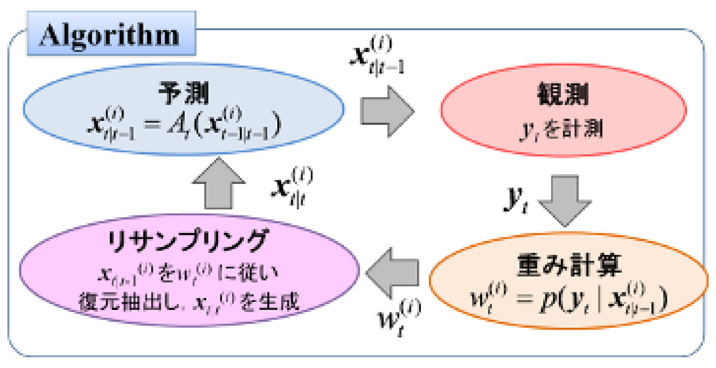

Portfolio Optimisation with Sequential Monte Carlo

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About

Portfolio optimisation is about efficiently allocating resources to achieve optimal investment objectives. Two usual objectives are maximising return and minimizing the returns.

This project views portfolio optimisation problem as a finite horizon control optimisation problem, and uses sequential monte carto (SMC) to search for the optimised control (investment decision).

Porfolio Optimisation

We begin to view the market consisting of N investible instruments. The price of an instrument S_i is assumed to follow an arithmetic Brownian motion as follows:

$$dS(t) = \mu dt + \sigma dW(t) \tag{1}$$

with initial condition $S_0 > 0$, μ is the average rate of return and σ is the volatility (risk).

Let $u(t)$ denote the number of shares of the asset one holds, then the value of the corresponding asset is a process $X(t)$ that evolves according to

$$dX(t) = \mu u(t)dt + \sigma u(t)dW(t) \tag{2}$$

The objective is to maximize the expected return over a fixed time interval $[0, T]$, at the same time minimizing the financial risk.

Stochastic Control

In control theory, performance is often expressed in terms of total reward (or total cost), J_x, which is an expectation of some function.

Decision maker cannot see the future, current decisions affect only future states and observations. Some possible settings include:

1. Total cost function (additive vs. multiplicative)

$$J_X^a = E_X[\sum_{n=1}^{T} j_i^a(X_n, U_n, Y_n)] \tag{3}$$

$$J_X^a = E_X[\prod_{n=1}^{T} j_n^a(X_n, U_n, Y_n)] \tag{4}$$

   where j_n^a is stage cost at time n.

2. Finite/infinite horizon T.
3. Open loop vs. closed loop (feedback).
4. Perfect vs. imperfect observation.

References

[1] A. Doucet, S. Godsill, C. Andrieu On Sequential Monte Carlo Sampling Methods for Bayesian Filtering. Statistics and Computing, vol. 10, no. 3, pp.197 -208, 2000

[2] N. Kantas, P. Del Moral, R. Vinter Particle methods for Stochastic Regulation: towards an application for power scheduling Technical Report, Imperial College, 2011

[3] M. Fischer and G. Livieri Continuous time mean-variance portfolio optimization through the mean field approach. Pre-print, 2014

[4] E. Ikonen Particle Filtering for Open-Loop Process Control 2006

Open loop control as Bayesian filtering

An example of non-linear time-varying dynamic system:

$$X_n = F_n(X_{n-1}, U_n)$$
$$Y_n = h_n(X_n)$$

where X_n are the markov chain state spaces, Y_n are the observations and U_n are the controls.

Let A_n and B_n be symmetric and semi-definite positive covariance matrices, a simple possible **finite horizon cost function** as follows:

$$J(U_{1:T}, Y_{1:T}) = \sum_{n=1}^{T} \|U_n\|_{A_n}^2 + \sum_{n=1}^{T} \|Y_n - Y^{ref}\|_{B_n}^2 \tag{5}$$

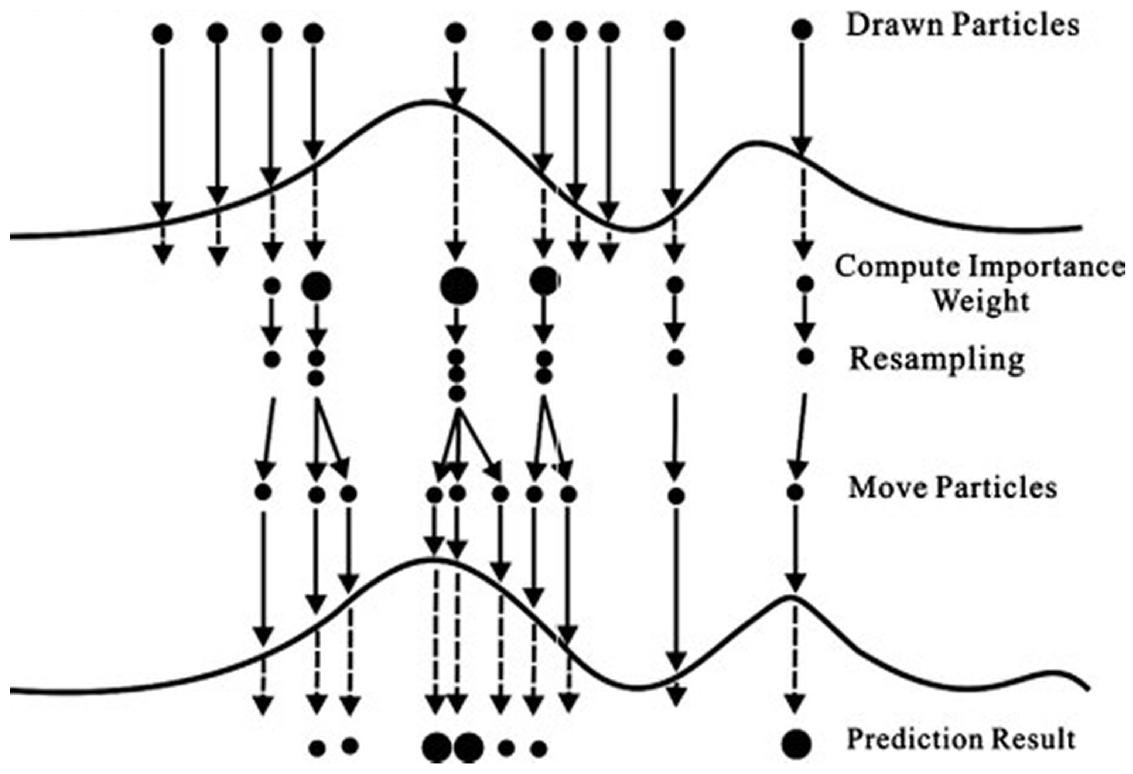
where Y^{ref} represent the reference target output, and $\|U\|_A^2 = U^T A^{-1} U$.

Objective: Search for an optimal policy (sequence of investment decisions) that minimize the cost control function, given some observations.

Sequntial Monte Carlo (SMC)

A swarm of particles (samples), that evolves towards the target distribution throughout sequence of intermediate distributions $\{\pi_n\}_{n \leq T}$.

The particles that approximates the intermediate distributions π_n are construsted based on the particles from π_{n-1} via sampling and re-sampling.



Optional step: MCMC step can be adeed to each iteration to improve diversity of the particles.

SMC Algorithm for estimating open loop controls

For each time step $n \in 1 \ldots T$,

1. For $i \in 1 \ldots N$, where N is number of particles,
   (a) If $n == 1$, initialize $x^i = x_0$, $y^i = h(x_0)$ and $J_T^i = 0$.
   (b) Generate random $u^i \sim N(0, A_n)$.
   (c) Store action to a list $u_n^i = u^i$.
   (d) Evaluate $J_y = \|y_n^{ref}\|_{B_n}^2$, $J_T^i = \|u^i\|_{A_n}^2$ and calculate the cost $J_T^i = J_T^i + J_Y^i + J_U^i$
   (e) Set weight $p_{un}^i = \exp(-\frac{\beta}{2} J_Y)$.
2. Normalize resampling probabilities: For $i \in 1 \ldots N$, $p^i = \frac{p_{un}^i}{\sum_{i=1}^{N} p_{un}^i}$
3. Resample: For $i \in 1 \ldots N$, select particles with replacement according to p^i.
4. Compute model for next time step n: $x^i = f(x^i, u^i)$ and $y^i = h(x^i)$.

Find $i^* = \arg\min_i J_T^i$. The solution for the optimal control is v^{i^*}.

Evaluation criteria

Compare the performance against other optimisation algorithms, e.g., genetic algorithms:

1. Performance of the resulting policy
2. Computation complexity (many subroutines here can execute in parallel.)