Solution 4: Perfect Competition

Problem 1 (*Competitive Equilibrium*)

(a) If a firm supplies an output $q > 0$, i.e. in case of an interior solution of its profit maximization problem, the firm's quantity must satisfy the following condition.

$$p = MC(q)$$
$$p = 2q \quad \Leftrightarrow \quad q = \frac{1}{2}p$$

The threshold price above which a firm's supply is $q > 0$ in the long run corresponds to the quantity for which marginal costs equal average total costs, i.e. where average total costs reach their minimum.

$$MC(q) = 2q = \frac{c^f}{q} + q = AC(q) \quad \Rightarrow \quad q = \sqrt{c^f}$$

Thus, the threshold price is

$$p = MC\left(\sqrt{c^f}\right) \quad \Rightarrow \quad p = 2\sqrt{c^f}.$$

Then, individual supply is

$$q(p) = \begin{cases} \frac{1}{2}p, & p \geq 2\sqrt{c^f} \\ 0, & p < 2\sqrt{c^f}, \end{cases}$$

and market supply is

$$Q^S(p) = \begin{cases} \frac{n}{2}p, & p \geq 2\sqrt{c^f} \\ 0, & p < 2\sqrt{c^f}. \end{cases}$$

In any competitive equilibrium, market demand equals market supply:

$$Q^D(p) = a - p = \frac{n}{2}p = Q^S(p).$$

The number of firms implying zero profits for each firm in equilibrium equalizes market demand and market supply at the threshold price.

$$Q^D\left(2\sqrt{c^f}\right) = a - 2\sqrt{c^f} = n\sqrt{c^f} = Q^S\left(2\sqrt{c^f}\right) \quad \Rightarrow \quad n = \frac{a}{\sqrt{c^f}} - 2$$

The number of firms must be a non-negative integer. Thus, the equilibrium number of firms as a function of a and c^f is

$$n^* = \max\left\{\left\lfloor \frac{a}{\sqrt{c^f}} - 2 \right\rfloor, 0\right\}.$$

This function is piecewise continuous.[1]

[1] The floor function $\lfloor x \rfloor$ gives the greatest integer less than or equal to x.

(b) Rearranging the condition for a competitive equilibrium and using n^* yields the equilibrium price

$$p^* = \frac{2a}{n^* + 2}.$$

Substituting the equilibrium price into market demand or market supply yields equilibrium output.

$$Q^* = \frac{an^*}{n^* + 2}$$

It follows that equilibrium output per firm is

$$q^* = \frac{a}{n^* + 2},$$

and equilbrium profit per firm is

$$\pi^* = \frac{a^2}{(n^* + 2)^2} - c^f.$$

(i) If $a = 120$ and $c^f = 100$, the equilibrium number of firms is $n^* = 10$, and the equilibrium price is $p^* = 20$. The corresponding equilibrium quantity is $Q^* = 100$, where each firm produces $q^* = 10$. It follows that profit per firm is $\pi^* = 0$.

(ii) If $a = 120$ and $c^f = 64$, the equilibrium number of firms is $n^* = 13$, and the equilibrium price is $p^* = 16$. The corresponding equilibrium quantity is $Q^* = 104$, where each firm produces $q^* = 8$. It follows that profit per firm is $\pi^* = 0$.

(iii) If $a = 126$ and $c^f = 100$, the equilibrium number of firms is $n^* = 10$, and the equilibrium price is $p^* = 21$. The corresponding equilibrium quantity is $Q^* = 105$, where each firm produces $q^* = 10.5$. It follows that profit per firm is $\pi^* = 10.25$.

(c) A firm's total cost plus tax payment is

$$C(q) + tq = \begin{cases} c^f + q^2 + tq, & q > 0 \\ 0, & q = 0. \end{cases}$$

If a firm supplies an output $q > 0$, i.e. in case of an interior solution of its profit maximization problem, the firm's quantity must satisfy the following condition.

$$\begin{aligned} p &= MC(q) + t \\ p &= 2q + t \quad \Leftrightarrow \quad q = \frac{1}{2}(p - t) \end{aligned}$$

The threshold price above which a firm's supply is $q > 0$ in the long run corresponds to the quantity for which marginal costs plus tax rate equal average total costs plus tax rate, i.e. where average total costs reach their minimum.

$$MC(q) + t = 2q + t = \frac{c^f}{q} + q + t = AC(q) + t \quad \Rightarrow \quad q = \sqrt{c^f}$$

Thus, the threshold price is

$$p = MC\left(\sqrt{c^f}\right) + t \quad \Rightarrow \quad p = 2\sqrt{c^f} + t.$$

Then, individual supply is

$$q(p) = \begin{cases} \frac{1}{2}(p - t), & p \geq 2\sqrt{c^f} + t \\ 0, & p < 2\sqrt{c^f} + t, \end{cases}$$

and market supply is

$$Q^S(p) = \begin{cases} \frac{n}{2}(p - t), & p \geq 2\sqrt{c^f} + t \\ 0, & p < 2\sqrt{c^f} + t. \end{cases}$$

In any competitive equilibrium, market demand equals market supply:

$$Q^D(p) = a - p = \frac{n}{2}(p - t) = Q^S(p).$$

The number of firms implying zero profits for each firm in equilibrium equalizes market demand and market supply at the threshold price.

$$Q^D\left(2\sqrt{c^f}+t\right) = a - 2\sqrt{c^f} - t = n\sqrt{c^f} = Q^S\left(2\sqrt{c^f}+t\right)$$

$$\Rightarrow \quad n = \frac{a-t}{\sqrt{c^f}} - 2$$

The number of firms must be a non-negative integer. Thus, the equilibrium number of firms as a function of a, c^f, and t is

$$n^* = \max\left\{\left\lfloor \frac{a-t}{\sqrt{c^f}} - 2\right\rfloor, 0\right\}.$$

(d) If $a = 126$, $c^f = 100$, and $t = 6$, the equilibrium number of firms is $n^* = 10$, and the equilibrium price is $p^* = 26$. The corresponding equilibrium quantity is $Q^* = 100$, where each firm produces $q^* = 10$. It follows that profit per firm is $\pi^* = 0$, tax revenue is $T = 600$, and the welfare loss of taxation is $WL = 15$[2].

[2] When $a = 126$, $c^f = 100$, and $t = 0$, the equilibrium number of firms is $n^* = 10$, and output is $Q^* = 105$ (see also (b) (iii)). Thus, the change in tax rate from $t = 0$ to $t = 6$ does not affect the equilibrium number of firms but reduces equilibrium output by 5 units implying a welfare loss of $WL = \frac{1}{2} \cdot 5 \cdot 6 = 15$. Graphically, the welfare loss is the area of a triangle with the output reduction as basis and the tax rate as height.

Problem 2-6 (*Competitive Equilibrium*)

If a firm supplies an output $q > 0$, i.e. in case of an interior solution of its profit maximization problem, the firm's quantity must satisfy the following condition.

$$\begin{aligned} p &= MC(q) \\ p &= 20 + \frac{1}{2}q \quad \Leftrightarrow \quad q = 2p - 40 \end{aligned}$$

The threshold price above which a firm's supply is $q > 0$ in the long run corresponds to the quantity for which marginal costs equal average total costs, i.e. where average total costs reach their minimum.

$$MC(q) = 20 + \frac{1}{2}q = \frac{25}{q} + 20 + \frac{1}{4}q = AC(q) \quad \Rightarrow \quad q = 10$$

Thus, the threshold price is

$$p = MC(10) \quad \Rightarrow \quad p = 25.$$

Then, individual supply is

$$q(p) = \begin{cases} 2p - 40, & p \geq 25 \\ 0, & p < 25, \end{cases}$$

and market supply is

$$Q^S(p) = \begin{cases} n(2p - 40), & p \geq 25 \\ 0, & p < 25. \end{cases}$$

Problem 2

The number of firms implying zero profits for each firm in equilibrium equalizes market demand and market supply at the threshold price.

$$Q^D(25) = 125 - 25 = n(2 \cdot 25 - 40) = Q^S(25) \quad \Rightarrow \quad n^* = 10$$

$\Rightarrow$ **(B)** is correct.

Problem 3

If the equilibrium number of firms is $n^* = 10$, the equilibrium price is $p^* = 25$ and the equilibrium quantity is $Q^* = 100$. Then, consumer surplus and producer surplus are given by

$$CS = \frac{1}{2} \cdot (125 - 25) \cdot 100 = 5{,}000 \quad \text{and} \quad PS = \frac{1}{2} \cdot (25 - 20) \cdot 100 = 250.$$

$\Rightarrow$ **(B)** is correct.

Problem 4

At the equilibrium price $p^* = 25$, each firm produces $q = 10$ and makes profits $\pi = 0$. A price ceiling below the equilibrium price causes losses for any firm that produces $q > 0$, so that no production will take place in the long run and thus no surplus is realized. Hence, a price ceiling at $p' = 20$ results in a welfare loss equal to total surplus in the equilibrium allocation without the price ceiling $TS = CS + PS = 5{,}000 + 250 = 5{,}250$.

$\Rightarrow$ **(D)** is correct.

Problem 5

If the equilibrium price is $p^* = 25$, the introduction of a price floor at $p'' = 20$ does not affect the allocation, so that the resulting welfare loss is 0.

$\Rightarrow$ **(A)** is correct.

Problem 6

A lump-sum subsidy $S = 24$ for each firm that produces $q > 0$ effectively lowers quasi-fixed costs, so that a firm's total costs minus the subsidy are

$$C(q) - S = \begin{cases} 1 + 20q + \frac{1}{4}q^2, & q > 0 \\ 0, & q = 0. \end{cases}$$

It follows that market supply is

$$Q^S(p) = \begin{cases} n(2p - 40), & p \geq 21 \\ 0, & p < 21. \end{cases}$$

The number of firms implying zero profits for each firm in equilibrium equalizes market demand and market supply at the threshold price.

$$Q^D(21) = 125 - 21 = n(2 \cdot 21 - 40) = Q^S(21) \quad \Rightarrow \quad n^* = 52$$

$\Rightarrow$ **(D)** is correct.