# Portfolio Optimisation with SMC

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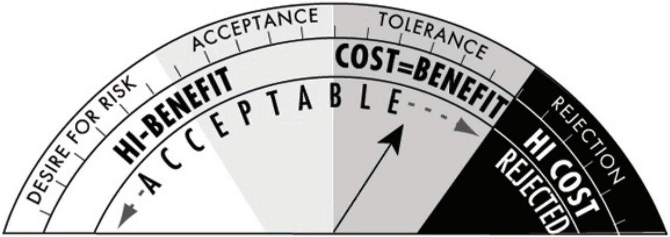
September 10, 2014

# Maximising Returns

# Minimising Risk

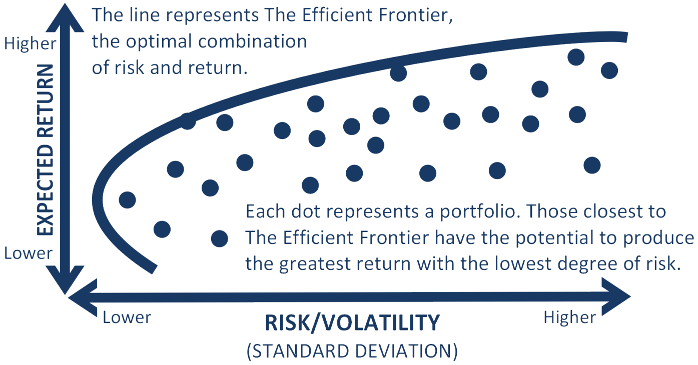


# All eggs in one basket?

# Diversifaction

# Portfolio Optimisation

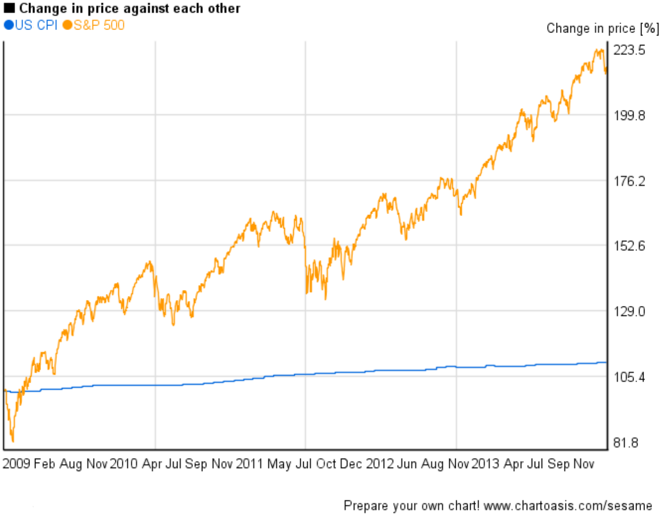




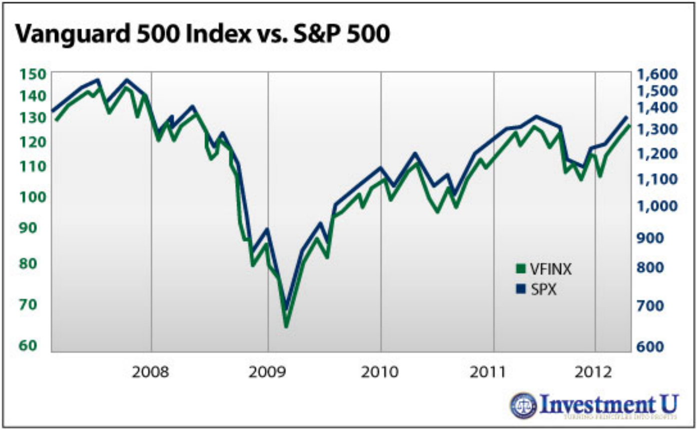
# A tale of two investment styles

# S&P Index for the last 5 Year





# Passive Index Fund



# Why Index Fund?

1. Lower fees
2. Less vulnerable to the change of managers
3. Typically more tax efficient
4. Comparable performance?

**Objective**

Minimising the tracking error, $\epsilon$ of index tracking fund:

$$\epsilon = \sqrt{Var[r_p - r_b]} \quad (1)$$

where $r_p$ is the return of the fund and $r_b$ is the return of the benchmark index.

# Methodology



## State Space Model

Adopt a conditional linear Gaussian Model

$$X_t = A_t(U_t)X_{t-1} + B_t(U_t)W_t + F_t(U_t)$$
$$Y_t = C_t(U_t)X_t + D_t(U_t)V_t + G_t(U_t)$$

where $W_t, V_t \sim \mathcal{N}(0, I)$.

## Trasition Density and conditional likelihood

$$\begin{aligned} p_t(u_t \mid u_{t-1}) &= \text{(any given form)} \\ f_t(x_t \mid x_{t-1}, u_t) &= \mathcal{N}(A_t(u_t)x_{t-1} + F_t(u_t), B_t(u_t)B_t(u_t)^T) \\ g_t(y_t \mid x_t, u_t) &= \mathcal{N}(C_t(u_t)x_t + G_t(u_t), D_t(u_t)D_t(u_t)^T) \end{aligned} \quad (2)$$

## Open loop problem regulation

Viewing portfolio optimisation as a stochastic control problem. Express performance using a reward (cost function) $J_{x_0}(u_{1:T})$, where $J_{x_0}(u_{1:T})$ is an expectation of some function

$$J(u_{1:T}, y_{1:T}^{ref}, x_0) = E_{x_0}\left[\exp\left(-\frac{1}{2}\sum_{t=1}^{T}\left(||y_t^{ref} - C_t(u_t)x_t - G_t(u_t)||^2\right.\right.\right. \tag{3}$$

where $Q(u_t) = D_t(u_t)D_t(u_t)^T$ and $L_t$ are assumed to known.

# Experiment 1: Tracking an oscillating wave

## The state space model

$$X_t = X_{t-1} + W_t + U_t, W_t \sim \mathcal{N}(0, I)$$
$$Y_t = X_t + V_t, V_t \sim \mathcal{N}(0, I)$$

## Target reference signal

$$y_t^{ref} = \cos(0.2\pi t + 0.3)$$

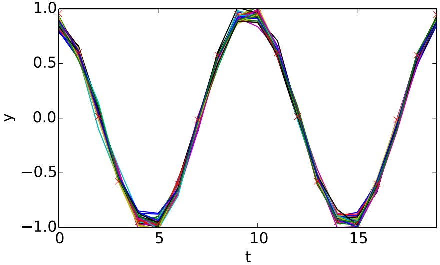
## Parameter Settings

1. Various time period length, $T$: 5, 10 and 20.
2. Various sample size, $N$: 100, 500, 1000, 5000 and 10000.
3. Resampling step: Always vs. Selectively with ES.
4. Resample-Move step with MCMC (random walk proposal).
5. Various $\gamma$ settings: Constant function of 1, 50, 100, 1000 and increasing function of time $t$, $10t$, $50t$ and $100t$.

# Experiment 1: Results and Discussion

## Result and Discussion

1. More samples helps — better distribution representation
2. ESS doesn't help — resampling pushes samples towards the mode
3. MCMC helps — pertubation helps to escape local optimal
4. $\gamma$ setting need tuning — optimisation vs. exploration




Setting: $\gamma$=100, N=1000, MCMC=0, T=20

# Application: Tracking the DAX Index

## the DAX Index

1. One of the major index of the world
2. 30 largest companies in Germany as the constituents
3. Data: Adjusted daily close from Jan 2014

## Experiments

1. DAX4
2. DAX with full replication
3. DAX with partial replication
4. DAX with MPC

# Experiment: Tracking the DAX Index



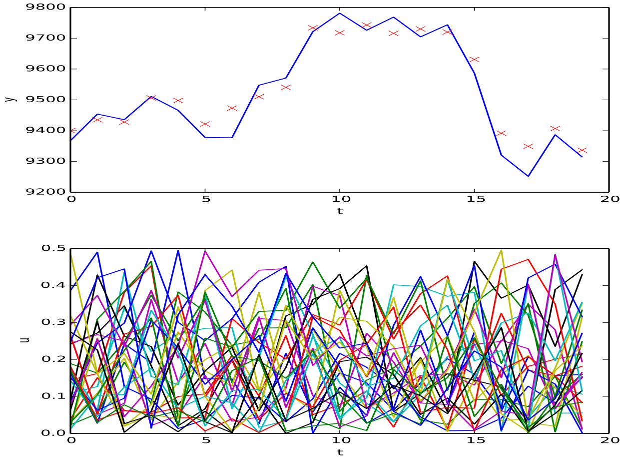
## The state space model

$$X_t = X_{t-1} + F_t(U_t) + W_t$$
$$Y_t = 30U_t^T X_t + 0.01V_t$$

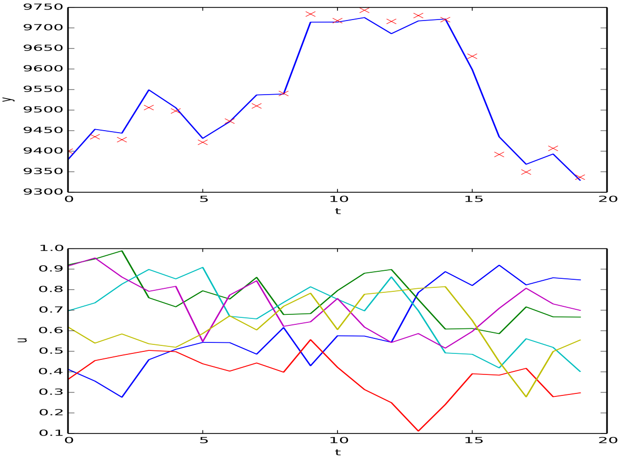
iwhere $W_t \sim \mathcal{N}(\mu_{t_0}, \Sigma_{t_0})$, $V_t \sim \mathcal{N}(0, I)$, $\{X_t\}_{t\geq 0}$ is a vector of stock price processes modelled as Arithmetic Brownian Motion with drift, $\{U_t\}_{t\geq 0}$ is a vector of control input processes, each component represents the position we have for each stock, $F_t(U_t)$ can be viewed as the market impact on price due to position changes and is set to be $0.0001U_t$ here, $\mu_{t_0}$ and $\Sigma_{t_0}$ are vector of the estimated mean of price changes and the estimated covariance matrix of the price changes and $\{Y_t\}_{t\geq 0}$ is the process represents the index level.

# Experiment 3:Full Replication





# Experiment 3:Partial Replication



# What we have achieved so far

### What the technique can do?

Track a *given* target reference index.

### Issues

1. We do not have a reference signal
2. Open loop policy, no feedback used

### Solution

1. use EWMA estimate for the target reference signal
2. use Model Predictive Control (MPC)

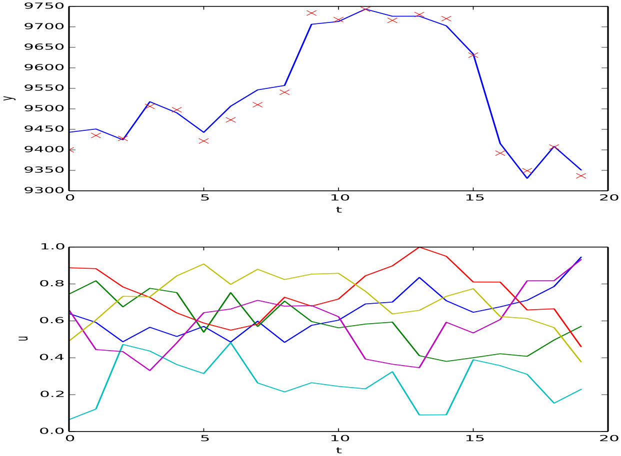
# MPC: Algorithm



**function** ModelPredictiveControl(T,H)
    Set $t = 1$.
    **while** $t \leq T$ **do**
        Search the optimal $u^*_{t:t+H}$ for the problem $t : t + H$.
        Apply the first set of the optimal control $u^*_t$.
        Update the model states with any new information.
        Set $t = t + 1$.
    **end while**
**end function**

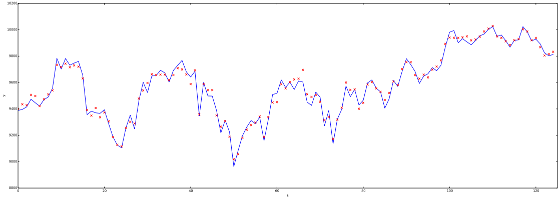
# MPC: Results and Discussion

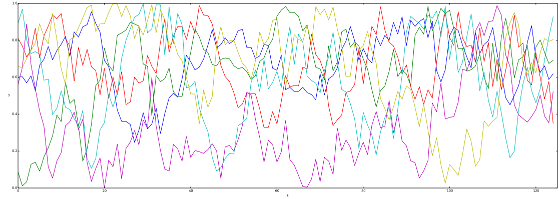




# MPC: Results and Discussion





# Contributions

In this thesis, it is found that SMC has the potential to be an effective means of searching the optimal strategy for index tracking funds. The following work has been carried out:

1. exploring the potential of SMC in searching the optimal strategy for minimising the tracking error and transaction costs of an index fund.
2. exploring the sensitivity of SMC in terms of the parameter settings, the trade-off between estimation accuracy and computational efforts numerically and providing suggestions for real-world problem.
3. introducing the Model Predictive Control (MPC) framework and demonstrate how to integrate SMC technique proposed into the MPC framework.

# Future Work

1. More realistic models
   - Geometric Brownian Model, Jump Diffusion model, etc.
   - Moving away from a conditional Gaussian model
     $\implies$ the Kalman Filter recursion is no longer optimal.
   - Try: SMC2.
2. Parallel computation — The nested SMC setup inevitably adds consideration amount of computation requirements.
3. More complex financial indices

# Questions?



Thank you.