Exercises for Chapter 3

- **3.1** Consider the time series (920, 980, 1.03, 950, 990).
 - a) Which stochastic and deterministic errors does this time series seem to contain?

stochastic errors: noise, deterministic errors: outlier 1.03

- b) What may be reasons for these errors? stochastic errors: measurement noise, deterministic errors: wrong format $1.030 \leftrightarrow 1030$
- c) Compute the output of an asymmetric median filter with window length 3 for this time series.
 (920, 950, 950)
- d) Which effect does this filter have on the observed errors? noise is reduced, outlier is removed
- **3.2** Which of these filters are FIR, IIR, or none of these?
 - a) $x_k + x_{k-1} + y_k = 0$ FIR, because it is linear but does not consider previous values of y
 - b) $x_k + x_{k-1} + x_{k-2} = 0$ none, because it does not contain y
 - c) $x_k + y_{k-1} + y_k = 0$ IIR, because it is linear and does consider previous values of y

- **3.3** Consider the IIR filter $y_k = 2y_{k-1} y_{k-2} + x_k + x_{k-1}$, $k = 3, 4, ..., y_1 = y_2 = 0$.
 - a) What is the filter output sequence y for the input sequence x = (0,0,1,0,0,0,0,0)?

$$y_3 = 2y_2 - y_1 + x_3 + x_2 = 0 - 0 + 1 - 0 = 1$$

$$y_4 = 2y_3 - y_2 + x_4 + x_3 = 2 - 0 + 0 + 1 = 3$$

$$y_5 = 2y_4 - y_3 + x_5 + x_4 = 6 - 1 + 0 + 0 = 5$$

$$y_6 = 2y_5 - y_4 + x_6 + x_5 = 10 - 3 + 0 + 0 = 7$$

$$y_7 = 2y_6 - y_5 + x_7 + x_6 = 14 - 5 + 0 + 0 = 9$$

$$y_8 = 2y_7 - y_6 + x_8 + x_7 = 18 - 7 + 0 + 0 = 11$$

$$y = (0, 0, 1, 3, 5, 7, 9, 11)$$

b) What is the filter output sequence y for the input sequence $x = (0,0,1,a,b,0,0,0), a,b \in \mathbb{R}$?

$$\begin{array}{l} y_3 = 2y_2 - y_1 + x_3 + x_2 = 0 - 0 + 1 - 0 = 1 \\ y_4 = 2y_3 - y_2 + x_4 + x_3 = 2 - 0 + a + 1 = 3 + a \\ y_5 = 2y_4 - y_3 + x_5 + x_4 = 6 + 2a - 1 + b + a = 5 + 3a + b \\ y_6 = 2y_5 - y_4 + x_6 + x_5 = 10 + 6a + 2b - 3 - a + 0 + b = 7 + 5a + 3b \\ y_7 = 2y_6 - y_5 + x_7 + x_6 = 14 + 10a + 6b - 5 - 3a - b + 0 + 0 = 9 + 7a + 5b \\ y_8 = 2y_7 - y_6 + x_8 + x_7 = 18 + 14a + 10b - 7 - 5a - 3b + 0 + 0 = 11 + 9a + 7b \\ y = (0, 0, 1, 3 + a, 5 + 3a + b, 7 + 5a + 3b, 9 + 7a + 5b, 11 + 9a + 7b) \end{array}$$

- c) Give a formula for the filter output y_k , k = 8, 9, 10, ..., for x = (0, 0, 1, a, b, 0, 0, 0, ...), $a, b \in \mathbb{R}!$ $y_k = 2k - 5 + (2k - 7)a + (2k - 9)b$
- d) For which finite values of a and b will the filter be unstable? $y_k = 2k(1+a+b) 5 7a 9b$ not finite for $k \to \infty$ if $1+a+b \neq 0$
- e) For which finite values of a and b will the filter converge to $\lim_{k\to\infty}y_k=0$? for $1+a+b=0 \Leftrightarrow b=-1-a$: $\lim_{k\to\infty}y_k=-5-7a-9b=4+2a=0$ for $a=-2,\ b=-1+2=1$