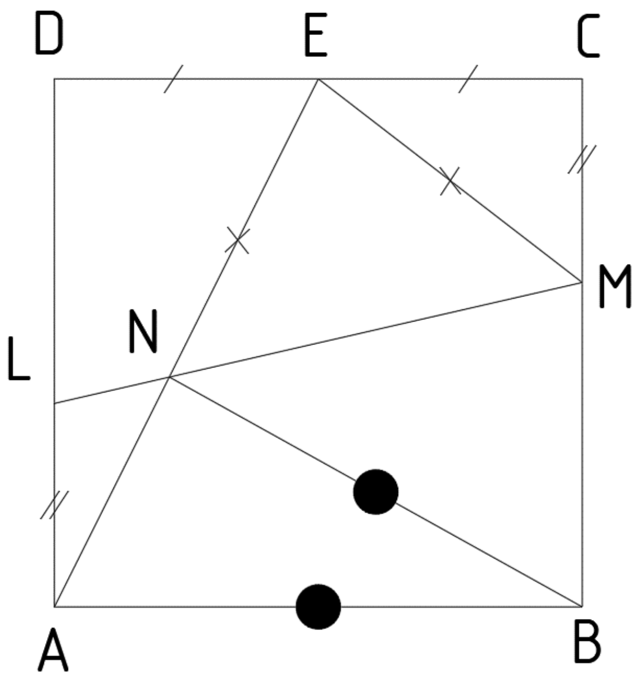


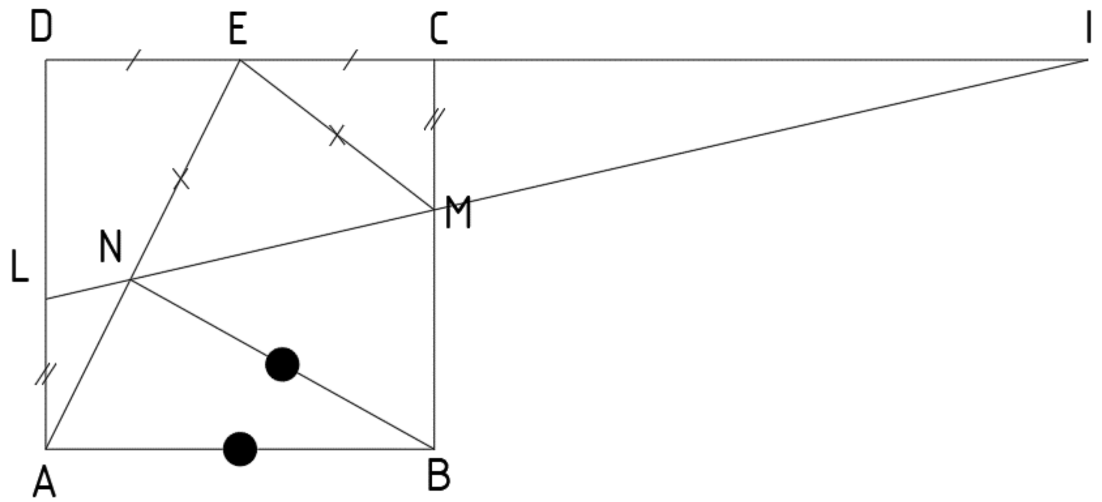
I have simplified the problem, now it can be announced that:

"Can 2 equations: $EN = EM$ and $AB = BN$ happen at the same time?"

1. Dicuss 1: When will $EN = EM$?

Let $AL = CM = x, 0 < x < a$ and extend LM that meets CD at I. There will be 2 cases:

Case 1: point C between point I and D (figure bellow)



Using Menelaus theorem for triangle DAE with secant L-N-I, we have:

$$\frac{AL}{DL}.\frac{EN}{AN}.\frac{DI}{EI} = 1$$

⇨ $\frac{EN}{AN} = \frac{EI}{DI}.\frac{DL}{AL} = \frac{EC+CI}{DC+CI}.\frac{DL}{AL} = \frac{\frac{a}{2}+CI}{a+CI}.\frac{a-x}{x}$

Then we have to find CI by a, x

Using Thales theorem in triangle DLI with CM // DL, we have:

$$\frac{CM}{DL} = \frac{CI}{DI} = \frac{CI}{a + CI}$$

⇨ $\frac{x}{a-x} = \frac{CI}{a+CI}$

⇨ $CI = \frac{ax}{a-2x}$, with condition $0 < x < a/2$ (*)

Then we find that $\frac{EN}{AN} = \frac{a^2-ax}{2ax-2x^2}$

We also have, $EN + AN = AE = \sqrt{AD^2 + DE^2} = \sqrt{a^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{5}}{2}a$

⇨ $EN = \frac{\sqrt{5}a(a^2-ax)}{2(a^2+ax-2x^2)}$

About EM, we have: $EM = \sqrt{EC^2 + CM^2} = \sqrt{\left(\frac{a}{2}\right)^2 + x^2} = \frac{\sqrt{a^2+4x^2}}{2}$

$EN = EM$ ➔ $\frac{\sqrt{5}a(a^2-ax)}{2(a^2+ax-2x^2)} = \frac{\sqrt{a^2+4x^2}}{2}$

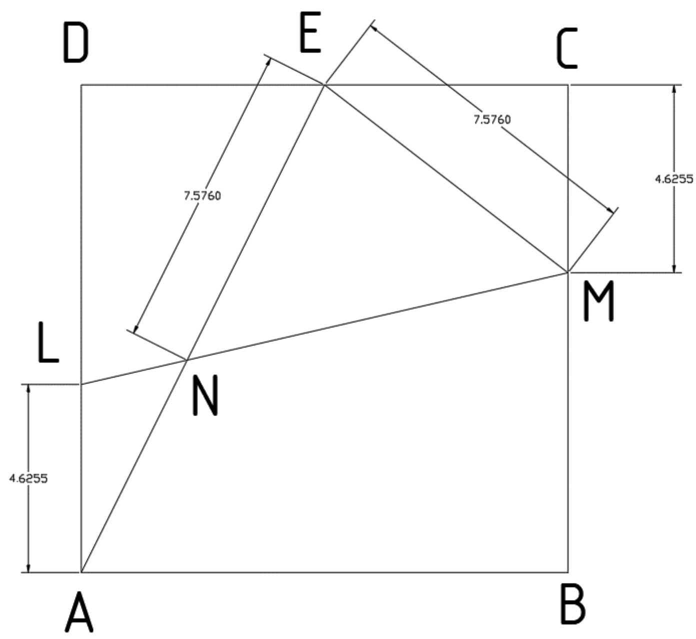
⇨ $\sqrt{5}a(a^2 - ax) = (a^2 + ax - 2x^2)\sqrt{a^2 + 4x^2}$

Square both sides and simplified it, we have equation bellow:

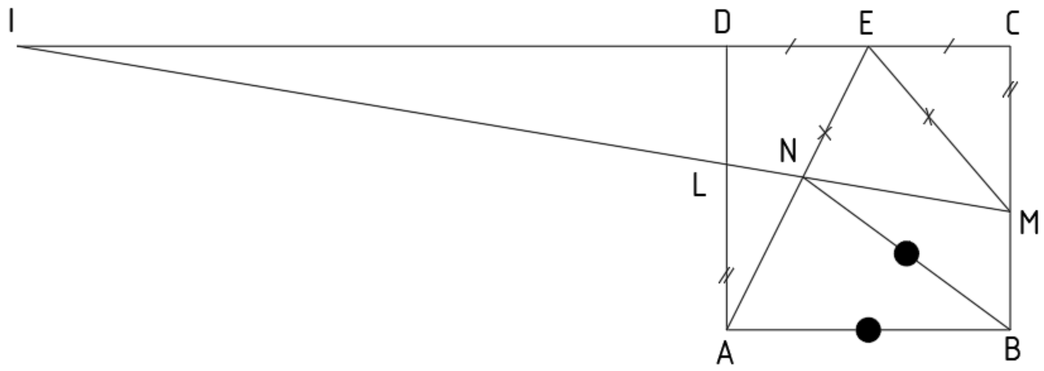
$$4x^6 - (4a)x^5 - (2a^2)x^4 + a^3x^3 - a^4x^2 + (3a^5)x - a^6 = 0 \text{ (1)}$$

That mean, if equation (1) is satisfied and root x is accepted by condition (*), eqution $EN = EM$ will be true.

In your problem, we already have $a = 12$, then we can calculate $x \approx 4{,}6255$ and check by drawing, we have:



Case 2: Point D between point I and C (figure bellow)



Using Menelaus theorem for triangle DAE with secant N-L-I, we have:

$$\frac{AL}{DL} \cdot \frac{EN}{AN} \cdot \frac{DI}{EI} = 1$$

⇨ $\frac{EN}{AN} = \frac{EI}{DI} \cdot \frac{DL}{AL} = \frac{ED+DI}{DI} \cdot \frac{DL}{AL} = \frac{\frac{a}{2}+DI}{DI} \cdot \frac{a-x}{x}$

Then we have to find DI by a, x

Using Thales theorem in triangle CMI with CM // DL, we have:

$$\frac{CM}{DL} = \frac{CI}{DI} = \frac{a+DI}{DI}$$

⇨ $\frac{x}{a-x} = \frac{a+DI}{DI}$

⇨ $DI = \frac{a^2-ax}{2x-a}$, with condition $\frac{a}{2} < x < a$ (+)

Then we find that $\frac{EN}{AN} = \frac{a}{2x}$

We also have, $EN + AN = AE = \sqrt{AD^2 + DE^2} = \sqrt{a^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{5}}{2}a$

⇨ $EN = \frac{\sqrt{5}a^2}{2(2x+a)}$

About EM, we have: $EM = \sqrt{EC^2 + CM^2} = \sqrt{\left(\frac{a}{2}\right)^2 + x^2} = \frac{\sqrt{a^2+4x^2}}{2}$

$EN = EM$ ➔ $\frac{\sqrt{5}a^2}{2(2x+a)} = \frac{\sqrt{a^2+4x^2}}{2}$

⇨ $\sqrt{5}a^2 = (2x+a)\sqrt{a^2+4x^2}$

Square both sides and simplified it, we have equation bellow:

$$4x^4 + (4a)x^3 + (2a^2)x^2 + a^3x - a^4 = 0 \quad (2)$$

That mean, if equation **(2)** is satisfied and root x is accepted by condition (+), eqution $EN = EM$ will be true.

In your problem, we already have $a = 12$, then we can calculate $x = -12 \; or \; x \approx 4{,}6255$. They are both rejected by condition (+) ➔ **No solution**

2. Dicuss 2: Will equation EN = EM true when combine with condition $BN = BA$?

We have:

$$\cos(DAE + EAB) = cos90° = 0$$

⇨ $cosDAEcosEAB - sinDAEsinEAB = 0$

⇨ $\frac{a}{\left(\frac{\sqrt{5}}{2}a\right)}cosEAB - \frac{\frac{a}{2}}{\left(\frac{\sqrt{5}}{2}a\right)}sinEAB = 0$

Because $EAB < 90°$➔ $cosEAB, sinEAB > 0$

⇨ $cosEAB = \frac{1}{\sqrt{5}}$

In triangle BAN:

$$cosBAN = \frac{BA^2 + AN^2 - BN^2}{2AB.AN} = \frac{AN}{2AB} = \frac{AN}{2a} = cosEAB$$

⇨ $AN = \frac{2a}{\sqrt{5}}$

⇨ $EN = AE - AN = \frac{\sqrt{5}}{2}a - \frac{2}{\sqrt{5}}a = \frac{\sqrt{5}}{10}a$

Like **Discuss 1**, we have $EM = \frac{\sqrt{a^2+4x^2}}{2}$

As the result, $\frac{\sqrt{5}}{10}a = \frac{\sqrt{a^2+4x^2}}{2}$

⇨ $\left(\frac{\sqrt{5}}{5}a\right)^2 = a^2 + 4x^2$

⇨ $x^2 = -\frac{1}{5}a^2 < 0$

⇨ **Impossible**

That mean, 2 equations in the title can not be true at the same time

But is there a ability that only equation $BA = BN$ be true with fixed condition $LA = CM$?

3. Discuss 3: When will $BA = BN$?

Case 1: Point C between point I and D

From **Dicuss 1_Case 1**, we have $EN = \frac{\sqrt{5}a(a^2-ax)}{2(a^2+ax-2x^2)}$

⇨ $AN = AE - EN = \frac{\sqrt{5}}{2}a - \frac{\sqrt{5}a(a^2-ax)}{2(a^2+ax-2x^2)} = \frac{\sqrt{5}(a^2x-ax^2)}{a^2+ax-2x^2}$

From **Dicuss 2**, we have $AN = \frac{2}{\sqrt{5}}a$

As the result, $\frac{\sqrt{5}(a^2x-ax^2)}{a^2+ax-2x^2} = \frac{2}{\sqrt{5}}a$ (3)

⇨ $\frac{\sqrt{5}(ax-x^2)}{a^2+ax-2x^2} = \frac{2}{\sqrt{5}}, a > 0$

Condition: $a^2 + ax - 2x^2 \neq 0$ (**)

⇨ $x \neq a, x \neq -\frac{a}{2}$

Transform eqution (3), we have:

$$5(ax - x^2) = 2(a^2 + ax - 2x^2)$$

⇨ $x^2 - (3a)x + 2a^2 = 0$

⇨ $x = a \; or \; x = 2a$ (both are rejected by condition $(*)$)

That mean equation $BA = BN$ can not be true

Case 2: Point D between point I and C

From **Dicuss 1_Case 2**, we have $EN = \frac{\sqrt{5}a^2}{2(2x+a)}$

⇨ $AN = AE - EN = \frac{\sqrt{5}}{2}a - \frac{\sqrt{5}a^2}{2(2x+a)} = \frac{\sqrt{5}ax}{2x+a}$

From **Dicuss 2**, we have $AN = \frac{2}{\sqrt{5}}a$

As the result, $\frac{\sqrt{5}ax}{2x+a} = \frac{2}{\sqrt{5}}a$

⇨ $\frac{\sqrt{5}x}{2x+a} = \frac{2}{\sqrt{5}}, a > 0$ (4)

Condition: $2x + a \neq 0$, always right because $\frac{a}{2} < x < a$

Transform eqution (4), we have:

$$5x = 2(2x + a)$$

⇨ $x = 2a$ (rejected by condition of x)

That mean equation $BA = BN$ can not be true

4. Sum up

With fixed condition $AL = CM$ and E is midpoint of CD, just only equation $EN = EM$ is accepted!