PROBLEMS FOR CHAPTER 5

2. Integrate $\frac{1}{\pi}\int_0^{\pi} e^{2\sin x}dx$

   using Trapezoidal rule with 4 intervals

Solution:

$\frac{1}{\pi}\ \int_0^{\pi} e^{2sinx}$ dx in 4 intervals

$f(x) = \frac{1}{\pi}\ e^{2sinx}$

n	0	1	2	3	4
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
fx	0.31831	1.309288	2.35201	1.309288	0.31831

I=$\int_0^{\pi} f(x)dx = \frac{h}{2}[f(x_o) + 2\sum_{n=1}^{3} f(x_n) + f(x_4)]$

=$\frac{\pi}{4*2}[0.31831 + 2x(1.3093 + 2.35201 + 1.3093) + 0.31381]$

=4.15389

12. Compute the value of I by Simpson's rule, considering 6 intervals.

$$I = \int_0^{\pi/2} \sqrt{1-0.162\sin^2 \theta} d\theta$$

Solution:

I= $\int_0^{\pi/2} \sqrt{1 - 0.162sin^2 0}\ d0$

=$f(x)\sqrt{1 - 0.162sin^2 0}$

n	0	1	2	3	4	5	6
x	0	π/12	π/6	π/4	π/3	π/12	π/2
fx	1	0.99456	0.9995	0.95865	0.93728	0.92133	0.91542

I = $\frac{h}{3}[f(x_0) + 4(f(x_1) + f(x_3) + f(x_5) + 2\big(f(x_2) + f(x_4)\big) + f(x_6)]$

=$\frac{\pi}{12*3}[0.99456 + 4(0.99456 + 0.9586 + 0.92133) + 2(0.9795 + 0.93728) + 0.9542$

=1.5051

13. Using the data of the given table.

x	0.5	0.6	0.7	0.8	0.9	1.0	1.1
y	.4804	.5669	.6490	.7262	.7985	.8658	.9281

Compute the following integrals using Simpson's rule.

(a) $\int_{0.5}^{1.1} xy\, dx$ (b) $\int_{0.5}^{1.1} y^2\, dx$

(c) $\int_{0.5}^{1.1} x^2 y\, dx$ (d) $\int_{0.5}^{1.1} y^3\, dx$

Solution:

x	0.5	0.6	0.7	0.8	0.9	1	1.1
y	0.4804	0.5669	0.6490	0.7262	0.7985	0.8658	0.9281

a) $\int_{0.5}^{1.1} xydx$ therefore, $f(x) = xy$

x	0.5	0.6	0.7	0.8	0.9	1	1.1
y	0.2402	0.34014	0.4543	0.58096	0.71865	0.8658	1.02091

I = $\frac{h}{3}[f(x_0) + 4(f(x_1) + f(x_3) + f(x_5) + 2\big(f(x_2) + f(x_4)\big) + f(x_6)]$

I = $\frac{0.1}{3}[0.2402 + 4(0.34014 + 0.58096 + 0.8658) + 2(0.4543 + 0.71865)]$

= 0.358487

b) $\int_{0.5}^{1.1} y^2 dx$ therefore, $f(x)y^2$

x	0.5	0.6	0.7	0.8	0.9	1.0	1.1
$f(x)$	0.23078	0.32137	0.4212	0.5274	0.6376	0.7496	0.86137

I= $\frac{0.1}{3}[0.23078 + 4(0.32137 + 0.5274 + 0.7496) + 2(0.4214 + 0.6376) + 0.86137]$

= 0.320106

c) $\int_{0.5}^{1.1} x^2 y\, dx$ therefore, $f(x) - x^2 y$

x	0.5	0.6	0.7	0.8	0.9	1	1.1
fx	0.1201	0.20408	0.31801	0.46476	0.64678	0.8658	1.123

I = $\frac{0.1}{3}[0.1201 + 4(0.20408 + 0.46476 + 0.8658) + 2(0.31801 + 0.64678) + 1.123]$

= 0.310376

d) $\int_{0.5}^{1.1} y^3\, dx$ therefore, $f(x) = y^3$

X	0.5	0.6	0.7	0.8	0.9	1	1.1
f(x)	0.11087	0.1822	0.27336	0.38297	0.50912	0.649	0.7994

I = $\frac{0.1}{3}[0.11087 + 4(0.1822 + 0.38297 + 0.649) + 2(0.27336 + 0.50912 + 0.7994)]$

= 0.244399

16. Solve the following by Simpson's rule, using 2 and 4 intervals.

a. I = $\int_{100}^{200} \frac{1}{\log_{10} x} dx$

b. I = $\int_{\pi/6}^{\pi/2} \log_{10}(\sin x) dx$

Solution:

a) I = $\int_{100}^{20} \frac{1}{log_{10}x} dx$ therefore, f(x) = $\frac{1}{log_{10}x}$

n	0	1	2	3	4
x	100	125	150	175	200
$f(x)$	0.5	0.476892	0.45954	0.445824	0.434588

2 intervals

I = $\frac{(x_2-x_0)}{3}[f(x_0) + 4f(x_2) + f(x_a)]$

I = $\left(\frac{150-100}{3}\right)[0.5 + 4(0.45954) + 0.434588$

= 46.21244

4 intervals

I = $\frac{(x_2-x_0)}{3}[f(x_0) + 4f(x_1) + f(x_3) + 2(fx_2) + f(x_4)]$

= $\frac{125-100}{3}[0.5 + 4(0.476892 + 0.445824) + 2(0.45954) + 0.434588]$

= 46.20443

b) I = $\int_{\pi/6}^{\pi/2} log_{10}(sinx)dx$ therefore, f(x) = $log_{10}(sinx)$

n	0	1	2	3	4
x	π/6	π/4	π/3	π/12	π/2
$f(x)$	-0.30103	-0.15051	-0.06247	-0.01506	0

2 intervals

I= $\frac{\pi}{6*3}[-0.30103 + 4(-0.06247) + 0] = -0.0961515$

4 intervals

I = $\frac{\pi}{12*3}[-0.30103 + 4(-0.15051 - 0.01506) + 2(-0.06247) + 0]$

= 0.094968