PART I SCHWESER'S QuickSheet™

CRITICAL CONCEPTS FOR THE 2021 FRM® EXAM

FOUNDATIONS OF RISK MANAGEMENT

Types of Risk

Key classes of risk include market risk, credit risk, liquidity risk, operational risk, legal and regulatory risk, business and strategic risk, and reputation risk.

- *Market risk* includes interest rate risk, equity price risk, foreign exchange risk, and commodity price risk.
- *Credit risk* includes default risk, bankruptcy risk, downgrade risk, and settlement risk.
- *Liquidity risk* includes funding liquidity risk and market liquidity risk.

Credit Risk Transfer Mechanisms

Credit default swaps (CDSs) enable investors to transfer credit risk on a loan product to an insurance company by paying a premium to buy downside protection.

Collateralized debt obligations (CDOs) enable loan originators to repackage loan products into large baskets of loans and then resell those bundles of loans to investors on the secondary markets.

Collateralized loan obligations (CLOs) are very similar to CDOs except they primarily hold underwritten bank loans as opposed to the mortgage bias of CDOs.

Systematic Risk

A standardized measure of systematic risk is beta:

$$\text{beta}_i = \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2}$$

Capital Asset Pricing Model (CAPM)

In equilibrium, all investors hold a portfolio of risky assets that has the same weights as the market portfolio. The CAPM is expressed in the equation of the *security market line* (SML). For any single security or portfolio of securities *i*, the expected return in equilibrium is

$$E(R_i) = R_F + \text{beta}_i[E(R_M) - R_F]$$

CAPM Assumptions

- Information is freely available.
- Markets are frictionless.
- Fractional investments are possible.
- There is perfect competition.
- Investors make their decisions solely based on expected returns and variances.
- Market participants can borrow and lend unlimited amounts at the risk-free rate.
- Expectations are homogenous.

Measures of Performance

The *Sharpe measure* is equal to the risk premium divided by the standard deviation, or total risk:

$$\text{Sharpe measure} = \left[\frac{E(R_P) - R_F}{\sigma_P}\right]$$

The *Treynor measure* is equal to the risk premium divided by beta, or systematic risk:

$$\text{Treynor measure} = \left[\frac{E(R_P) - R_F}{\beta_P}\right]$$

The *Jensen measure* (a.k.a. Jensen's alpha or just alpha), is the asset's excess return over the return predicted by the CAPM:

$$\text{Jensen measure} = \alpha_P = E(R_P) - \{R_F + \beta_P[E(R_M) - R_F]\}$$

The *information ratio* is essentially the alpha of the managed portfolio relative to its benchmark divided by the tracking error, where tracking error is the standard deviation of the difference between portfolio return and benchmark return.

$$IR = \left[\frac{E(R_P) - E(R_B)}{\text{tracking error}}\right]$$

The *Sortino ratio* is similar to the Sharpe ratio except we replace the risk-free rate with a minimum acceptable return, denoted R_{MIN}, and we replace the standard deviation with a type of downside deviation.

$$\text{Sortino} = \frac{R_P - R_{MIN}}{\text{downside deviation}}$$

Arbitrage Pricing Theory (APT)

APT describes expected returns as a linear function of exposures to common risk factors:

$$E(R_i) = R_F + b_{i1}RP_1 + b_{i2}RP_2 + \ldots + b_{ik}RP_k$$

where:

b_{ij} = *j*th factor beta for stock i

RP_j = risk premium associated with risk factor *j*

APT defines the structure of returns but does not define which factors should be used in the model.

The *CAPM* is a special case of APT with only one factor exposure: the market risk premium.

The *Fama-French three-factor model* describes returns as a linear function of the market index return, firm size, and book-to-market factors.

Risk Data Aggregation

Involves defining, gathering, and processing risk data for measuring performance against risk tolerance/appetite. Benefits include:

- Increases ability to anticipate problems
- Identifies routes to financial health.
- Improves resolvability in event of bank stress
- Increases efficiency, reduces chance of loss, and increases profitability

Enterprise Risk Management (ERM)

Integrated and centralized framework for managing firm risks in order to meet business objectives, minimize unexpected earnings volatility, and maximize firm value. Benefits include (1) increased organizational effectiveness, (2) better risk reporting, and (3) improved business performance.

Risk Factors for Financial Disasters

Interest rate risk: results from fluctuations in interest rate levels (measured using duration). Case study: the savings and loan (S&L) crisis in the 1980s.

Liquidity risk: potential for loss that results from short-term funding issues. Case studies: Lehman Brothers, Continental Illinois, and Northern Rock.

Hedging strategies: a firm must choose between a static hedge and a dynamic (rolling) hedge. Case study on dynamic hedge challenges: Metallgesellschaft Refining and Marketing (MGRM).

Model risk: can include making improper assumptions, measuring relationships the wrong way, and deploying the wrong model overall. Case studies: the Niederhoffer case and Long-Term Capital Management (LTCM).

Rogue trader: misleading reporting can cause the collapse of an entire organization. Case study: Barings Bank (Nick Leeson).

Financial engineering: involves the use of forwards, futures, swaps, options, and securitized products to hedge risk. Case studies on understanding the risks of these hedging tools: Bankers Trust, Orange County, and Sachsen Landesbank.

Reputational risk: the way in which the general public perceives the firm. Case study: Volkswagen.

Corporate governance: system of policies and procedures that direct how a firm is operated. Case study: Enron.

Cyber risk: risk of financial or reputational loss due to cyberattack on internal technology infrastructure. Case study: the SWIFT system.

Financial Crisis 2007–2009

Contributing factors: (1) banks relaxed their lending standards with move to originate-to-distribute model (increased subprime lending), (2) institutions increasingly funded themselves using short-term facilities (increased liquidity risk), and (3) the Lehman Brothers default caused a loss of confidence with banks refusing to lend to each other, and ultimately requiring central banks to provide liquidity support.

GARP Code of Conduct

Sets forth principles related to ethical behavior within the risk management profession.

It stresses ethical behavior in the following areas:

Principles

- Professional integrity and ethical conduct
- Conflicts of interest
- Confidentiality

Professional Standards

- Fundamental responsibilities
- Adherence to best practices

Violations of the Code of Conduct may result in temporary suspension or permanent removal from GARP membership. In addition, violations could lead to a revocation of the right to use the FRM designation.

QUANTITATIVE ANALYSIS

Probabilities

Unconditional probability (marginal probability) is the probability of an event occurring.

Conditional probability, P(A | B), is the probability of an event A occurring given that event B has occurred.

Bayes' Rule

Updates prior probability for an event in response to the arrival of new information.

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$

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Expected Value
Weighted average of the possible outcomes of a random variable, where the weights are the probabilities that the outcomes will occur.

$$E(X) = \sum P(x_i)x_i = P(x_1)x_1 + P(x_2)x_2 + \ldots + P(x_n)x_n$$

Properties of Expected Values
If *c* is any constant, then:

$$E(cX) = cE(X)$$

If *X* and *Y* are any random variables, then:

$$E(X + Y) = E(X) + E(Y)$$

Variance
Provides a measure of the extent of the dispersion in the values of the random variable around the mean. The square root of the variance is the standard deviation.

$$\sigma^2 = E\{[X - E(X)]^2\} = E[(X - \mu)^2]$$

Covariance
Expected value of the product of the deviations of two random variables from their respective expected values.

$$Cov(X,Y) = E\{[X - E(X)][Y - E(Y)]\}$$

Correlation
Measures the strength of the linear relationship between two random variables. It ranges from –1 to +1.

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sigma(X)\sigma(Y)}$$

Uniform Distribution
Distribution where the probability of *X* occurring in a possible range is the length of the range relative to the total of all possible values. Letting *a* and *b* be the lower and upper limits of the uniform distribution, respectively, then for $a \le x_1 < x_2 \le b$:

$$P(x_1 \le X \le x_2) = \frac{(x_2 - x_1)}{(b - a)}$$

Binomial Distribution
Evaluates a random variable with two possible outcomes over a series of *n* trials. The probability of “success” on each trial equals:

$$p(x) = [n!/(n - x)!x!]p^x(1 - p)^{n-x}$$

For a binomial random variable:

expected value = np

variance = np(1 – p)

Poisson Distribution
Poisson random variable *X* refers to the number of successes per unit. The parameter lambda (λ) refers to the average number of successes per unit. For the distribution, both its mean and variance are equal to the parameter, λ.

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Normal Distribution
The normal distribution is completely described by its mean and variance.

- 90% of observations fall within ± 1.65s.
- 95% of observations fall within ± 1.96s.
- 99% of observations fall within ± 2.58s.

Standardized Random Variables
A *standardized random variable* is normalized so that it has a mean of zero and a standard deviation of 1.

z-score: represents the number of standard deviations a given observation is from the population mean.

$$z = \frac{\text{observation} - \text{population mean}}{\text{standard deviation}} = \frac{x - \mu}{\sigma}$$

Chi-Squared Distribution
The *chi-squared test* is used for hypothesis tests concerning the variance of a normally distributed population.

$$\text{chi-squared test: } \chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

F-Distribution
The *F-test* is used for hypotheses tests concerning the equality of the variances of two populations.

$$\text{F-test: } F = \frac{s_1^2}{s_2^2}$$

Population and Sample Mean
The *population mean* sums all observed values in the population and divides by the number of observations in the population, *N*.

$$\mu = \frac{\sum_{i=1}^{N} X_i}{N}$$

The *sample mean* is the sum of all values in a sample of a population, ΣX, divided by the number of observations in the sample, *n*. It is used to make *inferences* about the population mean.

Population and Sample Variance
The *population variance* is defined as the average of the squared deviations from the mean. The *population standard deviation* is the square root of the population variance.

$$\sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$$

The *sample variance*, s^2, is the measure of dispersion that applies when we are evaluating a sample of *n* observations from a population. Using *n* – 1 instead of *n* in the denominator improves the statistical properties of s^2 as an estimator of σ^2.

$$s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

Sample Covariance

$$\text{covariance} = \sum_{i=1}^{n}\frac{(X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$$

Desirable Properties of an Estimator
- A point estimate should be a *linear* estimator when it can be used as a linear function of sample data.
- An *unbiased* estimator is one for which the expected value of the estimator is equal to the parameter you are trying to estimate.
- A *consistent* estimator is one for which the accuracy of the parameter estimate increases as the sample size increases.

Central Limit Theorem
When selecting simple random samples of size *n* from a population with mean μ and finite variance σ^2, the sampling distribution of sample means approaches the normal probability distribution with mean μ and variance equal to σ^2/n as the sample size becomes large.

Skewness and Kurtosis
Skewness, or skew, refers to the extent to which a distribution is not symmetrical. The skewness of a normal distribution is equal to zero.

- A *positively skewed* distribution is characterized by many outliers in the upper region, or right tail.
- A *negatively skewed* distribution has a disproportionately large amount of outliers that fall within its lower (left) tail.

Kurtosis is a measure of the degree to which a distribution is spread out compared to a normal distribution. Excess kurtosis = kurtosis – 3.

Hypothesis Testing
Null hypothesis (H_0): hypothesis the researcher wants to reject; hypothesis that is actually tested; the basis for selection of the test statistics.

Alternative hypothesis (H_A): what is concluded if there is significant evidence to reject the null hypothesis.

One-tailed test: tests whether value is greater than or less than another value. For example:

$$H_0: \mu \le 0 \text{ versus } H_A: \mu > 0$$

Two-tailed test: tests whether value is different from another value. For example:

$$H_0: \mu = 0 \text{ versus } H_A: \mu \ne 0$$

Standard Error
The *standard error of the sample mean* is the standard deviation of the distribution of the sample means. When the standard deviation of the population, σ, is known, the standard error of the sample mean is calculated as:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Confidence Interval
A range of values within which a researcher believes the true population parameter may lie:

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

T-Distribution
The *t-distribution* is a bell-shaped probability distribution that is symmetrical about its mean. It is the appropriate distribution to use when constructing confidence intervals based on small samples from populations with unknown variance and a normal, or approximately normal, distribution.

$$\text{t-test: } t = \frac{x - \mu}{s / \sqrt{n}}$$

Linear Regression

$$Y = \alpha + \beta \times (X) + \varepsilon$$

where:

Y = dependent or explained variable

X = independent or explanatory variable

α = intercept coefficient

β = slope coefficient

ε = error term

Linear Regression Assumptions
- The expected value of the error term, conditional on the independent variable, is zero.
- All (X, Y) observations are independent and identically distributed (i.i.d.).
- It is unlikely that large outliers will be observed in the data.

- The variance of X is strictly > 0.
- The variance of the errors is constant (i.e., homoskedasticity).

Multiple Regression

A *simple regression* is the two-variable regression with one dependent variable, Y_i, and one independent variable, X_i. A *multiple regression* has more than one independent variable.

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \varepsilon$$

Standard Error of the Regression (SER)

Measures the degree of variability of the actual *Y*-values relative to the estimated *Y*-values from a regression equation. The SER gauges the "fit" of the regression line. The smaller the standard error, the better the fit.

Total Sum of Squares

For the dependent variable in a regression model, there is a total sum of squares (TSS) around the sample mean.

total sum of squares (TSS) = explained sum of squares (ESS) + residual sum of squares (RSS)

$$\sum\left(Y_i - \bar{Y}\right)^2 = \sum\left(\hat{Y} - \bar{Y}\right)^2 + \sum\left(Y_i - \hat{Y}\right)^2$$

Coefficient of Determination

Represented by R^2, it is a measure of the *goodness of fit* of the regression.

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$$

In a simple two-variable regression, the square root of R^2 is the **correlation coefficient** (r) between X_i and Y_i. If the relationship is positive, then

$$r = \sqrt{R^2}$$

Adjusted R-Squared

Adjusted R^2 is used to analyze the importance of an added independent variable to a regression.

$$R_a^2 = 1 - \left[\left(\frac{n-1}{n-k-1}\right) \times \left(1 - R^2\right)\right]$$

Regression Assumption Violations

Heteroskedasticity occurs when the variance of the residuals is not the same across all observations in the sample.

Multicollinearity refers to the condition when two or more of the independent variables, or linear combinations of the independent variables, in a multiple regression are highly correlated with each other.

Covariance Stationary

A time series is covariance stationary if its mean, variance, and covariances with lagged and leading values are stable over time. Covariance stationarity is a requirement for using autoregressive (AR) models. Models that lack covariance stationarity are unstable and do not lend themselves to meaningful forecasting.

Autoregressive (AR) Process

The first-order autoregressive process [AR(1)] is specified as a variable regressed against itself in lagged form. Mean is 0 and variance is constant.

$$y_t = d + \Phi y_{t-1} + \varepsilon_t$$

where:

d = intercept term

y_t = the time series variable being estimated

y_{t-1} = one-period lagged observation of the variable being estimated

Φ = coefficient for the lagged observation of the variable being estimated

Trend Models

A *linear trend* is a time series pattern that can be graphed with a straight line:

$$y_t = \delta_0 + \delta_1 t + \varepsilon_t$$

A *nonlinear trend* is a time series pattern that can be graphed with a curve. It can be modeled using either quadratic or log-linear functions, respectively:

$$y_t = \delta_0 + \delta_1 t + \delta_2 t^2 + \varepsilon_t$$

$$\ln(y_t) = \delta_0 + \delta_1 t + \varepsilon_t$$

Seasonality

Seasonality in a time series is a pattern that tends to repeat from year to year.

There are two approaches for modeling and forecasting a time series impacted by seasonality: (1) regression analysis with seasonal dummy variables and (2) seasonal differencing.

Combining a trend with a pure seasonal dummy model produces the following model:

$$y_t = \beta_1(t) + \sum_{i=1}^{s} \gamma_i (D_{i,t}) + \varepsilon_t$$

Spearman's Rank Correlation

Step 1: Order the set pairs of variables *X* and *Y* with respect to set *X*.

Step 2: Determine the ranks of X_i and Y_i for each time period *i*.

Step 3: Calculate the difference of the variable rankings and square the difference.

$$\rho_S = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2 - 1)}$$

Where *n* is the number of observations for each variable and d_i is the difference between the ranking for period *i*.

Kendall's Tau (τ)

$$\tau = \frac{n_c - n_d}{n(n-1)/2}$$

Where the number of concordant pairs is represented as n_c (pair rankings in agreement), and the number of discordant pairs is represented as n_d (pair rankings not in agreement).

Simulation Methods

Monte Carlo simulations can model complex problems or estimate variables when there are small sample sizes. Basic steps are (1) specify data-generating process, (2) estimate unknown variable, (3) repeat steps 1 and 2 *N* times, (4) estimate quantity of interest, and (5) assess accuracy of standard error and increase *N* until required accuracy is achieved.

FINANCIAL MARKETS AND PRODUCTS

P&C Insurance Ratios

loss ratio = percentage of payouts versus premiums generated

expense ratio = percentage of expenses versus premiums generated

combined ratio = loss ratio + expense ratio

combined ratio after dividends = combined ratio + dividends

operating ratio = combined ratio after dividends – investment income

Net Asset Value (NAV)

Open-end mutual funds trade at the fund's NAV:

$$NAV = \frac{\text{Fund Assets} - \text{Fund Liabilities}}{\text{Total Shares Outstanding}}$$

Hedge Fund Strategies

Long/short equity: go long and short similar securities to exploit mispricings—decreases market risk and generates alpha.

Dedicated short: find company that is overvalued and then short sell the stock.

Distressed debt: purchase bonds of distressed company with the potential to turn things around.

Merger arbitrage: involves purchasing shares in a target firm and selling short shares in the purchasing firm.

Convertible arbitrage: investor purchases a convertible bond and sells short the underlying stock.

Fixed-income arbitrage: long/short strategy that looks for pricing inefficiencies between various fixed-income securities.

Emerging market: invests in developing countries' securities and/or sovereign debt.

Global macro: makes leveraged bets on anticipated price movements in broad equity and fixed-income markets, interest rates, foreign exchange, and commodities.

Managed futures: focuses on investments in commodity futures. Employs a high degree of leverage.

Option and Forward Contract Payoffs

The payoff on a call option to the option buyer is calculated as: $C_T = \max(0, S_T - X)$

The price paid for the call option, C_0, is referred to as the *call premium*. Thus, the profit to the option buyer is calculated as:

$$\text{profit} = C_T - C_0$$

The payoff on a put option is calculated as:

$$P_T = \max(0, X - S_T)$$

The payoff to a long position in a forward contract is calculated as:

$$\text{payoff} = S_T - K$$

where:

S_T = spot price at maturity

K = delivery price

Futures Market Participants

Hedgers: lock-in a fixed price in advance.

Speculators: accept the price risk that hedgers are unwilling to bear.

Arbitrageurs: interested in market inefficiencies to obtain riskless profit.

Central Counterparties (CCPs)

When trades are centrally cleared, a CCP becomes the seller to a buyer and the buyer to a seller.

Advantages of CCPs: loss mutualization, legal and operational efficiency, liquidity, standardized documentation, and increased transparency.

Disadvantages of CCPs: moral hazard, adverse selection, procyclicality, and credit risk.

Risks faced by CCPs: default risk, model risk, liquidity risk, operational risk, legal risk, and investment risk. Default of a clearing member and its flow through effects is the most significant risk for a CCP.

Basis
The basis in a hedge is defined as the difference between the spot price on a hedged asset and the futures price of the hedging instrument (e.g., futures contract). When the hedged asset and the asset underlying the hedging instrument are the same, the basis will be zero at maturity.

Optimal Hedge Ratio
The *hedge ratio* minimizes the variance of the combined hedge position. This is also the beta of spot prices with respect to futures contract prices.

$$HR = \rho_{S,F}\frac{\sigma_S}{\sigma_F}$$

Hedging With Stock Index Futures

$$\#\text{ of contracts} = \beta_P \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}\right)$$

Adjusting Portfolio Beta
If the beta of the capital asset pricing model is used as the systematic risk measure, then hedging boils down to a reduction of the portfolio beta.

$$\#\text{ of contracts} = (\text{target beta} - \text{portfolio beta})\frac{\text{portfolio value}}{\text{underlying asset}}$$

Foreign Exchange Markets
Purchasing power parity (PPP): changes in exchange rates should exactly offset the price effects of any inflation differential between the two countries.

$$\%\Delta S = \text{inflation(foreign)} - \text{inflation(domestic)}$$

Interest rate parity (IRP): in a competitive market, a firm should not be able to make excess profits from foreign investments.

$$\text{forward} = \text{spot} \times \left[\frac{(1 + r_{YYY})}{(1 + r_{XXX})}\right]^T$$

where:
r_{YYY} = quote currency rate
r_{XXX} = base currency rate

Forward Prices
Forward price when underlying asset does not have cash flows:

$$F = S(1 + r)^T$$

If $F > S(1 + r)^T$, arbitrageurs will profit by selling the forward and buying the asset with borrowed funds.
If $F < S(1 + r)^T$, arbitrageurs will profit by selling the asset, lending out the proceeds, and buying the forward.
Forward price when underlying asset has cash flows, *I*:

$$F = (S - I) \times (1 + r)^T$$

Forward price with continuous dividend yield, *q*:

$$F = S \times [(1 + r) / (1 + q)]^T$$

Forward price with storage costs, *U*:

$$F_{0,T} = (S_0 + U) \times (1 + r)^T$$

Forward price with lease rate, δ

$$F_{0,T} = S_0 \times [(1 + r) / (1 + \delta)]^T$$

Forward price with convenience yield, *Y*:

$$F_{0,T} \geq (S_0 + U) \times [(1 + r) / (1 + Y)]^T$$

Backwardation and Contango
- *Backwardation* refers to a situation where the futures price is below the spot price. For this to occur, there must be a significant benefit to holding the asset.
- *Contango* refers to a situation where the futures price is above the spot price. If there are no benefits to holding the asset (e.g., dividends, coupons, or convenience yield), contango will occur because the futures price will be greater than the spot price.

Option Pricing Bounds
Upper bound European/American call:

$$c \leq S_0;\ C \leq S_0$$

Upper bound European/American put:

$$p \leq PV(X);\ P \leq X$$

Lower bound European call on non-dividend-paying stock:

$$c \geq \max(S_0 - PV(X), 0)$$

Lower bound European put on non-dividend-paying stock:

$$p \geq \max(PV(X) - S_0, 0)$$

Put-Call Parity

$$p = c - S + PV(X)$$
$$c = p + S - PV(X)$$

Exercising American Options
- It is never optimal to exercise an American call on a non-dividend-paying stock before its expiration date.
- American puts can be optimally exercised early if they are sufficiently in-the-money.
- An American call on a dividend-paying stock may be exercised early if the dividend exceeds the amount of forgone interest.

Covered Call and Protective Put
Covered call: Long stock plus short call.
Protective put: Long stock plus long put. Also called portfolio insurance.

Option Spread Strategies
Bull spread: Purchase call option with low exercise price and subsidize the purchase with sale of a call option with a higher exercise price.
Bear spread: Purchase call with high strike price and short call with low strike price.
Investor keeps difference in price of the options if stock price falls. Bear spread with puts involves buying put with high exercise price and selling put with low exercise price.
Butterfly spread: Three different options: buy one call with low exercise price, buy another with a high exercise price, and buy short two calls with an exercise price in between. Butterfly spread buyer is betting the stock price will stay near the price of the written calls.
Calendar spread: Two options with different expirations. Sell a short-dated option and buy a long-dated option. Investor profits if stock price stays in a narrow range.
Diagonal spread: Similar to a calendar spread except the options can have different strike prices in addition to different expirations.
Box spread: Combination of bull call spread and bear put spread on the same asset. This strategy will produce a constant payoff that is equal to the high exercise price minus the low exercise price.

Option Combination Strategies
Long straddle: Bet on volatility. Buy a call and a put with the same exercise price and expiration date. Profit is earned if stock price has a large change in either direction.
Strangle: Similar to straddle except purchased option is out-of-the-money, so it is cheaper to implement. Stock price has to move more to be profitable.
Strips and straps: Add an additional put (strip) or call (strap) to a straddle strategy.

Exotic Options
Gap option: payoff is increased or decreased by the difference between two strike prices.
Forward start option: options that begin their existence at some time in the future.
Compound option: option on another option.
Chooser option: owner chooses whether option is a call or a put after initiation.
Barrier option: payoff and existence depend on price reaching a certain barrier level.
Binary option: pay either nothing or a fixed amount.
Lookback option: payoff depends on the maximum (call) or minimum (put) value of the underlying asset over the life of the option. This can be fixed or floating, depending on the specification of a strike price.
Asian option: payoff depends on average of the underlying asset price over the life of the option; less volatile than standard option.
Exchange option: exchange one currency with another.
Basket options: options to purchase or sell baskets of securities. These baskets may be defined specifically for the individual investor and may be composed of specific stocks, indices, or currencies.

Forward Interest Rates
Forward rates are interest rates implied by the spot curve for a specified future period. The forward rate between T_1 and T_2 can be calculated as:

$$R_{forward} = \frac{R_2T_2 - R_1T_1}{T_2 - T_1}$$
$$= R_2 + (R_2 - R_1) \times \left(\frac{T_1}{T_2 - T_1}\right)$$

Forward Rate Agreement (FRA)
An FRA is a forward contract obligating two parties to agree that a certain interest rate will apply to a principal amount during a specified future time. The T_2 cash flow of an FRA that promises the receipt or payment of R_K is:

$$\text{cash flow (if receiving } R_K) = L \times (R_K - R) \times (T_2 - T_1)$$

$$\text{cash flow (if paying } R_K) = L \times (R - R_K) \times (T_2 - T_1)$$

where:
L = principal
R_K = annualized rate on L
R = annualized actual rate
T_i = time *i* expressed in years

MBS Prepayments
Three general forms: (1) increasing frequency/amount of payments, (2) refinancing outstanding balance, (3) repaying outstanding balance when property is sold.
Prepayments are more likely when market interest rates fall and borrowers wish to refinance their existing mortgages at a new/lower rate.

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Conditional Prepayment Rate (CPR)
Annual rate at which a mortgage pool balance is assumed to be prepaid during the life of the pool. The *single monthly mortality* (SMM) rate is derived from CPR and used to estimate monthly prepayments for a mortgage pool:

$$\text{SMM} = 1 - (1 - \text{CPR})^{1/12}$$

Option-Adjusted Spread (OAS)
- Spread after the "optionality" of the cash flows is taken into account.
- Expresses the difference between price and theoretical value.
- When comparing two MBSs of similar credit quality, buy the bond with the higher OAS.

Treasury Bond Futures
In a T-bond futures contract, any government bond with more than 15 years to maturity on the first of the delivery month (and not callable within 15 years) is deliverable on the contract. The procedure to determine which bond is the *cheapest-to-deliver* (CTD) is as follows:

$$\text{cash received by the short} = (\text{QFP} \times \text{CF}) + \text{AI}$$

$$\text{cost to purchase bond} = \text{QBP} + \text{AI}$$

where:
QFP = quoted futures price
CF = conversion factor
QBP = quoted bond price
AI = accrued interest

The CTD is the bond that minimizes the following: QBP – (QFP × CF). This formula calculates the cost of delivering the bond.

Duration-Based Hedge Ratio
The objective of a *duration-based hedge* is to create a combined position that does not change in value when yields change by a small amount.

$$\#\text{ of contracts} = -\frac{\text{portfolio value} \times \text{duration}_P}{\text{futures value} \times \text{duration}_F}$$

Interest Rate Swaps
Plain vanilla interest rate swap: exchanges fixed for floating-rate payments over the life of the swap. At inception, the value of the swap is zero. After inception, the value of the swap is the difference between the present value of the remaining fixed- and floating-rate payments:

$$V_{\text{swap to pay fixed}} = B_{\text{float}} - B_{\text{fixed}}$$

$$V_{\text{swap to receive fixed}} = B_{\text{fixed}} - B_{\text{float}}$$

$$B_{\text{fixed}} = (\text{PMT}_{\text{fixed},t_1} / (1+r^{t_1}) + (\text{PMT}_{\text{fixed},t_2} / (1+r^{t_2}) + [(\text{notional} + \text{PMT}_{\text{fixed},t_n}) / (1+r^{t_n})$$

$$B_{\text{floating}} = [\text{notional} + (\text{notional} \times r_{\text{float}})] / (1+r^{t_1})$$

Currency Swaps
Exchanges payments in two different currencies (XXX and YYY); payments can be fixed or floating. If a swap has a
positive value to one counterparty, that party is exposed to credit risk.

$$V_{\text{swap}}(\text{YYY}) = B_{\text{YYY}} - (S_0 \times B_{\text{XXX}})$$

where:
S_0 = spot rate in YYY per XXX

VALUATION AND RISK MODELS

Value at Risk (VaR)
Minimum amount one could expect to lose with a given probability over a specific period of time.
Delta-normal VaR:

$$[\mu - (z)(\sigma)] \times \text{asset value}$$

Historical simulation VaR uses historical data to compute VaR. For example, to calculate the 5% daily VaR, you accumulate a number of past daily returns, rank the returns from highest to lowest, and then identify the lowest 5% of returns.

Expected Shortfall (ES)
Expected value of all losses greater than the VaR. Unlike VaR, ES has the ability to satisfy the coherent risk measure property of subadditivity.

$$\text{ES} = \mu + \sigma \frac{e^{-(z^2/2)}}{(1-x)\sqrt{2\pi}}$$

EWMA Model
The exponentially weighted moving average (EWMA) model assumes weights decline exponentially back through time. This assumption results in a specific relationship for variance in the following model:

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)r_{n-1}^2$$

where:
λ = weight on previous volatility estimate (λ is a positive constant between zero and one)

High values of λ will minimize the effect of daily percentage returns, whereas low values of λ will tend to increase the effect of daily percentage returns on the current volatility estimate.

GARCH Model
A GARCH(1,1) model incorporates the most recent estimates of variance and squared return and also includes a variable that accounts for a long-run average level of variance.

$$\sigma_n^2 = \omega + \alpha r_{n-1}^2 + \beta\sigma_{n-1}^2$$

where:
α = weighting on the previous period's return
β = weighting on the previous volatility estimate
ω = weighted long-run variance = γVL

V_L = long-run average variance = $\frac{\omega}{1-\alpha-\beta}$
$\alpha + \beta + \gamma = 1$
$\alpha + \beta < 1$ for stability so that γ is not negative

The EWMA is nothing other than a special case of a GARCH(1,1) volatility process, with $\omega = 0$, $\alpha = 1 - \lambda$, and $\beta = \lambda$.

Credit Ratings
At-the-point approach: goal is to predict the credit quality over a relatively short horizon of a few months or, more generally, a year.
Through-the-cycle approach: focuses on a longer time horizon and includes the effects of forecasted cycles.

Country Risk
Sources of country risk: (1) where the country is in the economic growth life cycle, (2) political risks, (3) the legal systems of a country, including both the structure and the efficiency of legal systems, and (4) economic structure, including the level of diversification.
Factors influencing sovereign default risk: (1) a country's level of indebtedness, (2) obligations such as pension and social service commitments, (3) a country's level of and stability of tax receipts, (4) political risks, and (5) backing from other countries or entities.

Expected Loss
The *expected loss* (EL) represents the decrease in value of an asset (portfolio) with a given exposure, subject to a positive probability of default.

EL = probability of default (PD) × loss given default (LGD)

Unexpected Loss
Unexpected loss represents the variability of potential losses and can be modeled using the definition of standard deviation.

Operational Risk
Operational risk is defined as "the risk of direct and indirect loss resulting from inadequate or failed internal processes, people, and systems or from external events."

Operational Risk Capital Requirements
- *Basic indicator approach*: capital charge measured on a firmwide basis as a percentage of annual gross income.
- *Standardized approach*: banks divide activities among business lines; capital charge equals sum for each business line. Capital for each business line determined with beta factors and annual gross income.
- *Advanced measurement approach*: banks use their own methodologies for assessing operational risk. Capital allocation is based on the bank's operational VaR.
- *Standardized measurement approach* (SMA): banks calculate a business indicator (BI), a BI component (% of BI), and a loss component (LC). The BI component is then multiplied by an internal loss multiplier (based on LC) to compute operational capital.

Loss Frequency and Loss Severity
Operational risk losses are classified along two independent dimensions:
Loss frequency: the number of losses over a specific time period (typically one year). Often modeled with the *Poisson distribution* (a distribution that models random events).
Loss severity: value of financial loss suffered. Often modeled with the *lognormal distribution* (distribution is asymmetrical and has fat tails).

Stress Testing
Stress testing shocks key input variables by large amounts to measure the impact on portfolio value. Key elements of effective governance and controls over stress testing include the governance structure, policies and procedures, documentation, validation and independent review, and internal audit.

Bond Valuation
There are three steps in the bond valuation process:
Step 1: Estimate the cash flows. For a bond, there are two types of cash flows: (1) the annual or semiannual coupon payments and (2) the recovery of principal at maturity or when the bond is retired.
Step 2: Determine the appropriate discount rate. The approximate discount rate can be either the bond's yield to maturity (YTM) or a series of spot rates.
Step 3: Calculate the PV of the estimated cash flows. The PV is determined by discounting the bond's cash flow stream by the appropriate discount rate(s).

Clean and Dirty Bond Prices

When a bond is purchased, the buyer must pay any accrued interest (AI) earned through the settlement date.

$$\text{AI} = \text{coupon} \times \left(\frac{\#\text{ of days from last coupon to the settlement date}}{\#\text{ of days in coupon period}} \right)$$

Clean price: bond price without AI.
Dirty price: includes AI; price the seller of the bond must be paid to give up ownership.

Compounding

Discrete compounding:

$$FV_n = PV_0\left(1 + \frac{r}{m}\right)^{m \times n}$$

where:
r = annual rate
m = compounding periods per year
n = years

Continuous compounding:

$$FV_n = PV_0 e^{r \times n}$$

Spot Rate

The rate earned on an investment when it is received at a single point in the future. The discount rate given the spot rate at time, *r(t)*, assuming semiannual compounding is:

$$d(t) = \left(1 + \frac{r(t)}{2}\right)^{-2t}$$

Forward Rates

Forward rates are interest rates that span future periods.

$$(1 + \text{forward rate})^t = \frac{(1 + \text{periodic yield})^{t+1}}{(1 + \text{periodic yield})^t}$$

Realized Return

The gross realized return for a bond is its end-of-period total value minus its beginning-of-period value divided by its beginning-of-period value.

$$R_{t-1,t} = \frac{BV_t + C_t - BV_{t-1}}{BV_{t-1}}$$

The net realized return for a bond is its gross realized return minus per-period financing costs.

Yield to Maturity (YTM)

The YTM of a fixed-income security is equivalent to its internal rate of return. The YTM is the discount rate that equates the present value of all cash flows associated with the instrument to its price. The yield to maturity assumes cash flows will be reinvested at the YTM and assumes that the bond will be held until maturity.

Relationship Between YTM and Coupon

The bond with the smaller coupon will be more sensitive to interest rate changes. With all else being equal,

- the lower the coupon rate, the greater the interest rate risk; and
- the higher the coupon rate, the lower the interest rate risk.

Dollar Value of a Basis Point

The DV01 is the change in a fixed-income security's value for every one basis point change in interest rates.

$$DV01 = -\frac{\Delta P}{\Delta y}$$

where:
ΔP = change in the value of the portfolio
Δy = size of a parallel shift in the interest rate term structure

Duration and Convexity

Duration: first derivative of the price-yield relationship; most widely used measure of bond price volatility; the longer (shorter) the duration, the more (less) sensitive the bond's price is to changes in interest rates; can be used for linear estimates of bond price changes.

$$D = -\frac{\Delta P / P}{\Delta y} = -\frac{\Delta P}{P \Delta y}$$

Convexity: measure of the degree of curvature (second derivative) of the price/yield relationship; accounts for error in price change estimates from duration. Positive convexity always has a favorable impact on bond price.

$$C = \frac{1}{P}\left[\frac{P^+ + P^- - 2P}{(\Delta y)^2}\right] = \left[\frac{P^+ + P^- - 2P}{P(\Delta y)^2}\right]$$

Bond Price Changes With Duration and Convexity

percentage bond price change ≈ duration effect + convexity effect

$$\Delta P = -D \times P \times \Delta y + \frac{1}{2} \times C \times P \times \Delta y^2$$

Bonds With Embedded Options

Callable bond: issuer has the right to buy back the bond in the future at a set price; as yields fall, bond is likely to be called; prices will rise at a *decreasing rate—negative convexity*.
Putable bond: bondholder has the right to sell bond back to the issuer at a set price.

Binomial Option Pricing Model

A *one-step binomial model* is best described within a two-state world where the price of a stock will either go up once or down once, and the change will occur one step ahead at the end of the holding period.
In the *two-period binomial model* and multiperiod models, the tree is expanded to provide for a greater number of potential outcomes.
Step 1: Calculate option payoffs at the end of all states.
Step 2: Calculate option values using risk-neutral probabilities.

$$\text{size of up move} = U = e^{\sigma\sqrt{T}}$$

$$\text{size of down move} = D = \frac{1}{U}$$

$$\pi_{up} = \frac{e^{rt} - D}{U - D};\ \pi_{down} = 1 - \pi_{up}$$

Step 3: Discount to today using risk-free rate.

π_{up} can be altered so that the binomial model can price options on stocks with dividends, stock indices, currencies, and futures.
Stocks with dividends and stock indices: replace e^{rT} with $e^{(r-q)T}$, where *q* is the dividend yield of a stock or stock index.
Currencies: replace e^{rT} with $e^{(r_{DC} - r_{FC})T}$, where r_{DC} is the domestic risk-free rate and r_{FC} is the foreign risk-free rate.
Futures: replace e^{rT} with 1 since futures are considered zero-growth instruments.

Black-Scholes-Merton Model

$$c = S_0 \times N(d_1) - Xe^{-rT}N(d_2)$$
$$p = Xe^{-rT}N(-d_2) - S_0N(-d_1)$$

where:

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left[r + 0.5 \times \sigma^2\right] \times T}{\sigma \times \sqrt{T}}$$

$d_2 = d_1 - \left(\sigma \times \sqrt{T}\right)$
T = time to maturity
S_0 = asset price
X = exercise price
r = risk-free rate
σ = stock return volatility
N(•) = cumulative normal probability

Greeks

Delta: estimates the change in value for an option for a one-unit change in stock price.

- Call delta between 0 and +1; increases as stock price increases.
- Call delta close to 0 for far out-of-the-money calls; close to 1 for deep in-the-money calls.
- Put delta between –1 and 0; increases from –1 to 0 as stock price increases.
- Put delta close to 0 for far out-of-the-money puts; close to –1 for deep in-the-money puts.
- The delta of a forward contract is equal to 1.
- The delta of a futures contract is equal to e^{rT}.
- When the underlying asset pays a dividend, *q*, the delta must be adjusted. If a dividend yield exists, delta of call equals $e^{-qT} \times N(d_1)$, delta of put equals $e^{-qT} \times [N(d_1) - 1]$, delta of forward equals e^{-qT}, and delta of futures equals $e^{(r-q)T}$.

Theta: time decay; change in value of an option for a one-unit change in time; more negative when option is at-the-money and close to expiration.
Gamma: rate of change in delta as underlying stock price changes; largest when option is at-the-money.
Vega: change in value of an option for a one-unit change in volatility; largest when option is at-the-money; close to 0 when option is deep in- or out-of-the-money.
Rho: sensitivity of option's price to changes in the risk-free rate; largest for in-the-money options.

Delta-Neutral Hedging

- To completely hedge a long stock/short call position, purchase shares of stock equal to delta × number of options sold.
- Only appropriate for small changes in the value of the underlying asset.
- Gamma can correct hedging error by protecting against large movements in asset price.
- Gamma-neutral positions are created by matching portfolio gamma with an offsetting option position.

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