POE Exercises Solutions Merged

Principles of Economics (Technische Universität München)

Solution 1: Specialization and Trade

Problem 1 (*Gains from Trade*)

(a) Jack can produce more fish and more rum per hour than Will. Thus, Jack can produce each of the two goods using less resources (hours) than Will.

kg per hour

	Fish	Rum
Jack	1	1/4
Will	1/5	1/5

Absolute Advantages

hours per kg

	Fish	Rum
Jack	1	4
Will	5	5

Absolute Advantages

$\Rightarrow$ Jack has absolute advantages in the production of both goods.

Jack can produce fish at lower opportunity costs than Will. Consequently, Will can produce Rum at lower opportunity costs than Jack.

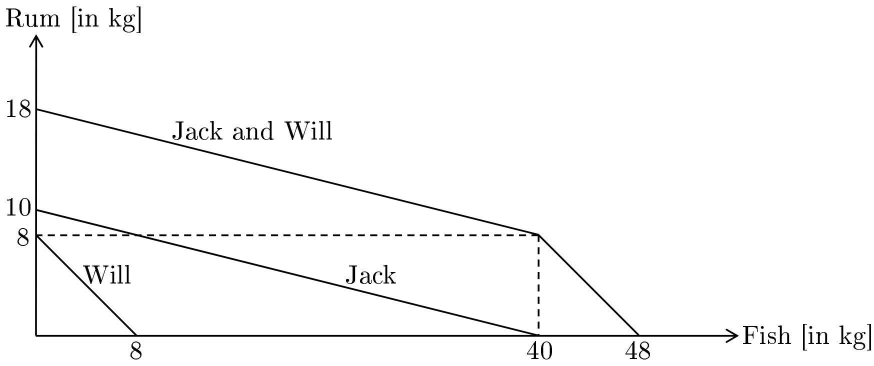
Opportunity Costs per kg

	Fish [in kg Rum]	Rum [in kg Fish]
Jack	1/4	4
Will	1	1

Comparative Advantages

$\Rightarrow$ Jack has a comparative advantage in the production of fish, while Will has a comparative advantage in the production of rum.

(b) Transformation Curves:



Transformation Curves

The axis intercepts and the corner point of the joint transformation curve can be determined as follows:

- ⇒ If both, Jack and Will, produce only fish, then the overall production is 48 kg fish and 0 kg rum.
- ⇒ If both, Jack and Will, produce only rum, then the overall production is 0 kg fish and 18 kg rum.
- ⇒ If Jack produces only fish and Will produces only rum, i.e. if both specialize completely according to their respective comparative advantage, the overall production is 40 kg fish and 8 kg rum.

(c) Under autarky, individual consumption equals individual production. In this case, each pirate produces 5 kg rum per week and spends the remaining time producing fish. This results in 20 kg fish produced by Jack and 3 kg fish produced by Will.
Under specialization and trade, individual consumption can differ from individual production. Will has a comparative advantage in the production of rum, of which he can maximally produce 8 kg. Since 10 kg rum are required for both pirates together, Will should produce only rum. The remaining 2 kg rum must be produced by Jack, even though Jack's comparative advantage lies in the production of fish. In the remaining time, Jack can produce 32 kg fish. If 1 kg rum is traded for 3 kg fish, Jack can consume 23 kg fish, while Will can consume 9 kg fish in addition to the 5 kg rum that each of them consumes.

Production & Consumption

	Fish	Rum
Jack	20	5
Will	3	5

Autarky

Production (Consumption)

	Fish	Rum
Jack	32 (23)	2 (5)
Will	0 (9)	8 (5)

Specialization & Trade

$\Rightarrow$ Specialization and trade is beneficial for both pirates.

(d) Liz joins Jack and Will.

kg per hour

	Fish	Rum
Jack	1	1/4
Will	1/5	1/5
Liz	1/5	1/2

Absolute Advantages

hours per kg

	Fish	Rum
Jack	1	4
Will	5	5
Liz	5	2

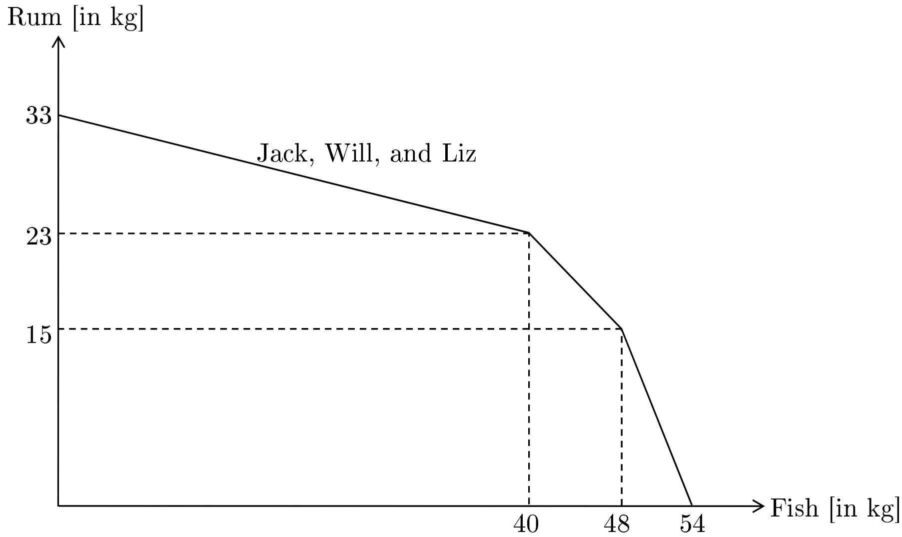
Absolute Advantages

Opportunity Costs per kg

	Fish [in kg Rum]	Rum [in kg Fish]
Jack	1/4	4
Will	1	1
Liz	5/2	2/5

Comparative Advantages

Joint transformation curve



Transformation Curve

The axis intercepts and corner points can be determined as follows:

- ⇒ If Jack, Will, and Liz produce only fish, then the overall production is 54 kg fish and 0 kg rum.
- ⇒ If Jack, Will, and Liz produce only rum, then the overall production is 0 kg fish and 33 kg rum.
- ⇒ If Jack produces only fish and Liz produces only rum, i.e. both completely specialize according to their comparative advantages, and if Will produces only rum, then the overall production is 40 kg fish and 23 kg rum.
- ⇒ If Jack produces only fish and Liz produces only rum, i.e. both completely specialize according to their comparative advantages, and if Will produces only fish, then the overall production is 48 kg fish and 15 kg rum.

(e) Under autarky, individual consumption equals individual production. In this case, each pirate produces 5 kg rum per week and spends the remaining time producing fish. This results in 20 kg fish produced by Jack, 3 kg fish produced by Will, and 4 kg fish produced by Liz.
Under specialization and trade, individual consumption can differ from individual production. Liz has a comparative advantage over Jack and Will in the production of rum, of which she can maximally produce 15 kg. Since 15 kg rum are required for all three pirates together, Liz should produce only rum. Then Jack and Will, who both have a comparative advantage over Liz in the production of fish, should produce only fish. This results in the production of 48 kg fish. If 1 kg rum is traded for $\frac{4}{5}$ kg fish, Jack can consume 36 kg fish, Will can consume 4 kg fish, and Liz can consume 8 kg fish in addition to the 5 kg rum that each of them consumes.

Production & Consumption

	Fish	Rum
Jack	20	5
Will	3	5
Liz	4	5

Autarky

Production (Consumption)

	Fish	Rum
Jack	40 (36)	0 (5)
Will	8 (4)	0 (5)
Liz	0 (8)	15 (5)

Specialization & Trade

$\Rightarrow$ Specialization and trade is beneficial for all three pirates.

Problems 2-6 (*Gains from Trade*)

Days per unit

	Car Bodies	Engines
Carl	16	4
Gottlieb	10	5

Absolute Advantages

Opportunity Costs per unit

	Car Bodies [in units of Engines]	Engines [in units of Car Bodies]
Carl	4	1/4
Gottlieb	2	1/2

Comparative Advantages

Problem 2

Gottlieb has both, an absolute and a comparative advantage in the production of car bodies.

$\Rightarrow$ **(B)** is correct.

Problem 3

To manufacture one vehicle, Carl needs $16 + 4 = 20$ days, while Gottlieb needs $10 + 5 = 15$ days. Under autarky, Carl can maximally manufacture $300 \div 20 = 15$ vehicles per year, while Gottlieb can maximally manufacture $300 \div 15 = 20$ vehicles per year.

$\Rightarrow$ **(C)** is correct.

Problem 4

Gottlieb should completely specialize according to his comparative advantage, and thus manufacture $300 \div 10 = 30$ car bodies. If Carl also completely specialized according to his comparative advantage, he would manufacture $300 \div 4 = 75$ engines. In this case, only 30 vehicles would be produced, with 45 engines remaining unused. Consequently, Carl should produce both, car bodies and engines. To begin with, he should produce 30 engines for Gottlieb's 30 car bodies to make 30 vehicles, for which he needs $30 \cdot 4 = 120$ days. In the remaining 180 days, he should produce engines and car bodies, resulting in $180 \div 20 = 9$ complete vehicles. Thus, if Carl and Gottlieb cooperate, they can maximally produce $30 + 9 = 39$ vehicles per year.

$\Rightarrow$ **(B)** is correct.

Problem 5

Carl and Gottlieb can realize mutual gains from trade if they agree on terms of trade lying in between their respective opportunity costs, i.e. in between $\frac{1}{4}$ and $\frac{1}{2}$ car bodies per engine, or equivalently, in between 2 and 4 engines per car body.

$\Rightarrow$ **(A)** is correct.

Problem 6

If they cooperate, Carl and Gottlieb can maximally produce 39 vehicles per year (see also problem 4), implying that the combination 39 car bodies and 39 engines is located on the two engineers' joint transformation curve. Consequently, the combination 39 car bodies and 50 engines is *not* located on their joint transformation curve but beyond it.

$\Rightarrow$ **(D)** is correct.

Solution 2: Consumption and Demand

Problem 1 (*Budget Restriction*)

(a) The individual's budget restriction with respect to time is $Z = L + F$, while his budget restriction with respect to earned income is $wL = pq$. Solving the former for L and substituting it into the latter yields $w(Z - F) = pq$. His budget restriction with respect to his potential income is thus

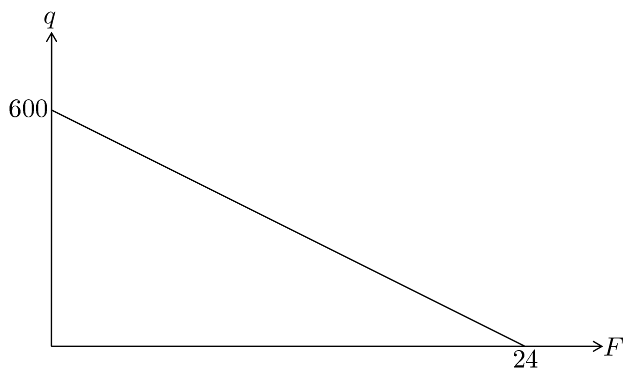
$$wZ = pq + wF.$$

Solving for q yields the budget line

$$q = \frac{wZ}{p} - \frac{w}{p}F.$$

Substituting $Z = 24$, $w = 25$, and $p = 1$ gives

$$q = 600 - 25F.$$



Budget line

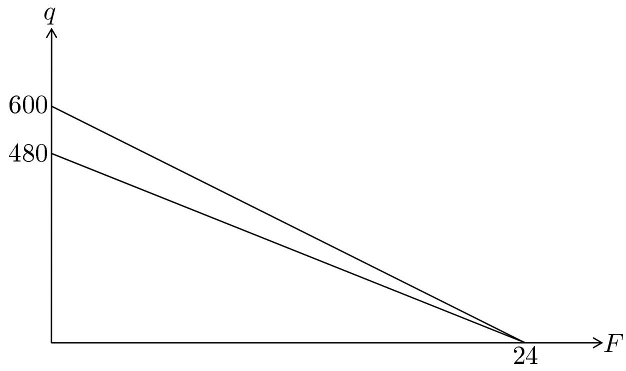
(b) The individual's budget line

(i) if an income tax reduces the wage rate to $(1-t)w = 20$ is

$$q = 480 - 20F,$$

(ii) if a consumption tax raises the price of the consumption good to $(1+\tau)p = 1.25$ is

$$q = 480 - 20F,$$



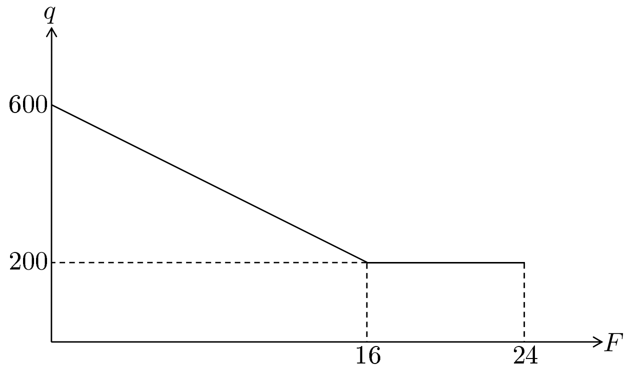
Budget line before and after taxes

(iii) if a social transfer

$$S = \begin{cases} 0, & wL \geq 200 \\ 200 - wL, & wL < 200 \end{cases}$$

subsidizes earned incomes is

$$q = \begin{cases} 600 - 25F, & F \leq 16 \\ 200, & F > 16. \end{cases}$$



Budget line with social transfer

Problem 2 (*Assumptions on Preferences*)

The individual's preferences regarding the consumption bundles $A = (8, 2)$, $B = (2, 8)$, and $C = (6, 6)$ involve the following relations: $A \sim B$ and $B \succ C$.[1]

- If her preferences are **complete**, she can compare the consumption bundles A and C such that $A \sim C$ or $A \succ C$ or $C \prec A$.

- If her preferences are **complete** and also **transitive**, $A \sim B$ together with $B \succ C$ implies $A \succ C$.

- If her preferences are **complete**, **transitive**, and also **convex**, they cannot be **monotonous**. To see this, consider a consumption bundle D that contains just as many apples as oranges and is located on a convex indifference curve connecting A and B. Such a consumption bundle can at most contain 5 apples and 5 oranges.[2] Given that $D \sim B$ and $B \succ C$ (and likewise given that $D \sim A$ and $A \succ C$), transitivity implies that $D \succ C$. This however would mean that „less is better".

- If her preferences are **complete**, **transitive**, and also **monotonous**, they cannot be **convex**. To see this, consider a consumption bundle E that contains just as many apples as oranges and is located on an indifference curve connecting A and B. Given that $E \sim B$ and $B \succ C$ (and likewise given that $E \sim A$ and $A \succ C$), transitivity implies that $E \succ C$. Monotonicity then requires that E must contain more of both goods than C. Hence, the indifference curve connecting A, E, and B must run to the northeast of C and must therefore be strictly concave.

Thus, the four assumptions cannot hold together in this case.

[1] The symbol $\sim$ represents the indifference relation, i.e. the individual is indifferent between A and B. The symbol $\succ$ represents the strong preference relation, i.e. the individual strictly prefers B to C.

[2] In case of linear indifference curves (i.e. indifference curves that are convex, but not strictly so), D is a convex combination of A and B and contains exactly 5 apples and 5 oranges. In case of strictly convex indifference curves, D must contain less than 5 apples and less than 5 oranges.

Problem 3 (*Individual Demand*)

(a) Individual demand for a particular good follows from utility maximization.

(i) The optimal consumption bundle must be located on the budget line, i.e. the budget restriction must bind.

$$y = p_1 q_1 + p_2 q_2$$

(ii) For any interior solution, the indifference curve through the optimal consumption bundle must be tangent to the budget line, i.e. the MRS must equal the price ratio.

$$\underbrace{\frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}}}_{\text{MRS}} = \frac{p_1}{p_2} \;\Rightarrow\; \frac{\frac{1}{2}q_1^{-\frac{1}{2}}}{\frac{1}{2}q_2^{-\frac{1}{2}}} = \frac{p_1}{p_2} \;\Leftrightarrow\; q_2 = \left(\frac{p_1}{p_2}\right)^2 q_1 \;\Leftrightarrow\; q_1 = \left(\frac{p_2}{p_1}\right)^2 q_2$$

Substituting (ii) into (i) yields

$$q_1 = \frac{yp_2}{p_1p_2 + p_1^2}, \quad q_2 = \frac{yp_1}{p_1p_2 + p_2^2}.$$

Thus, the individual demand for good $i \in \{1,2\}$ is

$$q_i(p_i, p_j, y) = \frac{yp_j}{p_ip_j + p_i^2},$$

where $j \in \{1,2\}$ and $j \neq i$.

(b) The demand for good $i \in \{1,2\}$ increases as income increases.

$$\frac{\partial q_i(p_i, p_j, y)}{\partial y} = \frac{p_j}{p_ip_j + p_i^2} > 0$$

$\Rightarrow$ Both goods are normal goods.

The demand for good $i \in \{1,2\}$ decreases if the respective price p_i increases.

$$\frac{\partial q_i(p_i, p_j, y)}{\partial p_i} = -\frac{yp_j(p_j + 2p_i)}{(p_ip_j + p_i^2)^2} < 0$$

$\Rightarrow$ Both goods are ordinary goods.

The demand for good $i \in \{1,2\}$ increases if the price p_j of the other good increases.

$$\frac{\partial q_i(p_i, p_j, y)}{\partial p_j} = \frac{yp_i^2}{(p_ip_j + p_i^2)^2} > 0$$

$\Rightarrow$ The goods are substitutes.

Problem 4 (*Substitution and Income Effects*)

(a) Optimal consumption

(i) The optimal consumption bundle must be located on the budget line, i.e. the budget restriction must bind.

$$y = p_1 q_1 + p_2 q_2$$

(ii) For any interior solution, the indifference curve through the optimal consumption bundle must be tangent to the budget line, i.e. the MRS must equal the price ratio.

$$\underbrace{\frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}}}_{\text{MRS}} = \frac{p_1}{p_2} \quad \Rightarrow \quad \frac{\frac{1}{2} q_1^{-\frac{1}{2}} q_2^{\frac{1}{2}}}{\frac{1}{2} q_1^{\frac{1}{2}} q_2^{-\frac{1}{2}}} = \frac{p_1}{p_2} \quad \Leftrightarrow \quad q_2 = \frac{p_1}{p_2} q_1 \quad \Leftrightarrow \quad q_1 = \frac{p_2}{p_1} q_2$$

Substituting (ii) into (i) yields

$$q_1 = \frac{y}{2p_1}, \quad q_2 = \frac{y}{2p_2}.$$

If $y = 600$, $p_1 = 25$, and $p_2 = 25$, the optimal consumption bundle C is

$$q_1 = 12, \quad q_2 = 12.$$

(b) If $y = 600$, $p_1 = 25$, and $p_2' = 100$, the optimal consumption bundle C' is

$$q_1' = 12, \quad q_2' = 3.$$

The total effect of the price change $C \to C'$ can be decomposed into the substitution effect $C \to \tilde{C}$ and the income effect $\tilde{C} \to C'$, where $\tilde{C}$ is a hypothetical consumption bundle. Given the new price ratio $\frac{p_1}{p_2'} = \frac{1}{4}$, the individual would choose $\tilde{C}$ if her income was compensated to the extent that she could obtain the initial level of utility $(12 \cdot 12)^{\frac{1}{2}} = 12$.

(i) $\tilde{C}$ must be located on the initial indifference curve through C.

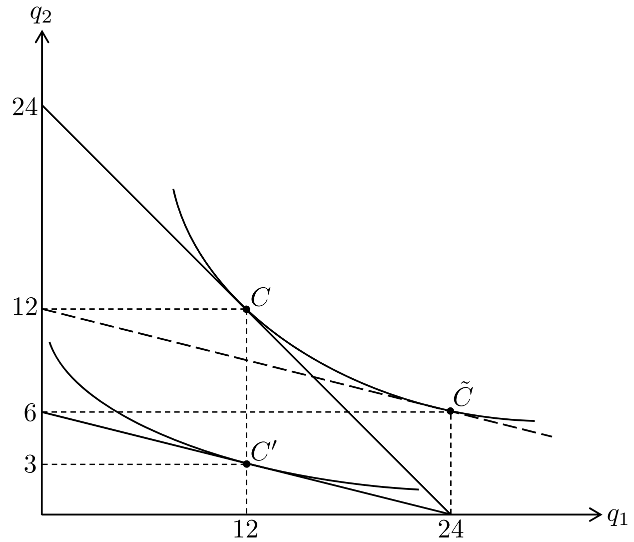
$$(q_1 \cdot q_2)^{\frac{1}{2}} = 12$$

(ii) $\tilde{C}$ must be located where the initial indifference curve is tangent to the hypothetical budget line parallel to the new budget line through C'.

$$q_2 = \frac{1}{4} q_1 \quad \Leftrightarrow \quad q_1 = 4 q_2$$

Substituting (ii) into (i) yields

$$\tilde{q}_1 = 24, \quad \tilde{q}_2 = 6.$$



Substitution- and Income Effect

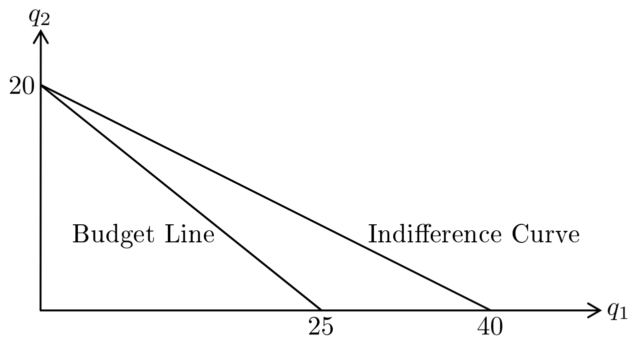
(c) The hypothetical income necessary after the price increase so that the individual could afford the hypothetical consumption bundle $\tilde{C}$ and therefore obtain the initial level of utility is

$$\tilde{y} = p_1\tilde{q}_1 + p_2'\tilde{q}_2 = 25 \cdot 24 + 100 \cdot 6 = 1{,}200.$$

Problem 5 (*Optimal Consumption*)

The marginal rate of substitution is always lower than the price ratio, i.e. the slope of the indifference curve is less steep than the slope of the budget line.

$$\underbrace{\frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}}}_{\text{MRS}} = \frac{1}{2} < \frac{4}{5} = \frac{p_1}{p_2}$$



Corner Solution

The individual's optimal consumption bundle is $q_1 = 0$ and $q_2 = 20$. Thus, the individual spends her entire budget on good 2.

$\Rightarrow$ **(B)** is correct.

Problem 6-10 (*Optimal Consumption*)

Optimal consumption

(i) The optimal consumption bundle must be located on the budget line, i.e. the budget restriction must bind.

$$y = p_1 q_1 + p_2 q_2$$

(ii) For any interior solution, the indifference curve through the optimal consumption bundle must be tangent to the budget line, i.e. the MRS must equal the price ratio.

$$\underbrace{\frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}}}_{\text{MRS}} = \frac{p_1}{p_2} \quad \Rightarrow \quad \frac{\frac{1}{4} q_1^{-\frac{3}{4}} q_2^{\frac{3}{4}}}{\frac{3}{4} q_1^{\frac{1}{4}} q_2^{-\frac{1}{4}}} = \frac{p_1}{p_2} \quad \Leftrightarrow \quad q_2 = \frac{p_1}{p_2} 3 q_1 \quad \Leftrightarrow \quad q_1 = \frac{p_2}{p_1} \frac{q_2}{3}$$

Substituting (ii) into (i) yields

$$q_1 = \frac{y}{4p_1}, \quad q_2 = \frac{3y}{4p_2}.$$

Substituting $y = 12$ and $p_1 = 1$ yields

$$q_1 = 3, \quad q_2 = \frac{9}{p_2}.$$

Problem 6

If $p_2 = 1$, the individual's optimal consumption bundle is $q_1 = 3$ and $q_2 = 9$.

$\Rightarrow$ **(D)** is correct.

Problem 7

If $p_2 = 3$, the individual's optimal consumption bundle is $q_1 = 3$ and $q_2 = 3$. Thus, the optimal consumption bundle causes expenses of 9 for good 2.

$\Rightarrow$ **(D)** is correct.

Problem 8

Regarding good 1, the total effect of a price increase of good 2 from $p_2 = 1$ to $p_2 = 3$ is zero (see also problems 6 and 7). The substitution effect induces the individual to consume more of good 1 as it becomes relatively less expensive. Thus, the income effect must work in the opposite direction, effectively neutralizing the substitution effect.

$\Rightarrow$ **(C)** is correct.

Problem 9

The income effect of the price increase of good 2 from $p_2 = 1$ to $p_2 = 3$ induces the individual to consume less of good 1 (see also problem 8). Thus, good 1 is a normal good. Regarding good 2, the total effect of the price increase of good 2 from $p_2 = 1$ to $p_2 = 3$ is negative (see also problems 6 and 7). Thus, good 2 is an ordinary good.

$\Rightarrow$ **(A)** is correct.

Problem 10

If $p_2 = 3$, the indifference curve through the consumption bundle $q_1 = 3$ and $q_2 = 3$ is tangent to the budget line as this is the optimal consumption bundle (see also problem 7).

$\Rightarrow$ **(B)** is correct.

Solution 3: Production and Supply

Problem 1 (*Production Function*)

(a) Positive and decreasing marginal products in both inputs

(i) Marginal Product of Labor:

$$\frac{\partial F(L,K)}{\partial L} = aL^{a-1}K^b > 0, \quad \frac{\partial^2 F(L,K)}{\partial L^2} = (a-1)aL^{a-2}K^b < 0$$

(ii) Marginal Product of Capital:

$$\frac{\partial F(L,K)}{\partial K} = bL^aK^{b-1} > 0, \quad \frac{\partial^2 F(L,K)}{\partial K^2} = (b-1)bL^aK^{b-2} < 0$$

(b) Returns to Scale

Provided that $L > 0$ and $K > 0$, a multiplication of both inputs by λ implies a multiplication of output by λ^{a+b}.

$$F(\lambda L, \lambda K) = (\lambda L)^a \cdot (\lambda K)^b = \lambda^{a+b}L^aK^b = \lambda^{a+b}q$$

Thus, the production function exhibits

(i) increasing returns to scale if $a + b > 1$,

(ii) constant returns to scale if $a + b = 1$,

(iii) decreasing returns to scale if $a + b < 1$.

(c) Strictly Convex Isoquants

The MRTS indicates the slope of an isoquant (in absolute value) for any input bundle.

$$\text{MRTS} = \frac{\frac{\partial F(L,K)}{\partial L}}{\frac{\partial F(L,K)}{\partial K}} = \frac{aL^{a-1}K^b}{bL^aK^{b-1}} = \frac{a}{b}\frac{K}{L}$$

The MRTS decreases when L is substituted for K at constant output.

$$\frac{d\text{MRTS}}{dL} = \frac{a}{b}\frac{\frac{dK}{dL}L - K}{L^2} < 0$$

Thus, the isoquant is strictly convex.

Problem 2 (*Cost Minimization*)

(a) Optimal input bundle

(i) The optimal input bundle must be located on the isoquant representing the given output level.

$$q = F(L, K) = L^{\frac{1}{2}} K^{\frac{1}{2}}$$

(ii) For any interior solution, the isocost line through the optimal input bundle must be tangent to the isoquant, i.e. the MRTS must equal the input price ratio.

$$\underbrace{\frac{\frac{\partial F}{\partial L}}{\frac{\partial F}{\partial K}}}_{\text{MRTS}} = \frac{w}{r} \quad \Rightarrow \quad \frac{\frac{1}{2} L^{-\frac{1}{2}} K^{\frac{1}{2}}}{\frac{1}{2} L^{\frac{1}{2}} K^{-\frac{1}{2}}} = \frac{w}{r} \quad \Leftrightarrow \quad K = \frac{w}{r} L \quad \Leftrightarrow \quad L = \frac{r}{w} K$$

Substituting (ii) into (i) yields

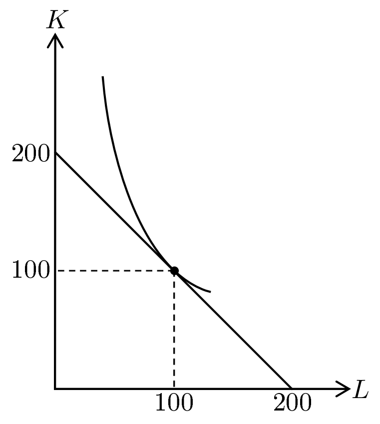
$$L = q \left(\frac{r}{w} \right)^{\frac{1}{2}}, \quad K = q \left(\frac{w}{r} \right)^{\frac{1}{2}}.$$

Given the output $q = 100$ and input prices $w = 2.5$ and $r = 2.5$, the optimal input bundle is

$$L = 100, \quad K = 100,$$

and the corresponding input costs are

$$c = wL + rK = 2.5 \cdot 100 + 2.5 \cdot 100 = 500.$$



Optimal Input Employment

(b) Now, the same output $q = 100$ shall be produced given a higher wage rate for labor $w' = 10$ and an unchanged rental rate for capital $r = 2.5$.

(i) At the new input price ratio, the input bundle $L = 100$ and $K = 100$ no longer minimizes costs. It causes input costs of

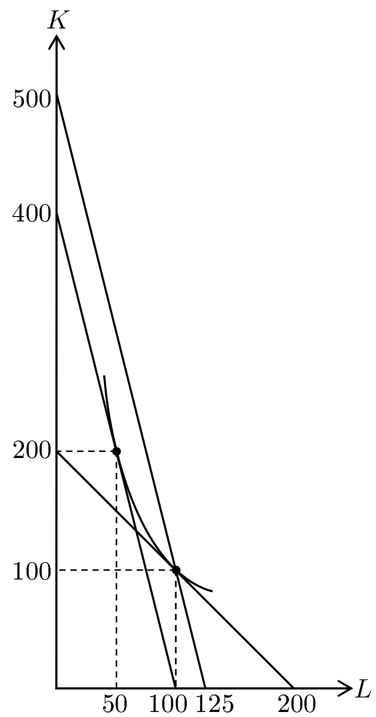
$$c = w'L + rK = 10 \cdot 100 + 2.5 \cdot 100 = 1{,}250.$$

(ii) The new cost minimizing input bundle is

$$L' = 50, \quad K' = 200,$$

and the corresponding input costs are

$$c' = w'L' + rK' = 10 \cdot 50 + 2.5 \cdot 200 = 1{,}000.$$



(Optimal) Input Employment

Problems 3-5 (*Cost Minimization*)

Problem 3

Provided that $L > 0$ and $K > 0$, a multiplication of both inputs by 4 implies a multiplication of output by 2.

$$F(4L, 4K) = (4L)^{\frac{1}{4}} \cdot (4K)^{\frac{1}{4}} = 4^{\frac{1}{2}} L^{\frac{1}{4}} K^{\frac{1}{4}} = 2L^{\frac{1}{4}} K^{\frac{1}{4}} = 2q$$

$\Rightarrow$ **(B)** is correct.

Optimal input bundle

(i) The optimal input bundle must be located on the isoquant representing the given output level.

$$q = F(L, K) = L^{\frac{1}{4}} K^{\frac{1}{4}}$$

(ii) For any interior solution, the isocost line through the optimal input bundle must be tangent to the isoquant, i.e. the MRTS must equal the input price ratio.

$$\underbrace{\frac{\frac{\partial F}{\partial L}}{\frac{\partial F}{\partial K}}}_{\text{MRTS}} = \frac{w}{r} \quad \Rightarrow \quad \frac{\frac{1}{4} L^{-\frac{3}{4}} K^{\frac{1}{4}}}{\frac{1}{4} L^{\frac{1}{4}} K^{-\frac{3}{4}}} = \frac{w}{r} \quad \Leftrightarrow \quad K = \frac{w}{r} L \quad \Leftrightarrow \quad L = \frac{r}{w} K$$

Substituting (ii) into (i) yields

$$L = q^2 \left(\frac{r}{w}\right)^{\frac{1}{2}}, \quad K = q^2 \left(\frac{w}{r}\right)^{\frac{1}{2}}.$$

Thus, variable costs (minimum input costs as a function of output) are

$$c(q) = w \cdot q^2 \left(\frac{r}{w}\right)^{\frac{1}{2}} + r \cdot q^2 \left(\frac{w}{r}\right)^{\frac{1}{2}} = 2(wr)^{\frac{1}{2}} q^2.$$

Problem 4

If the wage rate for labor is given by $w = 16$, and the rental rate for capital is given by $r = 4$, variable costs are $c(q) = 16q^2$.

$\Rightarrow$ **(D)** is correct.

Problem 5

If the wage rate for labor is given by $w = 18$, and the rental rate for capital is given by $r = \frac{1}{2}$, variable costs are $c(q) = 6q^2$.

$\Rightarrow$ **(C)** is correct.

Problems 6-8 (*Profit Maximization*)

$$\begin{aligned} \text{Marginal Costs:} \quad & MC(q) = 4q \\ \text{Average Total Costs:} \quad & AC(q) = \tfrac{200}{q} + 2q,\ \forall q > 0 \\ \text{Average Variable Costs:} \quad & ac(q) = 2q \end{aligned}$$

Problem 6

For $q = 20$, marginal costs are higher than average total costs.

$$MC(20) = 80 > 50 = AC(20)$$

$\Rightarrow$ **(B)** is correct.

Problem 7

If the firm produces $q > 0$, profit maximization requires $p = MC(q)$. For $p = 20$, this yields

$$20 = 4q \quad \Leftrightarrow \quad q = 5.$$

In the short run, the firm produces $q = 5$, because the price is higher than average variable costs at $q = 5$.

$$p = 20 > 10 = ac(5)$$

In the long run, the firm produces $q = 0$, because the price is lower than average total costs at $q = 5$.

$$p = 20 < 50 = AC(5)$$

Thus, if $p = 20$, the firm's supply is 5 in the short run and 0 in the long run.

$\Rightarrow$ **(B)** is correct.

Problem 8

The threshold price, above which the firm's supply is $q > 0$ in the long run, corresponds to the quantity for which marginal costs equal average total costs, i.e. where average total costs reach their minimum.

$$MC(q) = 4q = \frac{200}{q} + 2q = AC(q) \quad \Rightarrow \quad q = 10$$

Thus, the threshold price is

$$p = MC(10) = 40.$$

$\Rightarrow$ **(D)** is correct.

Solution 4: Perfect Competition

Problem 1 (*Competitive Equilibrium*)

(a) If a firm supplies an output $q > 0$, i.e. in case of an interior solution of its profit maximization problem, the firm's quantity must satisfy the following condition.

$$p = MC(q)$$
$$p = 2q \quad \Leftrightarrow \quad q = \frac{1}{2}p$$

The threshold price above which a firm's supply is $q > 0$ in the long run corresponds to the quantity for which marginal costs equal average total costs, i.e. where average total costs reach their minimum.

$$MC(q) = 2q = \frac{c^f}{q} + q = AC(q) \quad \Rightarrow \quad q = \sqrt{c^f}$$

Thus, the threshold price is

$$p = MC\left(\sqrt{c^f}\right) \quad \Rightarrow \quad p = 2\sqrt{c^f}.$$

Then, individual supply is

$$q(p) = \begin{cases} \frac{1}{2}p, & p \geq 2\sqrt{c^f} \\ 0, & p < 2\sqrt{c^f}, \end{cases}$$

and market supply is

$$Q^S(p) = \begin{cases} \frac{n}{2}p, & p \geq 2\sqrt{c^f} \\ 0, & p < 2\sqrt{c^f}. \end{cases}$$

In any competitive equilibrium, market demand equals market supply:

$$Q^D(p) = a - p = \frac{n}{2}p = Q^S(p).$$

The number of firms implying zero profits for each firm in equilibrium equalizes market demand and market supply at the threshold price.

$$Q^D\left(2\sqrt{c^f}\right) = a - 2\sqrt{c^f} = n\sqrt{c^f} = Q^S\left(2\sqrt{c^f}\right) \quad \Rightarrow \quad n = \frac{a}{\sqrt{c^f}} - 2$$

The number of firms must be a non-negative integer. Thus, the equilibrium number of firms as a function of a and c^f is

$$n^* = \max\left\{\left\lfloor \frac{a}{\sqrt{c^f}} - 2 \right\rfloor, 0\right\}.$$

This function is piecewise continuous.[1]

[1] The floor function $\lfloor x \rfloor$ gives the greatest integer less than or equal to x.

(b) Rearranging the condition for a competitive equilibrium and using n^* yields the equilibrium price

$$p^* = \frac{2a}{n^* + 2}.$$

Substituting the equilibrium price into market demand or market supply yields equilibrium output.

$$Q^* = \frac{an^*}{n^* + 2}$$

It follows that equilibrium output per firm is

$$q^* = \frac{a}{n^* + 2},$$

and equilbrium profit per firm is

$$\pi^* = \frac{a^2}{(n^* + 2)^2} - c^f.$$

(i) If $a = 120$ and $c^f = 100$, the equilibrium number of firms is $n^* = 10$, and the equilibrium price is $p^* = 20$. The corresponding equilibrium quantity is $Q^* = 100$, where each firm produces $q^* = 10$. It follows that profit per firm is $\pi^* = 0$.

(ii) If $a = 120$ and $c^f = 64$, the equilibrium number of firms is $n^* = 13$, and the equilibrium price is $p^* = 16$. The corresponding equilibrium quantity is $Q^* = 104$, where each firm produces $q^* = 8$. It follows that profit per firm is $\pi^* = 0$.

(iii) If $a = 126$ and $c^f = 100$, the equilibrium number of firms is $n^* = 10$, and the equilibrium price is $p^* = 21$. The corresponding equilibrium quantity is $Q^* = 105$, where each firm produces $q^* = 10.5$. It follows that profit per firm is $\pi^* = 10.25$.

(c) A firm's total cost plus tax payment is

$$C(q) + tq = \begin{cases} c^f + q^2 + tq, & q > 0 \\ 0, & q = 0. \end{cases}$$

If a firm supplies an output $q > 0$, i.e. in case of an interior solution of its profit maximization problem, the firm's quantity must satisfy the following condition.

$$\begin{aligned} p &= MC(q) + t \\ p &= 2q + t \quad \Leftrightarrow \quad q = \frac{1}{2}(p - t) \end{aligned}$$

The threshold price above which a firm's supply is $q > 0$ in the long run corresponds to the quantity for which marginal costs plus tax rate equal average total costs plus tax rate, i.e. where average total costs reach their minimum.

$$MC(q) + t = 2q + t = \frac{c^f}{q} + q + t = AC(q) + t \quad \Rightarrow \quad q = \sqrt{c^f}$$

Thus, the threshold price is

$$p = MC\left(\sqrt{c^f}\right) + t \quad \Rightarrow \quad p = 2\sqrt{c^f} + t.$$

Then, individual supply is

$$q(p) = \begin{cases} \frac{1}{2}(p - t), & p \geq 2\sqrt{c^f} + t \\ 0, & p < 2\sqrt{c^f} + t, \end{cases}$$

and market supply is

$$Q^S(p) = \begin{cases} \frac{n}{2}(p - t), & p \geq 2\sqrt{c^f} + t \\ 0, & p < 2\sqrt{c^f} + t. \end{cases}$$

In any competitive equilibrium, market demand equals market supply:

$$Q^D(p) = a - p = \frac{n}{2}(p - t) = Q^S(p).$$

The number of firms implying zero profits for each firm in equilibrium equalizes market demand and market supply at the threshold price.

$$Q^D\left(2\sqrt{c^f}+t\right) = a - 2\sqrt{c^f} - t = n\sqrt{c^f} = Q^S\left(2\sqrt{c^f}+t\right)$$

$$\Rightarrow \quad n = \frac{a-t}{\sqrt{c^f}} - 2$$

The number of firms must be a non-negative integer. Thus, the equilibrium number of firms as a function of a, c^f, and t is

$$n^* = \max\left\{\left\lfloor \frac{a-t}{\sqrt{c^f}} - 2\right\rfloor, 0\right\}.$$

(d) If $a = 126$, $c^f = 100$, and $t = 6$, the equilibrium number of firms is $n^* = 10$, and the equilibrium price is $p^* = 26$. The corresponding equilibrium quantity is $Q^* = 100$, where each firm produces $q^* = 10$. It follows that profit per firm is $\pi^* = 0$, tax revenue is $T = 600$, and the welfare loss of taxation is $WL = 15$[2].

[2]When $a = 126$, $c^f = 100$, and $t = 0$, the equilibrium number of firms is $n^* = 10$, and output is $Q^* = 105$ (see also (b) (iii)). Thus, the change in tax rate from $t = 0$ to $t = 6$ does not affect the equilibrium number of firms but reduces equilibrium output by 5 units implying a welfare loss of $WL = \frac{1}{2} \cdot 5 \cdot 6 = 15$. Graphically, the welfare loss is the area of a triangle with the output reduction as basis and the tax rate as height.

Problem 2-6 (*Competitive Equilibrium*)

If a firm supplies an output $q > 0$, i.e. in case of an interior solution of its profit maximization problem, the firm's quantity must satisfy the following condition.

$$\begin{aligned} p &= MC(q) \\ p &= 20 + \frac{1}{2}q \quad \Leftrightarrow \quad q = 2p - 40 \end{aligned}$$

The threshold price above which a firm's supply is $q > 0$ in the long run corresponds to the quantity for which marginal costs equal average total costs, i.e. where average total costs reach their minimum.

$$MC(q) = 20 + \frac{1}{2}q = \frac{25}{q} + 20 + \frac{1}{4}q = AC(q) \quad \Rightarrow \quad q = 10$$

Thus, the threshold price is

$$p = MC(10) \quad \Rightarrow \quad p = 25.$$

Then, individual supply is

$$q(p) = \begin{cases} 2p - 40, & p \geq 25 \\ 0, & p < 25, \end{cases}$$

and market supply is

$$Q^S(p) = \begin{cases} n(2p - 40), & p \geq 25 \\ 0, & p < 25. \end{cases}$$

Problem 2

The number of firms implying zero profits for each firm in equilibrium equalizes market demand and market supply at the threshold price.

$$Q^D(25) = 125 - 25 = n(2 \cdot 25 - 40) = Q^S(25) \quad \Rightarrow \quad n^* = 10$$

$\Rightarrow$ **(B)** is correct.

Problem 3

If the equilibrium number of firms is $n^* = 10$, the equilibrium price is $p^* = 25$ and the equilibrium quantity is $Q^* = 100$. Then, consumer surplus and producer surplus are given by

$$CS = \frac{1}{2} \cdot (125 - 25) \cdot 100 = 5{,}000 \quad \text{and} \quad PS = \frac{1}{2} \cdot (25 - 20) \cdot 100 = 250.$$

$\Rightarrow$ **(B)** is correct.

Problem 4

At the equilibrium price $p^* = 25$, each firm produces $q = 10$ and makes profits $\pi = 0$. A price ceiling below the equilibrium price causes losses for any firm that produces $q > 0$, so that no production will take place in the long run and thus no surplus is realized. Hence, a price ceiling at $p' = 20$ results in a welfare loss equal to total surplus in the equilibrium allocation without the price ceiling $TS = CS + PS = 5{,}000 + 250 = 5{,}250$.

$\Rightarrow$ **(D)** is correct.

Problem 5

If the equilibrium price is $p^* = 25$, the introduction of a price floor at $p'' = 20$ does not affect the allocation, so that the resulting welfare loss is 0.

$\Rightarrow$ **(A)** is correct.

Problem 6

A lump-sum subsidy $S = 24$ for each firm that produces $q > 0$ effectively lowers quasi-fixed costs, so that a firm's total costs minus the subsidy are

$$C(q) - S = \begin{cases} 1 + 20q + \frac{1}{4}q^2, & q > 0 \\ 0, & q = 0. \end{cases}$$

It follows that market supply is

$$Q^S(p) = \begin{cases} n(2p - 40), & p \geq 21 \\ 0, & p < 21. \end{cases}$$

The number of firms implying zero profits for each firm in equilibrium equalizes market demand and market supply at the threshold price.

$$Q^D(21) = 125 - 21 = n(2 \cdot 21 - 40) = Q^S(21) \quad \Rightarrow \quad n^* = 52$$

$\Rightarrow$ **(D)** is correct.

Solution 5: Market Failure

Problem 1 (*Monopoly*)

(a) Inverse market demand for sparkling wine per opera season is

$$p(Q) = 10 - \frac{1}{1{,}000}Q.$$

Thus, the seller's revenue is

$$R(Q) = 10Q - \frac{1}{1{,}000}Q^2.$$

The payment for the exclusive right to sell sparkling wine during one opera season is a quasi-fixed cost c^f. If the firm is awarded the contract, its total costs are

$$C(Q) = c^f + 2Q.$$

As a monopolist, the firm solves the following profit maximization problem.

$$\max_Q \quad \pi(Q) = 10Q - \frac{1}{1{,}000}Q^2 - c^f - 2Q$$

The necessary condition

$$\frac{d\pi(Q)}{dQ} = 8 - \frac{1}{500}Q = 0$$

yields the monopoly quantity

$$Q^M = 4{,}000.$$

The corresponding monopoly price is $p^M = 6$, and the monopoly profit is $\pi^M = 16{,}000 - c^f$. The firm is only interested in selling sparkling wine at the opera if this yields a non-negative profit. Thus, the firm's reservation price for the exclusive right of sale during one opera season is $c^f = 16{,}000$.

(b) If the city of Munich levies a tax on the seller of sparkling wine

(i) at the rate $t = 2$ per unit sold, the firm's profit maximization problem is

$$\max_{Q} \quad \pi(Q) = 10Q - \frac{1}{1{,}000}Q^2 - c^f - 2Q - \underbrace{2Q}_{tQ}.$$

The necessary condition

$$\frac{d\pi(Q)}{dQ} = 6 - \frac{1}{500}Q = 0$$

yields the monopoly quantity

$$Q^M = 3{,}000.$$

The corresponding monopoly price is $p^M = 7$, and the monopoly profit after taxes is $\pi^M = 9{,}000 - c^f$. The firm's reservation price for the exclusive right of sale during one opera season is $c^f = 9{,}000$.

(ii) at the rate $t = 0.25$ on profit, the firm's profit maximization problem is

$$\max_{Q} \quad \pi(Q) = (1 - 0.25)\left[10Q - \frac{1}{1{,}000}Q^2 - 2Q - c^f\right].$$

The necessary condition for an interior solution

$$\frac{d\pi(Q)}{dQ} = 0.75\left[8 - \frac{1}{500}Q\right] = 0$$

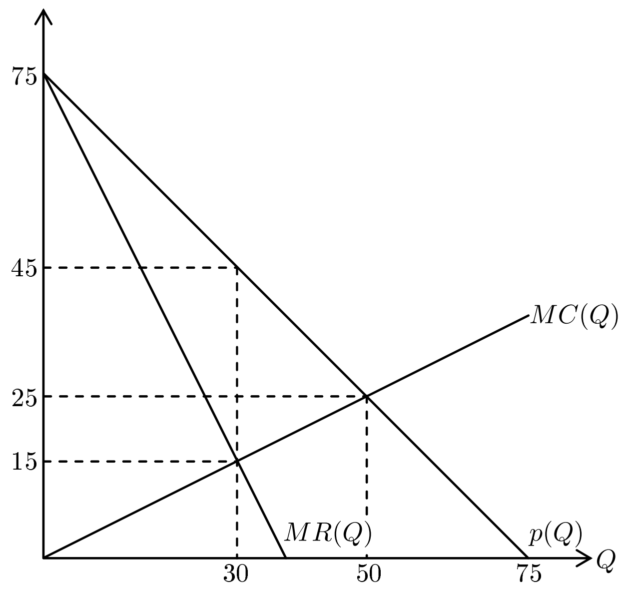
yields the monopoly quantity

$$Q^M = 4{,}000.$$

The corresponding monopoly price is $p^M = 6$, and the monopoly profit after taxes is $\pi^M = 0.75\left(16{,}000 - c^f\right)$. The firm's reservation price for the exclusive right of sale during one opera season is $c^f = 16{,}000$.[1]

[1] Note that for $c^f = 16{,}000$, the firm makes zero profits and thus pays no taxes.

Problems 2-5 (*Monopoly*)



Monopoly

The monopolist's profit maximization problem is

$$\max_{Q} \quad \pi(Q) = 75Q - Q^2 - \frac{1}{4}Q^2 - c^f.$$

The necessary condition for an interior solution

$$\frac{d\pi(Q)}{dQ} = 75 - 2\frac{1}{2}Q = 0$$

yields the monopoly quantity

$$Q^M = 30.$$

The corresponding monopoly price is $p^M = 45$, and the monopoly profit is $\pi^M = 1{,}125 - c^f$. For comparison, the welfare maximizing output for which inverse market demand equals marginal costs is $Q^* = 50$, and the corresponding price is $p^* = 25$.

Problem 2

The monopolist's profit is non-negative if quasi-fixed costs satisfy $c^f \leq 1{,}125$. Thus, the threshold regarding quasi-fixed costs, below which the monopolist's output is $Q > 0$ in the long run is $c^f = 1{,}125$.

$\Rightarrow$ **(C)** is correct.

Problem 3

If fixed costs are $c^f = 625$, the monopolist produces $Q^M = 30$. A welfare loss occurs because the monopoly output Q^M is smaller than the welfare maximizing output Q^*. Hence, for the output difference $Q^* - Q_M$, potential gains from trade exist but are not realized.

The welfare loss is

$$WL = \frac{1}{2} \cdot \left(Q^* - Q^M\right) \cdot \left(p(Q^M) - MC(Q^M)\right) = \frac{1}{2} \cdot (50 - 30) \cdot (45 - 15) = 300.$$

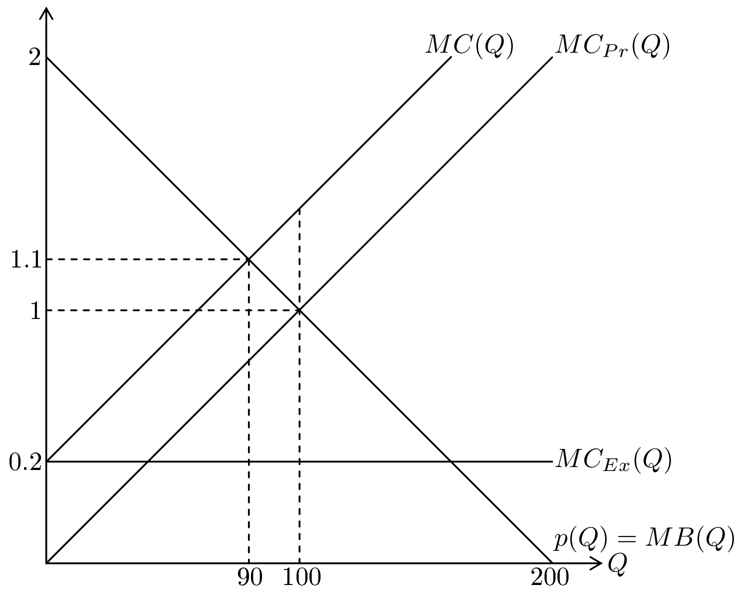
$\Rightarrow$ **(D)** is correct.

Problem 4

If fixed costs are $c^f = 0$, the monopoly output is $Q^M = 30$ and the monopoly price is $p^M = 45$. A price ceiling at $p' = 40$ will induce the monopolist to increase output to $Q^D(p') = 35$. This causes an increase in consumer surplus.

$\Rightarrow$ **(A)** is correct.

Problem 5-8 (*External Effects*)



External Effect

Problem 5

Individual profit maximization implies that the firms do not internalize the negative externalities they impose on each other.

The maximization problem of firm $i \in \{A, B\}$ is

$$\max_{q_i} \quad \pi_i(q_i) = p \cdot q_i - 15 - \frac{1}{100}q_i^2 - \frac{1}{5}q_j.$$

The necessary condition for an interior solution is:

$$\frac{d\pi_i(q_i)}{dq_i} = p - \frac{1}{50}q_i = 0 \quad \Rightarrow \quad p = MC_{Pr}(q_i)$$

Rearranging yields the supply of firm $i \in \{A, B\}$.

$$q_i(p) = 50p$$

For two identical firms, market supply is then

$$Q^S(p) = 100p.$$

The equilibrium price equalizes market demand and market supply.

$$Q^D(p^*) = 200 - 100p^* = 100p^* = Q^S(p^*)$$

Hence, the equilibrium price is $p^* = 1$, and the corresponding equilibrium quantity is $Q^* = 100$, where each firm produces $q^* = 50$, and profit per firm is $\pi^* = 0$.

$\Rightarrow$ **(A)** is correct.

Problem 6

Suppose, hypothetically, that the two firms collectively maximized profit and yet remained price takers. Collective maximization implies that the firms internalize the negative externalities they impose on each other. The resulting equilibrium quantity maximizes welfare as it aligns (social) marginal benefit to (social) marginal costs.

The collective maximization problem of firms A and B is

$$\max_{q_A, q_B} \quad \Pi(q_A, q_B) = p \cdot q_A + p \cdot q_B - 15 - \frac{1}{100}q_A^2 - \frac{1}{5}q_B - 15 - \frac{1}{100}q_B^2 - \frac{1}{5}q_A.$$

The necessary conditions for an interior solution are:

$$\frac{\partial \Pi(q_A, q_B)}{\partial q_A} = p - \frac{1}{50}q_A - \frac{1}{5} = 0 \quad \Rightarrow \quad p = MC_{Pr}(q_A) + MC_{Ex}(q_A)$$
$$\frac{\partial \Pi(q_A, q_B)}{\partial q_B} = p - \frac{1}{50}q_B - \frac{1}{5} = 0 \quad \Rightarrow \quad p = MC_{Pr}(q_B) + MC_{Ex}(q_B)$$

Rearranging yields the supply of firm $i \in \{A, B\}$.

$$q_i(p) = 50p - 10$$

For two identical firms, market supply is then

$$Q^S(p) = 100p - 20.$$

The equilibrium price equalizes market demand and market supply.

$$Q^D(\tilde{p}) = 200 - 100\tilde{p} = 100\tilde{p} - 20 = Q^S(\tilde{p})$$

Hence, the equilibrium price is $\tilde{p} = 1.1$, and the corresponding equilibrium quantity is $\tilde{Q} = 90$, where each firm produces $\tilde{q} = 45$, and profit per firm is $\tilde{\pi} = 5.25$.

Accordingly, the welfare maximizing total quantity is $Q_E = 90$.

$\Rightarrow$ **(C)** is correct.

Problem 7

A welfare loss occurs because the equilibrium quantity Q^*, resulting from individual profit maximization, exceeds the welfare maximizing quantity Q_E. Hence, for the exceeding quantity $Q^* - Q_E$, (social) marginal costs exceed (social) marginal benefits.

The welfare loss is

$$WL = \frac{1}{2} \cdot (Q^* - Q_E) \cdot MC_{Ex}(Q^*) = \frac{1}{2} \cdot (100 - 90) \cdot \frac{1}{5} = 1.$$

$\Rightarrow$ **(B)** is correct.

Problem 8

Since the rate of this (Pigouvian) tax equals external marginal costs, individual profit maximization implies that firms effectively internalize the negative externalities they impose on each other. The resulting equilibrium quantity maximizes welfare as it aligns (social) marginal benefit to (social) marginal costs.

The maximization problem of firm $i \in \{A, B\}$ is

$$\max_{q_i} \quad \pi_i(q_i) = p \cdot q_i - 15 + S - \frac{1}{100} q_i^2 - \frac{1}{5} q_j - \underbrace{\frac{1}{5} q_i}_{tq_i}.$$

The necessary condition for an interior solution is:

$$\frac{d\pi_i(q_i)}{dq_i} = p - \frac{1}{50} q_i - \underbrace{\frac{1}{5}}_{t} = 0 \quad \Rightarrow \quad p = MC_{Pr}(q_i) + \underbrace{MC_{Ex}(q_i)}_{t}$$

Rearranging yields the supply of firm $i \in \{A, B\}$

$$q_i(p) = 50p - 10.$$

For two firms, market supply is then

$$Q^S(p) = 100p - 20.$$

The equilibrium price equalizes market demand and market supply.

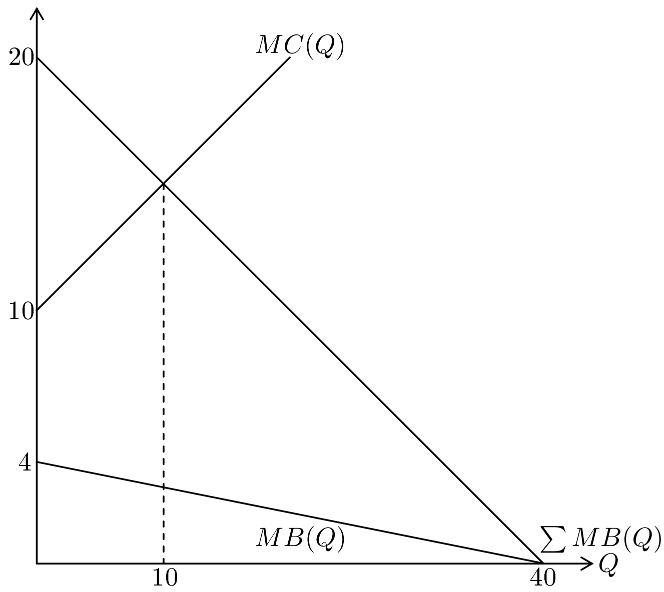
$$Q^D(p^*) = 200 - 100p^* = 100p^* - 20 = Q^S(p^*)$$

Hence, the equilibrium price is $p^* = 1.1$, and the corresponding equilibrium quantity is $Q^* = 90$, where each firm produces $q^* = 45$, and profit per firm is $\pi^* = S - 3.75$.

It follows that firms make zero profits if the lump-sum subsidy is $S = 3.75$.

$\Rightarrow$ **(A)** is correct.

Problem 9-10 (*Public Goods*)



Public Good

Problem 9

Suppose, hypothetically, that the five individuals collectively provided the public good in order to maximize total surplus, i.e. welfare.

The necessary condition for an interior solution of the collective maximization problem, i.e. the Samuelson condition, is:

$$\sum MB(Q_E) = MC(Q_E)$$
$$5 \cdot MB(Q_E) = MC(Q_E)$$

Substituting yields:

$$20 - \frac{1}{2}Q_E = 10 + \frac{1}{2}Q_E \quad \Rightarrow \quad Q_E = 10$$

$\Rightarrow$ **(B)** is correct.

Problem 10

Individual provision of the public good implies that each individual maximizes individual surplus.

The necessary condition for an interior solution of the individual maximization problem is:

$$MB(Q^*) = MC(Q^*)$$

Substituting shows that the necessary condition is never satisfied since

$$4 - \frac{1}{10}Q < 10 + \frac{1}{2}Q \quad \forall \quad Q \geq 0.$$

Hence, no individual contributes to the provision of the public good. Consequently, the resulting quantity is $Q = 0$ (corner solution).

A welfare loss occurs because individual maximization implies an underprovision of the public good; the public good is not provided at all, although, (social) marginal benefits exceed (social) marginal costs up to the welfare maximizing quantity Q_E.

The welfare loss is

$$WL = \frac{1}{2} \cdot Q_E \cdot (5 \cdot MB(0) - MC(0)) = \frac{1}{2} \cdot 10 \cdot 10 = 50.$$

$\Rightarrow$ **(C)** is correct.

Solution 6: Macroeconomic Indicators

Problem 1 (*Gross Domestic Product*)

(i) Output Method: GDP is the sum of market values of all final goods produced domestically in a given period of time (net of intermediate goods produced abroad or in previous periods).

$$\text{GDP}_R = \underbrace{600}_{\substack{\text{Market Value}\\\text{of Fresh Fish}}} + \underbrace{400}_{\substack{\text{Market Value}\\\text{of Dried Fish}}} = 1{,}000$$

$$\text{GDP}_F = \underbrace{800}_{\substack{\text{Market Value}\\\text{of Coconuts}}} + \underbrace{400}_{\substack{\text{Market Value}\\\text{of Coconut Oil}}} = 1{,}200$$

(ii) Income Method: GDP is the sum of incomes from domestic production in a given period of time.

$$\text{GDP}_R = \underbrace{200}_{\substack{\text{Capital Income}\\\text{of Friday}}} + \underbrace{800}_{\substack{\text{Labor Income}\\\text{of Robinson}}} = 1{,}000$$

$$\text{GDP}_F = \underbrace{500}_{\substack{\text{Capital Income}\\\text{of Friday}}} + \underbrace{100}_{\substack{\text{Capital Income}\\\text{of Robinson}}} + \underbrace{600}_{\substack{\text{Labor Income}\\\text{of Friday}}} = 1{,}200$$

(iii) Expenditure Method: GDP is the sum of expenditures on final goods produced domestically in a given period of time (net of expenses for intermediate goods produced abroad or in previous periods).

$$\text{GDP}_R = \underbrace{300 + 300}_{\substack{\text{Consumption of}\\\text{goods produced}\\\text{in } R \text{ and } F}} + \underbrace{400}_{\substack{\text{Investment of}\\\text{goods produced}\\\text{in } R}} + \underbrace{300}_{\substack{\text{Exports to}\\ F}} - \underbrace{300}_{\substack{\text{Imports from}\\ F}} = 1{,}000$$

$$\text{GDP}_F = \underbrace{300 + 300}_{\substack{\text{Consumption of}\\\text{goods produced}\\\text{in } F \text{ and } R}} + \underbrace{200 + 400}_{\substack{\text{Investment of}\\\text{goods produced}\\\text{in } F}} + \underbrace{300}_{\substack{\text{Exports to}\\ R}} - \underbrace{300}_{\substack{\text{Imports from}\\ R}} = 1{,}200$$

Problems 2-4 (*Price Level*)

Base Period: 2018

	Nominal GDP	Real GDP	Cost of base-period consumer basket
2018	550,000	550,000	550,000
2019	687,500	550,000	687,500
2020	643,500	495,000	1,100,000

$$\text{GDP-Deflator} = \frac{\text{Nominal GDP}}{\text{Real GDP}}$$

$$\text{CPI} = \frac{\text{Cost of base-period consumer basket at current prices}}{\text{Cost of base-period consumer basket at base-period prices}}$$

Problem 2

Price Indices in 2019:

$$\text{GDP-Deflator} = \frac{687{,}500}{550{,}000} = 1.25,$$
$$\text{CPI} = \frac{687{,}500}{550{,}000} = 1.25.$$

$\Rightarrow$ **(B)** is correct.

Problem 3

Price Indices in 2020:

$$\text{GDP-Deflator} = \frac{643{,}500}{495{,}000} = 1.3,$$
$$\text{CPI} = \frac{1{,}100{,}000}{550{,}000} = 2.$$

$\Rightarrow$ **(D)** is correct.

Problem 4

Inflation rates between 2019 and 2020

$$\text{based on GDP-Deflator:} \quad \frac{1.3 - 1.25}{1.25} = 0.04,$$
$$\text{based on CPI:} \quad \frac{2 - 1.25}{1.25} = 0.6.$$

$\Rightarrow$ **(B)** is correct.

Problems 5-6 (*Unemployment*)

The labor force participation rate is

$$e = \frac{L}{N} = \frac{E + U}{N}. \tag{1}$$

The unemployment rate is

$$u = \frac{U}{L} = \frac{U}{E + U}. \tag{2}$$

Problem 5

Rearranging (1) and substituting $N = 70$, $U = 2.1$, and $e = 0.5$ yields

$$E = e \cdot N - U \quad \Rightarrow \quad E = 0.5 \cdot 70 - 2.1 = 32.9.$$

$\Rightarrow$ **(A)** is correct.

Problem 6

Rearranging (2) and substituting $U = 2.1$ and $u = 0.05$ yields

$$L = \frac{U}{u} \quad \Rightarrow \quad L = \frac{2.1}{0.05} = 42.$$

Substituting $L = 42$ into (1) yields

$$e = \frac{42}{70} = 0.6.$$

$\Rightarrow$ **(C)** is correct.

Solution 7: Economic Growth

Problem 1 (*Golden-Rule Steady State*)

(a) The production function

$$Y = F(L, K) = L^{\frac{1}{2}} K^{\frac{1}{2}}$$

exhibits constant returns to scale; multiplication with $\frac{1}{L}$ yields

$$\frac{Y}{L} = F\left(1, \frac{K}{L}\right) = \left(\frac{K}{L}\right)^{\frac{1}{2}}.$$

Accordingly, output per worker as a function of capital per worker is

$$y = f(k) = k^{\frac{1}{2}}.$$

(b) In a steady-state, savings per worker equal break-even investment.

$$sf(k^*) = (\delta + n)k^*$$

Using the production function and the given parameter values yields

$$s\left(k^*\right)^{\frac{1}{2}} = \frac{k^*}{20} \quad \Leftrightarrow \quad \left(k^*\right)^{\frac{1}{2}} = 20s = y^*.$$

Hence, steady-state consumption per worker as a function of the saving rate is

$$c^*(s) = (1 - s)y^* = 20s - 20s^2.$$

(c) The golden-rule saving rate maximizes steady-state consumption per worker.

$$\max_s \quad c^*(s) = 20s - 20s^2$$

Necessary condition:

$$\frac{dc^*(s)}{ds} = 20 - 40s = 0 \quad \Leftrightarrow \quad s_{gold} = \frac{1}{2}$$

Problems 2-6 (*Golden-Rule Steady State*)

The production function

$$Y = F(L, K) = L^{\frac{1}{3}} K^{\frac{2}{3}}$$

exhibits constant returns to scale; multiplication with $\frac{1}{L}$ yields

$$\frac{Y}{L} = F\left(1, \frac{K}{L}\right) = \left(\frac{K}{L}\right)^{\frac{2}{3}}.$$

Accordingly, output per worker as a function of capital per worker is

$$y = f(k) = k^{\frac{2}{3}}.$$

In a steady-state, savings per worker equal break-even investment.

$$sf(k^*) = (\delta + n)k^*$$

Using the production function and the given parameter values yields

$$s\left(k^*\right)^{\frac{2}{3}} = \frac{k^*}{3},$$

which simplifies to

$$\left(k^*\right)^{\frac{2}{3}} = 9s^2. \tag{1}$$

Accordingly, the steady state output per worker is

$$y^* = 9s^2. \tag{2}$$

The golden-rule capital stock per worker must satisfy

$$f'(k^*_{gold}) = n + \delta.$$

Using the production function and the given parameter values yields

$$\frac{2}{3}\left(k^*_{gold}\right)^{-\frac{1}{3}} = \frac{1}{3},$$

which simplifies to

$$k^*_{gold} = 8. \tag{3}$$

Problem 2

Substituting the steady-state output $y^* = 1$ into equation (2) yields the corresponding saving rate.

$$1 = 9s^2 \quad \Leftrightarrow \quad s = \frac{1}{3}$$

$\Rightarrow$ **(C)** is correct.

Problem 3

Substituting the saving rate $s = \frac{1}{3}$ into equation (1) yields the corresponding steady-state capital stock per worker.

$$(k^*)^{\frac{2}{3}} = 1 \quad \Leftrightarrow \quad k^* = 1$$

Thus, at $k = 1$, the economy is in a steady state, implying that over time, all per-worker quantities are constant, and all aggregate quantities increase at a constant rate, namely the rate of population growth $n = \frac{1}{6}$.

$\Rightarrow$ **(B)** is correct.

Problem 4

Substituting equation (3) into equation (1) yields the golden-rule saving rate.

$$4 = 9s^2 \quad \Leftrightarrow \quad s_{gold} = \frac{2}{3}$$

$\Rightarrow$ **(D)** is correct.

Problem 5

Substituting $s_{gold} = \frac{2}{3}$ into equation (2) yields output per worker in the golden-rule steady state.

$$y^*_{gold} = 4$$

Thus, consumption per worker in the golden-rule steady state is

$$c^*_{gold} = (1 - s)y^* = \frac{4}{3}.$$

$\Rightarrow$ **(C)** is correct.

Problem 6

Any saving rate satisfying $s \in \left[0, \frac{1}{3}\right)$ is strictly below the golden-rule saving rate $s_{gold} = \frac{2}{3}$ and thus implies a dynamically efficient steady state.

$\Rightarrow$ **(A)** is correct.

Solution 8: Economic Fluctuations

Problem 1 (*General Equilibrium*)

(a) The goods market is in equilibrium if output Y equals demand Z.

$$Y = 200 + 0.75(Y - T) + 50 - 5r + G$$

Rearranging yields the IS-Curve.

$$Y = 1{,}000 - 3T - 20r + 4G \tag{1}$$

The financial market is in equilibrium if liquidity demand L equals money supply M.

$$Y - 80r = M$$

Rearranging yields the LM-Curve.

$$Y = M + 80r \tag{2}$$

Substituting equation (1) into equation (2) and solving yields the interest rate in general equilibrium as a function of taxes, government consumption, and money supply.

$$r^*(T, G, M) = 10 - 0.03T + 0.04G - 0.01M \tag{3}$$

(b) Substituting $T = 100$ into equation (3) yields

$$r^*(G, M) = 7 + 0.04G - 0.01M.$$

Total differentiation yields

$$\frac{dr^*}{dG} = 0.04 - 0.01\frac{dM}{dG} = 0 \quad \Rightarrow \quad \frac{dM}{dG} = 4.$$

Problems 2-7 (*General Equilibrium*)

The goods market is in equilibrium if output Y equals demand Z.

$$Y = 100 + 0.8(Y - T) + 100 - 8r + G$$

Rearranging yields the IS-Curve.

$$Y = 1{,}000 - 4T - 40r + 5G \tag{4}$$

The financial market is in equilibrium if liquidity demand L equals money supply M.

$$Y - 60r = M$$

Rearranging yields the LM-Curve.

$$Y = M + 60r \tag{5}$$

Substituting equation (4) into equation (5) and solving yields the interest rate in general equilibrium as a function of taxes, government consumption, and money supply.

$$r^*(T, G, M) = 10 - 0.04T + 0.05G - 0.01M \tag{6}$$

Substituting equation (6) into equation (4) or equation (5) yields output in general equilibrium as a function of taxes, government consumption, and money supply.

$$Y^*(T, G, M) = 600 - 2.4T + 3G + 0.4M \tag{7}$$

Problem 2

Differentiating equation (4) with respect to G yields the government-consumption multiplier.

$$\frac{\partial Y}{\partial G} = 5$$

$\Rightarrow$ **(C)** is correct.

Problem 3

Differentiating equation (4) with respect to T yields the tax multiplier.

$$\frac{\partial Y}{\partial T} = -4$$

$\Rightarrow$ **(A)** is correct.

Problem 4

Substituting $T = 200$, $G = 200$, and $M = 700$ into equation (7) yields general-equilibrium output.

$$Y^* = 1{,}000$$

$\Rightarrow$ **(A)** is correct.

Problem 5

In general equilibrium, investment equals savings. Thus, general-equilibrium savings $S^* = 60$ require

$$I^*(r) = 100 - 8r^* = 60 \quad \Leftrightarrow \quad r^* = 5.$$

Substituting $r^* = 5$ as well as $T = 200$ and $G = 300$ into equation (6) and solving yields the corresponding money supply.

$$M = 1{,}200$$

$\Rightarrow$ **(C)** is correct.

Problem 6

Ceteris paribus,

(A) an increase in taxes T (contractionary fiscal policy) shifts the IS-Curve to the left, while an increase in money supply M (expansionary monetary policy) shifts the LM-Curve downwards. This implies a decrease in the interest rate r^* and thus results in an increase in investment I^* and savings S^*, respectively, in general equilibrium.

(B) an increase in government consumption G (expansionary fiscal policy) shifts the IS-Curve to the right, while a decrease in money supply M (contractionary monetary policy) shifts the LM-Curve upwards. This implies an increase in the interest rate r^* and thus results in a decrease in investment I^* and savings S^*, respectively, in general equilibrium.

(C) a decrease in taxes T (expansionary fiscal policy) shifts the IS-Curve to the right, while an increase in money supply M (expansionary monetary policy) shifts the LM-Curve downwards. This implies an increase in output Y^* which – together with the given decrease in taxes – leads to an increase in disposable income $Y^* - T$ resulting in an increase in private consumption C^* in general equilibrium.

(D) a decrease in government consumption G (contractionary fiscal policy) shifts the IS-Curve to the left, while a decrease in money supply M (contractionary monetary policy) shifts the LM-Curve upwards. This implies a decrease in output Y^* which leads to a decrease in disposable income $Y^* - T$ resulting in a decrease in private consumption C^* in general equilibrium.

$\Rightarrow$ **(D)** is correct.

Problem 7

Goods Market: For any combination (Y, r) located to the left (right) of the IS-curve, the goods market is not in equilibrium; at the given interest rate r, output Y is too small (large), hence investment $I(r)$ exceeds (falls short of) savings $S(Y)$.

Financial Market: For any combination (Y, r) located below (above) the LM-curve, the financial market is not in equilibrium; at the given output Y, the interest rate r is too low (high), hence liquidity demand $L(Y, r)$ exceeds (falls short of) money supply M.

Accordingly, any combination (Y, r) located

(A) to the left of the IS-curve and below the LM-curve satisfies $I > S$ and $L > M$.

(B) on the IS-curve and above the LM-curve satisfies $I = S$ and $L < M$.

(C) to the right of the IS-curve and on the LM-curve satisfies $I < S$ and $L = M$.

(D) to the right of the IS-curve and above the LM-curve satisfies $I < S$ and $L < M$.

$\Rightarrow$ **(A)** is correct.