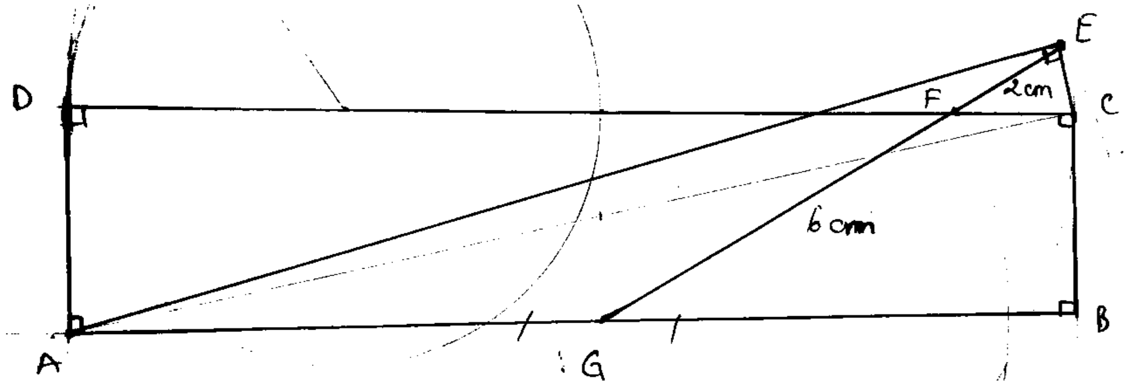
Given the figure bellow, calculate $\frac{AE}{EC}$?



1/Condition that the figure has solution

First, we define some variables:

$$AB = a, AD = b$$

$$EBC = \alpha, EGB = \beta$$

The condition for them:

- $a, b > 0; b = d_{G/CD}$ (distance from point G to line CD) $< GF = 6$ **(*)**
- $\alpha, \beta \in (0,90)$

Not hard to see that 5 points: A, D, E, C and B are lying on the same circle which AC is diameter

⇨ $EAC = EBC = \alpha$

In triangle EGB:

$$\frac{EB}{sinEGB} = \frac{EG}{sinEBG}$$

⇨ $\frac{EB}{(\frac{b}{6})} = \frac{8}{sin\,(90-\alpha)}$

⇨ $EB = \frac{4}{3}bcos\alpha = \frac{4}{3}b\sqrt{a^2+b^2}.\frac{1}{AE}$ (triangle AEC is right at E)

⇨ $EB.EA = \frac{4}{3}b\sqrt{a^2+b^2}$ **(1)**

Besides that, triangle EGB has EG is median:

$$EG^2 = \frac{2(EA^2+EB^2)-AB^2}{4}$$

⇨ $8^2 = \frac{2(EA^2+EB^2)-a^2}{4}$

⇨ $EA^2 + EB^2 = 128 + \frac{a^2}{2}$ **(2)**

Because $EAC = EBC = \alpha$

⇨ $cosEAC = cosEBC$

⇨ $\frac{AE}{\sqrt{a^2+b^2}} = \frac{EB^2+b^2-EC^2}{2bEB}$

⇨ $2b\left(\frac{4}{3}b\sqrt{a^2+b^2}\right) = \left(\sqrt{a^2+b^2}\right)(EB^2+b^2-EC^2)$ (from (1))

⇨ $EB^2 - EC^2 = \frac{5}{3}b^2$ (3)

(2) – (3)

⇨ $EA^2 + EC^2 = 128 + \frac{a^2}{2} - \frac{5}{3}b^2$

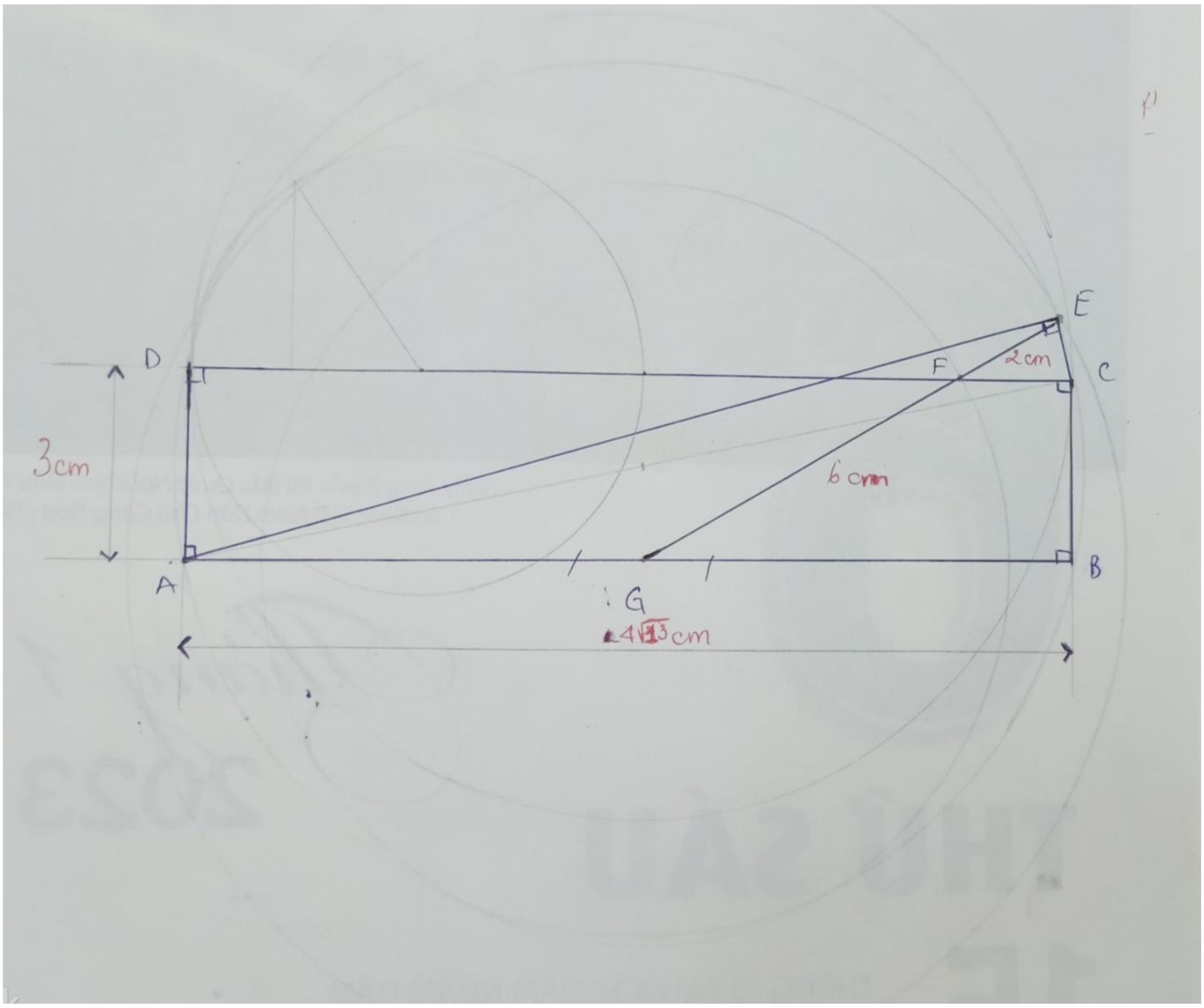
But $EA^2 + EC^2 = a^2 + b^2$

⇨ $a^2 + b^2 = 128 + \frac{a^2}{2} - \frac{5}{3}b^2$

⇨ $3a^2 + 16b^2 = 768$ (**)

Then we have: if conditions (*) and (**) are accepted at the same time, the figure has solution!

I have one solution that fits those conditions is $a = 4\sqrt{13}\ cm, b = 3\ cm$ (figure bellow)



2/Calculate AE/EC

We easily have that:

- $EA^2 + EB^2 = 256 - \frac{8}{3}b^2 = m, m > 0$
- $EA.EB = \frac{4}{3}b\sqrt{256 - \frac{13}{3}b^2} = n, n > 0$

⇨ $\begin{cases} EA + EB = \sqrt{m + 2n} \\ EA - EB = \sqrt{m - 2n} \end{cases}$

⇨ $AE = \frac{\sqrt{m+2n}+\sqrt{m-2n}}{2}$

⇨ $CE = \sqrt{256 - \frac{13}{3}b^2 - AE^2}$

⇨ $\frac{AE}{CE} = \cdots$

It looks so complicated and it depends on the value of a, b. So, I recommend that you should change the question, such as: *find the maximum area of rectangle ABCD, trianlge AEC,..*

The next part, I will show you one of my recommendations: *find the area of rectangle ABCD*

3/Another question (recommendation): find the maximum area of rectangle ABCD?

From part **1** and **2**, we have some information:

- The length of two sides is a and b ($a, b > 0$ and $b < 6$)
- The relationship between both sides is: $3a^2 + 16b^2 = 768$

The value we want to find:

$$Max\ S = ab\ (cm^2)$$

We have:

$$S = ab = a\sqrt{\frac{768 - 3a^2}{16}} = \frac{\sqrt{3}}{4}a\sqrt{256 - a^2}$$

Using Cauchy inequality, we have:

$$S = \frac{\sqrt{3}}{4}a\sqrt{256 - a^2} \leq \frac{\sqrt{3}}{4}.\frac{a^2 + \left(\sqrt{256 - a^2}\right)^2}{2} = 32\sqrt{3}$$

"=" occur ⇔ $a = \sqrt{256 - a^2}$

⇔ $a = 8\sqrt{2}\ cm$

⇨ $b = 2\sqrt{6}\ cm < 6\ cm$ (accepted)

Finally, the maximum of area rectangle ABCD is $32\sqrt{3}\ cm^2$, with $a = 8\sqrt{2}\ cm, b = 2\sqrt{6}\ cm$