PHYS 205-03
Electricity and Magnetism
Practice Exam

Instructions

- This is a closed book exam. You are not allowed to use any resources (formula sheet or any electronic devices, including smart wearables).
- Use proper notation and describe your work clearly. Provide proper units for your final answers.

Short answers

Provide short answers and proper descriptions. Providing mathematical equations/treatment is not necessary.

1. Gauss's law in magnetism states that the flux passing through a closed surface is zero. Why? **(2 marks)**
   Since there is no magnetic monopoles, the electric field lines that come out of a magnetic north pole and pass through any closed surface, will turn back into the magnetic south pole, making the net flux zero.

2. If we charge two capacitors C_1 and C_2, disconnect them from the battery and then connect them with opposite polarities, what happens to the energy stored in the system? Does it increase or decrease? Why? **(3 marks)**
   Since we connect the capacitors with opposite polarities, the net charge stored in the system decreases ($Q_{net} = Q_2 - Q_1$), so the energy decreases.

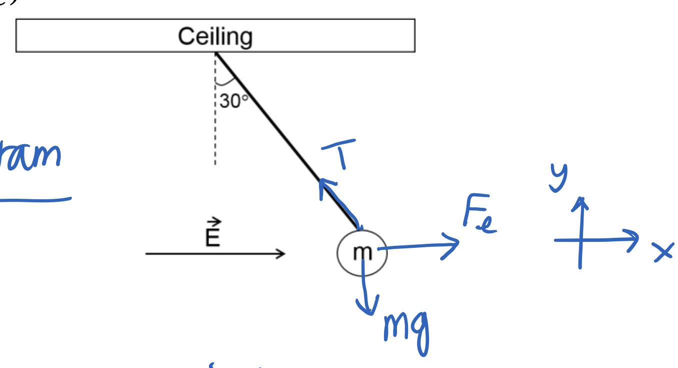
3. What is the effect of the inductor on the current, in a RL circuit with DC current, after a long time that the switch is closed? **(2 marks)**
   The inductor responds to change in magnetic flux, so its effect is observed immediately after the switch is closed or opened. After a long time that switch is closed, the current has already reached its steady-state value and remains constant. So the magnetic flux doesn't change and hence the inductor will have no effects.

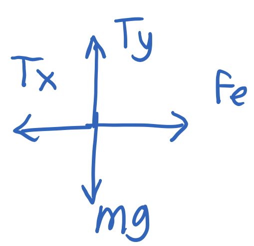
4. A permanent magnet is dropped and passes through a circular loop, North pole first. What is the direction of the induced current in the loop, as the magnet (a) approaches the loop and (b) goes away from the loop after passing through it? **(3 marks)**
When the magnet is approaching the loop with its North pole, the magnetic field inside the loop is pointing downwards and is increasing. This causes the magnetic flux to increase. According to the lenses law, the direction of the induced current should be such that the induced magnetic field opposes the change of magnetic flux, i.e. it should be in the opposite direction of the applied magnetic field. Using the right hand rule (RHR), the direction of the induced current will be **counter-clockwise** (looking from the top).
When the magnet passes through the loop and leaves it, the South pole of the magnet is getting away from the loop. So the magnetic field inside the loop is pointing downwards and decreasing which causes the magnetic flux to decrease. Using the same concept explained above, the induced magnetic field should be parallel to the applied magnetic field. Using RHR, the direction of the current in the loop should be **clockwise** (looking from the top).

Problems

1. A charge of 2μC (2×10^{-6} C) with mass of 100 g is suspended, making an angle of 30° with the vertical line, as shown in the figure below. Determine:
   a) The sign of the charge (positively charged or negatively charged?) **(2 points)**
   b) the magnitude of the electric field **(3 points)**
   Take the gravitational constant $g = 10\frac{m}{s^2}$
   Hint: The forces applied on the object are Tension (T) along the rope, Weight (mg), and Electric force (F_e).

free body diagram





$$\begin{cases} T_x = T\sin(30) \\ T_y = T\cos(30) \end{cases}$$

a) Since $\Sigma F_x = 0$, F_e should be to the right which means q is POSITIVE.

b)

$$\sum F_y = 0 \rightarrow T_y = mg \rightarrow T = \frac{mg}{\cos(30)}$$

$$\sum F_x = 0 \rightarrow F_e = T_x = T\sin(30) \rightarrow Eq = \frac{mg}{\cos(30)} \cdot \sin(30)$$

$$E = \frac{mg}{q}\tan(30) = \frac{(0.1)(10)}{2\times10^{-6}}\left(\frac{\sqrt{3}}{3}\right) = \boxed{\frac{\sqrt{3}}{6}\times10^{6}\ N/C}$$

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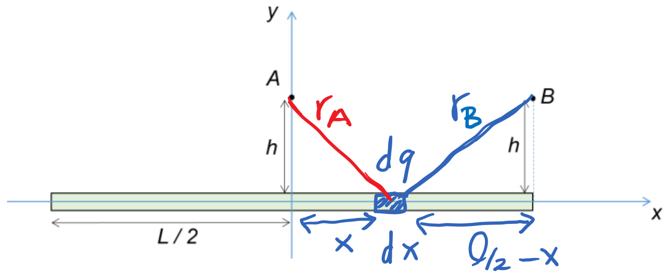
2. Figure below, shows a uniformly charged rod with total charge +Q, length L, and linear charge density λ. Determine:
   a) The electric potential at points A and B. Show your work properly. **(3 points)**
   b) The work required to move a charge +q from point A to point B. **(2 points)**



To find V_B:

$$dV_B = k_e\frac{dq}{r_B} = k_e\frac{\lambda\,dx}{\sqrt{(\frac{\ell}{2}-x)^2+h^2}} \rightarrow V_B = k_e\lambda\int_{-\frac{\ell}{2}}^{\ell/2}\frac{dx}{\sqrt{(\frac{\ell}{2}-x)^2+h^2}}$$

This integral will be given to you!
I don't expect you to know how to do this integral!
You should only know the power functions and exponents.

To find V_A:

$$dV_A = k_e\frac{q}{r_A} = k_e\frac{\lambda\,dx}{\sqrt{x^2+h^2}} \rightarrow V_A = k_e\lambda\int_{-\frac{\ell}{2}}^{\ell/2}\frac{dx}{\sqrt{x^2+h^2}}$$

b) $W = q(V_B - V_A)$

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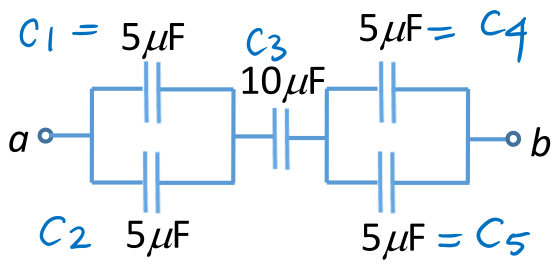
3. Consider the circuit branch below.
a) What is the equivalent capacitance between points a and b? **(3 points)**
b) If we connect a 10 V battery between points a and b, and wait a very long (infinite) time, how much energy is stored in one of the 5 μF capacitor? **(3 points)**



a) $C_{12} = C_1 + C_2 = 10\mu F$ parallel

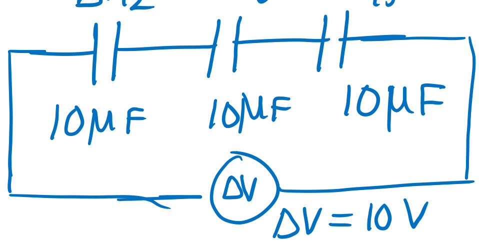
$$C_{123} = \frac{C_3 C_{12}}{C_3 + C_{12}} = \frac{(10)(10)}{10+10} = 5\mu F \text{ series}$$

$C_{45} = C_4 + C_5 = 10\mu F$ parallel

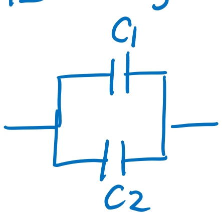
$$C_{12345} = \frac{C_{123} C_{45}}{C_{123} + C_{45}} = \frac{(10)(5)}{10+5} = \frac{50}{15} = \frac{10}{3}\mu F$$

$\Delta V_{12} = \Delta V_3 = \Delta V_{45} \rightarrow$ same capacitance

b) We have



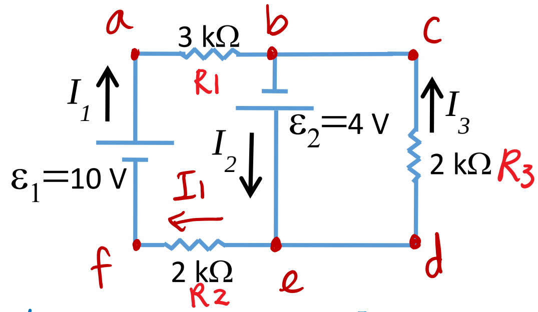
$$\Delta V_{12} = \Delta V_3 = \Delta V_{45} = \frac{10}{3} V$$



parallel $\rightarrow \Delta V_1 = \Delta V_2 = \frac{10}{3}$

$$E_1 = \frac{1}{2} C \Delta V_1^2 = \left(\frac{1}{2}\right)(5\times10^{-6})\left(\frac{10}{3}\right)^2 = \boxed{\frac{5}{18}\times10^{-4} J}$$

4. Calculate the values of the currents I_1, I_2, I_3. **(4 points)**



At Junction b: $I_1 + I_3 = I_2$

loop abefa: $-I_1R_1 + \mathcal{E}_2 - I_1R_2 + \mathcal{E}_1 = 0$

$$\mathcal{E}_1 + \mathcal{E}_2 - I_1(R_1+R_2) = 0$$

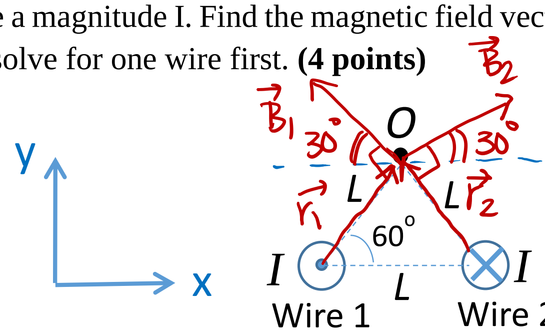
$$\to I_1 = \frac{\mathcal{E}_1+\mathcal{E}_2}{R_1+R_2} = \frac{10+4}{(3+2)\times 10^3} = \frac{14}{5\times 10^3} = \boxed{\frac{14}{5}\times 10^{-3}A}$$

loop bcdeb:

$$I_3R_3 - \mathcal{E}_2 = 0 \to I_3 = \frac{\mathcal{E}_2}{R_2} = \frac{4}{2\times 10^3} = \boxed{2\times 10^{-3}A}$$

$$I_1 + I_2 = I_3 \to I_3 = \left(\frac{14}{5}+2\right)\times 10^{-3} = \boxed{4.8\times 10^{-3}A}$$

5. Two infinite wires are parallel to each other as shown below. The current in wire 1 points in the + Z direction (out of page) and current in wire 2 is in the –Z direction (into the page). Both currents have a magnitude I. Find the magnetic field vector (magnitude and direction) at point O. **Hint:** solve for one wire first. **(4 points)**



$\vec B_1$ and $\vec B_2$ are found using RHR

$\vec B_1 \perp \vec r_1$ } Tangent to circle
$\vec B_2 \perp \vec r_2$ } radius

$$\vec B_{net} = \vec B_x + \vec B_y$$

$$\vec B_x = \vec B_{1x} + \vec B_{2x} = 0$$

$$\vec B_y = \vec B_{1y} + \vec B_{2y} = 2B_{1y}(\hat y)$$

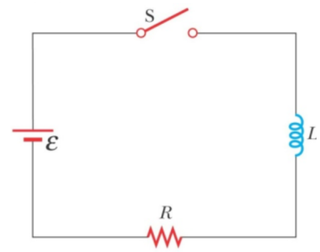
$$= \cancel{2}\frac{\mu_0 I}{2\pi l}\left(\frac{1}{\cancel{2}}\right)\hat y = \boxed{\frac{\mu_0 I}{2\pi l}}$$

$$B_1 = \frac{\mu_0 I}{2\pi l} = B_2$$

$$\vec B_{1x} = -\vec B_{2x} = B_1\cos(30)\,\hat x$$

$$\vec B_{1y} = \vec B_{2y} = B_1\sin(30)\,\hat y$$

6. For the RL circuit below, the switch is open for time t < 0, then the switch is closed at time t=0. If L= 1 mH, and R= 0.01Ω and Ɛ= 10 V, What is the current in the inductor at t= 0.05s ? **(3 points)**



Flux in: $I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$

$\tau = \frac{L}{R} = \frac{10^{-3}}{10^{-2}} = 0.1\,s$

$I(0.05s) = \frac{10}{10^{-2}}\left(1 - e^{-\frac{0.05}{0.1}}\right) = 10^3(0.39) = 390A$

which is very large!!!

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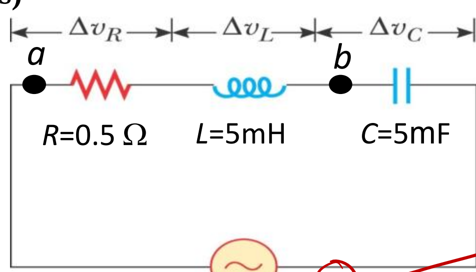
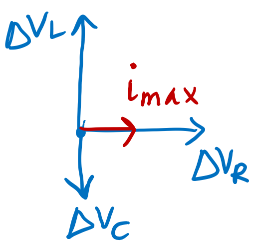
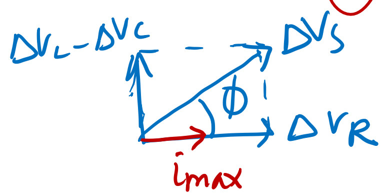
7. For the RLC circuit below $i(t) = 5A\sin(100t)$, (a) what is Δv_s the voltage across the source (time dependent function)? (b) What is Δv_{ab} the voltage across both the resistor and inductor? **(4 points)**



$i = 5A\sin(100t)$

$\omega = 100\ \frac{rad}{s}$





$X_L = \omega L = (100)(5\times10^{-3}) = 0.5\,\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{(5\times10^{-3})(100)} = 2\,\Omega$

$R = 0.5\,\Omega$

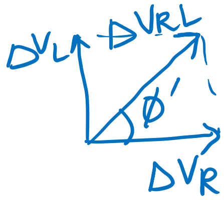
$Z = \sqrt{R^2 + (X_L - X_C)^2} = 1.58\,\Omega$

$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}(-3) = -1.25$ rad

a) $\Delta V_s = i_{max} Z \sin(\omega t + \phi) = 5(1.58)\sin(100t + (-1.25))$

$\boxed{\Delta V_s = 7.9\sin(100t - 1.25)}$

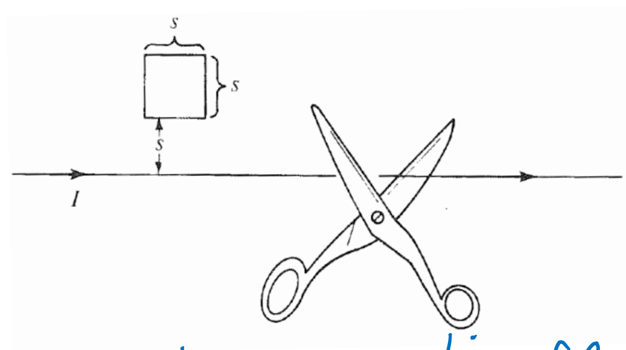
* Note that the phase angle is negative, which means that the $(\Delta V_s)_{max}$ lags i_{max}

b) 

$$\phi' = \tan^{-1}\left(\frac{X_L}{R}\right) = \frac{\pi}{4} \text{ rad}$$

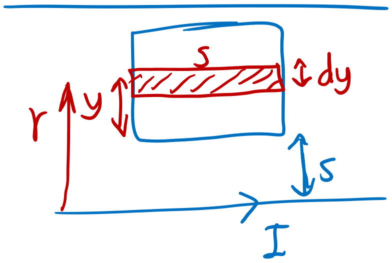
$$\boxed{\Delta V_s = i_{max} Z' \sin\left(100t + \frac{\pi}{4}\right)}, \quad Z' = \sqrt{R^2 + X_L^2}$$

8. An infinite wire carries a current I, and a square loop of wire sits a distance *s* away from the wire as shown below. Initially there is no current in the loop. Suddenly, you cut the infinite wire with a scissor. Explain why (with formulas and a few words) a current is induced in the square loop, and which way it flows. **(4 points)**



Initially there is a magnetic flux in the loop, which is constant. Cutting the wire, makes $I=0$ and hence $B=0$. So the flux decreases, which induces emf in the loop. Lens'z law and RHR gives: Counter clockwise I_{ind}

To find the emf:



$$d\phi_B = B\,dA = \frac{\mu_0 I}{2\pi r}(s\,dy) = \frac{\mu_0 I}{2\pi(s+y)} s\,dy$$

$$\phi_B = \frac{\mu_0 I s}{2\pi}\int_0^s \frac{dy}{s+y} = \frac{\mu_0 I s}{2\pi} \ln(s+y)\Big|_0^s$$

$$\Delta\phi_B = 0 - \phi_B = -\frac{\mu_0 I s}{2\pi}\ln(2)$$

$$\mathcal{E} = -\frac{\Delta\phi_B}{\Delta t}$$

$\Delta t \rightarrow$ Time it takes the current I to go to zero.

Not

9. High-power lasers in factories are used to cut through cloth and metal. One such laser has a beam diameter of 1 mm and generates an electric field having an amplitude of 0.700 MV/m at the target. Find
   a) the amplitude of the magnetic field produced **(2 points)**
   b) the intensity of the laser **(2 points)**
   c) the power delivered by the laser **(2 points)**

a) $B_{max} = \dfrac{E_{max}}{c} = \dfrac{7\times10^5 \frac{V}{m}}{3\times10^8 \frac{m}{s}} = \boxed{2.3\times10^{-3}\ T}$

b) $I = \dfrac{E_{max}^2}{2\mu_0} = \dfrac{(7\times10^5)^2}{2(4\pi\times10^{-7})} = \boxed{6.5\times10^8\ \frac{W}{m^2}}$

c) $I = \dfrac{P}{A} \rightarrow P = IA = (6.5\times10^8)\big(\underbrace{\pi\left(\tfrac{10^{-3}}{2}\right)^2}_{\pi r^2}\big) = \boxed{511\ W}$