ENGR 371 - May 2015

#1 $n=16$

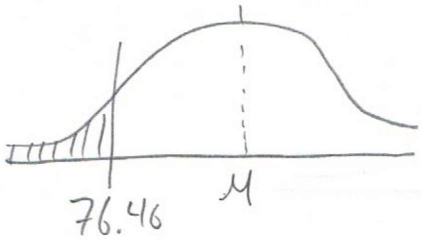
$X_i = \{78.3, 77.1, 71.3, 84.5, 87.8, 75.7, 64.8, 72.5, 78.2, 91.2,$
$86.2, 80.9, 82.1, 89.3, 89.4, 81.6\}$

(a) $\sum x_i = 1'290.9$ $\quad \bar{X} = 80.68$ $\quad S^2 = \dfrac{\sum x_i^2 - n\bar{X}^2}{n-1}$

$\sum x_i^2 = 104'967.61$ $\quad S = 7.38 \;\therefore$

$S^2 = 54.41$

(b)



$1-\alpha = 98\%$ $\quad \bar{x} = 80.68$

$\alpha = 0.02$ $\quad S = 7.38$

$n = 16$

$\mu > \bar{X} - T_{\alpha, n-1} \dfrac{S}{\sqrt{n}}$

$\mu > 80.68 - T_{0.02,15} \dfrac{7.38}{\sqrt{16}}$

$\mu > (80.68) - (T_{0.02,15})(1.845)$

$\mu > 80.68 - 2.288(1.845)$

$\mu > 76.46 \;\therefore$

$T_{0.02,15} = 0.02$ not on table

$\alpha_1 = 0.025 \quad \alpha_2 = 0.01$

$t_1 = 2.131 \quad t_2 = 2.602$

$\alpha = 0.02 \quad t = ?$

$\dfrac{\alpha_1 - \alpha_2}{t_1 - t_2} = \dfrac{\alpha - \alpha_2}{t - t_2}$

$\dfrac{0.025 - 0.01}{2.131 - 2.602} = \dfrac{0.02 - 0.01}{t - 2.602}$

$t = 2.288$

(c) $n=16$

$\bar{X} = 80.68$

$S = 7.38$

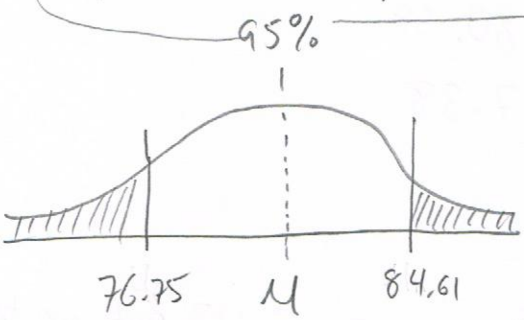
$\alpha = 0.05$

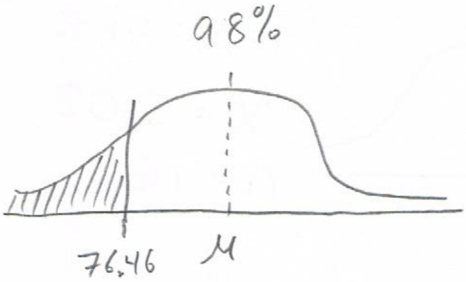
$$\bar{X} - T_{\frac{\alpha}{2},n-1}\frac{S}{\sqrt{n}} < \mu < \bar{X} + T_{\frac{\alpha}{2},n-1}\frac{S}{\sqrt{n}}$$

$$T_{\frac{\alpha}{2},n-1} = T_{0.025,15} = 2.131$$

$$80.68 - 2.131\left(\frac{7.38}{\sqrt{16}}\right) < \mu < 80.68 + 2.131\,(1.845)$$

$$76.75 < \mu < 84.61$$





98% CI is more confident than 95% CI so it's less.

(d) $n=16$

$\bar{X} = 80.68$

$S = 7.38$

$\alpha = 0.05$

$S^2 = 54.41$

$$\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2},n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}$$

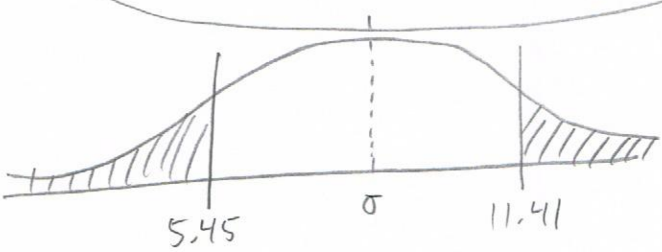
$$\chi^2_{\frac{\alpha}{2},n-1} = \chi^2_{0.025,15} = 27.49 \qquad \chi_{1-\frac{\alpha}{2},n-1} = \chi^2_{0.975,15} = 6.27$$

$$\frac{(15)(54.41)}{27.49} \le \sigma^2 \le \frac{(15)(54.41)}{6.27}$$

$$29.69 \le \sigma^2 \le 130.17$$

$$\sqrt{29.69} \le \sigma \le \sqrt{130.17}$$

$$5.45 \le \sigma \le 11.41$$



(e) H_0 $\mu = 80$

H_1 $\mu \neq 80$

$\alpha = 0.05$

$n = 16$

$\bar{x} = 80.68$

$S = 7.38$

$$T = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{80.68 - 80}{7.38/\sqrt{16}} = 0.369$$

$T_\alpha \rightarrow T_{0.4,15} < 0.369 < T_{0.25,15}$

$0.258 < 0.369 < 0.691$

$$\frac{\alpha_1 - \alpha_2}{t_1 - t_2} = \frac{\alpha - \alpha_2}{t - t_2}$$

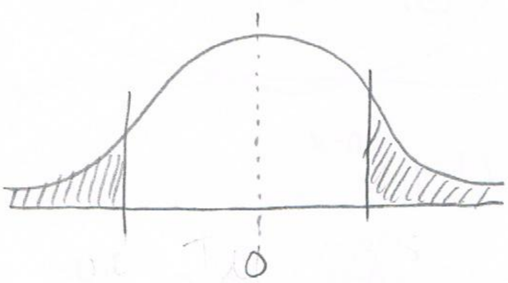
$\alpha_1 = 0.4$ $t_1 = 0.258$

$\alpha_2 = 0.25$ $t_2 = 0.691$

$\alpha = ?$ $t = 0.369$

$$\frac{0.4 - 0.25}{0.258 - 0.691} = \frac{\alpha - 0.25}{0.369 - 0.691}$$

$\alpha = 0.3615$



p-value $= 2\phi(-t_0)$

$= 2(0.369)$

$= 0.738$

$73.8\% > 5\%$ ∴

H_0 = accepted

H_1 = rejected.

(f) $\alpha = 0.05$

$n = 16$

$\bar{x} = 80.68$

$S = 7.38$

$T_{\frac{\alpha}{2}, n-1} = T_{0.025,15} = 2.131$

$$\bar{X} - T_{\frac{\alpha}{2},n-1} S\sqrt{1+\tfrac{1}{n}} < \bar{X}_{n+1} < \bar{X} + T_{\frac{\alpha}{2},n-1} S\sqrt{1+\tfrac{1}{n}}$$

$80.68 \pm (2.131)(7.38)(1.033) \lessgtr \bar{X}_{17}$

80.68 ± 16.246

$64.434 < X_{17} < 96.925$ ∴

#2 $n_1 = 16$ $\quad n_2 = 9$

$\mu_1 = 75$ $\quad \mu_2 = 70$ $\quad P(\bar{X}_1 - \bar{X}_2 > 4)$

$\sigma_1 = 8$ $\quad \sigma_2 = 12$

$$Z \quad \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma^2}{n_2}}} = \frac{(X_1 - X_2) - (5)}{\sqrt{\frac{8^2}{16} + \frac{12^2}{9}}} = \frac{X_1 - X_2 - 5}{\sqrt{20}} =$$

$X_1 - X_2 > 4$ $\quad P\left(Z > \frac{4-5}{\sqrt{20}}\right) = P(Z > -0.2236)$

$X_1 - X_2 = 4$

$P(Z > Z_2) = 1 - \phi(Z_2)$

$P = 1 - \phi(-0.2236) = 1 - 0.587 = 0.4129 = 41.3\%$ $\therefore$

#3 P-non-conforming = 0.01

P-conforming = 0.99

$\binom{n}{x} p^x (1-p)^{n-x}$

(a) x is # success in n trials

$n = ?$ $\quad x \geq 1$

$P(x \geq 1) = 0.9$

$1 - P(x=0) = 0.9$

$1 - \binom{n}{0} p^0 (1-p)^{n-0} = 0.9$

$(1)(1)(1-0.01)^n = 0.1$

$(0.99)^n = 0.1$

$n = \frac{\ln 0.1}{\ln 0.99} \approx 230$ $\therefore$

b) $\mu_x = 5 \quad \sigma_x = 2$

$\mu_y = 10 \quad \sigma_y = 1$

i) $E(x+2y) = E(x) + 2E(y)$
$= 5 + 2(10)$
$= 25$ ∴

ii) $V(x+2y) = V(x) + 4V(y)$
$= 2 + 4(1)$
$= 6$ ∴

iii) $E(x+x) = 2E(x) = 10$ ∴

iv) $V(x+x) = V(2x) = 4V(x)$
$= 4(2) = 8$ ∴

v) $E(xy) = E(x)E(y) = (5)(10) = 50$ ∴

#4) $D = 2$
$N = 75$

(a) x is the # deffective copies

$$\frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} = \frac{\binom{2}{x}\binom{75-2}{3-x}}{\binom{75}{3}}$$

$P(x=1) = ?$
$n = 3$
$N = 75$
$K = 2$
$x = 1$

$$\frac{\binom{2}{1}\binom{75-2}{3-1}}{\binom{75}{3}} = \frac{(2)(2628)}{(67'525)} = 0.078 = 7.8\%$$ ∴

(b) $P(x=2) = ?$

$$\frac{\binom{2}{2}\binom{75-2}{3-2}}{\binom{75}{3}} = \frac{(1)(73)}{(67'525)} = 0.00108 = 0.108\%$$ ∴

(c) $P(x=2) = ?$
$n = 73$
$K = 2$
$x = 2$
$N = 75$

$$\frac{\binom{2}{2}\binom{75-2}{73-2}}{\binom{75}{73}} = \frac{\binom{73}{71}}{\binom{75}{73}} = \frac{2'628}{2775} = 0.947$$

$= 94.7\%$ ∴

#5 X = temperature
Y = spectrum

$$f(x,y) = \begin{cases} 10xy^2 & 0 < x < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) $f_x(x) = \int_{\text{all } y} f(x,y)\,dy = \int_x^1 10xy^2\,dy = \frac{10xy^3}{3}\Big|_x^1 = 10x\left(\frac{1}{3} - \frac{x^3}{3}\right)$

$f_y(y) = \int_{\text{all } x} f(x,y)\,dx = \int_0^y 10xy^2\,dx = \frac{10y^2x^2}{2}\Big|_0^y = 10y^2\left(\frac{y^2}{2}\right)$

$f(y|x) = \frac{f(x,y)}{f_x(x)} = \frac{10xy^2}{10x\left(\frac{1}{3} - \frac{x^3}{3}\right)}$

(b) $P(y > \frac{1}{2} \mid x = \frac{1}{4}) = \int_{1/4}^{1/2} f(y \mid x = 0.25)\,dy$

$= \int_{1/4}^{1/2} \frac{10(\frac{1}{2})y^2}{10(\frac{1}{2})\left(\frac{1}{3} - \frac{(\frac{1}{2})^3}{3}\right)} = \int_{1/4}^{1/2} \frac{5y^2}{\frac{5}{3} - \frac{1}{24}} = \int_{1/4}^{1/2} \frac{5y^2}{1.625}$

$= \frac{40}{13}\int_{1/4}^{1/2} y^2\,dy = \frac{40}{13}\left[\frac{y^3}{3}\right]_{1/4}^{1/2} = \frac{40}{13}\left[\frac{1}{24} - \frac{1}{192}\right] = 0.112$

(#6) $\mu = 260$ min

$\sigma = 50$ min

(a) $P(x > 240) = ?$

⚠ change to z-domain $\quad z = \dfrac{x-\mu}{\sigma}$

$P\left(z > \dfrac{240-260}{50}\right) = P(z > -0.4)$

$= 1 - \phi(-0.4) = 1 - 0.344578 = 0.655$

$P(x > 240) = 0.655 \;\therefore$

(b) $P(x > a) = 0.25$

$P(x > b) = 0.75$

$P(x > a)\; z = \dfrac{a-\mu}{\sigma} = \dfrac{a-260}{50} = P\left(z > \dfrac{a-260}{50}\right)$

$1 - \phi\left(\dfrac{a-260}{50}\right) = 0.25$

$\phi\left(\dfrac{a-260}{50}\right) = 0.75 \longrightarrow (z(0.68) = 0.75)$

$\dfrac{a-260}{50} = 0.68$

$a = 294 \;\therefore \longrightarrow P(x > 294) = 25\%$

$P(x > b) = P\left(z > \dfrac{b-260}{50}\right) = 0.75$

$\phi\left(\dfrac{b-260}{50}\right) = 0.25 \longrightarrow (z(-0.67) = 0.25)$

$\dfrac{b-260}{50} = -0.67$

$b = 226.5 \;\therefore \longrightarrow P(x > 226.5) = 75\%$

(c) $P(x > c) = 0.95$

$P\left(z > \frac{x-\mu}{\sigma}\right) = P\left(z > \frac{c-260}{50}\right) = 0.95$

$1 - \phi\left(\frac{c-260}{50}\right) = 0.95$

$\phi\left(\frac{c-260}{50}\right) = 0.05 \longrightarrow \left(z(-1.64) = 0.05\right)$

$\frac{c-260}{50} = -1.64$

$c = 178$ ∴ $\longrightarrow P(x > 178) = 95\%$