ENGR 391 - NUMERICAL METHODS IN ENGINEERING

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PROBLEMS FOR CHAPTER 4

1. A calibration test on a proximity pickup gave the following results:

Displacement,	X	5	10	15
Voltage,	V	19.4	18.7	18.2

Displacement is expressed in thousands of an inch and voltage is in volts.

a) Obtain a least square approximation of the type

$$P(x) = c_0 + c_1 x + c_2 x^2$$

b) Obtain the Lagrange Polynomial for the given data.

Solution:

a) Approximation is a quadratic (degree 2).

$$P(x) = a_0 + a_1x^1 + a_2x^2$$

The matrix has the form:

$$\begin{bmatrix} n+1 & \sum_{i=0}^{n} x_i & \sum_{i=0}^{n} x_i^2 \ldots & \sum_{i=0}^{n} x_i^m \\ \sum_{i=0}^{n} x_i & \sum_{i=0}^{n} x_i^2 & \sum_{i=0}^{n} x_i^3 \ldots & \sum_{i=0}^{n} x_i^{m+1} \\ \sum_{i=0}^{n} x_i^2 & \sum_{i=0}^{n} x_i^3 & \sum_{i=0}^{n} x_i^4 \ldots & \sum_{i=0}^{n} x_i^{m+2} \\ \vdots & & & \\ \sum_{i=0}^{n} x_i^m & \sum_{i=0}^{n} x_i^{m+1} & \sum_{i=0}^{n} x_i^{m+2} & \sum_{i=0}^{n} x_i^{2m} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{n} f(x_i) \\ \sum_{i=0}^{n} x_i f(x_i) \\ \sum_{i=0}^{n} x_i^2 f(x_i) \\ \vdots \\ \sum_{i=0}^{n} x_i^{m} f(x_i) \end{bmatrix}$$

substituting the data gives

$$\begin{bmatrix} 3 & 30 & 350 \\ 30 & 350 & 4500 \\ 350 & 4500 & 61250 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 56.3 \\ 557 \\ 6450 \end{bmatrix}$$

Solving the matrix gives us

$$c_2 = 0.004$$

$$c_1 = -0.2$$

$$c_0 = 20.3$$

Hence $P(x) = 0.004x^2 - 0.2x + 20.3$

b) Lagrange Polynomial is

$$P(x) = \sum_{k=0}^{n} f(x_k) L_{n,k}$$

$$L_{n,k} = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}$$

k=0

$$L_{2,0} = \prod_{\substack{i=0 \\ i \neq 0}}^{n} \frac{(x - x_i)}{(x_k - x_i)} = \frac{(x - x_1)}{(x_0 - x_1)} * \frac{(x - x_2)}{(x_0 - x_2)} =$$

$$\frac{(x-10)(x-15)}{(5-10)(5-15)} = \frac{x^2 - 35x + 150}{50}$$

k=1

$$L_{2,1} = \prod_{\substack{i=0 \\ i \neq 0}}^{n} \frac{(x - x_i)}{(x_k - x_i)} = \frac{(x - x_1)}{(x_0 - x_1)} * \frac{(x - x_2)}{(x_0 - x_2)} =$$

$$\frac{(x-5)(x-15)}{(10-5)(10-15)} = \frac{x^2 - 25x + 75}{-25}$$

k=2

$$L_{2,2} = \prod_{\substack{i=0 \\ i \neq 0}}^{n} \frac{(x - x_i)}{(x_k - x_i)} = \frac{(x - x_1)}{(x_0 - x_1)} * \frac{(x - x_2)}{(x_0 - x_2)} =$$

$$\frac{(x-5)(x-10)}{(15-10)(15-5)} = \frac{x^2 - 15x + 50}{50}$$

$$P(x) = 19.4\left[\frac{x^2 - 35x + 150}{50}\right] + 18.7\left[\frac{x^2 - 25x + 75}{-25}\right] + 18.2\left[\frac{x^2 - 15x + 50}{50}\right]$$

$$P(x) = 0.004x^2 - 0.34x + 20.3$$

4. Find the best linear fit in the least squares sense to the data given below.

$$x_1 = 1 \quad x_2 = 2 \quad x_3 = 3$$

$$f_1 = 1 \quad f_2 = 3 \quad f_3 = 1$$

Solution

$$P(x) = a_0 + a_1x^1$$

The matrix has the form:

$$\begin{bmatrix} \sum_{i=0}^{3} x_i^0 & \sum_{i=0}^{3} x_i^1 \\ \sum_{i=0}^{3} x_i^1 & \sum_{i=0}^{3} x_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{3} f(x_i) \\ \sum_{i=0}^{3} f(x_i)x_i \end{bmatrix}$$

substituting the data gives

$$\begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix} \begin{bmatrix} a_o \\ a_1 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

Solving the matrix gives us

$$a_1 = 0$$

$$a_0 = 1.6667$$

Hence $P(x) = 1.6667$

Best linear fit to the given data is a horizontal line at y=1.6667

10. In modal testing procedure to obtain the modal parameters of a vibrating system, it is necessary to fit a circle to the measured mobility values. For the given data set fit a least squares circle and obtain its center point and radius. The equation of a general circle is of the form.

$$x^2 + y^2 + ax + by + c = 0$$

x	0.0789	0.3570	60.0	8.70	0.4360	0.1200
y	2.2194	4.700	0.100	-21.00	-5.200	-2.700

Solution: The matrix equation for circular regression is

$$\begin{bmatrix} \sum_{i=0}^{n} x_i^2 & \sum_{i=0}^{n} x_i^1 y_i^1 & \sum_{i=0}^{n} x_i^1 \\ \sum_{i=0}^{n} x_i^1 y_i^1 & \sum_{i=0}^{n} y_i^2 & \sum_{i=0}^{n} y_i^1 \\ \sum_{i=0}^{n} x_i^1 & \sum_{i=0}^{n} y_i^1 & \sum_{i=0}^{n} x_i^0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{n} x(x_i^2 + y_i^2) \\ \sum_{i=0}^{n} y(x_i^2 + y_i^2) \\ \sum_{i=0}^{n} (x_i^2 + y_i^2) \end{bmatrix}$$

For the given set of data

n	0	1	2	3	4	5	$\sum_{i=0}^{n}$
x	0,0789	0,357	60	8,7	0,436	0,12	69,6919
y	2,2194	4,7	0,1	-21	-5,2	-2,7	-21,8806
$x_i^1 y_i^1$	0,175111	1,6779	6	-182,7	-2,2672	-0,324	-177,438
x_i^2	0,006225	0,127449	3600	75,69	0,190096	0,0144	3676,028
y_i^2	4,925736	22,09	0,01	441	27,04	7,29	502,3557
$(x_i^2 + y_i^2)$	4,931962	22,21745	3600,01	516,69	27,230096	7,3044	4178,384
$x(x_i^2 + y_i^2)$	0,389132	7,931629	216000,6	4495,203	11,872322	0,876528	220516,9
$y(x_i^2 + y_i^2)$	10,946	104,422	360,001	-10850,49	-141,5965	-19,72188	-10536,4

Therefore

$$\begin{bmatrix} 3676.028 & -177.44 & 69.69 \\ -177.44 & 502.36 & -21.88 \\ 69.69 & -21.88 & 6 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 220516.9 \\ -10536.4 \\ 4178.38 \end{bmatrix}$$

Solving the matrices we get

$$a = 59.99030$$

$$b = 0.23601$$

$$c = 0.46997$$

$$x^2 + y^2 + 59.99030\, x + 0.23601\, y + 0.46997 = 0$$

where

$h = -a/2 = -29.99$

$k = -b/2 = -0.118$

$r = \sqrt{(4c + a^2 + b^2)}/2 = 30.003$

15. Fit a Lagrange polynomial to the data
    a) (1,2.8) , (2, 7.4), (3, 20.1).
    b) (1.0, 2.0) , (2.0,4.0), (3.0, 5.5).

Solution: a) Lagrange Polynomial is

$$P(x) = \sum_{k=0}^{n} f(x_k) L_{n,k}$$

$$L_{n,k} = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}$$

k=0

$$L_{2,0} = \prod_{\substack{i=0 \\ i \neq 0}}^{n} \frac{(x - x_i)}{(x_k - x_i)} = \frac{(x - x_1)}{(x_0 - x_1)} * \frac{(x - x_2)}{(x_0 - x_2)} =$$

$$\frac{(x-2)(x-3)}{(1-2)(1-3)} = \frac{x^2 - 5x + 6}{2}$$

k=1

$$L_{2,1} = \prod_{\substack{i=0 \\ i \neq 0}}^{n} \frac{(x - x_i)}{(x_k - x_i)} = \frac{(x - x_1)}{(x_0 - x_1)} * \frac{(x - x_2)}{(x_0 - x_2)} =$$

$$\frac{(x-1)(x-3)}{(2-1)(2-3)} = \frac{x^2 - 4x + 3}{-1}$$

k=2

$$L_{2,2} = \prod_{\substack{i=0 \\ i \neq 0}}^{n} \frac{(x - x_i)}{(x_k - x_i)} = \frac{(x - x_1)}{(x_0 - x_1)} * \frac{(x - x_2)}{(x_0 - x_2)} =$$

$$\frac{(x-1)(x-2)}{(3-2)(3-1)} = \frac{x^2 - 3x + 2}{2}$$

$$P(x) = 2.8\left[\frac{x^2 - 5x + 6}{2}\right] + 7.4\left[\frac{x^2 - 4x + 3}{-1}\right] + 20.1\left[\frac{x^2 - 3x + 2}{2}\right]$$

$$P(x) = 4.05x^2 - 7.55x + 6.3$$

b) Lagrange Polynomial is

$$P(x) = \sum_{k=0}^{n} f(x_k) L_{n,k}$$

$$L_{n,k} = \prod_{\substack{i=0\\ i\neq k}}^{n} \frac{(x-x_i)}{(x_k-x_i)}$$

k=0

$$L_{2,0} = \prod_{\substack{i=0\\ i\neq 0}}^{n} \frac{(x-x_i)}{(x_k-x_i)} = \frac{(x-x_1)}{(x_0-x_1)} * \frac{(x-x_2)}{(x_0-x_2)} =$$

$$\frac{(x-2)(x-3)}{(1-2)(1-3)} = \frac{x^2-5x+6}{2}$$

k=1

$$L_{2,1} = \prod_{\substack{i=0\\ i\neq 0}}^{n} \frac{(x-x_i)}{(x_k-x_i)} = \frac{(x-x_1)}{(x_0-x_1)} * \frac{(x-x_2)}{(x_0-x_2)} =$$

$$\frac{(x-1)(x-3)}{(2-1)(2-3)} = \frac{x^2-4x+3}{-1}$$

k=2

$$L_{2,2} = \prod_{\substack{i=0\\ i\neq 0}}^{n} \frac{(x-x_i)}{(x_k-x_i)} = \frac{(x-x_1)}{(x_0-x_1)} * \frac{(x-x_2)}{(x_0-x_2)} =$$

$$\frac{(x-1)(x-2)}{(3-2)(3-1)} = \frac{x^2-3x+2}{2}$$

$$P(x) = 2.0\left[\frac{x^2-5x+6}{2}\right] + 4.0\left[\frac{x^2-4x+3}{-1}\right] + 5.5\left[\frac{x^2-3x+2}{2}\right]$$

$$P(x) = -0.25x^2 + 2.75x + -0.5$$

Text Book: *"Numerical Analysis"*, R. Bhat & A. Kaushal, Alpha Science Publishers ISBN 978-1-78332-346-3