Christian B. Mendl, Pedro Hack, Keefe Huang, Irene López Gutiérrez

Exercise 1.1 (Complex number arithmetic)
This exercise should refresh your knowledge and proficiency with complex numbers. Given $a = 3 + 4i$ and $b = 2 - i$:

(a) Compute

- $a + b$
- ab (product of a and b)
- $1/a$
- a^* (complex conjugate of a)
- $|a|$ and $\arg(a)$ (argument), such that $a = |a|\,\mathrm{e}^{i\arg(a)}$
- the Euclidean length of the vector $\psi = \begin{pmatrix} a \\ b \end{pmatrix}$, denoted $\|\psi\|$

(b) Draw a in the complex plane, and interpret a^*, $|a|$ and $\arg(a)$ geometrically.

(c) How can one construct $a + b$ and ab geometrically in the complex plane?

Solution

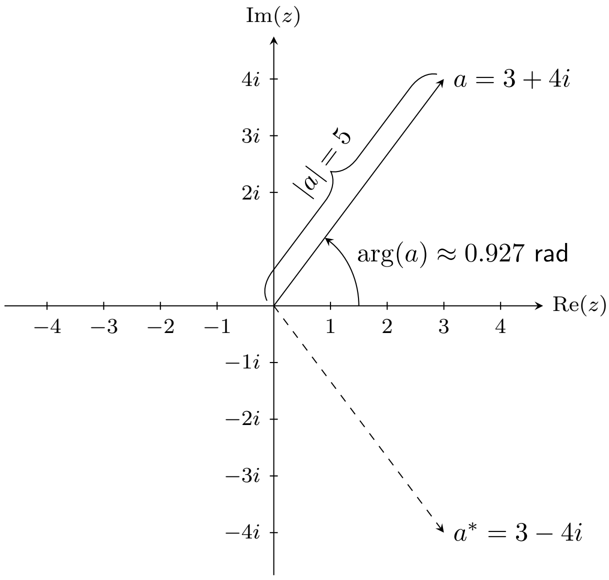
(a) Given $a = 3 + 4i$ and $b = 2 - i$, the following expressions are equal to

- $a + b = 5 + 3i$
- $ab = 10 + 5i$
- $1/a = \frac{a^*}{|a|^2} = \frac{3}{25} - \frac{4}{25}i$
- $a^* = 3 - 4i$
- $|a| = \sqrt{aa^*} = \sqrt{\mathrm{Re}(a)^2 + \mathrm{Im}(a)^2} = 5$, $\arg(a) = \arctan(4/3) \approx 0.927$ rad
- In general, the norm of a complex vector $\psi = (\psi_1, \psi_2, \dots, \psi_n)$ is defined as

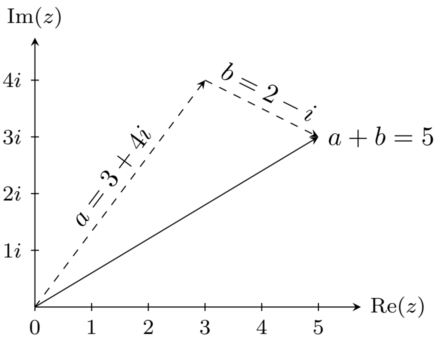
$$\|\psi\| = \sqrt{\sum_{i=1}^{n} |\psi_i|^2}.$$

Here $\|\psi\| = \sqrt{|a|^2 + |b|^2} = \sqrt{25 + 5} = \sqrt{30}$.

(b) Drawing a in the complex plane:



(c) Drawing $a + b$ in the complex plane:



Drawing ab in the complex plane: