PROBLEMS FOR CHAPTER 2

1) Find the smallest positive root of the equation:

$$1 - \frac{x^2}{6} + \frac{x^4}{120} + \frac{x^6}{5040} = 0$$

In the interval (0, 8), with an accuracy of 5 digits. Initially locate the root roughly with incremental search technique, and then refine it using Newton-Raphson method.

Solution:

$$1 - \frac{x^2}{6} + \frac{x^4}{120} + \frac{x^6}{5040} = 0$$

(0,8)

Starting with increment search of 1

x	0	1	2	3	4
f(x)	1	0.8414	0.4539	0.03036	-0.3461

Therefore, the root between 3 and 4

Now using the Newton – Raphson Method

$$x_{n-1} = x_n + \frac{f(x_n)}{f'(x_n)}$$

$f^1(x) = -\frac{x}{3} + \frac{x^3}{30} - \frac{x^5}{540} = 0$

n	x_n	f(x_n)	$f^1(x_{n+1})$	x_{n+1}
0	3	0.030357	-0.389286	3.077981
1	3.07781	2.53061×10^{-4}	-0.382860	3.08642
2	3.078642	1.165579×0^{-7}	-0.382808	3.0786423

Answer: 3.078642

4. Find the point where the following function has a maximum in the range $2 \leq x \leq 6$ using the Bisection method.

$$f(x) = cosx\ coshx - 1 = 0$$

With an accuracy of 3 digits.

Solution:

f(x) = cos x cosh x – 1 = 0

for maximum g(x) = $\frac{d\,f(x)}{dx}$ = 0 or cosx sinhx – coshx sinx = 0

using incremental search we get

x	2	3	4
g(x)	-4.49303	-11.3384	2.8291

we start between 3 and 4

a = 3 and b =4

f(a)=-11.3384 and f(b) = 2.8291

p = 3.5 and f(p)= -9.67799

therefore p is the new a. continuing we get

n	a	b	p	fa	fb	fp
1	3	4	3.5	-11.3384	2.829058	-9.67799
2	3.5	4	3.75	-9.67799	2.829058	-5.27746
3	3.75	4	3.875	-5.27746	2.829058	-1.75589
4	3.875	4	3.9375	-1.75589	2.829058	0.394922
5	3.875	3.9375	3.90625	-1.75589	0.394922	-0.7148
6	3.90625	3.9375	3.921875	-0.7148	0.394922	-0.16866
7	3.921875	3.9375	3.929688	-0.16866	0.394922	0.110936
8	3.921875	3.929688	3.925782	-0.16866	0.110954	-0.0294
9	3.921578	3.929688	3.925633	-0.17919	0.110954	-0.03471
10	3.925633	3.929688	3.927661	-0.03471	0.110954	0.037973
11	3.925633	3.927611	3.926622	-0.03471	0.036195	0.000706
12	3.925633	3.926622	3.926128	-0.03471	0.000706	-0.01701

Therefore, maximum is at 3.926

6. Determine the smallest positive root between (0,2) of the given equation using Bisection method with an error less than 0.02.

$$cosx = \frac{1}{x} 1n_x$$

Solution:

f(x) = cosx - $\frac{1}{x}$ 1n x

using incremental search we get

x	0	1	2
f(x)		0.540302	-0.76272

Therefore root lies between 1 and 2

a = 1 and b =2

f(a)=0.5403 and f(b) = -0.763

p = 1.5 and f(p)= -0.1996

Therefore p is the new b.

n	A	b	p	fa	fb	fp	er
1	1	2	1.5	0.540302	-0.76272	-0.19957	
2	1	1.5	1.25	0.540302	-0.19957	0.136808	0.2
3	1.25	1.5	1.375	0.136808	-0.19957	-0.03706	0.090909
4	1.25	1.375	1.3125	0.136808	-0.03706	0.048246	0.047619
5	1.3125	1.375	1.34375	0.048246	-0.03706	0.00522	0.023256
6	1.34375	1.375	1.359375	0.00522	-0.03706	-0.01601	0.011494

15. Use Newton-Raphson method to determine the positive root of the equation

$$sinx - 4x^2 + 1 = 0$$

Correct to 3 decimal places.

Solutuion:

sin(x) – $4x^2$ + 1 = 0

f’(x) = cos(x) – 8x

from incremental we get

(x)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
f(x)	1	1.0598	1.038669	0.9355	0.7494	0.4794	0.1246	-0.316

Therefore,

x_0 = 0.6

$f(x_0) = f(0.6) = \sin(0.6) – 4(0.6)^2 + 1 = 0.124642$

$f'(x_0) = f'(0.6) = \cos(0.6) – 8(0.6) = -3.97466$

$x_1 = x_0 – (\frac{f(x_0)}{f'(x_0)}) = 0.6 - (\frac{0.124642}{-3.97466}) = 0.63136$

Continuing in table form

i	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}	$\sum$
0	0.6	0.124642	-3.79466	0.63136	0.04967
1	0.63136	-0.0042	-4.24365	0.630366	0.001576
2	0.630366	$-4.2 * 10^{-6}$	-4.23512	0.630365	$1.59 * 10^{-6}$

x = 0.63036

25. Find cubic root of 8 using Secant method accurate to 4 digits.
(Hint: solve $x^3 - 8 = 0$).

Solution:
$x^3 - 8 = 0$
Cubic root of 8 is 2. Therefore we start at 1.9;
$x_n = 1.9$ and $x_{n-1} = 1.8$

$$f'(x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} = \frac{(-1.141) - (-2.168)}{1.9 - 1.8} = 10.27$$

$$x_{n+1} = x_n - \left(\frac{f(x_n)}{f'(x_n)}\right) = 1.9 - \left(\frac{-1.141}{10.27}\right) = 2.0111$$

Continuing in table form

n	x_{n-1}	x_n	$f(x_n)$	$f(x_{n-1})$	$f'(x_n)$	x_{n+1}
0	1.8	1.9	-1.141	-2.168	10.27	2.0111
1	1.9	2.0111	0.133944	-1.141	11.4756	1.999428
2	2.0111	1.999428	-0.00686	0.133944	12.0633	1.999997

Therefore,
$x = 1.999997$ or 2