Assignment 1: Module 2 - The LP Model

- Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.
  a. Clearly define the decision variables, b. What is the objective function?, c. What are the constraints?, d. Write down the full mathematical formulation for this LP problem.

ANS:

a. Clearly define the decision variables:

Decision variables are as follows:
$x1$ = Number of Collegiate backpack to be produced per week
$x2$ = Number of Mini Backpack to be produced per week

b. What is the objective function?

Objective function is as follows:
How many Quantity of each type of backpack to be produced to maximize the profit.
So, Z = 32 $x1$ + 24 $x2$

c. What are the constraints?

There are several constraints and they are as follows:
Nylon Material: Only 5000 square foot of nylon material is received each week and 3 square ft of nylon is required to make one Collegiate bag and 2 Sqare ft is required to make one Mini.
3 $x1$ + 2 $x2$ <= 5000

Maximum unit that can be sold
Collegiate : $x1$ <= 1000
Mini : $x2$ <= 1200

Time: only (35 * 40) 1400 hours available per week
45 $x1$ + 40 $x2$ <= 84000 (1400 Hours * 60)

d. Write down the full mathematical formulation for this LP problem.

Maximize Z = 32 $x1$ + 24 $x2$

ST:
3 $x1$ + 2 $x2$ <= 5000
45 $x1$ + 40 $x2$ <= 84000
and
$x1$ <= 1000, $x2$ <= 1200

- The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

  a. Define the decision variables

  b. Formulate a linear programming model for this problem.

ANS:

__Decision variables are as follows:__

A1, A2, A3 = Number of Large size products to be produced in plant 1, plant 2 and plant 3.
B1, B2, B3 = Number of Medium size products to be produced in plant 1, plant 2 and plant 3.
C1, C2, C3 = Number of small size products to be produced in plant 1, plant 2 and plant 3.

__Objective function:__

Z = 420(A1+A2+A3) + 360(B1+B2+B3) + 300(C1+C2+C3)

__Constraints:__

Plant Capacity: Plants 1, 2, and 3 only have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

A1 + B1 + C1 <= 750
A2 + B2 + C2 <= 900
A3 + B3 + C3 <= 450

Storage limitation: Plants 1, 2, and 3 only have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product and each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

20 A1 + 15 B1 + 12 C1 <= 13000
20 A2 + 15 B2 + 12 C2 <= 12000
20 A3 + 15 B3 + 12 C3 <= 5000

Maximum Sales: If available, only 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

$A1 + A2 + A3 <= 900$

$B1 + B2 + B3 <= 1200$

$C1 + C2 + C3 <= 750$

Full mathematical formulation for this LP problem:

Maximize $Z = 420(A1+A2+A3) + 360(B1+B2+B3) + 300(C1+C2+C3)$

ST:

$A1 + B1 + C1 <= 750$

$A2 + B2 + C2 <= 900$

$A3 + B3 + C3 <= 450$

$20 A1 + 15 B1 + 12 C1 <= 13000$

$20 A2 + 15 B2 + 12 C2 <= 12000$

$20 A3 + 15 B3 + 12 C3 <= 5000$

$A1 + A2 + A3 <= 900$

$B1 + B2 + B3 <= 1200$

$C1 + C2 + C3 <= 750$

AND

$A1, A2, A3 >= 0$,

$B1, B2, B3 >= 0$,

$C1, C2, C3 >= 0$