Department of Mathematics & Statistics		
Course	**Number**	**Section(s)**
Mathematics	204/4	All, except EC
Examination	**Date**	**Pages**
Final	April 2013	2
Instructors		**Course Examiner**
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Special Instructions: ▷ Only approved calculators are allowed.

Answer 10 questions. All questions have equal value.

1. Using the Gauss-Jordan method (i.e. reduced row echelon form method), find all the solutions of the following system of equations

$$\begin{aligned} 2x + 3y + 7z \quad\quad + 11v &= -2 \\ 3x + 3y + 9z - 6u \quad\quad &= -6 \\ 2x \quad\quad + 4z + u + 4v &= 5. \end{aligned}$$

2. Let $M = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 0 & 4 \end{bmatrix}$.

   a) Calculate M^{-1}.

   b) Find the matrix C such that $MC = B$.

3. a) Use Cramer's rule to solve the following system of equations

$$\begin{aligned} 2x \quad\quad + z &= -3 \\ y + z &= 6 \\ 3x + y \quad\quad &= -1. \end{aligned}$$

   (No marks given if you don't use Cramer's rule.)

   b) Calculate the determinant of the matrix $\begin{bmatrix} 3 & 1 & 0 & 1 \\ 0 & 1 & 3 & 2 \\ 3 & 0 & 2 & 3 \\ 1 & 2 & 1 & 0 \end{bmatrix}$.

4. a) Find parametric equations for the line that is the intersection of the planes $x - y + 2z + 1 = 0$ and $2x + 2y + z - 3 = 0$.

   b) Find the equation of the plane, containing the origin $(0, 0, 0)$, which is orthogonal to both of the planes of part (a).

5. Let $P_0 = (1, 2, 0)$, $P_1 = (1, 1, 2)$, $P_2 = (1, 1, 0)$ and $P_3 = (0, 2, 1)$.

   a) Find an equation of the plane containing P_1, P_2 and P_3.

   b) Find the volume of the parallelepiped determined by the vectors $\overrightarrow{P_0P_1}$, $\overrightarrow{P_0P_2}$ and $\overrightarrow{P_0P_3}$.

6. Let $\mathcal{L}$ be the line with parametric equations $x = 1 + 2t$, $y = 2 - 3t$, $z = 1 - t$, and let $\mathbf{v} = (2, -1, 0)$. Find vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ such that $\mathbf{v} = \mathbf{w}_1 + \mathbf{w}_2$, and such that $\mathbf{w}_1$ is parallel to $\mathcal{L}$ and $\mathbf{w}_2$ is perpendicular to $\mathcal{L}$.

7. a) Express the vector $(7, 2, -7)$ as a linear combination of the vectors $(3, 2, 1)$ and $(2, 2, 3)$.

   b) Prove that the set $\{(1, 1, 1), (1, 2, 1), (0, 1, 1)\}$ is a basis of $\mathbb{R}^3$.

8. Let

$$A = \begin{bmatrix} 1 & 5 & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 0 & -4 & 2 \\ 0 & 0 & 0 & 1 & 6 & 3 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \\ u \\ v \\ w \end{bmatrix}.$$

   Find a basis for the solution space of the homogeneous system of linear equations $AX = 0$.

9. Find the standard matrix for the composition of the following two linear operators on $\mathbb{R}^2$: A reflection about the line $y = x$, followed by a rotation counterclockwise of 60^o.

10. Let $A = \begin{bmatrix} 1 & -2 & 4 \\ -2 & 1 & 4 \\ 0 & 0 & 3 \end{bmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

11. Let $A = \begin{bmatrix} 7/3 & 2/3 \\ -4 & -1 \end{bmatrix}$. Calculate A^{100}.