

Programs and Proofs



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Cool Lean projects

- Video player
- Raytracer
- Webring generator
- HouLean
- SciLean
- Functorio
- Rupert
- Equational theories
- Mathlib
- Analysis textbook
- Erdős 707
- LeanTeX

History of formalized math

- 1910: Principia Mathematica

*54·43. $\vdash :: \alpha, \beta \in 1 . \supset : \alpha \cap \beta = \Lambda . \equiv . \alpha \cup \beta \in 2$

Dem.

$\vdash . *54·26 . \supset \vdash :: \alpha = \iota'x . \beta = \iota'y . \supset : \alpha \cup \beta \in 2 . \equiv . x \neq y .$

[*51·231] $\equiv . \iota'x \cap \iota'y = \Lambda .$

[*13·12] $\equiv . \alpha \cap \beta = \Lambda \quad (1)$

$\vdash . (1) . *11·11·35 . \supset$

$\vdash :: (\exists x, y) . \alpha = \iota'x . \beta = \iota'y . \supset : \alpha \cup \beta \in 2 . \equiv . \alpha \cap \beta = \Lambda \quad (2)$

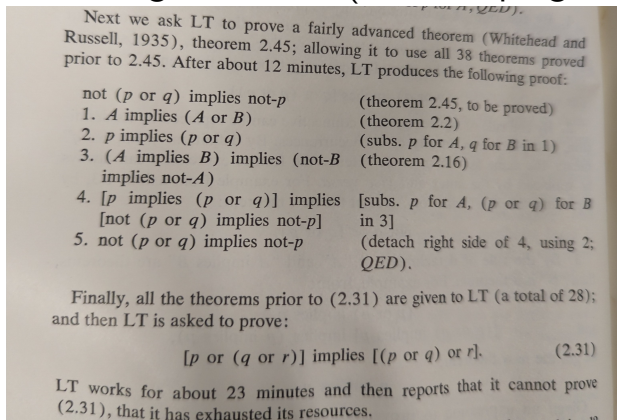
$\vdash . (2) . *11·54 . *52·1 . \supset \vdash . \text{Prop}$

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.

- 1931: Gödel's incompleteness theorems

History (cont.)

- 1936: Entscheidungsproblem proven undecidable
- 1956: Logic Theorist ("first AI program")



History (cont.)

- 1976: Four color theorem proved using brute force (verified in Rocq in 2005)

ITPs vs ATPs

- Two main paradigms
- ITP = Interactive theorem prover, uses tactics, ex: Rocq, Lean
- ATP = Automated ..., uses SMT, ex: Dafny
- ATPs are buggier, more brittle, require learning arcane SMT magic

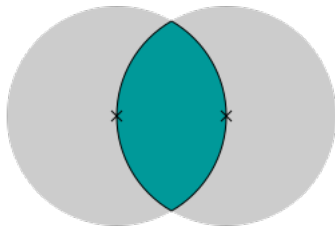
Foundations

- Set theory (Mizar, Metamath)
- Simple type theory (Isabelle/HOL)
- Dependent type theory (Lean, Rocq, Agda, Idris)

Lean bio

- 2013: Created by Leo de Moura at Microsoft, previously created Z3
- 2023: Lean 4 released, rewritten in Lean (except type checker)
- Not named after the drug

Is Lean practical?



- "Invisible math"
- Automated tactics: grind, hammer, canonical
- AI: **Harmonic's Aristotle**, AlphaProof