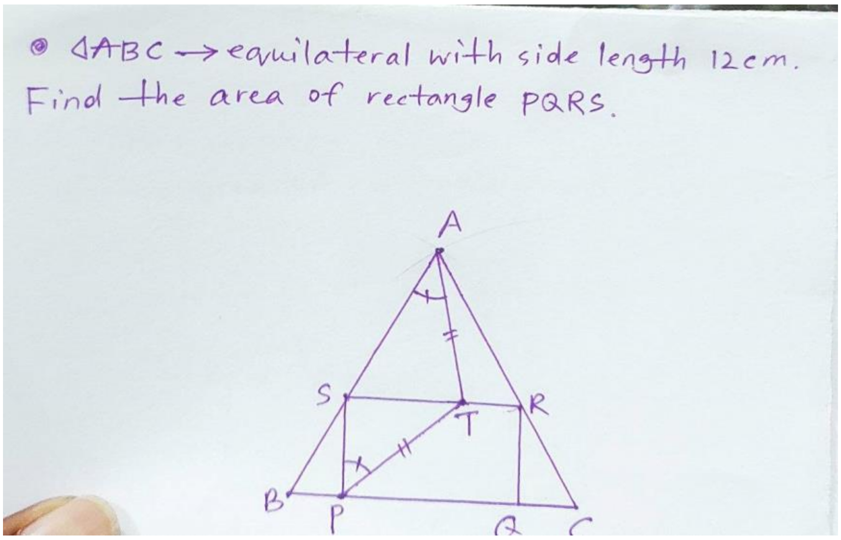

1/Condition that figure has solution

With $TA = TP$, we have triangle TAP is isosceles at T

⇨ $TAP = TPA$
⇨ $SAP = SPA$ $(because\ of\ SAT = SPT)$
⇨ Triangle SAP is isosceles at S
⇨ $SA = SP$ (1)
⇨ ST is lying on median line of $AP, or\ ST$ perpendicular with AP
⇨ AP perpendicular with BC

But SP is perpendicular with BC also (PQRS is a rectangle)

⇨ S, A, P is colinear
⇨ $S \notin AB$
⇨ **Conflict hypothesis**

Now, if we want to keep that hypothesis, we have to change the problem

Realize that point T is lies on median line of AP (through point S), that means T can not lie on SR

⇨ Point T can lie on another line, such as RP, RQ…

Now we discover one more information about rectangle PQRS

Not hard to see triangle ASR is equilateral

⇨ $SA = SR$ (2)

From (1) and (2), we have $SP = SR$

Combine with PQRS is a rectangle, that leads to PQRS is a square

In conclusion, just change the position of point T in the original problem!

2/Calculate area of square PQRS

Let define some variables:

- $AB = BC = CA = a, a > 0$
- $SP = SA = x, x \in (0, a)$ ➔ $SB = a - x$

Drawing altitude AH of triangle ABC from point A ($H \in BC$)

⇨ $AH = \frac{\sqrt{3}}{2}a$

Because SP and AH are perpendicular with BC

⇨ SP // AH

Using Thales theorem, we have:

$$\frac{SP}{AH} = \frac{SB}{AB}$$

⇨ $\frac{x}{\frac{\sqrt{3}}{2}a} = \frac{a-x}{a}$

⇨ $\frac{\sqrt{3}}{2}(a - x) = x$

⇨ $x = (2\sqrt{3} - 3)a$

⇨ $S_{PQRS} = x^2 = (21 - 12\sqrt{3})a^2$

In your problem, $a = 12\ cm$

⇨ $S_{PQRS} = (21 - 12\sqrt{3}).12^2 = (3024 - 1728\sqrt{3})\ cm^2 \approx 31.0162\ cm^2$

Check with drawing: