Department of Mathematics & Statistics

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Mathematics	203	All
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1.44

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Special Instructions

▷ **Only Sharp EL 531 or Casio FX 300 MS calculators are allowed.**

MARKS

[10] **1.** (a) Suppose $f(x) = \sqrt{1-x}$ and $g(x) = \cos^2 x$. Find $F = f \circ g$ and simplify it. Use F to find $g \circ f \circ g$.

(b) Find the inverse of the function $f(x) = e^{x^5} - 1$. Determine the domain and range of f and f^{-1}.

[8] **2.** Evaluate the limits (**Do not use l'Hôpital's rule.**):

(a) $\displaystyle\lim_{x\to 2} \frac{\sqrt{7x+2}-4}{2x^3-8x}$ (b) $\displaystyle\lim_{x\to\infty} \frac{4x^2(\sqrt{x}-1)^3}{(1-2x)^3(\sqrt{x}+1)}$

[10] **3.** (a) Consider the function $f(x) = \dfrac{|x+3|}{x^2+x-6}$.

Calculate both one-sided limits at the point(s) where the function is undefined.

(b) Find numbers a and b such that the function

$$f(x) = \begin{cases} 2x^2, & \text{if} \quad x \le -1 \\ ax+b, & \text{if} \quad -1 < x \le 0 \\ x^2-2, & \text{if} \quad x > 0 \end{cases}$$

will be continuous at every point.

Sketch the graph of this function.

[16] 4. Find derivatives of the functions (you do not have to simplify the answers):

(a) $f(x) = \dfrac{x^{2/5} - \sqrt[5]{x} + x^{-2/5}}{x^{1/5}}$;

(b) $f(x) = \left(x + \dfrac{1}{x}\right)^3 \sin(2x)$;

(c) $f(x) = \dfrac{(\arctan x)^2}{1 + x^2}$;

(d) $f(x) = \tan(x^2 + \ln(3x))$;

(e) $f(x) = (\arcsin x)^{\sqrt{x}}$ (use logarithmic differentiation).

[9] 5. Given the function $f(x) = \dfrac{x+1}{x-1}$,

(a) Use the definition of derivative to find $f'(x)$.

(b) Use the appropriate differentiation rule(s) to verify your answer in part (a).

(c) Find the differential dy and evaluate it when $x = 2$ and $dx = 0.05$.

(d) Find the linear approximation $L(x)$ to $f(x)$ at $a = 2$ and find $L(2.05)$. Explain the connection between this and your answer in part (c).

[16] 6. (a) The equation of a curve defined implicitly is $4(x^2 + y^2)^2 = 25\,xy^2$. Verify that the point $(1, 2)$ belongs to the curve. Find an equation of the tangent line to the curve at this point.

(b) The sides of a square decrease in length at a rate of 1 m/sec. At what rate is the area of the square changing when the sides are 5 m long?

(c) Use l'Hôpital's rule to evaluate $\lim\limits_{x \to 0} \dfrac{e^{x^2} - 1}{1 - \cos 2x}$.

[12] 7. (a) Find the absolute maximum and minimum values of the function $g(x) = x^{2/3}(2 - x)$ on the interval $[-1, 2]$.

(b) An airline policy states that all baggage must be box-shaped with the sum of the length, width and height not to exceed 192 cm. What are the dimensions of a box with a square base that has the largest volume acceptable by the airline, and what is the largest volume?

[6] 8. Let $f(x) = \dfrac{x}{x+2}$.

(a) Find the slope m of the secant line joining the points $(1, f(1))$ and $(4, f(4))$.

(b) Find the point(s) $x = c$ on the interval $[1, 4]$ such that $f'(c) = m$.

[13] 9. You are given the following information about the function f:

$$f(x) = \frac{10x^3}{x^2-1}; \quad f'(x) = \frac{10x^2(x^2-3)}{(x^2-1)^2}; \quad f''(x) = \frac{20x(x^2+3)}{(x^2-1)^3}$$

(a) Find the domain of f and check for symmetry. Find asymptotes (if any).

(b) Determine interval(s) where f is increasing, interval(s) where f is decreasing, and also find all local extrema (if any).

(c) Determine interval(s) where f is concave upward, interval(s) where f concave downward and inflection point(s) (if any).

(d) Sketch the graph of f.

[5] **Bonus Question**

Use the Mean Value Theorem to prove that if $x > 1$ then

(a) $\ln x < x - 1$, and

(b) $\dfrac{x-1}{x} < \ln x$.

[*Hint:* apply the Mean Value Theorem to the function ln on the interval $[1, x]$ for part (a), and again to the function ln on the interval $[1/x, 1]$ for part (b)].