Concordia University
Department of Computer Science & Software Engineering

SOEN331/W- Winter 2017
Introduction to Formal Methods for Software Engineering

Midterm 1 answers

Question 1 (1 points)

Given the following well-formed formulas:

1.1 $P \wedge (P \vee Q)$
1.2 $\neg P \wedge (P \vee (P \Rightarrow Q))$
1.3 $(P \Rightarrow Q) \Rightarrow (\neg P \vee Q)$

Which are tautologies? Use truth tables to prove your answer

Answer: 1.3

Question 2 (6 points)

Consider the argument given by the following sentences.

P1. *If the program does not terminate, then the alarm rings forever*

P2. *Either the computer is not intelligent or the program does not terminate.*

P3. *"the computer runs forever" is implied by the fact that it is not intelligent,*

Q. *Therefore, either the computer runs forever, or the alarm rings forever.*

2.1) **Formalize the statement in propositional logic**

2.2) **Prove that Q is a logical consequence of the premises P1, P2, P3 using proof by contradiction technique seen in class**

Answer:

T- program terminates
A-alarm rings forever
I-computer is intelligent
R-computer runs forever

P1. $\neg T \rightarrow A$

P2. $\neg I \vee \neg T$

P3. $\neg I \rightarrow R$

Q. $R \vee A$

Proof:

1. $T \vee A$ *(from P1)*
2. $\neg I \vee \neg T$ *(P2)*
3. $I \vee R$ *(from P3)*
4. $\neg R$ *(from* $\neg$ ***Q**.)*
5. $\neg A$ *(from* $\neg$ ***Q**.)*

---

6. T *(from 1 and 5)*
7. I *(from 3 and 4)*
8. $\neg T$ *(from 2 and 6)*
9. NIL ((*from 6 and 8)*

Question 3 (4 points)

Consider the following statements:

1.1 *Every student owns some disk space.*

1.2 *A student may erase the disk space he/she owns.*

1.3 *A student who receives the disk space from another student cannot erase the contents of the disk space.*

Formalize the statements in predicate logic.

Answer:

1.1 $\forall s \in Students \exists ds \in DiskSpace \bullet owns(s,ds)$

1.2 $\forall s \in Students \forall ds \in DiskSpace \bullet (owns(s,ds) \wedge decides\text{-}erase(s,ds) \rightarrow erase(s,ds))$

1.3 $\forall s,ss \in Students \forall ds \in DiskSpace \bullet (s \neq ss \wedge owns(s,ds) \wedge receives (s,ss,ds) \rightarrow \neg erase(ss,ds))$

Question 4 (2 points. Circle the right answer(s))

4.1 Which of the following predicates are valid formalizations of "***Somebody likes somebody***"?

(a) $\exists\, p : Person \bullet likes(p, q)$ **(b)** $\exists\, q : Person \bullet likes(p, q)$	**(c)** $\exists\, p : Person \bullet \exists\, q : Person \bullet likes\ (p, q)$ **(d)** $\forall\, p : Person \bullet \exists\, q : Person \bullet likes(p, q)$ **(e)** $\exists\, p : Person \bullet \neg \exists\, q : Person \bullet likes\ (p, q)$	**(f)** None of the previous.

Answer: c)

4.2 Which of the following predicates are valid formalizations of ***"some cats are sleepy"***?

Note: ***isAcat(x)*** is true if ***x*** is a cat.

(a) $\exists\, x : Animal \bullet isAcat(x) \rightarrow sleepy(x)$ **(b)** $\exists\, x : Animal \bullet isAcat(x)\ \&\ sleepy(x)$ **(c)** $\exists\, c : Cat \bullet isAcat(c) \rightarrow sleepy(c)$	**(d)** $isAcat(x)\ \&\ sleepy(x)$ **(e)** $isAcat(x) \rightarrow sleepy(x)$ **(f)** $isAcat(c)\ \&\ sleepy(c)$	**(g)** $\exists\, x : Animal \bullet sleepy(x)$ **(h)** $\exists\, c : Cat \bullet sleepy(c)$ **(i)** None of the previous.

Answer: b) and h)

Question 5 (2 points)

Is Propositional Logic sound (consistent)? complete? Explain your answer.

Propositional Logic is **Sound** —All provable statements are semantically true. That is, if a set of premises S syntactically entails a proposition P , then there is an interpretation in which P can be reasoned about from S.

$$\boxed{\text{if } S \vdash P, \text{ then } S \models P}$$

Propositional Logic is **Complete**—All semantically true statements are provable. That is, if a set of premises S semantically entails a proposition P , then P can be derived formally (syntactically) within the formalism

$$\boxed{\text{if } S \models P, \text{ then } S \vdash P}$$