GROUP PROBLEM 12
MATH 271 TAE UK YOU.

1, Evaluate Integrals.

(a) $\int e^{\cos t} \cdot \sin t \, dt$

$u = \cos t$
$du = -\sin t \, dt$
$\therefore dt = \frac{du}{-\sin t}$

$= \int e^u \cdot \sin t \cdot \frac{du}{-\sin t}$
$= -\int e^u du$
$= -e^u$

$\therefore -e^{\cos t} + C$

(b) $\int \frac{2^x}{2^x+3} dx$

$u = 2^x + 3$
$du = \ln(2)(2^x) \cdot dx$
$\therefore dx = \frac{du}{\ln 2(2^x)}$

$\int \frac{2^x}{u} \cdot \frac{du}{\ln(2)(2^x)}$
$= \int \frac{1}{\ln 2 u} \cdot du$
$= \frac{1}{\ln(2)} \cdot \int \frac{du}{u}$
$= \frac{1}{\ln(2)} \ln|u|$
$= \frac{1}{\ln(2)} \ln|2^x+3|$

$\therefore \frac{1}{\ln(2)} |2^x+3| + C.$

(c) $\int \frac{\cos(\ln(y))}{y} dy. = \int \frac{\cos(u)}{y} \cdot du \cdot y$
$= \sin(u)$
$= \sin(\ln(y))$

$u = \ln y$
$du = \frac{dy}{y}$
$\therefore dy = du \cdot y$

$\therefore \sin(\ln(y)) + C$

(d) $\int x\sqrt{x+2} \, dx. = \int x\sqrt{u} \cdot du$

$u = x+2$
$du = 1 \cdot dx$
$x = u-2$

$= \int (u-2) \cdot \sqrt{u} \cdot du$
$= \int u^{\frac{3}{2}} du - \int 2\sqrt{u} \cdot du$
$= \frac{u^{\frac{3}{2}+1}}{\frac{3}{2}+1} - 2 \cdot \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}$
$= \frac{2}{5} \cdot u^{\frac{5}{2}} - \frac{4}{3} u^{\frac{3}{2}}$

$\therefore \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

(e) $\int \frac{1+x}{1+x^2} dx$

$= \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx$
$= \arctan(x) + \frac{1}{2} \ln|1+x^2|$

$u = 1+x^2$
$\int \frac{1}{2u} \cdot du.$
$= \frac{1}{2} \int \frac{1}{u} \cdot du.$
$= \frac{1}{2} \ln|u|$
$= \frac{1}{2} \ln|1+x^2|$

$\therefore \arctan(x) + \frac{1}{2} \ln|1+x^2| + C$