1. A company is forming an interdisciplinary team of six engineers. It should contain two computer engineers, two industrial engineers and two building engineers. The engineering department at that company has seven computer engineers, four industrial engineers and five building engineers.

   a. How many ways can the team of six be formed? (2 marks)
   b. Two of the building engineers are very junior. It is not appropriate to put two junior engineers on the same project. If we include the constraint that the two building engineers on the team cannot both be junior, how many ways are there to form the team? (3 marks)

2. 80% of all customers that visit a particular e-commerce site end up buying something. Assume that whether or not different customers buy something is independent of each other.

   a. What is the probability that three of the next five customers buy something? (3 marks)
   b. If Y is the number of customers to visit the website before four of them buy something, give a distribution for Y. Be sure to state its range and all parameters. (3 marks)
   c. What is the probability that it will take seven or less visits to the website, until four sales are made? (4 marks)

3. Consider a continuous random variable X with the following pdf

$$f(x) = \begin{cases} mx & 2 < x < 3 \\ 0 & otherwise \end{cases}$$

   a. Determine m. (3 marks)
   b. Find the expected value of X. (3 marks)
   c. Find the variance of X. (3 marks)

4. Let X be an exponential random variable with mean 4.

   a. Calculate the probability that Y>3. (3 marks)
   b. Find r such that P(X>r) =0.5. (3 marks)

5. A privately owned store operates both a drive-in facility and a walk-in facility. On a randomly selected day, let X and Y, respectively, be the proportions of the time that the drive-in and walk-in facilities are in use, and suppose that the joint probability distribution function of these random variables is described by

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & elsewhere \end{cases}$$

a) Find the marginal densities *f(x)* and *f(y)*. (6 marks)
b) Find the probability that the drive-in facility is busy less than one-half of the time. (3 marks)
c) Find the probability that the walk-in facility is busy more than 70% of the time given that the drive-in facility is used 10% of the time. (4 marks)
d) Find the covariance of X and Y. (5 marks)
e) Are the random variables X and Y independent? (Justify your answer) (2 marks)

6. Based on the history, TOEFL score is 550 in average and 30 in standard deviation. Assume that the population (scores) follows a normal distribution.

a) If 9 students in today's TOEFL exam have been selected at random, the final scores of those students are: 500, 520, 530, 540, 550, 550, 560, 570 and 600. Find the sample mean $\overline{X}$ and the sample variance S^2. (3 marks)
b) If 16 students are randomly selected, find $P(|\overline{X} - 550| < 15)$. (5 marks)
c) It is known that both the sample mean $\overline{X}$ and the sample variance S^2 are examples of unbiased estimators. Briefly explain why *minimum-variance unbiased estimators* are considered efficient estimators. (2 marks)

7. Your employer develops energy efficient solutions for manufacturing sites. One of the key elements of these solutions is proper insulation. Your team leader has asked you to test a new product line, if the product line has a mean thermal insulation (TI) of at least 25 it will be used.

You receive 25 samples of insulating material and test the thermal isolation coefficient. Based on the testing you determine that the sample mean TI is 26.5. The TI is normally distributed with a standard deviation of 2. Show all of your work and state any reasonable assumptions.

a) Using a statistical test with a significance level of: $(\alpha = 0.05)$ and based on the results given above, should this product line be used? (8 Marks)
b) If the true population mean was 26, determine β (type II error) with the same information given above. (8 Marks)

c) If the cost of using the product when it should have been rejected is \$300,000 (due to recalls and refits), and the cost of not using the product when it should have been accepted is \$100,000 (due to delays), determine the probability of having to pay each of the above costs and which is more likely to occur. (2 Marks)

d) How could you improve (reduce) the type I and type II errors? (2 Marks)

8. In medical treatments it is very important to be aware of the dosage given to patients. Infusion pumps are tested regularly to insure that the dosage levels are properly regulated. You may assume that the dosage levels are normally distributed. A set of ten infusion pumps were tested and the dosage deliveries had a sample standard deviation of $s = 0.1$ doses/hour and a sample mean of 5 doses/hour assuming.

   a) Compute the 95% confidence interval population for the population standard deviation based on the above data. (4 Marks)

   b) Compute the 95% confidence interval for the population mean based on the above data with based on the above data. (4 Marks)

   c) If the dosage level is more than six doses per hour the patient is in danger. Comment on whether the patient is at risk according to your findings. (2 Marks)

TOTAL: 90 marks