Concordia University Department of Electrical and Computer Engineering

Course	Number	Section	
Probability and Statistics	ENGR 371	………………	
Examination	**Date**	**Time**	**# of pages**
Final Exam	December 4, 2013	3Hours	
Instructor(s)	**Student Name**		**ID#**
Drs. I. Gohar, S. Sharifi	………………………………………….		…………………………….

Materials allowed: [X] No [ ] Yes (Please specify)

Calculators allowed: [ ] No [X] Yes

- This is a closed-book exam. Formula Sheet is provided at the end of this booklet.
- Students are allowed to use approved calculators (with ENCS sticker) only.

CELL PHONES OR ANY ELECTRONIC DEVICES ARE NOT PERMITTED.

PLEASE READ CAREFULLY BEFORE YOU START

- Attempt all questions. Questions will not be answered. If there is ambiguity regarding the statement of the problem, you may state clearly your interpretation of the problem and corresponding assumptions and solve the problem accordingly.
- All answers are to be written in the question sheet **in the space provided below each section.**
- **You can use the blank pages on the back of the question sheets for your rough work. Also two blank pages have been added to the end of this booklet in case extra space is needed.**
- Show all of your work and put a box around your final answer. **Providing ONLY the Final answer (without any calculations) will earn ONLY a Zero in that problem.**
- **Write clear and legible. Submit a neat and professional presentation.**

Problem#1

Suppose that on a small tropical island there are only two types of weather; sunny days and rainy days. The probability that a sunny day is followed by a rainy day is 0.6, and the probability that a rainy day is followed by another rainy day is 0.8. The weather on any day depends upon the previous day's weather but not upon any earlier days. Find the probability that if Thursday is rainy then it will be sunny on Saturday.

Solution

T: denote the event that Thursday is sunny
F: denote the event that Friday is sunny
S: denote the event that Saturday is sunny

$P(S/T') = P(S \cap F/ T') + P(S \cap F'/T')$

$P(S \cap F/ T')= P(S/F\cap T')P(F/T')= P(sunny/sunny)P(sunny/rainy)= (1-0.6)(1-0.8)=0.08$

$P(S \cap F'/T')= P(S/F'\cap T')P(F'/T')= P(sunny/rainy)P(rainy/rainy) = (1-0.8)(0.8)= 0.16$

$P(S/T') =0.08+0.16= 0.24$

You can use the tree diagram to solve start with Thursday :

P(sunny/sunny)P(sunny/rainy)+
P(sunny/rainy)P(rainy/rainy)=(0.2)*(0.4)+(0.2)*(0.8)= 0.24

Problem#2:

In a quality control scheme at a factory, batches of components are accepted or rejected depending on the number of defective items counted in a sample. Rejected batches are inspected and all defective items are replaced with good ones. From the machine reliability statistics it has been calculated that the probabilities of three, four, five, six, and seven defective items in a rejected batch are 0.3, 0.4, 0.2, 0.08 and 0.02 respectively. Fifty rejected batches produced a total of 221 defective items. Does this suggest that the machines are producing more defective items than they should? Support your answer with the proper calculations.

Solution

X: denote the number of defective items in a rejected batch

x	3	4	5	6	7
f(x)	0.3	0.4	0.2	0.08	0.02

μ_x= 3(0.3)+4(0.4)+5(0.2)+6(0.08)+7(0.02)= 4.12
$\sigma^2_x= 3^2(0.3)+4^2(0.4)+5^2(0.2)+6^2(0.08)+7^2(0.02) - (4.12)^2$= 0.9856
$\sigma_x= \sqrt{0.9856}$= 0.9928
Let Y: denote the number of defective items in a 50 rejected batches
μ_y= 50 μ_x= 50(4.12)= 206
σ^2_y= 50 σ^2_x= 50 (0.9856) = 49.28
$\sigma_y= \sqrt{49.28}$= 7.02
P(Y ≥ 221) = 1- P(Y ≤ 221) = 1 – $P(Z \leq \frac{221-\mu_y}{\sigma_y})$

=1 – $P(Z \leq \frac{221-206}{7.02})$ = 1- P(Z≤2.137)= 1-0.9836= 0.0164

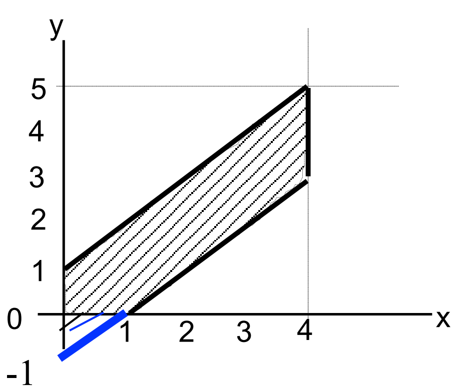
This probability is rather small, so the performance of the machines must come under investigation

Problem#3

Two methods of measuring surface smoothness are used to evaluate a paper product. The joined probability distribution of the two measurements is a uniform distribution over the region 0<x<4, and x-1<y<x+1. That is $f_{xy}(x,y)$= C(x+y) for x and y in the region. Determine,

a) The value of C such that *f* is a joined probability density function.
b) P(x<1.0,y<1.0)
c) Conditional probability distribution of y given x=2.
d) P(y<1.0/ x=2)

Solution



a) $\int_0^4 \int_{x-1}^{x+1} C(x+y)dy\,dx = 1$

$\int_0^4 C\,(xy + \frac{y^2}{2})\quad {}^{x+1}_{x-1}\, dx = 1$

$\int_0^4 C\,(4x) dx = 1$

32C=1…………C=1/32= 0.03125

b) P(x,1, y<1) = $\int_0^1 \int_{x-1}^1 C(x+y)dy\,dx$= $\int_0^1 C\,(3x - 1.5\,x^2)dx = 0.03125$

c) f_x(x) = $\int_{x-1}^{x+1} C(x+y)dy$ = C(4x)= $\frac{x}{8}$

$f_{y/x=2}(y) = \frac{f\quad_{xy}(x=2,y)}{f_x(x)} = \frac{C(2+y)}{C(8)} = \frac{1}{8}(2+y)$

d) P(y<1 /x=2) = $\int_{-1}^1 \frac{1}{8}(2+y)dy = 0.5$

Problem#4:

Three years ago, the mean price of a single family home was \$ 243,717. A real estate broker believes that the mean price has increased since then. The null and alternative hypotheses are H_0: μ=\$ 243,717, H_1: μ> \$243,717.

a) Explain what it would mean to make a type I error.
   (Type I error would occur in fact μ=\$ 243,717 , but the results of the sampling lead to the conclusion that μ> \$243,717)
b) Explain what it would mean to make a type II error.
   (Type II error would occur in fact μ>\$ 243,717 , but the results of the sampling fail to lead to the conclusion that μ> \$243,717)
c) Explain what it would mean to make a correct decision.
   (A correct decision will occur if μ=\$ 243,717 and the results of the sampling do not lead to the rejection of that fact, or if μ>\$ 243,717 and the results of the sampling lead to that conclusion)
d) Now suppose that the results of carrying out the hypothesis test lead to non-rejection of the null hypothesis. Classify that conclusion by error type or as a correct decision if in fact the mean price of a single home family equals \$ 234,717. (correct decision)
e) Suppose that the results of carrying out the hypothesis test lead to non-rejection of the null hypothesis. Classify that conclusion by error type or as a correct decision if in fact the mean price of a single home family exceeds \$ 234,717.
   (Type II error)
f) Which provides stronger evidence against the null hypothesis, a P-value of 0.02 or a P-value of 0.03? Explain your answer.
   (A P-value of 0.02 provides a stronger evidence than a P-value of 0.03

because the P-value is the probability of getting the data at least as inconsistent with the null hypothesis as the actual data obtained)

Problem#5

A random sample of 10 venture - capital investment of fiber optics business sector yielded the following data, in millions of dollars.(9.57, 5.63, 7.42, 9.75, 4.48, 5.53, 4.86, 3.28, 4.96, 3.76)

(a) Determine a 95% confidence intervals for the mean amount μ, for all venture - capital investments in fiber optics.

(b) Determine a 90% confidence intervals for the standard deviation σ, for all venture - capital investments in fiber optics.

(c) If the 95% confidence interval for μ has a length of at most 2.5. What sample size you recommend?

Solution

a) $X' = \Sigma X/n = 59.24/10 = 5.924$

$S^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} = \frac{397.1692 - 350.937}{9} = 5.137$

$S = 2.2665$

Use the T distribution

$\alpha = 1-0.95 = 0.05 \ldots\ldots\ldots\ldots \alpha/2 = 0.025$

$t_{\alpha/2,\, n-1} = t_{0.025,9} = 2.262$

$X' - t_{\alpha/2,\, n-1}\frac{S}{\sqrt{n}} \leq \mu \leq X' + t_{\alpha/2,\, n-1}\frac{S}{\sqrt{n}}$

$5.924 - 1.621 \leq \mu \leq 5.924 + 1.621$

$4.303 \leq \mu \leq 7.545$

b) $\alpha = 1-0.90 = 0.1$ $\alpha/2 = 0.05$,Use the Chi square distribution

$\chi^2_{0.05,9} = 16.92$ $\chi^2_{0.95,9} = 3.33$ from tables

$\frac{(9)(2.2665)^2}{16.92} \leq \sigma^2 \leq \frac{9\,(2.2665)^2}{3.33}$

$2.7324 \leq \sigma^2 \leq 13.8838$

$1.653 \leq \sigma \leq 3.7261$

c) Error = 2.5/2 = 1.25

$t_{\alpha/2,\, n-1}\frac{S}{\sqrt{n}} = \text{Error}$ $1.25 = 2.262(2.2665)/\sqrt{n}$ n= 17

OR Assume that the sample standard deviation is the population standard deviation

$Z_{\alpha/2} = 1.96$ $z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = \text{Error}$ $1.25 = 1.96\,(2.2665)/\sqrt{n}$ n= 13

Problem#6

Super-cavitations is a propulsion technology for undersea vehicles that can greatly increase their speed. It occurs above approximately 50 meters per second, when pressure drops sufficiently to allow the water to dissociate into water vapor, forming a gas bubble behind the vehicle. When the gas bubble completely encloses the vehicle, super-cavitations is said to occur. Eight tests were conducted on a scale model of an undersea vehicle in a towing basin with the average observed speed $x = 102.2$meters per second. Assume that speed is normally distributed with known standard deviation $\sigma = 4$meters per second.

a) Test the hypothesis: H_0: $\mu = 100$ versus: H_1: $\mu < 100$ using $\alpha = 0.05$.
b) What is the *P*-value for the test in part (a)?
c) Compute the power of the test if the true mean speed is as low as 95 meters per second.
d) What sample size would be required to detect a true mean speed as low as 95 meters per second if we wanted the power of the test to be at least 0.85?
e) Explain how the question in part (a) could be answered by constructing a one-sided confidence bound on the mean speed.

Solution

a) The parameter of interest is the true mean speed μ

The null hypothesis H_0: $\mu = 100$

The alternative hypothesis H_1: $\mu < 100$

Significant level $\alpha = 0.05$ $Z_\alpha = -1.65$

Z test = $Z_0 = \frac{X' - \mu}{\sigma/\sqrt{n}} = \frac{102.2 - 100}{4/\sqrt{8}} = 1.56$

Because 1.56>-1.65 Fail to reject the null hypothesis. There is insufficient evidence to conclude that the true mean speed is less than 100 at $\alpha = 0.05$.

b) $Z_0 = 1.56$, then the P-value = $\phi(Z_0) = 0.94$

c) $\beta = 1 - \phi\left(-Z_{0.05} - \frac{(95-100)}{4/\sqrt{8}}\right) = 1 - \phi(-1.65 - (-3.54)) =$

$1 - \phi(1.89) = 0.02938$

Power of the test= $1 - \beta = 0.97062$

d) n= $\frac{(Z_\alpha+Z_\beta)^2\sigma^2}{\delta^2} = \frac{(Z_{0.05}+Z_{0.15})^2 4^2}{(95-100)^2} = \frac{(1.65+1.03)^2 4^2}{(95-100)^2} = 4.597$

use sample size of n= 5

e) $X' - Z_\alpha \frac{\sigma}{\sqrt{n}} \leq \mu$ ==== $102.2 - 1.65\,(\frac{4}{\sqrt{8}}) \leq \mu$

$99.87 \leq \mu$

Because the 100 is included in the CI we do not have enough evidence to reject the null hypothesis.

Problem#7

The life of a semiconductor laser with a constant power is normally distributed with a mean of 7,000 hours and a standard deviation of 600 hours.

a) What is the probability that the laser fails before 5,800 hours?
b) What is the life in hours that 90% of the lasers exceed?
c) What should the mean life equal in order for 99% of the lasers to exceed 10,000 hours before failure?
d) A product contains three lasers, and the product fails if any of the three lasers fails. Assume the lasers fail independently. What should the mean life equal in order for 99% of the products to exceed 10,000 hours before failure?

Solution

Let X denote the life.

a) $P(X < 5800) = P(Z < \frac{5800-7000}{600}) = P(Z < -2) = 1 - P(Z \leq 2) = 0.023$

b) If P(X > x) = 0.9, then P(Z < $\frac{x-7000}{600}$) = -1.28. Consequently, $\frac{x-7000}{600}$ = -1.28 and x = 6232 hours.

c) If P(X > 10,000) = 0.99, then P(Z > $\frac{10{,}000-\mu}{600}$) = 0.99. Therefore, $\frac{10{,}000-\mu}{600}$ = -2.33 and $\mu = 11{,}398$.

d) The probability a product lasts more than 10000 hours is $[P(X > 10000)]^3$, by independence. If $[P(X > 10000)]^3$ = 0.99, then P(X > 10000) = 0.9967. Then, P(X > 10000) = $P(Z > \frac{10000-\mu}{600}) = 0.9967$. Therefore, $\frac{10000-\mu}{600}$ = -2.72 and $\mu = 11{,}632$ hours.

Problem#8

A computer software package was used to calculate some numerical summaries of a sample of data. The results are displayed below:

Variable	N	Mean	SE mean	St. Dev.	Variance	Sum	Sum of Squares
x	?	?	2.05	10.25	?	3761.70	?

a) Fill in all the missing quantities.
b) What is the estimate of the mean of the population from which this sample was drawn?
c) What is "unbiased estimator" mean?

Solution

a) $\frac{S}{\sqrt{N}} = \text{SE Mean} \rightarrow \frac{10.25}{\sqrt{N}} = 2.05 \rightarrow N = 25$

$\text{Mean} = \frac{3761.70}{25} = 150.468$

$\text{Variance} = S^2 = 10.25^2 = 105.0625$

$\text{Variance} = \frac{\text{Sum of Squares}}{n-1} \rightarrow 105.0625 = \frac{SS}{25-1} \rightarrow SS = 2521.5$

b) Estimate of mean of population = sample mean = 150.468

c) "unbiased estimator" means that the expected value of a certain parameter is equal to the true value of that parameter(no errors)