CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Section**
Mathematics	203	CA
Examination	**Date**	**Pages**
Final	August 2013	2
Instructor:		**Course Examiners**
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Special Instructions:	Only calculators approved by the Department are allowed	

MARKS

[12] **1.** **(a)** Sketch the graph of the function $f(x) = |\sqrt{-x+1} - 3|$ starting from the graph $f(x) = \sqrt{x}$ and using appropriate transformations.

(b) Given the function $f = x^2 - 2x + 1$ find the inverse function f^{-1}, if $x \geq 1$, and determine domain and range of f and f^{-1}.

(c) Let $f(x) = \sqrt{4 - x^2}$, and $g(x) = \dfrac{10}{5 - x^2}$. Find $h = g \circ f$ and determine the domain of h.

[12] **2.** Evaluate the limits. **Do not use l'Hôpital rule**:

(a) $\lim\limits_{x\to 2} \dfrac{x^2 - 7x + 10}{x - 2}$ **(b)** $\lim\limits_{x\to\infty} \dfrac{\sin(2x)}{\sqrt{x}}$ **(c)** $\lim\limits_{x\to-\infty} \dfrac{4x - 1}{\sqrt{x^2 + 2}}$

[5] **3.** Calculate the limit $\lim\limits_{x\to 0} \dfrac{2x - |x|}{|3x| - 2x}$ or show that it does not exists.

[17] **4.** Find the derivatives of the following functions (**You do not have to simplify!**):

(a) $f(x) = \dfrac{4x\sqrt{x} - x^{\frac{7}{4}} + 3\sqrt[7]{x^3}}{x^{\frac{3}{2}}}$

(b) $f(x) = e^{\cos(3x)}(1 + x^2)^3$

(d) $f(x) = \sqrt{2x + \sqrt{3x}}$

(e) $f(x) = (\cos(2x))^{\sin(3x)}$ (use logarithmic differentiation)

[15] **5.** **(a)** Verify that the point $(0,\ -1)$ belongs to the curve defined by the equation $y^2 \cos x = xy^5 + y + 2$, and find the equation of the tangent line to the curve at that point.

(b) What is the largest possible area for a right triangle whose hypotenuse is 5 cm long?

(c) Use l'Hôpital's rule to evaluate the $\lim\limits_{x\to 1}\left(\dfrac{1}{\ln x} - \dfrac{1}{x-1}\right)$.

Hint: first write this in the form $\lim\limits_{x\to 1}\dfrac{f(x)}{g(x)}$.

[11] **6.** Let $f(x) = x^3 + 2x - 4$.

(a) Find the slope m of the secant line joining the points $(1, f(1))$ and $(2, f(2))$.

(b) Find all points $x = c$ (if any) on the interval $[1,\ 2]$ such that $f'(c) = m$.

(c) Use the Intermediate Value Theorem to show that f has a root between 1 and 2.

[12] **7.** Consider the function $f(x) = \dfrac{2}{x+5}$.

(a) Use the **definition of the derivative** to find the formula for $f'(x)$.

(b) Use appropriate differentiation rules to verify (a).

(c) Write the linearization formula for f at $a = 5$.

(d) Use this linearization to approximate the value of $f(6)$.

[16] **8.** Given the function $f(x) = \dfrac{x^2}{x^2-1}$.

(a) Find the domain of f and check for symmetry. Find asymptotes of f (if any).

(b) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).

(c) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

(d) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question.** Given the function $f(x) = e^{3x}$, find a formula for $f^{(n)}(x)$.