Over all real numbers, find the minimum value of a positive real number y, such that;

y = sqrt((x + 2)^2 + 25) + sqrt((x - 6)^2 + 121)

$y = ((x + 6)^2 + 25)^{1/2} + ((x - 6)^2 + 121)^{1/2}$

$$\frac{dy}{dx} = \frac{((x+6)^2+25)-½ \ *(2(x+6))}{2} + \frac{((x-6)^2+121)-½ \ *(2(x-6))}{2}$$

$$\frac{dy}{dx} = \frac{(x+6)}{((x+6)^2+25)½} + \frac{(x-6)}{((x-6)^2+121)½}$$

Since for the minimum value of y, $\frac{dy}{dx} = 0$,

$$\frac{(x+6)}{((x+6)^2+25)½} + \frac{(x-6)}{((x-6)^2+121)½} = 0$$

Hence,

$$\frac{(x+6)}{((x+6)^2+25)½} = -\left(\frac{(x-6)}{((x-6)^2+121)½}\right)$$

Squaring both sides,

$$\frac{(x+6)^2}{((x+6)^2+25)} = \frac{(x-6)^2}{((x-6)^2+121)}$$

$(x + 6)^2(x - 6)^2 + 121(x + 6)^2 = (x - 6)^2(x + 6)^2 + 25(x - 6)^2$

$121(x^2 + 12x + 36) = 25(x^2 - 12x + 36)$

$121x^2 + 1452x + 4356 = 25x^2 - 300x + 900$

$96x^2 + 1752x + 3456 = 0$

Dividing through by 24, we have

$4x^2 + 73x + 144 = 0$

Solving the quadratic equation using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

a = 4, b = 73, c = 144

$$x = \frac{-73 \pm \sqrt{73^2 - 4(4)(144)}}{2(4)}$$

$$x = \frac{-73 \pm 55}{8}$$

$x_1 = -16$

$x_2 = -2.25$

Substituting for x in y

At x_1, $y = \sqrt{(-16+6)^2 + 25} + \sqrt{(-16-6)^2 + 121}$

$y_1 = 35.77$

At x_2, $y = \sqrt{(-2.25+6)^2+25} + \sqrt{(-2.25-6)^2+121}$

$y_2 = 20$

Hence, Over all real numbers, the minimum value for y is 20.