IB9KC Financial Econometrics Group Project Report

Group 1

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1. Part I: Portfolio Analysis and Asset Pricing Tests

Value and momentum are two well-known investment strategies extensively studied in the finance literature. The value strategy seeks to identify undervalued stocks relative to their fundamentals. In contrast, the momentum strategy focuses on stocks that have exhibited past solid performance and expect to continue in the future. This part of the project investigates the performance and risk characteristics of value and momentum portfolios across four markets: the United States; the United Kingdom; Europe; and Japan.

In this project, we will first examine the correlation structure to determine whether the risk-based explanation for value and momentum is more relevant than the behavioural one. Next, we will report the annualised Sharpe ratio of each market's value and momentum portfolios and compare the performance of an equal-weighted 50-50 combination of value and momentum within each market. Third, we will conduct a mean-variance analysis using all the value and momentum portfolios and the risk-free rate. Furthermore, using the Fama-Macbeth procedure, we will estimate the risk premium of the RMRF, SMB, HML, and UMD. Finally, we will conduct the GRS test for the asset pricing model containing the above four factors. Overall, this project provides a comprehensive analysis of the performance and risk characteristics of value and momentum portfolios across markets while also focusing on estimating the risk premium of the four factors.

For this project, we collected monthly return data on eight value and momentum portfolios across four global markets. The data covers 276 months, from January 1990 to December 2012. Additionally, we also collected monthly return data on the risk-free interest rate and four prevalent factors: RMRF (the excess return of the market portfolio over the risk-free rate), SMB (the small minus big), HML (the high minus low), and UMD (the up minus down) – as commonly used in the asset pricing literature (Asness, Moskowitz and Pedersen, 2013; Fama and French, 2012; Fama and French, 2018; Kewei, Chen and Lu, 2015). The data was sourced from reliable databases and underwent appropriate cleaning.

a. Analysis on Correlations in Value and Momentum Portfolios

We conducted a correlation analysis in Part 1a to distinguish between risk-based and behavioural explanations for value and momentum portfolios. A risk-based explanation seeks to explain the behaviour of financial assets based on their level of risk (Chih-Wen, Chun, and Sam, 2017). On the other hand, a behavioural explanation assumes that investors may not always act rationally and that their investment decisions are influenced by psychological biases or heuristics (Chih-Wen, Chun, and Sam, 2017). Therefore, if a strong correlation structure is observed between momentum and value portfolios, even in unrelated asset classes, it may suggest the presence of universal global risk factors that reward value and momentum premiums (Asness, Moskowitz and Pedersen, 2013). Figure 1 below displays the correlations between value and momentum portfolios.

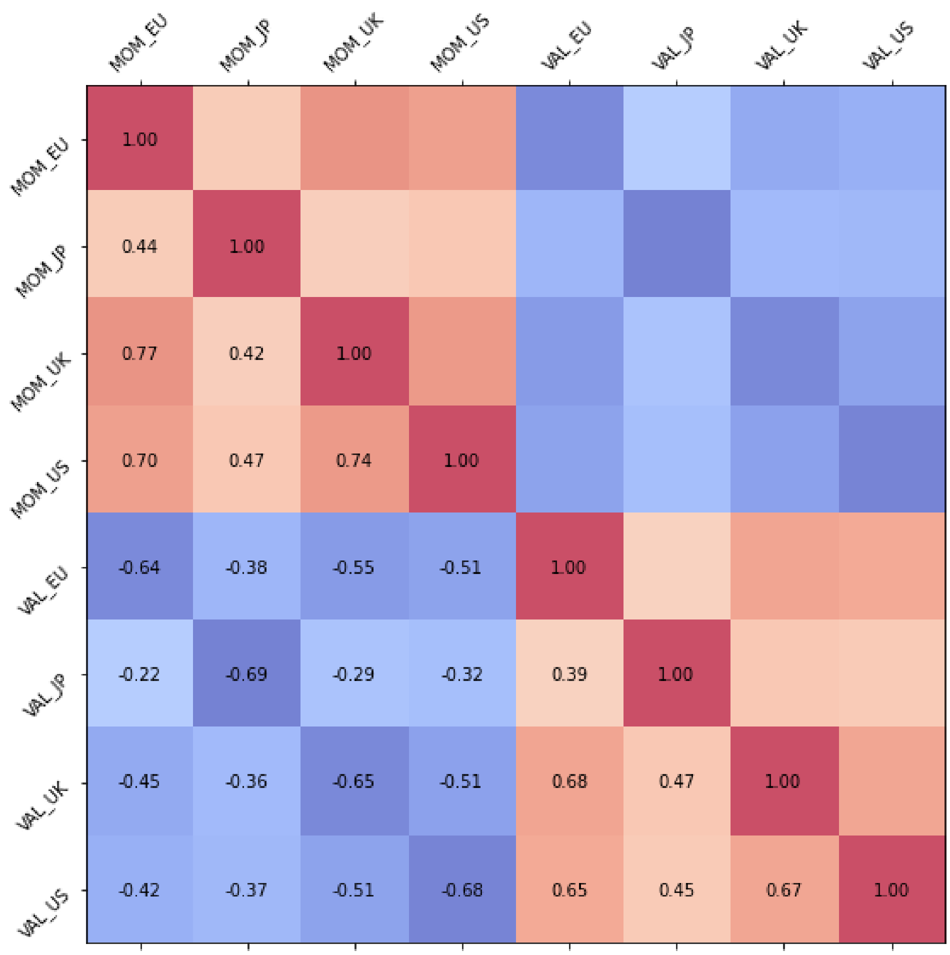

Figure 1. Correlations Between Value and Momentum Portfolios

Looking at the blue area at the bottom, value and momentum strategies tend to correlate negatively. These effects in different asset classes are difficult to explain using existing behavioural theories. The high return and Sharpe ratio of a globally diversified value and momentum portfolio are challenging for irrational behaviour-based models. These observations suggest that the correlation between value and momentum strategies may reflect common underlying risk factors not explicitly captured by behavioural explanation. Besides, the UK, US, and EU have relatively higher positive correlations within value and momentum portfolios. It may suggest that geographical factors and connections between different economies could be crucial. Overall, value and momentum effects in multiple asset classes argue for a more general framework to explain these anomalies. (Asness, Moskowitz, and Pedersen, 2013).

b. Sharpe Ratio of Value, Momentum, and Combined Portfolio

In Part 1b of this report, we aim to evaluate the performance of value and momentum portfolios in different markets by computing their annualised Sharpe ratios. We create an equal-weighted 50-50 combination of value and momentum within each market and calculate the annualised Sharpe ratio of the combined portfolios. We aim to compare the Sharpe ratios of the combination portfolios against those of the value and momentum portfolios and provide insights into their relative performance. These comparisons will give us more comprehensive insights into the performance of the value and momentum portfolios. Furthermore, we will apply two different methodologies to calculate Sharpe ratio.

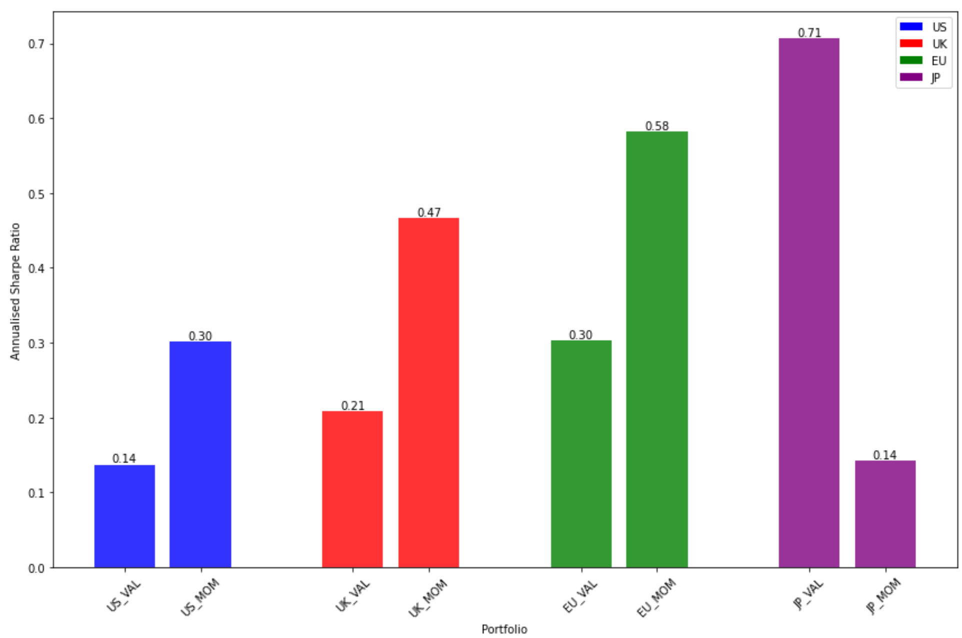

Figure 2. Annualised Sharpe Ratios of Momentum and Value Portfolios by Country

Figure 2 displays the “annualised Sharpe ratio” for the value and momentum portfolios using a monthly basis Sharpe ratio, which means it considers the volatility of the returns within each month. Then, the monthly basis Sharpe ratio is annualised by applying a scaling factor of the square root of 12 to retrieve the "annualised Sharpe ratio".

$$Annualised\ Sharpe\ ratio \ = \sqrt{12} \times \frac{E[return_{monthly}]}{SD(return_{monthly})}$$

The momentum portfolio had a higher “annualised Sharpe ratio” than the value portfolio in all other countries except Japan, where the momentum portfolio had a lower “annualised Sharpe ratio”. Japan's value portfolio had the highest “annualised Sharpe ratio” at 0.71, while the US value portfolio and Japan's momentum portfolio had the lowest “annualised Sharpe ratio” at 0.14.

Japan's momentum portfolio had a lower “annualised Sharpe ratio” than its value portfolio and others. This result is significant because past research has attempted to explain the underperformance of momentum in Japan (Chui, Titman, and Wei, 2010). However, these explanations must also account for the outstanding performance of value during the same period and the negative correlation (-0.69) between value and momentum in Japan over that time. Asness (2011) sought to explain Japan's momentum portfolio's relatively low “annualised Sharpe ratio” using the Fama-French three-factor model.

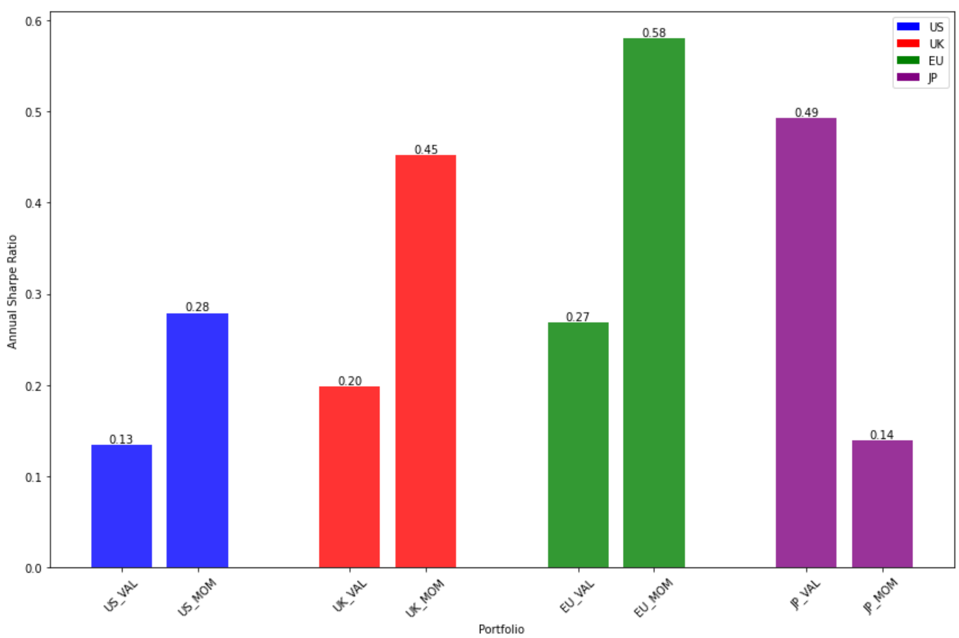

Figure 3. Annual Sharpe Ratios of Momentum and Value Portfolios by Country

Figure 3 shows the results of calculating the Sharpe ratio using an annual basis with the total period annual return. It is noteworthy that the “annualised Sharpe ratio” of Japan's value portfolio was 0.71 in the previous calculation but the "annual Sharpe ratio" fell to 0.49. The strong negative autocorrelations in the monthly returns could address this observation (Lo, 2002). The paper suggests that a monthly Sharpe ratio could not be annualised directly by multiplying a factor of the square root of 12 since autocorrelations exist within the time series of returns. On the one hand, if positive autocorrelations exist, the scaling factor should be higher than the square root of 12. On the other hand, if negative autocorrelations exist, the scaling factor should be lower than the square root of 12.

$$Annual\ Sharpe\ ratio\ = \frac{E(return_{annual})}{SD(return_{annual})}$$

It is important to note that the Sharpe ratio is only one measure of portfolio performance, and different calculation methods may produce different results. It is essential to use multiple performance measures and consider each measure's limitations when evaluating portfolio performance.

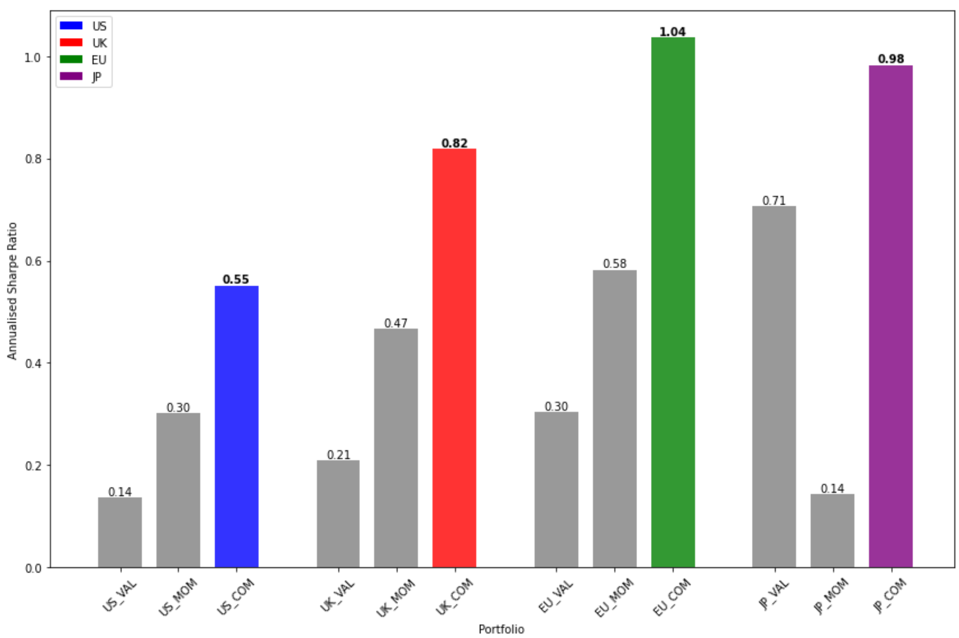

Figure 4. Annualised Sharpe Ratios of Combined Portfolios from Monthly Returns

Figure 4 shows the "annualised Sharpe ratio" of each country's value and momentum portfolios and the portfolios combined with equal weight. In all countries, the "annualised Sharpe ratio" of the combined portfolio tended to increase relative to the value and momentum portfolios, as it takes advantage of the diversification benefits of combining the two strategies. Value and momentum are two distinct investment strategies based on different factors that perform well in different market conditions. Value investing involves buying undervalued stocks that are trading below their intrinsic value, while momentum investing involves buying stocks that have performed well in the past and are expected to continue to perform well in the future. By combining these two strategies, investors can achieve a higher risk-adjusted return than either strategy alone, as the two strategies tend to have a low correlation. In other words, higher systematic returns on equivalent weighted portfolios than value and momentum portfolios arise from relatively high exposure to market, size, and value factors (Yuliya, Raman, and Grigory, 2012).

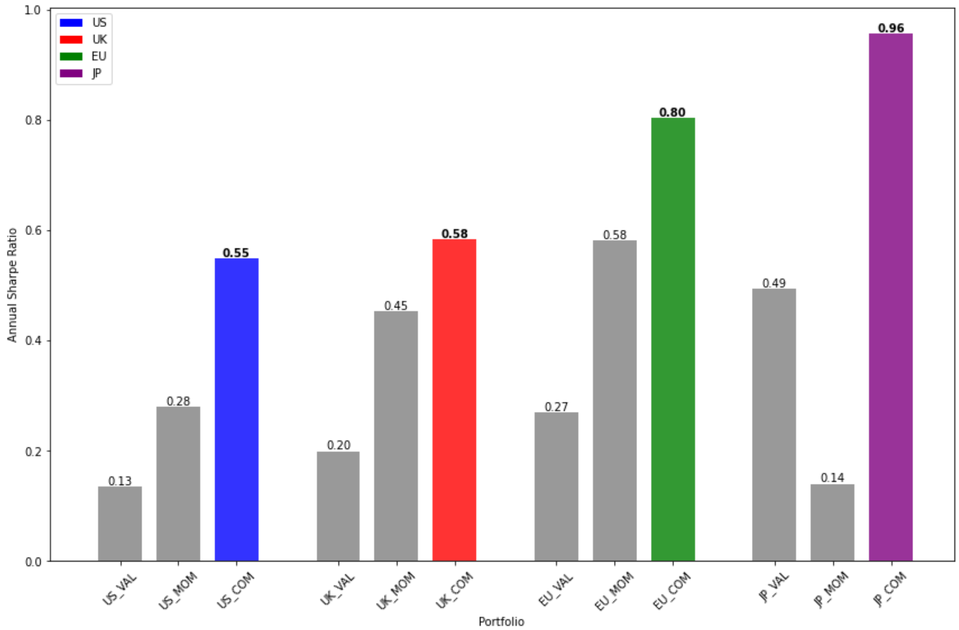

Figure 5. Annual Sharpe Ratios of Combined Portfolios from Monthly Returns

Figure 5 displays the "annual Sharpe ratio" calculated on an annual basis with the total period annual return. The "annual Sharpe ratio" is generally lower than those calculated on a monthly basis, which could be caused by negative autocorrelations (Lo, 2002).

In terms of comparing the combined portfolios across countries, we can see that Japan has the highest "annual Sharpe ratio" of 0.96 when calculated on an annual basis and "annualised Sharpe ratio" of 0.98 when calculated on a monthly basis, indicating that the combination of value and momentum strategies in Japan has performed well in terms of generating risk-adjusted returns. In contrast, the US has a lower "annualised Sharpe ratio" and "annual Sharpe ratio" of 0.55, indicating that the combination of value and momentum strategies in the US could have performed better in generating risk-adjusted returns compared to other countries.

Overall, it is essential to note that the Sharpe ratio is only one measure of portfolio performance, and different calculation methods may produce different results. It is essential to use multiple performance measures and consider each measure's limitations when evaluating portfolio performance.

c. Mean-variance Analysis and Efficient Frontier

In Part 1c, we performed a mean-variance analysis using all value and momentum portfolios and the risk-free interest rate. We then constructed the tangency and Global Minimum Variance (GMV) portfolios and plotted the efficient frontier. In this section, we will describe the composition of the tangency portfolio and include the individual value and momentum portfolios on the efficient frontier graph.

Portfolio	Expected Return	Standard Deviation
VAL_US	0.18%	4.66%
MOM_US	0.49%	5.65%
VAL_UK	0.27%	4.43%
MOM_UK	0.70%	5.18%
VAL_EU	0.26%	3.01%
MOM_EU	0.72%	4.26%
VAL_JP	0.80%	3.92%
MOM_JP	0.20%	4.79%
UK	0.48%	2.04%
US	0.34%	2.13%
EU	0.49%	1.64%
JP	0.50%	1.76%

Table 1. Statistics of Value, Momentum, and Combined Portfolios

To determine each portfolio's expected monthly return and standard deviation, we calculated the arithmetic mean of the monthly returns over the sample period. Specifically, we computed the mean of the monthly returns for each portfolio using the simple average of the monthly returns in the data set. The results of these calculations are presented in Table 1 above.

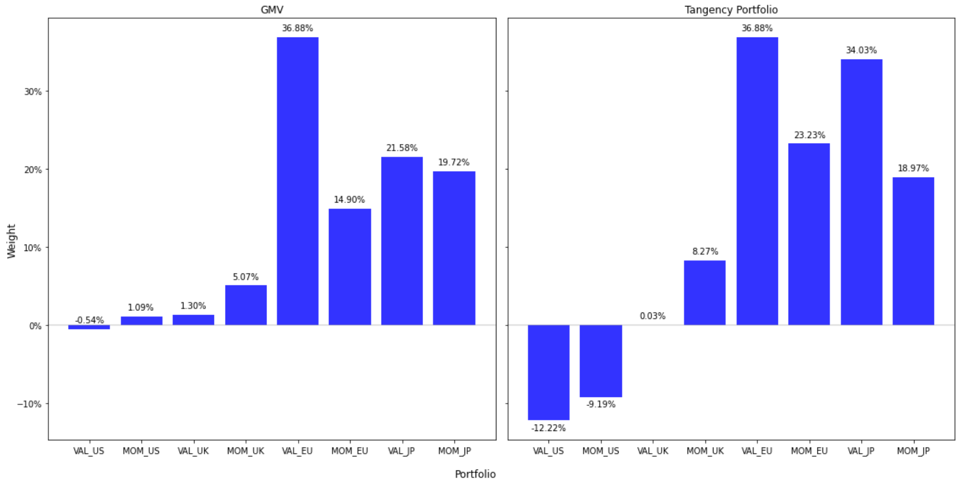

Figure 6. Weights of GMV and Tangency Portfolios

By minimising portfolio standard deviation and minimising the negative of Sharpe ratio respectively, we retrieved the weights of the Global Minimum Variance (GMV) and Tangency portfolios for the portfolios consisting only of each country's value and momentum portfolios. The weights of these portfolios are shown in Table 2 above.

The table shows that negative weights of the US value portfolio characterise the GMV portfolio. In contrast, the European value portfolio has the most significant positive weight in the GMV portfolio. Additionally, the Tangency portfolio is characterised by negative weights of the US value and momentum portfolios.

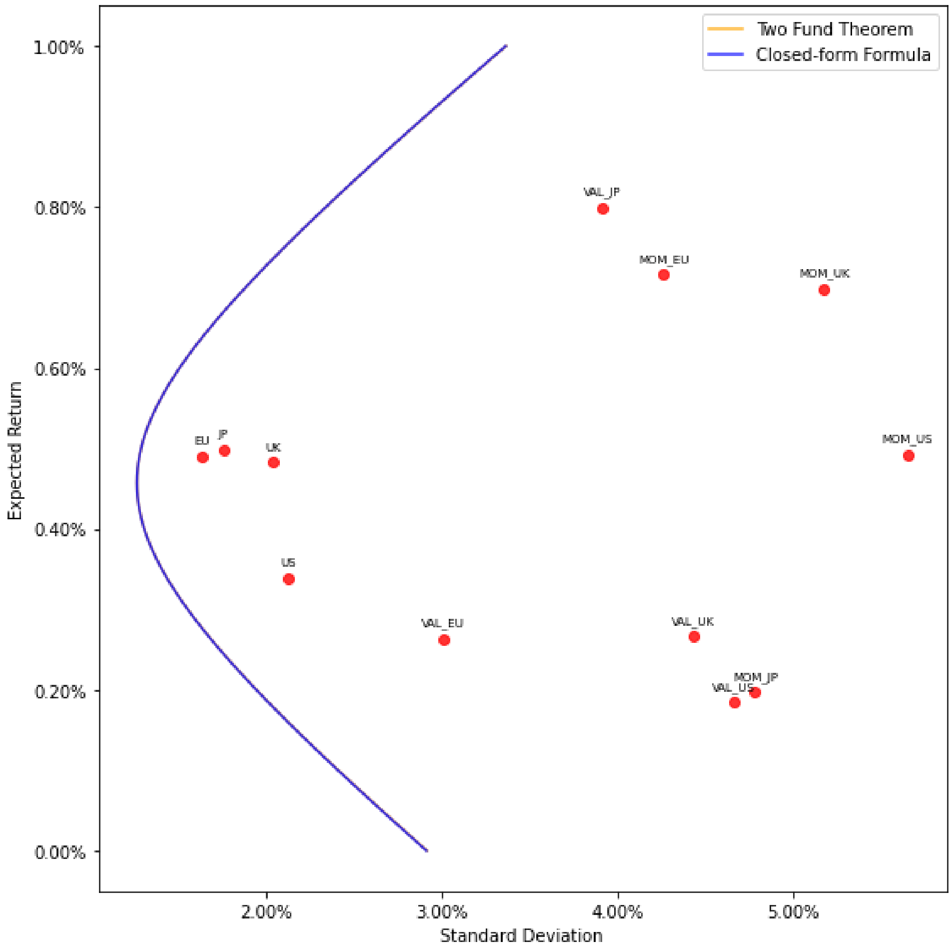

Figure 7. Efficient Frontier from Value and Momentum Portfolios

The efficient frontier in Figure 7 is a graphical representation of the optimal portfolio combinations for a given level of risk, standard deviation, and expected return. It is constructed in two ways. The first uses the Two Fund Theorem, and the second uses the closed-form formula. The Two Fund Theorem approach first involves finding two optimal portfolios on the efficient frontier. This study found the GMV portfolio and the portfolio with the highest Sharpe ratio (tangency portfolio). Then, the efficient frontier is found. This is achieved by varying the weights of the two portfolios until the desired risk-return tradeoff is achieved. On the other hand, the closed-form formula approach directly computes the expected return and standard deviation for each point on the efficient frontier without finding the weights of two portfolios. The approach uses a closed-form formula that considers the covariance matrix of the portfolio returns and the expected returns of the individual assets.

Our analysis found no significant difference between the Two Fund Theorem and the closed-form formula approaches in constructing the efficient frontier. Both approaches produced similar results, indicating that the optimal portfolio combinations were well distributed across the frontier. We also observed that the VAL_UK, MOM_US, and MOM_UK portfolios had relatively high standard deviations and expected returns compared to portfolios with a similar expected return. Besides, the combined portfolios showed relatively superior results to other value and momentum portfolios, even in the efficient frontier, which is, indeed, because they are more diversified than the individual value and momentum portfolios. The risk-adjusted return indicates that the combined portfolio has the potential to outperform the individual value and momentum portfolios.

d. Estimation of Risk Premium with Fama-Macbeth Procedure

Part 1d of the analysis involves using the Fama-Macbeth procedure to estimate the RMRF, SMB, HML, and UMD risk premium. We consider all the value and momentum portfolios in this part, excluding those 50-50 combinations computed in part 1b. After estimating the RMRF, SMB, HML, and UMD risk premium, we report the estimates and their standard errors. Then, we analyse whether the signs and magnitudes of the estimates are consistent with our expectations. This analysis helps us understand the factors that drive the returns of value and momentum portfolios and how they relate to well-known risk factors.

	VAL_US	MOM_US	VAL_UK	MOM_UK	VAL_EU	MOM_EU	VAL_JP	MOM_JP
const	0.002884	-0.001448	0.002480	0.003344	0.002230	0.004656	0.009018	-0.000401
RM-RF	-0.115604	0.064725	0.055745	-0.029453	0.109175	-0.132683	-0.140004	0.006883
SMB	-0.241576	0.312269	-0.121071	0.219751	0.001759	0.112584	-0.046882	0.126059
HML	0.884902	-0.091517	0.617903	-0.130155	0.428495	-0.061050	0.353583	-0.108915
UMD	-0.403475	0.985973	-0.261831	0.646539	-0.221902	0.544530	-0.194644	0.413116

Table 2a. Estimated Coefficients from the Fama-Macbeth Regression Analysis

	VAL_US	MOM_US	VAL_UK	MOM_UK	VAL_EU	MOM_EU	VAL_JP	MOM_JP
const	0.001431	0.001362	0.002213	0.002350	0.001430	0.001863	0.002184	0.002630
RM-RF	0.033835	0.032194	0.052323	0.055554	0.033810	0.044054	0.051648	0.062190
SMB	0.044427	0.042272	0.068702	0.072945	0.044394	0.057844	0.067816	0.081659
HML	0.048292	0.045950	0.074679	0.079291	0.048257	0.062877	0.073716	0.088763
UMD	0.029003	0.027596	0.044850	0.047620	0.028982	0.037762	0.044271	0.053309

Table 2b. Standard Error of Coefficients from the Fama-Macbeth Regression Analysis

Table 2 shows the estimated coefficients and the standard error of coefficients from the Fama-Macbeth regression analysis using the value and momentum portfolios. The coefficients for the market risk premium (RM-RF), size (SMB), value (HML), and momentum (UMD) factors indicate how sensitive the portfolio returns are to changes in those factors. A positive coefficient indicates that the portfolio performs better when that factor is high and vice versa. For the US portfolios, a 1% increase in the market risk premium (RM-RF) is associated with a decrease of 0.1156% in the return of the value portfolios and an increase of 0.0647% in the return of the momentum portfolios.

The coefficients for the other portfolios and factors show similar patterns, with some notable differences. Specifically, the SMB factor has negative coefficients for the value portfolios (VAL_US, VAL_UK and VAL_JP), indicating that smaller firms outperform larger ones in these markets. However, for the VAL_EU portfolio, the SMB factor shows a positive coefficient that is close to zero, suggesting that there is little difference in performance between large and small firms in the market. After discussing some findings in the SMB factor, we again put the focus back on the HML and UMD factors. On one hand, the HML factor has positive coefficients for all value portfolios, indicating that value stocks tend to outperform growth stocks in most markets. On the other hand, the UMD factor has consistently positive coefficients for all momentum portfolios, indicating that high-momentum stocks tend to outperform low-momentum stocks in all markets. The coefficients are generally statistically significant at the 5% level and have standard errors that are relatively small compared to their magnitudes, indicating that the estimates are precise.

	VAL_US	MOM_US	VAL_UK	MOM_UK	VAL_EU	MOM_EU	VAL_JP	MOM_JP
const	Rejected					Rejected	Rejected	
RM-RF	Rejected	Rejected			Rejected	Rejected	Rejected	
SMB	Rejected	Rejected		Rejected				
HML	Rejected	Rejected	Rejected		Rejected		Rejected	
UMD	Rejected	Rejected	Rejected	Rejected	Rejected	Rejected	Rejected	Rejected

Table 3. Results of Null Hypothesis of Coefficients Equal to 0 on a 95% Confidence

Table 3 shows the results of the null hypothesis test for each coefficient and constant, where the null hypothesis is that the actual value of the coefficient or constant is zero. If the confidence interval for a coefficient or constant does not contain zero, the null hypothesis is rejected, indicating that the coefficient or constant is statistically significant. We used the lower and upper bounds for each coefficient, and the constant is calculated using the standard errors and the critical value of 1.96 for a 95% confidence interval. The results show that all coefficients and constants are statistically significant for at least one of the portfolios. It is noteworthy that the UMD coefficients are statistically significant for all portfolios. Additionally, a statistically significant HML factor was observed for the value portfolios.

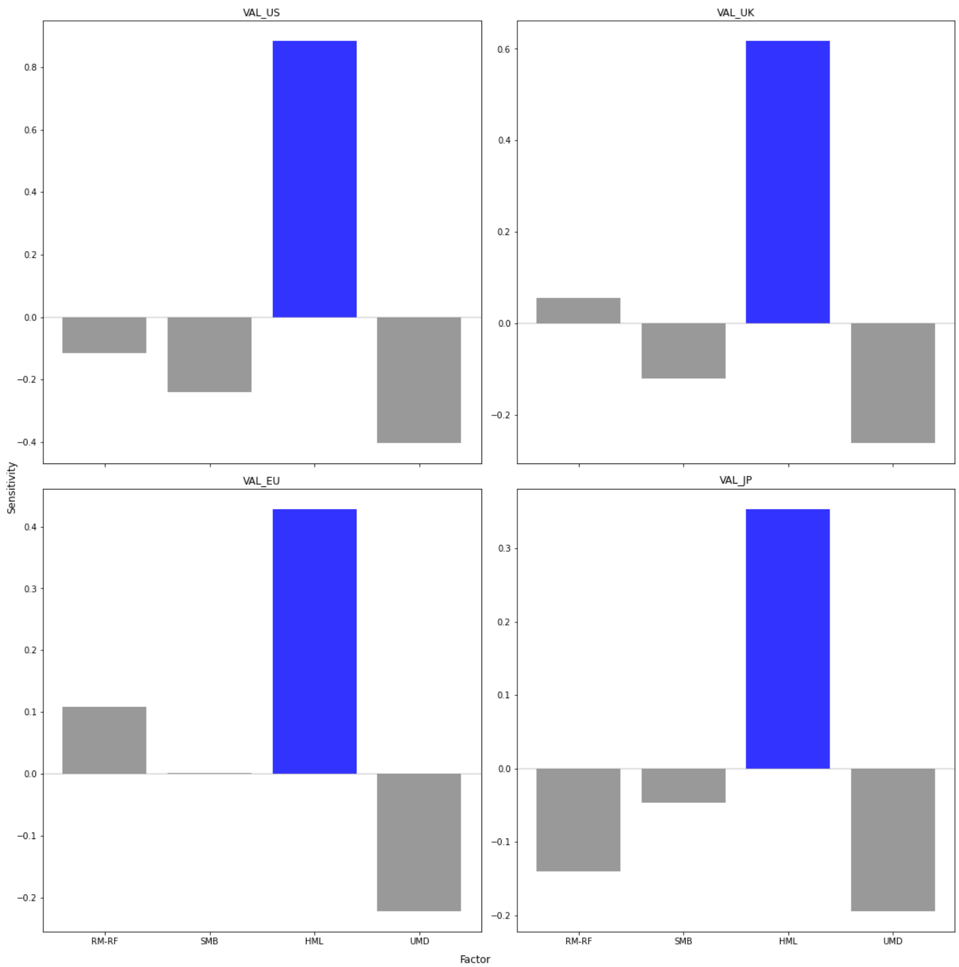

Figure 8. Factor Sensitivities of Value Portfolios to Different Factors

Furthermore, we plotted the factor sensitivities of the value portfolios to different factors for each country. The bar charts above show the sensitivity of the value portfolios to the different factors, with blue bars representing the HML factor and grey bars representing the other factors. We found that the HML factor positively impacted the value portfolios in all countries, contrary to the negative impact observed on the momentum portfolios, and will be shown in the later part. Specifically, the HML factor sensitivity was highest in the United States and the United Kingdom, with values of 0.8 and 0.6, respectively. In Europe and Japan, the HML factor sensitivity was also positive but lower, with values of 0.4 and 0.3.

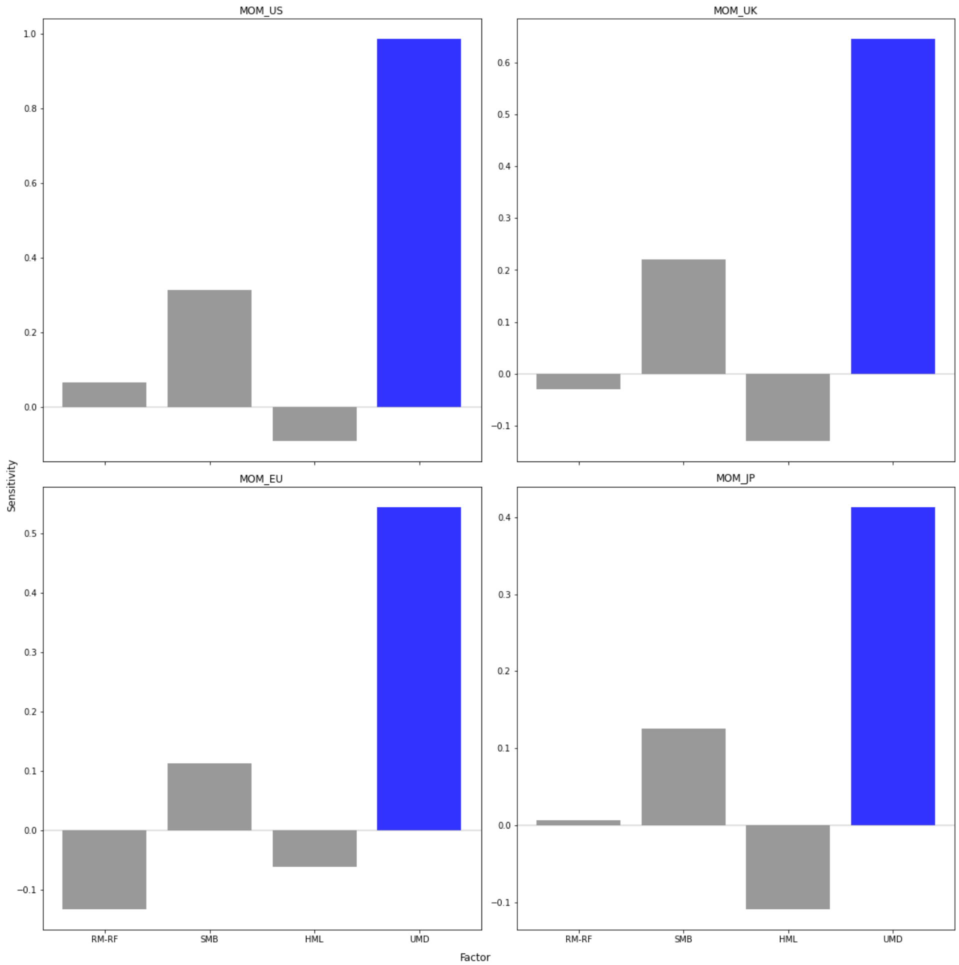

Figure 9. Factor Sensitivities of Momentum Portfolios to Different Factors

Based on the momentum portfolio analysis results, we observed that the UMD factor had a relatively high impact on the portfolio returns across all four countries. Specifically, we found that for the United States, the United Kingdom, Europe, and Japan, the UMD factor had a sensitivity of 1.0, 0.6, 0.5, and 0.4, respectively. These results suggest that changes in UMD factor values strongly influence momentum portfolio returns in the different markets.

The above results are the basis for the fact that Japan's momentum portfolio is working very well. Asness (2011) argued that both Japan's momentum and value portfolios are negatively correlated, as we observed. This is because value and momentum strategies have different strengths and weaknesses but tend to perform well under different market conditions (Asness, 2011).

In the factor sensitivity analysis of the value and momentum portfolios, we found that HML factors had a relatively high impact on the value portfolios. In contrast, UMD factors had negative values on the value portfolios and a high positive impact on the momentum portfolios.

This suggests that value and momentum are indeed negatively correlated. Furthermore, despite its seemingly poor performance, the relatively high sensitivity observed in the UMD factor for Japan's momentum portfolio suggests that there may be something unique about the Japanese market that makes momentum strategies particularly effective. The finding supports that momentum strategies in Japan have added considerable return over the period studied when viewed as a system along with negatively correlated value strategies and using a version of the Fama-French three-factor model.

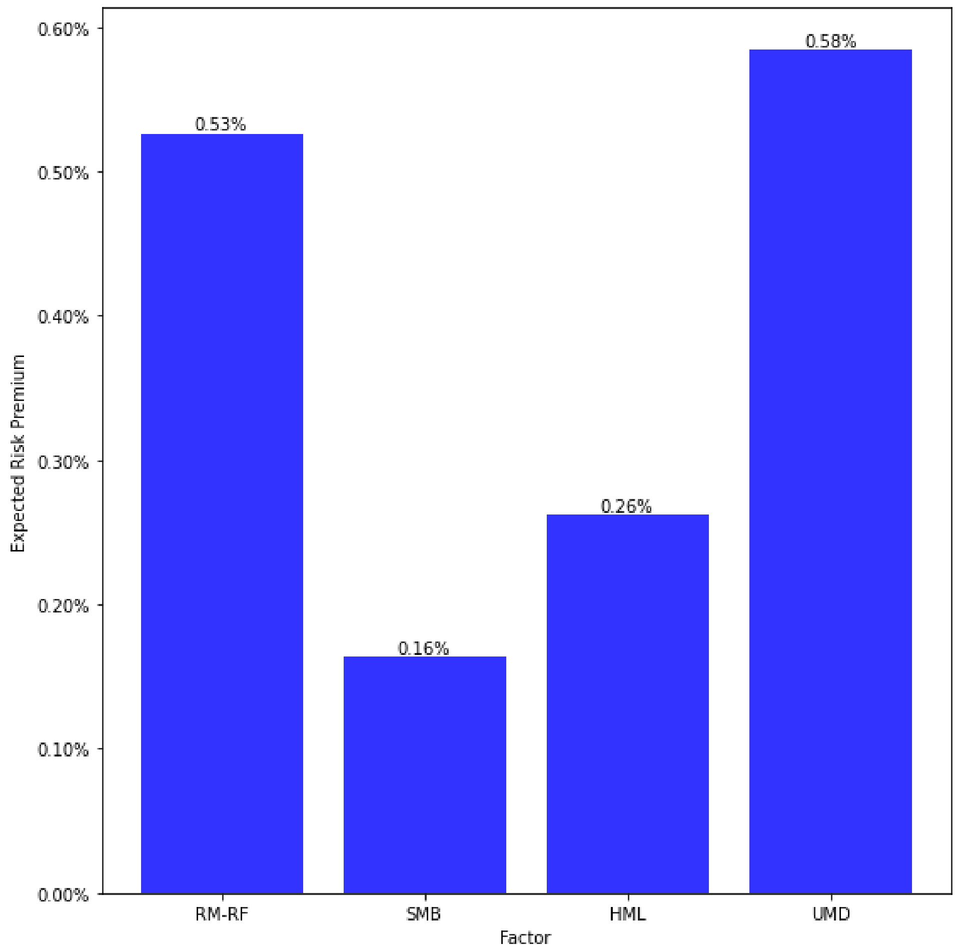

Figure 10. Risk Premium of the Four Prevalent Factors

We adopted two methods to calculate the risk premium. The first approach is taking the average of the factors for the whole time period. The second approach is constructing unit sensitivity portfolios using value portfolios. Note that the results should be the same for both methods. Figure 10 shows the risk premium of each of the factors. From the figure, RMRF and UMD have more significant risk premia than HML and SMB. The high risk premium for UMD may address the much higher expected returns for momentum portfolios for countries except Japan. Also, one thing to note is that the higher risk premium of HML than SMB aligns with the research of Fama-French that HML has a more substantial effect than SMB.

In this analysis, it was used to estimate the risk premiums for the RMRF, SMB, HML, and UMD factors in the value and momentum portfolios. The SMB and UMD factors have positive coefficients for the momentum portfolios, while the HML factor has positive coefficients for the value portfolios. The null hypothesis test results show that UMD coefficients are statistically significant for all portfolios, with the HML coefficients statistically significant for all the value portfolios. The factor sensitivity analysis reveals that the HML factor has a relatively high impact on the value portfolios, while the UMD factor has a negative impact on the value portfolios and a positive impact on the momentum portfolios. The results of the risk premiums show that RMRF and UMD have more significant risk premiums than HML and SMB. Overall, the signs and magnitudes of the estimates are consistent with the HML factor having a positive impact on value portfolios and the UMD factor having a positive impact on momentum portfolios.

e. GRS Test for the Asset Pricing Model

Part 1e of the analysis aims to conduct a Gibbons, Ross, and Shanken (GRS) test to assess the validity of the four-factor asset pricing model. The GRS test is a statistical test that examines whether a set of factors can adequately explain the cross-section of asset returns. In this case, we are using the four-factor model (RMRF, SMB, HML, and UMD) to explain the returns of the value and momentum portfolios. The GRS test allows us to test the null hypothesis that the model is valid or not and if it should be rejected in favour of an alternative model. Suppose the null hypothesis cannot be rejected, meaning there is a relatively high chance for all the intercepts to be zero. It further indicates that the model could explain most of the variations. The test also provides a p-value to help determine the statistical significance of the results. In this project, we conducted a GRS test using both the method presented in lecture notes and an alternative method from external sources to ensure the accuracy and robustness of our results.

```
intercepts = coefficients.loc['const']

T = factor_df.shape[0]
N = intercepts.shape[0]
L = mean_factors.shape[0]

residuals_cov = np.dot(residuals.T, residuals) / (T - L - 1)
factor_excess = (factor_df - factor_df.mean(axis=0)).drop('RF', axis=1)
factor_cov = np.dot(factor_excess.T, factor_excess) / (T - 1)

residuals_cov_inv = linalg.pinv(residuals_cov)
factor_cov_inv = linalg.pinv(factor_cov)

grs = np.dot(np.dot(intercepts, residuals_cov_inv), intercepts)
grs /= (1 + np.dot(np.dot(mean_factors, factor_cov_inv), mean_factors))
grs *= T / N
grs *= (T - N - L) / (T - L - 1)

f_dist = f(N, T - N - L)
p_val = 1 - f_dist.cdf(grs)
```

Code 1. Lecture Notes

First, it extracts the intercepts of the regression model from the coefficients DataFrame. Next, it calculates the number of observations (T), the number of intercepts (N), and the number of factors (L). Then, it computes the covariance matrices of the residuals and factors and takes the inverse of each. The GRS test statistic is then calculated using the formula by lecture note. Finally, the code calculates the p-value of the GRS statistic by comparing it to the F-distribution with degrees of freedom (N, T - N - L). The resulting p-value indicates the probability of observing a GRS statistic as significant or larger than the calculated value under the null hypothesis that the given set of factors cannot explain the cross-sectional variation in the portfolio returns.

```python
def grs_test(res_output, N, K, factors):
  T = res_output.nobs #number of time-series observations
  N = N  # number of portfolios
  K = K  # number of factors    # dividing the GRS equation into 3 sections a, b and c to simplyfy
  # Part a
  a = (T - N - K)/N

  # Part b
  # omega hat should be a K x K matrix (verified and True)
  E_f = factors.mean()
  omega_hat = (1/T)*(factors - E_f).T.dot(factors - E_f)    # b should be a scalar (verified and True)
  omega_hat_inv = linalg.pinv(omega_hat)  # pseudo-inverse
  b = 1+ ((E_f.T).dot(omega_hat_inv).dot(E_f))
  b_inv = b**(-1)

  # Part c
  # sigma hat should be a N x N matrix (verified and True)
  sigma_hat = res_output.std_errors
  sigma_hat = (sigma_hat).dot(sigma_hat.T)
  sigma_hat_inv =linalg.pinv(sigma_hat)  # pseudo-inverse
  alpha_hat = res_output.alphas
  c = alpha_hat.dot(sigma_hat_inv).dot(alpha_hat.T)    # Putting the 3 GRS parts together
  grs = a*b_inv*c
  print(f'GRS Test Statistic: {grs}')

  dfn= N
  dfd = T - N - K

  p_value = 1- f.cdf (grs, dfn, dfd)
  print(f'p-value: {p_value}')

  return grs, p_value
```

Code 2. External Resources

The second code further simplifies the GRS equation by breaking it into three distinct parts. Part (a) calculates a scaling factor, part (b) computes a factor based on the mean and covariance of the factors, and part (c) computes a factor based on the alphas and residual covariance matrix. The p-value is then calculated using the complement of the cumulative distribution function (CDF) of the F-distribution, equivalent to subtracting the CDF from 1.

	Code 1	Code 2
GRS Test Statistic	5.16	8.19
P-value	0.00	0.00

Table 4. Results of GRS Test

The results of the GRS test for two different codes are provided in the table. The first column shows the results for Code 1, and the second column shows the results for Code 2. The GRS test statistic value for Code 1 is 5.158, and for Code 2, it is 8.187. The reason why the difference arises could be due to the pseudo-inverse method used in Code 2 and also due to machine epsilon.

A small p-value indicates that the test's null hypothesis can be rejected, and it suggests that the four-factor model cannot adequately explain the cross-section of asset returns. The p-value for Code 1 is 5.43e-06, which is very low. The p-value for Code 2 is 6.80e-10, a minimal value. The results of Code 1 and Code 2 align. Both suggest that the four-factor model is invalid and cannot capture all variations.

According to Gibbons, Ross, and Shanken (1989), the null hypothesis in the GRS test is that the coefficients of the asset pricing model are equal to those of the market index. In the context of the GRS test, a p-value of zero indicates that the null hypothesis can be rejected at any significance level. If the p-value is zero, it suggests strong evidence against this null hypothesis and that the asset pricing model cannot explain the variation in the data.

However, we noted that some studies had shown p-values close to zero in the GRS test, even for widely accepted models in the literature. For example, studies by Asness, Moskowitz, and Pedersen (2013), Fama and French (2018), and Hou, Xue, and Zhang (2015) have all reported p-values close to zero for various asset pricing models.

One possible reason for a p-value close to zero is that the time horizon is large enough to capture periods where portfolios have different factor sensitivities. However, the test is based on constant sensitivities. In this case, the GRS test would indicate that the asset pricing model significantly differs from the market index since the actual sensitivities vary. Another reason for a p-value close to zero is that the asset pricing model being tested is a poor fit for the data, and the market index is a much better fit. In this case, the GRS test would indicate that the asset pricing model cannot explain the variation in the data and is therefore rejected.

2. Part II: Literature review

a. Summary of " Digesting anomalies: An investment approach" by Kewei Hou, Chen Xue, and Lu Zhang (HXZ 2015)

This paper examines and interprets six anomalies (including momentum, value-versus-growth, investment, profitability, intangibles and trading frictions) in financial markets, which are inconsistent with the predictions of traditional asset pricing models. It further proposes a new empirical q-factor model which effectively summarises the cross-section of average stock returns. Drawing inspiration from investment-based asset pricing, the model is built on the neoclassical q-theory of investment. Based on the 2-period stochastic general equilibrium model, investment and profitability are closely related to the expected returns of stocks. Considering the production side and the consumption side, the expected returns of a firm can be represented with the formula below:

$$E_0\left[r^s_{i1}\right] = \frac{E_0\left[\Pi_{i1}\right]}{1+a(\frac{I_{i0}}{A_{i0}})},$$

where $E_0\left[r^s_{i1}\right]$ is the expected date-1 stock return, Π_{i1} is the firm's stochastic date-1 profitability, and $\frac{I_{i0}}{A_{i0}}$ is the ratio between date-0 investment and date-0 assets with a constant parameter $a > 0$.

The formula implies a negative investment-return relation and a positive profitability-return relation, consistent with many cross-sectional patterns.

The authors construct the q-factor model based on the economic model with factor regression and compare its performance with two well-known models, namely the Fama-French (1993) three-factor model and Carhart (1997) four-factor model. The q-factor model explains the excess returns of stocks based on a market risk factor, a size factor, an investment factor and a profitability factor. Specifically, the market risk factor refers to market excess returns, and the size factor refers to the difference in returns between portfolios of small-size stocks and large-size stocks. The investment factor is constructed as the difference in returns between portfolios of low-investment stocks and high-investment stocks, using investment-to-assets (I/A) as an indicator. Profitability is measured as return on equity (ROE), with the factor constructed as the difference in returns between portfolios of high-profitability stocks and low-profitability stocks. To control for size, the q-factors are constructed by performing a triple 2-by-3-by-3 sort on size, I/A and ROE. A total of 18 portfolios are formed and rebalanced monthly with the most recent available data. By adding the size factor, the critical components for the q-factor model become similar to the Carhart model. Although the effect of the size factor is not large, the performance of the q-factor model is enhanced.

The q-factor model can be represented with the formula below:

$$E\left[r^i\right] - r_f = \beta^i_{MKT} E[MKT] + \beta^i_{ME} E\left[r_{ME}\right] + \beta^i_{IA} E\left[r_{IA}\right] + \beta^i_{ROE} E\left[r_{ROE}\right],$$

where $E[MKT]$, $E\left[r_{ME}\right]$, $E\left[r_{IA}\right]$, and $E\left[r_{ROE}\right]$ are the expected factor premiums, and β^i_{MKT}, β^i_{ME}, β^i_{IA}, and β^i_{ROE} are the corresponding factor sensitivities, respectively.

To test the q-factor model against other popular asset pricing models, the authors comprehensively examine 80 anomalies across various categories using regression. It reveals that around half of the anomalies are insignificant in the broad cross-section, which suggests many claims in the anomalies literature are likely exaggerated. Except for the R&D-to-market and operating accrual anomalies, the q-factor model outperforms the Fama-French and Carhart models for the remaining significant anomalies, including momentum, investment, Sharpe ratio and especially in profitability. According to the empirical findings, a significant number of anomalies are attributed to the influence of investment and profitability effects. Specifically, the investment factor accounts for anomalies in the value-versus-growth and investment categories, while the ROE factor is useful in explaining anomalies observed in the momentum (which is short-lived) and profitability categories. Additionally, other anomalies can be attributed to a joint impact of both the ROE and investment factors under the q-factor model. One of the drawbacks of the q-model is that it cannot fit the operate accrual anomalies as earnings will be abnormally higher than usual due to the effect of operate accrual.

b. Differences between the factors related to profitability

In "Digesting anomalies: An investment approach" (Hou, Xue and Zhang, 2015), return on equity (ROE) is used as a proxy of the expected profitability in the q-factor model, which is computed as income before extraordinary items divided by 1-quarter-lagged book equity. When constructing the q-factor model, the authors perform a triple 2-by-3-by-3 sort on size, I/A and ROE, forming 18 portfolios. Based on the ranked ROE values using the NYSE breakpoints, all stocks are sorted into three groups (low, middle and high ROE values), and rebalanced monthly. The ROE factor is constructed by taking a long-short position in six portfolios with high ROE and six portfolios with low ROE. It is calculated as the corresponding difference in average portfolio returns.

In "Choosing factors" (Fama and French, 2018), Fama and French proposed two profitability factors, namely robust-minus-weak operating profitability (RMW_o) and cash profitability (

RMW_c). The operating profitability is calculated as income net of interest expense, scaled by book equity, while the cash profitability is calculated similarly, except accruals are deducted from operating profit before scaling by book equity. Both operating and cash profitability are measured before R&D expenses. To construct the operating profitability factor, the stocks are first sorted into two size groups. They are further sorted on operating profitability, using accounting data for the fiscal year ending in the previous year. The cash profitability factor is constructed like operating profitability, except the second sort is on cash profitability.

The model performances are measured by the value of alphas (unexplained average returns) and GRS test in the first paper, while models are ranked on squared Sharpe ratio for model factors in the second paper. As the criteria for model selection between the two papers are different, the performance of ROE, RMW_o and RMW_c factors cannot be compared directly.

From an accounting perspective, the ROE factor used in the first paper measures the overall rate of return that a company generates on its equity, which is subjected to the accounting practices, such as treatments of depreciation and amortisation. In contrast, the operating and cash profitability in the second paper evaluates a company's profitability concerning its operations and operating cash flows, respectively. These factors exclude non-operational items and remain unaffected by accounting practices. Given that the ROE factor can be distorted by accounting practices and is more susceptible to manipulation, the operating and cash profitability factors may provide a more accurate representation of expected returns compared to the ROE factor.

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