Department of Computer Science and Software Engineering
Comp 232 Mathematics for Computer Science Fall 2020
Assignment 3 Due : November 14, 2020

1. a) Use set identities to prove that $A \cup (B - A) = A \cup B$

Proof : L.H. S. = $A \cup (B - A) = A \cup \left(B \cap \overline{A}\right)$ $(Definition\ of\ Set\ Difference)$

= $(A \cup B) \cap (A \cup \bar{A})$ (Distributive Law)

= $(A \cup B) \cap U$ (Complement Law)

= $(A \cup B)$ (Identity Law)

= R.H.S.

b) Use set identities to prove that $A \cap (B - A) = \phi$

Proof : L.H. S. = $A \cap (B - A) = A \cap (B \cap \bar{A})$ $(Definition\ of\ Set\ Difference)$

= $A \cap (\bar{A} \cap B)$ $(Commutative\ Law)$

= $(A \cap \bar{A}) \cap B$ $(Associative\ Law)$

= $\phi \cap B$ $(Complement\ Law)$

= ϕ (Domination Law)

= R.H.S.

c) Use set identities to prove that $\overline{A \cup (B \cap C)} = (\bar{C} \cup \bar{B}) \cap \bar{A}$

Proof : L.H. S. = $\overline{A \cup (B \cap C)}$ = $\bar{A} \cap (\overline{B \cap C})$ $\quad (De\ Morgan's\ Law)$

= $\bar{A} \cap (\bar{B} \cup \bar{C})$ (De Morgan's Law)

= $(\bar{B} \cup \bar{C}) \cap \bar{A}$ $\quad (Commutative\ Law)$

= $(\bar{C} \cup \bar{B}) \cap \bar{A}$ $\quad (Commutative\ Law)$

= R.H.S.

2. a) Prove or give a counterexample for the statement that if A and B are sets, then $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

Solution : $\mathcal{P}(A) \cap \mathcal{P}(B)$ consists of all sets which are subsets of both A and B, and these are precisely the subsets of $A \cap B$.

b) Let $A = \{0, 1, \phi\}$. $List\ the\ elements\ of\ \mathcal{P}(A)$.

Solution : $\mathcal{P}(A) = (\phi, \{0\}, \{1\}, \{\phi\}, \{0,1\}, \{0, \phi\}, \{1, \phi\}, \{0,1, \phi\}\}$

3. Give an example of a function f :N →N that is

a) one to one, but not onto

b) onto, but not one to one

Solution : a) Let f(n) = 3n+1 if n ≥ 0 and -3n if n < 0.

b) Let f(0) = 0, f(n) = n if n < 0, f(n) = n-1 if n > 0.

4. Give a proof by cases that $\lfloor 4x \rfloor = \lfloor x \rfloor + \lfloor x + ¼ \rfloor + \lfloor x + ½ \rfloor + \lfloor x + ¾ \rfloor$

Solution : For any real number x, $\lfloor x \rfloor = \mathrm{n} + \mathrm{r}$, where $0 \le \mathrm{r} < 1$.

Case A: $0 \le \mathrm{r} < ¼$

Case B: $¼ \le \mathrm{r} < ½$

Case C: $½ \le \mathrm{r} < ¾$

Case D: $¾ \le \mathrm{r} < 1$

Case	$\lfloor x \rfloor$	$\lfloor x + ¼ \rfloor$	$\lfloor x + ½ \rfloor$	$\lfloor x + ¾ \rfloor$	$\lfloor 4x \rfloor$
A	n	n	n	n	4n
B	n	n	n	n+1	4n+1
C	n	n	n+1	n+1	4n+2
D	n	n+1	n+1	n+1	4n+3

5. Give an example of two uncountable sets A and B such that A – B is

a) finite

Solution: Let A = [1,2] and B = (1, 2]. Then A- B = {1} is finite.

b) countably infinite

Solution: Let A = (2,3) $\cup Z$ and B = (2, 3). Then A- B = Z is countably infinite.

c) uncountable

Solution: Let A = [1, 3] and B = (2, 3]. Then A- B = [1, 2] is uncountable.

6. Use the Euclidean algorithm to find the following :

a) gcd(985, 408)

Solution : a) 985 = 2 x 408 + 169

408 = 2 x 169 + 70

169 = 2 x 70 + 29

70 = 2 x 29 + 12

29 = 2 x 12 + 5

12 = 2 x 5 + 2

5 = 2 x 2 + **1**

2 = 2 x 1 + 0

So the gcd(985, 408) = 1

b) gcd(7953, 5822)

7953 = 1 x 5822 + 2131

5822 = 2 x 2131 + 1560

2131 = 1 x 1560 + 571

1560 = 2 x 571 + 418

571 = 1 x 418 + 153

418 = 2 x 153 + 112

153 = 1 x 112 + 41

112 = 2 x 41 + 40

41 = 1 x 30 + 11

30 = 2 x 11 + 8

11 = 1 x 8 + 3

8 = 2 x 3 + 2

2 = 1 x 2 + **1**

2 = 2 x 1 + 0

So the gcd(7953, 5822) =1

c) gcd(38785, 16768) = 1

a) 38785 = 2 x 16768 + 5249
16768 = 3 x 5249 + 1021
5249 = 5 x 1021 + 144
1021 = 7 x 144 + 13
144 = 11 x 13 + **1**
13 = 13 x 1 + 0

7. a) Find the value of 10! **mod** 11

Solution : $10! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 = (1 \cdot 10) \cdot (2 \cdot 5) \cdot (3 \cdot 7) \cdot (4 \cdot 8) \cdot (6 \cdot 9)$

$$\equiv (-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) \equiv -1 \ (\boldsymbol{mod}\ 11)$$

b) Find the value of 12! **mod** 13

Solution : $12! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot 11 \cdot 12$

$= (1 \cdot 12) \cdot (2 \cdot 6) \cdot (3 \cdot 4) \cdot (7 \cdot 11) \cdot (9 \cdot 10) \cdot (5 \cdot 8)$

$\equiv (-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (1) \equiv -1 \ (\boldsymbol{mod}\ 13)$

c) Make a conjecture about the value of (p-1)! **mod** p, where p is a prime.

$$(p-1)! \equiv -1 \ (\boldsymbol{mod}\ p)$$

8. Prove that for any positive integer n, gcd(7n+2, 4n+1) = 1

Solution: 7n+2 =1 x 4n+1 + 3n+1

4n+1 = 1 x 3n+1 + n

3n+1 = 3 x n + 1

n = n x 1 + 0

So from the Euclidean algorithm we find that gcd(7n+2, 4n+1) = 1.

9. Show that if a, b, c, and d are integers and a ≠ 0, that if a | c and b | d then ab | cd.

Proof: If $a \mid c\ then\ \exists\ k\ \in Z\ such\ that\ ak = c.$ If $b \mid d\ then\ \exists\ l\ \in Z\ such\ that\ bl = d.$

Multiplying the two equations give us $akbl = cd$. From the associative law, ab(kl) = cd. Since kl $\in Z$, (by closure under multiplication) this means that $ab \mid cd$ by the definition of divides.

10. Prove that if n is an odd positive integer then $n^2 \equiv 1\ (mod\ 8)$.

Proof: (by cases): If n is an odd positive integer, then $n \equiv 1, 3, 5, or\ 7\ (mod\ 8)$

If $n \equiv 1\ (mod\ 8)\ then\ n^2 \equiv 1\ (mod\ 8)$

If $n \equiv 3\ (mod\ 8)\ then\ n^2 \equiv 9 \equiv 1\ (mod\ 8)$

If $n \equiv 5\ (mod\ 8)\ then\ n^2 \equiv 25 \equiv 1\ (mod\ 8)$

If $n \equiv 7\ (mod\ 8)\ then\ n^2 \equiv 49 \equiv 1\ (mod\ 8)$