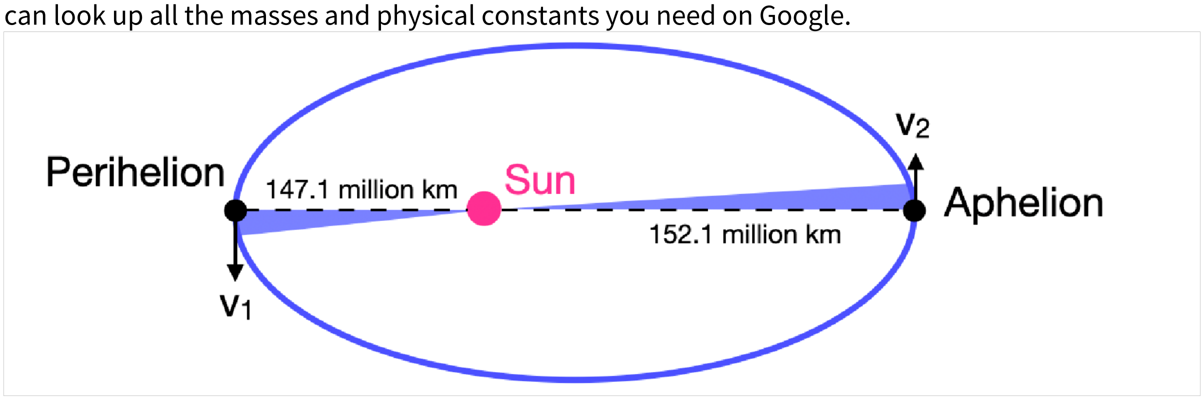
Exam 2 Take-Home Project

In this project you will implement a code to simulate planetary motion. You will use the Euler or Runge-Kutta 2nd order method to integrate the equation of motion.

The project is organized into five tasks, through which you'll develop a single program.

TASK 1 (20 pts). First a quick analytical calculation to set up initial conditions for our problem. We assume that the Sun is fixed at the origin (0,0) and that we know the perihelion and aphelion (Earth's nearest and farthest point from the Sun respectively) to be 147.1 million km and 152.1 million km. Make an analytical calculation of what the Earth's velocities are at these two points, v_1 and v_2. An exaggerated figure showing the elliptical orbit is shown below. The actual orbit is almost circular. You can solve it with two simple equations with a pen and calculator! Sketch out your steps in the code as a comment. You'll get to use v1 in Task 2.
Hint 1, Conservation of energy: Write down the total energy = kinetic energy plus gravitational potential energy ($-\frac{GMm}{r}$) at the two points and equate them.
Hint 2, Conservation of angular momentum, a.k.a. Kepler's Second Law: A line segment joining Earth and Sun sweeps out equal areas during equal intervals of time. Take a tiny sweep across an infinitesimally small time *dt* near the perihelion and aphelion, and equate the areas (blue areas in figure). You can look up all the masses and physical constants you need on Google.



TASK 2 (20 pts). Initialize the Earth at the Perihelion with velocity v1 pointing in the appropriate direction. It's easiest to set up everything in SI units. Implement the expression for the gravitational force acting on the Earth $\vec{F} = -\frac{GMm}{r^2}\hat{r} = -\frac{GMm}{r^3}\vec{r}$ and the corresponding acceleration in the function f(). Here $\vec{r}$ is the vector from the Sun to the Earth and r is its magnitude $r = |\vec{r}|$. Determine a timestep *dt* that is suitable for simulating the trajectory of Earth around the Sun for 1 year. *dt* only needs to be small compared to a year. If *dt* is too small it'll take forever to finish the calculation; if *dt* is too large you'll get large errors in the simulation, especially if you use Euler's method.

TASK 3 (20 pts). Simulate the Earth's trajectory for one year using an ODE solver of your choice. The resulting trajectory should be a closed orbit that passes through both the Perihelion and Aphelion.

TASK 4 (20 pts). Create a second plot to show the values of the energies (potential, kinetic, total) as a function of time. You can choose to plot this as a second subplot or in a separate figure.

TASK 5 (20 pts). Try to tweak Newton's law of gravitation a little bit by changing the exponent of *r* in $-\frac{G M m}{r^3} \vec{r}$ from –3 to –3.02. Simulate the Earth's trajectory for 200 years. Describe what you see.

Extra credits (20 pts).
Animate the Earth's trajectory in Task 5, using the animation module of matplotlib. You can refer to the two animation codes we developed in class and in Homework Week #7 as a template.