1. (1 point) Match each of the trigonometric expressions below with the equivalent non-trigonometric function from the following list. Enter the appropriate letter (A,B,C,D, or E) in each blank.

A. $\tan(\arcsin(x/5))$
B. $\cos(\arcsin(x/5))$
C. $(1/2)\sin(2\arcsin(x/5))$
D. $\sin(\arctan(x/5))$
E. $\cos(\arctan(x/5))$

___1. $\frac{x}{25}\sqrt{25-x^2}$
___2. $\frac{x}{\sqrt{25+x^2}}$
___3. $\frac{x}{\sqrt{25-x^2}}$
___4. $\frac{5}{\sqrt{25+x^2}}$
___5. $\frac{\sqrt{25-x^2}}{5}$

Correct Answers:

- C
- D
- A
- E
- B

2. (1 point) Let $f(x) = \tan^{-1}\left(\sqrt{6x^2-1}\right)$. Find $f'(x)$.
$f'(x) =$ ______________

Correct Answers:

- `1/(1+[sqrt(6*x^2-1)]^2)*1/[2*sqrt(6*x^2-1)]*6*2*x`

5. (1 point) A street light is at the top of a 14.0 ft. tall pole. A man 5.9 ft tall walks away from the pole with a speed of 4.5 feet/sec along a straight path. How fast is the tip of his shadow moving when he is 35 feet from the pole?
Your answer: ________
Hint: Draw a picture and use similar triangles.
Solution:
Solution: Let h be the height of the man, H the height of the street light, d the (horizontal) distance of the man from the light, and s the length of the man's shadow. By similar triangles we have

$$\frac{s}{h} = \frac{s+d}{H}.$$

Here, h and H are constant, and s and d are functions of time. Differentiating implicitly gives

$$\frac{s'}{h} = \frac{s'+d'}{H}.$$

Solving for s' gives

$$s' = \frac{d'h}{H-h}.$$

The tip of the shadow is moving at the combined speed v of the man and the speed at which the shadow is growing. Thus

$$v = \frac{d'h}{H-h} + d'.$$

Substituting

$$d' = 4.5, \quad h = 5.9, \quad \text{and} \quad H = 14$$

gives

$$v \approx 7.778 \text{ feet per second.}$$

Remarkably, the speed of the tip of the shadow is independent of the distance of the man from the lamp post. The information that $d = 35$ is a red herring.

Correct Answers:

- `7.77777777777778`

6. (1 point) You are blowing air into a spherical balloon at a rate of $\frac{4\pi}{3}$ cubic inches per second. (The reason for this strange looking rate is that it will simplify your algebra a little.) Assume the radius of your balloon is zero at time zero. Let $r(t)$, $A(t)$, and $V(t)$ denote the radius, the surface area, and the volume of your balloon at time t, respectively. (Assume the thickness of the skin is zero.) All of your answers below are expressions in t:
$r'(t) =$ ______ inches per second,
$A'(t) =$ ______ square inches per second, and
$V'(t) =$ ______ cubic inches per second.
Hint: The surface area A and the volume V of a sphere of radius r are given by

$$A = 4\pi r^2 \quad \text{and} \quad V = \frac{4\pi r^3}{3}.$$

Solution:
Solution: There are different ways to do this problem. Probably the easiest is to take the volume formula, solve for r as a function of t, and then go from there.

We have

$$V = \frac{4\pi}{3}t = \frac{4\pi r^3}{3}.$$

Thus

$$r^3 = t,$$

i.e.,

$$r(t) = t^{1/3}$$

and

$$r'(t) = \frac{1}{3}t^{-2/3}.$$

For the area we obtain:

$$A(t) = 4\pi r^2(t)$$

and hence

$$A'(t) = 8\pi r(t)r'(t) = 8\pi \times t^{1/3} \times \frac{1}{3}t^{-2/3} = \frac{8\pi}{3}t^{-1/3}.$$

Correct Answers:

- `t^(-2/3)/3`
- `8*3.14159265358979/3*t^(-1/3)`
- `4.18879020478639`

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7. (1 point) A spherical snowball is melting in such a way that its diameter is decreasing at rate of 0.1 cm/min. At what rate is the volume of the snowball decreasing when the diameter is 14 cm.

Your answer ________ (cubic centimeters per minute) should be a positive number.

Hint: The volume of a sphere of radius r is

$$V = \frac{4\pi r^3}{3}.$$

The diameter is twice the radius.

Solution:

Solution: The volume of our sphere is

$$V = \frac{4\pi r^3}{3} = \frac{4\pi d^3}{24} = \frac{\pi d^3}{6}.$$

Differentiating gives:

$$V' = \frac{3\pi d^2 d'}{6} = \frac{\pi d^2 d'}{2}.$$

Substituting

$$d = 14 \quad \text{and} \quad d' = 0.1$$

gives the answer

$$V' = 30.78760797.$$

Correct Answers:

- `30.78760797`

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8. (1 point) Water is leaking out of an inverted conical tank at a rate of 11600 cubic centimeters per minute at the same time that water is being pumped into the tank at a constant rate. The tank has height 14 meters and the diameter at the top is 4.5 meters. If the water level is rising at a rate of 21 centimeters per minute when the height of the water is 2.5 meters, find the rate at which water is being pumped into the tank in cubic centimeters per minute.

Your answer: ____________ cubic centimeters per minute.

Solution:

Solution: The volume V of a cone of diameter d and height h is

$$V = \frac{\pi d^2 h}{12}.$$

We might as well do this problem a little more generally than stated. (It actually makes things clearer to use variables throughout and substitute specific values only at the end.) So suppose the diameter of our tank is D and its height is H. Suppose the depth of the water is h and the radius of the surface is r. Then, by similar triangles,

$$\frac{r}{h} = \frac{R}{H},$$

or

$$r = \frac{Rh}{H}.$$

Substituting in the volume formula gives

$$V = \frac{\pi D^2 h^3}{12H^2}.$$

Differentiating gives:

$$V' = \frac{3\pi D^2 h^2 h'}{12H^2} = \frac{\pi D^2 h^2 h'}{4H^2}$$

This rate of change in volume would be the rate T at which water is being pumped into the tank if there was no leak. However, if L is the leak rate then

$$T = L + \frac{\pi D^2 h^2 h'}{4H^2}.$$

Note that since we have to have consistent t units, and the answer is in cubic centimeters per minute, the lengths given in meters have to be converted to centimeters.

Thus, substituting

$$L = 11600, \quad D = 450.0, \quad H = 1400, \quad h = 250, \quad h' = 21$$

we obtain

$$T \approx 118102 \text{ cubic centimeters per minute.}$$

Correct Answers:

- `118102.092885045`

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9. (1 point) A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 2.3 ft/s.

(a) How rapidly is the area enclosed by the ripple increasing when the radius is 4 feet?

The area is increasing at ________ ft^2/s.

(b) How rapidly is the area enclosed by the ripple increasing at the end of 9.6 seconds?

The area is increasing at ________ ft^2/s.

Correct Answers:

- `57.8053`
- `319.085`

10. (1 point) At noon, ship A is 30 nautical miles due west of ship B. Ship A is sailing west at 15 knots and ship B is sailing north at 23 knots. How fast (in knots) is the distance between the ships changing at 4 PM? (Note: 1 knot is a speed of 1 nautical mile per hour.)

Note: Draw yourself a diagram which shows where the ships are at noon and where they are "some time" later on. You will need to use geometry to work out a formula which tells you how far apart the ships are at time t, and you will need to use "distance = velocity * time" to work out how far the ships have travelled after time t.

Correct Answers:

- `26.9305949204144`