# Arbitrarily Sampled Signal Reconstruction Using Relative Difference Features

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## Motivations

- Estimating the target quantity for each point **separately** is prone to **noise.**
- The estimated **differences between points** can be used as extra hints to find the solution.
- Similar problem paradigm has been used in the phase unwrapping (PU) literature, and their methods could be extended to handle other applications as well.

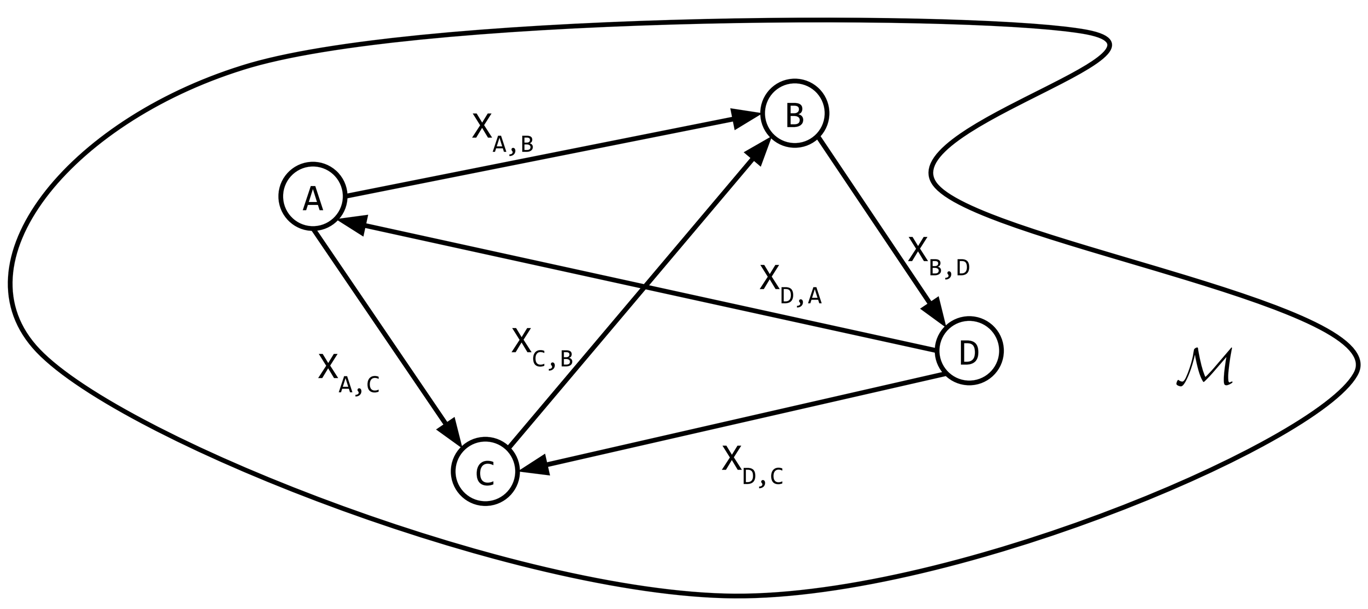
## Problem Definition

The task:

- Given N discrete points $\Theta = \{\theta_0, \dots, \theta_{N-1}\}$, estimate $f(\theta_i) : \theta_i \in \Theta$ of a continuous function $f(\theta)$ and $\theta \in \mathcal{M}$.

We have:

- A graph G consists of the nodes (a.k.a points) V(G) = {0,...,N-1} and M edges $E(G) = \{e_0, \dots, e_{M-1}\}$.
- We have close estimation of $X_{u,v} \cong f(\theta_v) - f(\theta_u)$ for $(u, v) \in E(G)$.




## Solution Using Linear Programming

We adapt the edgelist method [1] from the PU literature to our applications.

$$\min_{\mathbf{k},\mathbf{y}} \ \mathbf{w}^T \boxed{|\mathbf{k}|} \longleftarrow \text{Residuals}$$

$$s.t. \ [\mathbf{A} \quad \mathbf{I}] \begin{bmatrix} \mathbf{y} \\ \mathbf{k} \end{bmatrix} = \mathbf{x}$$

$$A_{ij} = \begin{cases} -1, & j = u \\ 1, & j = v \\ 0, & otherwise \end{cases} : (u, v) = e_i$$

$$\text{Targets} \longrightarrow \boxed{\mathbf{y}} = [f(\theta_0), \dots, f(\theta_{N-1})]$$

$$\mathbf{x} = [X_{e_0}, \dots, X_{e_{M-1}}]$$

## Applications

### Head-related transfer functions time-of-arrival estimation

Head-related transfer functions (HRTFs) encode the acoustic scattering patterns from the sound source to human ears in different directions. The time differences between ears are crucial spatial cues. Check out [2] for more evaluations on the proposed TOA estimation.

- $\mathcal{M} = S^2 \times$ {Left, Right}
- $X_{u,v}$ = time-shift of maximum correlation between the HRIRs at $\theta_u$ and $\theta_v$

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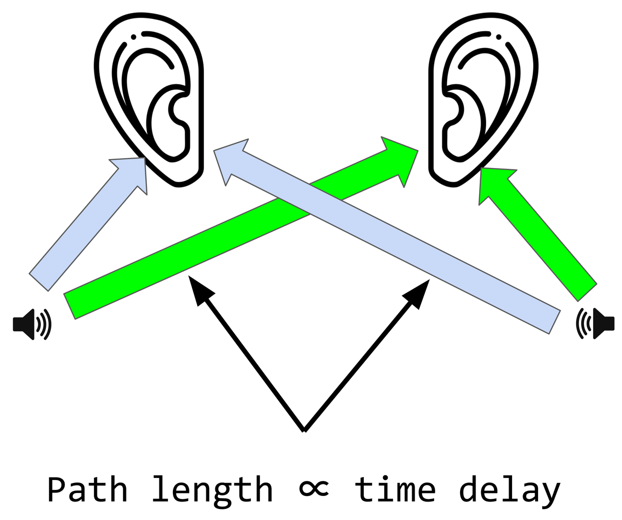
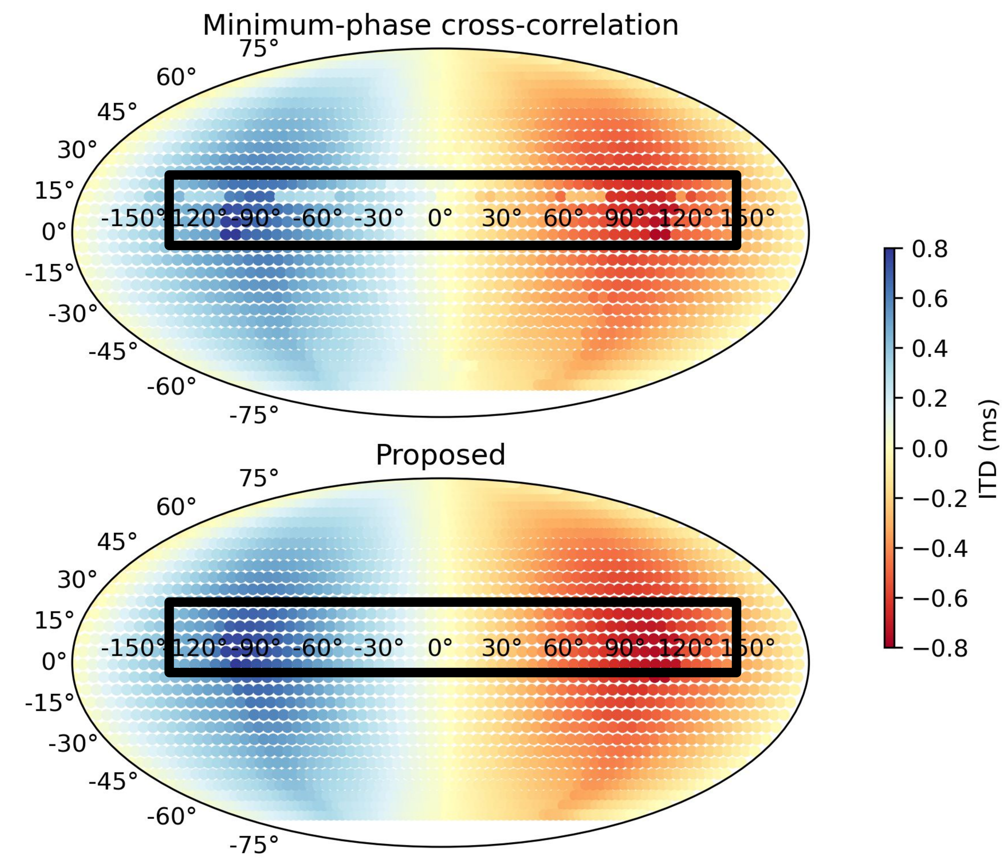
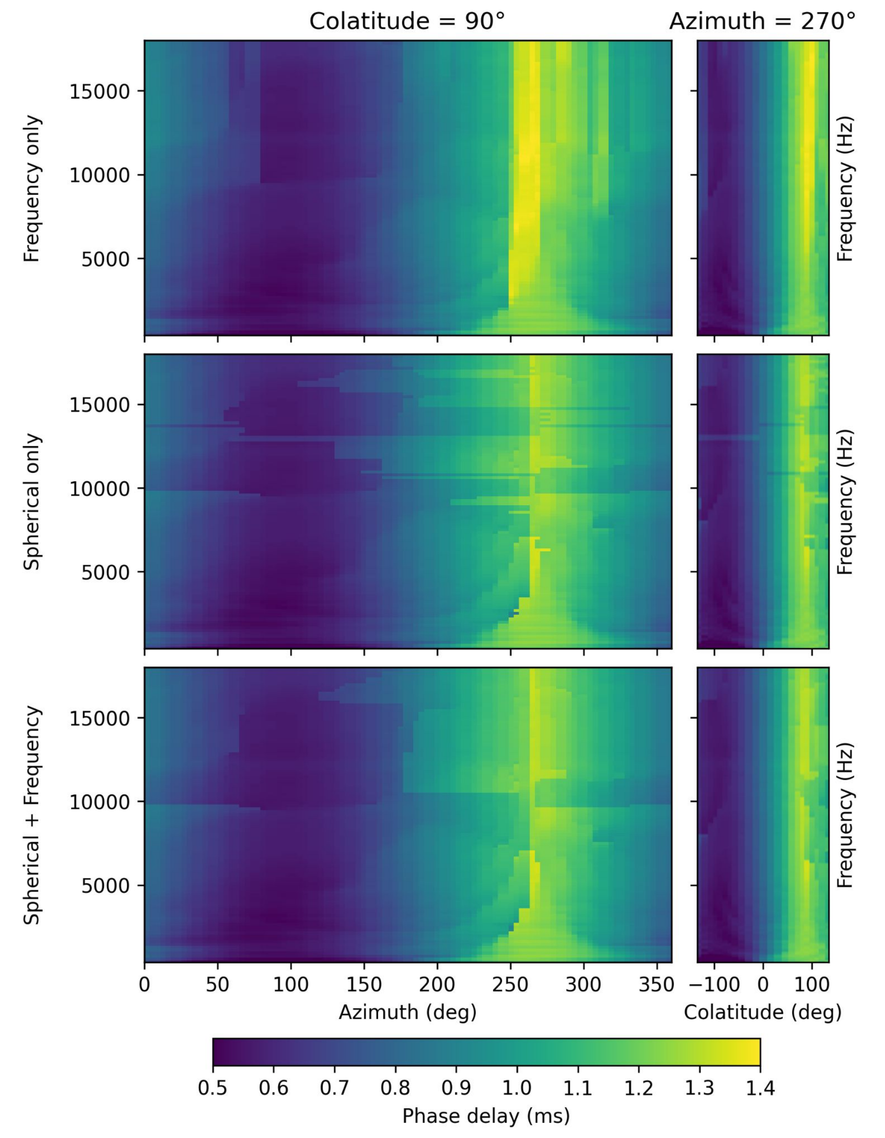


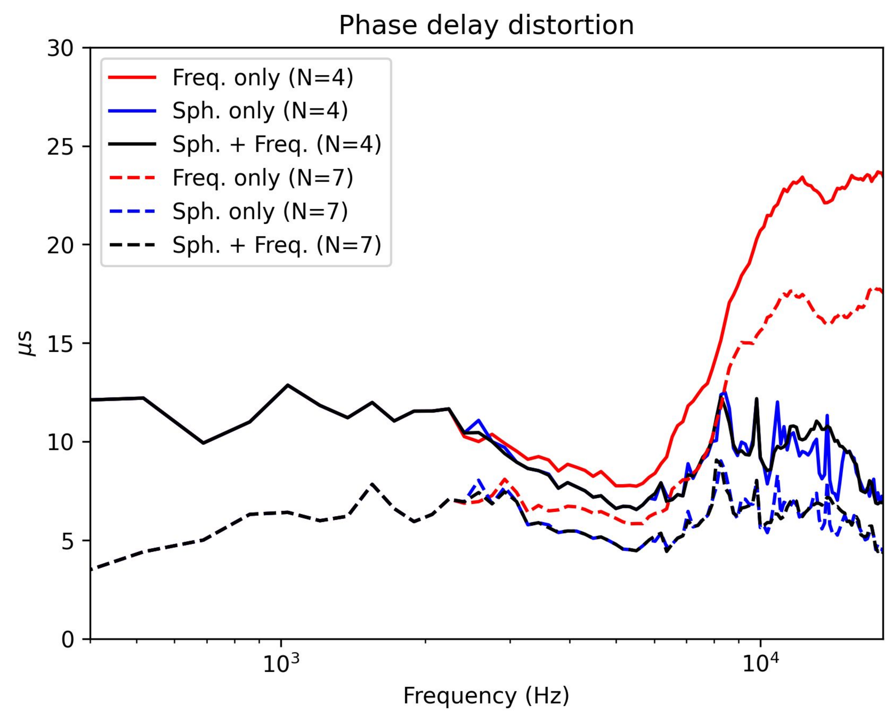




### Head-related transfer functions phase unwrapping

- $\mathcal{M} = S^2 \times [0, \pi]$
- $X_{u,v}$ = wrapped phase differences $\in [-\pi, \pi)$

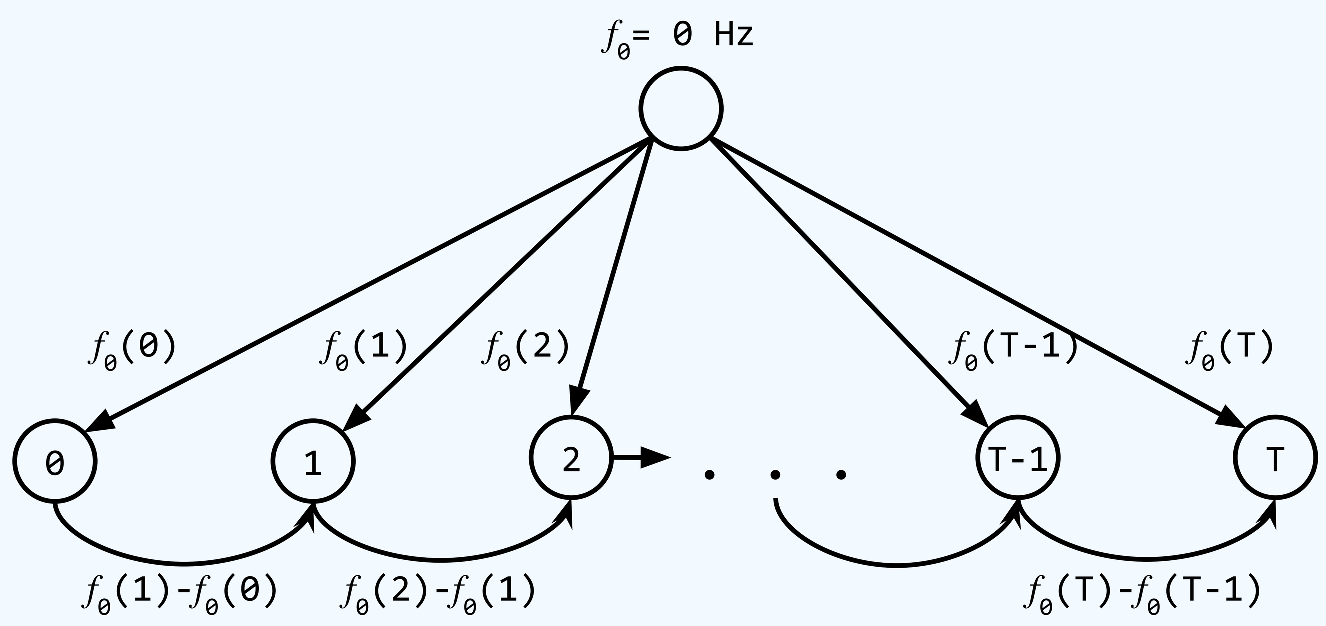
Check out [2] for more evaluations on the proposed phase unwrapping method.




### Can it be applied to fundamental frequency estimation?

- $\mathcal{M}$ = [0, T], $X_{u,v}$ = ?




## Conclusions and Future Works

- The ILP solution attain the smoothest results in HRTFs TOA estimation and PU, and is more noise robust than the previous state-of-the-art in TOA estimation.
- How to calculate pitch differences efficiently is crucial for adapting the proposed framework to fundamental frequency estimaion.
- Other optimisation methods from the PU literature could be explored as well.

## Reference


[1] A. P. Shanker and H. Zebker, "Edgelist phase unwrapping algorithm for time series InSAR analysis," JOSA A, vol. 27, no. 3, pp. 605-612, Mar. 2010.
[2] C.-Y. Yu, J. Pauwels, and G. Fazekas. "Time-of-arrival Estimation and Phase Unwrapping of Head-related Transfer Functions With Integer Linear Programming." arXiv preprint arXiv:2405.06804 (2024).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Code

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