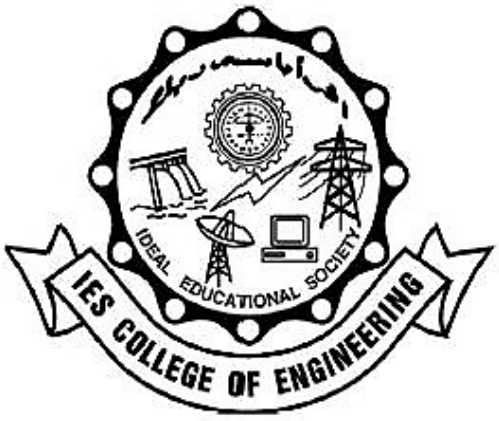
Vudoku - Visual Sudoku Solver

A final project report during the year 2020 - 2021,
submitted to APJ Abdul Kalam Technological University in partial fulfillment of the requirements for the award of the degree of

Bachelor of Technology
in
Computer Science and Engineering

by

Jovial Joe Jayarson (IES17CS016)



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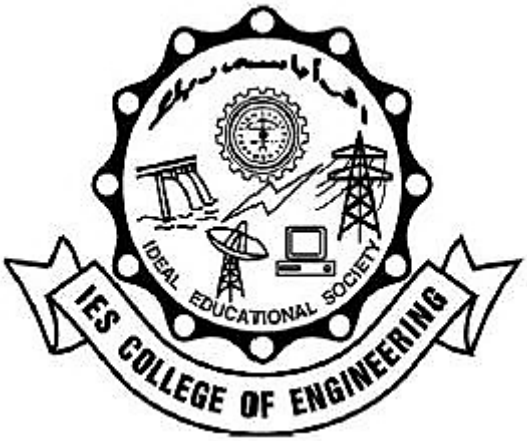

Certificate

This is to certify that the final project report entitled

Vudoku - Visual Sudoku Solver

submitted by

Jovial Joe Jayarson

in partial fulfilment of the requirements of the degree of *Bachelor of Technology in Computer Science and Engineering*, to APJ Abdul Kalam Technological University is a record of *bona fide* work done by him under my supervision and guidance and this work has not been submitted elsewhere for any degree or diploma.

		.....	.....
Place:	Thrissur	**Mr. Ebin P M**	**Dr. Kiruthiga G**
		Asst. Professor, CSE	Head of the Department
Date:	5th December 2020	(Project Guide)	CSE

Acknowledgement

I'm thrilled as I present final project report **Vudoku - Visual Sudoku Solver** in partial fulfillment of B.Tech in Computer Science and Engineering. Allow me to take this opportunity to first thank my **God Almighty** for providing His grace and guidance in this dispensation.

Sincere gratitude to **Dr. Brilly S Sangeetha**, Principal, for providing all the required facilities to make up to this stage of the project with a great success.

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Last but not the least, I convey my warm regards to my **family members**, **friends** and all well wishers, who had been there during the needed times.

May God bless us all.

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Abstract

It is no secret that AI is an upcoming titan. Even though people are stunned to hear that AI has been here for around a century, due to the advancement in computational methods and resources, today AI peaks like never before. As a tiny glimpse into the field of Digit Recognition, this project aims to understand the underlying cogs and wheels on which the neural networks spin. Ranging from core mathematical functions to flowery programming, this project will include all that is essential to recognize digits from an image. Even though the project is primarily research oriented, on the application part of it, this can be used to identify licence plate, read bank cheques, grade mathematics homework and as such. Currently the project tries to solve Sudoku drawn and written by hand. The paraphernalia for this project includes programming language: *Python3/Nim*; libraries: *OpenCV, Numpy, Keras/Tensorflow, Matplotlib, Streamlit*; datasets: *MNIST handwritten digit database*. Digit recognition is a classical problem which will introduce neurons, neural networks, connections hidden layers, weights, biases, activation functions like sigmoid, back-propagation and other related topics as well. Algorithm(s) used to solve Sudoku are also explored in this project.

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Chapter 1

Introduction

Digit Recognition is a classical machine learning problem. The objective is implicitly evident, it is to identify and classify (usually) handwritten digits. Machines are inherently incapable to understand, text, images and audio. These formats which carry information, needs to be converted into numbers, matrices or vectors which is a significant step, in any procedure, to solve problems using computers. Another vital stage for digit recognition is *machine learning*. With classical problem solving techniques, humans provide computers with both data and rules as input. In contrast, we provides data and presumed result, as input to a machine learning system. And then we let it find out some *rules*. A quick example would be to give three numbers x, y and z and tell a machine that - certain operation was performed on x and y to obtain z. Once a machine is fed with lots and lots of similar examples, it will be equipped find out some pattern behind it. At a later stage, it can very well predict, what operations might have been performed on x and y to result in z.

This report delves into a century long history of digit recognition, all the way from perceptron networks to recently released transformers. The means and ways, in which this problem was approached converges to SOTA (state of the art) models like in [1]. This report explores *The Transformer*, released in 2017. *Attention* mechanism in the Transformer [2] has sparked a lot of interest in the research community, not to mention the buzz associated with GPT-3, due to its mind-boggling capabilities flaunted all across the media. The very next year a preprint on *Image Transformer* [3] was released. Interestingly Google research just released another preprint which dubs the whole idea of manipulating images with this new technique as *Vision Transformer* [4]. Now at the core of this project, the digit recognition makes use of the Vision Transformer.

The project is named *Vudoku*, so the second part of the project involves solving classical computational problem. Sudoku is a mathematical problem and by default has 9×9 grids. There are other variant to it but to keep things simple, the project moves along with the default one. Sudoku is classified as Exact Cover or Hitting Set problem [5]. Various algorithms has been developed over the years to solve it, like Backtracking, Stochastic algorithms, Constraint programming and so on. Given that the output of the digit recognition system is as *value*, *position* pairs, this project will consider some of these algorithms to solve Sudoku and possibly visualize the steps.

Chapter 2

Overview and Prerequisites

2.1 Overview of the Topic

Complex problems exists all around us. Some of them are stumbled upon and other have be thoughtfully crafted. Solutions to these problems at times are intuitive and yet difficult to explain. Structured mathematical approach to problems has engineered some of the finest solutions to some of the toughest problem. In the era of computing, appetite to solve problems using machines has become a hobby. With technology at fingertips, it's no-brainer that upcoming generations have a choice to be way smarter.

Speaking of generations, there are puzzles and trivia question that people like to solve. These are not always driven by any particular motive. Sudoku is a famous mathematical puzzle. The rules to solve this can be summarized as: *All grids must be filled with numbers from 1 to 9 without repetition in either row, column or box.* Today any computer can solve this within seconds - if given with proper parameters. But computers by default do not possess visual perception. This is where image recognition steps in. This project can be thought of as an amalgamation of machine learning and classical computing. Instead of feeding a system with direct Sudoku *clues*, an image of Sudoku is provided. The computer has to derive the clues out of an image and then pass them along with correct positions to the second modules which then tries to solve Sudoku.

2.2 Prerequisites for the Project

Since this project strives to reach minimum research standard, reader is advised to have knowledge about related topics. This section tries to cover some of them but the rest is left at reader's discretion to follow up.

2.2.1 Mathematics to solve Sudoku

NP-Complete

In computational complexity theory, a problem is NP-complete when:

- A nondeterministic Turing machine can solve it in polynomial-time.
- A deterministic Turing machine can solve it in large time complexity classes and can verify its solutions in polynomial time.
- It can be used to simulate any other problem with similar solvability [6].

Combinatorics

Combinatorics is an area of mathematics primarily concerned with counting, both as a means and an end in obtaining results [7].

Group Theory

In mathematics and abstract algebra, group theory studies the algebraic structures known as groups. In mathematics, a group is a set equipped with a binary operation that combines any two elements to form a third element in such a way that four conditions called group axioms are satisfied, namely 'closure', 'associativity', 'identity' and 'invertibility' [8] [9].

2.2.2 Machine Learning and related Terminology

Gradient Descent

Gradient descent is a first-order iterative optimization algorithm for finding a local minimum of a differentiable function. The idea is to take repeated steps in the opposite direction of the gradient (or approximate gradient) of the function at the current point, because this is the direction of steepest descent. Conversely, stepping in the direction of the gradient will lead to a local maximum of that function; the procedure is then known as gradient ascent [10].

Fourier Transform

In mathematics, a Fourier transform (FT) is a mathematical transform that decomposes a function (often a function of time, or a signal) into its constituent frequencies, such as the expression of a musical chord in terms of the volumes and frequencies of its constituent notes. The term Fourier transform refers to both the frequency domain representation and the mathematical operation that associates the frequency domain representation to a function of time [11] This is used in image preprocessing step in CNN (5.2.1).

Activation Function

In artificial neural networks, the activation function of a node defines the output of that node given an input or set of inputs. A standard integrated circuit can be seen as a digital network of activation functions that can be 'ON' or 'OFF', depending on input [12].

Chapter 3

Literature Review

3.1 Multi-digit Number Recognition from Street View Imagery using Deep Convolutional Neural Networks

Abstract (as is): Recognizing arbitrary multi-character text in unconstrained natural photographs is a hard problem. In this paper, we address an equally hard sub-problem in this domain viz. recognizing arbitrary multi-digit numbers from Street View imagery. Traditional approaches to solve this problem typically separate out the localization, segmentation, and recognition steps. In this paper we propose a unified approach that integrates these three steps via the use of a deep convolutional neural network that operates directly on the image pixels. We employ the DistBelief [13] implementation of deep neural networks in order to train large, distributed neural networks on high quality images. We find that the performance of this approach increases with the depth of the convolutional network, with the best performance occurring in the deepest architecture we trained, with eleven hidden layers. We evaluate this approach on the publicly available SVHN dataset and achieve over 96% accuracy in recognizing complete street numbers. We show that on a per-digit recognition task, we improve upon the state-of-the-art, achieving 97.84% accuracy. We also evaluate this approach on an even more challenging dataset generated from Street View imagery containing several tens of millions of street number annotations and achieve over 90% accuracy. To further explore the applicability of the proposed system to broader text recognition tasks, we apply it to transcribing synthetic distorted text from a popular CAPTCHA service, reCAPTCHA. reCAPTCHA is one of the most secure reverse turing tests that uses distorted text as one of the cues to distinguish humans from bots. With the proposed approach we report a 99.8% accuracy on transcribing the hardest category of reCAPTCHA puzzles. Our evaluations on both tasks, the street number recognition as well as reCAPTCHA puzzle transcription, indicate that at specific operating thresholds, the performance of the proposed system is comparable to, and in some cases exceeds, that of human operators.

3.2 Handwritten Digit Recognition Using Machine Learning : A Review

Abstract (as is): The task for handwritten digit recognition has been troublesome due to various variations in writing styles. Therefore, we have tried to create a base for future researches in the area so that the researchers can overcome the existing problems. The existing methods and techniques for handwritten digit recognition were reviewed and understood to analyze the most suitable and best method for digit recognition. A number of 60,000 images were used as training sets of images with pixel size of 28×28. The images/training sets were matched with original image. It was found out after complete analysis and review that classifier ensemble system has the least error rate of just 0.32%. In this paper, review of different methods handwritten digit recognition were observed and analyzed.

3.3 An ensemble of simple convolutional neural network models for MNIST digit recognition

Abstract (as is): We report that a very high accuracy on the MNIST test set can be achieved by using simple convolutional neural network (CNN) models. We use three different models with 3×3, 5×5, and 7×7 kernel size in the convolution layers. Each model consists of a set of convolution layers followed by a single fully connected layer. Every convolution layer uses batch normalization and ReLU activation, and pooling is not used. Rotation and translation is used to augment training data, which is frequently used in most image classification tasks. A majority voting using the three models independently trained on the training data set can achieve up to 99.87% accuracy on the test set, which is one of the state-of-the-art results. A two-layer ensemble, a heterogeneous ensemble of three homogeneous ensemble networks, can achieve up to 99.91% test accuracy. The results can be reproduced by using the code at `https://github.com/ansh941/MnistSimpleCNN`.

3.4 An Image is Worth 16x16 Words : Transformers For Image Recognition at Scale

Abstract (as is): While the Transformer architecture has become the de-facto standard for natural language processing tasks, its applications to computer vision remain limited. In vision, attention is either applied in conjunction with convolutional networks, or used to replace certain components of convolutional networks while keeping their overall structure in place. We show that this reliance on CNNs is not necessary and a pure transformer applied directly to sequences of image patches can perform very well on image classification tasks. When pre-trained on large amounts of data and transferred to multiple mid-sized or small image recognition benchmarks (ImageNet, CIFAR-100, VTAB, etc.), Vision Transformer (ViT) attains

excellent results compared to state-of-the-art convolutional networks while requiring substantially fewer computational resources to train.

3.5 MDig: Multi-digit Recognition using Convolutional Nerual Network on Mobile

Abstract (as is): Multi-character recognition in arbitrary photographs on mobile platform is difficult, in terms of both accuracy and real-time performance. In this paper, we focus on the domain of hand-written multi-digit recognition. Convolutional neural network (CNN) is the state-of-the-art solution for object recognition, and presents a workload that is both compute and data intensive. To reduce the workload, we train a shallow CNN offline, achieving 99.07% top-1 accuracy. And we utilize pre-processing and segmentation to reduce input image size fed into CNN. For CNN implementation on the mobile platform, we adopt and modify DeepBeliefSDK to support batching fully-connected layers. On NVIDIA SHIELD tablet, the application processes a frame and extracts 32 digits in approximately 60ms, and batching the fully-connected layers reduces the CNN runtime by another 12%.

Chapter 4

Evolution of Neural Networks

It's of no surprise that the field of artificial intelligence has come a long way. This section deals with some of the famous models and how they are related to digit recognition. Starting from the ones released in 80's like *Recurrent Neural Network (RNN)* to the latest 2017 model *The Transformer*, much study has been conducted.

4.1 Recurrent Neural Network

Year: 1986

Recurrent neural networks are derived from feed-forward networks. The connections in a feed-forward network do not form a cycle. RNN is a class of neural network where the connections between nodes form a directed graph. This is always along a temporal sequence, so it exhibits a temporal dynamic behaviour. Dynamic systems are those that have a function which describes time dependence of a point in geometrical space. RNNs can use their internal state (or memory) to process variable length sequences of inputs [14]. Section (5.1) discusses more about vanilla networks.

Basic RNNs are a network of neuron-like nodes organized into successive layers. Each node in a given layer is connected with a directed (one-way) connection to every other node in the next successive layer. Each node (neuron) has a time-varying real-valued activation. Each connection (synapse) has a modifiable real-valued weight. Nodes are either input nodes (receiving data from outside of the network), output nodes (yielding results), or hidden nodes (that modify the data en route from input to output). Each sequence produces an error as the sum of the deviations of all target signals from the corresponding activations computed by the network. For a training set of numerous sequences, the total error is the sum of the errors of all individual sequences. The following equation (4.1) was proposed by **Michael Irwin Jordan** as part of his research on *Simple Recurrent Neural Network* [14]:

$$\begin{aligned} h_t &= \sigma_h(W_h x_t + U_h y_{t-1} + b_h) \\ y_t &= \sigma_y(W_y h_t + b_y) \end{aligned} \tag{4.1}$$

Where, x_t is input vector, h_t is hidden layer vector, y_t is output vector, W, U and b are parameter matrices and vector, respectively σ_h and σ_y are activation

functions The following figure (4.1) gives the internal schematics of Recurrent Neural Networks.

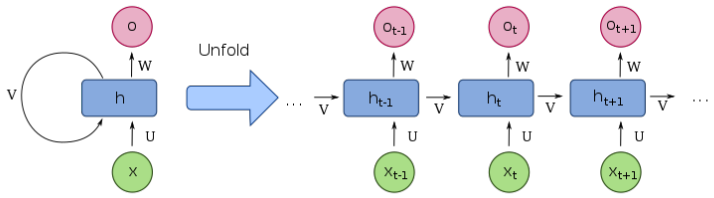

Figure 4.1: Recurrent Neural Networks Unfolded [14]

Recurrent neural networks models contain a self-connected hidden layer. One benefit of the recurrent connection is that a 'memory' of previous inputs remains in the network's internal state, allowing it to make use of past context. This is significant for handwriting recognition as mentioned in [15]. A major problem with gradient descent for standard RNN architectures is that error gradients vanish exponentially quickly with the size of the time lag between important events [14].

4.2 Long Short Term Memory

Year: 1997

Long short-term memory (LSTM) is a deep learning system that avoids the vanishing gradient problem. LSTM is normally augmented by recurrent gates called "forget gates". LSTM prevents backpropagated errors from vanishing or exploding. Instead, errors can flow backwards through unlimited numbers of virtual layers unfolded in space. That is, LSTM can learn tasks that require memories of events that happened thousands or even millions of discrete time steps earlier [14].

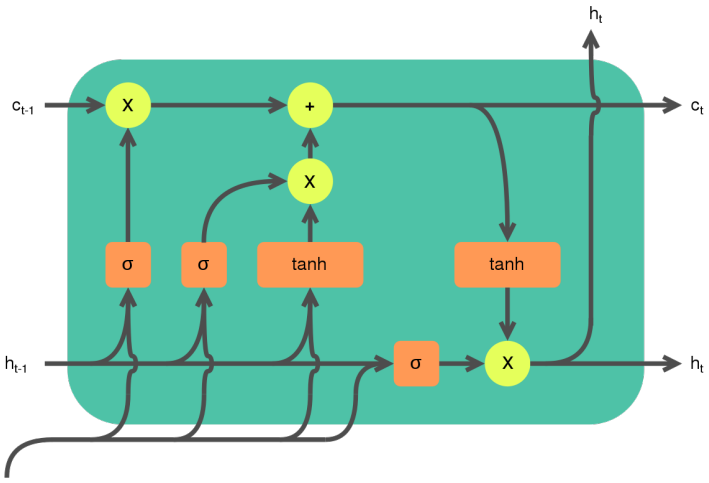

Figure 4.2: Long Short Term Memory Cell [16]

A common LSTM unit, as shown in figure (4.2), is composed of a cell, an *input gate*, an *output gate* and a *forget gate*. The cell remembers values over arbitrary time intervals and the three gates regulate the flow of information into and out of the cell. Some variations of the LSTM unit do not have one or more of these gates or maybe have other gates. For example, *Gated Recurrent Units* (GRUs) do not have an output gate. LSTM networks are hence well-suited for classifying time series data.

The compact forms of the equations for the forward pass of an LSTM unit with a forget gate are:

$$\begin{aligned} f_t &= \sigma_g(W_f x_t + U_f h_{t-1} + b_f) \\ i_t &= \sigma_g(W_i x_t + U_i h_{t-1} + b_i) \\ o_t &= \sigma_g(W_o x_t + U_o h_{t-1} + b_o) \\ \tilde{c}_t &= \sigma_c(W_c x_t + U_c h_{t-1} + b_c) \\ c_t &= f_t \circ c_{t-1} + i_t \circ \tilde{c}_t \\ h_t &= \sigma_t \circ \sigma_h(c_t) \end{aligned} \tag{4.2}$$

The initial values are $c_0 = 0$ and $h_0 = 0$ and the operator $\circ$ denotes the Hadamard element-wise product. Activations are σ_g - a sigmoid function, σ_c and σ_h - a hyperbolic tangent functions. The variable are [16]:

x_t : input vector to the LSTM unit
f_t : forget gate's activation vector
i_t : input or update gate's activation vector
o_t : output gate's activation vector
h_t : hidden state vector
$\tilde{c}_t$: cell input activation vector
c_t : cell state vector

4.3 Convolution Neural Networks

Year: 1998

A convolutional neural network (CNN), is a class of deep neural networks, most commonly applied to analyzing visual imagery. They are also known as shift invariant or *space invariant artificial neural networks* (SIANN). Figure (4.3) shows a common CNN architecture. CNNs have a wide variety of applications including, image and video recognition, recommender system, brain-computer interface etc. CNNs are regularized versions of multilayer perceptrons. Multilayer perceptrons usually mean fully connected networks, that is, each neuron in one layer is connected to all neurons in the next layer.

Convolutional networks were inspired by biological processes in that the connectivity pattern between neurons resembles the organization of the animal visual cortex. CNNs use relatively less pre-processing compared to other image classification algorithms. Independence from prior knowledge and human effort in feature design is a major advantage [17]. More about CNN in section (5.2).

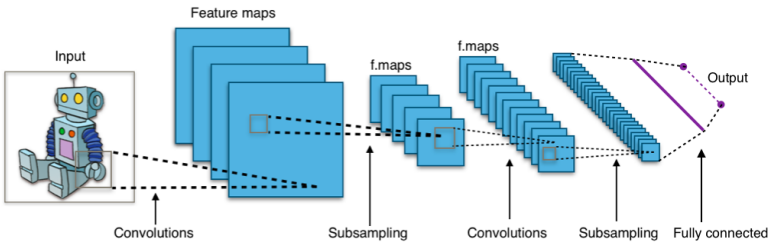

Figure 4.3: Typical CNN Architecture [17]

4.4 The Transformer Network

Year: 2017

The Transformer is a deep learning model primarily used in the field of natural language processing (NLP). Transformers are designed to handle sequential data, such as natural language unlike RNNs, Transformers do not require that the sequential data be processed in order. Thus Transformer allows for much more parallelization than RNNs and therefore reduced training hours. This has led to the development of pre-trained systems such as BERT (Bidirectional Encoder Representations from Transformers) and GPT (Generative Pre-trained Transformer), which have been trained with huge general language datasets, and can be fine-tuned to specific language tasks.

Transformer is an encoder-decoder architecture. The *encoder* consists of a set of encoding layers that processes the input iteratively one layer after another and the *decoder* consists of a set of decoding layers that does the same thing to the output of the encoder. Each encoder and decoder layer makes use of an *attention mechanism*, which for each input, weighs the relevance of every other input and draws information from them accordingly to produce the output. Each decoder layer also has an additional attention mechanism which draws information from the outputs of previous decoders, before the decoder layer draws information from the encodings. Both the encoder and decoder layers have a feed-forward neural network for additional processing of the outputs, and contain residual connections and layer normalization steps [18].

The basic building blocks of the Transformer are scaled dot-product attention units. When a sentence is passed into a Transformer model, attention weights are calculated between every token simultaneously. The attention unit produces embeddings for every token in context that contain information not only about the token itself, but also a weighted combination of other relevant tokens weighted by the attention weights. More about Transformer and Attention in section (5.3).

Chapter 5

Concurrent Systems

There are several existing systems or models which can perform digit recognition. Some of these even have state of the art accuracy claims. This section delves into two significant artificial neural network models, namely *The Perceptron* and *Convolutional Neural Network*. Even though these networks can perform a variety of tasks, the focus will be upon digit classification.

5.1 Vanilla ANN - The perceptron

Vanilla Artificial Neural Network (ANN) is a primitive model. They are computing systems vaguely inspired by the biological neural networks. An ANN is based on a collection of connected units or nodes called artificial neurons, which loosely model the neurons in a biological brain. Each connection, like the synapses in a biological brain, can transmit a signal to other neurons.

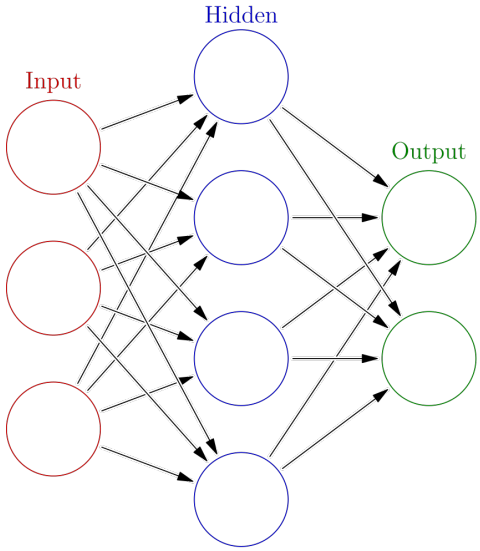

Figure 5.1: Simple Artificial Neural Network [19]

An artificial neuron that receives a signal then processes it and can signal neurons connected to it. The 'signal' at a connection is a real number, and the output of

each neuron is computed by some non-linear function of the sum of its inputs. The connections are called edges. Neurons and edges typically have a weight that adjusts as learning proceeds. The weight increases or decreases the strength of the signal at a connection. Neurons may have a threshold such that a signal is sent only if the aggregate signal crosses that threshold.

Typically, neurons are aggregated into layers. Different layers may perform different transformations on their inputs. Signals travel from the first layer (the input layer), to the last layer (the output layer), possibly after traversing the layers multiple times [19].

5.1.1 Neurons

ANNs are composed of artificial neurons, each artificial neuron has inputs and produce a single output which can be sent to multiple other neurons. To find the output of the neuron, first we take the weighted sum of all the inputs, weighted by the weights of the connections from the inputs to the neuron. We add a bias term to this sum. This weighted sum is then passed through a (usually nonlinear) activation function to produce the output [19].

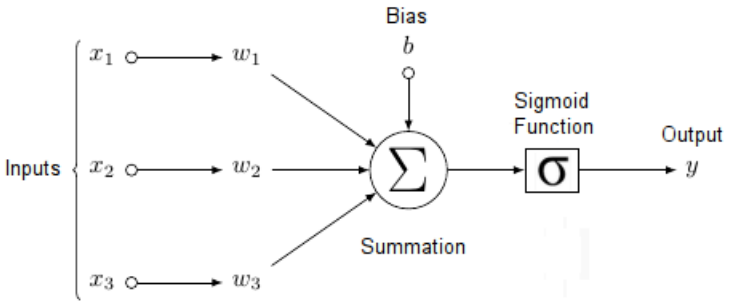

Figure 5.2: Artificial Neuron

5.1.2 Weights and Biases

The network consists of connections, each connection providing the output of one neuron as an input to another neuron. Each connection is assigned a *weight* w_i, that represents its relative importance. A given neuron can have multiple input and output connections. The *activation* function σ, computes the input to a neuron from the outputs of its predecessor neurons and their connections as a weighted sum as shown in figure (5.2) A *bias* b, term can be added to the result of the forward propagation [19].

$$a_i^{(j)} = \sigma(w_{i0}a_0^{(j)} + w_{i1}a_1^{(j)} + \ldots + w_{in}a_n^{(j)} + b_0) \tag{5.1}$$

The equation (5.1) is an example of an output from a single layer, it can be generalized and expressed neatly as in (5.2).

$$a^{(L)} = \sigma(W^{(L)}a^{(L-1)} + b^{(L)}) \tag{5.2}$$

5.1.3 Hyperparameters

A hyperparameter is a constant parameter whose value is set before the learning process begins. The values of parameters are derived via learning. Examples of hyperparameters include learning rate, the number of hidden layers and batch size. The values of some hyperparameters can be dependent on those of other hyperparameters. For example, the size of some layers can depend on the overall number of layers [19].

5.1.4 Learning or Training

Learning is the adaptation of the network to better handle a task by considering sample observations. Learning involves adjusting the weights (and optional thresholds) of the network to improve the accuracy of the result. This is done by minimizing the observed errors. Learning is complete when examining additional observations does not usefully reduce the error rate [19].

5.1.5 Backpropagation

Backpropagation is a method to adjust the connection weights to compensate for each error found during learning. The error amount is effectively divided among the connections. Technically, Backpropagation calculates the gradient (the derivative) of the cost function associated with a given state with respect to the weights. The weight updates can be done via stochastic gradient descent or other methods [19].

5.1.6 Digit Recognition

Since supervised model is used to create a a digit classifier, during the training dispensation the models is provided with correctly labeled outputs (say y). If errors arise, they need to be minimized. The cost function is calculated as the square of the difference between the expected and actual output.

$$C_0 = (a^{(L)} - y)^2 \tag{5.3}$$

The RHS of (5.2) can re-written as $\sigma(z^{(L)})$. To minimize cost function the derivative needs to be found, and here chain rule can be applied:

$$\frac{\partial C_0}{\partial w^{(L)}} = \frac{\partial z^{(L)}}{\partial w^{(L)}} \cdot \frac{\partial a^{(L)}}{\partial z^{(L)}} \cdot \frac{\partial C_0}{\partial a^{(L)}} \tag{5.4}$$

Therefore for a single training sample the minimized cost function will be as in (5.5) and the net cost ∇C will be as shown in (5.7).

$$\frac{\partial C_0}{\partial w^{(L)}} = a^{(L-1)} \cdot \sigma'(z^{(L)}) \cdot 2(a^{(L)} - y) \tag{5.5}$$

$$\frac{\partial C}{\partial w^{(L)}} = \frac{1}{n} \sum_{k=0}^{n-1} \frac{\partial C_k}{\partial w^{(L)}} \tag{5.6}$$

$$\nabla C = \begin{bmatrix} \frac{\partial C}{\partial w^{(0)}} \\ \frac{\partial C}{\partial w^{(1)}} \\ \vdots \\ \frac{\partial C}{\partial w^{(L)}} \end{bmatrix} \tag{5.7}$$

This is propagated backwards, errors are minimized after each sample and hence the digits are recognized correctly [20].

5.2 Digit Recognition with CNN

As stated in section (4.3), digit recognition is a powerful deep learning model mostly used with digital image manipulation tasks. It is unsurprisingly complex, yet beautiful in construct. The accuracy with which CNNs classifies objects is at par. No wonder why the current state-of-the-art is a CNN based model. This section tries to delve a little more deeper into what exactly CNNs does under the hood.

5.2.1 Object Recognition Fundamentals

General flow of digit recognition includes [21]:

1. **Image Acquisition :** This stage receives an input image which can from in any format from any source. They can be of image formats like: .png, .jpeg or capture from a camera or scanner.

2. **Preprocessing :** Here the input image undergoes a number of procedures which tries to minimize those factors which may cause the accuracy to drop. It usually works upon images to reduce and if possible remove distortions.

3. **Segmentation :** It is used to isolate focused parts of the input. Histogram process is use retrieve the entities from the image. Contents are processed in a tree like structure.

4. **Feature Extraction :** Converts an image into *n-dimensional array tensor*. It extracts those features from the sample which are most importance for classification. Various techniques include Principle Component Analysis (PCA), Chain Code (CC) etc.

5. **Classification :** Extracted characteristics is used to make decisions. Different classifier algorithms are used and they compare the input with the training sample. *Softmax* regressions is applied to rate the accuracy in terms of probability.

6. **Post Processing :** To correct misclassified results, shape recognition is used. Then along with the language knowledge, the errors can be minimized in case of handwriting recognition.

5.2.2 Demystifying CNNs

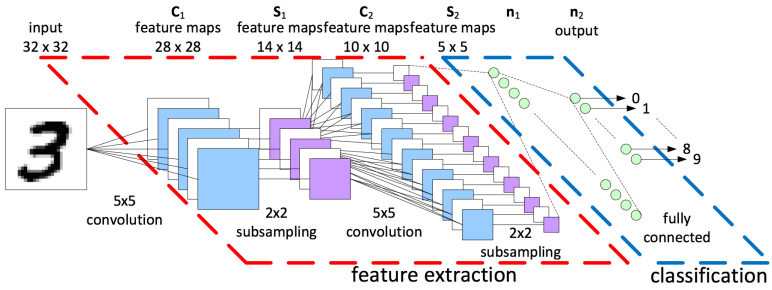

Figure 5.3: Layers in Convolutional Neural Network

Tensor

In mathematics, a tensor is an algebraic object that describes a (multilinear) relationship between sets of algebraic objects related to a vector space. Objects that tensors may map between, include vectors and scalars, and even other tensors. Tensors can take several different forms – for example: scalars and vectors etc [22].

Convolution

Convolutional layers convolve the input and pass its result to the next layer. The input is a tensor with shape (number of images) x (image height) x (image width) x (image depth). It becomes abstracted to a feature map, with shape (number of images) x (feature map height) x (feature map width) x (feature map channels) [17].

Pooling

Pooling layers reduce the dimensions of the data by combining the outputs of neuron clusters at one layer into a single neuron in the next layer. Local pooling combines small clusters, typically 2×2. They are applied with a stride S, of 2 downsamples at every depth slice in the input by 2 along both width and height, discarding 75% of the activations [17]:

$$f_{X,Y}(S) = \max_{a,b=0}^{1} S_{2X+a,2Y+b} \tag{5.8}$$

Fully Connected Layer

Fully connected layers connect every neuron in one layer to every neuron in another layer. It is in principle the same as the traditional multi-layer perceptron neural

network (MLP). The flattened matrix goes through a fully connected layer to classify the images [17] and thus recognize handwritten digits aptly.

5.3 Vision Transformer

On October 22, 2020 Google Research, Brain Team released an arXiv preprint on *Vision Transformer* [4]. A paper on *Image Transformer* was released in 2018 [3] which performed tasks like generative image modelling and super-resolution scaling. The latter model as expected diverted to fulfill different purpose. Inspired by the Transformer scaling successes in NLP, [4] experiments with applying a standard Transformer [2] directly to images, with the fewest possible modifications.

5.3.1 ViT Architecture

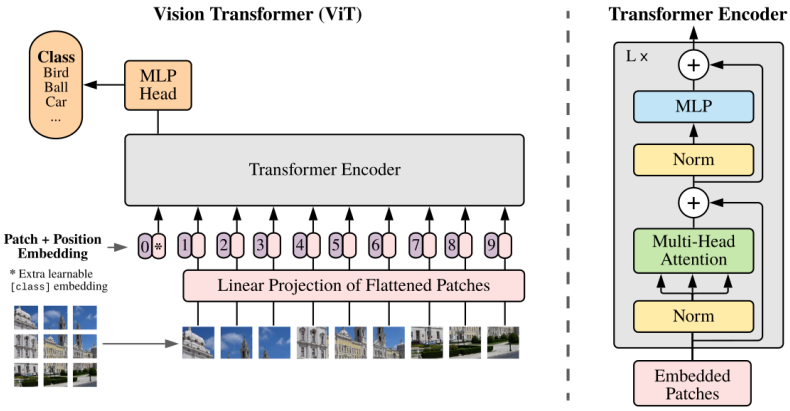

Figure 5.4: Vision Transformer Architecture [4]

The input image is split into fixed-size patches and each one of them is linearly embedded and position embeddings are added [23]. The resulting sequence of vectors is fed to a standard Transformer encoder [2]. In order to perform classification, ViT uses the standard approach of adding an extra learnable 'classification token' to the sequence.

The standard Transformer receives as input a 1D sequence of token embeddings. To handle 2D images, image $x \in \mathbb{R}^{H \times W \times C}$ is reshaped into a sequence of flattened 2D patches $x_p \in \mathbb{R}^{N \times (P^2 \cdot C)}$ where:

- (H, W) is the resolution of original image,
- C is the number of channels
- (P, P) is the resolution of each image patch and
- $N = HW/P^2$ is the resulting number of patches.

The Transformer uses constant latent vector size D throughout all of it layers fo the flatten patches are mapped to D dimensions, with a trainable linear projection.

This is termed as *patch embeddings*. A learnable embedded is pre-appended to the sequence embedding, whose state at the output of the Transformer encoder ($\mathbf{z}_L^0$) serves as the image representation y. Both during pre-training and fine-tuning, a classification head is attached to ($\mathbf{z}_L^0$). The classification head is implemented by a Multi-layer Perceptron (MLP) with one hidden layer at pre-training time and by a single linear layer at fine-tuning time [4].

5.3.2 Hybrid Architecture

As an alternative to raw image patches, the input sequence can be formed from feature maps of a CNN [24]. In this hybrid model, the patch embedding projection is applied to patches extracted from a CNN feature map. As a special case, the patches can have spatial size 1×1, which means that the input sequence is obtained by simply flattening the spatial dimensions of the feature map and projecting to the Transformer dimension. The classification input embedding and position embeddings are added as described above [4].

5.3.3 Fine Tuning ViT

Typically, ViT is pre-trained on large datasets, and fine-tuned to (smaller) downstream tasks. For this, the pre-trained prediction head is removed and a zero-initialized $D \times K$ feed-forward layer is attached, where K is the number of downstream classes. It is often beneficial to fine-tune at higher resolution than pre-training. When feeding images of higher resolution, the patch size is kept the same, which results in a larger effective sequence length. The Vision Transformer can handle arbitrary sequence lengths (up to memory constraints), however, the pre-trained position embeddings may no longer be meaningful. Therefore 2D interpolation of the pre-trained position embeddings is performed, according to their location in the original image [4].

Chapter 6

Proposed System

We've so far approached the project from machine learning perspective. Being a research oriented project, it focuses on the field of digit recognition. Yet to remind the reader of another part which is as relevant is Sudoku. Solving Sudoku might sound simple initially and that is quite natural. Our cognitive thinking skills tackles a problem with vigorous intuitiveness. But a machine needs to be told to solve a task in the sequence of steps, which is called an algorithm.

Various algorithms exists to solve Sudoku, some of which are 'backtracking', 'stochastic search', 'exact cover' and 'constrain programming' [25]. This project can be said to have two broad parts - *Artificial Intelligence* and *Sudoku Solver*. Figure (6.1) showcases an overly simplified project logic.

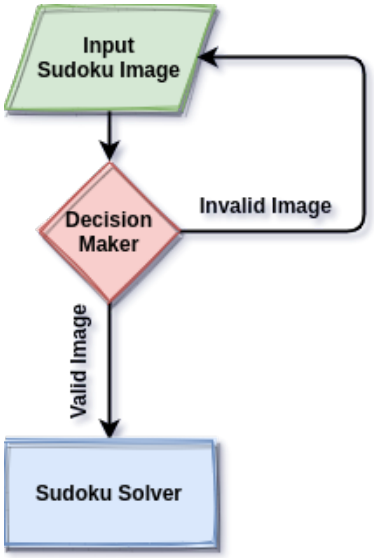

Figure 6.1: Vudoku: Simplified Architecture

The fundamental idea is to capture Sudoku pattern from images (even hand-written ones) using the latest Vision Transformer and solve them by an effective or optimal algorithm.

6.1 Project Modules

Applying divide and conquer method, this project can be broken in to different segments based on their functionality. Broadly speaking this project has a AI and non-AI module. The AI comprises of the *decision maker* and non-AI that of *Sudoku solver* as in figure (6.1).

6.1.1 Image Input Module

A little more expanded in architecture is shown in figure (6.2). The input image is split into patches, which is depicted by those yellow blocks, as discussed in (5.3.1). They are encoded and positional embeddings are provided. As it reaches the Transformer encoder, they are converted into computer readable vectors. A hypothesis is that the output in desired format can be extrapolated from the decoder.

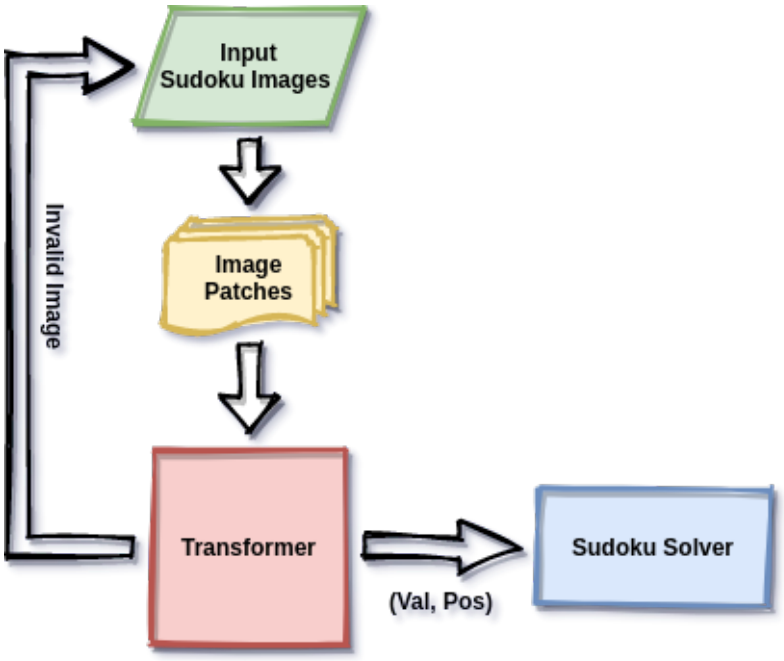

Figure 6.2: Vudoku: Expanded Architecture

6.1.2 Encoder and Decoder

The Vision Transformer (ViT) (5.3) tires to keep to the original Transformer architecture, hence similar encoder and decoder are used.

Encoder

The encoder is composed of a stack of $N = 6$ identical layers. Each layer has two sub-layers. The first is a multi-head self-attention mechanism, and the second is a simple, position-wise fully connected feed-forward network. A residual connection [26] around each of the two sub-layers, followed by layer normalization [27]. That is,

the output of each sub-layer is $LayerNorm(x + Sublayer(x))$, where $Sublayer(x)$ is the function implemented by the sub-layer itself [2].

Decoder

The decoder is also composed of a stack of $N = 6$ identical layers. In addition to the two sub-layers in each encoder layer, the decoder inserts a third sub-layer, which performs multi-head attention over the output of the encoder stack. Similar to the encoder, residual connections are employed around each of the sub-layers, followed by layer normalization. The self-attention sublayer is also modified in the decoder stack to prevent positions from attending to subsequent positions. This masking, combined with fact that the output embeddings are offset by one position, ensures that the predictions for position i can depend only on the known outputs at positions less than i [2].

6.1.3 Attention Mechanism

An attention function can be described as mapping a query and a set of key-value pairs to an output, where the query, keys, values, and output are all vectors. The output is computed as a weighted sum of the values, where the weight assigned to each value is computed by a compatibility function of the query with the corresponding key.

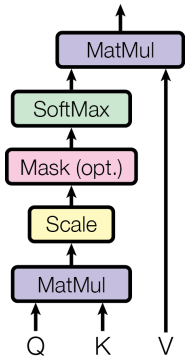

Figure 6.3: Scaled Dot-Product Attention [2]

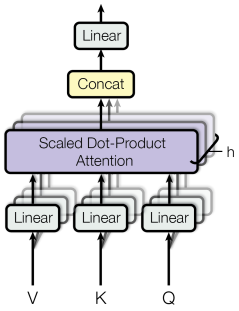

Figure 6.4: Masked Multi-head Attention [2]

The input consists of queries and keys of dimension d_k, and values of dimension d_v. The dot products of the query with all keys is computed then each one divided by $\sqrt{d_k}$. Finally a *softmax* function is applied to obtain the weights on the values. In practice, the attention function on a set of queries is computed simultaneously, packed together into a matrix Q. The keys and values are also packed together into matrices K and V see figures (6.2) and (6.3). The matrix of outputs is computed as [2]:

$$\text{Attention}(Q, K, V) = \text{softmax}(\frac{(QK^T)}{\sqrt{d_k}})V \tag{6.1}$$

6.1.4 Sudoku Solvability Checker

Since inputs can be noisy, constrains are applied during training to avoid garbage values from the Sudoku solver. Some of the basic conditions to solve Sudoku are hardcoded into this module. One of the simplest constrain is a puzzle should have at the minimum 17 clues and there is a solvable puzzle with at most 21 clues for every solved grid. rows, columns or boxes with repeated values is an invalid input. Digits with positions beyond the 9×9 gird automatically disqualifies a Sudoku.

6.1.5 Sudoku Solver

Once a correct set of inputs are received from the Transformer, this module works to solve them as effectively as possible. As stated in the beginning of (6), there are multiple algorithms available to solve Sudoku. Theoretically, this section tries to implement constrain programming and stochastic search approach or *algorithm* to arrive at a solutions. But in practice, the code will be more convoluted and inspired by [28]. Instead strictly following a single algorithm, it could be a mixture so as to get a optimal solution. The figure (6.5) displays the final proposed architecture of this mini dissertation.

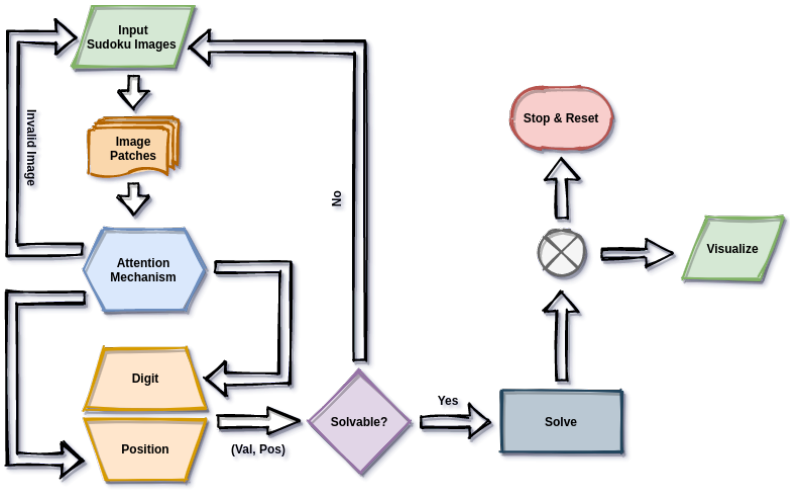

Figure 6.5: Vudoku: Final Architecture

6.1.6 Solution Visualizer

Solving Sudoku is one thing and visualizing it is another. Software libraries like matplotlib [29] and engines like manim [30] can be used to showcase how the Sudoku is being solved step by step before stopping and resetting the model state as shown in figure (6.5). This part of the project is educational as it gives a clearer picture of the process that takes places within the model. Note visualization is not quite easy as it sounds and hence this module will be considered as a peripheral bonus.

6.2 Project Requirements

This section spotlights paraphernalia required to create Vudoku. The followed tables provides a compact list of necessary items.

Hardware	Software
CPU: 2+Cores, 2.46+GHz	Code Editor: VSCode/OSS
RAM: 6 to 8GB	Language(s): Python/Nim
Disk: 5+GB (free space)	Libraries: OpenCV, TensorFlow etc.
Capture Device: Camera or Scanner	Dataset: MNIST

Some of the components mentioned are expensive, so those who desire to test the code, but hardware is not feasible, alternatives exist. Google provides and online platform called Colaboratory (Colabs for short) which is basically Jupyter Notebooks on a remote system. Another one Kaggle, which was later acquired by Google, is also a similar but far more powerful platform, commonly for data science and related tasks. High performance Tensor Processing Units (TPUs) are available to tinker with. Both of the platforms provide free access to massive computational resources.

For exhaustive development a version control system is inevitable. This project will be versioned using *Git* and the code is hosted on a GitHub repository at `https://github.com/joe733/vudoku`. The source code is distributed under MIT licence.

Chapter 7

Implementation

Theory remains as good as debunked until proved otherwise. This chapter concerns with groundwork of the project. It follows object oriented and scripting paradigm. Breaking it down the there are:

- three operational modules namely
  - *scavenger.py* - obtains Sudoku from image
  - *extractor.py* - grabs grid and digits from Sudoku
  - *solver.py* - solves the puzzle
  - *builder.py* - builds a mapped Sudoku
- a primary script
  - *main.py* - control script of the project
- neural network model
  - *classifier.py* - a dense CNN keras model

To follow along it's best to refer the repository. It has the following directory structure:

```
|- src/
    |- assets/
        |- digit_map/
        |- sample/
        |- screens/
    |- builder.py
    |- classifier.py
    |- extractor.py
    |- main.py
    |- scavenger.py
    |- solver.py
```

Note to the reader: *There has been and will be changes to the source code overtime, you may find this report outdated, if so, you're more than welcome to comment on the project's repository issue / discussion pages.*

7.1 Digit Recognition Module

Since Vudoku aims to solve Sudoku visually it takes an input image and processes it to recognize digits in it. This module *classifier.py* is not deployed on production system, instead it's used to generate classification models which are saved for later use. A sample of dataset, on which this model is trained, is shown in figure (7.1).

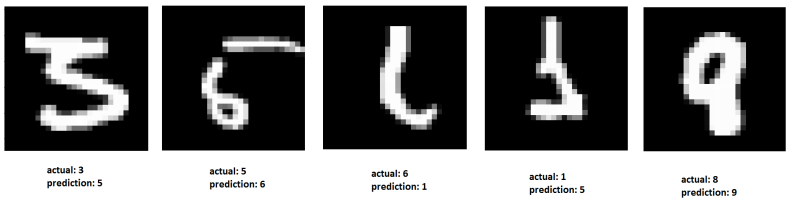

Figure 7.1: MNIST Digit Classification

The digit recognition model is structured as a sandwich of layers which is elucidated in the following sections.

7.1.1 Sequential Model

A Sequential model is appropriate for a plain stack of layers where each layer has exactly one input tensor and one output tensor [31]. Here every unit in the current layer is connected to every unit in the previous layer. This model consists of the following layers in order:

```
2D Convolution Layer with ReLu activation
+ another 2D Convolution Layer with ReLu activation
+ a 2D MaxPooling layer + Dropout Layer
+ Flatten Layer + Dense Layer with ReLu activation
+ Dropout Layer + Dense Layer with SoftMax activation
```

Convolution Layer 2D × 2

This layer creates a convolution kernel that is convoluted with the input layer to produce a tensor of outputs. When using this layer as the first layer in the model, a keyword argument *input_shape* is provided as a tuple (28, 28) which comes from the fact that MNIST dataset has 28 × 28 sized images only. There are two of these layer back-to-back which is because higher-level patterns cannot be seen with just one convolution layer.

MaxPooling layer 2D

Pooling is basically *downscaling* the image obtained from the previous layers. It can be compared to shrinking an image to reduce its pixel density. Down-scaling is performed using a hard-coded tensor operation such as *max*, rather than through a learned transformation, because the relatively expensive operation of learning the weights [32] is not needed. This way, useful spatial hierarchy at a fraction of the

cost is obtained. A good spatial hierarchies summarize the data substantially when moving from bottom to top.

Flatten and Dropout × 2

Flattening a tensor means to remove all of the dimensions except for one. This is exactly what the *Flatten* layer does.

Dropout layer randomly sets input units to 0 with a frequency of rate at each step during training time, which helps prevent over-fitting. Inputs not set to 0 are scaled up by 1/(1 - rate) such that the sum over all inputs is unchanged.

Dense Layer × 2

Dense layer is the regular deeply connected neural network layer. Which means each neuron in the dense layer receives input from all neurons of its previous layer. In the background, the dense layer performs a matrix-vector multiplication. Activation function is useful to introduce non linearity into the neural network so that the network can learn complex relationship between input and output data [33]. The final dense layer with a SoftMax activation is the output layer.

7.1.2 Losses and Optimization

Every machine learning model has errors, due to which the accuracy is lost. Such loss is calculated to realize an optimization which will give the best result.

Categorical Crossentropy Loss

The model calculates *categorical_ crossentropy loss*. It returns the sum of product of true values and log of output values for each observation or an average over all samples.

$$\text{loss}(p^{\text{true}}, p^{\text{predict}}) = -\sum_i p_i^{\text{true}} \log p_i^{\text{predict}}$$

Adelta Optimization

It is a stochastic gradient descent method that is based on adaptive learning rate per dimensions addresses two drawbacks: one *the continual decay of learning rates throughout training* and *the need for a manually selected global learning rate*. Adadelta is a more robust extension of Adagrad that adapts learning rates based on a moving window of gradient updates, instead of accumulating all past gradients.

7.2 Image Processing Modules

The input image is captured from a light sensitive device and then processed before passing it onto the classifier. This is achieved by *scavenger.py* and *extractor.py* modules. The figure (7.2) is an example input image which undergoes per-processing.

Figure 7.2: Sample Input

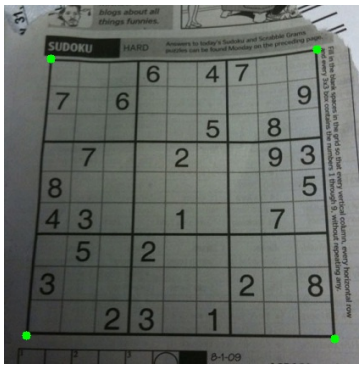

Figure 7.3: Marked Contours

7.2.1 Preprocessing and Transformation

The *scavenger.py* module imports powerful *OpenCV* library to perform *color to grayscale*, *gaussian blur*, *adaptive threshold*, *bitwise not* and *dilation* operations in order. This will remove the noise in image and will provide a grayscale image. Next it finds all the contours in the image and then finds those with maximum area, finally it create an approximate perimeter to obtain the contour points as seen in figure (7.3). This is then passed onto the perspective transform which expects four corners of the grid and calculates the euclidean distance which is mapped to an empty image of the same dimension resulting in figure (7.4).

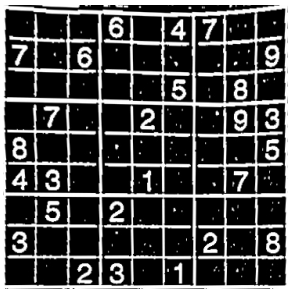

Figure 7.4: Transformed

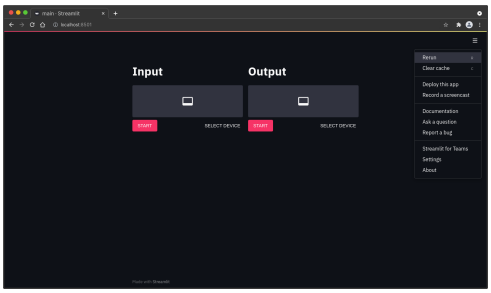

Figure 7.5: Wireframe UI

7.2.2 Digit Extraction

This submodule reads in the saved model (7.1) and accepts the transformed grid (7.4) as input. Then it uses a simple logic and the powerful *numpy* to split the image into a 9×9 grid. This method is not fool proof but efficient. Since the input image

can be of variable size it needs to be resized into a 28 × 28 dimension otherwise the model will reject the input image.

A caveat with the classification model is that it has been trained only on numbers and so its universal knowledge set consists only of images with digits. But Sudoku puzzle contains empty grids, it is observed that the classifier wrongly verdicts an empty cell as a digit. The current workaround is to skip the empty cells. The classifier spits out digits from the cells that it recognizes and they are encoded into an 81 bits long string.

7.3 Solver Module

The Sudoku from an image is encoded into an 81 bit long string with zeros indicating empty cells. This solver module performs a very simple backtracking operation. It first reshapes the input string into a 9 × 9 numpy array. Then for each row, at each empty cell, here 0, is replaced with a valid number.

The validity of a number is checked by traversing the current row, column and the box. If validated then the operation is repeated. If not then it traces back to the previous empty cell and replaces the current number with an incremented one.

7.4 Builder Module

Once Sudoku is solved it is highly recommended to verify it (which is marked in the project roadmap). To display the solved Sudoku grid, each numbers from 0 to 9 is mapped to a 28 × 28 image containing corresponding digit. The board is traversed and each of these images are stitched horizontally and vertically.

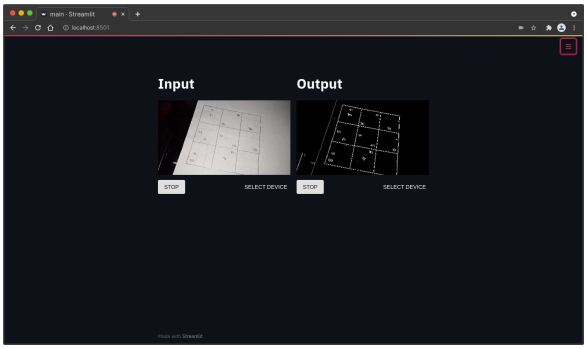

Figure 7.6: Live Image Processing

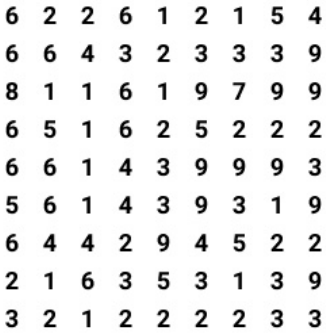

Figure 7.7: Stitched Image

Since images are considered as n-dimensions (here n = 3) numpy arrays, they can be concatenated. Stitching horizontally takes place along the ndarray's axis = 0 and vertically along ndarray's axis = 1, see (7.7)

7.5 Main Script

This is a small controls script which provides an overall finish to the project. It includes Streamlit [34] and Streamlit-Webrtc [35] for the frontend. They manage async operations with the UI. Rest of it is function calls to the imported local modules. Figure (7.6) shows the processing of a live image from left to right.

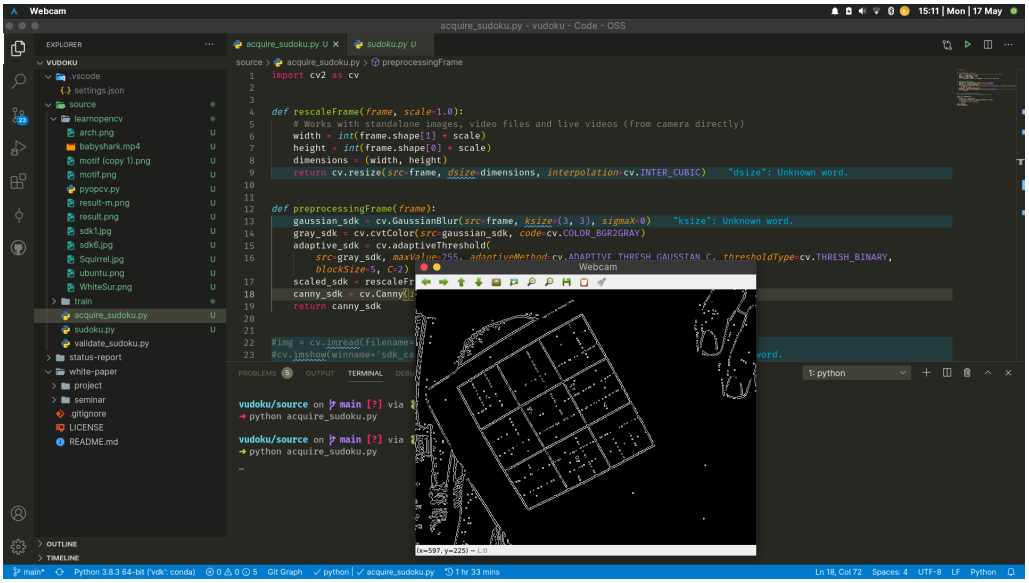

Figure 7.8: Testing Edge Detection

The above image (7.8) shows the author testing a live edge detector. After hours of work and with more than a hundred commits Vudoku became a web application with a beautiful interface as seen in (7.9).

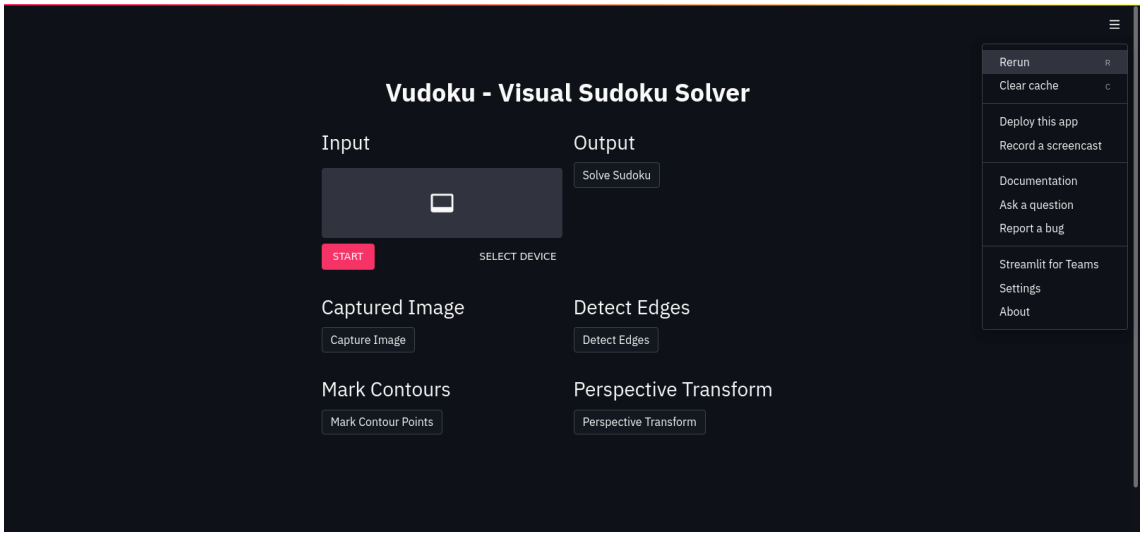

Figure 7.9: User Interface

Chapter 8

Conclusion

When the Transformer was first released back in 2017 by the Google-Brain team, it received a lot of traction from the research community. The OpenAI built three successive versions of GPT, which took the social media by the storm, was based on the same Transformer model. The scientific research landscape is shifting, it is at a crucial infliction point. Whatever we've learned so far from Recurrent Nets, Long Short Term Memory, Convolution Neural Networks etc. has paved way towards newer thinking and better approaches. Not a month ago Google-Research released their Vision Transformer (ViT), which was expected to be the at the heart of *Vudoku*.

I was exploring the ViT's Jupyter notebook, simultaneously trying to grab the Sudoku grid from an image using Hough lines transform, but since the latter was messed up I ended up enquiring the possibility of using ViT directly. Little did I know that the very structure of ViT couldn't be used for such a task (as it was stated in a comment on my thread), or is it... still an unfinished research.

Some of the libraries I've used, like Streamlit, Streamlit_Webrtc are very new and as of writing this, they are under heavy development. Hence it's likely that one will encounter hiccups while replicating this work. Another obstacle I found jutting out during development is the disparity in packages - one version of package is hard-bound to another version or a range of the same, updating them could be really messy. This is quite the reason that Streamlit hosting is failing. I've used *conda* as my primary package manage but due to the unavailability of certain packages I've used *pip* also.

Digit recognition is an interesting classical problem to embark the journey in the field of artificial intelligence. Sudoku, a mathematical puzzle is captivating beauty of the mystery that lies in various patterns and combinations in itself. This project could be an inspiring stepping stone for futures enthusiasts to work upon. On a personal note, I believe this project has been a crescendo of learning adventure.

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