Principles of Economics

Chapter 7: Economic Growth

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Agenda

7. Economic Growth
    - Steady State
    - Golden Rule
    - Technological Progress

Reading:

- Mankiw/Taylor (2020), Chapter 21
- Mankiw (2022), Chapters 8, 9

Model

Framework: Consider a closed economy in the long run, where all input and output prices are flexible.

- Output Y is determined by the production possibilities; the production function and the supply of inputs, i.e. the labor force L and the capital stock K.

$$Y = F(L, K)$$

- Output is used for consumption C and investment I. Investment equals savings sY, where $s \in [0, 1]$ denotes the saving rate.

$$Y = C + sY$$

- Savings are invested in the capital stock.

Production Function

Properties: The production function satisfies the following conditions.

- Constant returns to scale:

$$F(\lambda L, \lambda K) = \lambda F(L, K) \quad \forall \quad \lambda > 0,$$

- Positive but decreasing marginal products:

$$\frac{\partial F}{\partial L} > 0, \quad \frac{\partial^2 F}{\partial L^2} < 0$$
$$\frac{\partial F}{\partial K} > 0, \quad \frac{\partial^2 F}{\partial K^2} < 0$$

- Inada-Conditions:

$$\lim_{L \to 0} \left(\frac{\partial F}{\partial L} \right) = \lim_{K \to 0} \left(\frac{\partial F}{\partial K} \right) = \infty$$
$$\lim_{L \to \infty} \left(\frac{\partial F}{\partial L} \right) = \lim_{K \to \infty} \left(\frac{\partial F}{\partial K} \right) = 0$$

Production Function

Intensive Form: Representation in per-worker terms

- Let lower-case letters denote quantities per worker.
- Constant returns to scale imply:

$$y = f(k)$$

- Positive but decreasing marginal product:

$$f'(k) > 0, \quad f''(k) < 0$$

- Inada-Conditions:

$$\lim_{k \to 0} f'(k) = \infty, \quad \lim_{k \to \infty} f'(k) = 0$$

Supply of Inputs

Population Growth: In any period t, the labor force grows at a constant rate n.

$$L_{t+1} = (1+n)L_t$$

Capital Accumulation: In any period t, a fraction s of output is invested in capital, while capital depreciates at the rate $\delta \in [0,1]$.

$$K_{t+1} = K_t + sY_t - \delta K_t$$

Ceteris paribus, capital per worker increases with savings and decreases with depreciation and population growth.

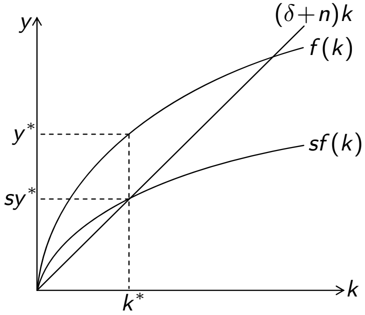
$$(1+n)k_{t+1} = k_t + sf(k_t) - \delta k_t$$

Steady State

Balanced Growth: The economy is in a steady state if the capital stock per worker is constant over time; $k^* = k_t = k_{t+1}$.

- Break-Even Investment: In a steady state, capital investment per worker exactly offsets the decrease in capital per worker due to depreciation and population growth.

$$sf(k^*) = (\delta + n)k^*$$



Steady State

Convergence: If capital per worker in period t

- is below the steady-state level, $k_t < k^*$, then savings exceed break-even investment, $sf(k_t) > (\delta + n)k_t$, and capital per worker increases.
- is above the steady-state level, $k_t > k^*$, then savings fall short of the break-even investment, $sf(k_t) < (\delta + n)k_t$, and capital per worker decreases.

Steady State

Comparative Statics: Ceteris paribus, an increase in

- the saving rate s implies an increase in steady-state capital per worker.
- the depreciation rate δ implies a decrease in steady-state capital per worker.
- the rate of population growth n implies a decrease in steady-state capital per worker.

Optimal Capital Accumulation

Optimization Problem: Maximize consumption per worker with respect to capital per worker in the steady state.

- In any period t, consumption per worker is

$$c_t = (1 - s)y_t = f(k_t) - sf(k_t).$$

- Substitution yields steady-state consumption per worker, i.e. the objective function.

$$\max_{k^*} \quad c^* = f(k^*) - (\delta + n)k^*$$

- Hence, the capital stock per worker which maximizes steady-state consumption per worker must satisfy

$$f'(k^*_{gold}) = \delta + n.$$

Optimal Capital Accumulation

Dynamic Inefficiency: No trade-off between present and future consumption

- If the saving rate exceeds its golden-rule level, $s > s_{gold}$, then a decrease in the saving rate implies an immediate increase in consumption as well as an increase in steady-state consumption per worker.

Dynamic Efficiency: Trade-off between present and future consumption

- If the saving rate falls short of its golden-rule level, $s < s_{gold}$, then an increase in the saving rate implies an immediate decrease in consumption but an increase in steady-state consumption per worker.

Model Extension

Production Function: Output is a function of the effective labor force $A \cdot L$ and the capital stock K.

- The effective labor force takes into account the number of workers L and workers' productivity A.

$$Y = F(A \cdot L, K)$$

Labor Productivity Growth: In any period t, workers' productivity grows at the rate g.

- In the steady state, the capital stock per worker also grows at the rate g.