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Bilateral Collaboration and the Emergence of Innovation Networks

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In this paper, we model the formation of innovation networks as they emerge from bilateral decisions. In contrast to much of the literature, here firms only consider knowledge production, and not network issues, when deciding on partners. Thus, we focus attention on the effects of the knowledge and information regime on network formation. The effectiveness of a bilateral collaboration is determined by cognitive, relational, and structural embeddedness. Innovation results from the recombination of knowledge held by the partners to the collaboration, and its success is determined in part by the extent to which firms' knowledge complement each other. Previous collaborations (relational embeddedness) increase the probability of a successful collaboration, as does information gained from common third parties (structural embeddedness). Repeated alliance formation creates a network. Two features are central to the innovation process: how firms pool their knowledge resources, and how firms derive information about potential partners. When innovation is decomposable into separate subtasks, networks tend to be dense; when structural embeddedness is important, networks become cliquish. For some regions in this parameter space, small worlds emerge.

Key words: networks; innovation; knowledge; collaborative R&D; embeddedness
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1. Introduction

One of the effects of the recent rapid technical advance is a change in the technological structure of many firms. As new technologies have emerged and been integrated into existing products and technology spaces, the need to incorporate many new types of expertise, both in production and in innovation, has created the "multitechnology firm" (see Powell et al. 1996, Grandstand 1996, Grandstand and Sjolander 1990, and Teece and Pisano 1989). This broadening of the technological base creates a difficulty as the knowledge and technology necessary for innovation may lie outside a firm's traditional core competencies. A common strategy for addressing this problem now is for a firm to form alliances with other firms and institutions. Interfirm cooperation can be extremely effective in increasing the circulation of tacit knowledge, and in creating possibilities for a firm to acquire knowledge outside its boundaries. Consequently, these cooperative agreements for R&D have grown dramatically in number since the 1970s.

In this paper, we are concerned with interfirm alliances as the mechanism through which industry networks are built. It has been observed that different industry networks (and the same industry at different points in time) have different structural properties (see, for instance, Powell et al. 2005). This could be explained by differences in the underlying knowledge and information environments in which the networks are being built. To explore this possibility, we develop a model of alliance formation and examine the nature of the networks that emerge under different knowledge and information structures.[1] We use the model to study how the nature of the innovation task and the nature of a firm's embeddedness affect network structure and innovation performance.

Empirical work on network formation has focussed on firms' network motivations, attempting to understand why firms seek certain positions, for example, trying to fill structural holes (Burt 1992), increase their

[1] To model strategic technological alliances in their entirety is far beyond the scope of this paper. See Narula (1999) and Oliver (1990) for a discussion of the many varied motivations of firms to form outsider relationships. We focus on a single effect, namely, the production of shared knowledge.

positions of centrality (Podolny 1993), or form clustered neighbourhoods (Coleman 1988, Walker et al. 1997). Firms use position in the network as a competitive tool, and something that can be manipulated to increase performance, profits, or control.[2] Network structure then emerges out of network-oriented activity, and is determined by firms' strategies.

Game-theoretic approaches to the endogenous formation of coalitions and networks in industrial organization have also been developed recently. Leaving aside coalitions, a stable network is one in which for each agent (or pair of agents) there is a payoff-maximizing decision about which link to form (or whether to form a link between them), and all these decisions are consistent. The network is then Nash (or pairwise) stable, and in general stable structures tend not to be efficient. Obtaining an exhaustive characterization of the stable structures can be very difficult, and when such a characterization is possible the stable structures can be quite stark: the complete network, the empty network, or the star. Jackson (2005) provides a nice survey of that literature, including an overview of research on R&D networks.[3] Also, that literature is often static and has rational agents, although there are some models of adaptive network evolution with myopic players (see Watts 2001 and her later work). One thing that is typically ignored is the (largely path-dependent) coevolution of network and individual attributes (recent work by Marsili et al. 2004 offers one approach). In addition, explicit network position is in general not part of agents' motivations (a recent exception is Goyal and Vega-Redondo 2005, where agents seek to create links increasing their betweenness centrality).

Even assuming that network position is not considered in link formation, the context of knowledge creation and diffusion immediately introduces two complicating issues. First, the value of a partnership depends on cognitive complementarity between partners. Second, forming a link to create knowledge will change a firm's properties (knowledge in this case), and thereby change how well it complements other firms in the future, which adds dynamic complexity.[4]

These issues are central in what follows. In this, our model resembles Carley's (1991) constructural theory model, where agents are attracted to partners due to similarities in their respective knowledge, but bilateral interactions change both agents' knowledge.[5] Carley's concern is with the stability of predefined groups, however, whereas ours is with the emergence of network structure.[6]

In this paper, we develop an agent-based model of network formation in which firms repeatedly form pairs to create new knowledge. We abstract from "network-oriented" strategic motives of firms, and focus instead on the effects of firms' short-term innovation concerns. Partnerships are embedded cognitively, structurally, and relationally, and this directly influences the firm's choice of partner.[7] Over time, this link formation process results in an emergent structure representing an industrial network.[8] We look explicitly at two driving forces that jointly determine both

[2] See the studies of Powell et al. (2005) on the biotech industry, Baum et al. (2003) on bank syndicates, or Ahuja (2000) on the international chemicals industry.

[3] In general, the microeconomic details have been thin, but a good example of a multiple-stage game in which firms compete after forming an R&D network is Goyal and Joshi (2003).

[4] Because the act of forming an alliance can change a firm's properties, changing its desirability to other potential partners, the model we develop here is more evolutionary than many of the game-theoretic models of network formation that assume that a firm's inherent properties are stable over time (although of course its value as a partner can change if it becomes more central in the network).

[5] See also Axelrod's (1997) model of cultural dissemination, in which the cultural traits born by individuals evolve, while network structure remains fixed.

[6] Recent sociology literature has turned to analysing networks as complex systems with the emergence of high-level structure from lower-level interactions. Guimera et al. (2005), for example, connect the emergent network structure in the Broadway musical industry to the intricacy of the task of creating a musical. They find empirically that as the task becomes more involved, clusters increase in size, and there is a phase change in the sudden emergence of a giant component as the size of clusters grows. Powell et al. (2005) and Robins et al. (2005), empirically and through simulation, respectively, examine how the types of connections pursued by individuals condition global structures. In these papers and others (for example, Uzzi and Spiro 2005 and Chang and Harrington 2005), concern is with network emergence from individual behaviour, and in this literature we see some of the standard interests of complexity theory: phase changes, coevolution (Guimera et al. 2005), and interplay among different levels of aggregation (Uzzi and Spiro 2005, Chang and Harrington 2005). In our results, we find also that the nature of the innovation task has an important effect on both the global networks that emerge, and on the types of ego networks that perform well.

[7] The common explanations for partner choice, namely, resource complementarity and social structural context, are spanned by these three forms of embeddedness. In this regard, our model is in the tradition of empirical work such as Gulati (1995) and Chung et al. (2000), which find support for both explanations.

[8] Recent empirical analysis of networks has focused on their structural properties, in particular, whether various networks are small worlds. In general, the answer is yes. Coauthorship in a variety of academic disciplines (Newman 2001), patent citation in the U.S. biotech industry (Johnson and Mareva 2002), interlocking corporate directorships in the United States (for example, Davis et al. 2003), the BRITE/EURAM network, and the 5th Framework Targeted Socio-Economic Research Programme network (Cowan and Jonard 2004) all have small-world properties, and there is a consensus that small worlds are pervasive. A second structure that has received attention recently is the scale-free network (Barabasi and Albert 2000), which Riccaboni and Pammolli (2002) find in the life sciences and information and communication technology industry networks.

degree and clustering of the network, and the growth and distribution of knowledge: One is the nature of the innovation process and its decomposability, through its effect on the way firms can effectively pool their knowledge. The other is the relative importance of relational versus structural embeddedness in determining the probability of success of a collaboration.

2. Firm Cooperation

In the past, markets and hierarchies have been dominant forms of economic organization, and both imply a well-defined boundary of the firm. As more firms have increased their nonmarket relations such as strategic alliances, researchers have begun to discuss the "networked organization" (see Powell 1990, for example). Within a network structure, firm boundaries are relatively porous, and a firm survives by having good contacts with other firms who hold complementary assets.

The strategic alliance is central to the process creating a network within an industry. However, for any firm seeking to expand its innovation capability through alliance formation, there is the question of choosing a partner. Selecting a partner, while many other firms are doing the same, becomes a strategic issue for the firm. Part of the analysis of this issue involves observing that alliances do not exist in a vacuum, but are embedded in a variety of ways. Research has focussed on three: *cognitive, relational,* and *structural embeddedness*. Each gives a partnership value, and plays an important role in determining the desirability of different partners.

In the context of finding an alliance partner, cognitive embeddedness refers to two firms' abilities to effectively integrate their respective knowledge. Empirical analyses of alliance formation conclude that firms look for partners that are complementary, in the sense of providing missing resources.[9] In this regard, one difficult issue has to do with how resources are combined, which in turn determines what complementary means, and in the model below we characterize knowledge so as to permit us to address this issue directly. In the literature there is a consensus that the effectiveness of cooperation has an inverted U-shape in cognitive distance. If firms are too close together, their knowledge overlaps too much and there is little point in sharing; if they are too far apart, they have difficulty understanding each other, and so sharing is too difficult. The arguments are very appealing intuitively (see, for example, Nooteboom 1999 or Grant 1996), and Mowery et al. (1998) find this effect empirically.

One consequence of a knowledge partnership is that partners will develop closer cognitive ties. That is, their knowledge profiles will become more similar. Mowery et al. (1998), using patent data, show that "technological overlap between joint venture partners after alliance formation is greater than their pre-alliance overlap" (p. 517).[10] This has the feature of increasing embeddedness, but after a time may make firms less attractive to each other because as they become similar, there is less to share. Our model permits us to examine how these effects depend on the nature of the innovation process and how firms integrate their competencies in that process.

Cooperation between firms is also risky, and marked by uncertainty regarding a partner's skills, goals, and reliability, as well as the pair's ability to work together (see Powell 1990, p. 318). This can be cast as an issue of incomplete information, and the most obvious way to reduce uncertainty is to improve the information used in choosing a partner. There are two possible sources: *experience* and *other firms*. The first relates to relational embeddedness, and the second to structural embeddedness (see Uzzi 1997, 1996).

Past experiences with a firm will both improve abilities to cooperate and yield information about that firm. Successful collaboration involves common knowledge, shared routines, similar ways of thinking, and tacit knowledge, all of which can be built through repeated cooperation (Garcia-Pont and Nohria 2002). In addition, it also creates trust, both in terms of motives and in terms of competencies (Dodgson 1996, Sako 1991). As a consequence, there is inertia in partnership formation, and stability in network structures: firms will, all else being equal, prefer partners with whom they have worked in the past.

The second source of information about potential partners is other firms (see Kogut et al. 1992, for example). Those who have worked with a firm will have experience that they can, in principle, share with others who might be considering working with that firm. This is captured by the idea that many alliances exhibit structural embeddedness: Firms tend to find partners close to them in network space. In the model we develop below, this source of information is included explicitly: A firm's perceived value to me as a partner increases if my previous partners have had good experiences with that firm. My network of immediate contacts is a source of information about possible future contacts. The entanglement of these different effects—learning about partners from shared history and learning from other partners—complicates analysis of network formation. Chung

[9] This idea has a long history, going back to Penrose (1959). See Chung et al. (2000), Gulati (1995), Hamel et al. (1989), Doz (1988), Teece (1986), and Richardson (1972), among others.

[10] Uzzi (1997) finds the same convergence (see also Dyer and Nobeoka 2000). The models of Axelrod (1997) and Carley (1991) also include this effect.

et al. (2000) and Gulati (1995), examining very different industries, find that both indirect ties and a history of direct interactions affect the probability that two firms will form a partnership in the future, but that the strength of the effect decreases (and turns negative for the indirect effect) as the number of (direct or indirect) interactions increases.[11] We have argued above that the decreasing strength of the former effect arises from the dynamics of strategic complementarity. As two firms continue to interact and learn from each other, their knowledge bases will eventually have too big an overlap, and strategic complementarity falls. Such an effect would explain a decrease in strength, and even a negative effect, of past interactions on the probability that two firms interact in the future. The model we develop displays this inverted-U effect, but it is not assumed directly; rather, it arises from the way we have modelled the more basic process of knowledge creation.

3. The Model

A schematic description of the model is as follows. Each period every firm attempts to innovate, either by itself or in collaboration with one other firm. When two firms collaborate, they pool their knowledge and use that as input into new knowledge production. The success of an alliance is not guaranteed, but is determined by the partners' structural and relational embeddedness. Being more tightly embedded will increase the probability of success. If the alliance is successful, new knowledge is created and added to both partners' existing knowledge. All partnerships then dissolve, and in the next period firms again form alliances, possibly with previous partners. Repeated alliance formation generates an economywide network; repeated innovation changes the knowledge endowments of firms and of the economy.

Four assumptions are worthy of comment.

First, firms are typically found to have more than one bilateral partnership simultaneously, whereas we assume that a firm has at most one partner per period. The assumption is made for technical reasons: With single partnerships the structure of relationships is unique in each period, whereas if firms can form more than one this uniqueness disappears, and thereby drives significant amounts of randomness into the model. This makes the model more difficult to control, and the results more difficult to understand and interpret.

[11] The two papers find an inverted-U in direct interactions, and a positive but weakening effect in the number of joint partners. Chung et al. (2000) suggest that their data display an inverted-U in the latter relationship, but this claim should be treated cautiously because the peak occurs at 185 alliances according to their calculations!

Second, in the algorithm we use to model pair formation, there is an implicit assumption that firms know all other firms' knowledge profiles completely. This assumption may be less aggressive than it first appears. Significant amounts of information are available through patent searches and firms' public activities such as advertising, hiring, and specific investments. In addition, regardless of the "complete information" assumption, there is always a risk that an innovation attempt fails.

Third, we assume that firms evaluate a partnership on the basis of immediate knowledge production, excluding other, and longer-term, objectives. Including longer-term objectives would not destroy the uniqueness of the pair formation, provided that both firms have the same evaluation of the partnership, but would, however, add significant complication. The focus on immediate knowledge production fails to capture some aspects of partnership formation, but is behaviourally adequate either when knowledge is advancing very rapidly, or when production requires a rapidly expanding knowledge base. In the former, firms have a dominant concern for innovation because failing to innovate implies a serious competitive disadvantage. In the latter, firms have a strong need for partnerships because they typically cannot produce needed knowledge in-house.

Finally, firms have no explicit network motivation. However, empirical results show that firms consider how particular partnerships would change their position in the network when forming an alliance. Focussing solely on knowledge-related motives implies a simpler model, but nevertheless, identifiable patterns in the resulting network structure do emerge. This has two possible implications. The methodological implication is that the types of networks that emerge empirically need not be driven by (nor be evidence for) firms adopting particular strategies vis-à-vis network position in their alliance formation. The second implication comes as a result of the model. While our firms focus on knowledge creation, as we see below, there are situations in which firms that have nonstandard network positions do better, in terms of knowledge accumulation, than others. This implies that there may be points in which firms should operate against what appears to be the natural tendency.

In the subsections that follow, we give formal presentations of each of the components of the model.

3.1. Knowledge Pooling and Production

We assume an even-sized finite population of firms $S = \{1, \ldots, n\}$. Each firm $i \in S$ is characterised by the amounts it holds of κ distinct types of knowledge. We represent this as a vector of length $\kappa \geq 2$, which allows us to think of each firm as located at a point in knowledge space. By innovating, a firm creates new knowledge and so moves to a new location in the knowledge space.

Because innovation is knowledge creation, when two firms innovate together, both will have more knowledge after the innovation than they had before, and their profiles will be more similar. To capture this, we model knowledge production as a three-step process, wherein joint innovation moves firms farther from the origin, but closer together in knowledge space.

First, when i and j collaborate, they pool their knowledge to create a joint knowledge vector. Formally, the pooling is done elementwise, and each element of the pooled vector is written as

$$\nu_{ij,l} = (1-\theta)\min\{\nu_{i,l}, \nu_{j,l}\} + \theta\max\{\nu_{i,l}, \nu_{j,l}\}, \qquad (1)$$

where $\nu_{i,l}$ is firm i's knowledge of type l, and ν_{ij} is the vector of pooled knowledge of the alliance ij.

Second, this pooled knowledge acts as an input into a knowledge production function. For simplicity, we use a constant elasticity of substitution function, so the new knowledge created by the collaboration is

$$\phi(\nu_{ij}) = A\left(\sum_l \nu_{ij,l}^{\gamma}\right)^{1/\gamma}. \qquad (2)$$

Third, if the innovation project is successful, new knowledge is created and added to both partners' knowledge. As the argument of the production function is the joint knowledge profile, it seems natural to let this joint profile also determine the type of knowledge produced.[12] Therefore, we assume that when new knowledge is created, the probability that it is of type m is

$$\frac{\nu_{ij,m}}{\sum_l \nu_{ij,l}}. \qquad (3)$$

If the collaboration fails, both firms get zero.

In Equation (1), θ scales the pooled knowledge between the minimum and the maximum level of the partners. This parameter can be related to the nature of the innovation process. If the process is made up of discrete tasks that can be done in isolation, partners will specialize, each doing the task at which he is more proficient, with the results brought together at the end to create the complete innovation. The pooled-knowledge vector should be the elementwise maximum of the individual vectors, and thus θ is close to one. By contrast, if the innovation process is more systemic, each partner will be involved in all aspects. Specialization is not possible, and the weaker partner will act as a bottleneck. The pooled-knowledge vector will be the elementwise minimum, and θ is close to zero.[13]

A second interpretation of θ is as a measure of the taste for dissimilar partners. If θ is close to zero, then if i's partner is worse than he anywhere, the joint profile will produce less knowledge than i would in isolation. This is reflexive, so, in the extreme, partnerships can only form between agents with identical knowledge stocks. By contrast, if θ is close to one, a partner cannot hurt you, so firms look for partners whose endowments complement their own because both can benefit from the other's strengths. Implicitly, they search for partners who are different from themselves in the sense of being good where they are bad.

In many models of joint innovation, a simple measure of distance in knowledge space determines the effectiveness of a collaboration (see, for example, Peretto and Smulders 2002, Nooteboom 2000, Mowery et al. 1998, or Alstyne and Brynjolfsson 1996). However, putting two firms' knowledge together involves considering deeper issues of complementarities, beyond the specification of an optimal distance. These we capture by the production function approach.

3.2. Innovative Success and Experience

Innovative success is not guaranteed. Firms therefore evaluate their possible partnerships on the basis of expected outcomes. Estimates of success probabilities are closely tied to relational and structural embeddedness (Gulati and Gargiulo 1999). If a potential partner is embedded relationally, a firm can use its past history with that partner as information. If a potential partner is embedded structurally, a firm can use information from common partners. Our concern is the relative importance of relational versus structural embeddedness as information sources about possible partners.

Formally, define the probability of success of ij in period t as p_{ij}. Define $t_{ij} < t$ to be the period in which the latest collaboration of i and j took place, and χ_{ij} to be the outcome of that collaboration (one for an innovative success, zero otherwise). Then, define the relational credit of the pair ij in period t to be

$$r_{ij} = \delta^{t-t_{ij}} \cdot \chi_{ij}, \qquad (4)$$

where $0 < \delta \leq 1$ is a discount factor. Partner-specific collaborative skills are developed in a successful collaborative attempt, with more recent success being

[12] We have explored other variants that allow firms in a partnership to create slightly different types of knowledge from each other. Operationally, for example, a share of the new knowledge is allocated according to the joint profile to a category common to both participants, while the remaining part is firm specific, and allocated individually according to each partner's profile. As long as the share of new knowledge allocated according to the joint profile is not negligible, the results are qualitatively unchanged.

[13] For an empirical study on the decomposability of innovation, see Sobrero and Roberts (2001).

more valuable. Similarly, define the structural credit of the pair ij in period t to be

$$s_{ij} = \sum_{k\neq i\neq j} r_{ik} \cdot r_{kj}, \tag{5}$$

where r_{ik} and r_{kj} are defined as earlier. A sketch of the way this works would be as follows: Suppose that firm i is interested in firm j. Firm i then looks through its list of past partners, and when it finds one (k) with whom it was successful ($\chi_{ik} = 1$), it asks, "In your latest interaction with j, were you successful [which determines χ_{kj}], and if so, when was that interaction [which determines $\delta^{t-t_{kj}}$]?" The answer to these questions determines r_{kj}. The sum over all its previous partners determines the structural credit between i and j.

In general, a firm will consider both relational and structural credit, so total credit is a weighted sum

$$c_{ij} = \alpha \cdot r_{ij} + (1-\alpha) \cdot \frac{s_{ij}}{C}, \tag{6}$$

where $C = \sum_{k\neq i} \chi_{ik}$.[14] The parameter $\alpha \in [0, 1]$ measures the importance of relational versus structural embeddedness in estimating success probabilities. This way, c_{ij} accounts for both direct and indirect information about potential partners.

We assume that success is always possible but never guaranteed, so its likelihood has a range of values $0 < \pi < \Pi < 1$. The probability that the next collaborative attempt is a success is assumed to be

$$\pi_{ij} = \pi + (\Pi - \pi) \cdot c_{ij}, \tag{7}$$

and the expected knowledge produced by the partnership ij is

$$E_{ij} = \pi_{ij} \cdot \phi(v_{ij}). \tag{8}$$

Firms that innovate in isolation have one source of risk removed, namely, that associated with having to work with a partner. This does not make autarchic innovation a sure thing, however, a firm innovating alone is successful with probability Π.

3.3. Pair Formation and Equilibrium

Each period, firms can collaborate with one other firm. To model the pair formation process, we draw on the matching literature (Gale and Shapley 1962). Because we consider a single population of firms, matching here is a roommate problem. In a roommate-matching problem, each individual i has a preference ordering over all other individuals. We generalize to include the possibility of self-matching by including a preference for isolation. In that event, the pooled vector is simply the vector of i, and production remains defined as it was above. Supposing that firms are risk neutral, preferences are determined by the expected output of the potential pair, that is, i prefers j to k if and only if $E_{ij} > E_{ik}$. A stable matching μ is then a partition of the population into q singletons and $(n-q)/2$ pairs, such that there is no pair of nonmatched agents, such that each prefers the other to his current partner in the matching.[15]

Because firms in a pair assign the same cardinal value to their match, a unique stable matching always exists. The intuition is that of all possible pairs of firms (including ii pairs), one pair produces the biggest innovation. The two firms in that pair will block any matching in which they are not together. Thus, that pair of firms must be in the stable match. A recursive argument generates the unique stable matching.

4. Numerical Experiment

The model just developed represents a complex dynamic process, which we study numerically. We examine how industry behaviour responds to changes in decomposability (θ) and in the relative importance of relational (α) versus structural ($1-\alpha$) embeddedness.[16]

We study a population of 100 firms and five knowledge types. At the outset, individual knowledge endowments $\nu_{i,l}$ are randomly drawn from a uniform distribution over $[1, 2]$, independently for every element in each firm's knowledge vector. Each period, firms form pairs (or stay alone), they innovate, absorb any new knowledge, and then separate. In each period, the network consists of isolated firms and disconnected pairs, as given by the stable matching. We record firms' partnership histories over a 200-round industry lifetime to produce a weighted network.

We set the minimum and maximum success probabilities to $\pi = 0.75$ and $\Pi = 0.95$, respectively. The discount factor is equal to $\rho = 0.98$, and the scale factor on the production function, A, is set to scale the growth rate of knowledge to an average 20% per period.[17] The two parameters θ and α take on values between zero and one, inclusive. We explore this space on a 25×25 grid of equally spaced points,

[14] C is the number of firms i asks for information about j. Thus, s_{ij}/C is the average response about j. The results are not fragile with regard to this assumption. We have explored different specifications of s_{ij}: sum of responses, responses weighted by discount factors, and so on. Qualitatively, the results are unchanged.

[15] A matching μ is stable if there is no pair $ij \notin \mu$ such that $E_{ij} > E_{i\mu(i)}$ and $E_{ij} > E_{j\mu(j)}$.

[16] The code was written in C++ using Borland C++Builder, and can be obtained from the authors on request. The data files are available at http://www.merit.unimaas.nl/rcnj.

[17] We have experimented with other growth rates and have found no significant variation.

averaging the results of 15 independent replications at each (θ, α) location.

5. Parameter Effects

Characteristics of the network, knowledge growth, and distribution are affected by three parameters: γ, which controls substitutability in knowledge production; α, the relative value of relational embeddedness; and θ, measuring the decomposability of innovation.

The parameter γ determines the ease of substitution between knowledge types in innovation.[18] When γ is small, output is determined by the value of the smallest input. Thus, firms have strong incentives to find partners because increasing the minimum element in the knowledge vector is very valuable. As γ increases, output is less and less determined by the smallest input, so incentives to find partners diminish because a firm can compensate for its weak knowledge types with its own strengths. Throughout the experiments reported below, γ is set to 0.1, a value that ensures active networking.

The parameter α measures the relative importance of relational versus structural embeddedness in estimating success probabilities with different partners. When the former dominates, there is a natural source of inertia in the search for partners. Relational embeddedness only contributes information about past partners; structural embeddedness is the only source of information about new partners. When structural embeddedness has no value ($\alpha = 1$), success is more likely with past partners because novel partners are assigned the minimum success probability. Thus, we would expect firms to have more partners as α falls. In addition, structural embeddedness introduces information about partners' partners, and so provides a natural avenue by which new partnerships create closed triangles. This effectively increases clustering in the network.

The parameter θ captures the decomposability of the innovation task, affecting in turn the number of potentially valuable partners. A single firm's weakness in one type of knowledge can be compensated for either by an internal strength in a different type or by a partner who is strong in that type. The feasibility of the former is determined by the elasticity of substitution in knowledge production γ; the feasibility of the latter is governed by the decomposability of innovation θ. Consider an ij partnership. If j dominates i in every knowledge type, there is clearly no advantage to collaboration from j's point of view. As this is reflexive, any willing partner of i will be worse than i in some category and better in some other. This trade-off (between the gain in one category and the loss in another) is evaluated differently depending on γ and θ. For fixed γ, as θ increases a smaller gain is needed to compensate for a given loss. Thus, as θ increases, the number of acceptable partners increases for any firm i.[19] Further, repeated interaction makes two firms' knowledge profiles more similar, which decreases their abilities to compensate for each others' weaknesses as just described. Because increases in θ increase the size of innovations, the convergence of partners' knowledge happens faster. This implies a stronger tendency to search for novel partners. In short, when θ is large, new partners are relatively easy to find, and firms switch partners more frequently. We expect, then, a positive correlation between degree and θ.

6. Results

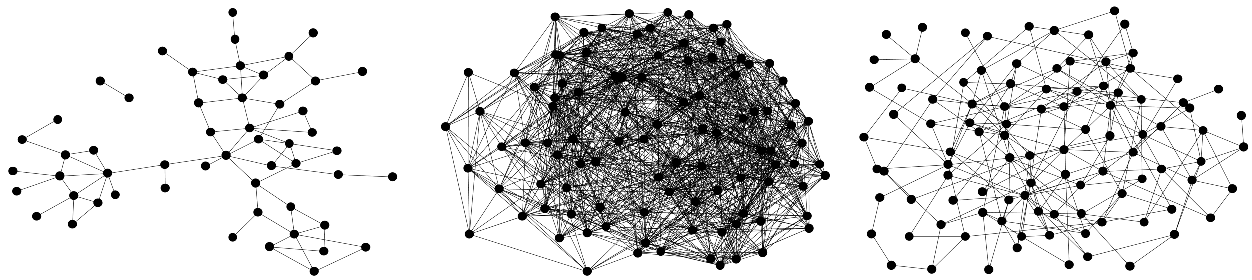
In the sections that follow, we examine how the emergent properties of the system respond to the parameters α and θ. First, however, to give a visual impression of the results, in Figure 1 we show three characteristic networks from different regions of the (α, θ)-space.[20] In the leftmost panel, we consider strong structural embeddedness with intermediate decomposability, $(\alpha, \theta) = (0.1, 0.4)$. Density is low, many firms innovate in isolation, and the largest connected component (54 firms) consists of several densely interconnected subgroups (with high levels of clustering) spanned by a small number of ties. When we consider the same level of structural embeddedness but high decomposability in the innovative tasks, $(\alpha, \theta) = (0.1, 0.8)$ in the central panel, the network is considerably denser, the largest component contains the entire industry, and clustering, once rescaled, does not suggest more structure than a random graph would show. Strong asymmetries in degree centrality also exist. Finally, with strong relational embeddedness and intermediate task decomposability $(\alpha, \theta) = (0.8, 0.6)$, we see a sparse connected network with little clustering and few central individuals having the feature of a random graph.

[18] Precisely, the elasticity of substitution is $1/(1-\gamma)$, and scales the production function between a linear function ($\gamma \to 1$), and a Leontieff (fixed coefficients) function ($\gamma \to -\infty$).

[19] For each firm i in the industry, there is an indifference frontier separating autarky from collaboration, running through i's location and obtained by constructing the fictitious individual ϵ such that $\phi(v_i) = \phi(v_{i\epsilon})$. Firm i is ready to accept any partner above the frontier and prefers autarky to any partner below. Using the implicit function theorem to differentiate $\phi(v_i) = \phi(v_{i\epsilon})$ with respect to θ shows that increasing decomposability lowers firm i's indifference frontier in any dimension, which in general will increase the number of its feasible partners.

[20] Precise definitions of the summary statistics are given in §§6.1–6.3. The network representations are obtained with the software NetDraw, using spring embedding with distance, node repulsion, and equal edge length as layout criteria.

Figure 1 Three Characteristic Networks from Different Parts of the Parameter Space



$(\alpha, \theta) = (0.1, 0.4)$	
GC	54
Den	0.0156
Cl	0.31
RCl	20.1
DegC	0.056

$(\alpha, \theta) = (0.1, 0.8)$	
GC	100
Den	0.2412
Cl	0.326
RCl	1.36
DegC	0.156

$(\alpha, \theta) = (0.8, 0.6)$	
GC	100
Den	0.0448
Cl	0.063
RCl	1.42
DegC	0.078

Note. GC: size of the giant component; Den: density; Cl: clustering coefficient; RCl: clustering rescaled against that of a random graph of identical degree; DegC: degree centralization.

In the next sections, we turn to a more detailed analysis. We begin by focusing on the individual firm's direct ties. Both decomposability and structural embeddedness increase the average firm's number of partners. The distribution of links over firms is relatively flat, except for high levels of decomposability and structural embeddedness, where we observe significant skewness in the distribution. In addition, in the same region of the parameter space we find a positive association between performance and degree, while the sign of the association is negative when networking is weak.

At a slightly higher level of aggregation, we turn to local neighbourhoods, i.e., indirect ties. Clustering increases as structural information $(1-\alpha)$ increases in importance, but has a nonmonotonic relationship with decomposability, increasing and then decreasing with θ, especially when rescaled. Performance has a positive correlation with clustering both when the overall network is itself highly clustered, and in one region where it is not.

Regarding global properties of the network, we find that connectivity demands a high level of decomposability, a condition under which closeness centrality and performance are positively associated, while the association is negative for low decomposability.

Finally, we move to aggregate results and discuss possible goals for innovation policy in terms of production and distribution of knowledge. Equity in the distribution of knowledge is seen to increase with decomposability. Industrywide specialization also increases in that same direction, but at the economy level specialization might decrease as θ rises.

6.1. Partnerships

To identify partnerships, it is first necessary to derive the distance between any pair of firms. We define the distance d_{ij} between i and j as the number of edges in the highest-frequency path linking them.[21] The number of partnerships held by firm i is then the number of firms at distance one from i. The density of the graph is the ratio of the average number of partnerships over the number of possible partnerships $n(n-1)/2$, and is shown in the top-left panel of Figure 2.[22] We observe that as innovation becomes more separable, and as structural embeddedness becomes more important, firms have more partners. This is in keeping with the discussion of parameter effects in §5.

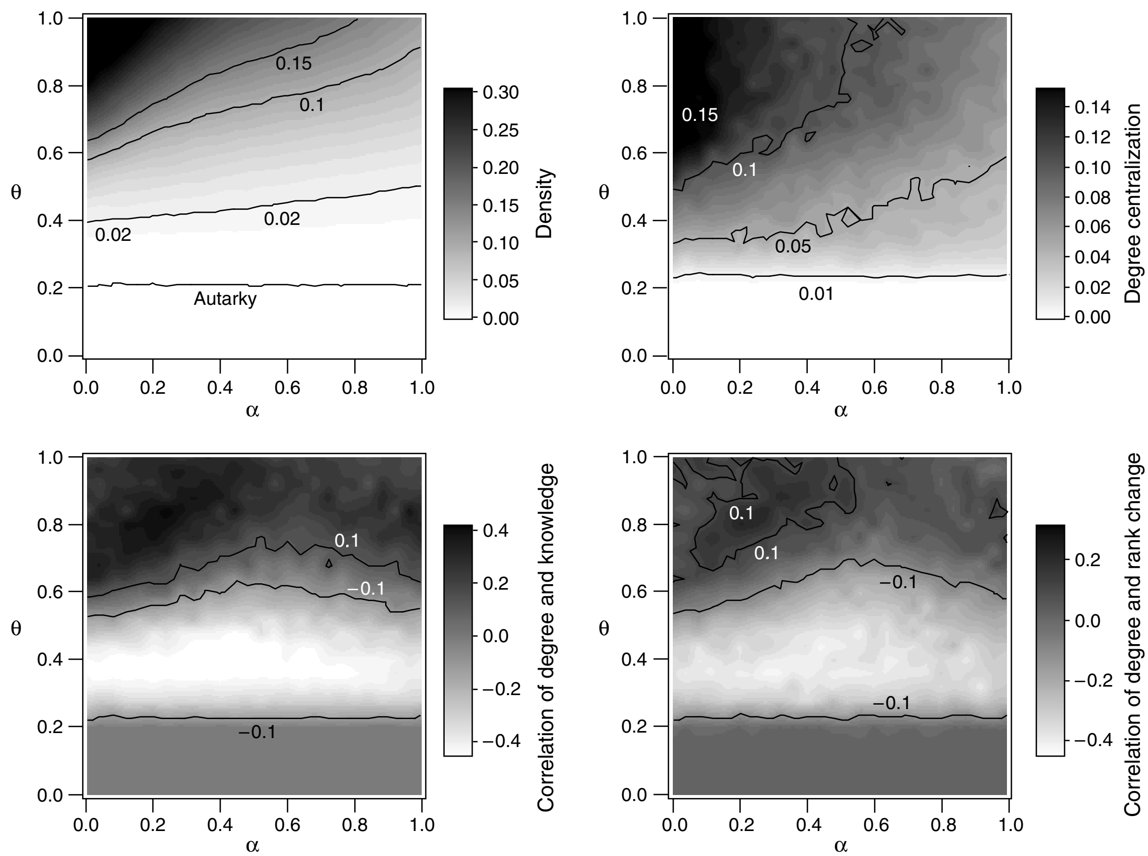
Do all firms engage in networking with equal intensity? The top-right panel of Figure 2 shows the normalized degree centralization index.[23] Degree centralization is low in general, implying a relatively

[21] The generalized distance between pairs of firms is obtained as follows. Define ω as the weighted graph of alliances over history, with ω_{ij} the frequency of partnership ij. There are typically many paths $s = (i = s_0, s_1, \ldots, s_z = j)$ between i and j, but different paths can be more or less common. Each path s has a length z and a probability of being activated equal to $p_s = \prod_{l=1,\ldots,z} \omega_{s_{l-1}s_l}$. The distance d_{ij} is the length of the most likely path s^* between i and j.

[22] In each of the results that follow we present a three-dimensional surface with α and θ in the (x, y) plane and the variable of interest on the z-axis. We use a filled contour plot, of the type seen in atlases. In each figure, darker shades of gray indicate larger values of the variable (higher points in the z-direction).

[23] The normalized degree centralization index is $\sum_i(\Delta - n_i)/[(n-1)(n-2)]$, with n_i the number of partnerships involving i and $\Delta = \max_i n_i$. Normalized centralization is the sum of individual deviations from the largest individual number of partnerships over the maximum possible value of that sum (obtained in the case of a star: $n-1$ firms having deviation equal to $n-2$ and one having

Figure 2 Density, Degree Centralization, and Correlations with Innovative Performance and Rank Change



even distribution of alliances across firms. Centralization is highest, however, when both task separability (θ) and structural embeddedness ($1-\alpha$) are high. There, some firms have many partnerships while many other firms have only a few.

A question that would be of interest to managers is whether these highly networked firms are better off. We measure knowledge by innovative potential: the innovation a firm could produce using only its own knowledge as input (as in Equation (2)).[24] Innovative potential expresses the absolute performance of a firm. To measure relative performance, we use the rank change of a firm's innovative potential in the population. The correlations between the firm's degree centrality (number of partners) and both its absolute and relative performance are displayed in the bottom panels of Figure 2. The patterns are very similar. In general, structural versus relational embeddedness has little effect, but correlations rise with innovation decomposability. Strikingly, when decomposability is low, correlation between networking and performance is negative. Here we are near the autarky frontier, and alliances are few. In this region, if a firm has high innovative potential, it has little incentive to seek a partner. Thus, firms forming many partnerships are the less knowledgeable ones. Here, partnering signals a weakness that the alliance hopes to attenuate, so in general those with partners are those with low innovative potential.

Correlations between degree centrality and performance are high when degree is high, so in that part of the parameter space a knowledge-focussed strategy is effective. However, there is a region (high θ and high α) when correlations are still strongly positive, yet degree is lower. Here, adding network considerations to a firm's partnering strategy could provide it with competitive advantage over firms that consider only short-term knowledge production. Of course, the strategy may be globally self-defeating: If all firms try to be stars, the net effect would be that degree increases without any firm becoming more central.

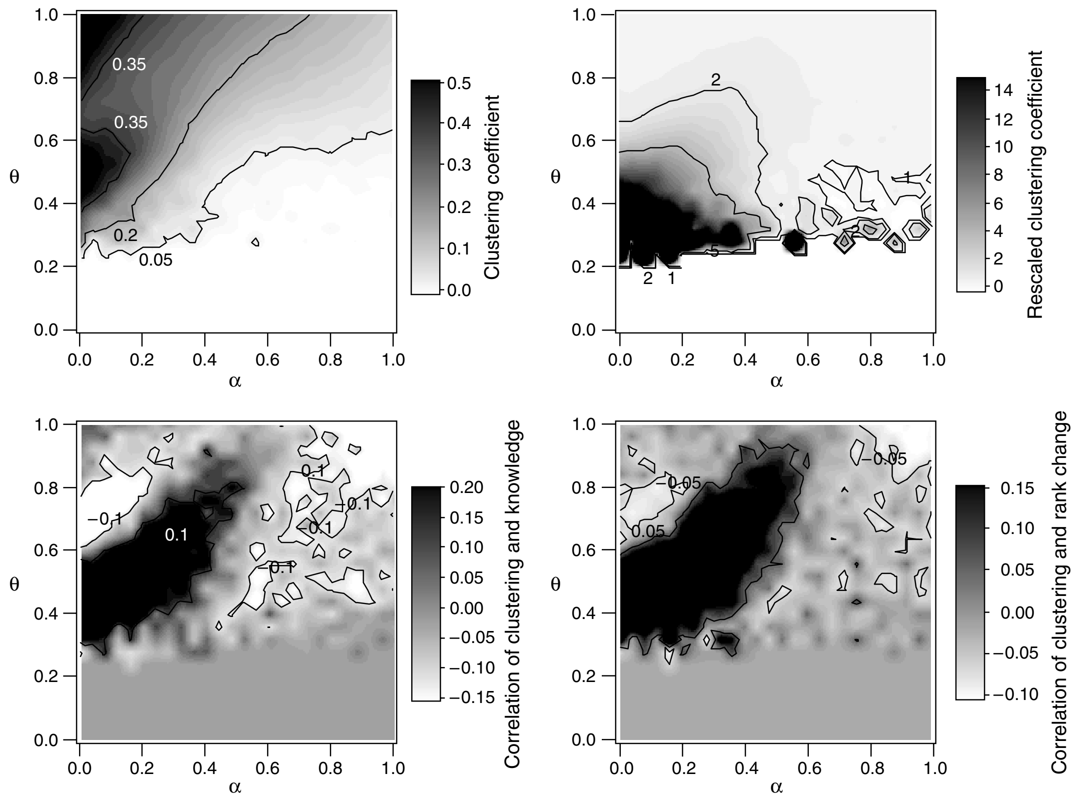
6.2. Local Structure

A single firm's clustering is defined as the average ratio of actual to possible alliances among its

zero deviation). The closer to one the centralization index is, the more unequal (centralized) the link distribution is.

[24] Because firms are driven by knowledge production, this measure of knowledge is more natural than other scalar measures such as a sum of its different types of knowledge.

Figure 3 Clustering Coefficient, Rescaled Clustering, and Correlations with Innovative Performance and Rank Change



partners.[25] A clustered neighbourhood for i corresponds to redundant ties, and thus a weak position in terms of spanned structural holes (see Burt 1992 and Ahuja 2000). In the top-left panel of Figure 3, we display the average of firms' clustering at each point in the (α, θ)-space.

In the autarky region ($\theta < 0.2$), firms innovate in isolation, which by convention gives a clustering coefficient of zero. As soon as partnering begins to take place, however, clustering increases monotonically with decomposability, and the gradient is higher when structural embeddedness is most important.

In general, though, this measure of clustering strongly correlates to the degree of the graph (which is observable in our results), and thus can be misleading. As firms acquire more links, even at random, the network becomes locally denser. In the top-right panel of Figure 3, we show rescaled clustering: Values significantly larger than one indicate a structure richer than a random graph.[26] Here there is a very clear effect of both θ and α. Clustered networks form when structural embeddedness is important (as expected, see §5), and when innovation is not decomposable. The latter effect is driven by the fact that near the autarky frontier, small isolated cliques form. The fall of rescaled clustering in the upper-left corner shows the erosion of structure when the network becomes denser, but with values in general greater than one, the emergent networks still have more structure than purely random graphs.

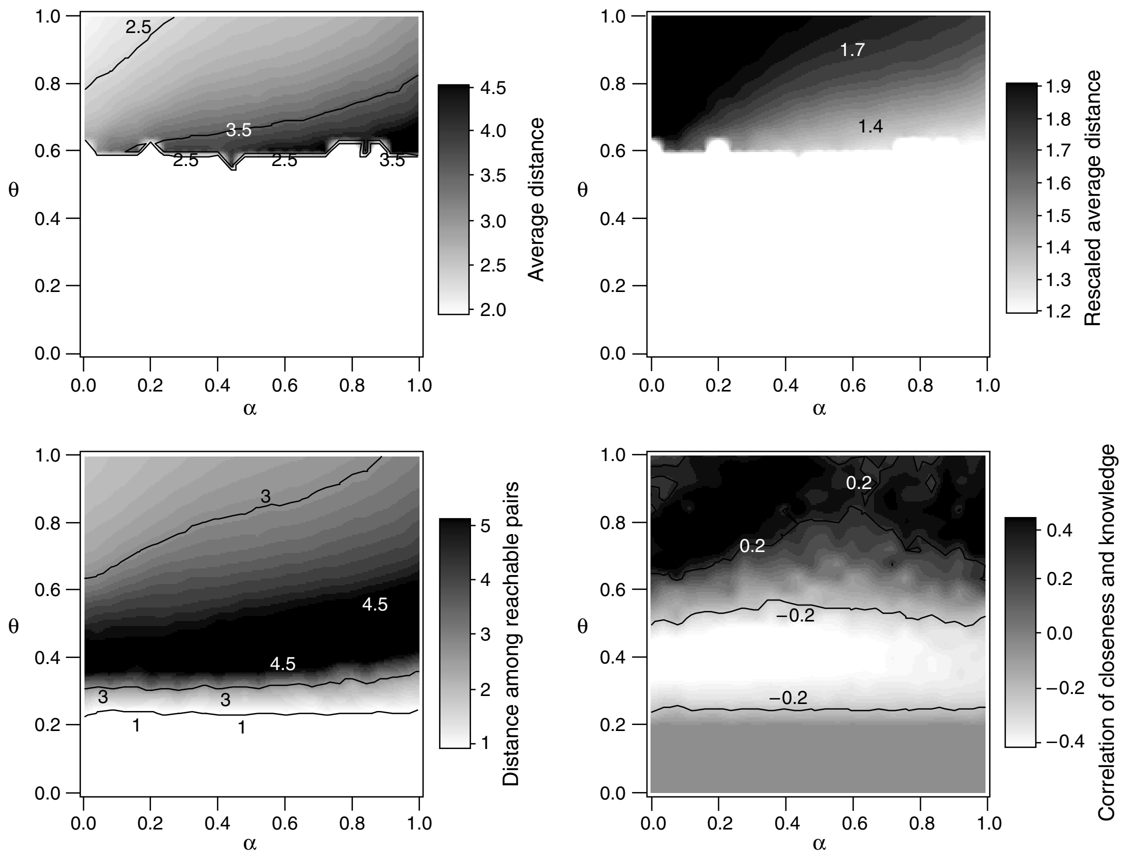
How do clustering and performance relate? As above, we compute the correlation coefficients between firms' clustering and both absolute (innovative potential) and relative (rank change) performance. These are shown in the bottom panels of Figure 3.

The correlation between clustering and knowledge performance is high when clustering is high. This suggests that managers following a knowledge-oriented strategy (as opposed to a network-oriented one) will not be led too far astray, as it naturally creates cliques in regions of the parameter space where clustering is good for performance. It also suggests, however, that if an industry occupies this space, a

[25] The clustering of firm i's neighbourhood is obtained by dividing the number of partnerships among firm i's partners by the number of possible partnerships. Formally, the numerator is $\sum_{j,\,l \in N_i} I(j, l)$, with N_i the list of firm i's partners, $\#N_i = n_i$, and $I(j, l)$ the indicator of $\{j \in N_l\}$. The denominator is $n_i(n_i - 1)/2$, the number of possible partnerships among i's partners. (This is equal to one minus the ratio of nonredundant contacts to total contacts (Burt 1992), which is a standard measure of structural holes.) The clustering coefficient is the average taken over all the firms.

[26] We rescale by dividing clustering by the clustering coefficient of a random graph of equivalent size and average degree, which is approximately $\sum_i n_i/n^2$.

Figure 4 Distance Measures and Correlation with Innovative Performance



firm that is for some reason not embedded in a clique may fall behind in the knowledge race, and so might actively pursue a clique-joining strategy. In these figures we can also see, however, a region of space where average clustering is lower, but the correlation between firm clustering and firm performance is still high. This occurs for moderate degrees of structural embeddedness and relatively high decomposability. Here a firm can gain both absolute and relative advantage by pursuing a strategy of strengthening its local cluster.

These results can also be interpreted in terms of structural holes. A firm in a neighbourhood with many structural holes will perform relatively well when both θ and α are high. Here, relational embeddedness dominates structural embeddedness, so the value of indirect links (which tend to eliminate structural holes) is minimal. In addition, task separability implies that dissimilar partners are valued. Because a firm and its partners become similar, and this similarity is transitory, a firm will, all else being equal, avoid partners of its partners, which implies that structural holes remain open.

6.3. Global Structure

The average distance between nodes in a network is often thought to affect how rapidly knowledge travels through it. The top-left panel in Figure 4 shows average distances in the connected network (distances in a disconnected network are infinite) at each point in the (α, θ)-space.

When θ is small, none of the networks connects, due to the high prevalence of autarchic innovation. When θ exceeds approximately 0.6, a single connected component consistently emerges. As soon as the network is connected, the pattern is driven by both θ and α and mirrors the density pattern, falling as we move toward the upper-left corner. Again, average distance is strongly correlated to the density of the graph, and thus in the top-right panel we present the average distance normalized by the average distance of an equivalent random graph.[27] Distances are always larger than they are in a random network, and become more so as both θ and $(1-\alpha)$ increase. This implies that the structure that differentiates our networks from random networks remains even as density increases.

The lower-left panel of Figure 4 shows average distance among reachable pairs.[28] Near the autarky fron-

[27] The random benchmark is equal to $\ln n$ over the logarithm of the average degree.

[28] One way to avoid the problem of infinite distances is to consider in the average only distances between pairs of agents who are separated by finite steps, i.e., who belong to the same component.

tier, distance are short because connected components are very small (pairs initially, and then small dense groups). As θ increases, from the point of view of a single firm, its connected component grows because firms are being added. Average distances increase. However, as θ continues to grow, degree increases, and this drives down average distances. There the pattern is driven entirely by the density of the network and replicates the top-left panel of Figure 2.

In the lower-right panel of Figure 4, we show the correlation between the closeness of a firm and its performance.[29] Comparing this correlation with that for degree (bottom-left panel of Figure 2), we see that first-order effects dominate: Closeness centrality is driven essentially by degree centrality.

These findings suggest the existence of several different regions in the (α, θ) space. Below the autarky frontier firms innovate in isolation. As we pass through the frontier, moving north in parameter space, isolated pairs appear. Continuing to move above the frontier, isolated caves of densely connected firms emerge, and the network has longer paths within its components, but is highly cliquish. Directly above that region, the caves connect, and a small-world structure may be present: clustered, yet with relatively short paths. Finally, we see a relatively densely connected, quasi-random network of the entire industry. This transition takes place much more quickly when relational embeddedness dominates, and the different regions are more visible when structural embeddedness dominates.

6.4. The Innovation System

Finally, we move to the aggregate level and consider two features of a knowledge system, the distribution of knowledge and the degree of specialization.[30]

The relative importance of relationally versus structurally acquired information (α) has little effect on the dispersion of knowledge, but there is a clear negative relationship between decomposability (θ) and equity (Figure 5, top-left panel). In the autarky region, inequality is driven by initial conditions: Firms with larger endowments in the initial assignment make larger innovations and grow faster, thereby magnifying initial differences. By contrast, as firms have more and more partners, which happens as θ increases, the distribution of innovative potential flattens out. Joint innovation implies that partners move toward each other in knowledge space, both in terms of where their expertise lies and (in relative terms) how much knowledge they possess. This effect is stronger when θ is larger, and this drives the pattern described.

Over time, innovation changes knowledge profiles and the extent of a firm's specialization. A firm knowing significantly more in one category than in the others can be called a *specialist*; a firm with roughly equal amounts of knowledge of each type is a *generalist*, and we can measure this using the coefficient of variation in the firm's different knowledge types.[31]

The top-right panel in Figure 5 shows individual specialization averaged over the population. The effect of θ is clear: Specialization falls as innovation becomes more separable.[32] When firms innovate as individuals, they become highly specialized because the type of knowledge produced is probabilistically the same as the knowledge input. In expected value, this will lead a firm to always innovate in the same knowledge type, and so drive extreme specialization. When alliances form in a more systematic way (larger θ), a firm will sometimes innovate in its speciality, sometimes in its partner's. This will smooth the firm's profile. This sort of variety in where a firm innovates produces much flatter profiles, and more so the more partners a firm has.

Are specialists better off? The bottom-left panel of Figure 5 shows the coefficient of correlation between specialization and innovative potential. We first observe that the correlation is always negative: More specialized firms perform worse in terms of knowledge accumulation than more generalized firms. Second, however, the negative correlation is less strong as θ increases. This is likely to be a mechanical effect: As (more) firms become (more) specialized, the penalty for specialization falls. What we notice, though, is that the effect is not monotonic: Around $\theta = 0.5$ the penalty increases again. This is the region in which networks start to connect, changing from isolated caves to a system of larger but less dense connected components. The likely explanation here is that being part of a large connected component implies being part of larger knowledge flows, and generally more partnerships, which creates generalist firms. Thus,

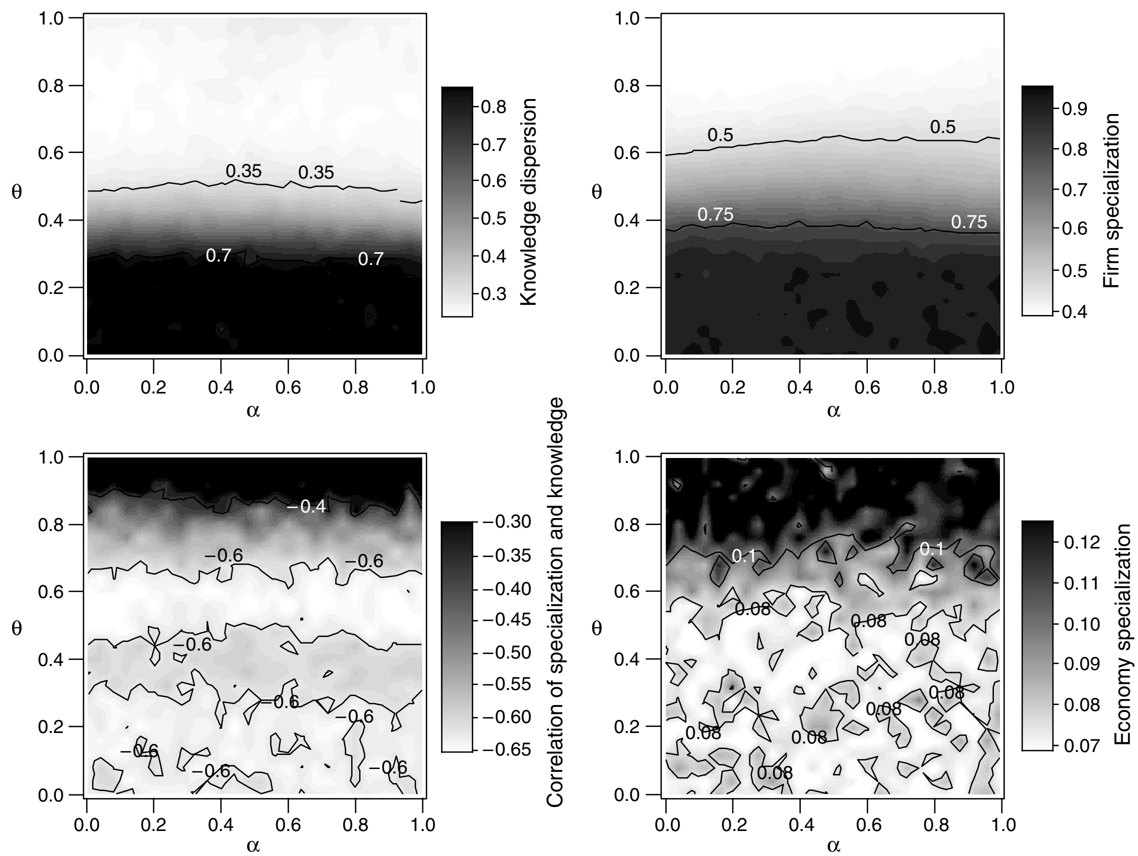
[29] The closeness of firm i is the reciprocal average distance to it, that is, the reciprocal of $\sum_{j \neq i} d_{ij}/(n-1)$.

[30] Knowledge levels in themselves convey little meaning because there is no way of normalizing the output to compare different parts of the parameter space. The pattern of knowledge levels is dominated by the effect of θ: When decomposability is stronger, inputs to innovation are in general larger, innovations are larger, and knowledge levels grow faster.

[31] With five elements, the coefficient of variation lies between zero and two, between a perfectly generalist firm and a fully specialized one.

[32] With a separable innovation task, specialization could be efficient. Extreme specialization would not be efficient in the context of the model, however, because there are five types of input to the production function, and partnerships are restricted to two members. Generalizing to permit larger coalitions would perhaps make specialization efficient in this region. It would be difficult to sustain, however, because of the nature of knowledge accumulation—all members of a partnership learn the same thing, which immediately dilutes the specialization for most of them.

Figure 5 Knowledge Distribution, Specialization at the Individual and Economy Level, and Correlation with Innovative Performance



this increase in the penalty for specializing is driven by the fact that the specialized firms are those trapped in relatively small components of the graphs, and are unable to connect to knowledge created in distant parts of the network.

Does the economy itself become specialized? Relative to individual specialization, the aggregation transform (however done) will tend to reduce variation, so we should expect that coefficients of variation at the economy level will be smaller than those at the individual level. This we observe in our data. How the degree of economy specialization responds to our parameters depends very much, however, on one's approach to economy-level innovation. If the economy's innovation system is seen merely as the sum of firm-level innovation activity, then the smoothing action of aggregation removes all structure. There is no pattern to economy specialization in our parameter space.[33] On the other hand, the innovation system could be treated more systemically, so that economy performance is determined not by the performance of firms as individuals (or pairs), but rather more fundamentally, by the economy's knowledge stocks. One way to address that is to aggregate firm knowledge vectors into an economy knowledge vector (by simple elementwise summation), and calculate the coefficient of variation on that vector. This is shown in the bottom-right panel of Figure 4. What we see is that aggregating on knowledge (as opposed to aggregating on innovation) generates a pattern in (α, θ)-space. Further, the pattern is the inverse of that at the firm level: The economy becomes less specialized as innovation becomes more decomposable. "Is this the optimal or most appropriate way to aggregate knowledge?" is an open question, and to answer it would demand a long excursus into innovation systems per se. The point remains, however, that whether or not innovation processes of the type we model create a more or less specialized economy depends heavily on how that specialization is measured: either in terms of innovation or in terms of knowledge.

We should note, however, that the possibility of firm and economy specialization moving in opposite directions has important policy implications. If there is an exogenous shift in the technological regime, the economy must respond. A specialized economy can be trapped in an inappropriate regime; a generalist

[33] This is the case whether we aggregate the actual innovations in the final period of our history, or whether we use firms' innovative potential to indicate how much and where they would innovate, and sum these to create the economy innovation vector.

economy can be more flexible, and therefore robust to outside shocks. However, even if the economy is relatively generalist, it could be that every individual firm is specialized. Firms can have difficulty responding, and so the economy, while appearing robust to shocks, is in fact brittle. One possible solution to this problem is the creation of cliques of diverse firms, and so creating flexibility at the intermediate level (this flexibility is one of the well-known advantages of network organizations). The important issue, however, is that even if an economy appears flexible at the macro level, policy makers must always be alert to the risk of a fallacy of (de-)composition.

7. Discussion and Conclusion

In the model developed here, firms are motivated solely by knowledge creation. In their decision making, they pay no attention to effects on, or consequences of, network position or structure. Nonetheless, networks form that are not random, but that have identifiable properties. One of the contributions of the paper is to provide a link between the knowledge regime and the network structure that emerges from bilateral cooperation in knowledge production. The decomposition of the innovation process into distinct subtasks implies that firms look for partners whose knowledge complements their own. However, repeated interactions generate similarity between partners' knowledge, reducing complementarity. Jointly, these effects imply that firms will have distinct partners over time. A second contribution concerns the importance of "secondhand" information about potential partners. When this is valuable, and firms can rely on their former partners for information about third parties, a firm has a larger number of credible potential partners because it can gather reliable information about more firms. This is a force favouring large numbers of partners. In addition, when information comes indirectly through former partners, triangles of firms tend to form, which increases local clustering. The dynamics that follow from these two aspects of the knowledge regime appear in our numerical results.

An interesting result regarding knowledge specialization is that patterns can differ at firm and economy levels. As task decomposability increases, firms specialize less, and become generalists over time. By contrast, aggregating knowledge to the economy level implies that specialization increases as innovation is more and more decomposed.

The effects of indirect information and task decomposability generate several parameter regimes in which different network structures are typical. When only direct information is valuable, we see isolated individuals or sparse random networks when decomposability is weak or strong, respectively. However, when structural embeddedness is important and indirect information becomes more valuable, increasing innovation decomposability changes the network structure from isolated individuals to small disconnected caves of densely connected firms, to a small world, and finally to an almost random (asymmetric) network.

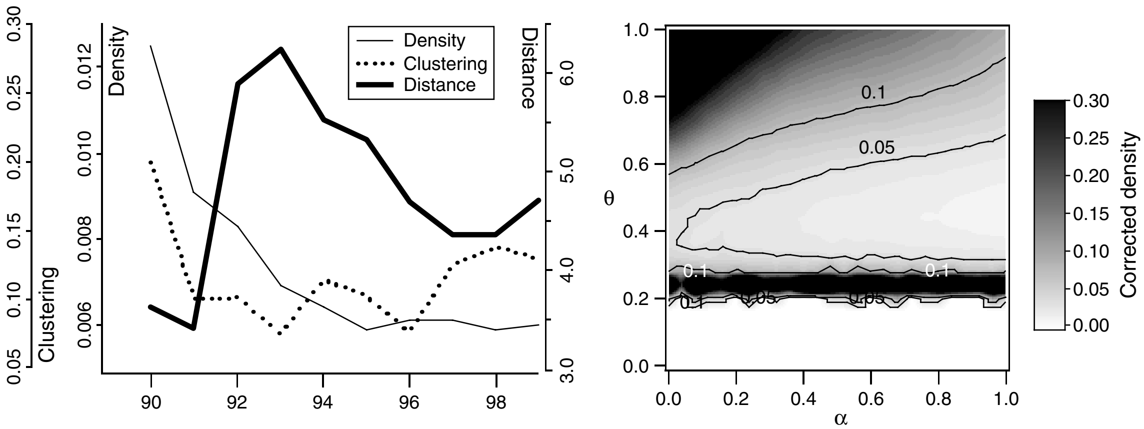
Correlations between network characteristics and absolute and relative performance provide some suggestions for both managers and policymakers. When innovation decomposability and the importance of structural embeddedness are such that stars form in the network, firms at the centre of the stars perform better than other firms (both in terms of absolute knowledge levels, and in terms of rank change in the industry). In this region, active pursuit by a firm of these central positions is an appropriate strategy because success would imply an improvement in competitive position relative to the rest of the industry. Throughout the parameter space, the correlation between local clustering and performance is positive, being especially strong when indirect information is important. Here it would be appropriate to devote resources to clique-building activities. Finally, policymakers are often interested in whether an economy is robust to external technological shocks. Two of our results are important. First, the level of aggregation matters. An economy that appears to be flexible in aggregate may be completely inflexible at the firm level because aggregate nonspecialization is generated by a population of firms, each a specialist, but in different areas. Active promotion of networking and joint knowledge production can alleviate this problem. In addition, however, if firm flexibility at the individual level is important, policies supporting knowledge codification may be valuable, as codification is likely to ease the separation of innovation into different tasks. In our model, it has the general effect of creating generalist firms over time.

As a heuristic test of the model, we link the results presented in the previous sections with data on firm R&D alliances in the biotech sector over the period 1989–1999. We make a loose mapping of the evolution of structure into changes in the innovation process and firms' embeddedness.[34]

Figure 6 displays the time evolution of three structural parameters in the biotech R&D alliance data: density, average distance among reachable pairs, and

[34] The SERD-Biotek database was provided by Andreas Pyka. It provides information on both (small and medium) biotech dedicated firms and established pharmaceutical companies. It includes data on patents, publications, and firm R&D collaborations for firms from the United States, France, England, and Germany.

Figure 6 Time Series of Density, Clustering, and Average Distance in the Biotech Industry and Corrected Density in the Experimental Data



clustering.[35] The left panel of Figure 6 shows the two-year moving average of the three time series.[36] The right panel of the figure displays the evolution of density with α and θ, once we correct for singletons.

Over time, in the biotech data (left panel) average distance among reachable pairs rapidly increases and then falls. Both corrected density and clustering exhibit monotonic behaviour, falling rapidly and then levelling out. Comparing these patterns with, respectively, the right panel of Figure 6, Figure 4, and Figure 3, suggests a move north, from a starting position near the autarky frontier, and a low α, followed by a move northeast, along a contour line of clustering and distance. This would produce the rise and subsequent decline in average distance, and the decline and levelling out of both corrected density and clustering. This admittedly bold interpretation suggests that over the life of these data, the biotech industry has seen innovation become more modularized, and partner selection become more relationally embedded. Both things are difficult to measure directly, and so provide an interesting empirical challenge.

We must acknowledge that the model presented here has a strong focus, and necessarily abstracts from various aspects of alliance creation. Thus, there are several directions in which the model could be extended. We have emphasized the absence of "network" motives on the part of firms. One obvious extension is to include them. A firm could consider the current state of the network and whether a particular alliance would create for it a clique-spanning, or a clique-building, tie, for example. Indeed, if different firms pursue different strategies, perhaps changing them over time, the model could be used to explore issues surrounding the value of different network strategies in different contexts, contributing to the debate over the value of structural holes and dense local information flows.

The tension we have explored, between information gathered in relational versus structurally embedded contexts, is a special case of information being distributed throughout the network, and firms being circumscribed in their abilities to look for it. A relatively simple generalization would be to permit firms to acquire information over longer and longer paths. Intuitively, it seems that regarding clustering the distance of two is critical because it is information at that distance that causes triangles to close. However, this is an issue that could be explored with little change to the model. Additionally, this could be endogenized, if search were costly, with costs increasing in distance. At each stage firms could decide how far afield to look. This enriches the behaviour set of the firms, but would demand much more detailed microeconomics.

The other parameter, regarding the decomposability of innovation, could also be endogenized. Trust regarding both motives and competence is necessary for one partner to permit the other to do part of the job in isolation. As trust between partners increases, they will be more willing to allow the tasks to be divided. The literature on trust argues that repeated interaction builds trust. What this suggests is that an endogenization along these lines would drive more inertia into network structures.

Discussions of networks that arise from alliance formation and empirical investigations of real networks emphasize the importance of embedded interactions. The formal model presented here shows a process by which the embeddedness of interactions translates into different network structures. Small worlds are commonly found in alliance networks, but they are

[35] Because the empirical data are about alliances, there are no singletons present, unlike in our networks for small θ. To make the simulation and biotech data comparable, we recalculate density excluding singletons. For clustering and distance among reachable pairs we use the simulation data as presented above in Figures 3 and 4, because for those measures singletons cause no problem.

[36] The averaging is done on the connection matrices rather than on the data because we have information only on starting, and not ending, dates of alliances.

not the only structure present in empirical results. The model we have developed can generate different network structures, depending on parameters, and in particular shows when small worlds can be expected, depending not only on the nature of the embedded relations, but also on the nature of the innovation task itself. Our results underline the observation that embedded relations are central in explaining the network structures that we observe, but they also show the importance of understanding how firms actually combine their knowledge to create innovations.

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