CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Section(s)**
Mathematics	204	All
Examination	**Date**	**Pages**
Final	December 2013	2
Instructors		**Course Examiner**
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Special Instructions

- ▷ Only approved calculators are allowed.
- ▷ Justify all your answers.
- ▷ All questions have equal value.

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1. Use the Gauss-Jordan method to find all the solutions of the system:

$$\begin{array}{rcrcrcr} 2x_1 & - & 3x_2 & + & 10x_3 & = & -2 \\ x_1 & - & 2x_2 & + & 3x_3 & = & -2 \\ -x_1 & + & 3x_2 & + & x_3 & = & 4 \end{array}$$

2. Let $M = \begin{pmatrix} 1 & -2 & 1 \\ -3 & 7 & -6 \\ 2 & -3 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ -3 & 4 \\ 5 & 6 \end{pmatrix}$.

   (a) Calculate M^{-1}.

   (b) Find the matrix C such that $MC = B$.

3. (a) Use Cramer's rule to solve the system of equations:

$$\begin{array}{rcrcrcr} 3x_1 & + & x_2 & & & = & 5 \\ -x_1 & + & 2x_2 & + & x_3 & = & -2 \\ & - & x_2 & + & 2x_3 & = & -1 \end{array}$$

   (b) Find the determinant of the matrix $\begin{pmatrix} -2 & 1 & 4 & -1 \\ 1 & 0 & -1 & 2 \\ 5 & -1 & 2 & 1 \\ 0 & 0 & 3 & -1 \end{pmatrix}$.

4. Let ℓ be the line with parametric equations

$$x = -2 + t,\ y = -1 + 3t,\ z = 1 + 5t,$$

and let $v = (1, 1, 1)$. Find w_1 and w_2 so that $v = w_1 + w_2$ and w_1 is parallel to ℓ and w_2 is perpendicular to ℓ.

5. Let $P_1(1, 1, 1)$, $P_2(1, 3, 4)$, $P_3(2, 1, 5)$.

   (a) Find the area of the triangle with vertices P_1, P_2, P_3.

   (b) Find the equation of the plane containing P_1, P_2 and P_3.

6. Let $P(1, 2, 3)$ be a point. Let $n = (1, 3, 4)$.

   (a) Find the point-normal equation of the plane through P with normal n.

   (b) Express the equation of the plane in the form $ax + by + cz + d = 0$.

7. Let $v_1 = (1, -2, 3)$ and $v_2 = (2, 0, 4)$.

   (a) Find numbers x and y so that $xv_1 + yv_2 = (0, -4, 2)$.

   (b) Find v_3 so that $\{v_1, v_2, v_3\}$ is a basis of $\mathbb{R}^3$.

8. Let $A = \begin{pmatrix} 1 & -3 & 0 & 0 & 2 & 5 \\ 0 & 0 & 1 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \\ z \\ u \\ v \\ w \end{pmatrix}$. Find a basis of the solution space of the homogeneous system of linear equations $AX = 0$.

9. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ -2 & -2 & -1 \\ 0 & 0 & -1 \end{pmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

10. Let $A = \begin{pmatrix} -3 & 4 \\ -2 & 3 \end{pmatrix}$. Compute A^{1000}.