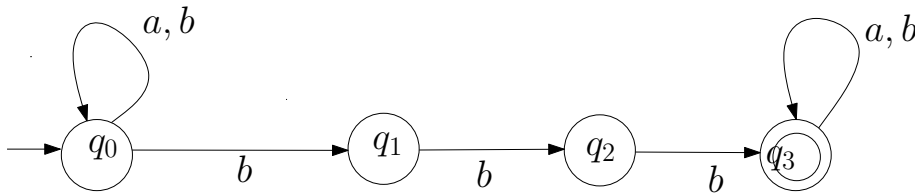


**Assignment 2**

**Submission through Moodle due on Thursday October 12 at 23:55**

1. (10 points) Give regular expressions for the following languages on the alphabet  $\{a, b\}$ .
  - (a) the set of all strings with an even number of  $a$ 's
  - (b) the set of strings in which all runs are of length  $< 3$ . (A run in a string is a non-extendable substring of length at least 2 which contains repetitions of the same symbol. For example (a) the string  $abab$  has no runs, (b) the string  $aabbbababbaaa$  has four runs, in order: a run of length 2 of  $a$ 's, a run of length 3 of  $b$ 's, a run of length 2 of  $b$ 's and a run of length 3 of  $a$ 's.)
2. (10 points) For each of the following regular expressions  $r$ , convert it to an NFA that accepts the language  $L(r)$ :
  - (a)  $((ba + b)^*aab)^*(a + \lambda)$
  - (b)  $ab(a + bbb)(b + aa)^*$

3. (10 points) Consider the following NFA  $M$ :




- (a) Convert it to a DFA  $M'$  such that  $L(M') = L(M)$ .
  - (b) Convert it to a right-linear grammar  $G$  such that  $L(G) = L(M)$ .
4. (10 points) Let  $L \subseteq \{a, b\}^*$  be the set of all strings with an odd number of  $a$ 's and an even number of  $b$ 's.
  - (a) Give a DFA  $M$  that accepts  $L$ .
  - (b) Convert  $M$  to a regular expression  $r$  such that  $L(M) = L(r)$ .
5. (10 points) Let  $L$  be a regular language defined on the alphabet  $\Sigma = \{a, b\}$ . Which of the following languages is regular? Prove your answer.
  - (a)  $f_2(L) = \{a_1a_1a_2a_2 \dots a_na_n : a_1a_2 \dots a_n \in L\}$
  - (b)  $f_3(L) = \{ww : w \in L\}$
6. (10 points) For each of the following languages, say whether or not it is regular. Prove your answer.
  - (a)  $L_1 = \{a^n b^n : n \bmod 3 = 0\}$
  - (b)  $L_3 = \{a^n : n \text{ is not a perfect square}\}$  (an integer  $n$  is a perfect square if  $n = i^2$  for some integer  $i$ ).

7. (10 points) Let  $L_1$  and  $L_2$  be any two languages over  $\Sigma^*$  where  $\Sigma = \{a, b\}$ . Prove or disprove:

(a) If  $L_1 \subseteq L_2$  and  $L_2$  is regular, then  $L_2 - L_1$  is regular.

(b) If  $L_1 = L_1 L_2$  and  $\lambda \notin L_2$ , then  $L_1 = \phi$

8. The purpose of the following question is to strengthen your skills in problem analysis and solving, and to evaluate the related CEAB Graduate Attribute "**Problem Analysis**" defined as: the ability to use appropriate knowledge and skills to identify, analyze, and solve complex problems in order to reach substantiated conclusions. (Please refer to the course outline for more details.)

THE QUESTION: Design a traffic light for an intersection between EW Street and NS Blvd. It is known that NS Blvd has a lot more traffic than EW Street. It is also known that in the morning hours, many vehicles turn left from NS Blvd onto EW Street. There is a bike lane on EW Street, and there are many pedestrians on the sidewalks for both roads. 

(a) (3 points) Write down a clear problem statement, making sure to specify every realistic assumption you make.

(b) (3 points) Consider and explain in English two approaches to solving the problem, one of them very simple, and another that is more complex.

(c) (4 points) Choose one of the approaches you considered, explaining your reasons, and the limitations and consequences of your solution, and draw the transition diagram for both the approaches.