2 + 2$ $E(x^2) = 5.8549$

$E(Y) = E(x)^2 + 2$ $E(Y) = 5.8549 + 2 = \boxed{7.8549}$

4. Your team is recruited to make a choice between two models of aircraft. The first model (Model A) has *two* engines with a reliability (i.e. probability of working properly for the entire flight) of 0.8. The second model of aircraft has *four* engines with a reliability of 0.7. You are required to determine which aircraft to choose if the criterion is based on *at least half* of the engines have to

A = .8

B = .7

be working properly for the entire flight. Show all of your work and put a box around your final answer.

a) Compute the probability that Model A will have at least half of the engines working properly for the entire flight. **(3 Marks)**

X= working engin

P(A)=.8

$P(X \geq 1) = P(X=1) + P(X=2)$

Binomial

$\binom{n}{x} p^x (1-p)^{n-x}$ $P(X=1) = \binom{2}{1}(.8)(.2) = .32$

$P(x \geq 1) = .32 + .64 = \boxed{.96}$

$P(x=2) = \binom{2}{2}(.8)^2 = .64$

b) Compute the probability that Model B will have at least half of the engines working properly for the entire flight. **(3 Marks)**

$P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$

$P(X=2) = \binom{4}{2}(.7)^2(.3)^2 = .2646$ $P(X=4) = \binom{4}{4}(.7)^4(.3)^0 = .2401$

$P(X=3) = \binom{4}{3}(.7)^3(.3)^1 = .4116$ $P(X \geq 2) = .2646 + .4116 + .2401 = \boxed{.9163}$

c) From the above information, select the model with the higher probability of of having at least half of the engines working. **(2 Marks)**

$\boxed{M_A = .96 = 96\%}$ factional Better alternative

$M_B = .9163 = 91.63\%$ factional

d) Compute the expected number of engines working for each model. Does this information support the same conclusion as part c? Justify your answer. **(2 Marks)**

$A = \binom{2}{2}(.8)^2(1-.8)^0 = .64$

$B = \binom{4}{4}(.7)^4(1-.7)^0 = .24$

Modle A is much more likly to have all engins working and this supports (c)