

ProbML

① Recap.

Prob. Distribution

Random process

Sample space → Event space

All possible events.



Mathematical object.

Describes random processes. Credibility Belief

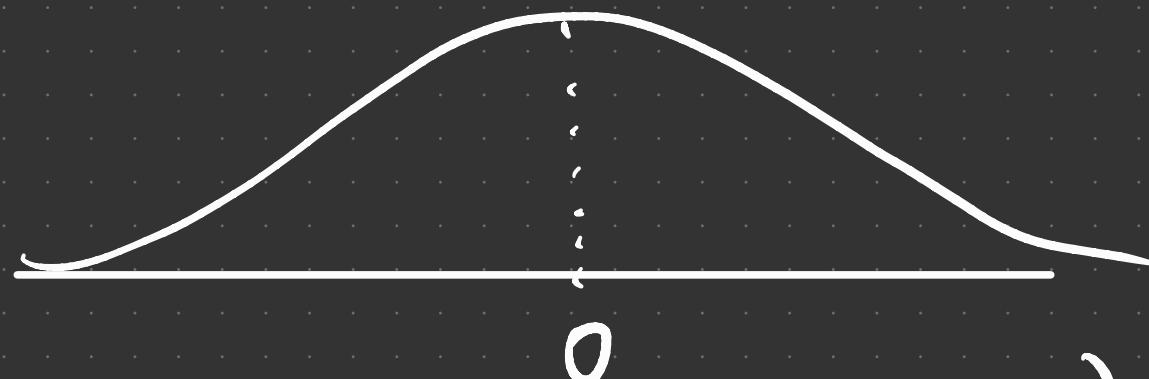
Actually occur.



Prob. Dist. $N(\mu, \sigma^2)$

↳ Random Variable.

Realisation of process.



$$3 \cdot 10^{-8}$$

0

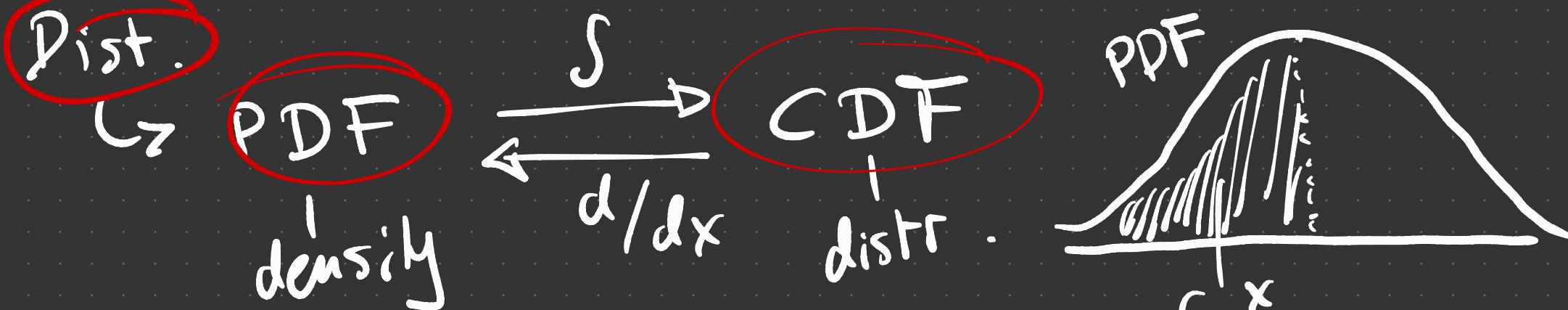
$$N(0, 4)$$

Prob. density (Mass)

↳ Apporitions prob. to events.

$$P \in (0, 1)$$

$$\Pr(\text{event}) = 0 \quad \text{---} \quad = 1.$$



$p(x) \rightarrow$ density.

$$N(\mu, \sigma^2) \rightarrow p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{|x-\mu|^2}{2\sigma^2}\right).$$

Dist.

$\Pr(E)$

PDF

\hookrightarrow Probability of an event.

$\Pr(E)$ → Prob. observing E Marginal

$\Pr(E_1 \text{ AND } E_2)$ → Prob. of BOTH
 E_1 AND E_2 .

$\Pr(E_1 | E_2)$ → Conditional Prob.

Rule Bayes Rule.

$$\Pr(A|B) = \frac{\Pr(B|A) \Pr(A)}{\Pr(B)}.$$

$$\Rightarrow \underbrace{Pr(A|B)}_{\text{Posterior}} = \underbrace{Pr(B|A)}_{\text{Likelihood}} \cdot \boxed{\frac{Pr(A)}{Pr(B)}} \underbrace{\text{"Bayes Factor"} \quad \quad}_{3B \times B} \quad \text{Reinventing } B's \text{ Rule.}$$

Data we observe \rightarrow Evidence.

↳ Generated by random process.

$$\Pr(\text{Cancer} \mid \text{Symptoms}) = \frac{\Pr(S \mid c) \Pr(c)}{\Pr(S)}$$

$\Pr(S \mid c)$ Likelihood.

$\Pr(c)$ → Observations.

$\Pr(S) \rightarrow \Pr(S \mid \neg c)$

Cursed
Bayesian
Denominator.

Denominator: Requires to $\sum S$ over
 "S over all possible models" ↳ Dimension.

② Uncertainty and Belief.

Prob → method to quantify our belief in sth.

Two types of uncertainty.

- Aleatoric u. "Alea iacta est"

"Dice"

↳ Unc. about the VALUE of a random variable.

- Epistemic u. → Unc. about which Dist.

"Science" → best specifies a random process.

Which can we reduce by more data?

G7 Epistemic

"In the infinite limit"

- ③ The probabilistic process → [Quantum
≡ Dist.]
- 3-step:
- ① Assign prior distribution to the process.
 - ② Observe evidence → modulates belief
 - ③ Update our beliefs.

Remark 1

Evidence \rightarrow

Posterior

Prior \rightarrow

"Interpolates" btw.
prior and evidence.

Remark 2:

\varnothing Data \rightarrow Evidence "overwhelms"
prior

$D \rightarrow$ Prior only.

4

Computational Bayes.

- ① Prior -
- ② Simulate data according to prior.
- ③ Evidence ← compare.
- ④ Posterior -