CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	204	ALL (except EC)
Examination	**Date**	**Pages**
Final	June 2016	2
Instructor		**Course Examiners**
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Special Instructions

- ▷ Only approved calculators are allowed.
- ▷ Justify all your answers.
- ▷ All questions have equal value.

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1. Using Gauss-Jordan method, find all solutions of the following system of equations:

$$\begin{array}{rcrcrcrcr} x_1 & + & 2x_2 & + & 2x_3 & & & = & 2 \\ x_1 & + & 8x_3 & + & 5x_4 & & & = & -6 \\ x_1 & + & x_2 & + & 5x_3 & + & 5x_4 & = & 3 \end{array}$$

2. Let $M = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

   (a) Find M^{-1}.

   (b) Calculate the matrix C so that $MC = B$, where $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$.

3. (a) Use Cramer's rule to solve the following system of equations:

$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 3x_3 & = & 2 \\ x_1 & & & + & x_3 & = & 3 \\ x_1 & + & x_2 & - & x_3 & = & 1 \end{array}$$

   (b) Evaluate the determinant of the matrix $A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 3 & 1 & 2 \\ -1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 4 \end{pmatrix}$

4. Let $\mathcal{L}$ be the line with parametric equations $x = 3 + t$, $y = 2 - 2t$, $z = 1 + 4t$ and let $v = (1, 2, 3)$. Find vectors w_1, w_2 such that $w_1 + w_2 = v$, and such that w_1 is parallel to $\mathcal{L}$ and w_2 is perpendicular to $\mathcal{L}$.

5. Let $P_1(1,0,2)$, $P_2(2,1,0)$, and $P_3(0,2,1)$ be 3 points

   (a) Find the area of a triangle with vertices $P_1,\ P_2,\ P_3$.

   (b) Find the equation of the plane containing $P_1,\ P_2,\ P_3$.

6. Let $\mathcal{L}$ be the line with parametric equations $x = 3 - t,\ y = 2 - t,\ z = 4 + t$ and let $\mathcal{P}$ be the plane $2x + 2y + 4z = 4$

   (a) Prove that $\mathcal{L}$ and $\mathcal{P}$ are parallel.

   (b) Find the distance between $\mathcal{L}$ and $\mathcal{P}$.

7. Let $v_1 = (1, 2, 0)$ and $v_2 = (2, 1, 0)$

   (a) Find scalars a and b such that $av_1 + bv_2 = (-2, 3, 0)$.

   (b) Find a vector v_3 such that v_1, v_2, v_3 are linearly independent.

8. Let $A = \begin{pmatrix} 1 & 2 & 0 & 0 & 3 & 7 \\ 0 & 0 & 1 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 & 1 & 2 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \\ z \\ u \\ v \\ w \end{pmatrix}$. Find a basis for the solution space of the homogeneous system $AX = 0$.

9. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$. Find all eigenvalues of A.

   Is A diagonalizable? If yes, find P so that $P^{-1}AP = D$ diagonal.

10. Let $A = \begin{pmatrix} 0 & --2 \\ 1 & 3 \end{pmatrix}$. Find A^{100}.