CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	209	All except EC
Examination	**Date**	**Pages**
Final	December 2015	2
Instructors		**Course Examiner**
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Special Instructions

- ▷ Ruled booklets to be used.
- ▷ Only approved calculators allowed.

[MARKS]

[14] **1.** (a) Find $\displaystyle\lim_{x\to -4} \frac{x^2+28x+96}{4x^2+7x-36}$.

(b) Find $\displaystyle\lim_{x\to 2} \frac{4-\sqrt{6x+4}}{3x^2-12}$.

(c) Give an example of a function f defined for all real numbers which has the property that $\displaystyle\lim_{x\to+\infty}$ and $\displaystyle\lim_{x\to-\infty}$ are both equal to $+\infty$.

[14] **2.** Find the derivatives of the following functions. YOU DO NOT HAVE TO SIMPLIFY.

(a) $f(x) = \dfrac{3^4}{\sqrt{x^7}} - 30x^2 - e^x$.

(b) $g(x) = \left(5\sqrt{x} - \dfrac{7}{x} + 6x - 8\right)(4x^5 - \ln(x) - e^2x)$.

(c) $h(x) = x^3\ln(x) - \dfrac{4}{5x} - e^{(-x^2+7)}$

[8] **3.** Use implicit differentiation to find $y' = dy/dx$

$$2x^5y^3 - 5x^3 + y^2\ln x = 7y + 4x + 7.$$

[6] **4.** Find dh if $h = x^{1.5}$, $x = 4$, and the change in the x is 0.1.

[14] **5.** Boyle's law for enclosed gases states that if the volume is kept constant, the pressure P and temperature T are related by the equation

$$\frac{P}{T} = k$$

where k is a constant. If the temperature is increasing at 3 kelvin per hour, what is the rate of change of pressure when the temperature is 250 kelvin and the pressure is 500 pounds per square inch?

[8] **6.** Use the price-demand equation $p = 60 - 0.02x$ to find the values of p for which the demand is elastic and for which the demand is inelastic.

[6] **7.** For $f(x) = x^3 - 6x^2 + 9x - 6$ find the absolute maximum and minimum, if either exists, on the interval $[-1, 2]$

[14] **8.** Graph the sales function $N(x) = 3x^3 - 0.25x^4 + 200$, over the interval $0 \le x \le 9$. Determine when N is increasing, when it is decreasing. Does N have a maximum? If so, find it. Does N have a point of inflection? If so, find it.

[6] **9.** Compute the following:

(a) $\int (4x^3 - 7x^5)\, dx$.

(b) $\int_2^3 \left(3x - \frac{6}{x} + 4e^x\right) dx$. Get the answer correct to three decimal places.

[10] **10.** Suppose that a country has Lorentz curve of the form $f(x) = x^a$ and a Gini index of 0.268. Find a.

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1 a) $\lim_{x\to -4} \frac{x^2+28x+96}{4x^2+7x-36} = \frac{(-4)^2+28(-4)+96}{4(-4)^2+7(-4)-36} = \frac{0}{0}$

$\lim_{x\to -4} \frac{\cancel{(x+4)}(x+24)}{(4x-9)\cancel{(x+4)}} = \lim_{x\to -4} \frac{x+24}{4x-9} = \frac{-4+24}{4(-4)-9} = \frac{20}{-25} = -\frac{4}{5}$

b) $\lim_{x\to 2} \frac{4-\sqrt{6x+4}}{3x^2-12} = \frac{4-\sqrt{6(2)+4}}{3(2)^2-12} = \frac{4-4}{0} = \frac{0}{0}$

$\lim_{x\to 2} \frac{(4-\sqrt{6x+4})(4+\sqrt{6x+4})}{(3x^2-12)(4+\sqrt{6x+4})} = \lim_{x\to 2} \frac{16-(6x+4)}{(3x^2-12)(4+\sqrt{6x+4})} = \lim_{x\to 2} \frac{12-6x}{\quad}$ ✓

$= \lim_{x\to 2} \frac{6(2-x)}{3(x-2)(x+2)(4+\sqrt{6x+4})} = \lim_{x\to 2} \frac{-6\cancel{(x-2)}}{3\cancel{(x-2)}(x+2)(4+\sqrt{6x+4})}$

$= \lim_{x\to 2} \frac{-6}{3(2+2)(4+\sqrt{6x+4})} = \frac{-6}{(3)(4)(4+4)} = \frac{-6}{12(8)} = -\frac{1}{16}$

c) Choose $f(x) = x^2$ then $\lim_{x\to +\infty} x^2 = +\infty$

$\lim_{x\to -\infty} x^2 = -\infty$

Sketch:

2. a) $f(x) = 3^4 (x^{-\frac{7}{2}}) - 30x^2 - e^x$

$f'(x) = 3^4(-\tfrac{7}{2})x^{-\frac{7}{2}-1} - 30(2)x^{2-1} - e^x$

$f'(x) = -81(\tfrac{7}{2})x^{-\frac{9}{2}} - 60x - e^x$

b) $g(x) = (5x^{\frac{1}{2}} - 7x^{-1} + 6x - 8)(4x^5 - \ln x - e^2x)$

$g'(x) = (5x^{\frac{1}{2}} - 7x^{-1} + 6x - 8)(20x^4 - \frac{1}{x} - e^2)$
$+ (4x^5 - \ln x - e^2x)(5(\tfrac{1}{2})x^{-\frac{1}{2}} + 7x^{-2} + 6)$

c) $h(x) = x^3 \ln x - \frac{4}{5}x^{-1} - e^{(-x^2+7)}$

$h'(x) = x^3(\tfrac{1}{x}) + (\ln x)(3x^2) - \tfrac{4}{5}(-1)x^{-2} - e^{(-x^2+7)}(-2x)$

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3. $2x^5y^3 - 5x^3 + y^2\ln x = 7y + 4x + 7$

$2((x^5)3y^2\frac{dy}{dx} + y^3 5x^4) - 15x^2 + y^2\frac{1}{x} + (\ln x)(2y\frac{dy}{dx}) = 7\frac{dy}{dx} + 4$

8 marks $\quad 6x^5y^2\frac{dy}{dx} + 2y\ln x\frac{dy}{dx} - 7\frac{dy}{dx} = -10y^3x^4 + 15x^2 - \frac{y^2}{x} + 4$

$\frac{dy}{dx}[6x^5y^2 + 2y\ln x - 7] =$ ✓

$\frac{dy}{dx} = \frac{-10y^3x^4 + 15x^2 - \frac{y^2}{x} + 4}{6x^5y^2 + 2y\ln x - 7}$

4 $\quad h = x^{1.5}$

$\frac{dh}{dx} = \frac{3}{2}x^{1.5-1}$

6 marks $\quad dh = \frac{3}{2}x^{.5}dx \implies dh\Big|_{\substack{x=4 \\ dx=.1}} = (\tfrac{3}{2})4^{\frac{1}{2}}(.1) = 3(.1) = 0.3$

when

5. $\frac{P}{T} = k \Rightarrow P = kT$

① $\frac{dP}{dt} = ?$ pound/hour

$\frac{dT}{dt} = 3$ Kelvins/hour

② $P = kT$

③ $\frac{d}{dt} P = \frac{d}{dt} kT$

$\frac{dp}{dt} = k \frac{dT}{dt}$

$\left.\frac{dp}{dt}\right|_{\frac{dT}{dt}=3} = k(3)$

Note: we can find k: $\frac{P}{T} = k$

$\frac{500}{250} = k$

$\Rightarrow k = 2$

$\Rightarrow$ Final Answer $\frac{dp}{dt} = 2(3)$

$\frac{dp}{dt} = 6$ pound/hour.

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6. ① $P = 60 - .02x$

$.02x = 60 - p$

$x = \frac{60}{.02} - \frac{1}{.02} P$

$x = 3000 - 50p$

② $\frac{d}{dp} x = \frac{d}{dp}(3000 - 50p)$

$\frac{dx}{dp} = -50$

③ $E = -\frac{P}{x} * \frac{dx}{dp}$ OR $\left(E = -P * \frac{f'(p)}{f(p)}\right)$

$E = \frac{-P}{3000-50p} * (-50)$

$E = \frac{50p}{3000-50p}$

④ Elastic $E > 1$ | In Elastic $0 < E < 1$

$\Rightarrow \frac{50p}{3000-50p} > 1$ | $\Rightarrow \frac{50p}{3000-50p} < 1$

$50p > 3000 - 50p$ | $50p < 3000 - 50p$

$100p > 3000$ | $100p < 3000$

$p > \frac{3000}{100}$

$p > 30\$$ | $0 < p < 30\$$

7. ① $f(x) = x^3 - 6x^2 + 9x - 6$

$f'(x) = 3x^2 - 12x + 9$

② let $f'(x) = 0$ $f'(x) = \frac{1}{0}$ No x here

$3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$(x-1)(x-3) = 0$

$x - 1 = 0$ | $x - 3 = 0$

$x = 1$ | $x = 3$ Not in given interval

③ $f(1) = 1^3 - 6(1)^2 + 9(1) - 6 = -2$ Absolute Max

$f(-1) = (-1)^3 - 6(-1)^2 + 9(-1) - 6 = -22$ Absolute Min

$f(2) = (2)^3 - 6(2)^2 + 9(2) - 6 = -$

8. $N'(x) = 3(3)x^2 - .25(4)x^3$

$N'(x) = 9x^2 - 1x^3$

$N'(x) = x^2(9-x)$

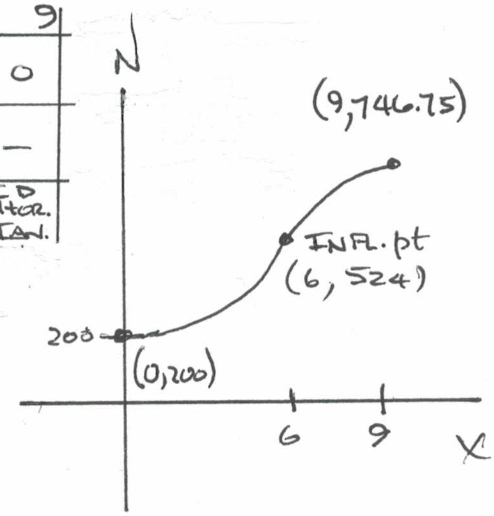
$N'(x)=0$	$N'(x)=\frac{1}{0}$
$x^2(9-x)=0$ $x^2=0$, $x=0$ | $9-x=0$, $x=9$	No x here

$N''(x) = 9(2)x - 3x^2$

$= 18x - 3x^2$

$N''(x)=0$	$N''(x)=\frac{1}{0}$
$18x-3x^2=0$ $3x(6-x)=0$ $3x=0$, $x=0$ | $6-x=0$, $x=6$	No x

X	0	0<x<6	6	6<x<9	9
$N'(x)$	0	$N'(1)=+$	+	+	0
$N''(x)$	+	$N''(1)=+$	0	$N''(7)=-$	−
$N(x)$	CU HOR. TAN.	INC, C.U.	INC INF.pt	INC. CD	CD HOR. TAN.



$N(0)=200$ MIN

$N(9)=746.75$ MAX

9. a) $\int (4x^3 - 7x^5)\,dx$

$\int 4x^3 dx - \int 7x^5 dx$

$4\int x^3 dx - 7\int x^5 dx$

$4\frac{x^4}{4} - 7\frac{x^6}{6} + C$

$x^4 - \frac{7}{6}x^6 + C$

b) $\int_2^3 3x\,dx + \int \frac{6}{x}\,dx + \int 4e^x dx$

$3\int_2^3 x\,dx + 6\int_2^3 \frac{1}{x}\,dx + 4\int_2^3 e^x dx$

$\left[3\frac{x^2}{2} + 6\ln x + 4e^x\right]_2^3$

$\left(\frac{3}{2}(3)^2 + 6\ln 3 + 4e^3\right) - \left(\frac{3}{2}(2)^2 + 6\ln 2 + 4e^2\right)$

10 Gini Index $= 2\int_0^1 (x - f(x))\,dx$

$.268 = 2\int_0^1 (x - x^a)\,dx$

$= 2\left[\int_0^1 x\,dx - \int_0^1 x^a dx\right]$

$= 2\left[\left(\frac{x^2}{2} - \frac{x^{a+1}}{a+1}\right)_0^1\right]$

$.268 = 2\left[\left(\frac{1^2}{2} - \frac{1^{a+1}}{a+1}\right) - \left(\frac{0^2}{2} - \frac{0^{a+1}}{a+1}\right)\right]$

$.268 = 1 - \frac{2}{a+1}$

$.268 - 1 = -\frac{2}{a+1} \Longrightarrow$

$-.732(a+1) = -2$

$-.732a = -2 + .732$

$a = 1.732$