CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Section(s)**
Mathematics	209/1	All
Examination	**Date**	**Pages**
Final	December 2013	3
Instructors		**Course Examiner**
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Special Instructions

- ▷ Ruled booklets to be used.
- ▷ Only approved calculators allowed.

MARKS

[6] **1.** Find the following limits:

(a) $\lim\limits_{x\to -2} \dfrac{2x^2+9x+10}{x+2}$ (b) $\lim\limits_{x\to 0} \dfrac{\sqrt{49+x}-7}{x}$ (c) $\lim\limits_{x\to\infty} \dfrac{2x^3+3x^2-9}{-\frac{1}{3}x^3+5x+7}$

[18] **2.** Find the derivative for each of the following (do not simplify):

(a) $y = 8x^3 + 7x^6 + 12$

(b) $y = \frac{2}{3}x^{-5} - 5\sqrt{x} + 5$

(c) $y = (2x^2+3)(x^3+x-2)^3$

(d) $y = \dfrac{-2x^3+x^2}{x^2+x+1}$

(e) $y = e^{\ln(7x)}$

(f) $y = \left(\ln(7x^3-4x)\right)\left(e^{3x^2+15x}\right)$

(g) $y = \ln(x^3+6)^2 \cdot e^{3x^2}$

(h) Find y' and evaluate at $(1,1)$: $2y + x\ln(y) = 2x^3$

[10] **3.** A company manufactures automatic transmissions for automobiles. The total weekly cost (in dollars) of producing x transmissions is given by

$$C(x) = 50000 + 600x - 0.75x^2,$$

(a) Find the marginal cost function.

(b) Find the marginal cost at a production level of of 200 transmissions per week and interpret the results.

(c) Find the exact cost at producing the 201st transmission.

[18] **4.** For the function $f(x) = x^4 + 4x^3$ find:

[Please list the following neatly]

(a) the intervals where $f(x)$ is increasing;

(b) the intervals where $f(x)$ is decreasing;

(c) the intervals where $f(x)$ is concave up;

(d) the intervals where $f(x)$ is concave down;

(e) the local maximum;

(f) the local minimum;

(g) the inflection point(s);

(h) $\lim_{x \to +\infty} f(x)$;

(i) $\lim_{x \to -\infty} f(x)$;

(j) Using the above results, sketch the graph of $f(x)$.

[6] **5.** Find the absolute extrema of $f(x) = x^4 - 4x^3 + 5$ on the interval $[0, 4]$.

[6] **6.** Find the equation of the line tangent to the $y = -3e^{x^2} + 5$ where $x = 0$.

[6] **7.** Evaluate the following; answers must be accurate to 3 decimals:

(a) $\displaystyle\int_0^3 4x^2\,dx$

(b) $\displaystyle\int_1^2 (2x + 3e^x - \frac{4}{x})\,dx$

(c) $\displaystyle\int_0^1 xe^{-x^2}\,dx$

[10] **8.** Compute the antiderivatives:

(a) $\displaystyle\int (4t^4 - t^3 + 5t)\,dt$

(b) $\displaystyle\int \left(-\frac{3}{x} - x^{-12} \right) dx$

(c) $\displaystyle\int xe^{-x^2}\,dx$

(d) $\displaystyle\int (x^3 + x)\,e^{(x^4+2x^2)}\,dx$

(e) $\displaystyle\int (x^2 - 2)(x + 3)\,dx$

[10] **9.** Find the area bounded by $y = x^3$ and $y = 4x$.

[10] **10.** The Gini index of a country is $\frac{1}{6}$. Its Lorenz curve has the form $f(x) = ax + \frac{1}{2}x^2$. Find a.

Mock Exam - Math 209

December 2013

① (a) $\lim_{x\to -2} \frac{2x^2+9x+10}{x+2} = \frac{2(-2)^2+9(-2)+10}{-2+2} = \frac{2(4)-18+10}{0} = \frac{0}{0}$ I.D.f

$2x^2+9x+10$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-9 \pm \sqrt{9^2-4(2)(10)}}{2(2)}$ $\frac{-9+1}{4} = -2$, $\frac{-9-1}{4} = -\frac{5}{2}$

$x=-2$ $(x+2)$ $(x+\frac{5}{2}=0)\times 2$ $(2x+5)=0$

$\lim_{x\to -2} \frac{\cancel{(x+2)}(2x+5)}{\cancel{(x+2)}} = 2x+5 = 2(-2)+5 = 1$

(b) $\lim_{x\to 0} \frac{\sqrt{49+x}-7}{x} = \frac{\sqrt{49+0}-7}{0} = \frac{0}{0}$ I.D.f

$\lim_{x\to 0} \frac{\sqrt{49+x}-7}{x} \cdot \frac{\sqrt{49+x}+7}{\sqrt{49+x}+7} = \frac{(49+x-49)}{x(\sqrt{49+x}+7)} = \frac{\cancel{x}}{\cancel{x}(\sqrt{49+x}+7)}$

$= \frac{1}{\sqrt{49+0}+7}$

$= \frac{1}{7+7} = \frac{1}{14}$

(c) $\lim_{x\to\infty} \frac{2x^3+3x^2-9}{-\frac{1}{3}x^3+5x+7} = \frac{\frac{2x^3}{x^3}+\frac{3x^2}{x^3}-\frac{9}{x^3}}{-\frac{1}{3}\frac{x^3}{x^3}+\frac{5x}{x^3}+\frac{7}{x^3}}$

$= \frac{2+\frac{3}{x}-\frac{9}{x^3}}{-1/3+\frac{5}{x^2}+\frac{7}{x^3}} = \frac{2+\frac{3}{\infty}-\frac{9}{\infty}}{-1/3+\frac{5}{\infty^2}+\frac{7}{\infty^3}}$

$= \frac{2+0-0}{-1/3+0+0} = -6$

(a) $y = 8x^3 + 7x^6 + 12$

$y' = 24x^2 + 42x^5$

(b) $y = \frac{2}{3}x^{-5} - 5\sqrt{x} + 5 = \frac{2}{3}x^{-5} - 5x^{1/2} + 5$

$y = -\frac{10}{3}x^{-6} - \frac{5}{2}x^{-1/2}$

(c) $y = (2x^2+3)(x^3+x-2)^3$

$u = 2x^2+3$ $\quad v = (x^3+x-2)^3$

$u' = 4x$ $\quad v' = 3(x^3+x-2)^2 \cdot (3x^2+1)$

$y' = 4x(x^3+x-2)^3 + 3(x^3+x-2)^2(3x^2+1)(2x^2+3)$

(d) $y = \frac{-2x^3+x^2}{x^2+x+1}$

$u = -2x^3+x^2$ $\quad v = x^2+x+1$

$u' = -6x^2+2x$ $\quad v' = 2x+1$

$$y' = \frac{(-6x^2+2x)(x^2+x+1) - (2x+1)(-2x^3+x^2)}{(x^2+x+1)^2}$$

(e) $y = e^{\ln 7x}$

$y = e^{\ln 7x} \cdot \left(\frac{1}{7x}\right)(7)$

(f) $y = \left(e^{3x^2+15x}\right)\left(\ln(7x^3-4x)\right)$

$u = e^{3x^2+15x}$ $\quad v = \ln(7x^3-4x)$

$u' = (6x+15)e^{3x^2+15x}$ $\quad v' = \frac{1}{7x^3-4x} \cdot (21x^2-4)$

$y' = (6x+15)e^{3x^2+15x}\ln(7x^3-4x) + \left(\frac{1}{7x^3 \cdot 4x}\right)(21x^2-4).$
$\left(e^{3x^2+15x}\right)$

(g) $y = e^{3x^2} \ln(x^3+6)^2$

$u = e^{3x^2}$ $\qquad$ $v = \ln(x^3+6)^2$

$u' = 6xe^{3x^2}$ $\qquad$ $v' = \frac{1}{(x^3+6)^2} \cdot 2(x^3\cancel{+6})(3x^2)$

$v' = \frac{6x^2}{(x^3+6)}$

$y' = 6xe^{3x^2}\ln(x^3+6)^2 + \frac{6x^2}{(x^3+6)} e^{3x^2}$

(h) $(1,1)$ $\qquad 2y + x\ln y = 2x^3$

$u = x \quad v = \ln y$

$u' = 1 \quad v' = \frac{1}{y} \cdot y'$

$\ln y + x\frac{y'}{y}$

$2y' + \ln y + x\frac{y'}{y} = 6x^2$

$2y' + x\frac{y'}{y} = 6x^2 - \ln y$

$y'\left(2 + \frac{x}{y}\right) = 6x^2 - \ln y$

$y' = \frac{6x^2 - \ln y}{2 + \frac{x}{y}} = \frac{6(1)^2 - \ln(1)}{2 + \frac{1}{1}}$

$= \frac{6}{3} = 2$

3 $\quad C(x) = 50{,}000 + 600x - 0.75x^2$

(a) $C'(x) = 600 - 1.5x$

(b) $C'(200) = 600 - 1.5(200) = 300$

At a production level of 200 transmissions per week, cost increases by \$300/week.

(c) $C(201) = 50{,}000 + 600(201) - 0.75(201)^2$

$C(201) = 140{,}299.25$

$C(200) = 50{,}000 + 600(200) - 0.75(200)^2$

$C(200) = 140{,}000$

$C(201) - C(200) = 140{,}299.25 - 140{,}000 = \$299.25$

(4) $f(x) = x^4 + 4x^3$

Increasing $]-3, +\infty[$

Decreasing $]-\infty, -3[$

$f'(x) = 4x^3 + 12x^2$

$4x^2(x+3)$

$-\infty$	-3	0	$+\infty$
$-$	$+$	$+$	
$\searrow$	$\nearrow$	$\nearrow$	

$4x^2(x+3) = 0$

$4x^2 = 0$ $\quad$ $x+3=0$

$\boxed{x=0}$ $\quad$ $\boxed{x=-3}$

$f(0) = 0^4 + 4(0)^3 = 0$ $\quad$ $(0,0)$ critical point

$f(-3) = (-3)^4 + 4(-3)^3 = -27$ $\quad$ $(-3,-27)$ local min

$f''(x) = 12x^2 + 24x$

$12x^2 + 24x = 0$

$12x(x+2) = 0$

$\boxed{x=0}$ $\quad$ $x+2=0$ $\quad$ $\boxed{x=-2}$

$-\infty$	-2	0	$+\infty$
$+$	$-$	$+$	
$\cup$	$\cap$	$\cup$	

$f(0) = 0^4 + 4(0)^3 = 0$ $\quad$ $(0,0)$ Inflection point

$f(-2) = (-2)^4 + 4(-2) = -16$ $\quad$ $(-2,-16)$ Inflection point

concave up $]-\infty,-2[\cup]0,+\infty[$

concave down $]-2,0[$

(h) $\lim_{x\to+\infty} f(x) = (+\infty)^4 + 4(+\infty)^3 = +\infty$

$\lim_{x\to-\infty} f(x) = (-\infty)^4 + 4(-\infty)^3 = +\infty$

4pni x-int: $y=0$

$x^4 + 4x^3 = 0$

$x^3(x+4) = 0$

$\boxed{x=0}$ $x+4=0$

$\boxed{x=-4}$

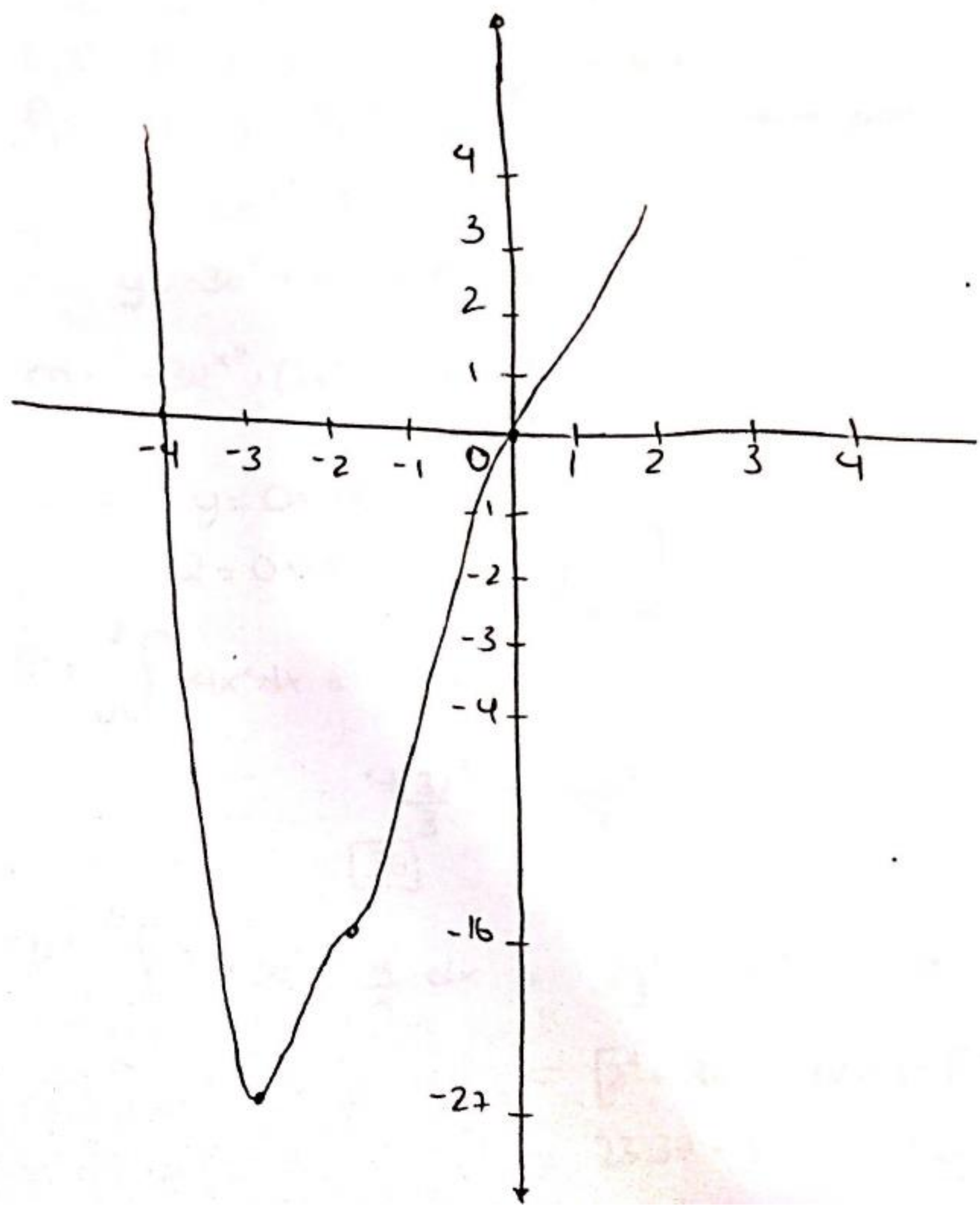
$(0,0)$ $(-4,0)$

y-int: $x=0$

$x^4 + 4x^3 = 0$

$0^4 + 4(0)^3 = 0$

$(0,0)$



$f'(x) = 4x^3 - 12x^2$

$4x^2(x-3) = 0$

$x = 0$

$x = 3$

$f(0) = 0^4 - 4(0)^3 + 5 = 5$ → local max

$f(3) = 3^4 - 4(3)^3 + 5 = -22$ – local min

$f(4) = 4^4 - 4(4)^3 + 5 = \cancel{763}\ 5$ – local ~~min~~ max

⑥ $y = -3e^{x^2} + 5$

$y = -3e^0 + 5 = -3 + 5 = 2$ $(0, 2)$

$m = -3e^{x^2} \cdot (2x) = -6xe^{x^2}$

$= -6(0)e^{0^2} = 0$

$y = 0x + b$

$2 = 0 + b$ $\boxed{y = 2}$

⑦ (a) $\int_0^3 4x^2 dx = \frac{4x^3}{3}\Big]_0^3$

$= \frac{4(3)^3}{3} - \frac{4(0)^3}{3}$

$= \boxed{36}$

(b) $\int_1^2 2x + 3e^x - \frac{4}{x}\, dx = \frac{2x^2}{2} + 3e^x - 4\ln x\Big]_1^2$

$= [2^2 + 3e^2 - 4\ln 2] - [1^2 + 3e^1 - 4\ln 1]$

$= 23.39 - 1 = \boxed{22.39}$

$$\int_0^1 xe^{-x^2}\,dx$$

$u = -x^2$

$du = -2\boxed{x\,dx}$

$\frac{du}{-2} = x\,dx$

$$\int_0^1 e^u \cdot \frac{du}{-2} = \frac{-1}{2}\int_0^1 e^u = \frac{-1}{2}\,e^{-x^2}\Big]_0^1$$

$$= \frac{-1}{2}e^{-1^2} - \left[\frac{-1}{2}e^{-0^2}\right]$$

$$= \frac{-1}{2}e^1 + \frac{1}{2}$$

⑧ (a) $\int 4t^4 - t^3 + 5t = \frac{4t^5}{5} - \frac{t^4}{4} + \frac{5t^2}{2} + C$

(b) $\int \left(\frac{-3}{x} - x^{-12}\right)dx = -3\ln x - \frac{x^{-11}}{-11} + C$

(c) $\int xe^{-x^2}\,dx$

$u = -x^2$

$du = -2x\,dx$

$\frac{du}{-2} = x\,dx$

$\int xe^{-x^{2.}}$

$u = -x^2$

$du = -2x\,d$

$\frac{du}{2} =$

$$\int e^u \cdot \frac{du}{-2} = \frac{-1}{2}\int e^u\,du = -\frac{1}{2}e^{-x^2} + C$$

$$\int (x^3 + x)\, e^{(x^4 + 2x^2)}\, dx$$

$u = x^4 + 2x^2$

$du = (4x^3 + 4x)\, dx$

$du = 4(x^3 + x)\, dx$

$\frac{du}{4} = (x^3 + x)\, dx$

$$\int e^u \cdot \frac{du}{4} = \frac{1}{4}\int e^u = \frac{1}{4} e^{x^4 + 2x^2} + C$$

(e) $\int (x^2 - 2)(x + 3) = x^3 + 3x^2 - 2x - 6$

$= \frac{x^4}{4} + \frac{3x^3}{3} - \frac{2x^2}{2} - 6x + C$

$= \frac{x^4}{4} + x^3 - x^2 - 6x + C$

Step 1

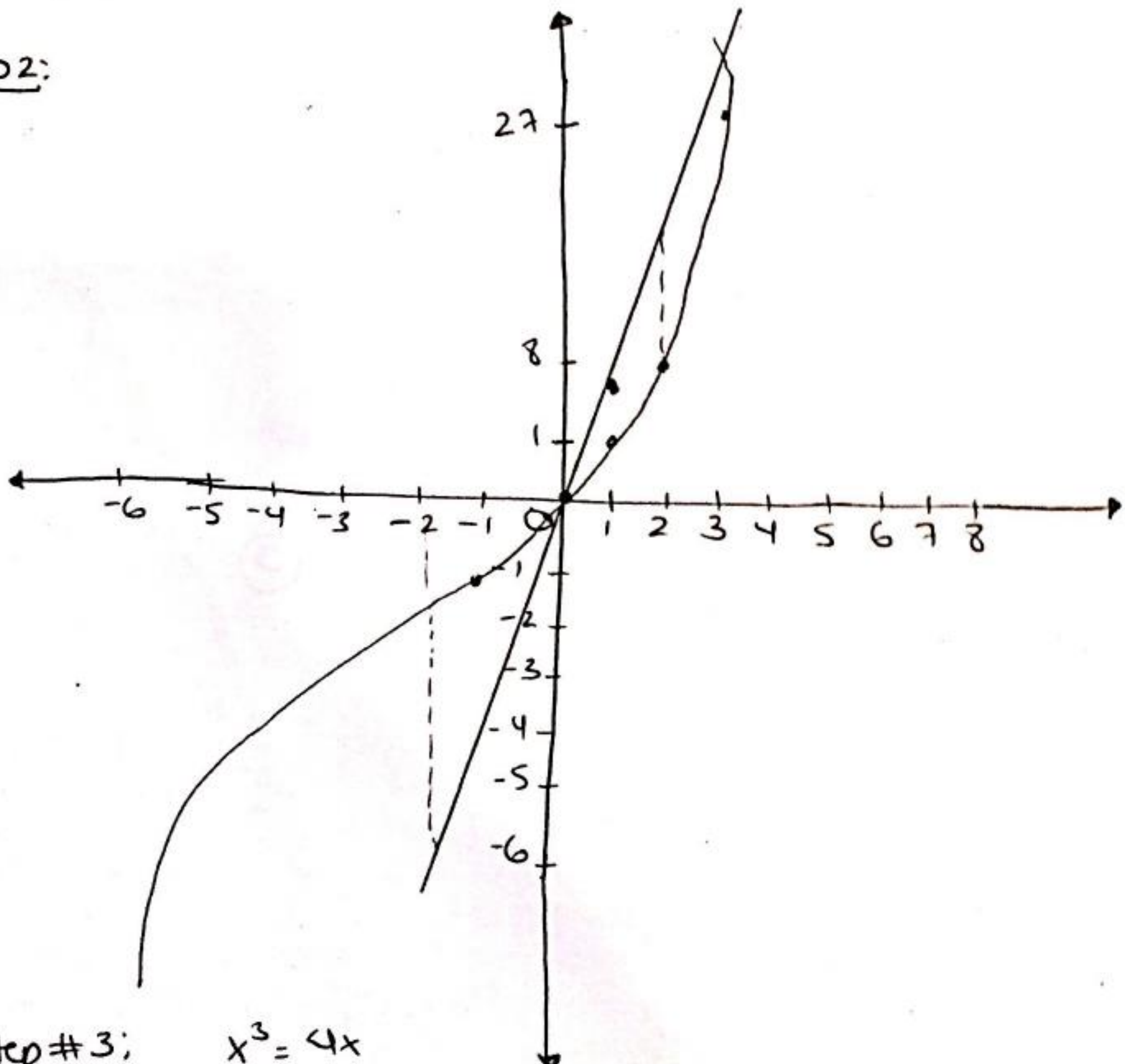
$y = x^3$

x	y
1	1
2	8
3	27
-1	-1

$y = 4x$

x	y
0	0
1	4

Step 2:



Step #3:

$x^3 = 4x$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x = 0 \quad (x-2)(x+2) \quad x = 2$

$x = -2$

$-2, 0, 2.$

$$\int_{-2}^{0} x^3-4x = \frac{x^4}{4} - \frac{4x^2}{2} = \frac{x^4}{4} - 2x^2 \Big]_{-2}^{0}$$

$$= \frac{0^4}{4} - 2(0)^2 - \left[\frac{-2^4}{4} - 2(-2)^2\right]$$

$$= -\left[4-8\right] = 4 \longrightarrow A_1$$

$$\int_{0}^{2} 4x-x^3 = \frac{4x^2}{2} - \frac{x^4}{4} \Big]_{0}^{2}$$

$$= \frac{4(2)^2}{2} - \frac{(2)^4}{4} - \left[\frac{4(0)^2}{2} - \frac{0^4}{4}\right]$$

$$= 8 - 4 = 4 \longrightarrow A_2$$

Total Area: $A_1 + A_2 = 4+4 = 8$

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$$G = 1 - 2\int_0^1 ax + \frac{1}{2}x^2$$

$\frac{1}{2}\frac{x^3}{3}$ $\frac{1}{6}x^3$

$\frac{0.5}{3}$

$$1/6 = 1 - 2\left[\frac{ax^2}{2} + \frac{0.5x^3}{3}\right]_0^1$$

$$1/6 = 1 - 2\left[\frac{a}{2}(1)^2 + 0.5\frac{(1)^3}{3} - \cancel{\left[\frac{a(0)^2}{2} + 0.5\frac{(0)^3}{3}\right]}\right]$$

$$1/6 = 1 - 2\left[\frac{a}{2} + \frac{0.5}{3}\right]$$

$$1/6 = 1 - 2\left[\frac{a}{2} + \frac{1}{6}\right]$$

$$1/6 = 1 - \frac{2a}{2} - \frac{2}{6}$$

$$1/6 = 1 - a - \frac{1}{3}$$

$$1/6 - 1 + \frac{1}{3} = -a$$

$$-a = -1/2 \quad a = 1/2$$