Solution 5: Market Failure

Problem 1 (*Monopoly*)

(a) Inverse market demand for sparkling wine per opera season is

$$p(Q) = 10 - \frac{1}{1{,}000}Q.$$

Thus, the seller's revenue is

$$R(Q) = 10Q - \frac{1}{1{,}000}Q^2.$$

The payment for the exclusive right to sell sparkling wine during one opera season is a quasi-fixed cost c^f. If the firm is awarded the contract, its total costs are

$$C(Q) = c^f + 2Q.$$

As a monopolist, the firm solves the following profit maximization problem.

$$\max_Q \quad \pi(Q) = 10Q - \frac{1}{1{,}000}Q^2 - c^f - 2Q$$

The necessary condition

$$\frac{d\pi(Q)}{dQ} = 8 - \frac{1}{500}Q = 0$$

yields the monopoly quantity

$$Q^M = 4{,}000.$$

The corresponding monopoly price is $p^M = 6$, and the monopoly profit is $\pi^M = 16{,}000 - c^f$. The firm is only interested in selling sparkling wine at the opera if this yields a non-negative profit. Thus, the firm's reservation price for the exclusive right of sale during one opera season is $c^f = 16{,}000$.

(b) If the city of Munich levies a tax on the seller of sparkling wine

(i) at the rate $t = 2$ per unit sold, the firm's profit maximization problem is

$$\max_{Q} \quad \pi(Q) = 10Q - \frac{1}{1{,}000}Q^2 - c^f - 2Q - \underbrace{2Q}_{tQ}.$$

The necessary condition

$$\frac{d\pi(Q)}{dQ} = 6 - \frac{1}{500}Q = 0$$

yields the monopoly quantity

$$Q^M = 3{,}000.$$

The corresponding monopoly price is $p^M = 7$, and the monopoly profit after taxes is $\pi^M = 9{,}000 - c^f$. The firm's reservation price for the exclusive right of sale during one opera season is $c^f = 9{,}000$.

(ii) at the rate $t = 0.25$ on profit, the firm's profit maximization problem is

$$\max_{Q} \quad \pi(Q) = (1 - 0.25)\left[10Q - \frac{1}{1{,}000}Q^2 - 2Q - c^f\right].$$

The necessary condition for an interior solution

$$\frac{d\pi(Q)}{dQ} = 0.75\left[8 - \frac{1}{500}Q\right] = 0$$

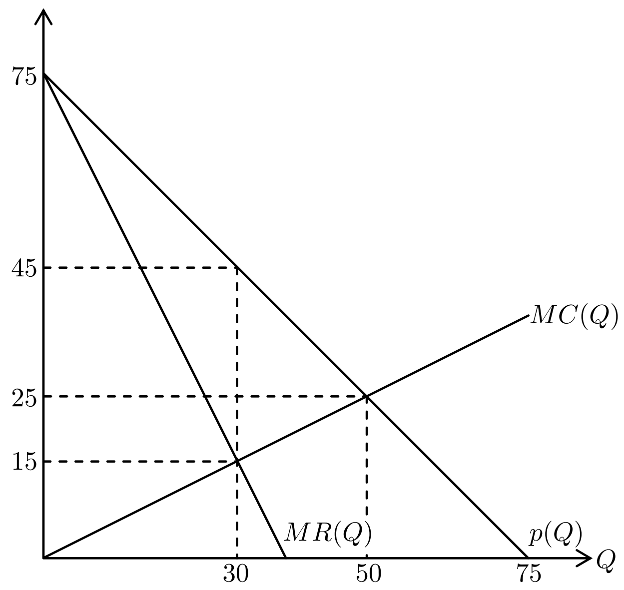
yields the monopoly quantity

$$Q^M = 4{,}000.$$

The corresponding monopoly price is $p^M = 6$, and the monopoly profit after taxes is $\pi^M = 0.75\left(16{,}000 - c^f\right)$. The firm's reservation price for the exclusive right of sale during one opera season is $c^f = 16{,}000$.[1]

[1]Note that for $c^f = 16{,}000$, the firm makes zero profits and thus pays no taxes.

Problems 2-5 (*Monopoly*)



Monopoly

The monopolist's profit maximization problem is

$$\max_{Q} \quad \pi(Q) = 75Q - Q^2 - \frac{1}{4}Q^2 - c^f.$$

The necessary condition for an interior solution

$$\frac{d\pi(Q)}{dQ} = 75 - 2\frac{1}{2}Q = 0$$

yields the monopoly quantity

$$Q^M = 30.$$

The corresponding monopoly price is $p^M = 45$, and the monopoly profit is $\pi^M = 1{,}125 - c^f$. For comparison, the welfare maximizing output for which inverse market demand equals marginal costs is $Q^* = 50$, and the corresponding price is $p^* = 25$.

Problem 2

The monopolist's profit is non-negative if quasi-fixed costs satisfy $c^f \leq 1{,}125$. Thus, the threshold regarding quasi-fixed costs, below which the monopolist's output is $Q > 0$ in the long run is $c^f = 1{,}125$.

$\Rightarrow$ **(C)** is correct.

Problem 3

If fixed costs are $c^f = 625$, the monopolist produces $Q^M = 30$. A welfare loss occurs because the monopoly output Q^M is smaller than the welfare maximizing output Q^*. Hence, for the output difference $Q^* - Q_M$, potential gains from trade exist but are not realized.

The welfare loss is

$$WL = \frac{1}{2} \cdot \left(Q^* - Q^M\right) \cdot \left(p(Q^M) - MC(Q^M)\right) = \frac{1}{2} \cdot (50 - 30) \cdot (45 - 15) = 300.$$

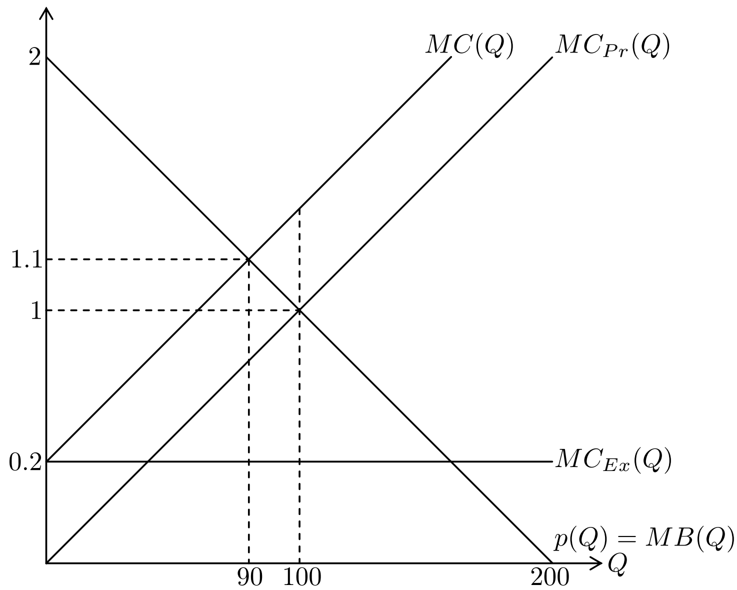
$\Rightarrow$ **(D)** is correct.

Problem 4

If fixed costs are $c^f = 0$, the monopoly output is $Q^M = 30$ and the monopoly price is $p^M = 45$. A price ceiling at $p' = 40$ will induce the monopolist to increase output to $Q^D(p') = 35$. This causes an increase in consumer surplus.

$\Rightarrow$ **(A)** is correct.

Problem 5-8 (*External Effects*)



External Effect

Problem 5

Individual profit maximization implies that the firms do not internalize the negative externalities they impose on each other.

The maximization problem of firm $i \in \{A, B\}$ is

$$\max_{q_i} \quad \pi_i(q_i) = p \cdot q_i - 15 - \frac{1}{100}q_i^2 - \frac{1}{5}q_j.$$

The necessary condition for an interior solution is:

$$\frac{d\pi_i(q_i)}{dq_i} = p - \frac{1}{50}q_i = 0 \quad \Rightarrow \quad p = MC_{Pr}(q_i)$$

Rearranging yields the supply of firm $i \in \{A, B\}$.

$$q_i(p) = 50p$$

For two identical firms, market supply is then

$$Q^S(p) = 100p.$$

The equilibrium price equalizes market demand and market supply.

$$Q^D(p^*) = 200 - 100p^* = 100p^* = Q^S(p^*)$$

Hence, the equilibrium price is $p^* = 1$, and the corresponding equilibrium quantity is $Q^* = 100$, where each firm produces $q^* = 50$, and profit per firm is $\pi^* = 0$.

$\Rightarrow$ **(A)** is correct.

Problem 6

Suppose, hypothetically, that the two firms collectively maximized profit and yet remained price takers. Collective maximization implies that the firms internalize the negative externalities they impose on each other. The resulting equilibrium quantity maximizes welfare as it aligns (social) marginal benefit to (social) marginal costs.

The collective maximization problem of firms A and B is

$$\max_{q_A, q_B} \quad \Pi(q_A, q_B) = p \cdot q_A + p \cdot q_B - 15 - \frac{1}{100}q_A^2 - \frac{1}{5}q_B - 15 - \frac{1}{100}q_B^2 - \frac{1}{5}q_A.$$

The necessary conditions for an interior solution are:

$$\frac{\partial \Pi(q_A, q_B)}{\partial q_A} = p - \frac{1}{50}q_A - \frac{1}{5} = 0 \quad \Rightarrow \quad p = MC_{Pr}(q_A) + MC_{Ex}(q_A)$$

$$\frac{\partial \Pi(q_A, q_B)}{\partial q_B} = p - \frac{1}{50}q_B - \frac{1}{5} = 0 \quad \Rightarrow \quad p = MC_{Pr}(q_B) + MC_{Ex}(q_B)$$

Rearranging yields the supply of firm $i \in \{A, B\}$.

$$q_i(p) = 50p - 10$$

For two identical firms, market supply is then

$$Q^S(p) = 100p - 20.$$

The equilibrium price equalizes market demand and market supply.

$$Q^D(\tilde{p}) = 200 - 100\tilde{p} = 100\tilde{p} - 20 = Q^S(\tilde{p})$$

Hence, the equilibrium price is $\tilde{p} = 1.1$, and the corresponding equilibrium quantity is $\tilde{Q} = 90$, where each firm produces $\tilde{q} = 45$, and profit per firm is $\tilde{\pi} = 5.25$.

Accordingly, the welfare maximizing total quantity is $Q_E = 90$.

$\Rightarrow$ **(C)** is correct.

Problem 7

A welfare loss occurs because the equilibrium quantity Q^*, resulting from individual profit maximization, exceeds the welfare maximizing quantity Q_E. Hence, for the exceeding quantity $Q^* - Q_E$, (social) marginal costs exceed (social) marginal benefits.

The welfare loss is

$$WL = \frac{1}{2} \cdot (Q^* - Q_E) \cdot MC_{Ex}(Q^*) = \frac{1}{2} \cdot (100 - 90) \cdot \frac{1}{5} = 1.$$

$\Rightarrow$ **(B)** is correct.

Problem 8

Since the rate of this (Pigouvian) tax equals external marginal costs, individual profit maximization implies that firms effectively internalize the negative externalities they impose on each other. The resulting equilibrium quantity maximizes welfare as it aligns (social) marginal benefit to (social) marginal costs.

The maximization problem of firm $i \in \{A, B\}$ is

$$\max_{q_i} \quad \pi_i(q_i) = p \cdot q_i - 15 + S - \frac{1}{100}q_i^2 - \frac{1}{5}q_j - \underbrace{\frac{1}{5}q_i}_{tq_i}.$$

The necessary condition for an interior solution is:

$$\frac{d\pi_i(q_i)}{dq_i} = p - \frac{1}{50}q_i - \underbrace{\frac{1}{5}}_{t} = 0 \quad \Rightarrow \quad p = MC_{Pr}(q_i) + \underbrace{MC_{Ex}(q_i)}_{t}$$

Rearranging yields the supply of firm $i \in \{A, B\}$

$$q_i(p) = 50p - 10.$$

For two firms, market supply is then

$$Q^S(p) = 100p - 20.$$

The equilibrium price equalizes market demand and market supply.

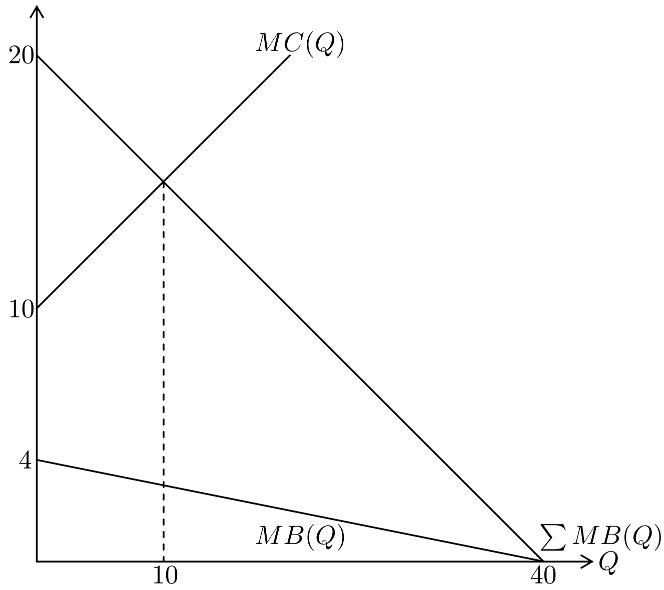
$$Q^D(p^*) = 200 - 100p^* = 100p^* - 20 = Q^S(p^*)$$

Hence, the equilibrium price is $p^* = 1.1$, and the corresponding equilibrium quantity is $Q^* = 90$, where each firm produces $q^* = 45$, and profit per firm is $\pi^* = S - 3.75$.

It follows that firms make zero profits if the lump-sum subsidy is $S = 3.75$.

$\Rightarrow$ **(A)** is correct.

Problem 9-10 (*Public Goods*)



Public Good

Problem 9

Suppose, hypothetically, that the five individuals collectively provided the public good in order to maximize total surplus, i.e. welfare.

The necessary condition for an interior solution of the collective maximization problem, i.e. the Samuelson condition, is:

$$\sum MB(Q_E) = MC(Q_E)$$
$$5 \cdot MB(Q_E) = MC(Q_E)$$

Substituting yields:

$$20 - \frac{1}{2}Q_E = 10 + \frac{1}{2}Q_E \quad \Rightarrow \quad Q_E = 10$$

$\Rightarrow$ **(B)** is correct.

Problem 10

Individual provision of the public good implies that each individual maximizes individual surplus.

The necessary condition for an interior solution of the individual maximization problem is:

$$MB(Q^*) = MC(Q^*)$$

Substituting shows that the necessary condition is never satisfied since

$$4 - \frac{1}{10}Q < 10 + \frac{1}{2}Q \quad \forall \quad Q \geq 0.$$

Hence, no individual contributes to the provision of the public good. Consequently, the resulting quantity is $Q = 0$ (corner solution).

A welfare loss occurs because individual maximization implies an underprovision of the public good; the public good is not provided at all, although, (social) marginal benefits exceed (social) marginal costs up to the welfare maximizing quantity Q_E.

The welfare loss is

$$WL = \frac{1}{2} \cdot Q_E \cdot (5 \cdot MB(0) - MC(0)) = \frac{1}{2} \cdot 10 \cdot 10 = 50.$$

$\Rightarrow$ **(C)** is correct.