COMM 225: MIDTERM REVIEW QUESTIONS

TOPIC: PROJECT MANAGEMENT

Q 1.1: Kozar International, Inc. begun marketing a new instant-developing film project. The estimates of R&D activity time (weeks) for Kozar's project are given in the table below. The project has two paths: A-C-E-F and A-B-D-F. Assume the activity times are independent.

a) What is the probability that the project will be completed between 35 and 45 days?
b) If the time to complete the path A-B-D-F is normally distributed, what is the probability that this path will take at least 38 weeks to be completed?

Activity	Predecessors	Time(weeks)			Mean	Variance
		Optimistic time	Probable time	Pessimistic time		
A	-	9	9	9	9	0.00
B	A	8	10	12	10	0.44
C	A	9	12	18	12.5	2.25
D	B	5	8	11	8	1.00
E	C	5	7	10	7.166	0.69
F	D, E	10	12	14	12	0.44

Solution:

(a) What is the probability that the project will be completed between 35 and 45 days?

The project has two paths:

A-C-E-F:
- Expected Duration = 9 + 12.5 + 7.166 + 12 = 40.66 weeks (Critical Path).
- Variance $\sigma^2 = 0 + 2.25 + 0.69 + 0.44 = 3.38$, Standard Deviation = $\sigma = 1.838$

For 35 weeks, $z_{35} = \dfrac{T - \text{Expected Duration}}{\sigma} = \dfrac{35 - 40.66}{=1.838}$, or z_{35} = -3.08 ➔ Prob (z ≤ z_{35}) = 0.001

For 45 weeks, $z_{45} = \dfrac{T - \text{Expected Duration}}{\sigma} = \dfrac{45 - 40.66}{=1.838}$, or z_{45} = 2.36 ➔ Prob (z ≤ z_{45}) = 0.9909

Probability that this path will be completed between 35 and 45 days is 0.9909-0.001 = 0.9899

A-B-D-F:
- Expected Duration = 9 + 10 + 8 + 12 = 39 weeks.
- Variance = 0 + 0.44 + 1.00 + 0.44 = 1.88, Standard Deviation = 1.371

For 35 weeks, $z_{35} = \dfrac{T - \text{Expected Duration}}{\sigma} = \dfrac{35 - 39}{=1.371}$, or z_{35} = -2.918 ➔ Prob (z ≤ z_{35}) = 0.0018

For 45 weeks, $z_{45} = \dfrac{T - \text{Expected Duration}}{\sigma} = \dfrac{45 - 39}{=1.371}$, or z_{45} = 4.376 ➔ Prob (z ≤ z_{45}) = 1

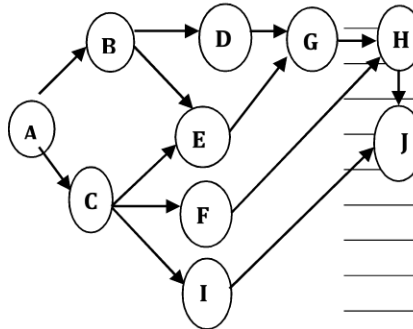Probability that this path will be completed between 35 and 45 days is 1 – 0.0018 = 0.9982

Hence, the probability of project completion between 35 and 45 days = 0.9899*0.9982= 0.9881 = 98.81%

(b) If the time to complete the path A-B-D-F is normally distributed, what is the probability that this path will take at least 38 weeks to be completed?

- The non-critical path A-B-D-F has an expected duration of 39 weeks and standard deviation of 1.371.
- This implies $Z_{38} = \frac{T - \text{Expected Duration}}{\sigma} = \frac{38-39}{1.371} = -0.73$.
- This z value corresponds to a probability of 0.2327.
- Probability that this path will take <u>less than 38 weeks</u> to be completed is 23.27%.
- Hence, the probability that this path will take <u>at least 38 weeks</u> to be completed is 76.73%.

Q 1.2: Given the following network and time & cost estimates, answer the following questions:
   (a) What is the project completion time?
   (b) What is the total cost required for completing this project on normal time?
   (c) Crash the network the maximum amount possible and compute the total crash cost.

Activity	Predecessor	Activity duration (weeks)		Activity cost ($)		Crash Cost / Week	# Days
		Normal	Crash	Normal	Crash		
A	-	6	4	10000	16000	3000	2
B	A	28	22	5000	9200	700	6
C	A	29	27	20000	20700	350	2
D	B	10	5	4000	6000	400	5
E	B,C	10	9	2500	3000	500	1
F	C	10	9	1000	7000	6000	1
G	D,E	15	14	1500	7500	6000	1
H	G,F	10	8	600	10600	5000	2
I	C	2	1	1000	2000	1000	1
J	H,I	10	8	900	8800	3950	2
TOTAL =				46,500			

Solution:
Parts (a & b)

The project has the following paths and their durations:
- A-B-D-G-H-J, Duration = 6 + 28 + 10 + 15 + 10 + 10 = 79
- A-B-E-G-H-J, Duration = 6 + 28 + 10 + 15 + 10 + 10 = 79
- **A-C-E-G-H-J, Duration = 6 + 29 + 10 + 15 + 10 + 10 = 80 (critical path)**
- A-C-F-H-J, Duration = 6 + 29 + 10 + 10 + 10 = 65
- A-C-I-J, Duration = 6 + 29 + 2 + 10 = 47

The project completion time = length of critical path = 80 weeks
Normal total cost = sum of the normal cost for all the activities = <u>**$46,500.**</u>

c)

Critical Path	Duration	It#1	It#2	It#3	It#4	It#5	It#6	It#7
A-B-D-G-H-J	79	79*	78*	77*	75*	73*	71*	70*
A-B-E-G-H-J	79	79*	78*	77*	75*	73*	71*	70*
A-C-E-G-H-J	80*	79*	78*	77*	75*	73*	71*	70*
A-C-F-H-J	65	64	64	63	61	59	57	57
A-C-I-J	47	46	46	45	43	41	41	41

It#	Critical path before crashing (length)	Activity crashed	Critical path after crashing (length)	Cumulative crashing cost
1	A-C-E-G-H-J (80)	C by 1	A-B-D-G-H-J (79)	$350
			A-B-E-G-H-J (79)	
			A-C-E-G-H-J (79)	
2	A-B-D-G-H-J (79)	D by 1	A-B-D-G-H-J (78)	$1,250
	A-B-E-G-H-J (79)	E by 1	A-B-E-G-H-J (78)	
	A-C-E-G-H-J (79)		A-C-E-G-H-J (78)	
3	A-B-D-G-H-J (78)	B by 1	A-B-D-G-H-J (77)	$2,300
	A-B-E-G-H-J (78)	C by 1	A-B-E-G-H-J (77)	
	A-C-E-G-H-J (78)		A-C-E-G-H-J (77)	
4	A-B-D-G-H-J (77)	A by 2	A-B-D-G-H-J (75)	
	A-B-E-G-H-J (77)		A-B-E-G-H-J (75)	$8,300
	A-C-E-G-H-J (77)		A-C-E-G-H-J (75)	
	A-C-F-H-J (77)		A-C-F-H-J (75)	
	A-C-I-J (77)		A-C-I-J (75)	
5	A-B-D-G-H-J (75)	J by 2	A-B-D-G-H-J (73)	
	A-B-E-G-H-J (75)		A-B-E-G-H-J (73)	$16,200
	A-C-E-G-H-J (75)		A-C-E-G-H-J (73)	
	A-C-F-H-J (75)		A-C-F-H-J (73)	
	A-C-I-J (75)		A-C-I-J (73)	
6	A-B-D-G-H-J (73)	H by 2	A-B-D-G-H-J (71)	
	A-B-E-G-H-J (73)		A-B-E-G-H-J (71)	$26,200
	A-C-E-G-H-J (73)		A-C-E-G-H-J (71)	
	A-C-F-H-J (73)		A-C-F-H-J (71)	
7		G by 1		
	A-B-D-G-H-J (71)		A-B-D-G-H-J (70)	$32,200
	A-B-E-G-H-J (71)		A-B-E-G-H-J (70)	
	A-C-E-G-H-J (71)		A-C-E-G-H-J (70)	

After crashing by 10 weeks, the duration of the project is **70 weeks** and the total cost for the crashed project is **$46,500+ $32,200 = $78,700.**

Q 1.3: The following table provides the necessary information for crashing a project. The project manager would like to crash the network by three weeks in the most economical way. Which activities should be crashed and by how many weeks?
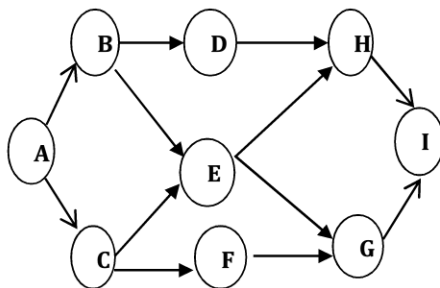
Activity	Activity duration (weeks)		Activity cost ($)		Crash Cost/ Week
	Normal	Crash	Normal	Crash	
A	4	3	4000	6000	2000
B	3	2	5000	6000	1000
C	2	1	2000	2800	800
D	5	3	4000	6000	1000
E	6	5	2500	3000	500
F	3	2	1000	2000	1000
G	4	3	2000	2900	900
H	4	3	1500	2600	1100
I	6	5	5000	12000	7000

Solution:

The project has the following paths and their durations (in weeks):
- A-B-D-H-I, Duration = 22
- **A-B-E-H-I, Duration = 23**
- **A-B-E-G-I, Duration = 23**
- A-C-E-H-I, Duration = 22
- A-C-E-G-I, Duration = 22
- A-C-F-G-I, Duration = 19

The normal duration of the project is 23 weeks and the normal total cost (sum of normal costs for all the activities) is **$27,000.**

Critical Path	Duration	It#1	It#2	It#3
A-B-D-H-I	22	22*	21*	20*
A-B-E-H-I	23*	22*	21*	20*
A-B-E-G-I	23*	22*	21*	20*
A-C-E-H-I	22	21	21*	20*
A-C-E-G-I	22	21	21*	20*
A-C-F-G-I	19	19	19	18

It#	CP Before Crashing	Activity Crashed	CP After Crashing	Cumulative cost
1	A-B-E-H-I (23) A-B-E-G-I (23)	E by 1	A-B-E-H-I (22) A-B-E-G-I (22) A-B-D-H-I (22)	$ 500
2	A-B-E-H-I (22) A-B-E-G-I (22) A-B-D-H-I (22)	B by 1	A-B-E-H-I (21) A-B-E-G-I (21) A-B-D-H-I (21)	$ 1500
3	A-B-E-H-I (21) A-B-E-G-I (21) A-B-D-H-I (21) A-C-E-H-I (21) A-C-E-G-I (21)	A by 1	A-B-E-H-I (20) A-B-E-G-I (20) A-B-D-H-I (20) A-C-E-H-I (20) A-C-E-G-I (20)	$ 3500

After crashing by 3 weeks, the duration of the project is **20 weeks** and the total cost of the crashed project is **$27,000 + $3500 = $30,500.**

Q 1.4: A company is planning to install a new computerized system for paying its employees. The management has determined the activities required for completing the project, the precedence relationships of the activities, and activity time estimates (in weeks) as given in the table below.

Activity	Preceding activity	Optimistic Time	Most likely Time	Pessimistic Time	Expected Time	Standard Deviation
A	--	7	9	14		1.167
B	A	2	2	8	3	1.000
C	A	8	12	16	12	
D	A	3	5	10	5.5	1.167
E	B	4	6	8	6	
F	B	6	8	10	8	0.667
G	C, F	2	3	4		
H	D	2	2	8	3	1.000
I	H	6	8	16	9	1.667
J	G, I	4	6	14	7	1.667
K	E, J	2	2	5		0.5

a) Calculate the missing expected times and standard deviations.
b) Find the critical path(s) and the expected project duration.
c) What is the earliest start time for activity E? What is the latest finish time for activity J?

Solution:

(a) Calculate the missing expected times and standard deviations.

Activity	Preceding activity	Optimistic Time	Most likely Time	Pessimistic Time	Expected Time	Standard Deviation	Variance
A	--	7	9	14	9.5	1.167	1.36
B	A	2	2	8	3	1	1
C	A	8	12	16	12	1.333	1.78
D	A	3	5	10	5.5	1.167	1.36
E	B	4	6	8	6	0.667	0.44
F	B	6	8	10	8	0.667	0.44
G	C, F	2	3	4	3	0.333	0.11
H	D	2	2	8	3	1	1
I	H	6	8	16	9	1.667	2.78
J	G, I	4	6	14	7	1.667	2.78
K	E, J	2	2	5	2.5	0.5	0.25

(b) Find the critical path(s) and the expected project duration.

1. A-B-E-K: 9.5+3+6+2.5=21
2. A-B-F-G-J-K: 9.5+3+8+3+7+2.5=33
3. A-C-G-J-K: 9.5+12+3+7+2.5=34
4. A-D-H-I-J-K: 9.5+5.5+3+9+7+2.5=36.5* (critical path and expected project duration is 36.5)

(c) What is the earliest start time for activity E? What is the latest finish time for activity J?

Activity	Earliest Start (ES)	Earliest Finish (EF)	Latest Start (LS)	Latest Finish (LF)
A	0	9.5	0	9.5
B	9.5	12.5	13	16
C	9.5	21.5	12	24
D	9.5	15	9.5	15
E	12.5	18.5	28	34
F	12.5	20.5	16	24
G	21.5	24.5	24	27
H	15	18	15	18
I	18	27	18	27
J	27	34	27	34
K	34	36.5	34	36.5

TOPIC: FORECASTING

Q2.1: Monthly sales for National Mixer, Inc. for a seven-month period were as follows:

Month (t)	Feb.	Mar.	Apr.	May	June	Jul.	Aug.
Sales (1000 UNITS)	19	18	15	20	18	22	20

Forecast the sales volume for September using each of the following methods:
a) 5-month moving average;
b) Weighted average, where the weights are: 0.60 (August), 0.30 (July), 0.10 (June)

c) Exponential smoothing with a smoothing constant equal to 0.20. Use the naïve approach to get the initial forecast.
d) Linear trend equation, Y = 16.86+0.5*T;

Solution:

a) *5-month moving average;*

Using the 5-month moving average, the forecast for September $= \dfrac{15+20+18+22+20}{5} = 19$

(in thousands of units)

b) *Weighted average, where the weights are - 0.60 (August), 0.30 (July), 0.10 (June)*

Using the weighted moving average, the forecast for September is $0.1*18+0.3*22+0.6*20 = 20.4$ (in thousands of units)

c) *Exponential smoothing with a smoothing constant equal to 0.20*

To get the method started, we use the naïve approach and we set the forecast for March to be equal to the actual demand in the previous period (i.e., February).

MONTH	SALES (1000 UNITS)	Forecast (alpha=0.2)	Error = (Sales-Forecast)
Feb.	19	-	
Mar.	18	19	-1
Apr.	15	18.8	-3.8
May	20	18.04	1.96
June	18	18.432	-0.432
Jul.	22	18.3456	3.6544
Aug.	20	19.07648	0.92352
September		19.26118	

The forecast for September = 19.2612 (in thousands of units)

d) *Linear trend equation, Y = 16.86+0.5*T;*
For the month of September, T = 8, as the data starts from Feb (T=1), and hence the forecast for September = 16.86+0.5*8 = 20.86 (in thousands of units).

Q 2.2: A fashion retailer buys fabric from several textile manufacturers. The demand for fabric across various locations in a city based on past historical data is shown in the table below. The manager would like to forecast the demand for the upcoming year in order to decide how many feet of fabrics they should buy.

a) Develop a trend adjusted exponential smoothing model using α=0.30 and β=0.20 and compute the adjusted exponentially smoothed forecasts for years 4 through 6.
b) Develop a trend line equation for the given data and use the resulting equation to forecast the sales for years 4 through 6.
c) Based on MAD values, which of the two forecasting models used above seems to be more accurate. Explain your answer.

Year (t)	1	2	3	4	5
Fabric (feet)	4260	4510	4050	3720	3900

Solution:

a) Trend adjusted exponential smoothing model with α=0.30 and β=0.20.

Initialization:

We will use the average of the first three periods to obtain an initial estimate for the smoothed average at the end of period 3:

$$S_3 = \frac{(4260+4510+4050)}{3} = 4273.33$$

The trend at the end of period 3 (T3) can be seen as the next change from period 1 to period 3, which is calculated as follows:

T_3 = (4050 - 4260)/(3-1) = - 105 (negative indicating a decreasing trend)

Hence, the trend adjusted forecast for period 4 is $TAF_4 = S_4 + T_3 = 4273.33 - 105 = 4168.33$

Updating: We use the update equations to find new values for S_t and T_t.

Period 4:
$S_4 = TAF_4 + \alpha(A_4 - TAF_4) = 4168.33 + 0.30(3720 - 4168.33) = 4033.831$
$T_4 = T_3 + \beta(S_4 - S_3 - T_3) = -105 + 0.20(4033.831 - 4273.33 + 105) = -131.90$
$TAF_5 = S_4 + T_4 = 4033.831 - 131.90 = 3901.93$

Period 5:
$S_5 = TAF_5 + \alpha(A_5 - TAF_5) = 3901.93 + 0.30(3900 - 3901.93) = 3901.351$
$T_5 = T_4 + \beta(S_5 - S_4 - T_4) = -131.90 + 0.20(3901.351 - 4033.831 + 131.90) = -132.016$
$TAF_6 = S_5 + T_5 = 3901.351 - 132.016 = 3769.335$
Hence, the trend adjusted forecast for period 6 is 3769.335 feet.

b) Trend line equation

Year (t)	Demand (y)	t.y	t^2	Forecast
1	4260	4260	1	
2	4510	9020	4	
3	4050	12150	9	
4	3720	14880	16	3937
5	3900	19500	25	3786
15	20440	59810	55	**3635**

The slop is given by: $b = \frac{5*59810-15*20440}{5*55-15^2} = -151$

The intercept is given as: $a = \frac{20440-(-151)(15)}{5} = 4541$

The equation is: $y_t = a + bt = 4541 - 151t$

Hence, the forecast for period 6 is: $y_6 = 4541 - 151*6 = 3635$

c) Based on MAD values, which of the two forecasting models used above seems to be more accurate. Explain your answer.

For Trend adjusted smoothing: $MAD = \frac{|3720-4168.33|+|3900-3901.93|}{2} = \frac{448.33+1.93}{2} = 225.13$

For linear trend method: $MAD = \frac{|3720-3937|+|3900-3786|}{2} = \frac{217+114}{2} = 165.5$

Based on just two observations, it seems that the linear trend method is more accurate since it has a lower MAD value.

Q2.3: Two independent methods of forecasting based on the managers experience have been prepared each month for the past 10 months. The forecasts and actual sales are as follows. Which forecast seem superior? Justify your answer with appropriate calculations using MAD, MSE & MAPE.

MONTH	FORECAST 1	FORECAST 2	ACTUAL SALES
1	771	769	770
2	785	787	789
3	790	792	794
4	784	798	780
5	770	774	768
6	768	770	772
7	761	759	760
8	771	775	775
9	784	788	786
10	788	788	790

Solution:

Method 1

Month	Actual Sales	F1	Error (A-F1)	|Error|	Error^2	(|Error|/actual) * 100
1	770	771	-1	1	1	0.130
2	789	785	4	4	16	0.507
3	794	790	4	4	16	0.504
4	780	784	-4	4	16	0.513
5	768	770	-2	2	4	0.260
6	772	768	4	4	16	0.518
7	760	761	-1	1	1	0.132
8	775	771	4	4	16	0.516
9	786	784	2	2	4	0.254
10	790	788	2	2	4	0.253
SUM	**7784**		**12**	**28**	**94**	**3.587%**

Method 2

Month	Actual Sales	F2	Error (A-F2)	|Error|	Error^2	(|Error|/actual) * 100
1	770	769	1	1	1	0.130
2	789	787	2	2	4	0.253
3	794	792	2	2	4	0.252
4	780	798	-18	18	324	2.308
5	768	774	-6	6	36	0.781
6	772	770	2	2	4	0.259
7	760	759	1	1	1	0.132
8	775	775	0	0	0	0.000
9	786	788	-2	2	4	0.254
10	790	788	2	2	4	0.253
SUM	**7784**		**-16**	**36**	**382**	**4.622%**

Method 1: $MAD = \dfrac{\sum |error|}{n} = 2.8$, $MSE = \dfrac{\sum error^2}{n} = 9.4$,

$$MAPE = \dfrac{\sum \left(\dfrac{Absolute\,Error}{Actual} \right)}{n} * 100 = \dfrac{3.587}{10} = 0.3587\%$$

Method 2: $MAD = \dfrac{\sum |error|}{10} = 3.6$, $MSE = \dfrac{\sum error^2}{10} = 38.2$

$$MAPE = \dfrac{\sum \left(\dfrac{Absolute\,Error}{Actual} \right)}{n} * 100 = \dfrac{4.622}{10} = 0.4622\%$$

All the three measures, MAD, MSE and MAPE are **smaller** for forecasting 1 and so that is **superior.**

Q 2.4: The following data are quarterly sales of natural gas in Saskatchewan by SaskEnergy (in peta joules ≈ 1 billion cubic feet) from Q1 of 2005 to Q3 of 2009.

Year	Q1	Q2	Q3	Q4
2005	49	24	18	37
2006	42	20	20	43
2007	48	24	20	40
2008	51	25	19	43
2009	51	24	15	

(a) Compute the seasonal relative for each quarter using the centred moving average method.
(b) Deseasonalize the data, fit an appropriate model to the deseasonalized data, extend the model four quarters, and reseasonalize these in order to forecast the sales of natural gas by SaskEnergy from Q4 2009 to Q3 2010.

Solutions:

a)

Year	Quarter	Sales	CMA$_4$	CMA$_2$	Sales/CMA$_2$
2005	1	49			
	2	24			
	3	18	32	31.125	0.578
	4	37	30.25	29.75	1.244
2006	1	42	29.25	29.5	1.424
	2	20	29.75	30.5	0.656
	3	20	31.25	32	0.625
	4	43	32.75	33.25	1.293
2007	1	48	33.75	33.75	1.422
	2	24	33.75	33.375	0.719
	3	20	33	33.375	0.599
	4	40	33.75	33.875	1.181
2008	1	51	34	33.875	1.506
	2	25	33.75	34.125	0.733
	3	19	34.5	34.5	0.551
	4	43	34.5	34.375	1.251
2009	1	51	34.25	33.75	1.511
	2	24	33.25		
	3	15			

Seasonal relatives	QUARTER				TOTAL
	1	2	3	4	
AVERAGE	1.466	0.702	0.588	1.242	3.999
Adjusted	**1.466**	**0.703**	**0.589**	**1.243**	4.000

b) Deseasonalizing demand through dividing the sales by the seasonal relatives for every season.

Quarter		Sales (peta joules)	Seasonal Relatives	Deseasonalized Sales
2005	1	49	1.466	33.4
	2	24	0.703	34.1
	3	18	0.589	30.6
	4	37	1.243	29.8
2006	1	42	1.466	28.6
	2	20	0.703	28.4
	3	20	0.589	34.0
	4	43	1.243	34.6
2007	1	48	1.466	32.7
	2	24	0.703	34.1
	3	20	0.589	34.0
	4	40	1.243	32.2
2008	1	51	1.466	34.8
	2	25	0.703	35.6
	3	19	0.589	32.3
	4	43	1.243	34.6
2009	1	51	1.466	34.8
	2	24	0.703	34.1
	3	15	0.589	25.5

Fitting the deseasonalized sales:

$y = 0.0569x + 31.977$