ENGR 371 FINAL 29 April, 2011

Probability and Statistics in Engineering

(1)

QUESTIONS

1. The world health organization wants to investigate the weights of children. A random sample of size 16 is collected and the weights are 20.5, 20, 20.8, 21, 19, 18, 23, 21.5, 22.5, 18.5, 19, 19.5, 25, 23, 21, and 24 kg. Assume that the population distribution is normal.
   (a) Test the hypothesis on the mean: H_0: μ =20 kg and H_1: μ >20 kg. Use α=0.05. **(10 marks)**
   (b) Test the hypothesis on the variance: H_0: σ^2 =2 kg^2, and H_1: $\sigma^2 \neq 2$ kg^2. Use α=0.05. **(10 marks)**

2. Final scores for ENGR 371 students follow a normal distribution with a standard deviation of 10. A random sample of 20 students is selected and it is found that the sample mean of the final scores is 65. Find the 95% confidence interval for the final scores of the population mean of ENGR371 grades. **(10 marks)**

3. The joint probability function of random variables X and Y is given by
   $f(x,y) = ke^{-2x-3y}$, 0<x<y.
   (a) Find k **(5 marks)**
   (b) Find P(1<X<2) **(5 marks)**
   (c) Find the mean of $Z = 2X + 3Y + 1$ **(5 marks)**
   (d) Are the random variables X and Y independent? Justify your answer **(5 marks)**

4. A product is made by three machines, and machine #1, #2 and #3 produce 50%, 25% and 25% of the product, respectively. Based on the previous knowledge, it is known that 2%, 1% and 3% of the products made by machine #1, #2 and #3, respectively are defective.
   (a) If a finished product is randomly selected, what is the probability that the selected product is defective? **(10 marks)**
   (b) If a finished product is randomly selected and is found to be defective, which machine is most likely to produce this defective product? **(10 marks)**

5. In hockey games, Concordia has a probability of 0.55 of beating McGill in each game that they play. Assume that the outcomes of games are independent of one another
   (a) Determine the probability that it takes Concordia 11 games until it has beaten McGill 6 times. **(5 marks)**
   (b) McGill and Concordia both reach the championships. There are 7 games and the team that wins 4 or more wins. What is the probability that Concordia wins the championship? Hint: be careful. **(5 marks)**

6. A random sample of size n_1=16 is selected from a normal population with a population mean of 75 and a population standard deviation of 8. A second random sample of size n_2=9 is taken from another normal population with population mean of 70 and population standard deviation of 12. Assume that individual samples from the different populations are independent of each other. Let $\bar{X}_1$ and $\bar{X}_2$ be the two sample means. Find the probability of $3.5 \leq \bar{X}_1 - \bar{X}_2 \leq 5.5$. **(5 marks)**

7. We measure the resistance of 2500 resistors that are each nominally 100 ohms. These come from a very large lot of resistors. Assume that these 2500 resistors are selected so that they are a random sample. We find that the sample mean is 99.97 ohms and the sample standard deviation is 1.1 ohms. Calculate a 95% confidence interval on the population mean of 100 ohm resistors from this lot. (**10 marks**)

8. Vehicles enter a certain on ramp to the highway according to a Poisson Process with λ=17 vehicles/hour. Determine the probability that there are less than 4 minutes between two consecutive vehicles. (**5 marks**)