Principles of Economics

# Chapter 3: Production and Supply

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# Agenda

3. Production and Supply
   - Cost Minimization
   - Profit Maximization
   - Individual Supply
   - Market Supply

**Reading:**

- Mankiw/Taylor (2020), Chapters 5, 10
- Varian (2014), Chapters 19-24

# Model

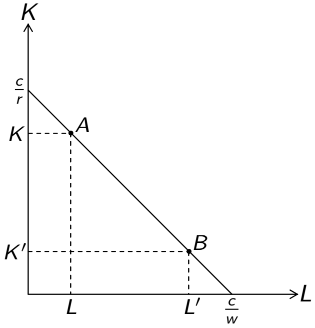
**Framework:** Consider a representative firm.

- The firm produces $q$ units of a particular good (output) employing two factors of production (inputs); $L$ denotes the quantity of labor, while $K$ denotes the quantity of capital.
- The firm is a price taker: It considers the output price $p$ as well as the input prices $w$ for labor and $r$ for capital as given.
- The firm's input costs are $c = wL + rK$.
- The firm's revenue is $R(q) = pq$.

# Input Costs

**Isocost Line:** Locus of all input bundles $(L, K)$ causing identical input costs

$$K = \frac{c}{r} - \frac{w}{r} L$$

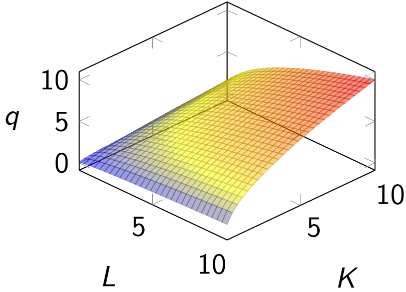



**Input Price Ratio:** Rate at which the firm can substitute one input for another at constant input costs

# Production

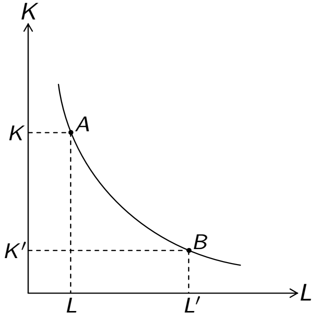
**Production Function:** The function $F(L, K)$ represents the firm's production technology; it expresses the maximum output $q$ the firm can produce from any given input bundle $(L, K)$.

**Example:** $q = F(L, K) = (L \cdot K)^{\frac{1}{2}}$



# Production

**Isoquant:** Locus of all input bundles $(L, K)$ that yield the same output $q = F(L, K)$




**Marginal Rate of Technical Substitution:** Rate at which the firm can substitute one input for another at constant output

$$\mathrm{MRTS}_{L,K} = \frac{\partial F/\partial L}{\partial F/\partial K}$$

# Assumptions on Technology

**Monotonicity:** If input bundle $A$ contains more of each input than input bundle $B$, then $A$ yields more output than $B$. If input bundle $A$ contains more of at least one input and not less of another, then $A$ yields at least as much output as $B$. If in the latter case, $A$ always yields more output than $B$, the technology is strictly monotonous.

- If the technology is strictly monotonous, then isoquants are negatively sloped.

**Convexity:** If input bundles $A$ and $B$ yield the same output, then any weighted average of $A$ and $B$ yields at least as much output as $A$ or $B$. If any weighted average of $A$ and $B$ yields more output than $A$ or $B$, the technology is strictly convex.

- If the technology is (strictly) convex, then isoquants are (strictly) convex.

# Extreme Cases of Technology

**Perfect Substitutes:** Two inputs that can be substituted for one another at a constant rate while output remains constant

- Linear isoquants

**Perfect Complements:** Two inputs that should be employed in fixed proportions

- Orthogonal isoquants

# Scaling of Production

**Returns to Scale:** If all inputs are multiplied by a constant $\lambda$, the resulting change in output can be proportional, more than proportional, or less than proportional. The production function $F(L, K)$ exhibits

- constant returns to scale if

$$F(\lambda L, \lambda K) = \lambda F(L, K) \quad \text{for any} \quad \lambda > 0,$$

- increasing returns to scale if

$$F(\lambda L, \lambda K) > \lambda F(L, K) \quad \text{for any} \quad \lambda > 1,$$

- decreasing returns to scale if

$$F(\lambda L, \lambda K) < \lambda F(L, K) \quad \text{for any} \quad \lambda > 1.$$

# Cost Minimum

**Optimization Problem:** The firm minimizes input costs with respect to input employment subject to a given output.

$$\min_{L,K} \quad c = wL + rK \quad \text{s.t.} \quad q = F(L, K)$$

Any interior solution of the minimization problem must satisfy the following conditions:

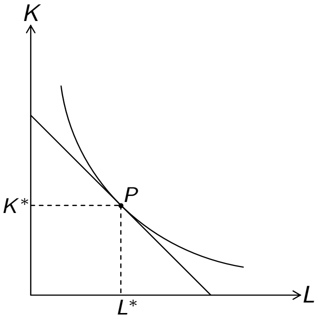
$$q = F(L, K),$$

$$\mathrm{MRTS}_{L,K} = \frac{\partial F / \partial L}{\partial F / \partial K} = \frac{w}{r}.$$

# Cost Minimum

**Interior Solution:** The rate at which the firm can substitute labor for capital at constant output must equal the rate at which it can substitute labor for capital at constant input costs.

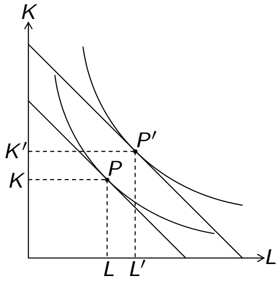
- In the optimal input bundle, the slope of the isoquant equals the slope of the isocost line.



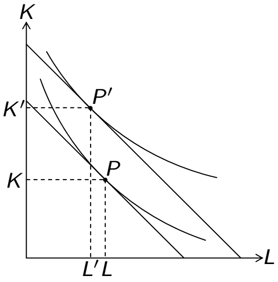
# Cost Minimum

**Change in Output:** An increase in output results in higher input costs and vice versa; $\frac{\partial c}{\partial q} > 0$.

- In order to increase output, more of at least one input must be employed.




More of both inputs




More Capital, less Labor

# Production Costs

**Total Costs:** Sum of fixed and variable costs

- Fixed costs $c^f$ are independent of output.
- Variable costs $c(q)$ represent minimum input costs as a function of output.

$$C(q) = c^f + c(q)$$

**Average Costs:** Costs per unit of output

- Let $AC(q)$ denote average total costs, and let $ac(q)$ denote average variable costs.

$$AC(q) = \frac{C(q)}{q} = \frac{c^f}{q} + \underbrace{\frac{c(q)}{q}}_{ac(q)}$$

# Production Costs

**Short-Run Total Costs:** In the short run, fixed costs are sunk costs.

- Sunk Costs: Incurred costs that cannot be recovered

$$C(q) = c^f + c(q), \quad q \geq 0$$

**Long-Run Total Costs:** In the long run, non-variable costs must be quasi-fixed costs.

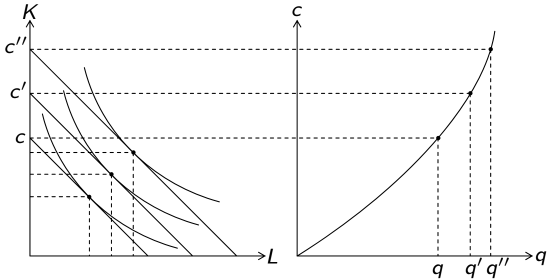
- Quasi-Fixed Costs: Costs that arise if the firm starts production but do not vary as output increases

$$C(q) = \begin{cases} c^f + c(q), & q > 0 \\ 0, & q = 0 \end{cases}$$

# Production Costs

**Variable Cost Curve:** Variable costs increase, as output increases and vice versa; $\frac{dc(q)}{dq} > 0$

- The price of capital is normalized to $r = 1$.




Cost Minimization & Variable Costs

# Marginal Effects of Production

**Marginal Costs:** Change in total costs resulting from a marginal change in output

$$MC(q) = \frac{dC(q)}{dq}$$

**Marginal Revenue:** Change in revenue resulting from a marginal change in output

$$MR(q) = \frac{dR(q)}{dq}$$

# Profit Maximum

**Optimization Problem**: The firm maximizes its profit $\pi(q)$ with respect to output $q$.

$$\max_{q} \quad \pi(q) = R(q) - C(q)$$

Any interior solution of the maximization problem must satisfy the following condition:

$$MR(q) = MC(q).$$

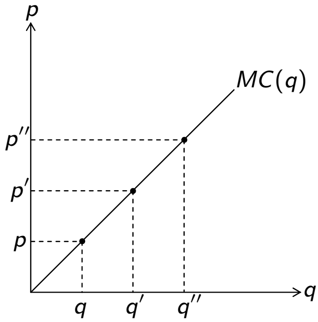
For a price-taking firm, marginal revenue equals the output price. Hence, an interior solution requires

$$p = MC(q).$$

# Profit Maximum

**Marginal Cost Curve:** Assume that costs are strictly convex. Hence, marginal costs increase as output increases and vice versa; $\frac{dMC(q)}{dq} > 0$.

- Interior Solution: An increase in the output price results in a higher profit maximizing output and vice versa; $\frac{\partial q}{\partial p} > 0$.




Profit Maximizing Output

# Individual Supply Curve

**Optimal Production:** The firm supplies either the output $q > 0$ satisfying the condition $p = MC(q)$ or the output $q = 0$.

- In the short run, the firm supplies $q > 0$ if and only if

$$R(q) \geq c(q) \quad \Leftrightarrow \quad p \geq ac(q).$$

- In the long run, the firm supplies $q > 0$ if and only if

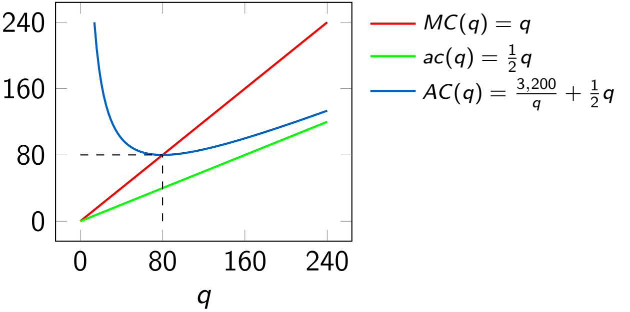
$$R(q) \geq C(q) \quad \Leftrightarrow \quad p \geq AC(q).$$

**Individual Supply Curve:** The short run (long run) individual supply curve corresponds to the segment of the marginal cost curve which runs above the average variable (average total) cost curve.

# Individual Supply Curve

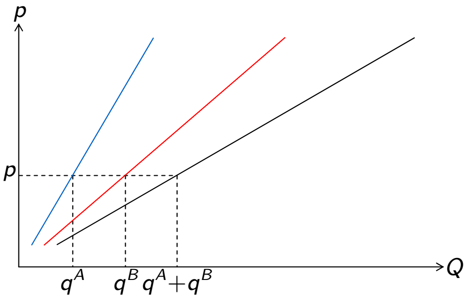
**Example:** Consider a firm with total costs $C(q) = 3{,}200 + \frac{1}{2}q^2$

- Short run individual supply: $q = p$
- Long run individual supply: $q = \begin{cases} p, & p \geq 80 \\ 0, & p < 80 \end{cases}$



# Market Supply Curve

**Market Supply:** Sum of individual supply quantities of a good; $Q = \sum q$




Individual & Market Supply Curves

**Law of Supply:** Empirical observation that, ceteris paribus, the market supply of a good increases when its price increases; $\frac{\partial Q}{\partial p} > 0$