7.1 a) $|a\rangle = \alpha|0\rangle + \beta|1\rangle \xrightarrow{H} \alpha\frac{|0\rangle+|1\rangle}{\sqrt{2}} + \beta\frac{|0\rangle-|1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^a|1\rangle\right)$ $\quad a \in \{0,1\}$

$|\beta_{ab}\rangle = |H|a\rangle, H|a\rangle \oplus b\rangle = \frac{1}{\sqrt{2}}\left(|0,b\rangle + (-1)^a|1, 1-b\rangle\right) \quad a,b \in \{0,1\}$

b) if $a=b=0$:

then clearly $|\beta_{ab}\rangle = |\beta_{00}\rangle$ as the circuit doesn't apply any transformation to the input state $|\beta_{00}\rangle$

if $a=0,\ b=1$:

$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle) \xrightarrow{X_1} \frac{1}{\sqrt{2}}(X|0\rangle|0\rangle + X|1\rangle|1\rangle) = \frac{1}{\sqrt{2}}(|10\rangle+|01\rangle) =$
$\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle) = |\beta_{01}\rangle$

if $a=1,\ b=0$:

$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle) \xrightarrow{Z_1} \frac{1}{\sqrt{2}}(Z|0\rangle|0\rangle + Z|1\rangle|1\rangle) = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) = |\beta_{10}\rangle$

if $a=1,\ b=1$:

$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle) \xrightarrow{X_1} |\beta_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle+|10\rangle) \xrightarrow{Z_1} \frac{1}{\sqrt{2}}(Z|0\rangle|1\rangle + Z|1\rangle|0\rangle) =$
$\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) = |\beta_{11}\rangle$

c) $\langle\beta_{ab}|E\otimes I|\beta_{ab}\rangle = \frac{1}{\sqrt{2}}(|0,b\rangle+(-1)^a|1,1-b\rangle)^\dagger\, E\otimes I\, \frac{1}{\sqrt{2}}(|0,b\rangle+(-1)^a|1,1-b\rangle)$

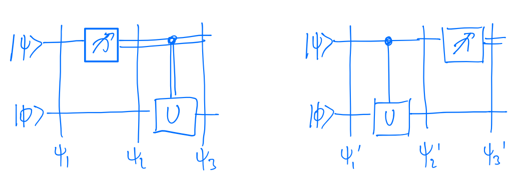
$= \frac{1}{2}\big(\langle 0,b|E\otimes I|0,b\rangle + (-1)^a\langle 0,b|E\otimes I|1,1-b\rangle + (-1)^a\langle 1,1-b|E\otimes I|0,b\rangle +$
$(-1)^a\langle 1,1-b|E\otimes I|1,1-b\rangle\big)$

$= \frac{1}{2}\big(\langle 0|E|0\rangle \overset{1}{\cancel{\langle b|I|b\rangle}} + (-1)^a\langle 0|E|1\rangle \overset{0}{\cancel{\langle b|I|1-b\rangle}} + (-1)^a\langle 1|E|0\rangle \overset{0}{\cancel{\langle 1-b|I|b\rangle}}$
$+ \underset{1}{\cancel{(-1)^a(-1)^a}}\langle 1|E|1\rangle \underset{1}{\cancel{\langle 1-b|I|1-b\rangle}}\big)$

$= \frac{1}{2}(\langle 0|E|0\rangle + \langle 1|E|1\rangle)$ regardless of the values of a and b.

Therefore nothing can be inferred with this setup, as the resulting state is independant of a, b.

7.2 a)



$|\psi_1\rangle = |\psi\phi\rangle$
$|\psi_2\rangle = |a,\phi\rangle \qquad |\psi\rangle$ measured as $a \in \{0,1\}$ with probabilities α^2, β^2 respectively
$|\psi_3\rangle = |a\rangle U^a|\phi\rangle$

$|\psi_1'\rangle = |\psi\phi\rangle$

$|\psi_2'\rangle = (\alpha|0\rangle + \beta|1\rangle)|\phi\rangle = \alpha|0\phi\rangle + \beta|1, U\phi\rangle$

$|\psi_3'\rangle = |0\phi\rangle$ with prob α^2

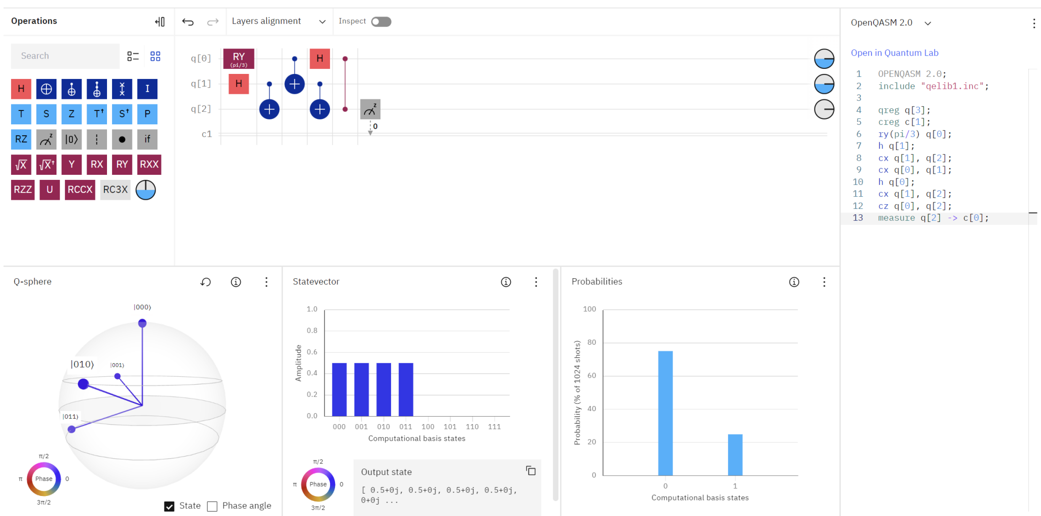
$|1, U\phi\rangle$ with prob β^2

which matches the left circuit

b) $R_y(\frac{\pi}{3}) = \begin{pmatrix} \cos(\frac{\pi}{6}) & -\sin(\frac{\pi}{6}) \\ \sin(\frac{\pi}{6}) & \cos(\frac{\pi}{6}) \end{pmatrix}$

$R_y(\frac{\pi}{3})|0\rangle = \begin{pmatrix} \cos(\frac{\pi}{6}) \\ \sin(\frac{\pi}{6}) \end{pmatrix} \rightarrow P(|0\rangle) = \cos(\frac{\pi}{6})^2 = 0.75$

$P(|1\rangle) = \sin(\frac{\pi}{6})^2 = 0.25$



c)

```
In [3]: import math
        import qiskit
        from qiskit import Aer

        c = qiskit.QuantumCircuit(3)

        c.ry(math.pi/3,0)
        c.h(1)
        c.cx(1,2)

        c.cx(0,1)
        c.h(0)
        c.cx(1,2)
        c.cz(0,2)

        simulator = Aer.get_backend('statevector_simulator')
        ex = qiskit.execute(c, simulator)
        res = ex.result()
        statevector = res.get_statevector(c)
        print(statevector)

Statevector([0.4330127+0.000000e+00j, 0.4330127+0.000000e+00j,
             0.4330127+0.000000e+00j, 0.4330127+0.000000e+00j,
             0.25     -3.061617e-17j, 0.25     -3.061617e-17j,
             0.25     -3.061617e-17j, 0.25     -3.061617e-17j],
            dims=(2, 2, 2))
```