CONCORDIA UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING

COMP 232: MATHEMATICS FOR COMPUTER SCIENCE: SECTION AA
SUMMER 2021

FINAL EXAMINATION

Total Time: 3 Hours **Total Marks: 100**

There are **TWENTY FIVE problems** in all, each carrying **4 marks**.

There are **THREE** types of problems:

1. For each of the problems 1 to 12, **indicate your choice** by mentioning **one of the letter (a) to (d) only**. There is no need to provide an explanation.

2. For each of the problems 13 to 22, **provide suitable text only for the blank space** so that the resulting statement is correct. There is no need to provide an explanation.

3. For each of the problems 23 to 25, provide a solution. You must show **all steps** of your solution.

Notation:

Z: set of integers, $\mathbf{Z}^+$: set of positive integers, **R**: set of real numbers.
The set of natural numbers includes 0.

PROBLEM 1. [4 MARKS]

This is about giving the **most simplified form** of the proposition

$$\neg[\ [\ p \wedge (\neg(\neg p \vee q))\ ] \vee (p \wedge q)\ ] \vee q.$$

Select **one** of the following choices:

(a) $p \longrightarrow q$.

(b) $p \wedge q$.

(c) $q \longrightarrow p$.

(d) $p \vee q$.

PROBLEM 2. [4 MARKS]

Consider the following statements, where the domain of each variable is **R**:

(1) $\forall x\ \exists y\ ((x + y = 2) \wedge (2x - y = 1))$.
(2) $\exists x\ \forall y\ (y \neq 0 \longrightarrow xy = 1)$.
(3) $\forall x\ \forall y\ \exists z\ (x + y = 2z)$.

Select **one** of the following choices:

(a) (1) and (2) are True, and (3) is False.

(b) (1) is False, and (2) and (3) are True.

(c) (1) and (2) are False, and (3) is True.

(d) (1) and (3) are False, and (2) is True.

PROBLEM 3. [4 MARKS]

Let p, q, and r be propositions.

Consider the following statements:

(1) For a, b, c, d, and m being integers with $m \geq 2$, if $ac \equiv bc \pmod{m}$, then $a \equiv b \pmod{m}$.
(2) $p \longrightarrow (q \longrightarrow r)$ and $p \longrightarrow (q \wedge r)$ are logically equivalent.

Select **one** of the following choices:

(a) (1) is True and (2) is False.

(b) (1) is False and (2) is True.

(c) (1) and (2) are True.

(d) (1) and (2) are False.

PROBLEM 4. [4 MARKS]

Let P(x, y, z) denote the statement $x^3 - y^3 = z$. Let the universe of discourse for x, y, and z be $\mathbf{Z}^+$.

Consider the following statements:

(1) $\forall x\ \forall z\ \exists y$ P(x, y, z).
(2) $\forall z\ \exists x\ \exists y$ P(x, y, z).

Select **one** of the following choices:

(a) (1) is True and (2) is False.
(b) (1) is False and (2) is True.
(c) (1) and (2) are True.
(d) (1) and (2) are False.

PROBLEM 5. [4 MARKS]

This problem is about the translation of a logical statement into an equivalent English statement.

Let C(x, y) be the statement "x and y have chatted over Zoom," where the domain for the variables x and y consists of all members in your group.

Select **one** of the following choices to indicate the English statement corresponding to

$$\forall y\ [\ \mathrm{C}(\mathrm{Mary}, y) \leftrightarrow (y \neq \mathrm{Robert})\ ].$$

(a) Robert has chatted with Mary only.
(b) Nobody has chatted with Robert or Mary.
(c) Mary has chatted with Robert but not with the others.
(d) Mary has chatted with everyone except Robert.

PROBLEM 6. [4 MARKS]

Consider the following statements:

(1) $(\sqrt{2}\cdot\sqrt{6}) / (\sqrt{18}\cdot\sqrt{24})$ is an irrational number.
(2) Let $S = \varnothing \times A$. Then, $|\ P(S)\ | = 0$, where $P(S)$ denotes the power set and $|\quad|$ denotes cardinality (that is, the number of elements).
(3) If n is a positive integer, then n is odd if and only if 5n + 6 is odd.

Select **one** of the following choices:

(a) (2) and (3) are True, (1) is False.
(b) (1) and (2) are True, (3) is False.
(c) (1) and (2) are False, (3) is True.
(d) (1) and (3) are True, (2) is False.

PROBLEM 7. [4 MARKS]

Let $A_i = \{\ldots, -2, -1, 0, 1, 2, \ldots, i\}$.
Let $B_n = A_1 \cup A_2 \cup \cdots \cup A_n$.
Let $C_n = A_1 \cap A_2 \cap \cdots \cap A_n$.

Consider the following statements:

(1) $B_n = C_n$.
(2) $B_n \subseteq C_n$.
(3) $C_n \subseteq B_n$.
(4) $|B_n - C_n| = n$, where $|\quad|$ denotes cardinality (that is, the number of elements).

Select **one** of the following choices:

(a) (2) is True and (4) is False.
(b) (3) is False and (4) is True.
(c) (1) and (4) are True.
(d) (3) is True and (4) is False.

PROBLEM 8. [4 MARKS]

Consider the following statements:

(1) If A is an uncountable set and B is a countable set, then $A - B$ can be a countable set.
(2) $\mathbf{Z}^+ \times \mathbf{Z}^+$ is a countable set.

Select **one** of the following choices:

(a) (1) is True and (2) is False.
(b) (1) is False and (2) is True.
(c) (1) and (2) are True.
(d) (1) and (2) are False.

PROBLEM 9. [4 MARKS]

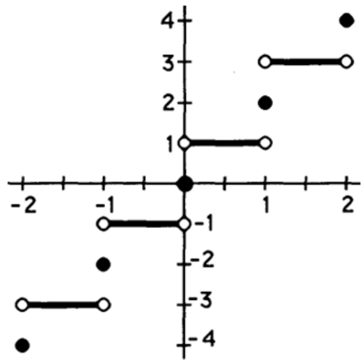
This problem is about giving an example of a function from $\mathbf{Z}^+$ to $\mathbf{Z}^+$ that is onto, but **not** one-to-one.

Select **one** of the following choices:

(a) $x + \lceil x \rceil - \lfloor x \rfloor$.
(b) $\lceil x/2 \rceil$.
(c) $2 \cdot \lfloor x/2 \rfloor + 1$.
(d) $|x|$.

PROBLEM 10. [4 MARKS]

Let the graph of a function $f(x)$ be as shown below:



Select **one** of the following choices, where x is a real number and $x \in [-2, 2]$:

(a) $f(x) = \lfloor x - 2 \rfloor + \lceil x + 2 \rceil$.

(b) $f(x) = \lfloor x + 2 \rfloor + \lceil x - 2 \rceil$.

(c) $f(x) = \lfloor x - 2 \rfloor - \lceil x + 2 \rceil$.

(d) $f(x) = \lfloor x + 1 \rfloor + \lceil x - 1 \rceil$.

PROBLEM 11. [4 MARKS]

A **Pythagorean prime number** is a prime number of the form 4n + 1, where n ≥ 1. A **Mersenne prime number** is a prime number of the form $2^n - 1$. A **perfect number** is a number that is equal to the sum of all of its divisors, including 1 but excluding the number itself.

Consider the following statements:

(1) Let m is the smallest Pythagorean prime number. Then, $2^m - 1$ is a Mersenne prime number.
(2) If m is a perfect number, then $2^m - 1$ is a Mersenne prime number.

Select **one** of the following choices:

(a) (1) is True and (2) is False.

(b) (1) is False and (2) is True.

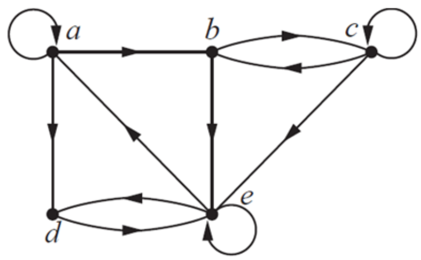
(c) (1) and (2) are True.

(d) (1) and (2) are False.

PROBLEM 12. [4 MARKS]

Consider the following statements:

(1) The "$x + y$ is a prime number" relation on $\mathbf{Z}^+$ is transitive.
(2) For $S = \{0, 1, 2, \ldots, 9\}$, $f : S \longrightarrow S$ be defined by $f(k) = (5k + 3)$ **mod** 10 is not invertible.
(3) The relation shown as a directed graph below is not a partial order.



Select **one** of the following choices:

(a) (2) and (3) are True, (1) is False.

(b) (1) and (2) are True, (3) is False.

(c) (1) and (3) are False, (2) is True.

(d) (1) and (3) are True, (2) is False.

PROBLEM 13. [4 MARKS]

(a) Let N be the number of rows in the truth table of $(p \longrightarrow r) \vee (s \longrightarrow \neg v) \vee (\neg u \longrightarrow p) \wedge (\neg r \longrightarrow \neg t)$.
Then, N = _____________.

(b) The prime factorization of N – 1 is _____________.

PROBLEM 14. [4 MARKS]

Let the universe of discourse for x, y, and z be $\mathbf{Z}$. Let $P(x, y, z)$ denote $xy^2 = z$. A **counterexample** to $\forall x\ \forall z\ \exists y\ P(x, y, z)$ is _________________________.

PROBLEM 15. [4 MARKS]

Let $A = \{x \mid x \text{ is a prime number and } 10 < x < 20\}$, $B = \{x \mid x \text{ is an odd number and } 10 < x < 20\}$, and $C = \{x \mid x \text{ is relatively prime to 18, and } 10 < x < 20\}$. Let $|\quad|$ denotes cardinality (that is, the number of elements). Then, $|A \cup B \cup C| =$ _____________.

PROBLEM 16. [4 MARKS]

Let $S = \{x \mid x \text{ is a prime number and } 1 < x < 10\}$. Let N be the number of **different** relations that are **both** reflexive and symmetric that can be defined on S. Then, N = _____________.

PROBLEM 17. [4 MARKS]

Let

$$\lfloor 1 \rfloor + \lfloor 1/2 \rfloor + \lfloor 1/3 \rfloor + \cdots + \lfloor 1/n \rfloor = \lceil 0/1 \rceil + \lceil 1/2 \rceil + \lceil 2/3 \rceil + \cdots + \lceil 1 - (1/n) \rceil$$

be given. Then, a value of n that satisfies the previous equation is _____________.

PROBLEM 18. [4 MARKS]

Let f be a function from **R** to **R** defined by $f(x) = x^2$. Let T denote the set $\{x \mid 0 < x \leq 1\}$. Then, $f^{-1}(T) =$ _____________.

PROBLEM 19. [4 MARKS]

Let $f(n)$ be defined recursively by

$$f(0) = 3, \text{ and}$$
$$f(n + 1) = 3^{f(n)/3}, \text{ for } n = 0, 1, 2, \ldots$$

Then, $f(10) =$ _____________.

PROBLEM 20. [4 MARKS]

Let the equation $\left| x - \gcd(11111, 111111) \right| = \gcd(96, 356) - \gcd(12, 15)$, where $\left| \ \right|$ denotes absolute value, be given. Then, integer values of x that satisfy the previous equation are _____________.

PROBLEM 21. [4 MARKS]

Let $A = \{2, 3, 4, 8, 9, 12\}$, and let the relation R on A be defined by

$$aRb \text{ if and only if } (a \mid b \wedge a \neq b).$$

Then, $R^3 =$ _________________________.

PROBLEM 22. [4 MARKS]

Let R be the relation $\{(a, b) \mid a \neq b\}$ on **Z**. Then, the reflexive closure of R is _____________.

PROBLEM 23. [4 MARKS]

Let $A = \{1, 2, \ldots, 12\}$. Let aRb mean $a \equiv b \pmod 5$.

(a) Give $[2]_R$, $[3]_R$, and $[5]_R$.

(b) Give $[2]_R \cap [12]_R$.

PROBLEM 24. [4 MARKS]

Let there be a sequence of numbers defined by $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$, for $n > 1$.

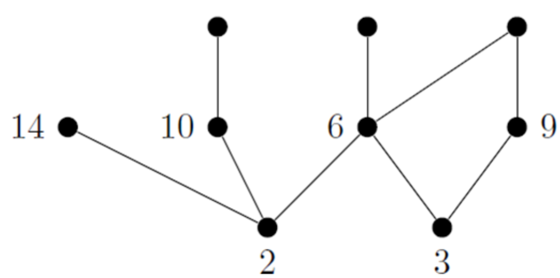
(a) For $n = 0$, give a value of f_{3n}.

(b) For $n > 0$, give an expression for f_{3n} in terms of f_{3n-2} and $f_{3(n-1)}$.

PROBLEM 25. [4 MARKS]

Let $A = \{2, 3, 6, 9, 10, 12, 14, 18, 20\}$ and let R be the partial order relation defined on A where xRy means x is a divisor of y.

(a) The following **partial** Hasse Diagram for R is given. Provide numbers for the **top three vertices** so that the Hasse Diagram is **complete and correct**.



(b) Find lub($\{3, 10\}$), if it exists or state that it does not exist.

(c) Find glb($\{14, 10\}$), if it exists or state that it does not exist.

(d) State whether the partially ordered set represented by the complete Hasse Diagram is a lattice.