1. а) $13^8 \bmod 7$

$13^8 \equiv 6^8 \pmod 7$

$6^8 \equiv (-1)^8 \pmod 7$

$(-1)^8 = 1$

$1 \bmod 7 = \boxed{1}$

б) $2^{152} \bmod 29 = (2^8)^{19} \bmod 29 = 256^{19} \bmod 29 \Rightarrow$

$2^5 = 32 \pmod{29} \Rightarrow 24^{19} \bmod 29 = (-5)^3 \cdot (-5)^{16} \bmod 29 =$

$= -125 \cdot (-5)^{16} \bmod 29 = -9 \cdot (-5^4)^4 \bmod 29 = -9 \cdot$

$\cdot (-625)^4 \bmod 29 = -9 \cdot 13^4 \bmod 29 = -9 (13^2)^2 \bmod 29 =$

$= -9 \cdot 24^2 \bmod 29 = -9 \cdot 576 \bmod 29 = -9 \cdot (-4) \bmod 29 =$

$= 36 \bmod 29 = \boxed{7}$

в) $36 \cdot (-12) \bmod 17$

$36 \bmod 17 = 2$

$-12 \bmod 17 = 5$

$5 \cdot 2 \bmod 17 = \boxed{10}$

г) $3^{701} \bmod 80$

$3^4 \equiv 81 \pmod{80}$

$81 \equiv 1 \pmod{80}$

$3 \cdot (3^4)^{175} \equiv 3 \cdot (1^4)^{175} \pmod{80} = \boxed{3}$

д) $27^{17} \bmod 15$

$27 \pmod{15} = 12$

$12^{17} \equiv 27^{17} \pmod{15} = \boxed{12}$

е) $414 \cdot 463 \bmod 413$

$414 \pmod{413} = 1$

$463 \pmod{413} = 50$

$1 \cdot 50 \pmod{413} = \boxed{50}$

2. а) $(421, 111)$

$4 = 1 \cdot 3 + \boxed{1}$
$19 = 4 \cdot 4 + 3$
$23 = 1 \cdot 19 + 4$
$88 = 3 \cdot 23 + 19$
$111 = 1 \cdot 88 + 23$
$421 = 3 \cdot 111 + 88$

$1 = 4 - 1 \cdot 3 = 4 - 1 \cdot (19 - 4 \cdot 4) = 5 \cdot 4 - 1 \cdot 19 =$

$= -1 \cdot 19 + 5(23 - 1 \cdot 19) = -6 \cdot 19 + 5 \cdot 23 =$

$= 5 \cdot 23 + 6(88 - 3 \cdot 23) = 23 \cdot 23 - 6 \cdot 88 =$

$= -6 \cdot 88 + 23(111 - 1 \cdot 88) = -29 \cdot 88 + 23 \cdot 111 =$

$= 23 \cdot 111 - 29(421 - 3 \cdot 111) = 110 \cdot 111 - 29 \cdot 421$

$\begin{array}{r|l}
421 & 111 \\
-333 & 3 \\
\hline
111 & 88 \\
-88 & 1 \\
\hline
88 & 23 \\
-69 & 3 \\
\hline
23 & 19 \\
-19 & 1 \\
\hline
19 & 4 \\
-16 & 4 \\
\hline
4 & 3 \\
-3 & 1 \\
\hline
3 & \boxed{1} - HCD \\
-3 & 3 \\
\hline
0 &
\end{array}$

б) $(959, 791)$

$$959 = 1 \cdot 791 + 168$$
$$791 = 4 \cdot 168 + 119$$
$$168 = 1 \cdot 119 + 49$$
$$119 = 2 \cdot 49 + 21$$
$$49 = 2 \cdot 21 + \boxed{7}$$

$7 = 49 - 2 \cdot 21 = 49 - 2(119 - 2 \cdot 49) =$
$= \mathbf{5} \cdot 49 - 2 \cdot 119 = -2 \cdot 119 + 5(168 - 1 \cdot 119) =$
$= -7 \cdot 119 + 5 \cdot 168 = 5 \cdot 168 - 7 \cdot (791 - 4 \cdot 168) =$
$= 33 \cdot 168 - 7 \cdot 791 = -7 \cdot 791 + 33(959 - 1 \cdot 791) =$
$= -40 \cdot 791 + 33 \cdot 959$

$$
\begin{array}{r|l}
959 & 791 \\
-791 & 1 \\
\hline
791 & 168 \\
-672 & 4 \\
\hline
168 & 119 \\
-119 & 1 \\
\hline
119 & 49 \\
-98 & 2 \\
\hline
49 & 21 \\
-42 & 2 \\
\hline
21 & \boxed{7} - \text{НСД} \\
-21 & 3 \\
\hline
0 &
\end{array}
$$

3. а) $8^{-1} \bmod 13$

$(8, 13) = 1$

$$
\begin{array}{r|l}
13 & 8 \\
-8 & 1 \\
\hline
8 & 5 \\
-5 & 1 \\
\hline
5 & 3 \\
-3 & 1 \\
\hline
3 & 2 \\
-2 & 1 \\
\hline
2 & \boxed{1} - \text{НСД} \\
-2 & 2 \\
\hline
0 &
\end{array}
$$

		-1	-1	-1	-1	
0	1	-1	2	-3	⑤	
					a^{-1}	

$5 \bmod 13$

1) $(8 \cdot 5 - 1) : 13 = 3 \in \mathbb{Z}$

2) $8 \cdot 5 \bmod 13 = 1$

б) $51^{-1} \bmod 71$

$(51, 71) = 1$

$$
\begin{array}{r|l}
71 & 51 \\
\underline{51} & 1 \\
\end{array}
$$

$$
\begin{array}{r|l}
51 & 20 \\
\underline{40} & 2 \\
\end{array}
$$

$$
\begin{array}{r|l}
20 & 11 \\
\underline{11} & 1 \\
\end{array}
$$

$$
\begin{array}{r|l}
11 & 9 \\
\underline{9} & 1 \\
\end{array}
$$

$$
\begin{array}{r|l}
9 & 2 \\
\underline{8} & 4 \\
\end{array}
$$

$$
\begin{array}{r|l}
2 & \boxed{1} - \text{НОД} \\
\underline{2} & 2 \\
0 & \\
\end{array}
$$

		-1	-2	-1	-1	-4
0	1	-1	3	-4	7	$\boxed{-32}$

a^{-1}

$-32 \bmod 71$

$39 \bmod 71$

1) $(51 \cdot 39 - 1) : 71 = 28 \in \mathbb{Z}$

2) $51 \cdot 39 \bmod 71 = 1.$

в) $43^{-1} \bmod 65$

$(43, 65) = 1$

$$
\begin{array}{r|l}
65 & 43 \\
\underline{43} & 1 \\
\end{array}
$$

$$
\begin{array}{r|l}
43 & 22 \\
\underline{22} & 1 \\
\end{array}
$$

$$
\begin{array}{r|l}
22 & 21 \\
\underline{21} & 1 \\
\end{array}
$$

$$
\begin{array}{r|l}
21 & \boxed{1} \\
\underline{21} & 21 \\
0 & \\
\end{array}
$$

		-1	-1	-1
0	1	-1	2	$\boxed{3}$

a^{-1}

$-3 \bmod 65$

$62 \bmod 65$

1) $(43 \cdot 62 - 1) : 65 = 41 \in \mathbb{Z}$

2) $43 \cdot 62 \bmod 65 = 1$

4. a) $5x \equiv 12 \mod 19$

$x_0 = 12 \cdot 5^{-1} \mod 19 = 12 \cdot 4 \mod 19 = 10$

$(5, 19) = 1$

$(4 \cdot 5 - 1) : 19 = 1 \in \mathbb{Z}$

$$\begin{array}{r|r} -19 & 5 \\ 15 & 3 \end{array}$$
$$\begin{array}{r|r} -5 & 4 \\ 4 & 1 \end{array}$$
$$\begin{array}{r|r} -4 & ① - \text{НСД} \\ 4 & 4 \\ \hline 0 & \end{array}$$

		-3	-1	-4
0	1	-3	④	-13

$x = x_0 + km, \; k \in \mathbb{Z}$

$x = 10 + 19k, \; k \in \mathbb{Z}$

б) $45x \equiv 5 \mod 55 \;|\; d = 5$

$9x \equiv 1 \mod 11$

$x_0 = 1 \cdot 9^{-1} \mod 11 = 1 \cdot 5 \mod 11 =$
$= 5$

$(11, 9) = 1$

		-1	-4	-2
0	1	-1	⑤	-11
			a_{-1}	

$x = x_0 + km, \; k \in \mathbb{Z}$

$x = 5 + 11m, \; k \in \mathbb{Z}$

$$\begin{array}{r|r} -11 & 9 \\ 9 & 1 \end{array}$$
$$\begin{array}{r|r} -9 & 2 \\ 8 & 4 \end{array}$$
$$\begin{array}{r|r} -2 & ① - \text{НСД} \\ 2 & 2 \\ \hline 0 & \end{array}$$

в) $12x \equiv 6 \mod 37$

$x_0 = 6 \cdot 12^{-1} \mod 37 = 6 \cdot 34 \mod 37 = 19$

$(37, 12) = 1$

$$\begin{array}{r|r} & 37 \;|\; 12 \\ -36 & 3 \end{array}$$
$$\begin{array}{r|r} 12 & ① \; \text{НСД} \\ -12 & 12 \\ \hline 0 & \end{array}$$

		-3
0	1	③
		a_1

$(-3 \cdot 12 - 1) : 37 = -1 \in \mathbb{Z}$

$x = 19 + 37k, \; k \in \mathbb{Z}$

г) $14x \equiv 21 \mod 98 \;|\; d = 7$

$2x \equiv 3 \mod 14$

$x_0 = 3 \cdot 2^{-1} \mod 14 =$ $x \in \emptyset$

$(14, 2) \neq 1$

$$\begin{array}{r|r} 14 & 2 \\ -14 & 7 \\ \hline 0 & \end{array}$$

g) $7x \equiv 6 \mod 129$

$x_0 = 6 \cdot 7^{-1} \mod 129 = 6 \cdot 20 \mod 129 = 120$

$(129, 7) = 1$

		-18	-5	-1	-2
0	1	-18	91	$\boxed{-109}$	

a^{-1}

$$\begin{array}{r|l} 129 & 7 \\ -126 & \overline{18} \end{array}$$

$$\begin{array}{r|l} 17 & 3 \\ -15 & \overline{5} \end{array}$$

$$\begin{array}{r|l} 3 & 2 \\ -2 & \overline{1} \end{array}$$

$$\begin{array}{r|l} 2 & \boxed{1} \text{ — нод} \\ -2 & \overline{2} \\ \hline 0 & \end{array}$$

$(-109 \cdot 7 - 1) \vdots 129$

$x = 120 + 129k, \ k \in \mathbb{Z}$