PHYS 204 - Practice Final - Solutions

1. (c) Regardless of their motion along the x-axis (horizontal), the ball and the bullet go through free fall motion with $\left(v_y\right)_i = 0$ along the y-axis (vertical). Hence, they both hit the ground at the same time (but at different locations).

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2. (c) Velocity of the particles can be found from the slope (rise over run) of $x - t$ graph. Speed is the absolute value of the velocity (in 1D). From the graph, particle C has the largest speed: $v_C = \frac{8}{6} > v_B = \frac{6}{6} > v_A = \frac{4}{6}$

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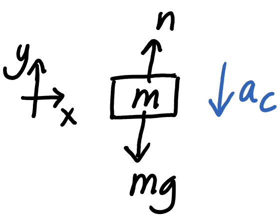
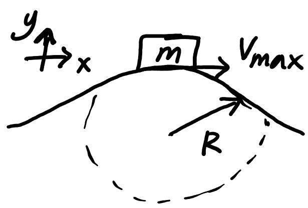
3. (b) At max speed, the normal force is zero and the car is at the verge of leaving the surface.



$$\sum F_y = ma_c$$
$$\rightarrow \cancel{n}^{\,0} - mg = -m\frac{v^2}{R}$$
$$\cancel{m}g = \cancel{m}v^2/R \rightarrow v = \sqrt{Rg} \simeq \boxed{10\ m/s}$$



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4. (b) This is a completely inelastic collision, in which the momentum of the system conserves but the mechanical energy does not:

$$p_i = p_f$$
$$mv = (m+m)v' \rightarrow v' = \frac{v}{2}$$
$$\frac{K_f}{K_i} = \frac{\frac{1}{2}(2m)v'^2}{\frac{1}{2}mv^2} = \frac{1}{2} = 50\%$$

The rest of the kinetic energy of the first block is spent to fuse the two blocks together, and also comes out as heat/sound.

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5. (b) Airbags increase the time during which passengers' speed changes from v (speed of the car before accident) to 0. Considering Impulse momentum principle, passengers experience a smaller force.

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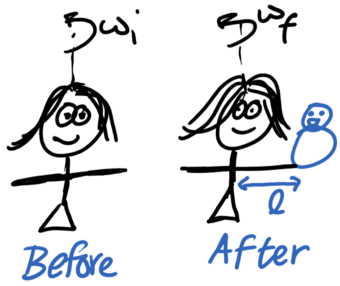
6. **(b)** Taking the system as the girl and the bird together, there is no external torque and hence the system is isolated. Using conservation of angular momentum:



$$L_i = L_f \rightarrow I_i \omega_i = I_f \omega_f$$

$$\rightarrow I_{girl}\,\omega_i = (I_{girl} + M_{bird}\,\ell^2)\,\omega_f$$

$$\rightarrow \frac{\omega_f}{\omega_i} = \left[\frac{I_{girl}}{(I_{girl} + M_{bird}\ell^2)}\right] < 1$$

Her angular velocity decreases.

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7. **(b)** The ball and the box both have the same energy initially (gravitational potential energy). As the box slides down, this energy transforms completely into translational kinetic energy at the bottom of the inclined surface. Since the ball rolls, the potential energy converts into translational and rotational kinetic energy. Hence the **box** will have a larger velocity of center of mass:

Box:

$$mgh = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2gh}$$

Ball:

$$mgh = \frac{1}{2}mv'^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv'^2 + \frac{1}{2}I\left(\frac{v'}{R}\right)^2 = \frac{1}{2}\left(m + \frac{I}{R^2}\right)v'^2$$

$$v' = \sqrt{\left(\frac{m}{m + \frac{I}{R^2}}\right)2gh} < \sqrt{2gh}$$

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8. **(c)** They will both reach the same height. Regardless of where the bullet enters, the linear momentum of the system conserves. Hence, both boxes with the bullets embedded inside them will have the same velocity after this completely inelastic collision. Using conservation of energy, they will both reach the same height.
You can watch the interesting demonstration of this problem here:
http://demo.webassign.net/sercp11/html/bullet_block/index.html?__cf_chl_jschl_tk__=bcdcb00b7efaefbd3d2469c2dc9e4042ba22d261-1606926230-0-ATUVyN8pcZwAOcD57spMwBfSIB1Gu16Axhhw3TPfQyV9wdfypTlHhG0u9EPmxvXqaWBhj94KWdhfMmSFcPd4aM79SGLGROxJWUJcad37pfdcvc3IBqctJZ3Pw6dfFiTvoAxfB-EdybgBwt50KJxqsTtOq8J0rVuUcsXznw0IHcvttUNuGxV77qKEfJjPireirLpy7IlVzpUGRKPJQtPAFWFrddh9NlPKdsMbWts8vsJjUOaMN1Z9uwhKcFs-RRWmmyEtW51oxLq7JAah_45xbWu6I9yYI1Qkv6ibOadg-MofGPT9wyIFei5o214ZlZE6RxMbl-oAmG7GM7YeolhdpagkJ2Fuglb4qNhueOg0l8bzIrvgzY8yKLHX3Ed8v-9hT9zT2rPWBFIKYGL67QhP35tEhFFvo_GbQV6bxQVo0CGs

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9. **(c)** This is a perfectly inelastic collision in rotational motion.
As the angular momentum of this system conserves:

$$L_i = L_f$$

$$I_i\omega = I_f\omega' \rightarrow \frac{\omega'}{\omega} = \frac{I_i}{I_f} = \frac{\frac{1}{12}mL^2}{\frac{1}{12}(2m)(2L)^2} = \frac{1}{8}$$

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10. **(c)** The velocity is given by:

$$v(t) = -\omega A\sin(\omega t)$$

From the $x(t)$ equation: $A = 3$ and $\omega = 2\pi$

$$\omega = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{\omega} = 1\ s$$

$$v(t) = -6\pi\sin(2\pi t)$$

$$v\left(\frac{T}{4}\right) = v\left(\frac{1}{4}\right) = -6\pi\sin\left(\frac{2\pi}{4}\right) = -\,6\pi$$

11. **(d)** Considering conservation of energy ($\Delta K + \Delta U_g = 0$), all three balls have the same kinetic energy when released from the same height and they all land on the floor. Since the change in their gravitational potential energy is the same, the change in their kinetic energy must be the same as well. Hence, they all hit the ground with the same speed.

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12. **(a)** Since the forces are the same, the situation in which the lever arm d is larger lever will have a larger applied torque.
(a) $d = R$
(b) $d = 0$
(c) $d < R$
(d) $d = 0$
(e) $d = 0$

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13. **(e)** Using conservation of energy:

$$m_A g\left(h - \frac{h}{4}\right) = K_A \qquad m_B g\left(h - \frac{h}{4}\right) = K_B \qquad \frac{K_B}{K_A} = \frac{m_B}{m_A} = 4$$

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14. **(b)** $K = \frac{1}{2}mv^2;\quad p = mv \rightarrow K = \frac{p^2}{2m},\quad p = \sqrt{2mK};\qquad K_1 = K_2$

$\frac{p_1}{p_2} = \sqrt{\frac{2m_1K}{2m_2K}} = \sqrt{\frac{m_1}{m_2}}$ which is unknown

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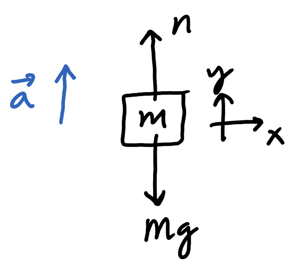
15. **(b)** The reading of the scale is the Normal Force.
Using Newton's 2nd law:

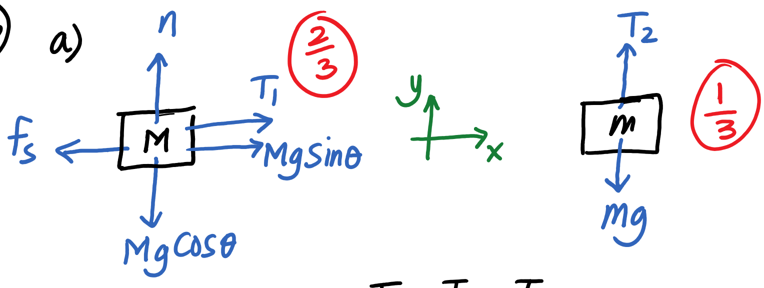


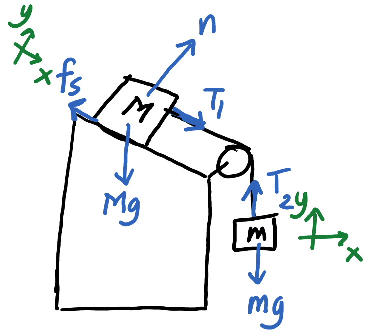
$$\sum F_y = ma_y$$

$$n - mg = ma$$

$$n = mg + ma \rightarrow n = m(g+a) > mg$$

16 a)





$$T_1 = T_2 = T$$

b)

$\boxed{M}$

(2/7)

$$\Sigma F_y = 0$$

$$n - Mg\cos\theta = 0$$

$$\boxed{n = Mg\cos\theta}$$

(3/7)

$$\Sigma F_x = 0$$

$$T + Mg\sin\theta - f_s = 0$$

$$\mu_s n = T + Mg\sin\theta$$

$$\rightarrow \mu_s Mg\cos\theta = mg + Mg\sin\theta$$

$$\rightarrow \boxed{\mu_s = \frac{m + M\sin\theta}{M\cos\theta}}$$ (1/7)

$\boxed{m}$

(1/7)

$$\Sigma F_y = 0$$

$$T - mg = 0$$

$$\boxed{T = mg}$$

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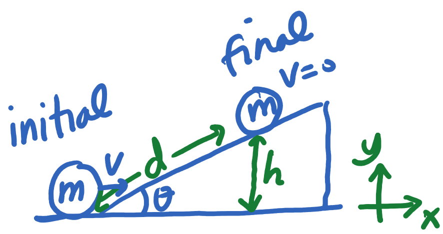
17. It is best to use the work-energy principle to solve this problem. Since the cylinder rolls without slipping, we should consider the kinetic energy of rolling motion:

System = Cylinder + Earth

Environment = Inclined surface



(4/10)

$$W_{f_k} = \Delta E = E_f - E_i$$

$$W_{f_k} = (\cancel{K_f}^{\,0} + U_{gf}) - (K_i + \cancel{U_{gi}}^{\,0})$$

$$-f_k d = mgh - \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)$$

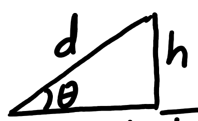
$$\omega = \frac{v}{R},\quad I = \frac{1}{2}mR^2,\quad f_k = \mu_k mg\cos\theta,\quad h = d\sin\theta$$

(3/10)

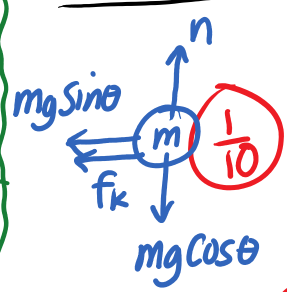
$$-\mu_k \cancel{m}g\,d\cos\theta = \cancel{m}g\,d\sin\theta - \left(\frac{1}{2}mv^2 + \frac{1}{4}m\cancel{R^2}\frac{v^2}{\cancel{R^2}}\right)$$

$$\frac{3}{4}\cancel{m}v^2$$

$$\boxed{d = \frac{3/4\, v^2}{g(\sin\theta + \mu_k\cos\theta)} = 2.05\ m}$$ (1/10)



$$\boxed{h = d\sin\theta}$$



(1/10)

$$\Sigma F_y = 0$$

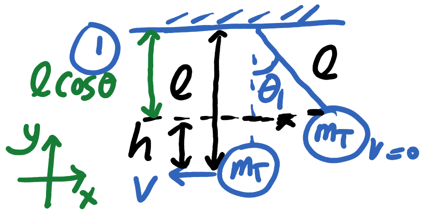
$$n = mg\cos\theta$$

$$\boxed{f_k = \mu_k n = \mu_k mg\cos\theta}$$

18. To do this problem, we should:

1) Find the speed of Tarzan before reaching Jane from conservation of energy
2) Find the speed of Tarzan and Jane after their inelastic collision
3) Find the maximum angle through conservation of energy

$m_T = 70\ kg\ ,\quad m_J = 50\ kg,\ \theta_1 = 60^\circ,\ \ell = 5\ m$



① Conservation of energy:

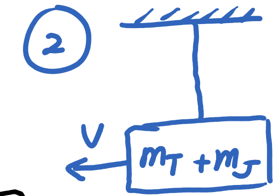
3/10

$h = \ell - \ell\cos\theta = \ell(1-\cos\theta)$

$\Delta E = 0 \rightarrow \Delta k + \Delta U_g = 0$

$k_f - \cancel{k_i} + \cancel{U_{gf}} - U_{gi} = 0 \rightarrow \frac{1}{2}\cancel{m_T}v^2 = \cancel{m_T}gh \rightarrow \boxed{v = \sqrt{2gh}}$

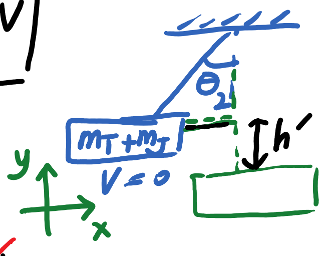
$0 \qquad 0$



② conservation of momentum:

3/10

$P_i = P_f \rightarrow m_T v = (m_T + m_J)v' \rightarrow \boxed{v' = \left(\frac{m_T}{m_T + m_J}\right)v}$



③ Conservation of energy:

3/10

$\Delta E = 0 \rightarrow \Delta k + \Delta U_g = 0 \rightarrow \cancel{k_f} - k_i + U_{gf} - \cancel{U_{gi}} = 0$

$0 \qquad 0$

$(m_T + m_J)gh' = \frac{1}{2}(m_T + m_J)v'^2 = \frac{1}{2}\cancel{(m_T + m_J)} \cdot \frac{m_T^2}{(m_T + m_J)^{\cancel{2}}} v^2 \rightarrow 2gh$

$h' = \frac{1}{\cancel{2}}\left(\frac{m_T}{m_T + m_J}\right)^2 \frac{\cancel{2}\cancel{g}h}{\cancel{g}} = \left(\frac{m_T}{m_T + m_J}\right)^2 h$

$\begin{cases} h' = \ell(1-\cos\theta') \\ h = \ell(1-\cos\theta) \end{cases}$

$\cancel{\ell}(1-\cos\theta') = \left(\frac{m_T}{m_T + m_J}\right)^2 \cancel{\ell}(1-\cos\theta)$

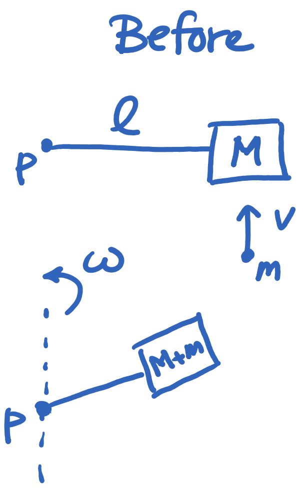
$\cos\theta' = 1 - \left(\frac{m_T}{m_T + m_J}\right)^2(1-\cos\theta) = 0.83$

$\boxed{\theta' = \cos^{-1}(0.83) = 33.9^\circ}$

1/10

19. This problem involves collision and rotation of the system after collision. We should use conservation of angular momentum, since there is no external torque on the system (Bullet + Block). 1/10

2/10 $L_i = L_f$, $\begin{cases} L_i = \vec{r}\times\vec{p} = mv\ell \quad 2/10 \\ L_f = I\omega = (m+M)\ell^2\omega \quad 2/10 \end{cases}$



$\rightarrow mv\cancel{\ell} = (m+M)\ell^{\cancel{2}}\omega$ 2/10

$\rightarrow \boxed{\omega = \left(\frac{m}{m+M}\right)\frac{v}{\ell} = 4.5\ rad/s}$ 1/10

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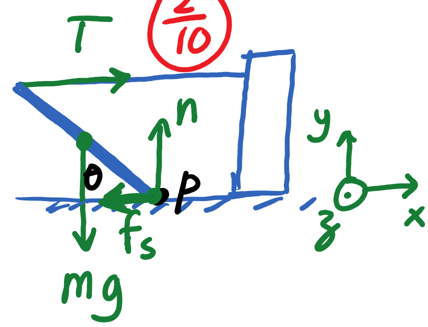
20. This is a static equilibrium problem, as Bob is not moving or rotating. We should set up Newton's laws of motion for linear and rotational motion:

f_s is max, since we're looking for θ_{min}

(a) $\sum F_x = 0$ 1/10 $\rightarrow f_s = T \rightarrow \boxed{\mu_s n = T}$ ①

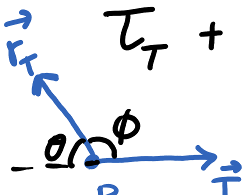
$\sum F_y = 0$ 1/10 $\rightarrow \boxed{n = mg}$ ②

①,② $\rightarrow \boxed{T = \mu_s mg = 245\ N}$ 1/10



(b) $\sum \vec{\tau} = 0$ about P

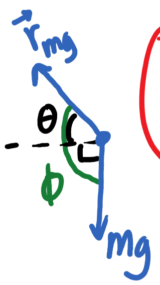
$\vec{\tau}_T + \vec{\tau}_{mg} + \cancel{\vec{\tau}_n}^{\,0} + \cancel{\vec{\tau}_{f_s}}^{\,0} = 0$ $(r=0)$ 1/10



1/10 $\vec{\tau}_T = \vec{r}_T \times \vec{T} = \ell T \sin\phi\,(-\hat{k})$ right-hand-rule (or turning clockwise)

$\phi = 180° - \theta \rightarrow \sin\phi = \sin\theta \rightarrow \boxed{\vec{\tau}_T = \ell T \sin\theta\,(-\hat{k})}$ ③

1/10



1/10 $\vec{\tau}_{mg} = \vec{r}_{mg} \times m\vec{g} = \frac{\ell}{2} mg \sin\phi = \frac{\ell}{2} mg \sin(90+\theta)\,(\hat{k}) \rightarrow$

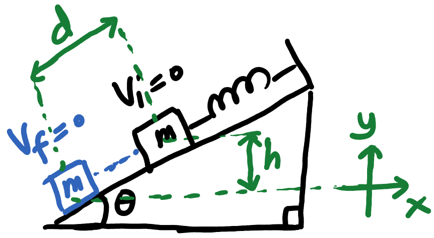
$\boxed{\vec{\tau}_{mg} = \frac{\ell}{2} mg \cos\theta\,(\hat{k})}$ ④

③,④ $\rightarrow \cancel{\ell} T \sin\theta = \frac{\cancel{\ell}}{2} mg \cos\theta \rightarrow \tan\theta = \frac{mg}{2T}$

$\rightarrow \boxed{\theta = \tan^{-1}(mg/2T) = 45°}$ 1/10

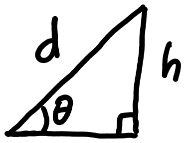
21) $m = 2\,kg$, $y_i = h$, $v_i = 0$

$k = 100\ \frac{N}{m}$, $y_f = 0$, $v_f = 0$



a) Using conservation of energy:

System = m + spring + Earth



1/5

$$\boxed{h = d\sin\theta}$$

1/5 $\Delta E = 0 \rightarrow \Delta K + \Delta U_g + \Delta U_s = 0$

2/5 $= \frac{1}{2}m(\cancel{v_f}^{2\,0} - \cancel{v_i}^{2\,0}) + mg(\cancel{y_f}^{0} - \cancel{y_i}^{h}) + \frac{1}{2}k(\cancel{x_f}^{2\,d} - \cancel{x_i}^{2\,0}) = 0$

$-mgh + \frac{1}{2}kd^2 = 0 \rightarrow \frac{1}{2}kd^{\cancel{2}} = mg\underbrace{\cancel{d}\sin\theta}_{h}$

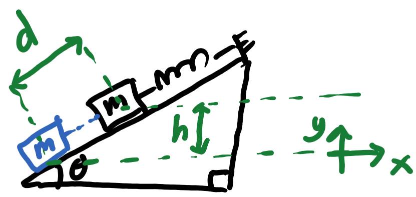
$\rightarrow d = 2\ \frac{mg\sin\theta}{k} = (2)\ \frac{(\cancel{2})(9.8)(\cancel{1/2})}{100}$ $\boxed{= 0.196\ m}$ 1/5

b) $v_i = 0$

$x_i = 0$, $y_i = h$

$x_f = d$, $y_f = 0$

$v_f = ?$



$h = d\sin\theta$

Conservation of energy:

system = m + spring + Earth

$\Delta E = 0 \rightarrow \Delta K + \Delta U_g + \Delta U_s = 0$ 1/5

$\rightarrow \frac{1}{2}m(v_f^2 - \cancel{v_i}^{2\,0}) + mg(\cancel{y_f}^{0} - \cancel{y_i}^{h}) + \frac{1}{2}k(\cancel{x_f}^{2\,d} - \cancel{x_i}^{2\,0}) = 0$ 1/5

$\rightarrow \frac{1}{2}mv_f^2 = mg\underbrace{d\sin\theta}_{h} - \frac{1}{2}kd^2$ 2/5

$\rightarrow v_f = \sqrt{2gd\sin\theta - \frac{k}{m}d^2} = \sqrt{(2)(9.8)(\frac{5}{100})(\frac{1}{5}) - \frac{100}{2}(\frac{5}{100})^2}$

$\rightarrow v_f = \sqrt{0.365}$ $\boxed{= 0.6\ \frac{m}{s}}$ 1/5