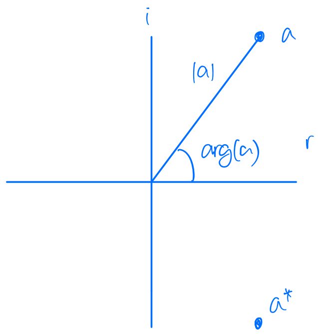
1.1 a)

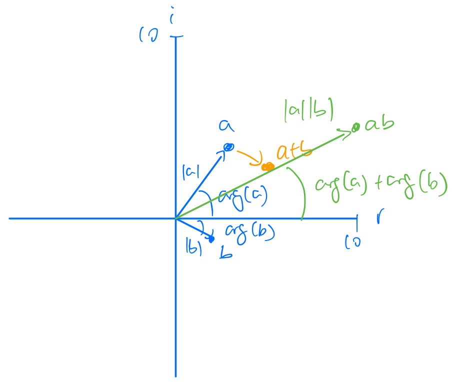
- $a+b = 3+4i+2-i = 5+3i$
- $ab = (3+4i)(2-i) = 6+8i-3i-4i^2 = 6+5i+4 = 10+5i$
- $1/a = \frac{1}{3}+\frac{1}{4}i$
- $a^* = 3-4i$
- $a = |a|e^{i\arg(a)} \Rightarrow |a| = \sqrt{\bar{a}a} = \sqrt{\underbrace{3^2}_{\text{real}}+\underbrace{4^2}_{\text{imaginary}}} = 5$

  $\arg(a) = \tan^{-1}\left(\frac{4}{3}\right) = \sim 53°$
- $c = \begin{pmatrix} a \\ b \end{pmatrix}$ $\quad ||c|| = \sum_i^2 c_i c_i^* = (3+4i)(3-4i)+(2-i)(2+i) = 9-16i^2+4-i^2 = 15$

b)



c)



$a+b$: just like a summation of real vectors

ab : $\arg(ab) = \arg(a)+\arg(b) \rightarrow$ angle

$|ab| = |a||b| \rightarrow$ length

1.2 a)

$$\begin{pmatrix} 2 & -i & 5 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ i \\ -3 \end{pmatrix} = \begin{pmatrix} -6 \\ 9 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 7 \\ 3 & 1+2i \end{pmatrix}\begin{pmatrix} 5 & -4 \\ 6i & 0 \end{pmatrix} = \begin{pmatrix} -10+42i & 8 \\ 3+6i & -12 \end{pmatrix}$$

b) normal: $A^*A = AA^*$ $\Rightarrow$ in the real domain any non-symmetric matrix

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad A^* = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

$$AA^* = \begin{pmatrix} 5 & 11 \\ 11 & 25 \end{pmatrix} \neq A^*A = \begin{pmatrix} 10 & 14 \\ 14 & 20 \end{pmatrix}$$

c) $A = \begin{pmatrix} 0 & 3/5 & 4/5 \\ -3/5 & 0 & 0 \\ -4/5 & 0 & 0 \end{pmatrix}$ $\quad A^* = \begin{pmatrix} 0 & -3/5 & -4/5 \\ 3/5 & 0 & 0 \\ 4/5 & 0 & 0 \end{pmatrix}$

$A^*A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 9/5 & 12/5 \\ 0 & 12/5 & 16/5 \end{pmatrix} = AA^* \Rightarrow A$ is normal

$\det(A-\lambda I) = \det\begin{pmatrix} -\lambda & 3/5 & 4/5 \\ -3/5 & -\lambda & 0 \\ -4/5 & 0 & -\lambda \end{pmatrix} = (-\lambda)^3 - \left(\frac{16}{25}\lambda + \frac{9}{25}\lambda\right) = -\lambda^3 - \lambda$ characteristic polynomial

$-\lambda^3 - \lambda \overset{!}{=} 0 \rightarrow -\lambda(\lambda^2+1) = 0 \rightarrow \lambda = 0$
$\quad \hookrightarrow \lambda^2 + 1 = 0 \rightarrow \pm i$ } eigenvalues

$\lambda_1 = 0 \rightarrow Av_1 = 0v_1 \rightarrow Av_1 = 0 \rightarrow v_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ eigenvector

d) A unitary $\Leftrightarrow A^*A = AA^* = I$

$A = \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}$ $\quad A^* = \begin{pmatrix} \cos\theta & -i\sin\theta \\ -i\sin\theta & \cos\theta \end{pmatrix}$

$A^*A = \begin{pmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I = AA^*$

e) $\det(U^+U) = \det(I) = 1$
$\det(U^+U) = \det(U^+)\det(U) = \overline{\det(U)}\det(U) = |\det(U)|^2$ }

$|a| = \sqrt{\bar{a}a}$

$\Rightarrow |\det(U)|^2 = 1 \rightarrow |\det(U)| = 1$