FACULTY OF ENGINEERING AND COMPUTER SCIENCE
FINAL EXAMINATION FOR APPLIED DIFFERENTIAL EQUATIONS
ENGR 213 - SAMPLE

Special instructions: Do all problems
Only Faculty approved calculators are allowed
ALL PROBLEMS CARRY THE SAME WEIGHT

PROBLEM No. 1. Solve the following equation by the separation of variables method.

$$(y^2 + xy)dx + x^2 dy = 0$$

(**Hint: use the idea of homogeneous functions**)

PROBLEM No. 2. Find the solution to the following equations by the exact differentials method:

(a) $(x + y)^2 dx + (2xy + x^2 - 1)dy = 0$

(b) $(y^2 \cos x - 3x^2 y - 2x)dx + (2y \sin x - x^3 + \ln y)dy = 0$

PROBLEM No. 3. Solve the following linear differential equations using the integrating factor method:

(a) $xy' + y = \dfrac{2}{x^3}$

(b) $y' + 5y = e^{-2x}$

PROBLEM No. 4. Solve the following Bernoulli equation,

$$x^2 y' - 2xy = y^4$$

PROBLEM No. 5. Give the general solutions of the following differential equations:

(a) $y^{(4)} + y'' - 12y = 0$

(b) $y^{(7)} - 6y^{(6)} + 20y^{(5)} - 56y^{(4)} + 112y^{(3)} - 160y'' + 192y' - 128y = 0$

Note:

$$m^7 - 6m^6 + 20m^5 - 56m^4 + 112m^3 - 160m^2 + 192m - 128 = (m-2)^3(m^2+4)^2$$

PROBLEM No. 6. Give the general solutions of the following differential equations:

(a) $y''+6y'+8y=\sin 3x$

(b) $y''''+2y''+y=x^2$

PROBLEM No. 7. The Space Shuttle lands in Kennedy Space Center. The spacecraft touches down at $t=0$ with a velocity of 100 m/sec. The spacecraft chute is deployed at $t=4$ sec. Between the touch down and the deployment of the chute $(0\le t\le 4)$ the velocity of the spacecraft is constant. After the deployment of the chute the velocity is governed by the equation:

$$\frac{dV}{dt}=-0.002V^2$$

Determine when the velocity of the spacecraft reaches 20 m/sec.

PROBLEM No. 8. Solve the differential equation by the method of variation of parameters:

$$y''+10y'+25y=e^x$$

PROBLEM No. 9. Solve the following system of differential equations using the method of you preference:

$$\frac{dx}{dt}=2(x-y)$$

$$\frac{dy}{dt}=y-x$$

PROBLEM No. 10. Use the power series method to solve the differential Equation:

$$y''+x^2y=0$$

(write the first 6 terms of the power series solution)