CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	203	All
Examination	**Date**	**Pages**
Final	April 2014	3
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Special Instructions:	Only calculators approved by the Department are allowed **Show your work for full marks**	

MARKS

[9] **1.** **(a)** Let $f(x) = \ln(1 + x^2)$ and $g(x) = \sqrt{4 + x}$. Find $f \circ g$ and $g \circ f$ and determine the domain of each of these composite functions.

(b) Find the range of the function $f = e^{2x} + 3$, the inverse function f^{-1}, and the range of f^{-1}.

[12] **2.** Evaluate the limits **Do not use l'Hôpital rule**:

(a) $\lim_{x\to 2} \dfrac{x-2}{x^2+x-6}$ **(b)** $\lim_{x\to 1} \dfrac{\sqrt{x^2+5x-5}-x}{x-1}$ **(c)** $\lim_{x\to\infty} \ln\left(\dfrac{1+x+2x^3}{3+2x+x^3}\right)$

[6] **3.** Calculate both one-sided limits of $f(x) = \dfrac{|x^2-9|}{x+3}$ at the point(s) where the function f is discontinuous.

[15] **4.** Find the derivatives of the following functions:

(a) $f(x) = \dfrac{\sqrt{x^7} + x^{5/2}}{x^3}$

(b) $f(x) = \ln(x^4 \cdot \sqrt{x+3}\,) + \ln e$

(c) $f(x) = \dfrac{\arctan(2x)}{\tan(x)}$

(d) $f(x) = \sin[x^2 \cos(e^x)]$

(e) $f(x) = (1+2x)^{x^2}$ (use logarithmic differentiation)

[15] **5.** **(a)** Verify that the point (2,1) belongs to the curve defined by the equation $xy + 2\sqrt{3+y^2} = x^3 - 2$, and find the equation of the tangent line to the curve at this point.

(b) Two cars start simultaneously moving away from the intersection of two orthogonal streets at the speeds $v_1 = 12$ m/s going west, and $v_2 = 16$ m/s going north. How fast is the distance between the cars increasing at the instant $t = 5$ seconds after they start moving from the intersection?

(c) Use the l'Hôpital's rule to evaluate the $\lim_{x\to 0} \frac{e^{x^2}-1}{\cos(2x)-1}$.

[6] **6.** Let $f(x) = 3 + x + 3\,x^2 - x^3$.

(a) Find the slope m of the secant line joining the points $(0, f(0))$ and $(3, f(3))$.

(b) Find all points $x = c$ (if any) on the interval [0,3] such that $f'(c) = m$.

[9] **7.** Consider the function $f(x) = \sqrt{2x+1}$.

(a) Use the **definition of the derivative** to find the formula for $f'(x)$.

(b) Write the linearization formula for f at $a = 4$

(c) Use this linearization to approximate the value of $f(3) = \sqrt{7}$

[12] **8.** **(a)** Find the absolute extrema of $f(x) = \dfrac{x}{x^2 - x + 1}$ on the interval $[0, 3]$.

(b) A box with a square base is to be constructed with a volume of 54 m^3. The material for the box costs \$2/m^2, and the material for the top costs \$6/m^2. Find the dimensions that minimize the cost of the box.

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[16] **9.** Given the function $f(x) = 2x^2 - x^4$.

(a) Find the domain of f and check for symmetry. Find asymptotes of f (if any).

(b) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).

(c) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

(d) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question:** Let f be a function which is monotonically decreasing (strictly) and differentiable everywhere on the real axis. Let also $g = x^2 + 1$. Prove that the composite function $h = f \circ g$ has one and only one local extremum and determine whether it corresponds to a maximum or minimum of $h(x)$.