



ABCD is a square, PQRS is rhombus, calculate $[CQR]$?

Let $PQ = QR = RS = SP = x, CQ = y$ ($0 < x, y < a$)

Not hard to see that:

$$DP = \frac{CQ}{\sqrt{2}} = \frac{y}{\sqrt{2}}, AP = PQ = x$$

$$\text{Then, } AD = a = AP + PD = x + \frac{y}{\sqrt{2}}, DP = \sqrt{x^2 - \frac{y^2}{2}}$$

$$\Rightarrow x = a - \frac{y}{\sqrt{2}}$$

$$\Rightarrow x^2 = a^2 - \sqrt{2}ay + \frac{y^2}{2}$$

Using Thales theorem with $SC \parallel AB$, we have:

$$\frac{SC}{AB} = \frac{CQ}{QA}$$

$$\Rightarrow \frac{a - \sqrt{x^2 - \frac{y^2}{2}}}{a} = \frac{y}{\sqrt{2}a - y}$$

$$\Rightarrow \sqrt{2}a^2 - 2ay = \sqrt{2x^2 - y^2} \left(a - \frac{y}{\sqrt{2}} \right) = x\sqrt{2x^2 - y^2}$$

$$\Rightarrow 2a^4 - 4\sqrt{2}a^3y + 4a^2y^2 = 2x^4 - x^2y^2 = 2 \left(a^2 - \sqrt{2}ay + \frac{y^2}{2} \right)^2 - \left(a^2 - \sqrt{2}ay + \frac{y^2}{2} \right) y^2$$

$$\Rightarrow 2a^4 - 4\sqrt{2}a^3y + 4a^2y^2 = 2a^4 - 4\sqrt{2}a^3y + 5a^2y^2 - \sqrt{2}ay^3$$

$$\Rightarrow y = \frac{a}{\sqrt{2}}$$

$$\Rightarrow Q \text{ is the center of } ABCD, S \equiv D$$

$$\Rightarrow \textbf{Impossible}$$

That means, there's no solution!