ENGR371 Probability and Statistics for Engineering

Note: start a new page for each question

1. It was reported that the percent protein from peanut has a Normal distribution. Now 16 samples are randomly selected and the percentages are 78.3, 77.1, 71.3, 84.5, 87.8, 75.7 64.8 72.5 78.2 91.2 86.2 80.9 82.1 89.3 89.4 81.6.
   (a) Find the sample mean and standard deviation (5 marks)
   (b) Find a 98% lower one-sided confidence interval on mean percentage (5 marks)
   (c) Find a 95% two-sided confidence interval on mean percentage. Explain why the lower end-point of the interval is or is not the same as in (a) (5 marks)
   (d) Find a 95% two-sided confidence interval on standard deviation of percentage. (6 marks)
   (e) Is there sufficient evidence to support a claim that the mean percentage is not equal to 80? Use a level of significance 0.05 (6 marks)
   (f) Find a 95% prediction interval for the percentage of a new peanut protein. (6 marks)

2. A random sample of size 16 is selected from a normal population with mean of 75 and standard deviation of 8. A second random sample of size 9 is taken from another normal population with mean of 70 and standard deviation of 12. Let $\overline{X_1}$ and $\bar{X}_2$ are the two sample means. Find the probability that $\overline{X_1} - \bar{X}_2$ exceeds 4. (8 marks).

3. (a) An assembly line has just received a lot with 10,000 items. Assume that 99% of the items are conforming. What sample size is needed so that the probability of choosing at least one nonconforming item in the sample is at least 0.9? (6 marks)
   (b) Assume E(X) = 5, V(X) = 2, E(Y) = 10 and V(Y) = 1. Assume X and Y are independent. Calculate the following (10 marks)
   (i) E(X+2Y), (ii) Var(X+2Y), (iii) E(X+X), (iv) Var(X+X), (v) E(XY)

4. It is known that two defective copies of a commercial software program were erroneously sent to a shipping lot that now has a total of 75 copies of the program. A sample of copies will be selected from the lot without replacement. (5 marks for each)
   (a) If three copies of the software are inspected, determine the probability that exactly one of the defective copies will be found.
   (b) If three copies of the software are inspected, determine the probability that both defective copies will be found.
   (c) If 73 copies are inspected, determine the probability that both copies will be found

5. The joint density for the random variables (X,Y), where X is the temperature change and Y is the proportion of the spectrum that shifts in a certain particle, is

$$f(x,y) = \begin{cases} 10xy^2, & 0 < x < y < 1 \\ 0, & elsewhere \end{cases}$$

   a) Find the marginal densities functions $g(x), h(y)$, and the conditional density $f(y|x)$. *(8 marks)*
   b) Find the probability that the spectrum shifts more than half of the total observations, given that the temperature is increased by 0.25 units. (5 marks)

6. The time until recharge for a battery in a laptop computer under common conditions is normally distributed with a mean of 260 minutes and a standard deviation of 50 minutes. (5 marks for each)
   a) What is the probability that a battery lasts more than four hours?
   b) What are the quartiles (the 25% and 75% values) of battery life?
   c) What value of life in minutes is exceeded with 95% probability?