CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	203	All
Examination	**Date**	**Pages**
Final	December 2014	3
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Special Instructions:	Only approved calculators are allowed **Show all your work for full marks.**	

MARKS

[11] **1.** (a) Solve for x : $\ln(4x^2) + 2\ln(x) = 2\ln(6x)$.

(b) Sketch the graph of the function $f(x) = |(x-1)^2 - 4|$. (Suggestion: start from the graph of standard parabola, then use appropriate transformations.)

(c) Given the function $f(x) = \ln(1 + e^{2x})$, find the inverse function $f^{-1}(x)$, the range of $f(x)$ and the range of $f^{-1}(x)$.

[7] **2.** Find the limit if it exists (**Do not use l'Hôpital's rule.**) :

(a) $\displaystyle\lim_{x\to-3} \frac{|x+3|}{x^2+2x-3}$ (b) $\displaystyle\lim_{x\to\infty} \frac{2x(x^2+\sqrt{1+x^2+4x^4})}{1+x^2-3x^3}$

[6] **3.** Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{9 + 2\cdot 3^x}{3^x - 9}$$

[15] **4.** Find the derivatives of the following functions (you don't need to simplify your final answer, but you must show how you calculate it):

(a) $f(x) = \arctan x + (x^{3/2} + 2x^{-1/2})\sqrt{x}$

(b) $f(x) = \ln\dfrac{x^3}{x+3}$

(c) $f(x) = \dfrac{e^{-x}\tan x}{1+e^x}$

(d) $f(x) = \ln[e^{x\sin x} + x\sin(e^x)]$

(e) $f(x) = (1+\cos x)^{x^2}$ (use logarithmic differentiation)

[12] 5. (a) Use the definition of derivative as the limit of difference quotient to find dy/dx for $y = \sqrt{5 + x^2}$.

(b) Find the linearization $L(x)$ of the function $\tan(x)$ at $a = \pi/4$

(c) Use $L(x)$ found in (b) to approximate $\tan(x)$ at $x = \frac{\pi}{3}$ $(= \frac{\pi}{4} + \frac{\pi}{12})$.

[7] 6. Let $f(x) = x^3 - 3x^2 - x - 3$.

(a) Find the slope m of the secant line joining the points $(2, f(2))$ and $(0, f(0))$.

(b) Find all points $x = c$ (if any) on the interval [0,2] such that the rate $f'(c)$ of instantaneous change of $f(x)$ is equal to the slope m of the secant line in (a).

[17] 7. (a) Verify that the point (3,1) belongs to the curve defined by the equation $y^3 + x^3 - 2x^2y^2 = 10$, and find an equation of the tangent line to the curve at that point.

(b) The length of a rectangle is increasing at the rate of 8 cm/s and its width is increasing at the rate of 5 cm/s. When the length is 20 cm and the width is 12 cm, how fast is the area of the rectangle increasing at that instant?

(c) Use l'Hôpital's rule to evaluate the $\lim\limits_{x \to 0} \dfrac{e^{2x} + e^{-2x} - 2}{x \sin x}$.

[11] 8. (a) Find the point (x_0, y_0) on the line $y + 2x = 2$ that is closest to the point $(5, 2)$.

(b) A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola $y = 3 - x^2$. Find the dimensions of such rectangle with the maximum possible area.

[14] **9.** Given the function $f(x) = \ln(1 + x^2)$.

(a) Find the domain of $f(x)$, check for symmetry, and also find asymptotes (if any).

(b) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).

(c) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

(d) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question.** If $y = f(u)$ and $u = g(x)$, where f and g are twice differentiable functions, use the Chain rule to derive the following formula for the second derivative:

$$\frac{d^2y}{dx^2} = \frac{d^2f}{du^2}\left(\frac{dg}{dx}\right)^2 + \frac{df}{du}\frac{d^2g}{dx^2}$$