

Concordia University
Applied Ordinary Differential Equations, ENGR 213
Final Exam
20 April 2009
D. Korotkin, A. Stancu, S. Hashtrudi Zad

Evaluation out of 100. Only admissible calculators are allowed.
Time allotted: Three hours.

1. (15) (a) Find the general solution of the differential equation

$$\frac{dy}{dx} = \left(\frac{2y+3}{4x+5} \right)^2 .$$

You may leave the solution in implicit form.

- (b) Solve the initial value problem

$$x \frac{dy}{dx} + y = e^x , \quad y(1) = 2$$

2. (10) Find the general solution (explicit or implicit) of the equation

$$(y^2 \cos x - 3x^2 y - 2x) dx + (2y \sin x - x^3 + \ln y) dy = 0 .$$

3. (10) Find the general solution of the equation using an appropriate substitution:

$$\frac{dy}{dx} = \tan^2(x+y) .$$

You may leave the solution in implicit form.

4. (10) The Space Shuttle lands in Kennedy Space Center. The spacecraft touches down at $t = 0$ with a velocity of 100 m/sec. The spacecraft chute is deployed at $t = 4$ sec. Between touch down and deployment of chute ($0 \leq t \leq 4$), the velocity of the spacecraft $V(t)$ (in m/sec) is governed by:

$$\frac{dV}{dt} = 0$$

and after the deployment of chute by:

$$\frac{dV}{dt} = -0.002V^2$$

Determine when the spacecraft velocity reaches 20 m/sec.

5. (10) Find the general solution of the following differential equations using the method of *undetermined coefficients*.

(a) $y'' + 6y' + 8y = \sin 3x$

(b) $y'' + 10y' + 25y = e^x$

6. (10) Find the general solution of the differential equation

$$2x^2y'' + 5xy' + y = x^2 - x$$

by *variation of parameters*.

7. (12) Solve the following system of differential equations by any method you wish (systematic elimination, undetermined coefficients, variation of parameters, or diagonalization).

$$\begin{aligned}\frac{dx}{dt} &= 2x + 3y - e^{2t} \\ \frac{dy}{dt} &= -x - 2y + e^{2t}.\end{aligned}$$

8. (11) Find the power series solution about the ordinary point $x = 0$ for the initial value problem

$$y'' - 3xy' - y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

It suffices to give only the constant term and those of x , x^2 and x^3 .

9. (12) Given the LRC-circuit with $L = \frac{5}{3}$ henries, $R = 10$ ohms, $C = \frac{1}{30}$ farads, and $E(t) = 50 \cos t$ volts, the charge $q(t)$ satisfies the linear second order ordinary differential equation

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = E(t).$$

- (a) Find the charge $q(t)$ if $q(0) = 100$ coulombs and $q'(0) = 0$ amperes.
 (b) Identify in $q(t)$ the transient terms and, respectively, the steady state terms. Is the circuit overdamped, underdamped, or critically damped?

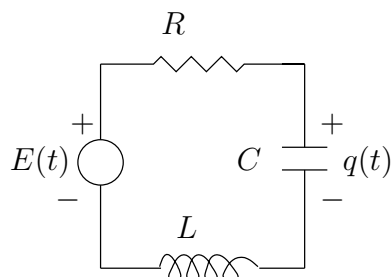


Figure 1: Problem 9.