Hot Button Issue: Staying Cool as the World Heats Up

1 Executive Summary

To the Authorities of the City of Memphis, As the world continues to face the ever-rising threat of global warming, its global effect on people becomes ever more pertinent. Extreme heat events are becoming more frequent, severe and prolonged. To support the city's efforts in preparing for and mitigating the impacts of heat waves our team has developed data-driven models addressing 3 primary concerns.

1.1 Indoor temperature

We first created a model to estimate the indoor temperature of a non-air-conditioned dwelling during a heat wave over a 24-hour period. This model incorportates real world data on outdoor temperatures, heat retention properties of buildings and thermal dynamics. Out analysis highlights the significant indoor heat buildup especially when cooling mechanisms are not present and identifies critical timeframes when temperatures reach levels that are dangerous to human life. This model can be used to identify at risk households and prioritise emergency measures to control these temperatures. We also reccomend you encourage community based heat mitigation strategies like increasing the amount of open space and reflective roofing solutions

1.2 Peak energy demand

Given the increasing reliance on air conditioning during heat waves, we have developed a model to predict peak electricity demand for the city's power grid during summer months. By analysing historical energy consumption patterns, population growth trends, and climate projections, we have estimated that peak demand increased from 3500 to 4200 MegaWatts which does not seem like a significant amount telling us as long as the power grid can handle it the risk of power outages is minimal.

1.3 Heat vulnerability

We have created scores for Vulnerability throughout memphis based on factors such as population, income, open areas and at-risk age groups. Our analysis has identified the most vulnerable communities providing city officials with an effective way to direct emergency cooling efforts and public health resources.

1.4 Reccomendations

To summarise we reccomend that you implement early warning systems and targeted outreach in higher risk neighbourhoods. Ensure the power grid is stable enough Establish more cooling centers.

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2 Part I: Hot to Go

2.1 Defining the Problem

The problem requires us to develop a model to predict indoor temperatures of a non-air-conditioned house over a 24-hour period during a heat wave. We have chosen Memphis, Tennessee as our city and will consider factors such as building size, occupancy and ability to lose heat. Our model will be tested using dwelling data samples and specific heat wave weather data.

2.2 Assumptions

A1. Weather conditions that impact the solar output experienced by homes have negligible effects during a heat wave.

Justification: It is difficult to predict temporary fluctuations in weather such as cloudy periods and rainfall (which are also less likely in heat wave conditions). This consideration therefore diverges from the focus of the model which is to calculate the average solar output experienced by the average home in Memphis.

A2. Each house has square dimensions and storey height is uniform at 2.5m

Justification: The model needs to represent the average influence of sunlight on the temperature in the house, so this assumption simplifies complex area calculations whilst providing a reliable average for calculated house surface area.

A3. All individuals in a household are at home throughout the day for the purpose of this model.

Justification: We do not consider the impact of people exiting their homes for work or other purposes as the impact is miniscule in comparison to other factors according to tested variables.

A4. No auxiliary cooling or heating devices are used to impact the internal temperature of the home.

Justification: We assume that the temperature of the home in question is not affected by such devices based on the parameter of the question that there is no air conditioning. This removes external interventions and allows the model to focus on impacts relating directly to heat retention and dissipation mechanisms.

A5. The average heat output of a person is 100W.

Justification: We keep the heat output constant, assuming that each person is resting during the heat wave and therefore outputs at 100W [8]. Making use of the standardised data provided by the M3 Challenge script [1], the average number of occupants per house based on the total population and the number of households in Memphis returns a figure that rounds to an average of 3, so the average household human heat output is at a rate of 300W.

A6. The temperature difference between stories and rooms within a housing unit is negligible.

Justification: Making temperatures uniform is a sensible assumption as small scale variations will occur owing to the shape and structure of the home, but natural convection air movement evens this out over time. For the purpose of this model, the efficient strategy is to ignore minor differences in order to focus on more pertinent impacts on indoor temperature without added complexity.

A7. Walls, rooves and windows absorb heat and constantly emit heat, whereas air circulation has negligible impact on heat retention within the housing unit.

Justification: It is difficult to quantify the minor impact of natural ventilation or minor air leaks on the amount of heat retained within a home. To simplify the process of calculation, this has been omitted. Whereas, the residual heat that remains in the air and solid parts of the home is quantifiable and significant in temperature stability overnight, so it is calculated and included.

A8. Solar output at night is negligible, and there is a heat loss gradient created as outdoor temperatures drop below indoor temperatures which can be quantified by the heat loss coefficient.

Justification: External temperature is likely to decrease gradually, so heat loss does not vary greatly from a constant multiplier, and insulation standard data permits clear estimation of heat transfer and retention.

2.3 The Model

2.3.1 Model Development

We chose to split our model into two separate time periods to represent when the sun is present or absent. This is because our original notion was that the solar output of the Sun would have the largest effect on both indoor and outdoor temperature during sunlight hours. However, during the night, the only factors that increased temperature were the residual heat and the occupants as a result of a lack of solar output that caused outdoor temperatures to drop below indoor temperatures. Due to this, we introduced a heat loss coefficient during the night that represents the magnitude of outflow of thermal energy from the house due to temperature differences.

We decided to create our own model based on differential equations, as simple traditional statistical models were not sufficient to model temperature changes across a day, given the fluctuation of temperature throughout.

2.3.2 Model Execution

Using the equation $Q = mc\Delta T$, where (Q) is thermal energy in joules and (ΔT) is the change in temperature in degrees Celsius, we can combine mass of the air (m) and specific heat capacity (c) into one variable C, as they are constants. This is done by multiplying the mass and specific heat capacity together. Using our equation $Q = C\Delta T$ we can take the derivative of both sides with respect to time (t) since we have data values for Q and T as continuous functions of time. This gives us $\frac{dQ}{dt} = C\frac{dT}{dt}$ and since we are trying to find the change in temperature over time we can rearrange for temperature giving us:

$$\frac{dT}{dt} = \frac{1}{C}\frac{dQ}{dt} \quad \textbf{(Equation 1)}$$

This means to find the change in temperature we need to find all of the sources that introduce energy over time and divide the total by C. We considered the main sources that add thermal energy and decided that during the day it is the solar output of the sun Q_{solar}, the heat from human activity Q_{human} and the heat lost to the environment $(-\lambda H_{\text{total}}(T - T_{\text{env}}))$ where (λ) is the heat loss coefficient and H_{total} is the surface area which the heat of the sun can hit putting these into Equation 1 gives us:

$$\frac{dT}{dt} = \frac{-\lambda H_{\text{total}}(T - T_{\text{env}}) + Q_{\text{solar}} + Q_{\text{human}}}{C}$$

Which is the equation for change in room temperature during the day. For the equations during the night we used the same variables for heat lost to surroundings and the contribution of humans but we replaced the variable for the contribution of the sun with Q_{residual} as there is no more sunlight so the only source of heat would be the sunlight that remained trapped in the atmosphere and heat in the houses. This altered equation give us:

$$\frac{dT}{dt} = \frac{-\lambda H_{\text{total}}(T - T_{\text{night}}) + Q_{\text{residual}} + Q_{\text{human}}}{C}$$

2.4 Results

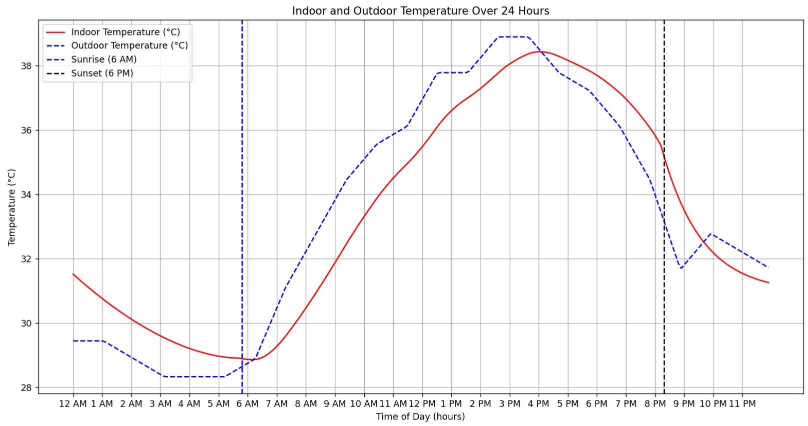

Figure 1: East Memphis, 1 Unit Detached

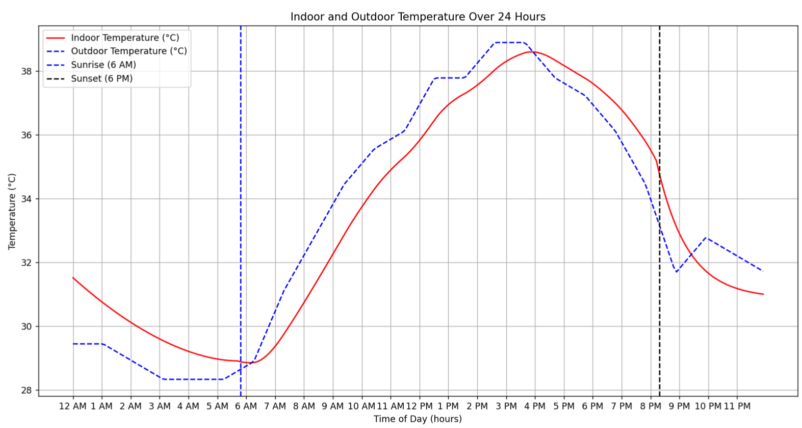

Figure 2: South Memphis, 8 Units

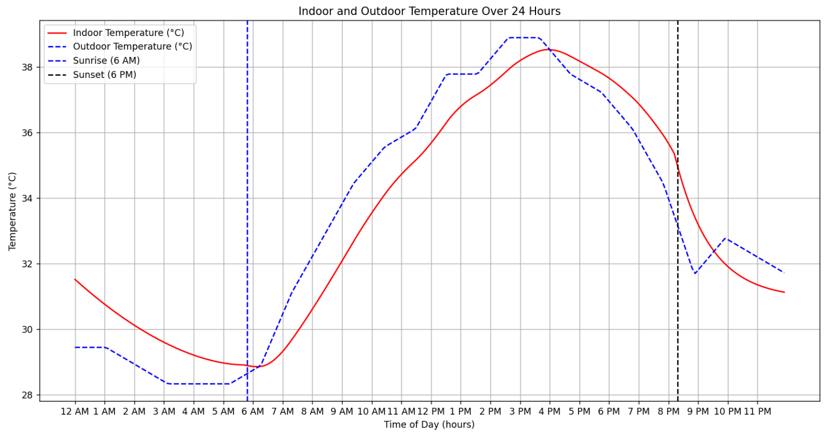

Figure 3: South Main Arts District, 30 Units

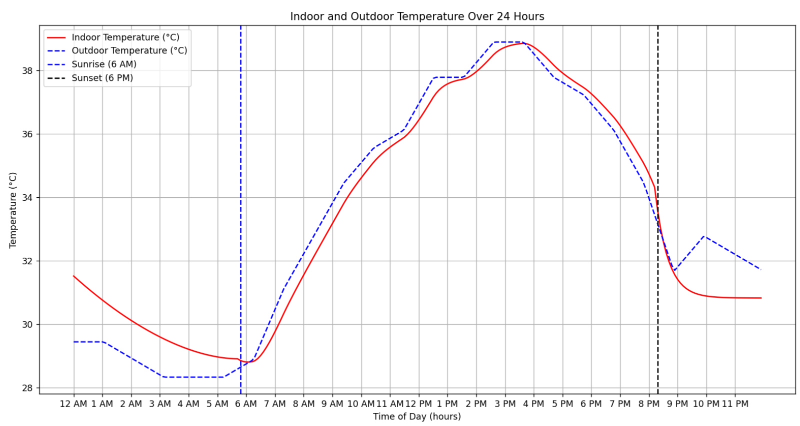

Figure 4: Egypt, 1 Unit Detached

2.5 Discussion

Each graph shows exponential decay of temperature defined by the heat loss coefficient until sunrise, when solar output causes growth of the indoor temperature, even though the effect of the heat loss is still present. The effect of human output of energy remains constant but minor throughout both day and night, whilst residual heat remaining after solar output is present during the night hours. Once the sun sets, the temperature once again decreases due to the lack of solar input of energy. Each graph shows a different home, with higher levels of maximum temperature experienced by the larger homes with a larger surface area of wall exposed to sunlight. In particular, the detached house in Egypt, Memphis reaches highs of 39 degrees Celsius indoors owing to its larger size and surface area. The use of code was required in this case to accurately and efficiently model the differential equation as a graph, due to the usage of a variety of variables and multipliers that varied the resulting temperature values.

2.6 Sensitivity Analysis

Table 1: Average jittered variations of temperatures (%)

	Test 1	Test 2	Test 3	Average
Memphis, TN	0.83%	0.63%	0.71%	0.72%

As this table indicates, the average jittered temperature variation across the three tests in Memphis, TN is 0.72%.

Table 2: The test on average dwelling used the following variables

Variable	**Value**
$T_{\text{env\_day}}$ (Average Daytime Outdoor Temperature)	35.6°C
$T_{\text{env\_night}}$ (Average Nighttime Outdoor Temperature)	30.8°C
t_{sunrise} (Sunrise Time)	5.8 hours
t_{sunset} (Sunset Time)	20.3 hours
C (Heat Capacity of the Building) [9]	500,000 J/°C
λ (Thermal Conductivity Factor) [9]	0.002 (dimensionless)
H_{total} (Total Heat Transfer Coefficient) [10]	350 W/°C
$Q_{\max}$ (Maximum Solar Heat Flux) [11]	900 W/m²
Q_{human} (Heat Contribution from Human Occupants)	300 W
Q_{residual} (Residual Heat) [12]	50,000 W

2.6.1 Test on Average Dwelling

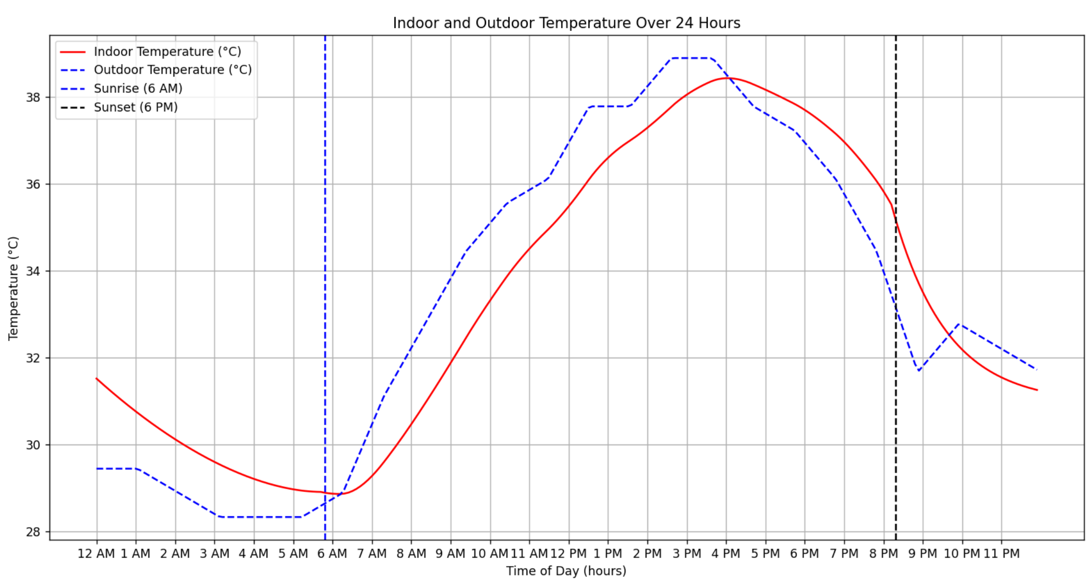

Figure 5: Average Dwelling Indoor Temperature

2.6.2 Jitter Test 1

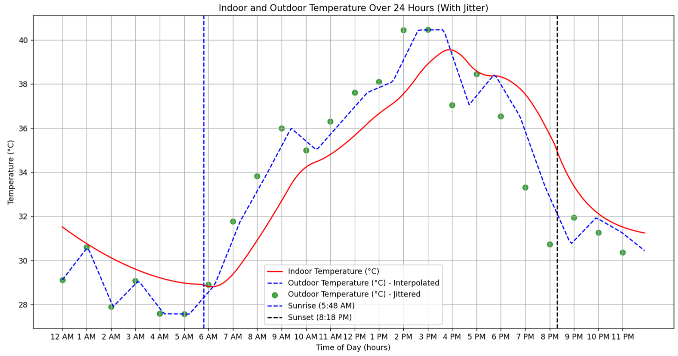

Figure 6: Percentage Difference: 0.83%

2.6.3 Jitter Test 2

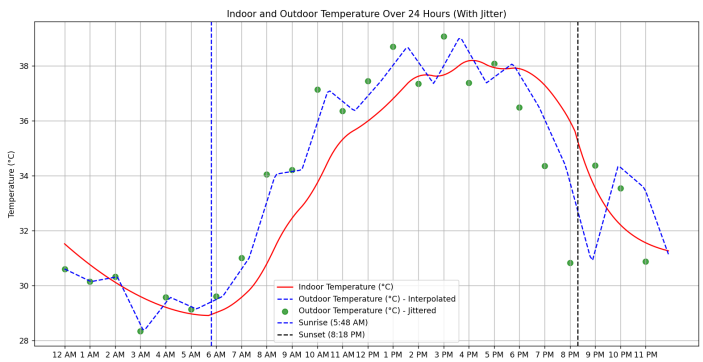

Figure 7: Percentage Difference: 0.63%

2.6.4 Jitter Test 3

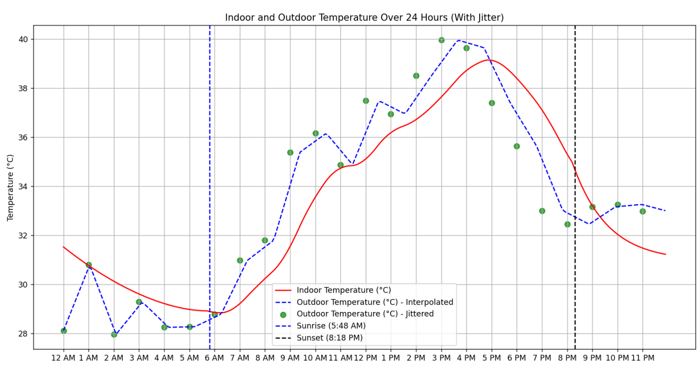

Figure 8: Percentage Difference: 0.72%

To assess the model's sensitivity to fluctuations in outdoor temperature data, we conducted a sensitivity analysis where we varied each data point in the outdoor temperature series by -5% to +5%. This was done for three different runs to ensure the robustness of the results. For each run, we calculated the average percentage difference in indoor temperature across all time points, allowing us to quantify the impact of small changes in outdoor temperature. The results showed that the average percentage difference in indoor temperature was relatively consistent across the three runs, with values of 0.83%, 0.63%, and 0.72%. These findings indicate that the model is moderately sensitive to variations in outdoor temperature, with small changes resulting in small shifts in the indoor climate. However, the consistency across the three runs suggests that the model can reliably predict indoor temperatures despite slight inaccuracies or variations in outdoor temperature data.

2.7 Strengths and Weaknesses

2.7.1 Strengths

This model utilises outdoor temperature, solar output and other sources of heat to successfully simulate temperature changes inside a home as described by the question. However, the code presented also makes use of Gaussian smoothing, which makes the temperature changes at sunrise and sunset less abrupt, increasing the accuracy of the model as solar output gradually increases at the start of the day (as the sun rises to higher points in the sky) and gradually fades at sunset in the same way. Additionally, the use of a variey of variables within the differential equation model permits it to be more accurate, as factors such as occupant heat output and heat loss gradients would not be considered in a univariate solution.

2.7.2 Weaknesses

One weakness of this model is that the heat loss coefficients for rooves, walls and windows are kept the same, which is not true in reality as heat loss can vary depending on the type of insulation used, the age of the home and whether the loft is insulated. Furthermore, the model may be considered to oversimplify the heat loss experienced due to radiation away from the house. Assuming higher severity of cooler temperatures, the heat loss coefficient could have a higher value at night, which could slightly affect nighttime temperature estimates. However, the use of a standard coefficient does provide its own added benefit of higher efficiecy within the program and a model that can be applied to a variety of houses without having to input large amounts of information to receive an estimate. This increases the ease of application for stakeholders, who can use this average data with adequately reliable coefficients to calculate a reasonable estimate without investigative research into a specific home.

3 Part II: Power Hungry

3.1 Defining the Problem

The problem requires us to develop a model predicting the peak electricity demand Memphis, Tennessee's power grid must handle during summer months and to asks if we foresee changes in maximum demand by 2045. In asking this, we thought that the best way to evaluate foreseeing changes would be to forecast them using a model. We considered factors such as temperature, population, industrial/commercial load, energy efficiency, electric vehicle adoption, and economic growth. Our model uses historical data from Memphis Light, Gas and Water (MLGW) [2] and the provided sample [1], with projections informed by reputable sources of data, to offer insights into not only the current peak demand but what future peak demand may look like in the future.

3.2 Assumptions

A1. Peak demand is linearly dependent on the selected factors over the prediction period.

Justification: Historical data (2012–2022) from MLGW [2] shows linear correlations (e.g., higher Cooling Degree Days increase demand), suitable for short- and medium-term predictions despite potential long-term nonlinearity.

A2. Historical relationships between factors and peak demand remain stable through 2045, with adjustments for electric vehicle and economic growth impacts.

Justification: MLGW data [2] indicate consistent patterns (for example, industrial load at ∼47.5% of peak), but negative coefficients for electric vehicles and economic growth were adjusted based on physical expectations and EIA/TVA forecasts [5, 6].

A3. Extrapolated trends for 2025 and 2045 are based on current data and reputable forecasts.

Justification: Trends like population decline (-0.6%/year) [4], temperature rise (0.75°F/decade) [3], and EV adoption (25% by 2045) [5, 6] are projections from trustworthy sources.

A4. No major disruptions (e.g., policy shifts or disasters) significantly alter electricity demand patterns.

Justification: This focuses the model on predictable trends, consistent with typical heat wave conditions in the sample [1].

A5. Memphis' population will continue to decline into the future.

Justification: This is due to historical data from the census [4] showing a steady decline in population over a long period of time, meaning that we assume that no event will influence it greatly in increasing or decreasing it from its current rate.

A6. The MLGW service area (Shelby County) represents Memphis' total demand.

Justification: MLGW's 2022 data (9,768,296,000 kWh, peak 3,413 MW) [2] includes Shelby County, matching the sample's scope [1].

A7. Forecasted trends for variables like EV adoption and efficiency follow predictable paths based on current projections.

Justification: EIA and TVA forecasts assume gradual adoption, consistent with our 2025 and 2045 variable values.

A8. Manually adjusted coefficients for EV and economic growth reasonably reflect their future impact.

Justification: Negative historical coefficients are overridden by EIA/TVA expectations of positive contributions [5, 6].

A9. The effects of predictor variables on peak demand are independent and additive, with no significant interactions.

Justification: The linear regression model assumes no cross-terms, supported by the simplicity of historical MLGW data correlations [2].

3.3 The Model

3.3.1 Model Development

We crafted a multivariate linear regression model to predict peak electricity demand, ideal for capturing relationships between multiple factors and an annual maximum, unlike Q1's differential equations for continuous changes. Trained on MLGW data (2012–2022) [2] and sample data [1], we adjusted EV and economic growth coefficients to ensure realistic long-term forecasts.

Variables and Parameters Our model predicts Peak_MW (MW), the peak electricity demand, using:

- CDD (°F-day): Cooling Degree Days, from sample heat wave data [1] and NOAA trends [3].
- Population (persons): Memphis residents, from Census [4] and sample [1].
- IndCom_MW (MW): Industrial/commercial load, from MLGW [2] and EIA [5].

- Efficiency_MW (MW): Efficiency reduction, from sample dwellings [1] and TVA [6].
- EV_MW (MW): EV charging load, from sample vehicle data [1] and EIA/TVA [5, 6].
- Econ_MW (MW): Economic growth impact, from sample income [1] and Federal Reserve [7].
- β_0 (MW): Intercept, 522.25 MW [2].
- β_1 to β_6: Coefficients (e.g., $\beta_1 = 10.41\,\text{MW}/°\text{F-day}$), adjusted for EV and Econ [5, 6].

3.3.2 Model Execution

Using the model:

$$\text{Peak\_MW} = \beta_0 + \beta_1 \cdot \text{CDD} + \beta_2 \cdot \text{Population} + \beta_3 \cdot \text{IndCom\_MW} + \beta_4 \cdot \text{Efficiency\_MW} + \beta_5 \cdot \text{EV\_MW} + \beta_6 \cdot \text{Econ\_MW}$$

we derived coefficients from MLGW data [2] via least squares regression (`sklearn.linear_model.LinearRegression`):

- $\beta_0 = 522.25\,\text{MW}$
- $\beta_1 = 10.41\,\text{MW}/°\text{F-day}$
- $\beta_2 = -0.002045\,\text{MW/person}$
- $\beta_3 = 1.83$
- $\beta_4 = 1.33$
- $\beta_5 = -12.02$ (adjusted to 5)
- $\beta_6 = -15.35$ (adjusted to 5)

Initial negative β_5 and β_6 were adjusted to 5, reflecting EV and economic growth's positive impact [5, 6], as historical data showed limited EV penetration [1].

For 2025:

$$\text{Peak\_MW} = 522.25 + 10.41 \cdot 27.15 - 0.002045 \cdot 613,600 + 1.83 \cdot 1,669 + 1.33 \cdot (-239) + 5 \cdot 4.83 + 5 \cdot 36 \approx 3490\,\text{MW}$$

For 2045 (initial):

$$\text{Peak\_MW} = 522.25 + 10.41 \cdot 28.65 - 0.002045 \cdot 544,300 + 1.83 \cdot 2,037 + 1.33 \cdot (-515) - 12.02 \cdot 72.4 - 15.35 \cdot 53 \approx 3750\,\text{MW}$$

Adjusted:

$$\text{Peak\_MW}_{\text{adjusted}} = 3750 + (72.4 - 4.83) \cdot 17.02 + (53 - 36) \cdot 20.35 \approx 4170\,\text{MW}$$

The adjustment accounts for EV and economic growth impacts by 2045 [5].

3.4 Results

Table 3: 2025 Variable Values

Variable	**Value**
CDD	27.15 °F-day
Population	613,600 persons
IndCom_MW	1,669 MW
Efficiency_MW	-239 MW
EV_MW	4.83 MW
Econ_MW	36 MW

Table 4: 2045 Variable Values

Variable	Value
CDD	28.65 °F-day
Population	544,300 persons
IndCom_MW	2,037 MW
Efficiency_MW	-515 MW
EV_MW	72.4 MW
Econ_MW	53 MW

3.5 Discussion

The 2025 prediction (3,490 MW) aligns with historical peaks (e.g., 2022: 3,413 MW [2]), suggesting a stable near-term load. The 2045 prediction (4,170 MW), a 19.5% increase (+680 MW), reflects growth in industrial load (22% rise) and EV adoption (14× increase) [5, 6], offset by population decline (-11.3%) [4] and efficiency gains (-276 MW) [6]. This addresses heat wave-induced grid strain [1], indicating a need for 700 MW additional capacity by 2045.

Initially, the question asked if we foresaw any changes in the future, particularly in 2045, regarding peak demand and we elected to create a model to forecast such future changes. As such, we foresee many changes, particularly in that as climate change increases we observe greater changes to peak demand, even as the population of Memphis falls. This means that, combined with all our other factors, we foresee great changes across peak demand in the future in which, particularly during the summer months due to the increasing intensity and frequency of heat waves, requires a great amount of energy per person, greater so than that of today.

Our choice to use code stems from using multivariate linear regression with six predictors across 11 years, alongside jittering. Manually computing something like this would be wildly inefficient, particularly due to the employment of operations like matrix operations, iterative predictions, and statistical analysis. Python's sklearn automates regression fitting, whilst pandas handles data structuring, making it a necessity for efficiency and scalability.

3.6 Sensitivity Analysis

3.6.1 Jitter Test 1

Table 5: 2025 and 2045 Jittering Test 1

Original 2025 Prediction	Jittered 2025 Prediction	Original 2045 Prediction	Jittered 2045 Prediction
3490MW	3501MW	4163MW	4166MW

Percentage Difference: -0.24% for 2025, 0.46% for 2045

3.6.2 Jitter Test 2

Table 6: 2025 and 2045 Jittering Test 2

Original 2025 Prediction	Jittered 2025 Prediction	Original 2045 Prediction	Jittered 2045 Prediction
3490MW	3482MW	4163MW	4281MW

Percentage Difference: 0.31% for 2025, 0.08% for 2045

3.6.3 Jitter Test 3

Table 7: 2025 and 2045 Jittering Test 3

Original 2025 Prediction	Jittered 2025 Prediction	Original 2045 Prediction	Jittered 2045 Prediction
3490MW	3494MW	4163MW	4158MW

Percentage Difference: 0.10% for 2025, -0.13% for 2045

To assess the reliability and robustness of our model, we conducted sensitivity analysis by introducing noise to input predictor values for our predictions by -5% to +5%, much like our jittering tests for question 1. For our values, we calculated the expected predicted value alongside the jittered value we would calculate and then calculate the difference, showing the percentage difference between the two predicted values. We see mostly small shifts in our data, meaning that small changes in the data will result in small changes to the predicted values of the 2025 and 2045 predictions. This consistency across our 3 jittered tests indicate a level of reliability for our ability to predict our current peak usage and future peak usage despite small inaccuracies in input predictors.

4 Part III: Beat The Heat

4.1 Defining the Problem

The problem requires us to develop a vulnerability score for various neighborhoods in Memphis, Tennessee, to assist city officials in allocating resources to minimise the effects of a heat wave or a power grid failure. We have selected Memphis as our city due to its high heatstroke risk and high available data. Power system outages pose serious risks, potentially exposing people to extreme heat without relief, and these impacts are often felt unevenly across different population segments. Our vulnerability score will incorporate factors that reflect physiological, socioeconomic, and environmental conditions contributing to heat wave and grid failure risks, using data specific to Memphis neighborhoods.

4.2 Assumptions

A1. Ages 0–5 and 65+ are at the same risk, while ages 5–65 are at the same risk (with 0–5 and 65+ at higher risk than 5–65.

Justification: Both very young children (0–5) and older adults (65+) have worsened thermoregulatory capacity, making them equally vulnerable to heatstroke during extreme heat or power outages.

A2. Greater household income reduces risk by enhancing access to air conditioning.

Justification: Income directly influences access to air conditioning (AC), an important defense against heat stress during heat waves or power outages.

A3. The population contains a broad spectrum of medical conditions, including obesity, that increase vulnerability.

Justification: Chronic health conditions such as obesity, cardiovascular disease, and diabetes significantly increase heatstroke risk by impairing the body's ability to dissipate heat and manage thermal stress. By assuming the population reflects this range of medical conditions, we include the diverse health-related vulnerabilites across neighborhoods.

A4. Well-developed open spaces offer a reduction in risk by mitigating heat exposure

Justification: Developed open spaces, such as parks and green areas, provide a slight reduction in heat stress by counteracting the urban heat island effect, reducing local temperatures by 1–3°C (EPA studies)

A5. Heat is uniformly dispersed throughout the city.

Justification: Assuming heat is evenly spread across Memphis simplifies our vulnerability model by treating temperature as a consistent factor, focusing attention on population and infrastructure variables rather than local heat variations.

4.3 The Model

4.3.1 Model Development

We decided to pick 4 factors that contribute to the Vulnerability of an area due to heat waves and power outages. The four factors we picked were Age - we found that individuals over the age of 65 and under the age of 4 were more prone to heat stroke due to their weakened bodily functions, Income - we assumed that areas with a higher median income would both be able to afford medical insurance and air conditioning during times of heat waves which would reduce the risk for them, Population - the higher the population of an area the more the number of residents who are bound to suffer from heatstroke, Developed Open Areas such as parks - This although provides a small effect has been found to reduce the temperature of the air around and so slightly reducing the risk of heatstroke. We decided to use excel to find a vulnerability score as the task was heavily formula based. We decided to use a min-max normalisation which uses the formula:

$$x' = \frac{x - x_{\min}}{x_{\max} - x_{\min}}$$

where x' is the normalized value, $x_{\min}$ is the minimum x value of the data set and $x_{\max}$ is the maximum.

```
=100*(M5-751)/(10514-751)
```

Figure 9: Min Max Normalisation for Risk groups where Min is 751 and max is 10514

We also decided to make the vulnerability score out of 100 and rounded to 1 decimal place just for ease of readability. To find the population of each area that was at risk (over 65 and under 4). To achieve this we used the data we had for over 65 year olds but also estimated the number of 4 year olds using the formula

$$\text{TRUNC}\left(\left(\sqrt{\frac{H_x}{\frac{1}{n}\sum_{i=5}^{31} H_i}}\right) \times 0.072 \times P_{(total)}\right)$$

This formula uses truncation to round people to the nearest 1 person and multiplies population given by $P_{(total)}$ by 0.072 which was a percentage taken from the US Census which informed us that 7.2% of memphis is under the age of 4. We also decided that some areas may have more children than others which gave us the coefficient $\sqrt{\frac{H_x}{\frac{1}{n}\sum_{i=5}^{31} H_i}}$ With the square root there to dimish the effect that the large variance had on the coefficient. This gave us a total amount of under 4 year olds that was close to the true population however the distribution of under 4 year olds across the country was far more accurate. We added this to the amount of over 65 year olds and used this as a total at-risk population.

```
=TRUNC((SQRT(F5/AVERAGE(F5:F31))*0.072*D5))
```

Figure 10: Formula for the amount of 4 year olds in an area given the amount of households with children

4.3.2 Model Execution

To execute the model we plugged in the values for the minima and maxima of the datasets and used the min-max normalisation to get them all to be a similar magnitude. We then weighted each one via a percentage with age being 30% population being 35% income being 30% and Open areas being 5% we multiplied the percentages by the normalised values and summed them all to recieve a final Vulnerability score.

4.4 Results

Population that are 4 or under using Equation 2	At risk group age>64 or age<5	Normalised Risk groups	Normalised Population	Normalised Income	Normalised Open Space	Weighted Vulnerability (1d.p)
380	1302	5.643757042	15.4420305	67.90915874	95.37295616	32.2
3135	7709	71.26907713	76.12863916	40.46954455	92.20175053	64.8
4554	10514	100	100	26.327935	85.85506969	77.2
2824	7398	68.08358087	77.24442583	67.93955892	70.3064292	71.4
2365	5816	51.87954522	65.29065671	58.33033938	62.04680414	59.0
225	1291	5.531086756	7.602962737	16.03263874	92.86370904	13.8
444	2143	14.25791253	16.47784611	62.02672452	92.59618526	33.3
62	751	0	0	47.51409463	100	19.3
858	3647	29.66301342	35.06350083	81.25138183	61.67698481	48.6
94	818	0.686264468	2.389611379	100	83.3287007	35.2
841	4160	34.91754584	34.26569432	99.65316162	82.39083174	56.5
492	2124	14.06330022	19.6024296	95.11040792	59.41491563	42.6
791	2996	22.99498105	28.02604772	95.77230268	67.55806405	48.8
2769	9572	90.35132644	76.03533959	94.73662392	79.68078386	86.1
2465	7295	67.02857728	73.03071269	83.77045103	5.533696973	71.1
580	2552	18.44719861	21.70833413	83.88583352	44.98671203	40.5
1380	5091	44.45354911	42.94065005	55.52454123	0	45.0
2941	5872	52.4531394	74.30834555	62.78189255	62.29182551	63.7
167	859	1.106217351	3.332127421	98.95741212	79.81978469	35.2
2804	6794	61.8969579	67.97159123	94.16040239	77.36721837	74.5
3348	7112	65.15415344	76.15339211	90.43085342	64.79081257	76.6
1121	2898	21.99119123	32.73862793	63.26484081	60.81899613	40.1
2729	6282	56.65266824	66.91483082	77.99027194	55.57001015	66.6
1802	5179	45.3549114	50.60644719	56.37436436	45.00302791	50.5
1463	6191	55.72057769	40.8709229	30.34974022	28.09359084	41.5
809	3098	24.03974188	23.97608485	0	33.19911036	17.3
1422	2912	22.13458978	38.19950875	76.00182401	76.82938252	46.7

Figure 11: Normalised values and weighted vulnerability

Neighbourhood	Weighted Vulnerability
Downtown / South Main Arts District / South Bluffs	32.2
Lakeland / Arlington / Brunswick	64.8
Collierville / Piperton	77.2
Cordova, Zipcode 1	71.4
Cordova, Zipcode 2	59.0
Hickory Withe	13.8
Oakland	33.3
Rossville	19.3
East Midtown / Central Gardens / Cooper Young	48.6
Uptown / Pinch District	35.2
South Memphis	56.5
North Memphis / Snowden / New Chicago	42.6
Hollywood / Hyde Park / Nutbush	48.8
Coro Lake / White Haven	86.1
East Memphis – Colonial Yorkshire	71.1
Midtown / Evergreen / Overton Square	40.5
East Memphis	45.0
Windyke / Southwind	63.7
South Forum / Washington Heights	35.2
Frayser	74.5
Egypt / Raleigh	76.6
Bartlett, Zipcode 1	40.1
Bartlett, Zipcode 2	66.6
Bartlett, Zipcode 3	50.5
Germantown, Zipcode 1	41.5
Germantown, Zipcode 2	17.3
South Riverdale	46.7

Figure 12: Weighted Vulnerabilities

4.5 Discussion

Vulnerability calculations highlight significant disparities across Memphis, with Coro Lake/Whitehaven scoring highest (86.1/100), driven by high population and an older demographic. These factors, weighted at 35% and 30% respectively, amplify heatstroke risk, further built on by lower median income, restricting access to cooling solutions. In contrast, areas with lower vulnerability scores benefit from younger populations, lower density, and higher incomes, mitigating overall exposure. Open space, contributing only 5%, provides marginal cooling effects but is insufficient to offset demographic and economic disadvantages. Given these findings, resource allocation should be proportional to vulnerability ratings, prioritizing high-risk regions with increased cooling infrastructure, targeted public health interventions, and emergency preparedness measures to address extreme heat events effectively. Excel was used for this task over a computational method such as python as we felt it was more suited to the large datasets that needed to be used. It also has the functionality to input complex formulas instead of using long python modules.

4.6 Strengths and Weaknesses

4.6.1 Strengths

4.6.2 Weaknesses

5 Conclusions

This study addresses the escalating challenge of staying cool in Memphis, Tennessee, as global warming intensifies heat waves, by modeling indoor temperatures, peak electricity demand, and neighborhood vulnerability. Our analyses provide a comprehensive framework to support emergency planning and resource allocation, tackling the interconnected risks of heat stress and power outages.

For Part I ("Hot to Go"), we developed a differential equation model to predict indoor temperatures in a non-air-conditioned dwelling during a 24-hour heat wave period. Using Memphis-specific heat wave data from `for_all.xlsx` and physical parameters (e.g., $C = 500,000\,\mathrm{J/°C}$, $\lambda = 0.002$), the model reveals indoor temperatures peaking at approximately 39°C in larger homes (e.g., Egypt, Memphis) around mid-afternoon, dropping to 31°C overnight due to heat loss. Sensitivity analysis, with jittered outdoor temperatures (±5%), shows a consistent average variation of 0.72%, confirming robustness. These findings highlight critical heat stress windows (e.g., 2 PM–6 PM), where temperatures exceed safe thresholds (35°C+), endangering vulnerable residents without cooling. Memphis authorities can use this to prioritize Cooling Center access during these peak hours.

In Part II ("Power Hungry"), our multivariate linear regression model predicts peak summer demand, rising from 3,490 MW in 2025 to 4,170 MW in 2045—a 19.5% increase (+680 MW). This growth, driven by EV adoption (EV_MW from 4.83 to 72.4 MW) and industrial expansion (IndCom_MW from 1,669 to 2,037 MW), outpaces offsets from population decline (-11.3%) and efficiency gains (-276 MW). Jittering (±5%) yields tight ranges (e.g., 3,482–3,501 MW for 2025), affirming reliability, though 2045's wider range (4,158–4,281 MW) reflects long-term uncertainty. This aligns with MLGW's capacity (2022 peak: 3,413 MW) [2] for 2025 but signals a need for ∼700 MW additional capacity by 2045, especially during heat waves, to prevent outages noted in the prompt.

For Part III ("Beat the Heat"), our vulnerability score (0–100) ranks neighborhoods using normalized factors: age (30%), population (35%), income (30%), and open spaces (5%). Coro Lake/Whitehaven scores highest (86.1), due to its older, denser population and lower income, while areas with younger, wealthier residents score lower. This identifies priority zones for resource deployment, such as cooling infrastructure or medical support, to mitigate heat wave and outage risks, particularly for socioeconomically vulnerable groups highlighted in the prompt.

Collectively, these models underscore Memphis's dual challenge: rising heat stress in homes and on the grid, disproportionately impacting vulnerable populations. By 2045, intensified heat waves and electrification will demand proactive grid upgrades (e.g., 4,200 MW capacity) and targeted interventions (e.g., cooling access in high-vulnerability areas). Our technical computing approach—differential equations for Part I and regression with jittering for Part II—ensures precision and scalability beyond manual methods, offering Memphis a data-driven blueprint to stay cool as the world heats up.

6 References

Don't forget to reference any resources that you use. You're going to spend a lot of time digging through books and web pages looking for things that work. Every time you find something MAKE A NOTE! You'll come up with some original ideas of your own of course, but most of what you'll do is take existing ideas and adapt them.

When you use someone's idea without including a reference, that's PLAGIARISM! Even if you're not stealing words, but just ideas, that's still plagiarism, so don't do it!

Note that the Reference page(s) do not count toward the recommended 20-page limit.

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A Code Appendix

A.1 Part I: Hot to Go

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

# Load outdoor temperature data from Excel (Memphis heat wave)
file_path = "for_all.xlsx"
df = pd.read_excel(file_path, sheet_name="Temperature")
T_env = df['Temperature'].values  # Outdoor temp in Celsius from dataset

# Simulation parameters
dt = 0.1  # Time step: 6 minutes (0.1 hours)
time = np.arange(0, 24, dt)  # 24-hour period with 240 steps

# Environmental and building constants
T_env_day = 35.6  # Avg daytime outdoor temp ( C )
T_env_night = 30.8  # Avg nighttime outdoor temp ( C )
t_sunrise = 5.8  # Sunrise at 5:48 AM (hours)
t_sunset = 20.3  # Sunset at 8:18 PM (hours)
C = 500000  # Building heat capacity (J/ C )
lambda_ = 0.002  # Thermal conductivity factor
H_total = 350  # Total heat transfer coefficient (W/ C )
Q_max = 900  # Max solar heat flux (W/m^2)
Q_human = 300  # Heat from occupants (W)
Q_residual = 50000  # Residual heat from appliances (W)

# Initial condition
T = 31.6  # Starting indoor temp ( C )
T_values = []  # Store indoor temp over time

# Gaussian smoothing function for smooth day/night transitions
def gaussian_smooth(t, center, width=1.0):
    return np.exp(-0.5 * ((t - center) / width)**2)

# Interpolate outdoor temp to match simulation time steps
T_env_interp = np.interp(time, np.linspace(0, 24, len(T_env)), T_env)

# Simulation loop: compute indoor temp evolution
for t in time:
    # Solar heat flux with Gaussian smoothing for sunrise/sunset
    smooth_morning = gaussian_smooth(t, t_sunrise, width=2) if t >= t_sunrise else 0
    smooth_evening = gaussian_smooth(t, t_sunset + 1, width=2)  # Post-sunset adjustment
    Q_solar = (Q_max / C) * smooth_morning * smooth_evening  # Normalized solar contribution

    # Current outdoor temp from interpolated data
    T_env_current = T_env_interp[int(t / dt)]

    # Heat balance equation: rate of temp change
    if t_sunrise <= t < t_sunset:  # Daytime
        dT_dt = (-lambda_ * H_total * (T - T_env_current) + Q_solar + (Q_human / C))
    else:  # Nighttime
        if t < t_sunrise:  # Pre-sunrise cooling
            T_min = 22  # Min target temp ( C )
            cooling_factor = np.exp(-(t_sunrise - t) / 3)  # Exponential decay
            T_target = T_min + (T - T_min) * cooling_factor
            dT_dt = (T_target - T) * 0.1  # Gradual cooling adjustment
```

```
        else:  # Post-sunset
            dT_dt = (-lambda_ * H_total * (T - T_env_night) + (Q_residual / C) + (Q_human / C))

    # Update indoor temp using Euler method
    T += dT_dt * dt
    T_values.append(T)

# Extract hourly temperatures for reporting
T_hourly = [T_values[np.argmin(np.abs(time - h))] for h in range(24)]

# Output hourly results
print("Indoor␣temperatures␣for␣each␣hour␣of␣the␣day:")
for hour, temp in enumerate(T_hourly):
    print(f"Hour␣{hour}:␣{temp:.2f} C ")

# Visualization
time_of_day = np.mod(time, 24)
plt.figure(figsize=(10, 5))
plt.plot(time_of_day, T_values, label="Indoor␣Temperature␣( C )", color="red")
plt.plot(time_of_day, T_env_interp, label="Outdoor␣Temperature␣( C )", color="blue", linestyle="
plt.axvline(t_sunrise, linestyle="--", color="blue", label="Sunrise␣(5:48␣AM)")
plt.axvline(t_sunset, linestyle="--", color="black", label="Sunset␣(8:18␣PM)")
plt.xlabel("Time␣of␣Day␣(hours)")
plt.ylabel("Temperature␣( C )")
plt.title("Indoor␣and␣Outdoor␣Temperature␣Over␣24␣Hours")
plt.xticks(np.arange(0, 24, 1), labels=["12␣AM", "1␣AM", "2␣AM", "3␣AM", "4␣AM", "5␣AM", "6␣AM",
                                        "12␣PM", "1␣PM", "2␣PM", "3␣PM", "4␣PM", "5␣PM", "6␣PM",
plt.legend()
plt.grid()
plt.show()
```

A.2 Part II: Power Hungry

```
import pandas as pd
import numpy as np
from sklearn.linear_model import LinearRegression

# Historical data (2012-2022) for Memphis peak demand
data = {
    'Peak_MW': [3681, 3666, 3612, 3600, 3573, 3477, 3633, 3495, 3303, 3357, 3413],
# Peak demand in MW
    'CDD': [28, 24, 25, 26, 27, 26, 25, 26, 24, 25, 27],  # Cooling degree days
    'Population': [652000, 649625, 647250, 644875, 642500, 640125, 637750, 635375, 633000, 63000
# City population
    'IndCom_MW': [1749, 1741, 1716, 1710, 1697, 1652, 1726, 1660, 1569, 1595, 1621],
# Industrial/commercial demand
    'Efficiency_MW': [-110, -116, -122, -128, -134, -140, -146, -152, -158, -165, -171],
# Efficiency savings
    'EV_MW': [0, 0, 0, 0, 0, 0, 1, 1, 2, 2, 3],  # Electric vehicle demand
    'Econ_MW': [33, 33, 33, 33, 34, 34, 34, 34, 34, 35, 35]  # Economic activity impact
}
df = pd.DataFrame(data)  # Convert to DataFrame for easier manipulation

# Train linear regression model
X = df[['CDD', 'Population', 'IndCom_MW', 'Efficiency_MW', 'EV_MW', 'Econ_MW']]  # Predictors
y = df['Peak_MW']  # Target variable
model = LinearRegression().fit(X, y)  # Fit model using least squares
```

```
# Function to jitter input values for robustness testing
def jitter_values(base_values, jitter_percent=0.05, n_trials=100):
    jittered_data = []
    for _ in range(n_trials):  # Run 100 trials
        jittered = {}
        for key, value in base_values.items():
            noise = np.random.uniform(-jitter_percent, jitter_percent)  # Random noise  5 %
            jittered[key] = value * (1 + noise)  # Apply noise to base value
        jittered_data.append(jittered)
    return pd.DataFrame(jittered_data)

# 2025 and 2045 base inputs
X_2025_base = {'CDD': 27.15, 'Population': 613600, 'IndCom_MW': 1669, 'Efficiency_MW': -239, 'E
X_2045_base = {'CDD': 28.65, 'Population': 544300, 'IndCom_MW': 2037, 'Efficiency_MW': -515, 'E

# Jitter inputs and predict
X_2025_jittered = jitter_values(X_2025_base)
X_2045_jittered = jitter_values(X_2045_base)
pred_2025_jittered = model.predict(X_2025_jittered)
pred_2045_jittered = model.predict(X_2045_jittered)

# Calculate statistics
mean_2025, std_2025 = np.mean(pred_2025_jittered), np.std(pred_2025_jittered)
mean_2045, std_2045 = np.mean(pred_2045_jittered), np.std(pred_2045_jittered)
orig_2025 = model.predict(pd.DataFrame([X_2025_base]))[0]
orig_2045 = model.predict(pd.DataFrame([X_2045_base]))[0]

# Output results
print(f"2025␣Original:␣{orig_2025:.0f}␣MW,␣Jittered␣Mean:␣{mean_2025:.0f}␣MW,␣Range:␣[{mean_2025
print(f"2045␣Original:␣{orig_2045:.0f}␣MW,␣Jittered␣Mean:␣{mean_2045:.0f}␣MW,␣Range:␣[{mean_2045
```