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Tutorial 8 (Experimental Bell inequality)

As mentioned in the lecture, the violation of Bell inequality has been experimentally verified. Many of these experiments make use of photons as the qubits.[1] Fig. 1 shows a schematic version of the most common setup used to test the Bell inequality with photons. The source generates a pair of entangled photons, one of which is sent to Alice, one of which is sent to Bob. The coincidence monitor counts the instances with simultaneous measurements on Alice's and Bob's side in order to discard detections of environment photons not created by the source.

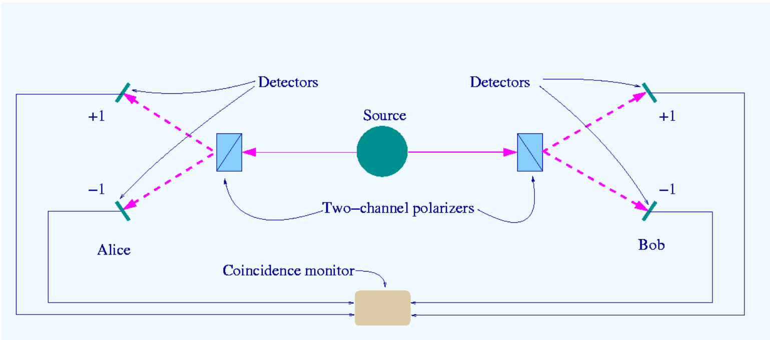

Figure 1: Photon analyzer (Source: https://commons.wikimedia.org/w/index.php?curid=641329)

The actual setup, of course, contains many more elements, like in the experiment of M. Giustina et al.:

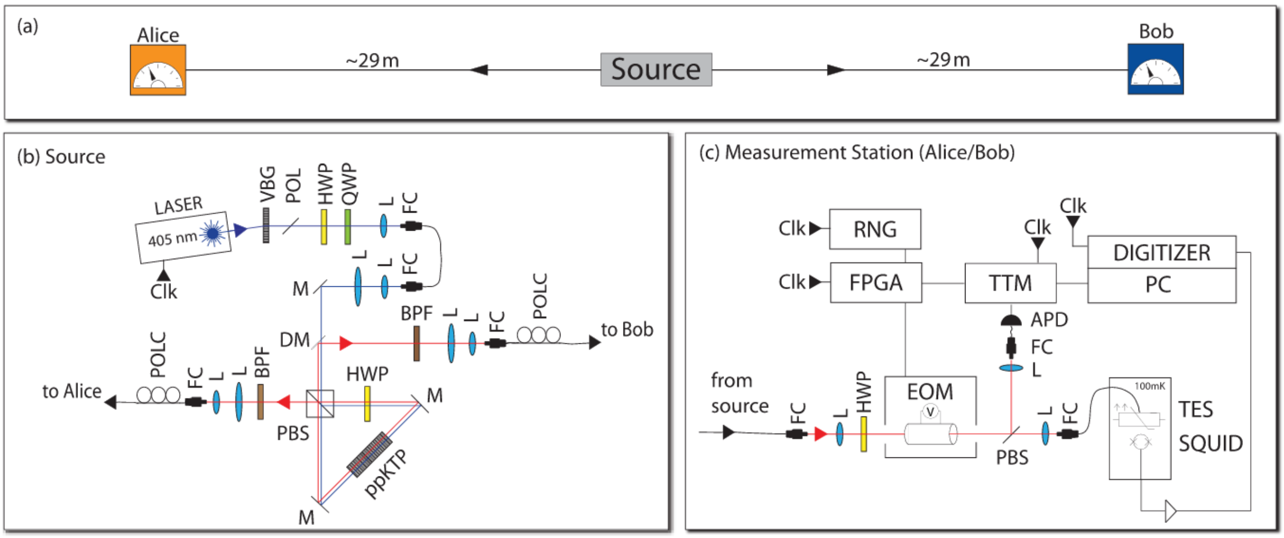

Figure 2: (a) Schematic setup. (b) Source of the entangled photons. (c) Detection station of one of the receivers.

(a) Discuss the role of the polarizers in these experiments. For simplicity, you can focus on the schematic representation from Fig. 1.

If we hypothesize that the experiment can be described by hidden variables, denoted λ, then the probability that Alice measures a and Bob measures b is given by:

$$P(a,b) = \sum_{x,y,\lambda} P(a|x,\lambda)P(b|y,\lambda)P(x,y)P(\lambda),$$

where x is the choice of the measurement basis used by Alice, and y is the one used by Bob. This expression is not able to reproduce all results from experiments according to Bell's inequality, so the consensus is that hidden variable theories are incorrect. However, there are *two* additional implicit assumptions leading to this expression:

[1] see, .e.g.,
M. Giustina et al.: *Significant-loophole-free test of Bell's theorem with entangled photons*. Phys. Rev. Lett. 115, 250401 (2015)
The BIG Bell Test Collaboration: *Challenging local realism with human choices*. Nature 557, 212-216 (2018)
D. Rauch et al.: *Cosmic Bell test using random measurement settings from high-redshift quasars*. Phys. Rev. Lett. 121, 080403 (2018)

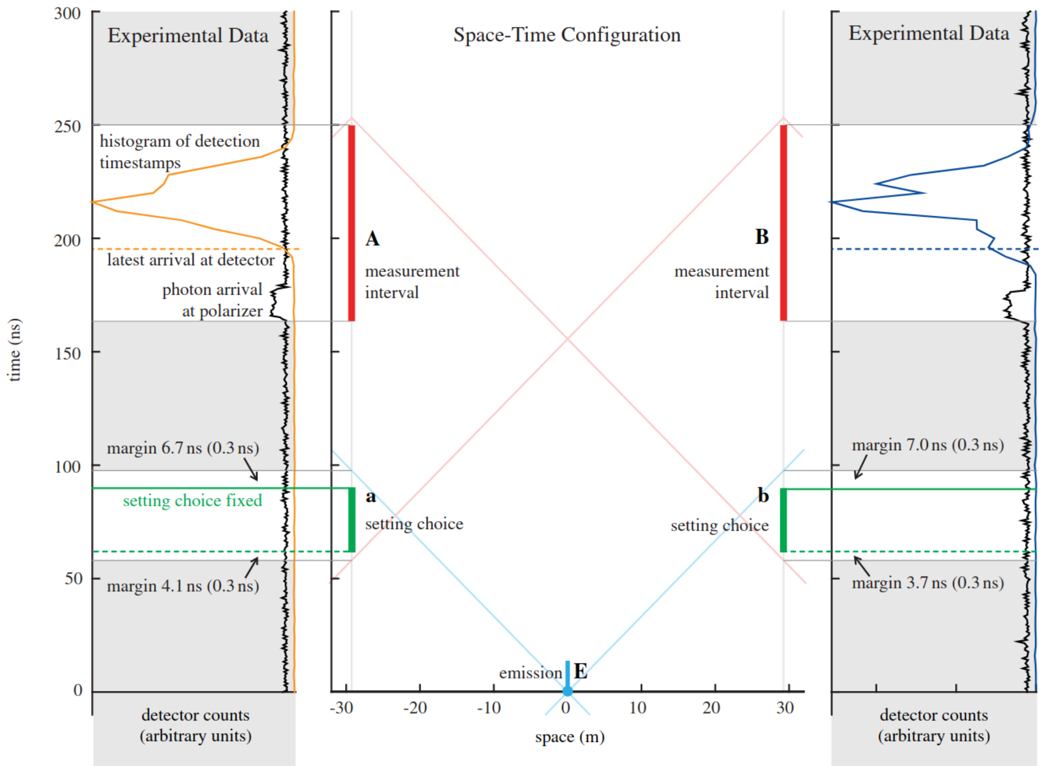

Figure 3: Space and time configuration of the experiment by M. Giustina et al. [Phys. Rev. Lett. 115, 250401 (2015)]

(b) One of these assumptions is locality. In a lab experiment the photon detection is not instantaneous. Why could this create a loophole, and what can be done to close this loophole? As an example, Fig. 3 shows the space-time configuration and specific times each experimental step takes.

(c) Identify the remaining assumption and discuss how one can make sure that it holds during the experiment.

Solution

(a) The role of the polarizers is to set the measurement basis. The two-channel polarizers only allow photons with a specific polarization to travel through them, and the orthogonal polarization is reflected. Consider a photon in state

$$|\psi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\rightarrow\rangle\,,$$

where the arrows represent the polarization direction. If $|\psi\rangle$ encounters a vertical polarizer, the photon will be transmitted with probability $|\alpha|^2$ and reflected with probability $|\beta|^2$. A similar thing will happen, if

$$|\psi\rangle = \alpha\,|\nearrow\rangle + \beta\,|\searrow\rangle$$

encounters a diagonal polarizer. Furthermore, if $|\psi\rangle = |\uparrow\rangle$ encounters a diagonal polarizer, it will be transmitted or reflected with equal probability. At this point you can probably notice the equivalence with the qubit basis we usually use in the course. For example, we could represent the vertical and horizontal polarization as the $\{|0\rangle\,, |1\rangle\}$ basis, in which case the diagonal polarization would correspond to the $\{|+\rangle\,, |-\rangle\}$ basis.

(b) The locality assumption corresponds to the fact that the probability of measuring a is independent from b and y, and the probability of b is independent from a and x. However, if Alice's and Bob's measurements are not instantaneous, they could hypothetically influence each other. To ensure that this is not the case, Alice's and Bob's detectors are separated in the experiment by a distance large enough such that the time needed by light to travel from Alice to Bob is longer than the time to complete the detection.

In Fig. 3 we can see that ~ 200ns elapse between the choice of the measurement basis until the measurement is completed. 200 light nanoseconds correspond to ~ 59m, which is approximately the physical distance between Alice and Bob in the experiment.

(c) The other assumption is that the choice for measurement basis is independent of λ, i.e. the choice of measurements is not dependent on the hidden variables. The most common way to ensure that this assumption holds is to use random number generators to select the basis. However, random number generators are also affected by the underlying physics of the system that generates them, which is why strictly speaking one cannot rule out their dependence on λ. In the publication *Challenging local realism with human choices* [Nature 557, 212-216 (2018)], human choices (with no connection to the experiment) were used instead to select the measurement basis.

Superdeterminism is a (rather exotic) theory stating that all events in the universe are affected by the hidden variables λ. In this setting, even the choices from the participants in the above experiment would be influenced by λ. The study *Cosmic Bell test using random measurement settings from high-redshift quasars* [Phys. Rev. Lett. 121, 080403 (2018)] used the light from two quasars ~ 7.8 billion light years away to determine the basis for their measurements. They still found a Bell inequality violation. Therefore, if superdeterminism was correct, λ would have had to be fixed more that 7.8 billion years ago.