Supporting Information

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SI Text

Alternative Form for Confidence Expression, *C*. As noted in the *Materials and Methods*, we have adopted a simple form for system confidence C that captures liquidity health at a given time by the proportion of interbank loans in the system that have not been withdrawn. However, systemic shortening of lending maturities—even in the absence of any outright withdrawals—may also play a significant role in system health by making banks more vulnerable to subsequent liquidity outflows. To incorporate this, we use an alternative form:

$$\mathrm{C} = \tfrac{1}{2}(E + V)A, \qquad \text{[S1]}$$

where, at a given time, V is the proportion of initially long-term interbank lending that remains long-term. As in the main text, A is the sum of all remaining assets in the system (at the current market price) as a proportion of its initial value, and E is the fraction of interbank loans not withdrawn.

Thus, if all loans are shortened (i.e., $V = 0$) and none withdrawn (i.e., $E = 1$), then $C = 0.5A$, but if all loans have also been withdrawn (i.e., additionally $E = 0$), then $C = 0$. Fig. S2*A* repeats the simulations presented in Fig. 2, illustrating qualitatively similar results. Fig. S2*B* additionally compares this framework with the baseline model from the main text, plotting (as in Fig. 1) the extent of contagion when the value of a single asset class of a randomly selected big bank is set to zero. Because loan shortening precedes outright withdrawal, system-level confidence deteriorates more quickly when it also depends on the extent of shortening, and the amount of contagion is concomitantly increased.

Interbank Lending Network: Preferential Mixing Between Banks. Here we relax the assumption that the interbank lending network is a random web. A study of the Fedwire payments system (1, 2) suggests that the financial lending network may in fact be disassortative, that is with low-degree nodes (equivalently, small banks) tending to connect more to high-degree nodes (i.e., large banks) than other low-degree nodes. We consider both this and its converse, the assortative case.

In particular, write N_{B}, N_{S} for the number of large and small banks, respectively, and z_{B}, z_{S} for the mean number of loans made by large and small banks, respectively. We now assume that a proportion P of all loans are made between banks of different size classes: that is, the loans made by big banks to small ones, or vice versa. It is straightforward to show, under the size distribution described in the main text and parametrized by λ, that $P = 1/2$ corresponds to a random web; $P < 1/2$ yields an assortative network; and $P > 1/2$ yields a disassortative one. Fig. S2 *C* and *D* show results for the latter two cases, illustrating that essential results shown in Fig. 2 are qualitatively unchanged.

Non-Poisson Bank Size Distributions. Here we relax the assumption that there are two distinct sizes of banks (as parametrized by λ in the main text). Fig. S3*A* shows data relating to the United States banking sector, drawn from the Federal Deposit Insurance Corporation (3). Shown is the closest fit to a Pareto distribution. Originally developed to describe the distribution of wealth among individuals in a society, this distribution has the cumulative function:

$$\mathrm{P}(X < x) = 1 - (x_0/x)^{\alpha} \qquad \text{[S2]}$$

defined for all $x \geq x_0$, for a given $x_0 > 0$. With the caveat that only three points are available here, the data suggest a Pareto shape parameter α of 0.83. Consistent with the main text, we take a system of 200 banks with the smallest banks making five interbank loans (recalling our assumption that this constitutes 20% of their balance sheet; Table S1). Thus, we take $x_0 = 5$ for the interbank degree distribution. Moreover, we assume a cut-off on bank sizes such that the biggest bank is no more than 10^4 times larger than the smallest.

This approach necessitates some minor adaptations of the plotting scheme used in Fig. 2. First, for given parameters, the present approach has some variability in the realized composition of bank sizes from one simulation to the next. To accumulate the results of successive simulations on a single plot, therefore, we measure the size of the index bank, not in absolute terms but as a proportion of the system's initial total assets. Second, in the present framework the index bank can be so large as to account for a significant proportion of the total system's assets by itself. Accordingly, we measure "impact" here as the proportion lost of total initial assets excluding the index bank. Similar to Fig. 2, we select a random index bank (in this case, of any size) and force it to fail by setting its capital to zero, without affecting the remainder of its balance sheet.

Fig. S3 *B*–*D* plots results from this approach. Here, different combinations of channels are shown on different plots to provide a better illustration of respective sets of individual simulation outcomes (dots in gray), that give rise to the mean impact plotted. The full model (Fig. S3*B*) illustrates strong nonlinearities: although index banks smaller than about 15% of the whole system do not initiate contagion, those above this threshold are liable to bring down the whole system. Indeed, the intermediate points in the figure represent outcomes where only a single bank remains, always being the largest bank originally present. Fig. S3 *C* and *D* explore less extreme cases, where either asset shocks or liquidity hoarding are deactivated. Here it is possible for system failure to be only partial (e.g., Fig. S3*D*). Nonetheless, the essential result is in accordance with that shown in Fig. 2: the effect of index bank collapse scales more than linearly with index bank size.

1. National Research Council (2007) *New Directions for Understanding Systemic Risk: A Report on a Conference Cosponsored by the Federal Reserve Bank of New York and the National Academy of Sciences* (National Academies Press, Washington, DC).
2. May RM, Levin SA, Sugihara G (2008) Ecology for bankers. *Nature* 451:893–895.
3. FDIC Quarterly Banking Profile (2011, First Quarter). Available at http://www2.fdic.gov/qbp/2011mar/qbp.pdf. Accessed August 3, 2012.

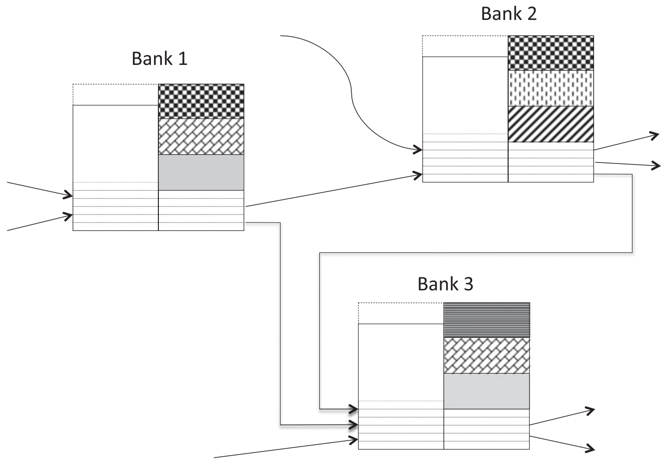

Fig. S1. A schematic illustrating two different modes of connectivity in the model, showing for simplicity the case where all banks have the same size. Arrows represent loans in the interbank network, pointing from lender to borrower. Independently of this network, banks may also hold external assets in common (shown hatched in different patterns). For example, bank 1 holds one asset in common with bank 2 and two assets in common with bank 3.

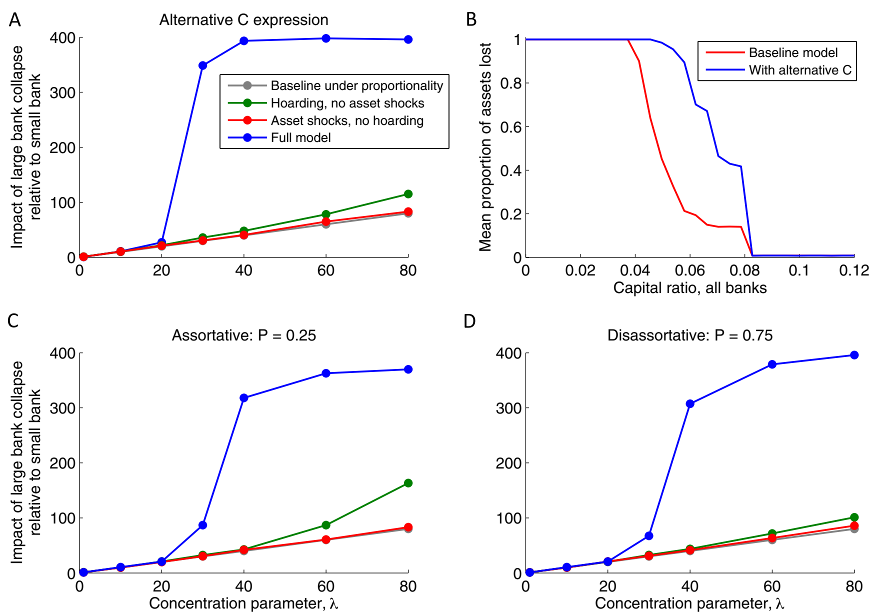

Fig. S2. Repeating Fig. 2*C*, under an alternative expression for system confidence (*A* and *B*) (see also *SI Text*) and allowing a nonrandom lending network between big and small banks (*C* and *D*) (see also *SI Text*). (B) Comparison of the extent of contagion from the model in *A* with the "baseline" model in the main text. In analogy to Fig. 1*B*, the initiating shock is applied to a randomly selected large bank, setting the value of one of its external asset classes to zero without affecting other banks holding the same asset.

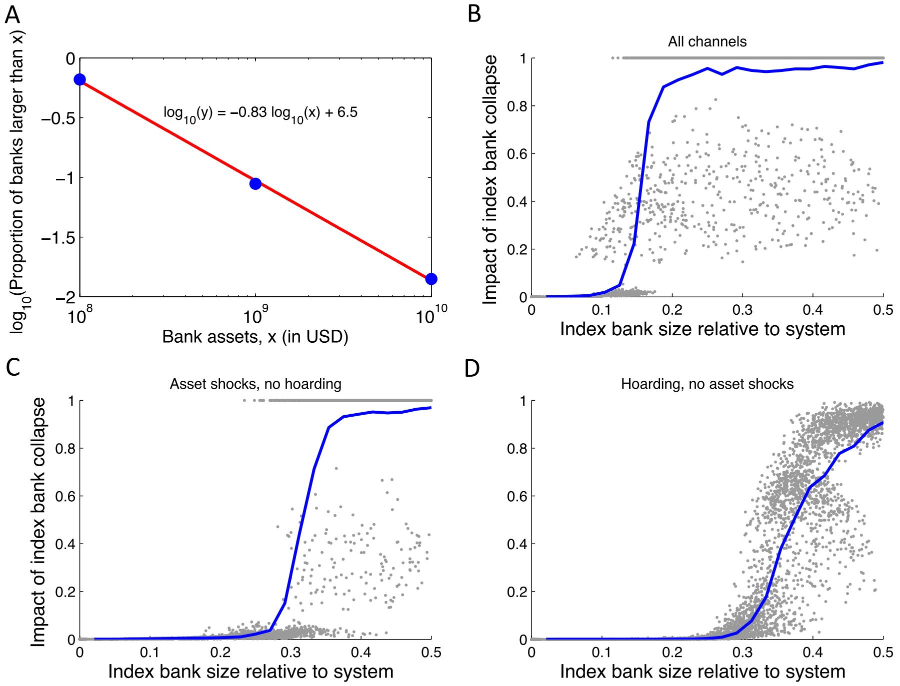

Fig. S3. Model results in the case of a Pareto distribution for bank sizes (see *SI Text*). (*A*) Cumulative distribution of bank sizes in the United States banking sector (3), and its closest fit with a Pareto distribution, implying a shape parameter of 0.83. (*B*) Impact of index bank collapse according to the full model, as a function of the index bank size. Gray points indicate individual simulation outcomes, and the blue line shows the mean outcome. Note that "system impact" here refers to the proportion of the initial system's assets, excluding the index bank, that is ultimately lost as a result of index bank collapse. Intermediate points all correspond to the survival of a single large bank, as described in the *SI Text*. (*C* and *D*) As for *B*, but individual channels acting alone (as indicated).

Table S1. Parameters and their default values

Category	Symbol	Meaning	Default value
Global	n	Total number of banks	200
	λ	Bank size disparity	24
	π_S	Initial proportion loans being short-term	0.5
Balance sheet	z	Number of outgoing loans, small banks	5
	θ	Proportion of total assets initially in interbank lending	0.2
	l_i	Proportion of bank *i*'s total assets initially liquid	0.01
	γ_i	Bank *i*'s initial capital (to assets) ratio	0.04
Asset class	Γ	Total number of distinct asset classes in the system	200
	g	Number of banks sharing an asset class	10
	n_B	Number of asset classes held by each big bank	10
	n_S	Number of asset classes held by each small bank	10