Division of Engineering Science
UNIVERSITY OF TORONTO

ROB501: Computer Vision for Robotics

Assignment #4 Addendum: Triangulation for Stereo VO

Fall 2020

Triangulation and Landmark Uncertainty Estimation

The authors of the Cheng (2005) paper are not particularly careful with their notation regarding vectors and matrices (including Jacobians). The purpose of this addendum is to more clearly define the equations used in the computing the estimated 3D position of each landmark, and the landmark uncertainty. Instead of using the subscripts '1' and '2', we will use 'L' and 'R' to denote the left and right cameras (rays, segments, etc.).

The Assignment #4 handout had already provided the equation to compute the ray $\hat{\mathbf{r}}_L$ from known image plane coordinates. We will also require the Jacobian matrix of the ray direction with respect to the image plane coordinates, $\mathbf{J}_L$, which is 3×2 in size (you can use the quotient rule to compute the Jacobian).

As noted in the Cheng paper, the coordinates of the line segment endpoints used to find the 3D landmark position are

$$\mathbf{p}_L = \mathbf{c}_L + \hat{\mathbf{r}}_L m_L,$$
$$\mathbf{p}_R = \mathbf{c}_R + \hat{\mathbf{r}}_R m_R,$$

where $\mathbf{c}_L$ and $\mathbf{c}_R$ are the positions of the left and right camera optical centres, respectively, and m_L and m_R are scalar lengths. These lengths are defined as

$$m_L = \frac{(\mathbf{b}^T\hat{\mathbf{r}}_L) - (\mathbf{b}^T\hat{\mathbf{r}}_R)(\hat{\mathbf{r}}_L^T\hat{\mathbf{r}}_R)}{1 - (\hat{\mathbf{r}}_L^T\hat{\mathbf{r}}_R)^2},$$
$$m_R = (\hat{\mathbf{r}}_L^T\hat{\mathbf{r}}_R)\, m_L - (\mathbf{b}^T\hat{\mathbf{r}}_R)$$

where $\mathbf{b}$ is the baseline between the cameras (a 3×1 vector). In the equations above, we have avoided the use of dot products to make the derivation of the full Jacobian matrix more clear (and we have also fixed a couple of typos in the Cheng paper). Note that the 'hat' refers to a unit vector.

We are now required to compute the Jacobian of m_L (and also of m_R) with respect to *all four image plane coordinates*. Thus, the Jacobian in this case will be 1×4 in size; this Jacobian is identified as m_1' in the Cheng paper. The expression is complicated—let's first break it down into a series of subexpressions. Let

$$u = (\mathbf{b}^T\hat{\mathbf{r}}_L) - (\mathbf{b}^T\hat{\mathbf{r}}_R)(\hat{\mathbf{r}}_L^T\hat{\mathbf{r}}_R),$$
$$v = 1 - (\hat{\mathbf{r}}_L^T\hat{\mathbf{r}}_R)^2.$$

The Jacobians of u and v with respect to all four image plane coordinates are then

$$\begin{aligned}
\mathbf{J}_u &= \left(\mathbf{b}^T\left[\mathbf{J}_L \;\; \mathbf{0}_{3\times2}\right]\right) - \left(\mathbf{b}^T\left[\mathbf{0}_{3\times2} \;\; \mathbf{J}_R\right]\right)\left(\hat{\mathbf{r}}_L^T\hat{\mathbf{r}}_R\right) - \left(\mathbf{b}^T\hat{\mathbf{r}}_R\right)\left(\hat{\mathbf{r}}_R^T\left[\mathbf{J}_L \;\; \mathbf{0}_{3\times2}\right] + \hat{\mathbf{r}}_L^T\left[\mathbf{0}_{3\times2} \;\; \mathbf{J}_R\right]\right)\\
\mathbf{J}_v &= -2\left(\hat{\mathbf{r}}_L^T\hat{\mathbf{r}}_R\right)\left(\hat{\mathbf{r}}_R^T\left[\mathbf{J}_L \;\; \mathbf{0}_{3\times2}\right] + \hat{\mathbf{r}}_L^T\left[\mathbf{0}_{3\times2} \;\; \mathbf{J}_R\right]\right).
\end{aligned}$$

We these expressions in hand, we may now write the expression for the Jacobian of m_L with respect to all four image plane coordinates (for which we use the identifier $\mathbf{M}_L$) as

$$\mathbf{M}_L = \frac{\mathbf{J}_u v - \mathbf{J}_v u}{v^2}.$$

Note that the resulting matrix is 1×4. A similar procedure can be followed to compute $\mathbf{M}_R$. Finally, determining the full landmark Jacobian (Equation 10 in the paper) is straightforward; the full Jacobian will be 3×4 in size.