Advanced Machine Learning – Deep Generative Models Exercise Sheet 01

Normalizing Flows

Problem 1: Consider the following transformation $f\colon \mathbb{R}^3 \to \mathbb{R}^3$

$$f(\boldsymbol{z}) = \begin{bmatrix} 2z_1 \\ e^{z_1} z_2 \\ e^{-z_1 - z_2} z_3 \end{bmatrix}.$$

Prove or disprove whether the transformation f is invertible.

Problem 2: Consider the following transformation $f\colon \mathbb{R}^2 \to \mathbb{R}^2$:

$$f(\boldsymbol{z}) = \begin{bmatrix} z_1^2 z_2 \\ z_2^3 \end{bmatrix}.$$

Prove or disprove whether the transformation f is invertible.

Problem 3: Consider the tranformation $f(\boldsymbol{z}) = \boldsymbol{A}\boldsymbol{z} + \boldsymbol{b}$ from $\mathbb{R}^2$ to $\mathbb{R}^2$, where $\boldsymbol{A} \in \mathbb{R}^{2\times 2}$ and $\boldsymbol{b} \in \mathbb{R}^2$. Under what conditions on $\boldsymbol{A}$ and $\boldsymbol{b}$ is this tranformation invertible? Justify your answer.

Problem 4: We consider the following forward tranformation $f\colon \mathbb{R}^3 \to \mathbb{R}^3$

$$\boldsymbol{x} = f(\boldsymbol{z}) = \begin{bmatrix} z_1 \\ e^{z_1} z_2 \\ \sqrt[3]{e^{-z_1} z_3 + z_1^2} \end{bmatrix}.$$

We assume a uniform base distribution $p_1(\boldsymbol{z}) = U([0,2]^3)$. Evaluate the density $p_2(\boldsymbol{x})$ at the points $\boldsymbol{x}^{(1)} = \begin{bmatrix} 0 \\ 1 \\ \frac{1}{3} \end{bmatrix}$ and $\boldsymbol{x}^{(2)} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$.

Problem 5: We consider the following forward transformation $x = f(z) = \sum_{k=1}^{K} \sigma(kz)$ from $\mathbb{R}$ to $(0, K)$ with $\sigma(z) = \frac{1}{1+e^{-z}}$. We assume a Gaussian base distribution $p_1(z) = \mathcal{N}(0,1)$. We sampled one point from the base distribution $z^{(1)} = 0$. Compute the corresponding sample $x^{(1)}$ from the transformed distribution and evaluate its density $p_2(x^{(1)})$.