Testing the Power and the Size of the Andrews and Ploberger-test by using Simulation on Generated Data

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Motivation

1. In econometrics, a structural break is an unexpected shift in a time series.
2. Causes :
   - Global shifts in capital and labor, changes in resource availability or a change in political system.
3. Examples :
   - Crisis: subprime mortgage crisis (2008), great depression (1929), oil crisis (1973, 1979) ; wars ; trade or financial openness.
4. Structural change is possible because of the dynamic nature of the economic system.
5. Consequences:
   - Huge forecasting errors and unreliability of the model in general

Motivation

1. Several studies which have been conducted by organizations (FED, IMF, NBER,...) show that the series are stationary on the sub-periods, but unstable over the whole sample.
2. AP (1994) revolutionizes the detection of a structural break at an “unknown” period of stationary series.
   - While Chow (1960) , Cusum (Evans, Brown, Durbin 1975), it was assumed "known".
3. Bai and Perron (1998) offering a broader method namely the detection of several breaks in one optimization algorithm.
4. Bruce E. Hansen (2003) proposes a method asymptotically robust to heteroskedasticity from bootstrap simulation.
   - He gives a new p-value for SupF initially submitted by AP.

Model

For computing this simulation (monte-carlo), we use a stationary first-order autoregression AR(1) because the test can be applied only to stationary series :

$$Y_t = c + \rho Y_{t-1} + \varepsilon_t \tag{1}$$

$|\rho| < 1$
$\varepsilon_t \sim iid(E(\varepsilon_t) = 0, V(\varepsilon_t) = \sigma^2)$
$E(Y_t) = \mu = \frac{c}{1-\rho}$

Andrews and Ploberger-test

AP used a model like this :

$$Y = \begin{cases} X \cdot \beta + \varepsilon & \text{under } H_0 \\ X \cdot \beta + X \cdot \theta_t + \varepsilon & \text{under } H_1 \end{cases} \tag{2}$$

Under H_0 we find up the unconditional variance :

$$\sigma^2 = \frac{\sum_{i=1}^{T} e_i^2}{T - m} \tag{3}$$

For any fixed t, this regression under H_1 can be estimated by OLS yielding estimates $\left(\widehat{\beta},\ \widehat{\theta}_t\right)$ and giving conditional variance :

$$\sigma_t^2 = \frac{\sum_{i=1}^{T} e_{ti}^2}{T - 2m} \tag{4}$$

Andrews and Ploberger-test

Therefore, with conditional and unconditional variance, we will estimate the test statistic of AP like that *ExpF*, *AveF* ,*MeanF*, *SupF*. However, for the simulation, we will use *SupF* given by Hansen :

$$F_t = \frac{(T-m)\,\hat{\sigma}^2 - (T-2m)\,\hat{\sigma}_t^2}{\hat{\sigma}_t^2} \tag{5}$$

$$SupF = max(F_t) \tag{6}$$

Design

- Step 1. Generate Y_t as specified in equation (1) for t = 1,....,T ;
- Step 2. Estimate the model with OLS to obtain $\widehat{\beta}$ and compute *SupF*, as in equation (3) ;
- Step 4. Compute the percentage of *SupF* rejections when compared to the asymptotic critical values to Hansen (1997) (for 1% is 16.4, 5% is 12.9, 10% is 11.2) ;
- Step 3. Repeat steps 1 to 3, Routine= 5000 times for many sample sizes (100, 250, 500 and 1000 observations) ;
- Step5. The resulting percentage is the empirical test size.

Results of simulation analysis of test size

$$\textit{For example } Y_t = 0.8Y_{t-1} + \varepsilon_t \tag{7}$$

Table 1:Size of the test for $\rho = 0.8$ when H_0 is true

Nominal Size	1%	5%	10%
T=100	0.16	0.70	1.72
T=250	0.10	0.62	1.48
T=500	0.12	0.72	1.40
T=1000	0.10	0.56	1.20

Design

- For $\{Y_t\}_{t=1}^{T}$ where Y_t is real value such as :

$$Y_t = \begin{cases} \rho_1 Y_{t-1} + \varepsilon_t & \text{if } t \leq t_0 \\ \rho_2 Y_{t-1} + \varepsilon_t & \text{otherwise} \end{cases} \tag{8}$$

where t_0 represent the breakpoint date

For data generating, we use a loop and suppose that ρ_1 and ρ_2 are known $(\rho_1 \neq \rho_2)$ and also $\varepsilon_t \sim N(0,1)$

- As the Type I error, we will repeat the same steps 1 to 5;
- However, we will constantly change the breakpoint date t_0 (like 25%T, 50%T and 75%T).

Results of simulation analysis of power of the test

$$\rho^a = \begin{cases} \rho_1 = 0.8 & \text{if } t \leq t_0 \\ \rho_2 = 0.2 & else \end{cases} \tag{9}$$

Table 2: Power of the test for T=100 and ρ^a when H_0 is false

Nominal Size	1%	5%	10%
$t_0 = 0.25 * T$	1.86	0.42	0.08
$t_0 = 0.50 * T$	21.88	8.64	4.64
$t_0 = 0.75 * T$	72.38	53.32	41.38

Results of simulation analysis of power of the test

Table 3: Power of the test for T=1000 and ρ^a when H_0 is false

Nominal Size	1%	5%	10%
$t_0 = 0.25 * T$	0.00	0.00	0.00
$t_0 = 0.50 * T$	0.00	0.00	0.00
$t_0 = 0.75 * T$	0.00	0.00	0.00

Results of simulation analysis of power of the test

$$\rho^d = \begin{cases} \rho_1 = 0.6 & \text{if } t \leq t_0 \\ \rho_2 = 0.8 & \textit{otherwise} \end{cases} \tag{10}$$

Table 13: Power of the test for T=100 and ρ^d when H_0 is false

Nominal Size	1%	5%	10%
$t_0 = 0.25 * T$	78.48	67	59.6
$t_0 = 0.50 * T$	88.64	80.9	75.78
$t_0 = 0.75 * T$	96	92.18	89.54

Results of simulation analysis of power of the test

Table 4: Power of the test for T=1000 and ρ^d when H_0 is false

Nominal Size	1%	5%	10%
$t_0 = 0.25 * T$	0.00	0.00	0.00
$t_0 = 0.50 * T$	0.52	0.12	0.1
$t_0 = 0.75 * T$	11.76	10.52	7.28

US inflation rate and DE GDP

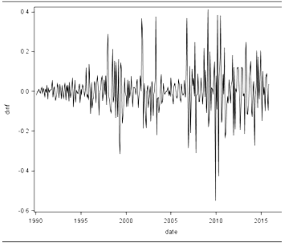

Figure 2: Inflation Rate after differenciation

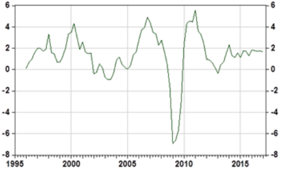

Figure: US Inflation Rate after differenciation and DE GDP

US inflation rate and DE GDP

US inflation rate :

- The series is highly volatile (it looks like the missing)
- Presumption of non- stationary
- We make it stationary by differentiating it
- No break, SupF=2.8507

Germany GDP :

- The serie seems stationary but we all see breakpoint in 2009
- The AP-test detect the break because Sup is more than 25,6755

Conclusion and remarks

For the type I error, all value are closed to 0 so we can say that the test has a good size,

- When $\mathbf{T}\nearrow$, the % of rejections are relatively the same

For the type II error, the test is most powerful for the break on the begining of the series than the others parts,

- More power in the middle than in the end
- When $\mathbf{T} > 250$, all values are closed to 0 for all breakpoints

Conclusion and remarks

- whatever the direction of ρ, AP-test totally shows his power when $\mathbf{T} \longrightarrow \infty$,
- when ρ is high in the first sub-period and becomes small after the breakpoint, the test is less powerful than the inverse,
- AP-test is less powerful when the shock on the parameter is very weak,
- With an intercept into the model and if there is a break on this one whatever the importance of the shock and the size of the sample (even if T=100), the test finds his real power.

References I

- Andrews, W.K., (1993), Tests for Parameter Instability and Structural Change With Unknown Change Point, Econometrica, 61, 821-856
- Andrews, W.K., Ploberger W., (1994), Optimal Tests when a Nuisance Parameter is Present only Under the Alternative, Econometrica, 62, 1383-1414.
- Andrews, W.K., Lee, I., Ploberger, W., (1996), Optimal change point tests for normal linear regression, Journal of Econometrics, 70, 9-38.
- Bai, J., Perron P., (2003), Computation and Analysis of Multiple Structural Change Models, Journal of Applied Econometrics, 18, 1-22.

References II

- Hansen E. Bruce (1997) Approximate Asymptotic P Values for Structural-Change Tests, Journal of Business & Economic Statistics, vol 15, 60-67 70
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THANK YOU FOR YOUR ATTENTION