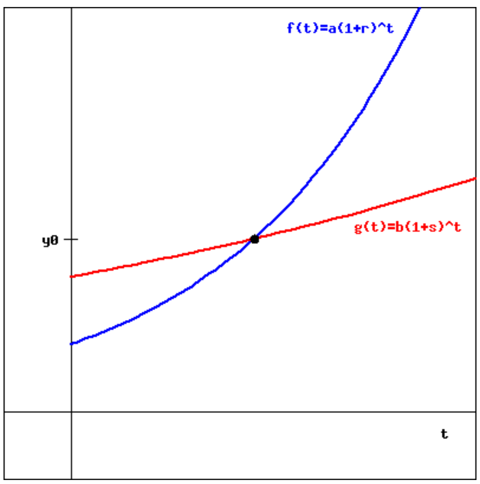
Jan Mikhail Alexei Ong **E**

WeBWorK assignment due : 04/16/2020 at 11:59pm EDT.

1. (1 point) Suppose y_0 is the y-coordinate of the point of intersection of the graphs below. Complete the statement below in order to correctly describe what happens to y_0 if the value of a (in the blue graph of $f(t) = a(1+r)^t$ below) is increased, and all other quantities remain the same.

As a increases, the value of y_0

- A. increases
- B. decreases
- C. remains the same



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Solution:

SOLUTION

As a is increased, the y-intercept of the blue graph increases. The entire blue graph will be shifted up, and the point of intersection shifts to the left and down, so the y-coordinate decreases.

Correct Answers:

- `B`

2. (1 point) The population of a colony of rabbits grows exponentially. The colony begins with 15 rabbits; 5 years later there are 330 rabbits.

(a) Express the population of the colony of rabbits, P, as a function of time, t, in years.

$P(t) =$ ______

(b) Use the graph to estimate how long it takes for the population of rabbits to reach 1000 rabbits.

It will take ______ years. (round to nearest 0.01 year)

Solution:

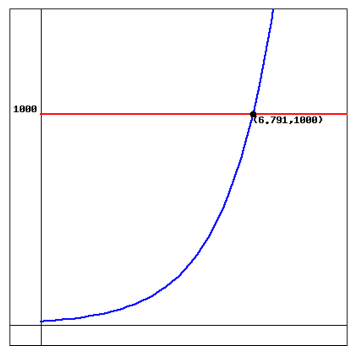
SOLUTION

a) Since the population grows exponentially, it can be described by $P = ab^t$, where P is the number of rabbits and t is the number of years which have passed. We know that a represents the initial number of rabbits, so $a = 15$ and $P = 15(b)^t$. After 5 years, there are 330 rabbits so

$$330 = 15(b)^5$$
$$22 = b^5$$
$$(b^5)^{1/5} = 22^{1/5}$$
$$b \approx 1.856$$

From this, we know that $P(t) = 15(1.856)^t$.

b) We want to find t when $P = 1000$. Using a graph of $P = 15(1.856)^t$, we see (figure below) that the line $P = 1000$ and $P = 15(1.856)^t$ intersect when $t \approx 6.791$ years.



Correct Answers:

- `15*1.856^t`
- `6.791`

3. (1 point)

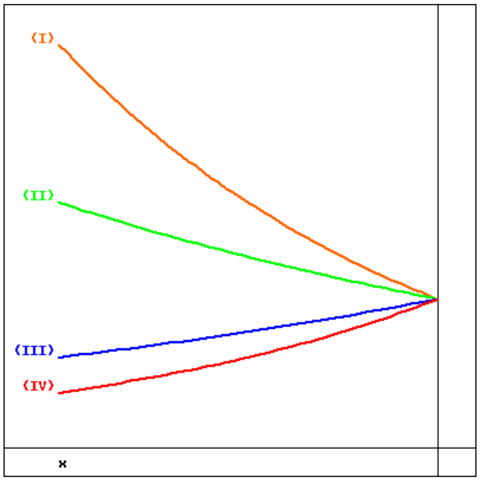
Without using a calculator, match each exponential function with its graph.

$\boxed{?}$ $e^{-0.3x}$

$\boxed{?}$ e^{-x}

$\boxed{?}$ $e^{0.3x}$

$\boxed{?}$ e^{x}



(Click on graph to enlarge)

Solution:
SOLUTION

The functions given in (b) and (a) represent exponential decay while the functions given in (c) and (d) represent exponential growth. Thus, (b) and (a) correspond to (I) and (II) (not necessarily in that order) while (c) and (d) correspond to (III) and (IV) (not necessarily in that order).

The function in (d) grows faster than the function in (c) since it grows continuously at a rate of 1% while the function in (d) grows continuously by 0.3%. Since (d) has a higher continuous growth rate, its graph must decrease the fastest to 0 as $x \to -\infty$, thus it has graph IV and (c) has graph III.

Graphs (I) and (II) correspond to the exponential decay formulas, with graph (I) decaying at a more rapid rate (since it increases to ∞ quicker as $x \to -\infty$ it similarly decreases to 0 faster as $x \to +\infty$). Thus formula I corresponds to graph (b) and formula (a) corresponds to graph II. We have:

a) II
b) I
c) III
d) IV

Correct Answers:

- `II`
- `I`
- `III`
- `IV`

---

4. (1 point) If $f(x) = 8^x$ and $g(x) = \dfrac{11x}{x+11}$, find a simplified formula for:
$g(f(x)) =$ ____________

Solution:
SOLUTION

We solve by substituting the expression $f(x) = 8^x$ in for x in $g(x) = \dfrac{11x}{x+11}$:

$$g(f(x)) = \frac{11f(x)}{f(x)+11} = \frac{11 \cdot 8^x}{8^x+11}.$$

Correct Answers:

- `11*8^x/(8^x+11)`

---

5. (1 point) If $f(x) = e^{x/3}$, $g(x) = 7x+7$, and $h(x) = \sqrt{x}$, Find a simplified formula for:
$f(g(x))h(x) =$ ____________

Solution:
SOLUTION

We start by computing $f(g(x)) = e^{3g(x)} = e^{3(7x+7)} = e^{21x+21}$. Next we compute $f(g(x))h(x)$ by multiplying the expression above by $h(x) = \sqrt{x}$:

$$f(g(x))h(x) = e^{21x+21} \cdot \sqrt{x}.$$

Correct Answers:

- `e^(7/3*x+7/3)*sqrt(x)`

---

6. (1 point) Let $f(x) = x^{4/3}$, $g(x) = \frac{(5x-2)^3}{8}$, and $h(x) = \tan(4x)$. Find values for the constants A and P which result in the simplified expression for $h(x)/f(g(x))$ equal to the combination of functions below:

$$\frac{h(x)}{f(g(x))} = \frac{A\tan(4x)}{(5x-2)^P}$$

$A =$ ____, and
$P =$ ____

Solution:
SOLUTION

We start by computing $f(g(x)) = (g(x))^{4/3} = \left(\dfrac{(5x-2)^3}{8}\right)^{4/3} = \dfrac{(5x-2)^4}{(2^3)^{4/3}} = \dfrac{(5x-2)^4}{16}$.

Thus $\dfrac{h(x)}{f(g(x))} = \dfrac{\tan(4x)}{(5x-2)^4/16} = \dfrac{16\tan(4x)}{(5x-2)^4}$.

Correct Answers:

- `16`
- `4`

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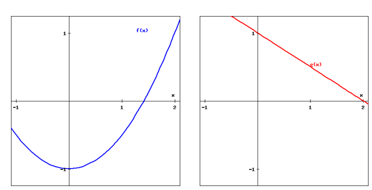
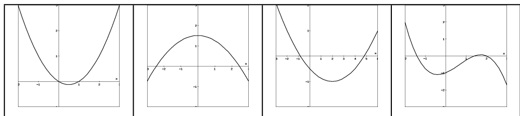
7. (1 point) The figures below shows the graph of $f(x)$ in blue and the graph of $g(x)$ in red:



Based on the figure above, graph the function $f(g(x))$.

Which of the graphs below labeled A-E most accurately matches your graph? ___ (enter the corresponding letter)



A	B	C	D	None of these graphs E

Solution:
SOLUTION
We can evaluate $f((g(x))$ at several points:

$$f(g(0)) = f(1) \approx -0.5$$
$$f(g(1)) = f(0.5) \approx -0.8$$
$$f(g(2)) = f(0) = -1.$$

Also observe $g(x)$ is linear and $f(x)$ is a quadratic function. Substituting the x's in the formula for $f(x)$ with the linear equation for $g(x)$ will result in a quadratic expression. Thus the graph of $f(g(x))$ must be parabola which goes through the points above. Thus we can see that graph C is the correct graph.

Correct Answers:

- `C`

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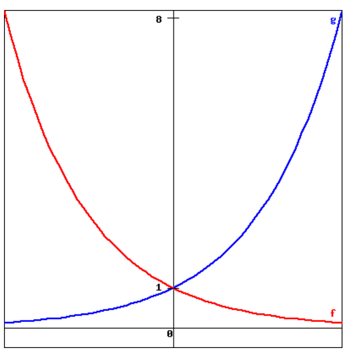
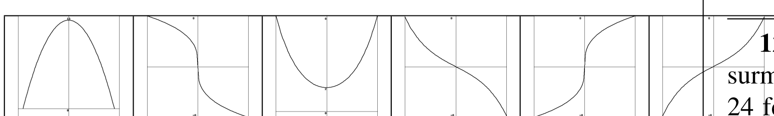
8. (1 point) Consider the graphs of $y = f(x)$ and $y = g(x)$ sketch in red and blue respectively on the graph below:



For each function, enter the letter of the graph A - F which represents it. Clearly not all of the graphs are matched with one of the equations.

$f(x)+g(x)$	___
$f(x)-g(x)$	___
$g(x)-f(x)$	___



A	B	C	D	E	F

(click on each individual graph to enlarge)

Solution:
SOLUTION

$f(x)+g(x)$	C
$f(x)-g(x)$	D
$g(x)-f(x)$	F

Correct Answers:

- `C`
- `D`
- `F`

---

9. (1 point) Is the following statement true or false?
If $f(x)\cdot g(x)$ is an odd function, then both $f(x)$ and $g(x)$ must be odd functions. Be sure you can explain your answer.

- A. True
- B. False

Solution:
SOLUTION
The statement is false. For example, if $f(x) = x$ and $g(x) = x^2$, then $f(x)\cdot g(x) = x^3$. In this case, $f(x)\cdot g(x)$ is an odd function, but $g(x)$ is an even function.

Correct Answers:

- `B`

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10. (1 point) Let $f(x) = \frac{x+1}{x^2+14x+49}$. Use interval notation to indicate the domain of $f(x)$.
Note: You should enter your answer in **interval notation**. If the set is empty, enter "" without the quotation marks.
Domain = ______________

Correct Answers:

- `(-infinity,-7) U (-7,infinity)`

---

11. (1 point) Find an expression for the function $f(x)$ whose graph is given by the bottom half of the parabola

$$x + (y-7)^2 = 0.$$

$f(x)$ = ____________

Correct Answers:

- `-sqrt(-x) + 7`

---

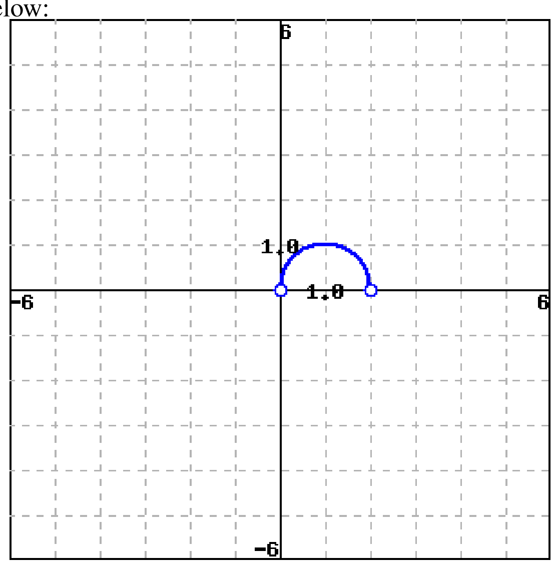
12. (1 point) A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 24 feet, express the area A of the window as a function of the width x (across the base) of the window.

$A(x)$ = ________________

Correct Answers:

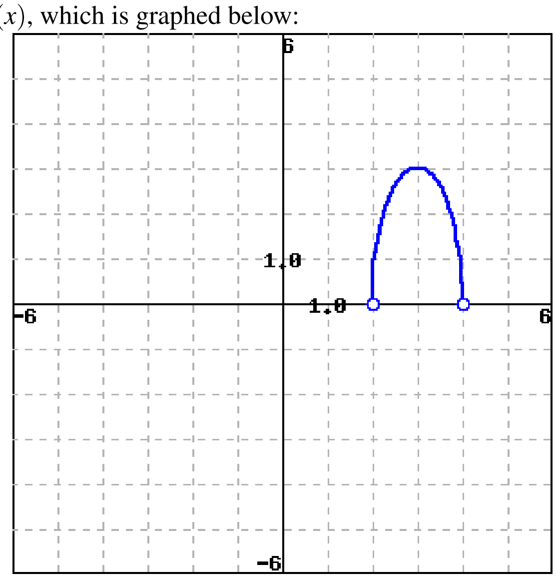
- `x*(24-x-pi*x/2)/2 + pi*x^2/8`

13. (1 point) The function $f(x) = \sqrt{2x - x^2}$ is given graphed below:



Note: Click on graph for larger version in new browser window.

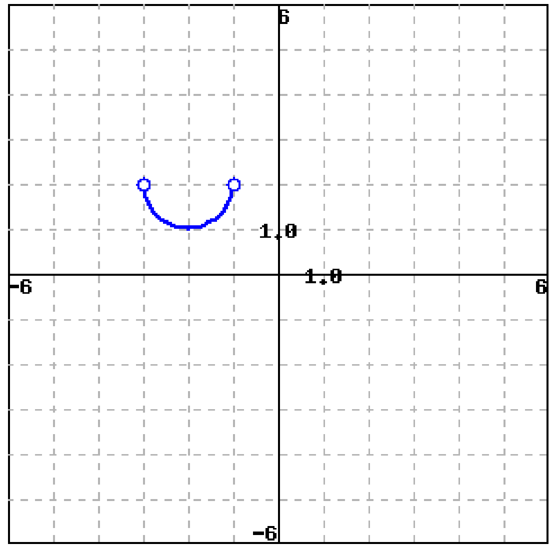
(A) Starting with the formula for $f(x)$, find a formula for $g(x)$, which is graphed below:



Note: Click on graph for larger version in new browser window.

$g(x)$ = ________________

(B) Starting with the formula for $f(x)$, find a formula for $h(x)$, which is graphed below:



Note: Click on graph for larger version in new browser window.

$h(x)$ = ________________

Correct Answers:

- `3*(2*(x-2)-(x-2)**2)**(1/2)`
- `2+-1*(2*(x--3)-(x--3)**2)**(1/2)`

14. (1 point) Suppose that

$$f(x) = \sqrt{8x - 8} \quad \text{and} \quad g(x) = 2x^2 - 7.$$

For each function h given below, find a formula for $h(x)$ and the domain of h. Enter the domains using **interval notation**.

(A) $h(x) = (f \circ g)(x)$.

$h(x)$ = ____________
Domain = ____________

(B) $h(x) = (g \circ f)(x)$.

$h(x)$ = ____________
Domain = ____________

(C) $h(x) = (f \circ f)(x)$.

$h(x)$ = ____________
Domain = ____________

(D) $h(x) = (g \circ g)(x)$.

$h(x)$ = ____________
Domain = ____________

Correct Answers:

- `sqrt(8*2*x^2+8*(-7)+(-8))`
- `(-infinity,-2] U [2,infinity)`

- `2*(8*x+(-8))+(-7)`
- `[1,infinity)`
- `sqrt(8*sqrt(8*x+(-8))+(-8))`
- `[1.125,infinity)`
- `2*(2*x^2+(-7))^2+(-7)`
- `(-infinity,infinity)`

---

15. (1 point) The expression $\left(5^{-7}\right)^{4x}$ can be written as $5^{f(x)}$, where $f(x)$ is a function of x. Find $f(x)$.

$f(x) =$ ______

Correct Answers:

- `(-7)*(4)*x`

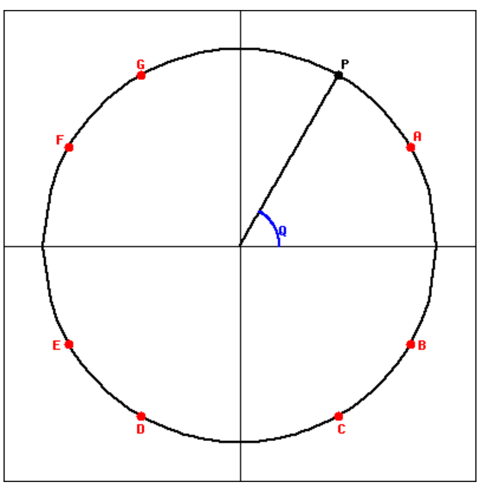
---

16. (1 point)

Consider the angle θ, which is labeled as Q in blue on the graph, with corresponding point P on the circle.

Sketch each of the angles given below, then select the point on the circle that best corresponds to the angle.

Angle	Point
$\frac{\pi}{2}+\theta$	?
$\frac{\pi}{2}-\theta$	?
$\pi+\theta$	?
$\pi-\theta$	?



(Click on graph to enlarge)

Solution:

SOLUTION

(a) F

(b) A

(c) D

(d) G

Correct Answers:

- F
- A
- D
- G

---

17. (1 point) **Part 1 of 3:**

In this multi-part problem, we will use algebra to verify the identity

$$\frac{\sin(t)}{1-\cos(t)} = \frac{1+\cos(t)}{\sin(t)}.$$

First, using algebra we may rewrite the equation above as

$$\sin(t) = \left(\frac{1+\cos(t)}{\sin(t)}\right)\cdot\left(\text{\_\_\_\_\_\_}\right)$$

Solution:

SOLUTION

Alternately, we can directly derive this by multiplying the denominator by $1+\cos(t)$ to get $\sin^2(t)$:

$$\begin{aligned}\frac{\sin(t)}{1-\cos(t)} &= \frac{\sin(t)}{1-\cos(t)}\,\frac{(1+\cos(t))}{(1+\cos t)}\\ &= \frac{\sin(t)(1+\cos(t))}{1-\cos^2 t}\\ &= \frac{\sin(t)(1+\cos(t))}{\sin^2(t)}\\ &= \frac{1+\cos(t)}{\sin(t)}.\end{aligned}$$

Correct Answers:

- `1-cos(t)`

---

18. (1 point) Find the inverse function (if it exists) of $h(x) = 4x^4-5$. If the function is not invertible, enter **NONE**.

$h^{-1}(x) =$ ______

(Write your inverse function in terms of the independent variable x.)

Solution:

SOLUTION

This function is not invertible. The graph does not pass the horizontal line test. If we try to solve for x in $y = 4x^4-5$, we get the following:

$$\begin{aligned} y &= 4x^4-5\\ y+5 &= 4x^4\\ x^4 &= \frac{y+5}{4}\\ x &= \pm\sqrt[4]{\frac{y+5}{4}}\end{aligned}$$

Since this is not one to one, we say the inverse $h^{-1}(x)$ does not exist.

Correct Answers:

- `NONE`

19. (1 point) If $t = g(v)$ represents the time in hours it takes to drive to the next town at velocity v mph.
Which if the following statement(s) correctly explain the meaning of $g^{-1}(t)$? Check all that may apply.

- A. The velocity in mph of the car if it takes t minutes to drive to the next town.
- B. How many hours it takes to reach a velocity of t mph.
- C. The velocity in mph of the car if it takes t hours to drive to the next town.
- D. The number of hours it takes to drive t miles.
- E. The velocity in mph of the car after you have driven for t miles.
- F. None of the above

Solution:
SOLUTION
The inverse function $g^{-1}(t)$ represents the velocity needed for a trip of t hours. Its units are mph.
Correct Answers:

- C

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20. (1 point)
If you are given the graph of f, how do you find the graph of f^{-1}?

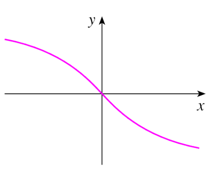
(a) Reflect it over the x-axis.
(b) Reflect it over the y-axis.
(c) Reflect it over $y = x$.
(d) Reflect it over $y = -x$.

______
Correct Answers:

- c

---

21. (1 point)
A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one. If it is one-to-one, enter "y" below. If not, enter "n" below.

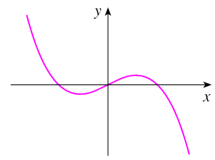


______
Correct Answers:

- y

---

22. (1 point)
A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one. If it is one-to-one, enter "y" below. If not, enter "n" below.



______
Correct Answers:

- n

---

23. (1 point)
Find a formula for the inverse of the function.

$f(x) = \frac{4x-1}{2x+3}$.
$f^{-1}(x) =$ ______
Correct Answers:

- (3x+1)/(4-2x)

---

24. (1 point)
Find a formula for the inverse of the function.

$f(x) = e^{x^3}$.
$f^{-1}(x) =$ ______
Correct Answers:

- (ln(x))^(1/3)

---

25. (1 point)
Find a formula for the inverse of the function.

$f(x) = \ln(x+8)$.
$f^{-1}(x) =$ ______
Correct Answers:

- e^x-(8)

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26. (1 point)
Express the given quantity as a single logarithm.

$\ln x + 2\ln y - 7\ln z$

______

Correct Answers:

- ln(x*y^{2}/ z^{7})

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27. (1 point)
Simplify $\cos(\sin^{-1} x)$.
______
Correct Answers:

- sqrt(1-x^2)

28. (1 point)
Let $f(x) = x^3 - 14x$. Calculate the difference quotient $\frac{f(2+h)-f(2)}{h}$ for
$h = .1$__________
$h = .01$__________
$h = -.01$__________
$h = -.1$ __________
If someone now told you that the derivative (slope of the tangent line to the graph) of $f(x)$ at $x = 2$ was an integer, what would you expect it to be?__________

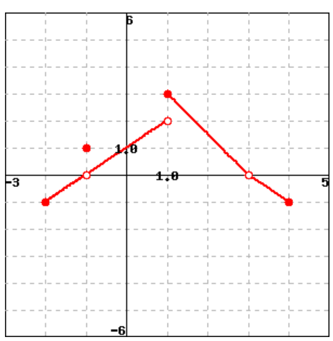
Correct Answers:

- `-1.39000000000003`
- `-1.93989999999999`
- `-2.05990000000007`
- `-2.59`
- `-2`

---

29. (1 point) Let F be the function whose graph is shown below. Evaluate each of the following expressions.
(If a limit does not exist or is undefined, enter "DNE".)

1. $\lim_{x\to -1^-} F(x) =$ ___
2. $\lim_{x\to -1^+} F(x) =$ ___
3. $\lim_{x\to -1} F(x) =$ ___
4. $F(-1) =$ ___
5. $\lim_{x\to 1^-} F(x) =$ ___
6. $\lim_{x\to 1^+} F(x) =$ ___
7. $\lim_{x\to 1} F(x) =$ ___
8. $\lim_{x\to 3} F(x) =$ ___
9. $F(3) =$ ___



The graph of $y = F(x)$.

Correct Answers:

- `0`
- `0`
- `0`
- `1`
- `2`
- `3`
- `DNE`
- `0`
- `DNE`

---

30. (1 point) Let

$$f(x) = \begin{cases} 13 & \text{if } x < -9 \\ -x+4 & \text{if } -9 \le x < 2 \\ -2 & \text{if } x = 2 \\ 4 & \text{if } x > 2. \end{cases}$$

Sketch the graph of this function and find the following limits, if they exist.
(*If a limit does not exist, enter* **DNE**.)

1. $\lim_{x\to -9^-} f(x) =$ ___
2. $\lim_{x\to -9^+} f(x) =$ ___
3. $\lim_{x\to -9} f(x) =$ ___
4. $\lim_{x\to 2^-} f(x) =$ ___
5. $\lim_{x\to 2^+} f(x) =$ ___
6. $\lim_{x\to 2} f(x) =$ ___

Correct Answers:

- `13`
- `13`
- `13`
- `2`
- `4`
- `DNE`

---

31. (1 point) Evaluate the limit

$$\lim_{x\to -6} \frac{x^2+7x+6}{x+6}$$

____

Correct Answers:

- `-5`

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32. (1 point) Evaluate the limit

$$\lim_{a\to 1} \frac{a^3-1}{a^2-1}$$

____

Correct Answers:

- `1.5`

---

33. (1 point) Evaluate the limit

$$\lim_{x\to\infty} \frac{4+5x}{5-6x}$$

Enter **I** for ∞, **-I** for $-\infty$, and **DNE** if the limit does not exist.
Limit = ________

Correct Answers:

- `-0.833333333333333`

34. (1 point) Evaluate the limit

$$\lim_{x\to\infty} \frac{10x+9}{5x^2-5x+11}$$

Enter **I** for ∞, **-I** for $-\infty$, and **DNE** if the limit does not exist.
Limit = ______

Correct Answers:

- 0

35. (1 point) The horizontal asymptotes of the curve

$$y = \frac{4x}{(x^4+1)^{\frac{1}{4}}}$$

are given by
$y_1 =$ ______ and
$y_2 =$ ______
where $y_1 > y_2$.

Correct Answers:

- 4
- -4

36. (1 point)
Evaluate the following limits.
(a) $\lim_{x\to\infty} \frac{8}{e^x+6} =$ ______
(b) $\lim_{x\to-\infty} \frac{8}{e^x+6} =$ ______
[NOTE: If needed, enter INF or infinity for ∞ and -INF or -infinity for $-\infty$.]
Correct Answers:

- 0
- 1.33333

37. (1 point)
Evaluate the following limits. If needed, enter INF for ∞ and MINF for $-\infty$.
(a)

$$\lim_{x\to\infty}\left(\sqrt{x^2+9x+1}-x\right) =$$

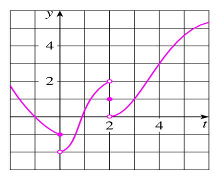
______
(b)

$$\lim_{x\to-\infty}\left(\sqrt{x^2+9x+1}-x\right) =$$

______
Correct Answers:

- 4.5
- INF

38. (1 point)
For the function g whose graph is given, state the value of the given quantity, if it exists. If it does not exist, enter "n" below.
(a) $\lim_{t\to 0^-} g(t)$
(b) $\lim_{t\to 0^+} g(t)$
(c) $\lim_{t\to 0} g(t)$
(d) $\lim_{t\to 2^-} g(t)$
(e) $\lim_{t\to 2^+} g(t)$
(f) $\lim_{t\to 2} g(t)$
(g) $g(2)$
(h) $\lim_{t\to 4} g(t)$



(a) ______
(b) ______
(c) ______
(d) ______
(e) ______
(f) ______
(g) ______
(h) ______

Correct Answers:

- -1
- -2
- n
- 2
- 0
- n
- 1
- 3

39. (1 point)
Guess the value of the limit (if it exists) by evaluating the function at values close to where the limit is to be done. If it does not exist, enter "n" below. If the answer is infinite, use "i" to represent infinity.

$$\lim_{x\to 0} \frac{\cos x - 1}{x}$$

______

Correct Answers:

- 0

40. (1 point)
Evaluate the following limit. If the answer is positive infinite, type "I"; if negative infinite, type "N"; and if it does not

exist, type "D".

$$\lim_{x\to\infty}\sqrt{\frac{12x^3-5x+4}{1+9x^2+3x^3}}$$

Correct Answers:

- `2`

---

41. (1 point) Let $f(x) = 11x^2 - 7x$.

(a) Use the limit process to find the slope of the line tangent to the graph of f at $x = 3$.

Slope at $x = 3$: _____

(b) Find an equation of the line tangent to the graph of f at $x = 3$.

Tangent line: $y =$ ____________________

Correct Answers:

- `59`
- `59*(x-3)+78`

---

42. (1 point) Let $f(x) = 5x^4 - 2$.

(a) Use the limit process to find the slope of the line tangent to the graph of f at $x = 2$.

Slope at $x = 2$: _____

(b) Find an equation of the line tangent to the graph of f at $x = 2$.

Tangent line: $y =$ ____________________

Correct Answers:

- `160`
- `160*(x-2)+78`

---

43. (1 point) Let f and g be functions that satisfy $f'(2) = -8$ and $g'(2) = 1$. Find $h'(2)$ for each function h given below:

(A) $h(x) = 10f(x)$.

$h'(2) =$ _____

(B) $h(x) = -5g(x)$.
$h'(2) =$ _____

(C) $h(x) = 6f(x) + 13g(x)$.

$h'(2) =$ _____

(D) $h(x) = 10g(x) - 7f(x)$.

$h'(2) =$ _____

(E) $h(x) = 12f(x) + 7g(x) - 4$.

$h'(2) =$ _____

(F) $h(x) = -3g(x) - 5f(x) + 4x$.

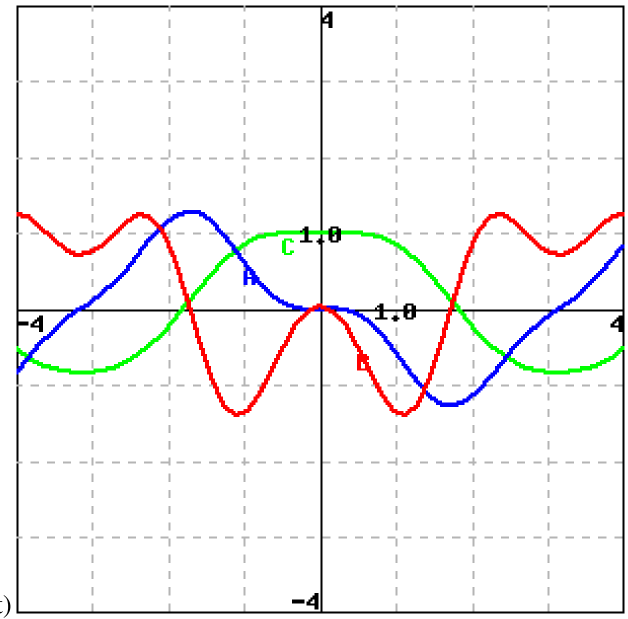
$h'(2) =$ _____

Correct Answers:

- `-80`
- `-5`
- `-35`
- `66`
- `-89`
- `41`

---

44. (1 point)



Identify the graphs A (blue), B(red) and C (green) as the graphs of a function $f(x)$ and its derivatives $f'(x)$ and $f''(x)$. (Clicking on the sketch will give you a version of the picture in a separate window.)

___ is the graph of the function, $f(x)$.
___ is the graph of the function's first derivative, $f'(x)$.
___ is the graph of the function's second derivative, $f''(x)$.

Hint: Remember that $f'(x)$ is itself a function, and we can find the derivative of the function $f'(x)$, which is called the second derivative of the function $f(x)$ and denoted by $f''(x)$.

Correct Answers:

- `C`
- `A`
- `B`

---

45. (1 point) Let

$$f(x) = \sqrt{2+2x}$$

$f'(3) =$ __________

Correct Answers:

- `0.353553390593274`

46. (1 point) Let $f(x) = \sqrt{29-x}$
The slope of the tangent line to the graph of $f(x)$ at the point $(4,5)$ is _____.
The equation of the tangent line to the graph of $f(x)$ at $(4,5)$ is $y = mx+b$ for
$m =$_____
and
$b =$_____.
Hint: the slope at $x = 4$ is given by
$m = \lim_{h\to 0} \frac{f(4+h)-f(4)}{h}$

Correct Answers:

- -0.1
- -0.1
- 5.4

---

47. (1 point) A function $f(x)$ is said to have a **removable** discontinuity at $x = a$ if:
1. f is either not defined or not continuous at $x = a$.
2. $f(a)$ could either be defined or redefined so that the new function is continuous at $x = a$.

Let $f(x) = \begin{cases} \frac{9}{x} + \frac{-8x+18}{x(x-2)}, & \text{if } x \neq 0,2 \\ 2, & \text{if } x = 0 \end{cases}$
Show that $f(x)$ has a removable discontinuity at $x = 0$ and determine what value for $f(0)$ would make $f(x)$ continuous at $x = 0$.
Must redefine $f(0) =$_______.
Hint: Try combining the fractions and simplifying.
The discontinuity at $x = 2$ is not a removable discontinuity, just in case you were wondering.

Correct Answers:

- -0.5

---

48. (1 point)
A rock is thrown off of a 100 foot cliff with an upward velocity of 50 m/s. As a result its height after t seconds is given by the formula:
$h(t) = 100 + 50t - 5t^2$
What is its height after 5 seconds?_______
What is its velocity after 5 seconds?_______
(Positive velocity means it is on the way up, negative velocity means it is on the way down.)

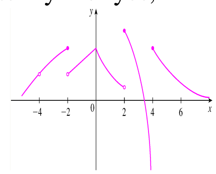
Correct Answers:

- 225
- 0

---

49. (1 point)

(a) From the graph of f, state whether or not f is continuous (enter "y" for yes, "n" for no below) at:



Part (a):
(1) _____ (2) _____ (3) _____
(4) _____ (5) _____

(b) For each of the numbers in part (a), determine if f is continuous from the right, or from the left, or neither (enter "r", "l", or "n", respectively below. In the case of continuity at a point from both sides, enter "b".)

Part (b):
(1) _____ (2) _____ (3) _____
(4) _____ (5) _____

Correct Answers:

- n
- n
- y
- n
- n
- n
- l
- b
- r
- r

---

50. (1 point)
If f and g are continuous functions with $f(0) = 3$ and $\lim_{x\to 0} f(x)g(x) = 6$, find $g(0)$.

$g(0) =$ _____

Correct Answers:

- 2

---

51. (1 point)
(a) Find the slope of the tangent line to the curve $y = \frac{2}{x+3}$ at the point where $x = a$.

(b-d) Find the slopes of the tangent lines at the points whose x-coordinates are: (b) -1, (c) 0, and (d) 1.

(a) __________
(b) __________
(c) __________
(d) __________

Correct Answers:

- -2/(a+3)^2
- -0.5

- -0.222222222222222
- -0.125

---

52. (1 point)
If $f(x) = 3x^2 - 5x$, find $f'(2)$
$f'(2) =$ ______________
Use it to find an equation of the tangent line to the parabola $y = 3x^2 - 5x$ at the point (2,2).
$y =$ ______________
Correct Answers:

- `7`
- `7 x - 12`

---

53. (1 point)
The following limit represents the derivative of some function f at some number a.

$$\lim_{h \to 0} \frac{(1+h)^{10} - 1}{h}$$

What are f and a?

$f(x) =$ ______________
$a =$ ______________
Correct Answers:

- `x^10`
- `1`

---

54. (1 point) If

$$f(x) = \frac{4x}{1+x^2}$$

find $f'(4)$.

$f'(4) =$ ______________

Use this to find the equation of the tangent line to the curve $y = \frac{4x}{1+x^2}$ at the point $(4, \frac{16}{17})$. The equation of this tangent line can be written in the form $y = mx + b$.
The equation of the tangent line is $y =$ ______________.
Correct Answers:

- `4*(1-4^2)/[(1+4^2)^2]`
- `0.941176--0.207612*4`

---

55. (1 point) Let $f(t) = \frac{\sqrt{2}}{t^7}$. Find $f'(t)$.
$f'(t) =$ ______________
Find $f'(3)$.
$f'(3) =$ ______________
Correct Answers:

- `-(1.41421*7*t^6/[(t^7)^2])`
- `-0.00150884`

---

56. (1 point) Let $f(x) = 6x^5\sqrt{x} + \frac{7}{x^3\sqrt{x}}$.

$f'(x) =$ ______________

[NOTE: Your answer should be a function in terms of the variable 'x' and not a number!]
Correct Answers:

- `6*(5+1/2)*x^(5-1/2)-7*(3+1/2)/[x^(3+3/2)]`

---

57. (1 point) If

$$f(x) = \frac{3-x^2}{2+x^2}$$

find $f'(x)$.

$f'(x) =$ ______________

Find $f'(1)$.

$f'(1) =$ ______________

Correct Answers:

- `[-2*x*(2+x^2)-(3-x^2)*2*x]/[(2+x^2)^2]`
- `[-2*1*(2+1^2)-(3-1^2)*2*1]/[(2+1^2)^2]`

---

58. (1 point) If $f(x) = \cos x - 2\tan x$, then find:

$f'(x) =$ ______________

$f'(1) =$ ______________

Correct Answers:

- `-[sin(x)]-2*[sec(x)]^2`
- `-[sin(1)]-2/([cos(1)]^2)`

---

59. (1 point) Let $f(x) = \frac{5\sin(x)}{2+\cos(x)}$. Find the following:

1. $f'(x)$ = ______________
2. $f'(3)$ = ______________

Correct Answers:

- `[5*cos(x)*[2+cos(x)]+5*sin(x)*sin(x)]/([2+cos(x)]^2)`
- `-4.80331`

---

60. (1 point) Let $f(x) = \frac{4\tan(x)}{x}$. Find the following:

1. $f'(x)$ = ______________
2. $f'(5)$ = ______________

Correct Answers:

- `(4*[sec(x)]^2*x-4*tan(x))/(x^2)`
- `10.4832`

61. (1 point) Let $f(x) = 9x(\sin(x)+\cos(x))$. Find the following:

1. $f'(x)$ = ____________
2. $f'(-\frac{\pi}{4})$ = ____________

Correct Answers:

- `9*[sin(x)+cos(x)]+9*x*[cos(x)-sin(x)]`
- `-9.99649`

---

62. (1 point) If $f(x) = 3x\sin(x)\cos(x)$, find $f'(x)$.
$f'(x) =$ ____________

Find $f'(3)$.
$f'(3) =$ ____________

Correct Answers:

- `3*sin(x)*cos(x)+3*x*([cos(x)]^2-[sin(x)]^2)`
- `8.22241`

---

63. (1 point) Let

$$f(x) = (4x^2-6)^5(8x^2-2)^9$$

$f'(x) =$ ____________

Correct Answers:

- `(4*x^2+-6)^4 * (8*x^2+-2)^8 * (896*x^3 + -944*x)`

---

64. (1 point) Let

$$f(x) = \ln(x^7)$$

$f'(x) =$ ____________

$f'(e^2) =$ ____________

Correct Answers:

- `7/x`
- `7*e^-2`

---

65. (1 point) At noon, ship A is 50 nautical miles due west of ship B. Ship A is sailing west at 18 knots and ship B is sailing north at 23 knots. How fast (in knots) is the distance between the ships changing at 7 PM? (Note: 1 knot is a speed of 1 nautical mile per hour.)

____

Note: Draw yourself a diagram which shows where the ships are at noon and where they are "some time" later on. You will need to use geometry to work out a formula which tells you how far apart the ships are at time t, and you will need to use "distance = velocity * time" to work out how far the ships have travelled after time t.

Correct Answers:

- `28.8054898194543`

---

66. (1 point) A plane flying with a constant speed of 4 km/min passes over a ground radar station at an altitude of 7 km and climbs at an angle of 25 degrees.

At what rate is the distance from the plane to the radar station increasing 5 minutes later?

The distance is increasing at ____________ km/min.

Hint:

Hint: The law of cosines for a triangle is

$$c^2 = a^2+b^2-2ab\cos(\theta)$$

where θ is the angle between the sides of length a and b.

Correct Answers:

- `3.8555`

---

67. (1 point) Suppose $xy = 2$ and $\frac{dy}{dt} = -4$. Find $\frac{dx}{dt}$ when $x = -2$.

$\frac{dx}{dt} =$ ____________

Correct Answers:

- `8`

---

68. (1 point) Let $f(x) = 4\log_9(x)$

$f'(x) =$ ____________

$f'(3) =$ ____________

Correct Answers:

- `4/(2.19722*x)`
- `0.606826`

---

69. (1 point) A street light is at the top of a 12 ft tall pole. A woman 6 ft tall walks away from the pole with a speed of 5 ft/sec along a straight path. How fast is the tip of her shadow moving when she is 35 ft from the base of the pole?

____

Note: You should draw a picture of a right triangle with the vertical side representing the pole, and the other end of the hypotenuse representing the tip of the woman's shadow. Where does the woman fit into this picture? Label her position as a variable, and label the tip of her shadow as another variable. You might like to use similar triangles to find a relationship between these two variables.

Correct Answers:

- `10`

---

70. (1 point) A spherical snowball is melting so that its diameter is decreasing at rate of 0.2 cm/min.

At what is the rate is the volume of the snowball changing when the diameter is 11 cm?

The volume is changing at a rate of ____________ cm^3/min.

Correct Answers:

- `-38.0133`

71. (1 point) Let $f(x) = \tan^{-1}(\sin(2x))$. Find $f'(x)$.

$f'(x) =$ ____________________

Correct Answers:

- `1/(1+[sin(2*x)]^2)*2*cos(2*x)`

72. (1 point) Let $f(x) = 3\sec^{-1}(4x)$. Find $f'(x)$.
$f'(x) =$ ____________________
Find $f'(1)$.
$f'(1) =$____________________

Correct Answers:

- `3*4*1/[|4*x|*sqrt((4*x)^2-1)]`
- `0.774597`

73. (1 point) Let $f(x) = x^3\tan^{-1}(7x)$

$f'(x) =$ ________________

Solution: **Solution:** Using the product and chain rules, we see $f'(x) = 3x^2\tan^{-1}(7x) + x^3\dfrac{7}{1+49x^2}$

Correct Answers:

- `3*x^2*atan(7*x)+x^3*7/(1+49*x^2)`

74. (1 point) Suppose $y = \sinh(x^2+x)$. Find $D_x y$.
Answer: $D_x y =$________.

Hint: Recall the derivative of $\sinh(x)$ and use the chain rule.
Solution:
Solution: $D_x\sinh(x^2+x) = (2x+1)(\sinh(x^2+x))$ by the chain rule.

Correct Answers:

- `(2*x+1)*cosh(x**2+x)`

75. (1 point) If $f(x) = 6\arctan(7\sin(2x))$, find $f'(x)$.

____

Correct Answers:

- `6*7*2*cos(2*x)/(1+7*7*(sin(2*x))**2)`

76. (1 point) a) Find
$\tan\left(\sin^{-1}(\frac{1}{6}) + \cos^{-1}(\frac{5}{9})\right) =$________________________________.
(Make sure your answer is an algebraic expression with square roots but without trigonometric or inverse trignometric functions.)
b) Express in terms of x:
$\sin\left(2\tan^{-1}(x)\right) =$________________________________.

Correct Answers:

- `(1/sqrt(2*1*5+5**2)+sqrt(2*5*4+4**2)/5)
  /(1-1/sqrt(2*1*5+5**2)*sqrt(2*5*4+4**2)/5)`
- `2*x/(1+x**2)`

77. (1 point) a) Let $f(x) = \dfrac{1}{6}\tan^{-1}\left(\dfrac{x}{6}\right)$.
Find $f'(x) =$ ________.
b) Let $g(x) = \frac{x}{2}\sqrt{36-x^2} + 18\sin^{-1}\left(\dfrac{x}{6}\right)$.
Find $g'(x) =$ ________.

Correct Answers:

- `1/(6^2+x^2)`
- `sqrt(6^2-x^2)`

78. (1 point) Let $f(x) = \left(x^2+5\right)^{\ln(x)}$. Find $f'(x)$.
$f'(x) =$ ____________________

Correct Answers:

- `[1/x*ln(x^2+5)+1/(x^2+5)*2*x*ln(x)]*exp(ln(x)*ln(x^2+5))`

79. (1 point) Let $f(x) = x^{2x}$. Find $f'(x)$.
$f'(x) =$ ____________________

Correct Answers:

- `[2*ln(x)+2*x*1/x]*exp(2*x*ln(x))`

80. (1 point) Let $y = 4x^2+5x+4$.
Find the differential dy when $x = 3$ and $dx = 0.3$ ________
Find the differential dy when $x = 3$ and $dx = 0.6$ ________

Correct Answers:

- `8.7`
- `17.4`

81. (1 point) Let $y = 2x^2$.
Find the change in y, Δy when $x = 3$ and $\Delta x = 0.1$ ________
Find the differential dy when $x = 3$ and $dx = 0.1$ ________

Correct Answers:

- `1.22`
- `1.2`

82. (1 point) Use linear approximation, i.e. the tangent line, to approximate $\sqrt[3]{1.03}$ as follows. Let $f(x) = \sqrt[3]{x}$ and find the equation of the tangent line to $f(x)$ at $x = 1$ in the form $y = mx+b$.

Note: The values of m and b are rational numbers which can be computed by hand. You need to enter expressions which give m and b *exactly* . You may not have a decimal point in the answers to either of these parts.

$m =$ ____________

$b =$ ____________

Using these values, find the approximation.

$\sqrt[3]{1.03} \approx$ ____________

Note: You can enter decimals for the last part, but it will has to be entered to very high precision (correct for 6 places past the decimal point).

Correct Answers:

- 1/(3*1^2)
- 1-1/(3*1^2)
- 1.01

---

83. (1 point) The linearization at $a = 0$ to $\sin(7x)$ is $A+Bx$. Compute A and B.

$A =$ ______
$B =$ ______

Correct Answers:

- 0
- 7

---

84. (1 point) Answer the following questions for the function

$$f(x) = x\sqrt{x^2+16}$$

defined on the interval $[-7,5]$.

a.) $f(x)$ is concave down on the region ______.

b.) $f(x)$ is concave up on the region ______.

c.) The minimum for this function occurs at ____.

d.) The maximum for this function occurs at ____.

Note: Your answer to parts **a** and **b** must be given in **interval notation** .

Correct Answers:

- (-7,0)
- (0,5)
- -7
- 5

---

85. (1 point) The function $f(x) = -2x^3+36x^2-210x+5$ has one local minimum and one local maximum.
This function has a local minimum at x equals ___ with value ______
and a local maximum at x equals ___ with value ______

Correct Answers:

- 5
- -395
- 7
- -387

---

86. (1 point) Consider the function $f(x) = 12x^5+30x^4-160x^3+3$. For this function there are four important intervals: $(-\infty,A]$, $[A,B]$,$[B,C]$, and $[C,\infty)$ where A, B, and C are the critical numbers.
Find A ____
and B ____
and C ____
At each critical number A, B, and C does $f(x)$ have a local min, a local max, or neither? Type in your answer as LMIN, LMAX, or NEITHER.
At A ____
At B ____
At C ____

Correct Answers:

- -4
- 0
- 2
- LMAX
- NEITHER
- LMIN

---

87. (1 point) Find the linear approximation of $f(x) = \ln x$ at $x = 1$ and use it to estimate $\ln(1.37)$.
$L(x) =$ ____
$\ln 1.37 \approx$ ____

Correct Answers:

- x-1
- 0.37

---

88. (1 point) Find the absolute maximum and absolute minimum of

$$g(t) = t\sqrt{3-t}$$

on the interval $(0.1, 2.3)$.

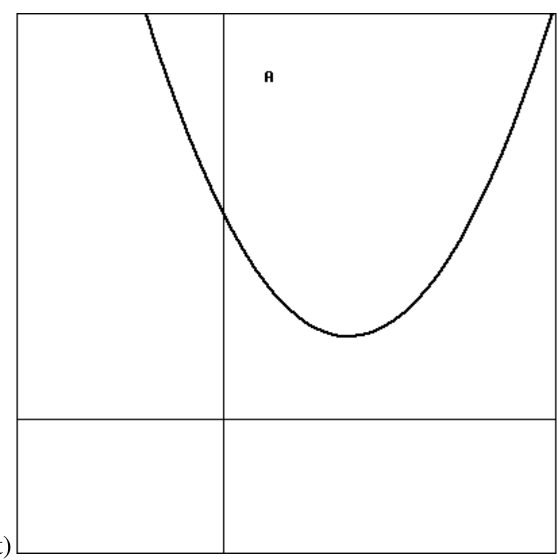
Enter *DNE* if the absolute maximum or minimum does not exist.

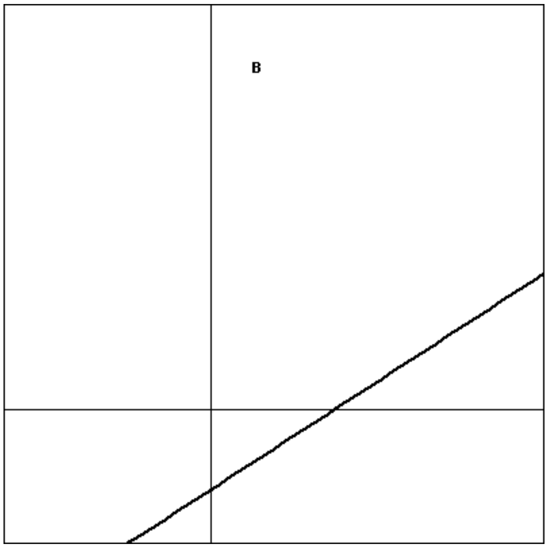
The absolute max occurs at $t =$ ______
The absolute min occurs at $t =$ ______

Correct Answers:

- 6/3
- DNE

---

89. (1 point) 



Graphs A and B are approximate graphs of f and f' for $f(x) = x^2 - 6x + 19$.
So f is increasing (and f' is positive) on the interval (a, ∞) for $a =$______.

Correct Answers:

- `3`

---

90. (1 point) Consider the function $f(x) = 12x^5 + 30x^4 - 160x^3 + 5$.
$f(x)$ has inflection values at (reading from left to right) $x = D$, E, and F
where D is ________
and E is ________
and F is ________
For each of the following intervals, tell whether $f(x)$ is concave up (type in CU) or concave down (type in CD).
$(-\infty, D]$: ________
$[D, E]$: ________
$[E, F]$: ________
$[F, \infty)$: ________

Correct Answers:

- `-2.88600093632938`
- `0`
- `1.38600093632938`
- `CD`
- `CU`
- `CD`
- `CU`

---

91. (1 point) Consider the function $f(x) = 4(x-3)^{2/3}$. For this function there are two important intervals: $(-\infty, A)$ and (A, ∞) where A is a critical number.
Find A ____
For each of the following intervals, tell whether $f(x)$ is increasing (type in INC) or decreasing (type in DEC).
$(-\infty, A)$: ____
(A, ∞): ____
For each of the following intervals, tell whether $f(x)$ is concave up (type in CU) or concave down (type in CD).
$(-\infty, A)$: ____
(A, ∞): ____

Correct Answers:

- `3`
- `DEC`
- `INC`
- `CD`
- `CD`

---

92. (1 point) Suppose that

$$f(x) = \ln(4 + x^2)$$

(A) Use interval notation to indicate where $f(x)$ is concave up.
Note: When using interval notation in WeBWorK, you use **I** for ∞, **-I** for $-\infty$, and **U** for the union symbol. If there are no values that satisfy the required condition, then enter "" without the quotation marks.
Concave up: __________

(B) Use interval notation to indicate where $f(x)$ is concave down.
Concave down: __________

(C) Find all inflection points of f. If there are no inflection points, enter -1000. If there are more than one, enter them separated by commas.
Inflection point(s) at $x =$ _____

Correct Answers:

- `(-2,2)`
- `(-infinity,-2) U (2,infinity)`
- `-2, 2`

---

93. (1 point) Let $f(x) = x^2 - 3x - 40$. Find the open intervals on which f is concave up (down). Then determine the x-coordinates of all inflection points of f.

1. f is concave up on the intervals ____________
2. f is concave down on the intervals ____________
3. The inflection points occur at $x =$ ____________

Notes: In the first two, your answer should either be a single interval, such as (0,1), a comma separated list of intervals, such as (-inf, 2), (3,4), or the word "none".
In the last one, your answer should be a comma separated list of x values or the word "none".

Correct Answers:

- `(-infinity,infinity)`
- `NONE`
- `NONE`

94. (1 point) Let $f(x) = x^3 - 5x^2 + 7x + 5$. Find the open intervals on which f is concave up (down). Then determine the x-coordinates of all inflection points of f.

1. f is concave up on the intervals ________
2. f is concave down on the intervals ________
3. The inflection points occur at $x =$ ________

Notes: In the first two, your answer should either be a single interval, such as (0,1), a comma separated list of intervals, such as (-inf, 2), (3,4), or the word "none".

In the last one, your answer should be a comma separated list of x values or the word "none".

Correct Answers:

- `(1.66667,infinity)`
- `(-infinity,1.66667)`
- `1.66667`

---

95. (1 point) Let $f(x) = \dfrac{1}{5x^2+6}$. Find the open intervals on which f is concave up (down). Then determine the x-coordinates of all inflection points of f.

1. f is concave up on the intervals ________
2. f is concave down on the intervals ________
3. The inflection points occur at $x =$ ________

Notes: In the first two, your answer should either be a single interval, such as (0,1), a comma separated list of intervals, such as (-inf, 2), (3,4), or the word "none".

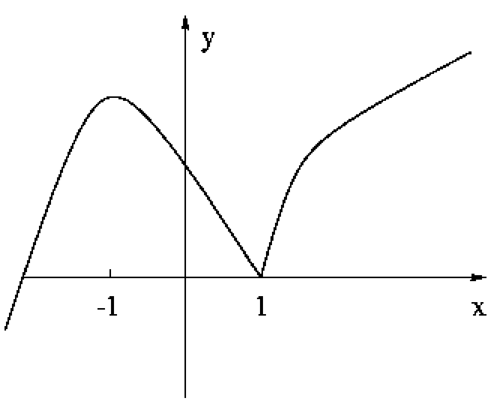
In the last one, your answer should be a comma separated list of x values or the word "none".

Correct Answers:

- `(-infinity,-0.632456), (0.632456,infinity)`
- `(-0.632456,0.632456)`
- `-0.632456, 0.632456`

---

96. (1 point)



For the function f given above, determine whether the following conditions are true. Input T if the condition is true, otherwise input F.

(a) $f'(-1) = 0$; ____
(b) $f'(1)$ does not exist; ____
(c) $f'(x) < 0$ if $|x| < 1$; ____
(d) $f'(x) > 0$ if $|x| > 1$; ____
(e) $f''(x) < 0$ if $x \neq 1$. ____.

Correct Answers:

- `T`
- `T`
- `T`
- `T`
- `T`

---

97. (1 point) Answer the following questions for the function

$$f(x) = x\sqrt{x^2+16}$$

defined on the interval $[-7, 4]$.

a.) $f(x)$ is concave down on the region ________.

b.) $f(x)$ is concave up on the region ________.

c.) The minimum for this function occurs at ____.

d.) The maximum for this function occurs at ____.

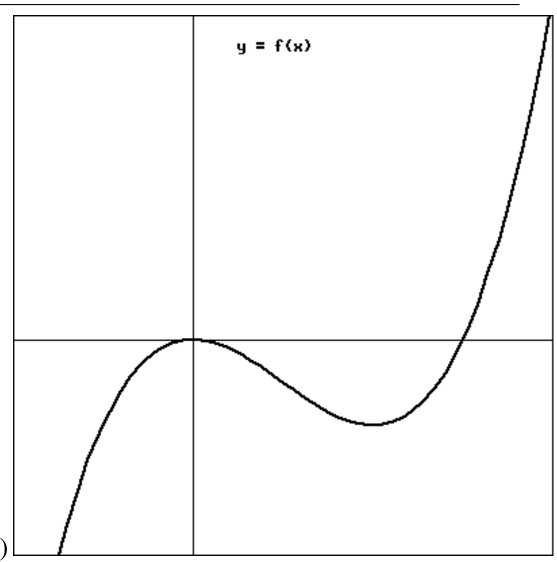
Note: Your answer to parts **a** and **b** must be given in **interval notation**.

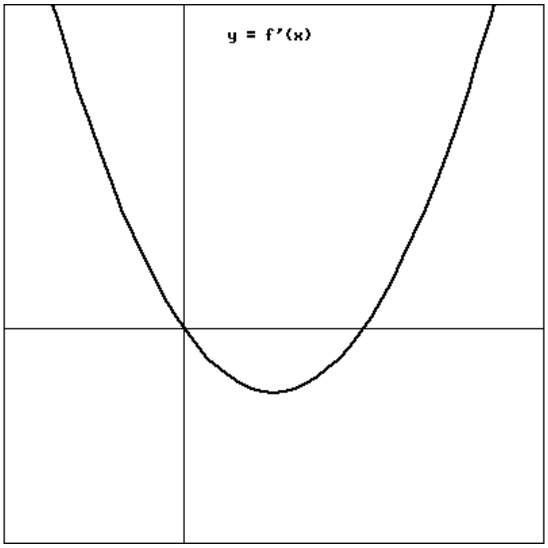
Correct Answers:

- `(-7,0)`
- `(0,4)`
- `-7`
- `4`

---

98. (1 point)





The graphs above are approximate graphs of f and f' for $f(x) = x^2(x-15)$.
So f is decreasing (and f' is negative) on the interval $(0,a)$ for $a =$_______.

Correct Answers:

- `10`

---

99. (1 point)
Find the limit. Use l'Hospital's Rule if appropriate. Use INF to represent positive infinity, NINF for negative infinity, and D if the limit does not exist.

$$\lim_{x\to -10} \frac{x^2-100}{x+10} = \_\_\_\_\_\_\_\_$$

Correct Answers:

- `-20`

---

100. (1 point)
Find the limit. Use l'Hospital's Rule if appropriate. Use INF to represent positive infinity, NINF for negative infinity, and D for the limit does not exist.

$$\lim_{t\to 0} \frac{e^{-2t}-1}{-1t} = \_\_\_\_\_\_\_\_$$

Correct Answers:

- `2`

---

101. (1 point)
Find the limit. Use l'Hospital's Rule if appropriate. Use INF to represent positive infinity, NINF for negative infinity, and D for the limit does not exist.

$$\lim_{\theta\to \pi/2} \frac{-9+9\sin\theta}{3\csc\theta} = \_\_\_\_\_\_\_\_$$

Correct Answers:

- `0`

---

102. (1 point)
Find the limit. Use l'Hospital's Rule if appropriate. Use INF to represent positive infinity, NINF for negative infinity, and D for the limit does not exist.

$$\lim_{x\to \infty} \frac{4e^x}{9x} = \_\_\_\_\_\_\_\_$$

Correct Answers:

- `INF`

---

103. (1 point)
Find the limit. Use l'Hospital's Rule if appropriate. Use INF to represent positive infinity, NINF for negative infinity, and D for the limit does not exist.

$$\lim_{x\to 0^+} \frac{3\ln x}{1x} = \_\_\_\_\_\_\_\_$$

Correct Answers:

- `INF`

---

104. (1 point)
Find the limit. Use l'Hospital's Rule if appropriate. Use INF to represent positive infinity, NINF for negative infinity, and D for the limit does not exist.

$$\lim_{x\to \infty} \frac{-2\ln(\ln x)}{6x} = \_\_\_\_\_\_\_\_$$

Correct Answers:

- `0`

---

105. (1 point)
Find the limit. Use l'Hospital's Rule if appropriate. Use INF to represent positive infinity, NINF for negative infinity, and D for the limit does not exist.

$$\lim_{t\to 0} \frac{5^t-4^t}{7t} = \_\_\_\_\_\_\_\_$$

Correct Answers:

- `0.0318776501877442`

---

106. (1 point)
Find the limit. Use l'Hospital's Rule if appropriate. Use INF to represent positive infinity, NINF for negative infinity, and D for the limit does not exist.

$$\lim_{x\to 1} \frac{8\ln x}{3\sin(\pi x)} = \_\_\_\_\_\_\_\_$$

Correct Answers:

- `-0.848826363156776`

---

107. (1 point)
Find the limit. Use l'Hospital's Rule if appropriate. Use INF to represent positive infinity, NINF for negative infinity, and D for the limit does not exist.

$$\lim_{x\to 0} \frac{4e^x-4-4x}{6x^2} = \_\_\_\_\_\_\_\_$$

Correct Answers:

- `0.333333333333333`

108. (1 point)

Find the limit. Use l'Hospital's Rule if appropriate. Use INF to represent positive infinity, NINF for negative infinity, and D for the limit does not exist.

$\lim_{x\to 0} \frac{2\arcsin x}{7x} =$ ______

Correct Answers:

- `0.285714285714286`

109. (1 point) Suppose that

$$f(x) = \frac{6}{x^2-16}.$$

(A) List all critical numbers of f. If there are no critical numbers, enter 'NONE'.
Critical numbers = ______

(B) Use interval notation to indicate where $f(x)$ is increasing.
Note: Use 'INF' for ∞, '-INF' for $-\infty$, and use 'U' for the union symbol.

Increasing: ______

(C) Use interval notation to indicate where $f(x)$ is decreasing.
Decreasing: ______

(D)List the x-coordinates of all local maxima of f. If there are no local maxima, enter 'NONE'.
x values of local maxima = ______

(E) List the x-coordinates of all local minima of f. If there are no local minima, enter 'NONE'.
x values of local minima = ______

(F) Use interval notation to indicate where $f(x)$ is concave up.
Concave up: ______

(G) Use interval notation to indicate where $f(x)$ is concave down.
Concave down: ______

(H) List the x values all inflection points of f. If there are no inflection points, enter 'NONE'.
Inflection points = ______

(I) List all horizontal asymptotes of f. If there are no horizontal asymptotes, enter 'NONE'.
Horizontal asymptotes $y =$ ______

(J) List all vertical asymptotes of f. If there are no vertical asymptotes, enter 'NONE'.
Vertical asymptotes $x =$ ______

(K) Use all of the preceding information to sketch a graph of f. When you're finished, enter a "1" in the box below.
Graph Complete: ______

Correct Answers:

- `0`
- `(-infinity,-4) U (-4,0)`
- `(0,4) U (4,infinity)`
- `0`
- `none`
- `(-infinity,-4) U (4,infinity)`
- `(-4,4)`
- `none`
- `0`
- `-4, 4`
- `1`

110. (1 point) Suppose that

$$f(x) = \frac{5x^2}{x^2+36}.$$

(A) List all critical numbers of f. If there are no critical numbers, enter 'NONE'.
Critical numbers= ______

(B) Use interval notation to indicate where $f(x)$ is increasing.
Note: Use 'INF' for ∞, '-INF' for $-\infty$, and use 'U' for the union symbol.

Increasing: ______

(C) Use interval notation to indicate where $f(x)$ is decreasing.
Decreasing: ______

(D) List the x-coordinates of all local maxima of f. If there are no local maxima, enter 'NONE'.
x values of local maxima = ______

(E) List the x-coordinates of all local minima of f. If there are no local minima, enter 'NONE'.
x values of local minima = ______

(F) Use interval notation to indicate where $f(x)$ is concave up.
Concave up: ______

(G) Use interval notation to indicate where $f(x)$ is concave down.
Concave down: ______

(H) List the x values of all inflection points of f. If there are no inflection points, enter 'NONE'.
x values of inflection points = ______

(I) List all horizontal asymptotes of f. If there are no horizontal asymptotes, enter 'NONE'.
Horizontal asymptotes $y =$ ______

(J) List all vertical asymptotes of f. If there are no vertical asymptotes, enter 'NONE'.
vertical asymptotes $x =$ ______

(K) Use all of the preceding information to sketch a graph of f. When you're finished, enter a "1" in the box below.

Graph Complete: ______

Correct Answers:

- `0`
- `(0,infinity)`
- `(-infinity,0)`
- `NONE`
- `0`

- `(-3.46410161513775,3.46410161513775)`
- `(-infinity,-3.46410161513775) U (3.46410161513775,infinity)`
- `-3.46410161513775, 3.46410161513775`
- `5`
- `NONE`
- `1`

---

111. (1 point) Suppose that

$$f(x) = 3x^2 - x^3 - 2.$$

(A) Find all critical numbers of f. If there are no critical numbers, enter 'NONE'.
Critical numbers = ______

(B) Use interval notation to indicate where $f(x)$ is increasing.
Note: Use 'INF' for ∞, '-INF' for $-\infty$, and use 'U' for the union symbol.

Increasing: ______

(C) Use interval notation to indicate where $f(x)$ is decreasing.
Decreasing: ______

(D) List the x-coordinates of all local maxima of f. If there are no local maxima, enter 'NONE'.
x values of local maxima = ______

(E) List the x-coordinates of all local minima of f. If there are no local minima, enter 'NONE'.
x values of local minima = ______

(F) Use interval notation to indicate where $f(x)$ is concave up.
Concave up: ______

(G) Use interval notation to indicate where $f(x)$ is concave down.
Concave down: ______

(H)List the x values of all inflection points of f. If there are no inflection points, enter 'NONE'.
x values of inflection points = ______

(I) Use all of the preceding information to sketch a graph of f. When you're finished, enter a "1" in the box below.
Graph Complete: ______

Correct Answers:

- `0, 2`
- `(0,2)`
- `(-infinity,0) U (2,infinity)`
- `2`
- `0`
- `(-infinity,1)`
- `(1,infinity)`
- `1`
- `1`

---

112. (1 point) Suppose that

$$f(x) = \frac{5x-7}{x+5}.$$

(A) Find all critical values of f, compute their average, and enter it below.
Note: If there are no critical values, enter -1000.
Average of critical values = ______

(B) Use interval notation to indicate where $f(x)$ is increasing.
Note: Enter 'I' for ∞, '-I' for $-\infty$, and 'U' for the union symbol.
If you have extra boxes, fill each in with an 'x'.
Increasing:

______

(C) Use interval notation to indicate where $f(x)$ is decreasing.
Decreasing:

______

(D) Find the x-coordinates of all local maxima of f, compute their average, and enter it below.
Note: If there are no local maxima, enter -1000.
Average of x values = ______

(E) Find the x-coordinates of all local minima of f, compute their average, and enter it below.
Note: If there are no local minima, enter -1000.
Average of x values = ______

(F) Use interval notation to indicate where $f(x)$ is concave up.
Concave up:

______

(G) Use interval notation to indicate where $f(x)$ is concave down.
Concave down:

______

(H) Find all inflection points of f, compute their average, and enter it below.
Note: If there are no inflection points, enter -1000.
Average of inflection points = ______

(I) Find all horizontal asymptotes of f, compute the average of the y values, and enter it below.
Note: If there are no horizontal asymptotes, enter -1000.
Average of horizontal asymptotes = ______

(J) Find all vertical asymptotes of f, compute the average of the x values, and enter it below.
Note: If there are no vertical asymptotes, enter -1000.
Average of vertical asymptotes = ______

(K) Use all of the preceding information to sketch a graph of f. When you're finished, enter a "1" in the box below.

Graph Complete: ______

Correct Answers:

- `-1000`
- `(-infinity,-5) U (-5,infinity)`
- `x`
- `-1000`
- `-1000`
- `(-infinity,-5)`
- `(-5,infinity)`
- `-1000`
- `5`
- `-5`
- `1`

---

113. (1 point) Suppose that

$$f(x) = (x^2+10)(4-x^2).$$

(A) Find all critical numbers of f. If there are no critical values, enter 'NONE'.
Critical numbers = ________

(B) Use interval notation to indicate where $f(x)$ is increasing.
Note: Use 'INF' for ∞, '-INF' for $-\infty$, and use 'U' for the union symbol.
Increasing: __________

(C) Use interval notation to indicate where $f(x)$ is decreasing.
Decreasing: __________

(D) Find the x-coordinates of all local maxima of f. If there are no local maxima, enter 'NONE'.
x values of local maxima = ________

(E) Find the x-coordinates of all local minima of f. If there are no local minima, enter 'NONE'.
x values of local minima = ________

(F) Use interval notation to indicate where $f(x)$ is concave down.
Concave down: __________

(G) List the x values of all inflection points of f. If there are no inflection points, enter 'NONE'.
x values of inflection points = ________

(H) Find all horizontal asymptotes of f. If there are no horizontal asymptotes, enter 'NONE'.
Horizontal asymptotes $y =$ _____

(J) Find all vertical asymptotes of f. If there are no vertical asymptotes, enter 'NONE'.
Vertical asymptotes $x =$ _____

(K) Use all of the preceding information to sketch a graph of f. When you're finished, enter a "1" in the box below.
Graph Complete: _____

Correct Answers:

- `0`
- `(-infinity,0)`
- `(0,infinity)`
- `0`
- `NONE`
- `(-infinity,infinity)`
- `NONE`
- `NONE`
- `NONE`
- `1`

---

114. (1 point) Suppose that

$$f(x) = 7x^6 - 3x^5.$$

(A) Find all critical numbers of f. If there are no critical numbers, enter 'NONE'.
Critical numbers = ________

(B) Use interval notation to indicate where $f(x)$ is increasing.
Note: Use 'INF' for ∞, '-INF' for $-\infty$, and use 'U' for the union symbol.
Increasing: ______________

(C) Use interval notation to indicate where $f(x)$ is decreasing.
Decreasing: __________

(D) Find the x-coordinates of all local maxima of f. If there are no local maxima, enter 'NONE'.
x values of local maxima = ________

(E) Find the x-coordinates of all local minima of f. Note: If there are no local minima, enter 'NONE'.
x values of local minima = ________

(F) Use interval notation to indicate where $f(x)$ is concave up.
Concave up: __________

(G) Use interval notation to indicate where $f(x)$ is concave down.
Concave down: ______________

(H) List the x values of all inflection points of f. If there are no inflection points, enter 'NONE'.
x values of inflection points = ________

(I) Find all horizontal asymptotes of f. If there are no horizontal asymptotes, enter 'NONE'.
Horizontal asymptotes $y =$ ________

(J) Find all vertical asymptotes of f. If there are no vertical asymptotes, enter 'NONE'.
Vertical asymptotes $x =$ ________

(K) Use all of the preceding information to sketch a graph of f. When you're finished, enter a "1" in the box below.
Graph Complete: _____

Correct Answers:

- `0, 0.357142857142857`
- `(0.357142857142857,infinity)`
- `(-infinity,0.357142857142857)`
- `NONE`
- `0.357142857142857`
- `(-infinity,0) U (0.285714285714286,infinity)`
- `(0,0.285714285714286)`
- `0, 0.285714285714286`
- `NONE`

- `NONE`
- `1`

---

115. (1 point) Suppose that

$$f(x) = 2x^5 - 5x^4.$$

(A) Find all critical numbers of f. If there are no critical values, enter 'NONE'.
critical numbers = ________
(B) Use interval notation to indicate where $f(x)$ is increasing.
Note: Use 'INF' for ∞, '-INF' for $-\infty$, and use 'U' for the union symbol.
Increasing: ____________
(C) Use interval notation to indicate where $f(x)$ is decreasing.
Decreasing: ____________
(D) Find the x-coordinates of all local maxima of f. If there are no local maxima, enter 'NONE'.
x values of local maxima = ________
(E) Find the x-coordinates of all local minima of f. If there are no local minima, enter 'NONE'.
x values of local minima = ________
(F) Use interval notation to indicate where $f(x)$ is concave up.
Concave up: ____________
(G) Use interval notation to indicate where $f(x)$ is concave down.
Concave down: ____________
(H) Find all x values of the inflection points of f. If there are no inflection points, enter 'NONE'.
x values of inflection points = ________
(I) Find all horizontal asymptotes of f, If there are no horizontal asymptotes, enter 'NONE'.
Horizontal asymptotes $y ==$ ________
(J) Find all vertical asymptotes of f. If there are no vertical asymptotes, enter 'NONE'.
Vertical asymptotes $x =$ ________
(K) Use all of the preceding information to sketch a graph of f. When you're finished, enter a "1" in the box below.
Graph Complete: ____

Correct Answers:

- `0, 2`
- `(-infinity,0) U (2,infinity)`
- `(0,2)`
- `0`
- `2`
- `(1.5,infinity)`
- `(-infinity,1.5)`
- `1.5`
- `NONE`
- `NONE`
- `1`

---

117. (1 point) Find constants a and b in the function $f(x) = axe^{bx}$ such that $f(\frac{1}{3}) = 1$ and the function has a local maximum at $x = \frac{1}{3}$.
$a =$ ____________
$b =$ ____________

Solution:
SOLUTION
Using the product rule on the function $f(x) = axe^{bx}$, we have $f'(x) = ae^{bx} + abxe^{bx} = ae^{bx}(1+bx)$. We want $f(\frac{1}{3}) = 1$, and since this is to be a maximum, we require $f'(\frac{1}{3}) = 0$. These conditions give

$$f(1/3) = \frac{a}{3}e^{b/3} = 1$$

and

$$f'(1/3) = ae^{b/3}(1+b/3) = 0.$$

Since $ae^{b/3}$ is non-zero, we can divide both sides of the second equation by $ae^{b/3}$ to obtain $1 + \frac{b}{3} = 0$. This implies $b = -3$. Plugging $b = -3$ into the first equation gives us $a(\frac{1}{3})e^{-1} = 1$, or $a = 3e$. How do we know we have a maximum at $x = \frac{1}{3}$ and not a minimum? Since $f'(x) = ae^{bx}(1+bx) = ()e^{-3x}(1-3x)$, and $(3e)e^{-3x}$ is always positive, it follows that $f'(x) > 0$ when $x < \frac{1}{3}$ and $f'(x) < 0$ when $x > \frac{1}{3}$. Since f' is positive to the left of $x = \frac{1}{3}$ and negative to the right of $x = \frac{1}{3}$, $f(\frac{1}{3})$ is a local maximum.

Correct Answers:

- `3*e`
- `-1*3`

---

118. (1 point) Find a formula for a curve of the form $y = e^{-(x-a)^2/b}$ for $b > 0$ with a local maximum at $x = 6$ and points of inflection at $x = 2$ and $x = 10$.
$y =$ ____________

Solution:
SOLUTION
The maximum of $y = e^{-(x-a)^2/b}$ occurs at $x = a$. (This is because the exponent $-(x-a)^2/b$ is zero when $x = a$ and negative for all other x-values. The same result can be obtained by taking derivatives.) Thus we know that $a = 6$.
Points of inflection occur where d^2y/dx^2 changes sign, that is, where $d^2y/dx^2 = 0$. Differentiating gives $\frac{dy}{dx} = -\frac{2(x-6)}{b}e^{-(x-6)^2/b}$, so $\frac{d^2y}{dx^2} = -\frac{2}{b}e^{-(x-6)^2/b} + \frac{4(x-6)^2}{b^2}e^{-(x-6)^2/b} = \frac{2}{b}e^{-(x-6)^2/b}\left(-1 + \frac{2}{b}(x-6)^2\right)$. Since $e^{-(x-6)^2/b}$ is never zero, $d^2y/dx^2 = 0$ where $-1 + \frac{2}{b}(x-6)^2 = 0.$. We know $d^2y/dx^2 = 0$ at $x = 10$, so substituting $x = 10$ gives $-1 + \frac{2}{b}(10-6)^2 = 0$. Solving for b gives $b = 32$.
Since $a = 6$, the function is

$$y = e^{-(x-6)^2/32}.$$

You can check that at $x = 6$, we have $\frac{d^2y}{dx^2} = \frac{2}{32}e^{-0}(-1+0) < 0$ so the point $x = 6$ does indeed give a maximum.

Correct Answers:

- e^(-1*(x-6)^2/32)

---

119. (1 point) The number, N, of people who have heard a rumor spread by mass media at time, t, is given by

$$N(t) = a(1-e^{-kt}).$$

There are 150000 people in the population who hear the rumor eventually. 23 percent of them heard it on the first day. Find a and k , assuming t is measured in days.

$a =$ ________________
$k =$ ________________

Solution:
SOLUTION

Since as $t \to \infty$, $N \to a$, we have $a = 150000$. Note that while $N(t)$ will never actually reach 150000, it will become arbitrarily close to 150000. Since N represents the number of people, it makes sense to round up long before $t \to \infty$. When $t = 1$, we have $N = 0.23(150000) = 34500$ people, so plugging into our formula gives

$$N(1) = 34500 = 150000\left(1-e^{-k(1)}\right).$$

Solving for k gives $0.23 = 1-e^{-k}$, so that $k = -\ln(1-0.23) \approx 0.261$.

Correct Answers:

- 150000
- -1*ln(1 - 23/100)

---

120. (1 point) Answer the following questions for the function

$$f(x) = x\sqrt{x^2+6x+18}+3\sqrt{x^2+6x+18}$$

defined on the interval $[-8,1]$. All answers refer to x-coordinates.

A. $f(x)$ is concave down on the region _____ to _____
B. $f(x)$ is concave up on the region _____ to _____
C. The inflection value for this function is _____
D. The minimum for this function occurs at _____
E. The maximum for this function occurs at _____

Correct Answers:

- -8
- -3
- -3
- 1
- -3
- -8
- 1

---

121. (1 point) Answer the following questions for the function

$$f(x) = \frac{x^3}{x^2-36}.$$

Enter "INF" for ∞ and "-INF" for $-\infty$.

A. The function $f(x)$ has two vertical asympototes:
$x =$ __________

B. $f(x)$ has one local maximum and one local minimum occuring at x values :
$x_{\max} =$ _____ and $x_{\min} =$ _____

C. For each interval, tell whether $f(x)$ is increasing (type in INC) or decreasing (type in DEC).
$(-\infty, \max)$ _____
$(\max,-6)$ _____
$(-6,0)$ _____
$(0,6)$ _____
$(6,\min)$ _____
$(\min,+\infty)$ _____

D. $f(x)$ is concave up on the open intervals: ________________________.

E. The inflection point for this function occurs at $x =$ _____

F. Sketch the graph of $f(x)$ and bring it to class.

Correct Answers:

- -6, 6
- -10.3923
- 10.3923
- INC
- DEC
- DEC
- DEC
- DEC
- INC
- (-6,0), (6,infinity)
- 0

---

122. (1 point) Consider the function $f(x) = 3x+3x^{-1}$. For this function there are four important intervals: $(-\infty,A]$, $[A,B)$,(B,C), and $[C,\infty)$ where A, and C are the critical numbers and the function is not defined at B.
Find A _____
and B _____
and C _____
For each of the following intervals, tell whether $f(x)$ is increasing (type in INC) or decreasing (type in DEC).
$(-\infty,A]$: _____
$[A,B)$: _____
$(B,C]$: _____
$[C,\infty)$: _____

Correct Answers:

- -1
- 0
- 1
- INC
- DEC
- DEC
- INC

123. (1 point) Please answer the following questions about the function

$$f(x) = \ln(11x^2 + 6).$$

Instructions: If you are asked to find x- or y-values, enter either a number, a list of numbers separated by commas, or *None* if there aren't any solutions. Use **interval notation** if you are asked to find an interval or union of intervals, and enter $\{\}$ if the interval is empty.

(a) Find the critical numbers of f, where it is increasing and decreasing, and its local extrema.
Critical numbers $x =$ __________
Increasing on the interval __________
Decreasing on the interval __________
Local maxima $x =$ __________
Local minima $x =$ __________

(b) Find where f is concave up, concave down, and has inflection points.
Concave up on the interval __________
Concave down on the interval __________
Inflection points $x =$ __________

(c) Find any horizontal and vertical asymptotes of f.
Horizontal asymptotes $y =$ __________
Vertical asymptotes $x =$ __________

(d) The function f is $\boxed{?}$ because $\boxed{?}$ for all x in the domain of f, and therefore its graph is symmetric about the $\boxed{?}$

(e) Sketch a graph of the function f without having a graphing calculator do it for you. Plot the y-intercept and the x-intercepts, if they are known. Draw dashed lines for horizontal and vertical asymptotes. Plot the points where f has local maxima, local minima, and inflection points. Use what you know from parts (a) and (b) to sketch the remaining parts of the graph of f. Use any symmetry from part (d) to your advantage. Sketching graphs is an important skill that takes practice, and you may be asked to do it on quizzes or exams.

Correct Answers:

- `0`
- `(0,infinity)`
- `(-infinity,0)`
- `NONE`
- `0`
- `(-0.738548945875996,0.738548945875996)`
- `(-infinity,-0.738548945875996) U (0.738548945875996,infini`
- `-0.738548945875996, 0.738548945875996`
- `NONE`
- `NONE`
- `even`
- `f(-x) = f(x)`
- `y-axis`

124. (1 point) Please answer the following questions about the function

$$f(x) = 5x^2 \ln(x), \quad x > 0.$$

Instructions: If you are asked to find x- or y-values, enter either a number, a list of numbers separated by commas, or *None* if there aren't any solutions. Use **interval notation** if you are asked to find an interval or union of intervals, and enter $\{\}$ if the interval is empty.

(a) Find the critical numbers of f, where it is increasing and decreasing, and its local extrema.
Critical numbers $x =$ __________
Increasing on the interval __________
Decreasing on the interval __________
Local maxima $x =$ __________
Local minima $x =$ __________

(b) Find where f is concave up, concave down, and has inflection points.
Concave up on the interval __________
Concave down on the interval __________
Inflection points $x =$ __________

(c) Find any horizontal and vertical asymptotes of f.
Horizontal asymptotes $y =$ __________
Vertical asymptotes $x =$ __________

(d) The function f is $\boxed{?}$ because $\boxed{?}$ for all x in the domain of f, and therefore its graph is symmetric about the $\boxed{?}$

(e) Sketch a graph of the function f without having a graphing calculator do it for you. Plot the y-intercept and the x-intercepts, if they are known. Draw dashed lines for horizontal and vertical asymptotes. Plot the points where f has local maxima, local minima, and inflection points. Use what you know from parts (a) and (b) to sketch the remaining parts of the graph of f. Use any symmetry from part (d) to your advantage. Sketching graphs is an important skill that takes practice, and you may be asked to do it on quizzes or exams.

Correct Answers:

- `0.606530659712633`
- `(0.606530659712633,infinity)`
- `(0,0.606530659712633)`
- `NONE`
- `0.606530659712633`
- `(0.22313016014843,infinity)`
- `(0,0.22313016014843)`
- `0.22313016014843`
- `NONE`
- `NONE`
- `neither`
- `not applicable`
- `not applicable`

---

125. (1 point) Suppose that

$$f(x) = x^{1/3}(x+3)^{2/3}$$

(A) Find all critical values of f. If there are no critical values, enter **None** . If there are more than one, enter them separated by commas.
Critical value(s) = ______

(B) Use interval notation to indicate where $f(x)$ is increasing.

Note: When using interval notation in WeBWorK, you use **I** for ∞, **-I** for $-\infty$, and **U** for the union symbol. If there are no values that satisfy the required condition, then enter "" without the quotation marks.

Increasing:

______

(C) Use interval notation to indicate where $f(x)$ is decreasing.
Decreasing:

______

(D) Find the x-coordinates of all local maxima of f. If there are no local maxima, enter **None** . If there are more than one, enter them separated by commas.

Local maxima at x = ______

(E) Find the x-coordinates of all local minima of f. If there are no local minima, enter **None** . If there are more than one, enter them separated by commas.

Local minima at x = ______

(F) Use interval notation to indicate where $f(x)$ is concave up.
Concave up:

______

(G) Use interval notation to indicate where $f(x)$ is concave down.
Concave down:

______

(H) Find all inflection points of f. If there are no inflection points, enter **None** . If there are more than one, enter them separated by commas.
Inflection point(s) at x = ______

(I) Use all of the preceding information to sketch a graph of f. When you're finished, enter a **1** in the box below.

Graph Complete: ______

Correct Answers:

- `0, -3, -1`
- `(-infinity,-3) U (-1,infinity)`
- `(-3,-1)`
- `-3`
- `-1`
- `(-infinity,-3) U (-3,0)`
- `(0,infinity)`
- `0`
- `1`

---

126. (1 point) Suppose that

$$f(x) = \ln(3+x^2)$$

(A) Find all critical values of f. If there are no critical values, enter **None** . If there are more than one, enter them separated by commas.
Critical value(s) = ______

(B) Use interval notation to indicate where $f(x)$ is increasing.

Note: When using interval notation in WeBWorK, you use **I** for ∞, **-I** for $-\infty$, and **U** for the union symbol. If there are no values that satisfy the required condition, then enter "" without the quotation marks.

Increasing:

______

(C) Use interval notation to indicate where $f(x)$ is decreasing.
Decreasing:

______

(D) Find the x-coordinates of all local maxima of f. If there are no local maxima, enter **None** . If there are more than one, enter them separated by commas.

Local maxima at x = ______

(E) Find the x-coordinates of all local minima of f. If there are no local minima, enter **None** . If there are more than one, enter them separated by commas.

Local minima at x = ______

(F) Use interval notation to indicate where $f(x)$ is concave up.

Concave up:

______

(G) Use interval notation to indicate where $f(x)$ is concave down.
Concave down:

______

(H) Find all inflection points of f. If there are no inflection points, enter **None** . If there are more than one, enter them separated by commas.
Inflection point(s) at x = ______

(I) Use all of the preceding information to sketch a graph of f. When you're finished, enter a **1** in the box below.

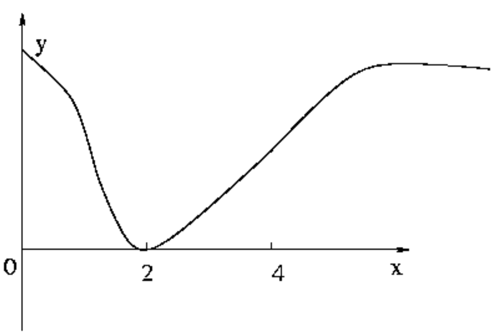
Graph Complete: ______

Correct Answers:

- `0`
- `(0,infinity)`
- `(-infinity,0)`
- `None`
- `0`
- `(-1.73205080756888,1.73205080756888)`
- `(-infinity,-1.73205080756888) U (1.73205080756888,infinity)`
- `-1.73205080756888, 1.73205080756888`
- `1`

---

127. (1 point)



For the function f given above, determine whether the following conditions are true. Input T if the condition is ture, otherwise input F .

(a) $f'(x) < 0$ if $0 < x < 2$; ______
(b) $f'(x) > 0$ if $x > 2$; ______
(c) $f''(x) < 0$ if $0 \leq x < 1$; ______
(d) $f''(x) > 0$ if $1 < x < 4$. ______
(e) $f''(x) < 0$ if $x > 4$; ______
(f) Two inflection points of $f(x)$ are, the smaller one is $x =$ ______ and the other is $x =$ ______

Correct Answers:

- `T`
- `F`
- `T`
- `T`
- `T`
- `1`
- `4`

---

128. (1 point)
Find the limit. Use l'Hospital's Rule if appropriate. Use INF to represent positive infinity, NINF for negative infinity, and D for the limit does not exist. Assume $b \neq 0$.

$\lim_{x \to 1} \frac{x^a - 1}{x^b - 1} =$ ______

Correct Answers:

- `a/b`

---

129. (1 point) For the given cost function $C(x) = 54\sqrt{x} + \frac{x^2}{421875}$ find
a) The cost at the production level 1800 ______
b) The average cost at the production level 1800 ______
c) The marginal cost at the production level 1800 ______
d) The production level that will minimize the average cost. ______
e) The minimal average cost. ______

Solution:
Solution:
Part (a)
To calculate the cost at the production level 1800, we need only plug 1800 into the cost function $C(x)$ to calculate $C(1800) = 54\sqrt{1800} + \frac{(1800)^2}{421875} = 2298.7060$.

Part (b)
The average cost at the production level 1800 is the average cost of producing each unit. In part (a) we saw that the total cost was 2298.7060, and we know that we are producing 1800 units. Hence, the average cost per unit is $\frac{2298.7060}{1800} = 1.2771$

Part (c)
The marginal cost function is the derivative of the cost function. Hence, we differentiate the cost function $C(x)$ and get $C'(x) = \frac{54}{2\sqrt{x}} + \frac{2x}{421875}$. Finally, to to determine the marginal cost at production level 1800, we plug 1800 into $C'(x)$ and get $C'(1800) = \frac{54}{2\sqrt{1800}} + \frac{2(1800)}{421875} = 0.6449$.

Part (d)
From part (b), the average cost function is $A(x) = \frac{C(x)}{x}$ where x is the production level. To minimize this, we find the derivative and solve for zero. Simplifying, we have $A(x) = \frac{54}{\sqrt{x}} + \frac{x}{421875}$. Hence, the derivative is $A'(x) = -\frac{(54)}{2\sqrt{x^3}} + \frac{1}{421875}$. Solving for

zero, we get:

$$A'(x) = 0$$

$$-\frac{(54)}{2\sqrt{x^3}} + \frac{1}{421875} = 0$$

$$\frac{(54)}{2\sqrt{x^3}} = \frac{1}{421875}$$

$$2\sqrt{x^3} = (54)(421875)$$

$$x^{3/2} = \frac{(54)(421875)}{2}$$

$$x = \left(\frac{(54)(421875)}{2}\right)^{2/3} = 50625.0000$$

Part (e)
Finally, the minimal average cost is found by plugging the value for x which we just found into the average cost function $A(x)$. This yields $A(50625.0000) = \frac{54}{\sqrt{50625.0000}} + \frac{50625.0000}{421875} = 0.3600$.

Correct Answers:

- `2298.70597104441`
- `1.27705887280245`
- `0.644929436401226`
- `50625`
- `0.36`

---

130. (1 point) A manufacture has been selling 1200 television sets a week at \$360 each. A market survey indicates that for each \$11 rebate offered to a buyer, the number of sets sold will increase by 110 per week.

a) Find the function representing the demand $p(x)$, where x is the number of the television sets sold per week and $p(x)$ is the corresponding price.
$p(x) =$ ______

b) How large rebate should the company offer to a buyer, in order to maximize its revenue? ______ dollars

c) If the weekly cost function is $72000 + 120x$, how should it set the size of the rebate to maximize its profit? ______ dollars

Correct Answers:

- `(1200-x)/10 +360`
- `120`
- `60`

---

131. (1 point) A fence 5 feet tall runs parallel to a tall building at a distance of 7 feet from the building. We want to find the the length of the shortest ladder that will reach from the ground over the fence to the wall of the building.

Here are some hints for finding a solution:
Use the angle that the ladder makes with the ground to define the position of the ladder and draw a picture of the ladder leaning against the wall of the building and just touching the top of the fence.

If the ladder makes an angle 1.03 radians with the ground, touches the top of the fence and just reaches the wall, calculate the distance along the ladder from the ground to the top of the fence.

______

The distance along the ladder from the top of the fence to the wall is

______

Using these hints write a function $L(x)$ which gives the total length of a ladder which touches the ground at an angle x, touches the top of the fence and just reaches the wall.
$L(x) =$ ______.

Use this function to find the length of the shortest ladder which will clear the fence.

The length of the shortest ladder is ______ feet.

Correct Answers:

- `5.83227096153733`
- `13.5970158598381`
- `5/sin(x)+7/cos(x)`
- `16.8914833600749`

---

132. (1 point) Let $Q = (0,2)$ and $R = (7,6)$ be given points in the plane. We want to find the point $P = (x,0)$ on the x-axis such that the sum of distances $PQ + PR$ is as small as possible. (Before proceeding with this problem, draw a picture!)
To solve this problem, we need to minimize the following function of x:
$f(x) =$ ______
over the closed interval $[a,b]$ where $a =$ ____ and $b =$ ____.
We find that $f(x)$ has only one critical number in the interval at $x =$ ______
where $f(x)$ has value ______
Since this is smaller than the values of $f(x)$ at the two endpoints, we conclude that this is the minimal sum of distances.

Correct Answers:

- `sqrt(x**2+2**2)+sqrt((7-x)**2+6**2)`
- `0`
- `7`
- `1.75`
- `10.6301458127346`

133. (1 point) Centerville is the headquarters of Greedy Cablevision Inc. The cable company is about to expand service to two nearby towns, Springfield and Shelbyville. There needs to be cable connecting Centerville to both towns. The idea is to save on the cost of cable by arranging the cable in a Y-shaped configuation. Centerville is located at $(7,0)$ in the xy-plane, Springfield is at $(0,5)$, and Shelbyville is at $(0,-5)$. The cable runs from Centerville to some point $(x,0)$ on the x-axis where it splits into two branches going to Springfield and Shelbyville. Find the location $(x,0)$ that will minimize the amount of cable between the 3 towns and compute the amount of cable needed. Justify your answer.

To solve this problem we need to minimize the following function of x:
$f(x) =$____________________
We find that $f(x)$ has a critical number at $x =$_________
To verify that $f(x)$ has a minimum at this critical number we compute the second derivative $f''(x)$ and find that its value at the critical number is _________, a positive number.
Thus the minimum length of cable needed is _________

Correct Answers:

- `2*sqrt(x**2+5**2)+7 -x`
- `2.88675134594813`
- `0.259807621135332`
- `15.6602540378444`

---

134. (1 point) A Norman window has the shape of a semicircle atop a rectangle so that the diameter of the semicircle is equal to the width of the rectangle. What is the area of the largest possible Norman window with a perimeter of 48 feet? ______________

Solution:
Solution:

To solve this maximization problem, we must find a formula for the area of the window in terms of one of its dimensions. Let h be the height of the rectangular portion of the window, and let x be half of the width of the rectangular portion (which is then the radius of the semicircular top). Then the area is given by $2xh + \frac{\pi x^2}{2}$ (since we have half of a circle on top of the rectangle).

Now, we need to find an expression for the height of the rectangular portion of the window in terms of the width, $2x$. This is where we use the information that the entire window should have a perimeter of 48 feet. The perimeter of the window is given by $2x+2h+\pi x$, again since we are only using half of a circle. Setting this equation equal to 48 and solving for h, we get:

$$2x+2h+\pi x = 48$$
$$2h = 48-2x-\pi x$$
$$h = \frac{48-2x-\pi x}{2}$$

Plugging this into our expression, we get a formula for the area of the window in terms of the width of the rectangular portion. That is:

$$a(x) = 2x\left(\frac{48-2x-\pi x}{2}\right) + \frac{\pi x^2}{2} = 48x - 2x^2 - \frac{\pi}{2}x^2$$

Taking the derivative of this and solving for 0, we get:

$$a'(x) = 45-4x-\pi x$$
$$a'(x) = 0$$
$$45-4x-\pi x = 0$$
$$4x+\pi x = 45$$
$$x(4+\pi) = 45$$
$$x = \frac{45}{4+\pi} \approx 6.3011$$

Checking will show that this is in fact a local maximum, so that the maximum area of the window can be found by plugging this value of x into the formula for area. That is:

$$a(6.3011) \approx 161.31$$

Correct Answers:

- `161.309`

---

135. (1 point) How would you divide a 19 inch line into two parts of length A and B so that $A+B = 19$ and the product AB is maximized? (Assume that $A \leq B$.
A = _____

B = _____

Correct Answers:

- `9.5`
- `9.5`

---

136. (1 point) Find two positive numbers A and B (with $A \leq B$) whose sum is 44 and whose product is maximized.
A = _____

B = _____

Correct Answers:

- `22`
- `22`

---

137. (1 point) Find the length L and width W (with $W \leq L$) of the rectangle with perimeter 60 that has maximum area, and then find the maximum area.
L = _____

W = _____

Maximum area = _____

Correct Answers:

- `15`
- `15`

- 225

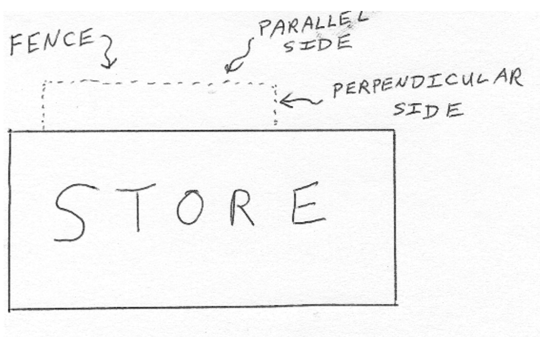
138. (1 point) An open box is to be made out of a 8-inch by 20-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. Find the dimensions of the resulting box that has the largest volume.

Dimensions of the bottom of the box: ____ x ____
Height of the box: ____

Correct Answers:

- `16.4785 x 4.47853`
- `1.76073`

139. (1 point) The owner of a used tire store wants to construct a fence to enclose a rectangular outdoor storage area adjacent to the store, using part of the side of the store (which is 230 feet long) for part of one of the sides. (See the figure below.) There are 450 feet of fencing available to complete the job. Find the length of the sides parallel to the store and perpendicular that will maximize the total area of the outdoor enclosure.



Note: you can click on the image to get a enlarged view.

Length of parallel side(s) = ________

Length of perpendicular sides = ________

Correct Answers:

- `225`
- `112.5`

140. (1 point) A small resort is situated on an island that lies exactly 5 miles from P, the nearest point to the island along a perfectly straight shoreline. 10 miles down the shoreline from P is the closest source of fresh water. If it costs 2.1 times as much money to lay pipe in the water as it does on land, how far down the shoreline from P should the pipe from the island reach land in order to minimize the total construction costs?

Distance from P = ____

Correct Answers:

- `2.70765180536941`

141. (1 point) Find two numbers differing by 40 whose product is as small as possible.

Enter your two numbers as a comma separated list, e.g. 2, 3.
The two numbers are ________.

Correct Answers:

- `-20, 20`

142. (1 point) Find the dimensions of the rectangle with area 361 square inches that has minimum perimeter, and then find the minimum perimeter.

1. Dimensions: ____
2. Minimum perimeter: ____

Enter your result for the dimensions as a comma separated list of two numbers. Do not include the units.

Correct Answers:

- `19, 19`
- `76`

143. (1 point) An open box is to be made out of a 6-inch by 14-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. Find the dimensions of the resulting box that has the largest volume.

Dimensions of the bottom of the box: ____ x ____
Height of the box: ____

Correct Answers:

- `11.3885 x 3.38851`
- `1.30575`

144. (1 point) A parcel delivery service will deliver a package only if the length plus the girth (distance around, taken perpendicular to the length) does not exceed 104 inches. Find the maximum volume of a rectangular box with square ends that satisfies the delivery company's requirements.

Maximum Volume = ________ in^3.

Correct Answers:

- `10415.4`

145. (1 point) A fence is to be built to enclose a rectangular area of 280 square feet. The fence along three sides is to be made of material that costs 3 dollars per foot, and the material for the fourth side costs 14 dollars per foot. Find the dimensions of the enclosure that is most economical to construct.

Dimensions: ____ x ____

Correct Answers:

- `28.1662 x 9.941`

146. (1 point) A rectangle has its two lower corners on the x-axis and its two upper corners on the parabola $y = 8 - x^2$.

What are the dimensions of such a rectangle with the greatest possible area?

1. Width = ________
2. Height = ________

Correct Answers:

- `3.26599`
- `5.33333`

147. (1 point) A rancher wants to fence in an area of 500000 square feet in a rectangular field and then divide it in half with a fence down the middle, parallel to one side.

What is the shortest length of fence that the rancher can use?

Length of fence = ________ feet.

Correct Answers:

- 3464.1

148. (1 point) A Union student decided to depart from Earth after graduation to find work on Mars. Careful calculations made regarding the space shuttle to be built used the following mathematical model for the velocity (in ft/sec) of the shuttle from liftoff at $t = 0$ seconds until the solid rocket boosters are jettisoned at $t = 62.6$ seconds:

$$v(t) = 0.00125483t^3 - 0.08905t^2 + 30.21t + 3.25$$

Using this model, consider the acceleration of the shuttle. Find the absolute maximum and minimum values of acceleration between liftoff and the jettisoning of the boosters.

1. Absolute maximum of acceleration = ________
2. Absolute minimum of acceleration = ________

Correct Answers:

- 33.8131
- 28.1035

149. (1 point) The linear approximation at $x = 0$ to $\sqrt{4+3x}$ is $A + Bx$

where $A =$ ________

and where $B =$ ________

Correct Answers:

- 2
- 0.75

150. (1 point) Let $f(x) = 4x^3 - 7$. Find the open intervals on which f is increasing (decreasing). Then determine the x-coordinates of all relative maxima (minima).

1. f is increasing on the intervals ________
2. f is decreasing on the intervals ________
3. The relative maxima of f occur at x = ________
4. The relative minima of f occur at x = ________

Notes: In the first two, your answer should either be a single interval, such as (0,1), a comma separated list of intervals, such as (-inf, 2), (3,4), or the word “none”.

In the last two, your answer should be a comma separated list of x values or the word “none”.

Correct Answers:

- (-infinity,infinity)
- NONE
- NONE
- NONE

151. (1 point) Suppose that

$$f(x) = \frac{5x}{x^2 - 25}.$$

(A) List all critical numbers of f. If there are no critical numbers, enter ’NONE’.

Critical numbers = ______

(B) Use interval notation to indicate where $f(x)$ is decreasing.

Note: Use ’INF’ for ∞, ’-INF’ for $-\infty$, and use ’U’ for the union symbol.

Decreasing: ________

(C)List the x-values of all local maxima of f. If there are no local maxima, enter ’NONE’.

x values of local maxima = ______

(D) List the x-values of all local minima of f. If there are no local minima, enter ’NONE’.

x values of local minima = ______

(E) List the x values of all inflection points of f. If there are no inflection points, enter ’NONE’.

Inflection points = ______

(F) Use interval notation to indicate where $f(x)$ is concave up.

Concave up: ________

(G) Use interval notation to indicate where $f(x)$ is concave down.

Concave down: ________

(H) List all horizontal asymptotes of f. If there are no horizontal asymptotes, enter ’NONE’.

Horizontal asymptotes $y =$ ______

(I) List all vertical asymptotes of f.

If there are no vertical asymptotes, enter ’NONE’.

vertical asymptotes $x =$ ______

(J) Use all of the preceding information to sketch a graph of f. When you’re finished, enter a ”1” in the box below.

Graph Complete: ______

Correct Answers:

- NONE
- (-infinity,-5) U (-5,5) U (5,infinity)
- NONE
- NONE
- 0
- (-5,0) U (5,infinity)
- (-infinity,-5) U (0,5)
- 0
- -5, 5
- 1