Engr 233, Section QA; Class test, Monday, November 15, 2021

Instructor A. Kokotov

Time: 60 min

Answer all questions. Closed book

1. **(10 points)** Reverse the order of integration

$$\int_0^1 \left(\int_{2-\sqrt{1-y^2}}^{2+\sqrt{1-y^2}} f(x,y)\,dx \right) dy$$

2. **(10 points)** Let $A = (-1, 1, -1)$ and $B = (0, 1, 1)$. Prove that the following line integral is path independent and compute it:

$$\int_A^B (y^2 + y + yz)dx + (2xy + x + xz)dy + xy\,dz\,.$$

3. Compute the following line integral (the circle of integration is oriented counterclockwise)

$$\oint_{(x-1)^2+(y+3)^2=4} [\arctan(1+x^2) + (y+3)^3]dx - [\cos(1+y^3) + (x-1)^3]dy\,.$$

4. **(10 points)** Compute the double integral

$$\iint_\Omega (y^3 + yx^2)\,dx\,dy$$

where the domain Ω is defined by the inequalities

$$y \geq 0; \quad x^2 + y^2 \geq 1; \quad x^2 + y^2 \leq 9$$

5. The cone $z^2 = x^2 + y^2$ cuts the sphere $x^2 + y^2 + z^2 = 1$ into three parts Ω_1, Ω_2 and Ω_3. Find the area of Ω_1, Ω_2 and Ω_3.