ENGR 391 – Numerical Methods in Engineering
Final Exam (Winter,2012)

Student's Name: _____

I.D.: _____

- Read carefully all the questions
- Closed Book Exam; **Double sided** 8 ½ x 11 formula sheet allowed
- You must **show all your steps** leading to the solution(s)
- Give the answers in the area provided.
- You can use the back of the papers to make additional calculations (will not be corrected)
- Please do not write in red (colour used for correction)
- Write all your answers in the foreseen places
- Everything not readable will NOT be corrected
- No questions will be answered during exam : answer to your best knowledge

1	
2	
3	
4	
5	
6	
Total	

PROBLEM 1 [Solving Nonlinear Equations] [10 marks]

a) Solve the following nonlinear equation using the Secant method with $x_0 = 0$ and $x_1 = 1$. Show 3 iterations.

$$f(x) = x \cdot e^{2x} - 1$$

b) Solve the same nonlinear equation using Newton Raphson's method with $x_0 = 1$. Show 3 iterations.

c) Based on your calculations, estimate the errors of a) and b) and give the solution of equation (1) with as many correct significant digits as you can guarantee.

PROBLEM 2 [Linear equations] [10 marks]

Solve the following system of equation using the LU decomposition:

$$x_1 + 2x_2 + 4x_3 + 3x_4 = -4$$
$$x_1 - x_2 + x_3 - x_4 = 0$$
$$2x_1 + 5x_2 - 3x_3 - x_4 = 11$$
$$2x_2 + x_3 - 4x_4 = 1$$

$-4x_1 +$

$x_1 =$ _____ $x_3 =$ _____

$x_2 =$ _____ $x_4 =$ _____

PROBLEM 3 [Numerical Integration] [10 marks]

Compute the following integral:

$$I = \int_{-1}^{3} \frac{2}{x^2 + 4} dx$$

a) using multi-segment composite Trapezoidal rule with a step *h* equal to 2 (2 segments)

I = _____

b) using multi-segment (composite) Simpson's 1/3 rule with a step *h* equal to 2 (2 segments) and 1 (4 segments).

I = _____

c) Estimate the error based on the results from a) and b)

PROBLEM 4 [Interpolation] [10 marks]

Given the following data set of travelled distance of a horse, *x,* in miles versus the time *t* in seconds

Distance x	***Time t***
0	0
0.5	49.4
1.0	96.4
1.25	119.4

a) Find the highest degree of Newton's polynomial P(x) that interpolates this data set (8 pts).

P(x) = _____

b) Estimate the elapsed time *t* at *x* = 0.75 (2 pts).

PROBLEM 5 [Numerical Differentiation] [5 marks]

Given a function $f(x)$ and five equally spaced points:
$x_0 - 2h,\ x_0 - h,\ x_0,\ x_0 + h$ and $x_0 + 2h$

a) Derive a finite difference approximation formula for the third derivative $f'''(x_0)$ by using $f(x_0 - 2h),\ f(x_0 - h),\ f(x_0),\ f(x_0 + h)$, and $f(x_0 + 2h)$ (4 pts).
b) Find the order of the truncation error (1 pts).

PROBLEM 6 [Solving Ordinary Differential Equations] [10 marks]

We aim to solve an ordinary differentiation equation in the form of:

$$\frac{dy}{dx} = g(x,y) \quad \text{when} \quad y(x_0) = y_0 \qquad a \le x \le b$$

a) Given the 2nd order Taylor method:

$$y_{i+1} = y_i + g(x_i, y_i)h + \frac{1}{2!}g'(x_i, y_i)h^2 \quad \text{(Eq. 1)}$$

and the 2nd order Runge-Kutta method:

$$y_{i+1} = y_i + (c_1k_1 + c_2k_2)h \qquad \text{(Eq. 2)}$$

where $h = x_{i+1} - x_i$

$k_1 = g(x_i, y_i)$

$k_2 = g(x_i + a_2h, y_i + b_{21}k_1h)$

Find the values of the constants c_1, a_2 and b_{21} when $c_2 = 1/2$, so that Eq. (2) becomes the modified Euler method. **Show all the intermediate steps to get full mark.** (5 pts)

Hint: 1) Transform Eq. (1) and (2) by using Taylor series expansions and compare the resultant equations.

2) The modified Euler method is $y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)h$ *with* $k_1 = g(x_i, y_i)$ *and* $k_2 = g(x_i + h, y_i + k_1h)$

c_1 = _____

a_2 = _____

b_{21} = _____

b) Solve the following 2nd order ordinary differential equation by the classical fourth-order Runge-Kutta Method when $t = 1.0$.

$$\frac{d^2y}{dt^2} - 0.5t + y = 0$$

when $y(0) = 2$; $y'(0) = 0$; and the step size $h = 0.5$

Solve for $y(1.0)$ (keep five significant figures for decimal numbers, 5 pts).

y(1.0) = _____