Concordia University
ENGR 371 - Probability and Statistics

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Final Exam December 7, 2005

NOTES

- The exam consists of two parts, one that has 15 multiple choice questions and the second has 3 regular questions. You are asked to attempt ALL questions.
- For the multiple choice part, circle only **ONE** answer for each question. Otherwise, your answer will not be considered.
- Each multiple choice question is worth TWO points.
- Feel free to guess an answer if you are not sure about the correct answer. Wrong answers do not deduct from the correct answers.
- Your final answers **MUST** be put on this cover page as this is the **ONLY** page that will be looked at during grading.
- You will be given a separate booklet to use for the three regular questions.

Name:_____

Student ID:_____

1)	(a)	(b)	(c)	(d)
2)	(a)	(b)	(c)	(d)
3)	(a)	(b)	(c)	(d)
4)	(a)	(b)	(c)	(d)
5)	(a)	(b)	(c)	(d)
6)	(a)	(b)	(c)	(d)
7)	(a)	(b)	(c)	(d)
8)	(a)	(b)	(c)	(d)
9)	(a)	(b)	(c)	(d)
10)	(a)	(b)	(c)	(d)
11)	(a)	(b)	(c)	(d)
12)	(a)	(b)	(c)	(d)
13)	(a)	(b)	(c)	(d)
14)	(a)	(b)	(c)	(d)
15)	(a)	(b)	(c)	(d)

Multiple Choice Questions

1) Let S^2 be the variance of a random sample of size n taken from a normal population having the variance σ^2, which of the following statement is true?
   a) S^2 is a biased estimator of σ^2.
   b) S^2 has a chi-squared distribution with n degrees of freedom.
   c) S^2 has a chi-squared distribution with n-1 degrees of freedom.
   d) $\frac{(n-1)S^2}{\sigma^2}$ has a chi-squared distribution with n-1 degrees of freedom.

2) A random variable X has mean $\mu = 10$ and variance $\sigma^2 = 4$. Using Chebyshev's theorem, find the value of the constant c such that $P(|X - 10| \geq c) < 0.04$
   a) $c = 10$ b) $c = 5$
   c) $c = 20$ d) $c = 12$

3) If X_1 and X_2 are independent random variables with the same probability density function *f(x)*

$$f(x) = \begin{cases} 0.25, & 0 \leq x \leq 4 \\ 0, & elsewhere \end{cases}$$

   What is the expected value of $Z = X_1 X_2$?
   a) 2 b) 4
   c) 6 d) 8

4) In a certain photographic process, the developing time of prints have the normal distribution with $\mu = 16.20$ seconds and $\sigma = 0.52$ second. Find the probabilities that the time it takes to develop one of the prints will be at least 17 seconds.
   a) 0.9382 b) 0.5
   c) 0.0668 d) 0.0618

5) $X_1, X_2, \cdots, X_n$ are independent random variables with identical distributions. Let σ^2 be their variance and $Y = \frac{\sum_{i=1}^{n} X_i}{n}$. What is the variance of *Y*?
   a) σ^2 b) $n\sigma^2$
   c) $\frac{\sigma^2}{n}$ d) $\frac{\sigma^2}{n^2}$

6) If we consider all possible unbiased estimators of some parameter θ, the most efficient estimator of θ is:
   a) the one with the smallest mean
   b) the one with the smallest standard deviation
   c) the one with the largest mean
   d) the one with the smallest median.

7) If P and Q are two events having positive probability in the sample space S such that $P \cap Q = \varnothing$, then all of the following pairs are independent EXCEPT:
   a) $\varnothing$ and P b) P and Q
   c) $\varnothing$ and the complement of $P \cap Q$ d) P and S.

8) For the joint density function

$$f(x,y)=\begin{cases}\dfrac{x(1+3y^2)}{4} & 0<x<2, 0<y<1\\ 0 & \text{elsewhere}\end{cases}$$

the value of P(1/4 < X < ½|Y=1/3) is given by:

a) 1/60 b) 2/65
c) 3/64 d) 5/72

9) The number of ways a committee of 3 people can be formed from a group of 5 people is:

a) 10 b) 8
c) 6 d) 4

10) In a true-false test with 100 questions, if a person guesses on each question, the expected number of correct answers is:

a) 60 b) 70
c) 50 d) 40

11) An urn contains two black balls and three white balls. Two balls are selected at random from the urn without replacement and the sequence of colors is noted. Find the probability that both are black.

a) 1/4 b) 2/5
c) 1/10 d) 3/5

12) The probability that a certain machine will produce a defective item is 0.20. If a random sample of 6 items is taken from the output of this machine, what is the probability that there will be 5 or more defectives in the sample?

a) 0.0154 b) 0.0015
c) 0.2458 d) 0.0016

13) The Gallup Poll has decided to increase the size of its random sample of Canadian voters from about 1500 people to about 4000 people. The effect of this increase is to:

a) increase the standard error of the estimate.
b) reduce the variability of the estimate.
c) increase the confidence interval width for the parameter.
d) have no effect since the population size is the same.

14) Which of the following statements about confidence intervals is INCORRECT?

a) A confidence interval for a mean always contains the sample mean.
b) If we keep the confidence coefficient fixed, the confidence interval gets narrower as we increase the sample size.
c) If the population standard deviation increases, the confidence interval decreases in width.
d) If the confidence intervals for two means do not overlap very much, there is evidence that the two population means are different.

15) If $P(A) = 0.20$, $P(B) = 0.30$ and $P(A \text{ and } B) = 0.06$, then A and B are:

a) dependent events
b) independent events
c) mutually exclusive events
d) complementary events

Regular Questions

1) (**10 points**)

a) Three dice were rolled. Given that no two faces were the same, what is the probability that there was an ace?

b) An oil well is to be drilled in a certain location where the soil is either rock (probability .53), clay (probability .21), or sand. If it is a rock, a geological test gives a positive result with 35% accuracy, if it is clay, this test gives a positive result with 45% accuracy, and if it is sand, the test gives a positive result with 75% accuracy. Given that the test is positive, what is the probability that the soil is rock?

c) You are to set up a code of 2 digit words using the digits 1, 2, 3, 4 without using any digit more than once. What is the maximum number of words in such a language?

2) (**10 points**) Suppose that a computer system contains a certain brand of hard disk whose time in years to failure is given by T. The random variable T is modeled nicely by the exponential distribution with mean time to failure $\beta = 6$.

a) What is the probability that a hard disk fails in the first 4 years?

b) If 80 of these hard disks are installed in different computer systems, what is the probability that at least 40 hard disks fail during the first 4 years?

3) (**10 points**) A quality control supervisor in a cannery knows that the exact amount each can contains will vary, since there are certain uncontrollable factors that affect the amount of fill. To estimate the amount of fill at the cannery, the supervisor randomly selects 10 cans and weighs the contents of each. The following weights (in ounces) are obtained:

7.96 7.90 7.98 8.01 7.97 7.96 8.03 8.02 8.04 8.02

a) Find the sample standard deviation.

b) Assuming that we know the variance $\sigma^2 = 0.002$, find the 96% confidence interval for the mean μ.