Concordia University

PHYS 205-Section 03
Electricity and Magnetism - Winter 2018
Assignment 6 – Solution

1. A solenoid having an inductance of $6.30\ mH$ is connected in series with a $1.20\ k\Omega$ resistor. (a) If a $14\ V$ battery is connected across the pair, how long will it take for the current through the resistor to reach 80% of its final value? **(3 points)**
(b) What is the current through the resistor at time $t = 1\,\tau_L$? **(2 points)**

Solution:

(a) If the battery is switched into the circuit at $t = 0$, then the current at a later time t is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$. Our goal is to find the time at which $i = 0.800\varepsilon/R$. This means

$$0.800 = 1 - e^{-t/\tau_L} \Rightarrow e^{-t/\tau_L} = 0.200 .$$

Taking the natural logarithm of both sides, we obtain $-(t/\tau_L) = \ln(0.200) = -1.609$. Thus,

$$t = 1.609\,\tau_L = \frac{1.609\,L}{R} = \frac{1.609\,(6.30\times10^{-6}\ \mathrm{H})}{1.20\times10^{3}\ \Omega} = 8.45\times10^{-9}\ \mathrm{s}.$$

(b) At $t = 1.0\,\tau_L$ the current in the circuit is

$$i = \frac{\varepsilon}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\ \mathrm{V}}{1.20\times10^{3}\Omega}\right)\left(1 - e^{-1.0}\right) = 7.37\times10^{-3}\ \mathrm{A}.$$

2. An alternating source drives a series RLC circuit with an emf amplitude of $6\ V$, at a phase angle of $+30.0°$. When the potential difference across the capacitor reaches its maximum positive value of $+5\ V$, what is the potential difference across the inductor (sign included)? **(5 points)**

Solution:
Drawing the phase diagram, like the one in your textbook leads to:

$$V_L - V_C = (6.00\ \mathrm{V})\sin(30^\circ) = 3.00\ \mathrm{V}.$$

With the magnitude of the capacitor voltage at 5.00 V, this gives a inductor voltage magnitude equal to 8.00 V. Since the capacitor and inductor voltage phasors are 180° out of phase, the potential difference across the inductor is $-8.00\ \mathrm{V}$.

3. An air conditioner connected to a 120 $V\ rms$ AC line is equivalent to a 12 Ω resistance and a 1.30 Ω inductive reactance in series. Calculate
(a) the impedance of the air conditioner **(3 points)**
(b) the average rate at which energy is supplied to the appliance **(2 points)**

Solution:
(a)

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(12.0\ \Omega)^2 + (1.30\ \Omega - 0)^2} = 12.1\ \Omega.$$

(b) The average rate at which energy has been supplied is

$$P_{avg} = \frac{\varepsilon_{rms}^2 R}{Z^2} = \frac{(120\ V)^2 (12.0\ \Omega)}{(12.07\ \Omega)^2} = 1.186 \times 10^3\ W \approx 1.19 \times 10^3\ W.$$

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4. A plane electromagnetic wave of intensity $6\frac{W}{m^2}$, moving in the x direction, strikes a small perfectly reflecting pocket mirror, of area 40 cm^2, held in the yz plane.
(a) What momentum does the wave transfer to the mirror each second? **(0 points)**
(b) Find the force the wave exerts on the mirror. **(0 points)**

Solution:
(a) The magnitude of the momentum transferred to the assumed totally reflecting surface in time interval Δt is (from Equation 34.29)

$$\Delta p = \frac{2T_{ER}}{c} = \frac{2SA\Delta t}{c}$$

Then the momentum transfer is

$$\Delta\vec{\mathbf{p}} = \frac{2\vec{\mathbf{S}}A\Delta t}{c} = \frac{2(6.00\ \hat{\mathbf{i}}\ \mathrm{W/m^2})(40.0\times 10^{-4}\ \mathrm{m^2})(1.00\ \mathrm{s})}{3.00\times 10^8\ \mathrm{m/s}}$$

$$\Delta\vec{\mathbf{p}} = \boxed{1.60\times 10^{-10}\ \hat{\mathbf{i}}\ \mathrm{kg\cdot m/s\ each\ second}}$$

(b) The force is

$$\vec{\mathbf{F}} = PA\hat{\mathbf{i}} = \frac{2SA}{c}\hat{\mathbf{i}} = \frac{2(6.00\ \mathrm{W/m^2})(40.0\times 10^{-4}\ \mathrm{m^2})(1.00\ \mathrm{s})}{3.00\times 10^8\ \mathrm{m/s}}$$

$$= \boxed{1.60\times 10^{-10}\ \hat{\mathbf{i}}\ \mathrm{N}}$$

(c) The answers are the same. Force is the time rate of momentum transfer.
