COMP233 PROBABILITY AND STATISTICS

TERM TEST February 29, 2016

NAME:	ID:	Total Points:

INSTRUCTIONS

You may only use a Concordia approved calculator.

Other electronic equipment, books, and notes are not allowed.

This exam has 11 Pages, with 10 Problems. Check that your copy is complete !

Selected formulas are listed on Page 11.

Except for the formula page, do not attach or detach any pages !

Use a *dark black* or *dark blue* pen. Writing in pencil will be ignored.

Do your calculations in the open space and on the back of the pages.

Where provided, put your final answer in the appropriate box.

Numerical answers must be correct to two significant decimal digits.

All questions have equal value, namely 8 points.

Table for Instructor's use

Problem	Points	Problem	Points
1		6	
2		7	
3		8	
4		9	
5		10	

Problem 1.

Two balls are selected at random from a bag with three white balls and one black ball. For each of the following enter the probability in the accompanying box.

- The probability the first ball is white.

- The probability the second ball is white.

- The probability the second ball is white, given that the first ball is white.

- The probability both balls are white.

Problem 2.

- An insurance company has these data: The probability of an insurance claim in a period of one year is 3% for persons under age 30 and 2% for persons over age 30. It is also known that 35% of the targeted population is under age 30. What is the probability of an insurance claim in a period of one year for a randomly chosen person from the targeted population? Enter your answer in the box below.

- Suppose 1 in 1000 persons has a certain disease. A test detects the disease in 99.5 % of diseased persons. The test also "detects" the disease in 0.1 % of healthly persons. With what probability does a positive test diagnose the disease? Enter your answer in the box below.

Problem 3. Given the joint probability mass function in the Table below on the left:

Probability mass function $p_{X,Y}(x,y)$

	$Y=0$	$Y=1$	$Y=2$	$p_X(\cdot)$
$X=0$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	
$X=1$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	
$X=2$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{12}$	
$p_Y(\cdot)$				

Probability distribution function $F_{X,Y}(x,y)$

	$Y \le 0$	$Y \le 1$	$Y \le 2$	$F_X(\cdot)$
$X \le 0$				
$X \le 1$				
$X \le 2$				
$F_Y(\cdot)$				

- Fill in the marginal probability mass function values.
- Fill in the probability distribution Table.

Problem 4. Given the Tables in Problem 3, determine the following:

- Are X and Y independent?

- $P(X \leq 1)$

- $P(X \leq 1\ ,\ 0 < Y \leq 2)$

- $P(X = 2 \mid Y = 2)$

Problem 5. Given the Tables in Problem 3, determine the following:

- $E[X]$ and $E[Y]$

- $Var[X]$

- $E[XY]$

- $cov(X, Y)$

Problem 6. A die is rolled five times. A roll is considered a "success" if the die lands with a six up. An example of an outcome is then `11010` , where "1" denotes "success" and "0" "failure".

- What is the probability of the outcome `11010` ?

- What is the probability of exactly three successes?

- What is the probability of no successes?

- What is the probability of five or more successes?

Problem 7. In each of the four parts of this problem your answer may be given in the form of a *numerical expression* , rather than the final numerical probability:

Suppose that customers arrive at a counter at the rate of 6 per hour.

Assuming that the arrivals have a Poisson distribution, what is the probability that :

- six customers arrive in an hour?

- At least one customer arrives in an hour?

Also give the above probabilities using the binomial random variable, after by dividing the hour into 12 intervals of five minutes :

- six customers arrive in an hour?

- At least one customer arrives in an hour?

Problem 8. For the random variable X with probability density function

$$f(x) = \begin{cases} x+1\,, & -1 < x \leq 0 \\ 1-x\,, & 0 < x \leq 1 \\ 0\,, & \text{otherwise} \end{cases}$$

- Draw an accurate graph of $f(x)$.

- Show that $\int_{-1}^{1} f(x)\,dx = 1$.

- What is the value of $E[X]$?

- Determine the distribution function $F(x)$.

Problem 9. For the random variable X in Problem 8 :

- Draw the graph of $F(x)$.

- What is $P(X \leq \frac{1}{2})$?

- Compute $E[X^2]$.

- Compute the standard deviation $\sigma(X)$.

Problem 10.

Use the method of moments to compute the mean and the variance of the exponential random variable X with density function

$$f(x) = e^{-x}, \text{ for } x \geq 0 .$$

Show all details of your work.

BASIC FORMULAS

Name	Formula
Bayes	$P(F\|E) = P(E\|F)P(F) \ / \ [\ P(E\|F)P(F) + P(E\|F^c)P(F^c)\ ]$
Method of Moments	If $\psi(t) = E[e^{tX}]$ then $\psi'(0) = E[X]$ and $\psi''(0) = E[X^2]$
Markov's Inequality	For continuous, nonnegative X, $c > 0$: $P(\ X \geq c\ ) \ \leq \ E[X] \ / \ c$
Chebyshev's Inequality	$P(\ \|X - \mu\| \geq k\sigma\ ) \ \leq \ 1 \ / \ k^2$

Name	Probability Mass Function	Domain	Mean	Std. Dev.
Bernoulli	$P(X = 1) = p$, $P(X = 0) = 1 - p$	$X = 0$, 1	p	$\sqrt{p(1-p)}$
Binomial	$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$	$0 \leq k \leq n$	np	$\sqrt{np(1-p)}$
Poisson	$P(X = k) = e^{-\lambda}\lambda^k / k!$	$k = 0, 1, 2, \cdots$	λ	$\sqrt{\lambda}$