Solution 2: Consumption and Demand

Problem 1 (*Budget Restriction*)

(a) The individual's budget restriction with respect to time is $Z = L + F$, while his budget restriction with respect to earned income is $wL = pq$. Solving the former for L and substituting it into the latter yields $w(Z - F) = pq$. His budget restriction with respect to his potential income is thus

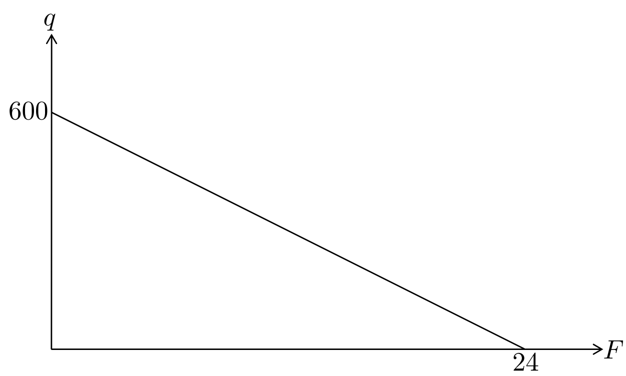
$$wZ = pq + wF.$$

Solving for q yields the budget line

$$q = \frac{wZ}{p} - \frac{w}{p}F.$$

Substituting $Z = 24$, $w = 25$, and $p = 1$ gives

$$q = 600 - 25F.$$



Budget line

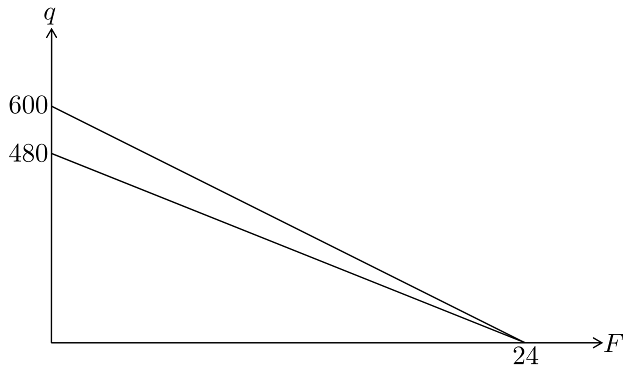
(b) The individual's budget line

(i) if an income tax reduces the wage rate to $(1-t)w = 20$ is

$$q = 480 - 20F,$$

(ii) if a consumption tax raises the price of the consumption good to $(1+\tau)p = 1.25$ is

$$q = 480 - 20F,$$



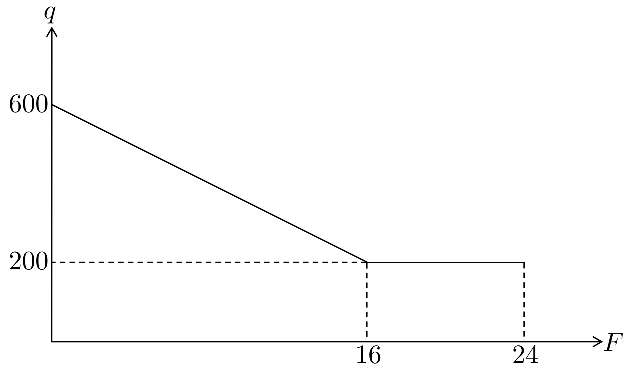
Budget line before and after taxes

(iii) if a social transfer

$$S = \begin{cases} 0, & wL \geq 200 \\ 200 - wL, & wL < 200 \end{cases}$$

subsidizes earned incomes is

$$q = \begin{cases} 600 - 25F, & F \leq 16 \\ 200, & F > 16. \end{cases}$$



Budget line with social transfer

Problem 2 (*Assumptions on Preferences*)

The individual's preferences regarding the consumption bundles $A = (8, 2)$, $B = (2, 8)$, and $C = (6, 6)$ involve the following relations: $A \sim B$ and $B \succ C$.[1]

- If her preferences are **complete**, she can compare the consumption bundles A and C such that $A \sim C$ or $A \succ C$ or $C \prec A$.

- If her preferences are **complete** and also **transitive**, $A \sim B$ together with $B \succ C$ implies $A \succ C$.

- If her preferences are **complete**, **transitive**, and also **convex**, they cannot be **monotonous**. To see this, consider a consumption bundle D that contains just as many apples as oranges and is located on a convex indifference curve connecting A and B. Such a consumption bundle can at most contain 5 apples and 5 oranges.[2] Given that $D \sim B$ and $B \succ C$ (and likewise given that $D \sim A$ and $A \succ C$), transitivity implies that $D \succ C$. This however would mean that „less is better“.

- If her preferences are **complete**, **transitive**, and also **monotonous**, they cannot be **convex**. To see this, consider a consumption bundle E that contains just as many apples as oranges and is located on an indifference curve connecting A and B. Given that $E \sim B$ and $B \succ C$ (and likewise given that $E \sim A$ and $A \succ C$), transitivity implies that $E \succ C$. Monotonicity then requires that E must contain more of both goods than C. Hence, the indifference curve connecting A, E, and B must run to the northeast of C and must therefore be strictly concave.

Thus, the four assumptions cannot hold together in this case.

[1] The symbol $\sim$ represents the indifference relation, i.e. the individual is indifferent between A and B. The symbol $\succ$ represents the strong preference relation, i.e. the individual strictly prefers B to C.

[2] In case of linear indifference curves (i.e. indifference curves that are convex, but not strictly so), D is a convex combination of A and B and contains exactly 5 apples and 5 oranges. In case of strictly convex indifference curves, D must contain less than 5 apples and less than 5 oranges.

Problem 3 (*Individual Demand*)

(a) Individual demand for a particular good follows from utility maximization.

(i) The optimal consumption bundle must be located on the budget line, i.e. the budget restriction must bind.

$$y = p_1 q_1 + p_2 q_2$$

(ii) For any interior solution, the indifference curve through the optimal consumption bundle must be tangent to the budget line, i.e. the MRS must equal the price ratio.

$$\underbrace{\frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}}}_{\text{MRS}} = \frac{p_1}{p_2} \;\Rightarrow\; \frac{\frac{1}{2}q_1^{-\frac{1}{2}}}{\frac{1}{2}q_2^{-\frac{1}{2}}} = \frac{p_1}{p_2} \;\Leftrightarrow\; q_2 = \left(\frac{p_1}{p_2}\right)^2 q_1 \;\Leftrightarrow\; q_1 = \left(\frac{p_2}{p_1}\right)^2 q_2$$

Substituting (ii) into (i) yields

$$q_1 = \frac{yp_2}{p_1p_2 + p_1^2}, \quad q_2 = \frac{yp_1}{p_1p_2 + p_2^2}.$$

Thus, the individual demand for good $i \in \{1, 2\}$ is

$$q_i(p_i, p_j, y) = \frac{yp_j}{p_ip_j + p_i^2},$$

where $j \in \{1, 2\}$ and $j \neq i$.

(b) The demand for good $i \in \{1, 2\}$ increases as income increases.

$$\frac{\partial q_i(p_i, p_j, y)}{\partial y} = \frac{p_j}{p_ip_j + p_i^2} > 0$$

$\Rightarrow$ Both goods are normal goods.

The demand for good $i \in \{1, 2\}$ decreases if the respective price p_i increases.

$$\frac{\partial q_i(p_i, p_j, y)}{\partial p_i} = -\frac{yp_j(p_j + 2p_i)}{(p_ip_j + p_i^2)^2} < 0$$

$\Rightarrow$ Both goods are ordinary goods.

The demand for good $i \in \{1, 2\}$ increases if the price p_j of the other good increases.

$$\frac{\partial q_i(p_i, p_j, y)}{\partial p_j} = \frac{yp_i^2}{(p_ip_j + p_i^2)^2} > 0$$

$\Rightarrow$ The goods are substitutes.

Problem 4 (*Substitution and Income Effects*)

(a) Optimal consumption

(i) The optimal consumption bundle must be located on the budget line, i.e. the budget restriction must bind.

$$y = p_1 q_1 + p_2 q_2$$

(ii) For any interior solution, the indifference curve through the optimal consumption bundle must be tangent to the budget line, i.e. the MRS must equal the price ratio.

$$\underbrace{\frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}}}_{\text{MRS}} = \frac{p_1}{p_2} \quad \Rightarrow \quad \frac{\frac{1}{2} q_1^{-\frac{1}{2}} q_2^{\frac{1}{2}}}{\frac{1}{2} q_1^{\frac{1}{2}} q_2^{-\frac{1}{2}}} = \frac{p_1}{p_2} \quad \Leftrightarrow \quad q_2 = \frac{p_1}{p_2} q_1 \quad \Leftrightarrow \quad q_1 = \frac{p_2}{p_1} q_2$$

Substituting (ii) into (i) yields

$$q_1 = \frac{y}{2p_1}, \quad q_2 = \frac{y}{2p_2}.$$

If $y = 600$, $p_1 = 25$, and $p_2 = 25$, the optimal consumption bundle C is

$$q_1 = 12, \quad q_2 = 12.$$

(b) If $y = 600$, $p_1 = 25$, and $p_2' = 100$, the optimal consumption bundle C' is

$$q_1' = 12, \quad q_2' = 3.$$

The total effect of the price change $C \to C'$ can be decomposed into the substitution effect $C \to \tilde{C}$ and the income effect $\tilde{C} \to C'$, where $\tilde{C}$ is a hypothetical consumption bundle. Given the new price ratio $\frac{p_1}{p_2'} = \frac{1}{4}$, the individual would choose $\tilde{C}$ if her income was compensated to the extent that she could obtain the initial level of utility $(12 \cdot 12)^{\frac{1}{2}} = 12$.

(i) $\tilde{C}$ must be located on the initial indifference curve through C.

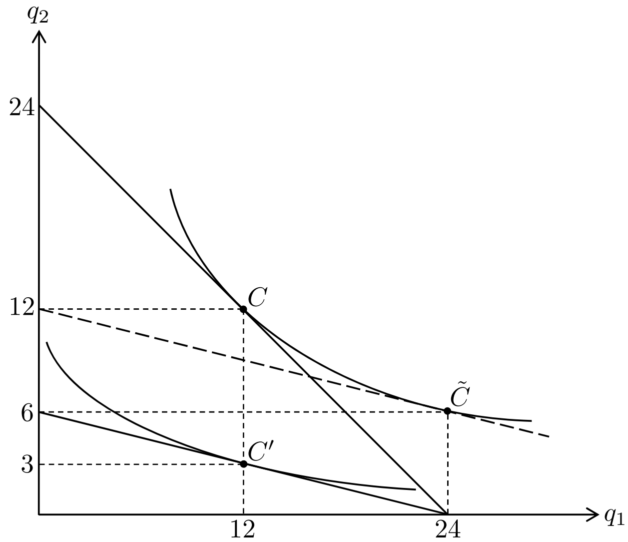
$$(q_1 \cdot q_2)^{\frac{1}{2}} = 12$$

(ii) $\tilde{C}$ must be located where the initial indifference curve is tangent to the hypothetical budget line parallel to the new budget line through C'.

$$q_2 = \frac{1}{4} q_1 \quad \Leftrightarrow \quad q_1 = 4q_2$$

Substituting (ii) into (i) yields

$$\tilde{q}_1 = 24, \quad \tilde{q}_2 = 6.$$



Substitution- and Income Effect

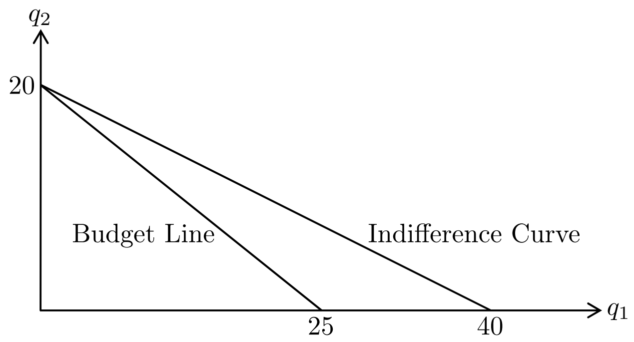
(c) The hypothetical income necessary after the price increase so that the individual could afford the hypothetical consumption bundle $\tilde{C}$ and therefore obtain the initial level of utility is

$$\tilde{y} = p_1\tilde{q}_1 + p_2'\tilde{q}_2 = 25 \cdot 24 + 100 \cdot 6 = 1{,}200.$$

Problem 5 (*Optimal Consumption*)

The marginal rate of substitution is always lower than the price ratio, i.e. the slope of the indifference curve is less steep than the slope of the budget line.

$$\underbrace{\frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}}}_{\text{MRS}} = \frac{1}{2} < \frac{4}{5} = \frac{p_1}{p_2}$$



Corner Solution

The individual's optimal consumption bundle is $q_1 = 0$ and $q_2 = 20$. Thus, the individual spends her entire budget on good 2.

$\Rightarrow$ **(B)** is correct.

Problem 6-10 (*Optimal Consumption*)

Optimal consumption

(i) The optimal consumption bundle must be located on the budget line, i.e. the budget restriction must bind.

$$y = p_1 q_1 + p_2 q_2$$

(ii) For any interior solution, the indifference curve through the optimal consumption bundle must be tangent to the budget line, i.e. the MRS must equal the price ratio.

$$\underbrace{\frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}}}_{\text{MRS}} = \frac{p_1}{p_2} \quad \Rightarrow \quad \frac{\frac{1}{4} q_1^{-\frac{3}{4}} q_2^{\frac{3}{4}}}{\frac{3}{4} q_1^{\frac{1}{4}} q_2^{-\frac{1}{4}}} = \frac{p_1}{p_2} \quad \Leftrightarrow \quad q_2 = \frac{p_1}{p_2} 3 q_1 \quad \Leftrightarrow \quad q_1 = \frac{p_2}{p_1} \frac{q_2}{3}$$

Substituting (ii) into (i) yields

$$q_1 = \frac{y}{4p_1}, \quad q_2 = \frac{3y}{4p_2}.$$

Substituting $y = 12$ and $p_1 = 1$ yields

$$q_1 = 3, \quad q_2 = \frac{9}{p_2}.$$

Problem 6

If $p_2 = 1$, the individual's optimal consumption bundle is $q_1 = 3$ and $q_2 = 9$.

$\Rightarrow$ **(D)** is correct.

Problem 7

If $p_2 = 3$, the individual's optimal consumption bundle is $q_1 = 3$ and $q_2 = 3$. Thus, the optimal consumption bundle causes expenses of 9 for good 2.

$\Rightarrow$ **(D)** is correct.

Problem 8

Regarding good 1, the total effect of a price increase of good 2 from $p_2 = 1$ to $p_2 = 3$ is zero (see also problems 6 and 7). The substitution effect induces the individual to consume more of good 1 as it becomes relatively less expensive. Thus, the income effect must work in the opposite direction, effectively neutralizing the substitution effect.

$\Rightarrow$ **(C)** is correct.

Problem 9

The income effect of the price increase of good 2 from $p_2 = 1$ to $p_2 = 3$ induces the individual to consume less of good 1 (see also problem 8). Thus, good 1 is a normal good. Regarding good 2, the total effect of the price increase of good 2 from $p_2 = 1$ to $p_2 = 3$ is negative (see also problems 6 and 7). Thus, good 2 is an ordinary good.

$\Rightarrow$ **(A)** is correct.

Problem 10

If $p_2 = 3$, the indifference curve through the consumption bundle $q_1 = 3$ and $q_2 = 3$ is tangent to the budget line as this is the optimal consumption bundle (see also problem 7).

$\Rightarrow$ **(B)** is correct.