CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	203	All
Examination	**Date**	**Pages**
Final	April 2019	3
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Special Instructions:	Only calculators approved by the Department are allowed **Show your work for full marks**	

MARKS

[9] **1.** **(a)** Solve for x: $3^{2x} + 2 \cdot 3^{x+1} = 4^2$.

(b) Given the function $f(x) = \ln(1 + e^{2x})$, find the inverse function $f^{-1}(x)$, the range of $f(x)$ and the range of $f^{-1}(x)$.

[12] **2.** Evaluate the limit if it exists, otherwise explain why the limit does not exist.

(a) $\displaystyle\lim_{x\to 2} \frac{|x^2-4|}{x^2+x-6}$ **(b)** $\displaystyle\lim_{x\to-\infty} (\sqrt{x^2+5x+1} + x)$ **(c)** $\displaystyle\lim_{x\to\infty} \ln \frac{1+x+2x^3}{x\,(3+2x+x^2)}$

[6] **3.** Find **(a)** all horizontal, and **(b)** all vertical asymptotes of the function

$$f(x) = \frac{3^{x+1} + 2 \cdot 4^x}{4^x - 16}$$

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[15] **4.** Find the derivatives of the following functions:

(a) $f(x) = x^{1/2}(\sqrt{x} - x^{-3/2})\, 2^x$

(b) $f(x) = \ln(x^4 \cdot \sqrt{x+3}\,) + \ln e^2$

(c) $f(x) = \dfrac{\arctan(2x)}{1 + \tan(x)}$

(d) $f(x) = \sin[\sqrt{x^2+1} \cdot \cos(e^x)]$

(e) $f(x) = (1+2x)^{x^2}$ (use logarithmic differentiation)

[15] **5.** **(a)** Verify that the point (2,1) belongs to the curve defined by the equation $xy + 2\sqrt{3+y^2} = x^3 - 2$, and find the equation of the tangent line to the curve at this point.

(b) The length of a rectangle is increasing at the rate of 8 cm/s and its width is increasing at the rate of 5 cm/s. When the length is 20 cm and the width is 12 cm, how fast is the area of the rectangle increasing at that instant?

(c) Use the l'Hôpital's rule to evaluate the $\lim\limits_{x\to 0} \dfrac{e^{x^2}-1}{\cos(2x)-1}$.

[6] **6.** Let $f(x) = 3 + x + 3\,x^2 - x^3$.

(a) Find the slope m of the secant line joining the points $(0, f(0))$ and $(3, f(3))$.

(b) Find all points $x = c$ (if any) on the interval [0,3] such that $f'(c) = m$.

[9] **7.** Consider the function $f(x) = \sqrt{2x+1}$.

(a) Use the **definition of the derivative** to find the formula for $f'(x)$.

(b) Write the linearization formula for f at $a = 4$

(c) Use this linearization to approximate the value of $f(3) = \sqrt{7}$

[14] **8.** **(a)** Find the absolute extrema of $f(x) = \dfrac{x}{x^2 - x + 1}$ on the interval $[0, 3]$.

(b) A box with a square base is to be constructed with a volume of 54 m^3. The material for the box costs $\$2/m^2$, and the material for the top costs $\$6/m^2$. Find the dimensions that minimize the cost of the box.

(c) Let $f(x) = \dfrac{(a^2 + x^2)^2}{x^3}$ where a is a real number. Find $f'''(1)$.

[14] **9.** Given the function $f(x) = 2x^2 - x^4$.

(a) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).

(b) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

(c) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question:** Let f be a function which is monotonically decreasing (strictly) and differentiable everywhere on the real axis. Let also $g = x^2 + 1$. Prove that the composite function $h = f \circ g$ has one and only one critical point, and determine whether it corresponds to a maximum, minimum or inflection point of $h(x)$.