

GROUP PROBLEM 12  
MATH 1271 TAKE ONE YOU.

1. Evaluate Integrals.

$$(a) \int e^{\cos t} \cdot \sin t \, dt$$

$$\begin{aligned} u &= \cos t \\ du &= -\sin t \, dt \\ \therefore dt &= \frac{du}{-\sin t} \end{aligned}$$

$$\therefore -e^{\cos t} + C$$

$$\begin{aligned} &= \int e^u \cdot \sin t \cdot \frac{du}{-\sin t} \\ &= - \int e^u \, du \\ &= -e^u \end{aligned}$$

$$(b) \int \frac{2x}{2^x+3} \, dx$$

$$\begin{aligned} u &= 2^x + 3 \\ du &= \ln(2^x) \cdot dx \\ \therefore dx &= \frac{du}{\ln(2)} \end{aligned}$$

$$\begin{aligned} &\int \frac{2x}{u} \cdot \frac{du}{\ln(2^x)} \\ &= \int \frac{1}{\ln(2)u} \, du \\ &= \frac{1}{\ln(2)} \cdot \int \frac{du}{u} \\ &= \frac{1}{\ln(2)} \ln(u) \\ &= \frac{1}{\ln(2)} \ln(2^x+3) \end{aligned}$$

$$\therefore \frac{1}{\ln(2)} \ln(2^x+3) + C$$

$$(c) \int \frac{\cos(\ln(y))}{y} \, dy = \int \frac{\cos(u)}{y} \, du \cdot y$$

$$\begin{aligned} u &= \ln y \\ du &= \frac{dy}{y} \\ \therefore dy &= du \cdot y \end{aligned}$$

$$\begin{aligned} &= \sin(u) \\ &= \sin(\ln(y)) \end{aligned}$$

$$\therefore \sin(\ln(y)) + C$$

$$(d) \int x \sqrt{x+2} \, dx = \int x \sqrt{u} \cdot du$$

$$\begin{aligned} u &= x+2 \\ du &= 1 \cdot dx \\ x &= u-2 \end{aligned} \quad \begin{aligned} &= \int (u-2) \cdot \sqrt{u} \cdot du \\ &= \int u^{\frac{1}{2}} du - 2 \int u^{\frac{1}{2}} du \\ &= \frac{u^{\frac{3}{2}}}{\frac{3}{2}+1} - 2 \cdot \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} \\ &= \frac{2}{5}u^{\frac{5}{2}} - \frac{4}{3}u^{\frac{3}{2}} \end{aligned}$$

$$\therefore \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$$

$$(e) \int \frac{1+x}{1+x^2} \, dx$$

$$\begin{aligned} &= \int \frac{1}{1+x^2} \, dx + \int \frac{x}{1+x^2} \, dx \rightarrow \int \frac{1}{2u} \cdot du \\ &= \arctan(x) + \frac{1}{2} \ln|1+x^2| = \frac{1}{2} \int \frac{1}{u} \, du \\ &= \frac{1}{2} \ln|1+x^2| \\ &= \frac{1}{2} \ln|1+x^2| \end{aligned}$$

$$\therefore \arctan(x) + \frac{1}{2} \ln|1+x^2| + C$$