Summer RA work

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Abstract

2 country, 2 stage, snake GVC trade model abstract from Sposi, Yi, and Zhang (2020).

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Contents

1 Model

2 countries, 2 stages GVC model, Fréchet on production chains.

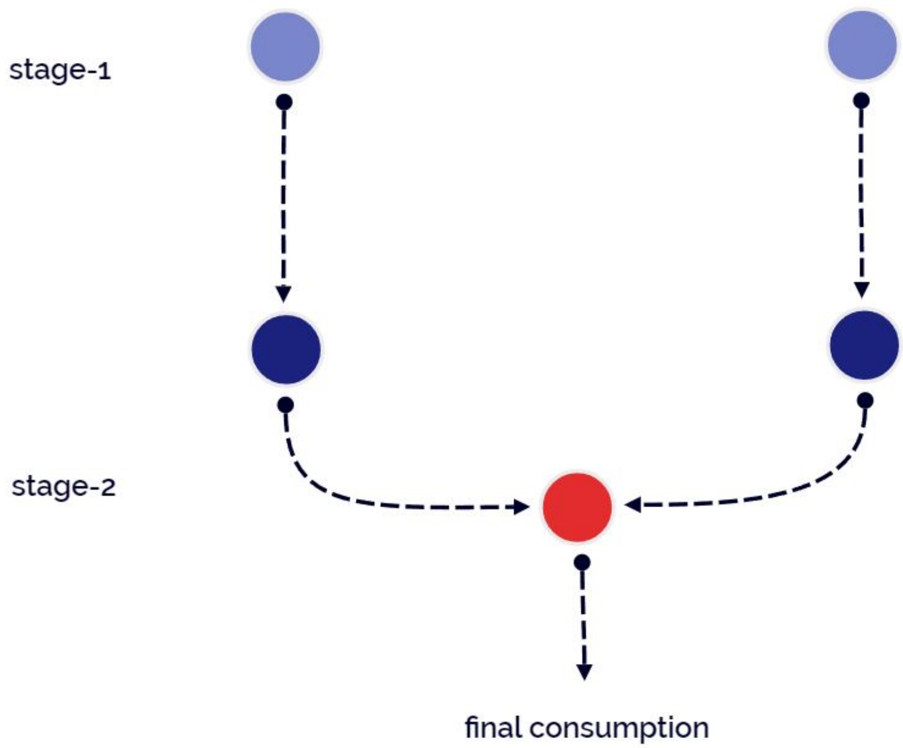

Figure 1:

1.1 Basic Setup

For given chain $\ell = \begin{pmatrix} \ell^1 \\ \ell^2 \end{pmatrix}$ from $\mathcal{C} = \{\mathcal{C}_1; \mathcal{C}_2; \mathcal{C}_3; \mathcal{C}_4\} = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 \\ 2 \end{pmatrix}; \begin{pmatrix} 2 \\ 1 \end{pmatrix}; \begin{pmatrix} 2 \\ 2 \end{pmatrix} \right\}$

$\gamma^1 \equiv 1, \widetilde{\gamma}^s \equiv \prod_{s\prime=s+1}^{S} (1-\gamma^{s\prime})$; $\sum_{s=1}^{S} \gamma^s \widetilde{\gamma}^s \equiv 1; s \in \{1,\ 2\}$

Production of each variety $\upsilon \in [0,1]$ involves multiple stages S.

The production chain is denoted by the sequence $\ell = (\ell^1, \cdots, \ell^S)$. ℓ^s : the location that production of stage s occurs.

For each variety υ, # of possible path $\ell : N^S$.

γ^s: valued added share of stage s production, $\gamma^1 \equiv 1$.

$\widetilde{\gamma}^s = \prod_{s\prime=s+1}^{S} (1-\gamma^{s\prime})$: share of stage-s gross output in the gross value of finished good, $\widetilde{\gamma}^S \equiv 1$.

$\gamma^s \widetilde{\gamma}^s$: share of stage-s valued added in the gross value of finished good, $\sum_{s=1}^{S} \gamma^s \widetilde{\gamma}^s \equiv 1$.

1.2 Preference

A representative household in each country owns primary factors of production: labor L_n. The household supplies labor inelastically to domestic firms at competitive rate w_n. She spends all factor income on consumption C_n. C_n consumers in country n derive utility from the continuum of final-good varieties, following a CES aggregation:

$$C_n = \left(\int_0^1 (c_n(\upsilon))^{\frac{(\eta-1)}{\eta}} d\upsilon \right)^{\frac{\eta}{\eta-1}}$$

and

$$P_n C_n = W_n L_n + D_n = I_n$$

1.3 Production Technology

The production at stage s has the form:

$$y^s_{\ell^s}(\upsilon) = a_\ell(\upsilon) \left(A^s_{\ell^s} L^s_{\ell^s}(\upsilon) \right)^{\gamma^s} \left(m^s_{\ell^s}(\upsilon) \right)^{1-\gamma^s}$$

The input quantity in stage s must equal output from previous stage with adjustments for trade costs:

$$m^s_{\ell^s}(\upsilon) \, d^s_{\ell^{s-1},\ell^s} = y^{s-1}_{\ell^{s-1}}(\upsilon)$$

All finished varieties, potentially traded internationally, assembled into a non-traded composite retail good:

$$Q_n = \left(\int_0^1 (q_n(\upsilon))^{\frac{(\eta-1)}{\eta}} d\upsilon \right)^{\frac{\eta}{\eta-1}}$$

where $q_n(\upsilon)$ is the quantity of stage S, or finished variety purchased by country n.
The unit cost of factors: $u^s_n = B^s w_n$, and $B^s = 1$.
Consider a chain ℓ demanded by country n, each country involved in chain ℓ adds to the cost of the finished good. The factor cost contributed in stage s as $\mathcal{U}^s_{\ell,n}$:

$$\mathcal{U}^s_{\ell,n} = \left[\left(\frac{u^s_{\ell^s}}{A^s_{\ell^s}} \right)^{\gamma^s} d^s_{\ell^s,\ell^{s+1}} \right]^{\widetilde{\gamma}^s}$$

where $\ell^{S+1} = n$.
The price of chain ℓ in country n for finished variety υ is given:

$$p_{\ell,n}(\upsilon) = \frac{1}{a_\ell(\upsilon)} \prod_{s=1}^{S} \mathcal{U}^s_{\ell,n}$$

The lead firm for any variety sources input using the chain with the lowest price:

$$p_n(v) = \min_{\ell \in \mathfrak{C}} \{p_{\ell,n}(v)\}$$

1.4 Price

$$P_n = \zeta \left[\sum_{\ell \in \mathfrak{C}} \prod_{s=1}^{S} \left(\left(\frac{u^s_{\ell^s}}{A^s_{\ell^s}} \right)^{\gamma^s} d^s_{\ell^s,\ell^{s+1}} \right)^{-\theta \widetilde{\gamma}^s} \right]^{-\frac{1}{\theta}} \tag{1}$$

where $\zeta = \Gamma \left(\frac{1+\theta-\eta}{\theta} \right)^{\frac{1}{1-\eta}}$; $u^s_n = w_n$; $d^s_{\ell^S,\ell^{S+1}} = d^s_{\ell^S,n}$

$$P_1 = \zeta \times \left[\left(\left(\frac{w_1}{A^1_1} \right)^{-\theta\gamma^1\widetilde{\gamma}^1} \right) \left(\left(\frac{w_1}{A^2_1} \right)^{-\theta\gamma^2\widetilde{\gamma}^2} \right) + \left(\left(\left(\frac{w_1}{A^1_1} \right)^{\gamma^1} d^1_{1,2} \right)^{-\theta\widetilde{\gamma}^1} \right) \left(\left(\left(\frac{w_2}{A^2_2} \right)^{\gamma^2} d^2_{2,1} \right)^{-\theta\widetilde{\gamma}^2} \right) + \left(\left(\left(\frac{w_2}{A^1_2} \right)^{\gamma^1} d^1_{2,1} \right)^{-\theta\widetilde{\gamma}^1} \right) \left(\left(\frac{w_1}{A^2_1} \right)^{-\theta\gamma^2\widetilde{\gamma}^2} \right) + \left(\left(\frac{w_2}{A^1_2} \right)^{-\theta\gamma^1\widetilde{\gamma}^1} \right) \left(\left(\left(\frac{w_2}{A^2_2} \right)^{\gamma^2} d^2_{2,1} \right)^{-\theta\widetilde{\gamma}^2} \right) \right]^{-\frac{1}{\theta}}$$

$$P_2 = \zeta \times \left[\left(\left(\frac{w_1}{A^1_1} \right)^{-\theta\gamma^1\widetilde{\gamma}^1} \right) \left(\left(\left(\frac{w_1}{A^2_1} \right)^{\gamma^2} d^2_{1,2} \right)^{-\theta\widetilde{\gamma}^2} \right) + \left(\left(\left(\frac{w_1}{A^1_1} \right)^{\gamma^1} d^1_{1,2} \right)^{-\theta\widetilde{\gamma}^1} \right) \left(\left(\frac{w_2}{A^2_2} \right)^{-\theta\gamma^2\widetilde{\gamma}^2} \right) + \left(\left(\left(\frac{w_2}{A^1_2} \right)^{\gamma^1} d^1_{2,1} \right)^{-\theta\widetilde{\gamma}^1} \right) \left(\left(\left(\frac{w_1}{A^2_1} \right)^{\gamma^2} d^2_{1,2} \right)^{-\theta\widetilde{\gamma}^2} \right) + \left(\left(\frac{w_2}{A^1_2} \right)^{-\theta\gamma^1\widetilde{\gamma}^1} \right) \left(\left(\frac{w_2}{A^2_2} \right)^{-\theta\gamma^2\widetilde{\gamma}^2} \right) \right]^{-\frac{1}{\theta}}$$

1.5 Spending Share

The fraction of finished varieties that country n sources through a particular path ℓ:

$$\lambda_{\ell,n} = \frac{\prod_{s=1}^{S} \left(\left(\frac{u^s_{\ell^s}}{A^s_{\ell^s}} \right)^{\gamma^s} d^s_{\ell^s,\ell^{s+1}} \right)^{-\theta\widetilde{\gamma}^s}}{\sum_{\ell' \in \mathfrak{C}} \prod_{s=1}^{S} \left(\left(\frac{u^s_{\ell'^s}}{A^s_{\ell'^s}} \right)^{\gamma^s} d^s_{\ell'^s,\ell'^{s+1}} \right)^{-\theta\widetilde{\gamma}^s}} = \left[\frac{P_n}{\zeta} \right]^{\theta} \prod_{s=1}^{S} \left(\left(\frac{u^s_{\ell^s}}{A^s_{\ell^s}} \right)^{\gamma^s} d^s_{\ell^s,\ell^{s+1}} \right)^{-\theta\widetilde{\gamma}^s} \tag{2}$$

$\forall n \in 0,\ 1$

$$\{\lambda_{\ell,n}\}_{\ell=\mathcal{C}_1} == \left[\frac{P_n}{\zeta}\right]^\theta \times \left(\left(\left(\frac{u_1^1}{A_1^1}\right)^{\gamma^1}\right)^{-\theta\widetilde{\gamma}^1}\right)\left(\left(\left(\frac{u_1^2}{A_1^2}\right)^{\gamma^2} d_{1,n}^2\right)^{-\theta\widetilde{\gamma}^2}\right)$$

$$\{\lambda_{\ell,n}\}_{\ell=\mathcal{C}_2} = \left[\frac{P_n}{\zeta}\right]^\theta \times \left(\left(\left(\frac{u_1^1}{A_1^1}\right)^{\gamma^1} d_{1,2}^1\right)^{-\theta\widetilde{\gamma}^1}\right)\left(\left(\left(\frac{u_2^2}{A_2^2}\right)^{\gamma^2} d_{2,n}^2\right)^{-\theta\widetilde{\gamma}^2}\right)$$

$$\{\lambda_{\ell,n}\}_{\ell=\mathcal{C}_3} = \left[\frac{P_n}{\zeta}\right]^\theta \times \left(\left(\left(\frac{u_2^1}{A_2^1}\right)^{\gamma^1} d_{2,1}^1\right)^{-\theta\widetilde{\gamma}^1}\right)\left(\left(\left(\frac{u_1^2}{A_1^2}\right)^{\gamma^2} d_{1,n}^2\right)^{-\theta\widetilde{\gamma}^2}\right)$$

$$\{\lambda_{\ell,n}\}_{\ell=\mathcal{C}_4} = \left[\frac{P_n}{\zeta}\right]^\theta \times \left(\left(\left(\frac{u_2^1}{A_2^1}\right)^{\gamma^1}\right)^{-\theta\widetilde{\gamma}^1}\right)\left(\left(\left(\frac{u_2^2}{A_2^2}\right)^{\gamma^2} d_{2,n}^2\right)^{-\theta\widetilde{\gamma}^2}\right)$$

1.6 Firms input and product

Define $\Lambda_n^s \equiv \{\ell \in \mathcal{C} : \ell^s = n\}$ as the set of chains that country n occupies position s, firms demand inputs so that marginal products of all inputs are equated:

$$w_n L_n^s = \gamma^s \widetilde{\gamma}^s \sum_{\ell \in \Lambda_n^s} \sum_{i=1}^{N} X_i \lambda_{\ell,i} \tag{3}$$

where $P_i Q_i \equiv X_i$, $\Lambda_n^s \equiv \{\ell \in \mathcal{C} : \ell^s = n\}$.

$$\sum_{s=1}^{S} L_n^s = L_n \tag{4}$$

so $L_1^1 + L_1^2 = L_1$; $L_2^1 + L_2^2 = L_2$.

$$\Lambda_{n=1}^{s=1} \equiv \{\mathcal{C}_1; \mathcal{C}_2\} = \left\{\begin{pmatrix}1\\1\end{pmatrix}; \begin{pmatrix}1\\2\end{pmatrix}\right\}; \qquad \Lambda_{n=1}^{s=2} \equiv \{\mathcal{C}_1; \mathcal{C}_3\} = \left\{\begin{pmatrix}1\\1\end{pmatrix}; \begin{pmatrix}2\\1\end{pmatrix}\right\}$$

$$\Lambda_{n=2}^{s=1} \equiv \{\mathcal{C}_3; \mathcal{C}_4\} = \left\{\begin{pmatrix}2\\1\end{pmatrix}; \begin{pmatrix}2\\2\end{pmatrix}\right\}; \qquad \Lambda_{n=2}^{s=2} \equiv \{\mathcal{C}_2; \mathcal{C}_4\} = \left\{\begin{pmatrix}1\\2\end{pmatrix}; \begin{pmatrix}2\\2\end{pmatrix}\right\}$$

$$w_1 L_1^1 = \gamma^1 \widetilde{\gamma}^1 \left\{\sum_{i=1}^{N} X_i \left(\lambda_{\ell=\mathcal{C}_1,i} + \lambda_{\ell=\mathcal{C}_2,i}\right)\right\}$$

$$w_1 L_1^2 = \gamma^2 \widetilde{\gamma}^2 \left\{\sum_{i=1}^{N} X_i \left(\lambda_{\ell=\mathcal{C}_1,i} + \lambda_{\ell=\mathcal{C}_3,i}\right)\right\}$$

$$w_2 L_2^1 = \gamma^1 \widetilde{\gamma}^1 \left\{\sum_{i=1}^{N} X_i \left(\lambda_{\ell=\mathcal{C}_3,i} + \lambda_{\ell=\mathcal{C}_4,i}\right)\right\}$$

$$w_2 L_2^2 = \gamma^2 \widetilde{\gamma}^2 \left\{ \sum_{i=1}^{N} X_i \left(\lambda_{\ell=\mathcal{C}_2,i} + \lambda_{\ell=\mathcal{C}_4,i} \right) \right\}$$

1.7 Trade balance

Final Demand $\{P_i Q_i\}_{i=1}^N$ and the corresponding shares along each chain $\{\lambda_{\ell,i}\}_{i=1}^N$ imply trade flow across countries.
Total exports (imports) is the sum of trade flows along all chains:

$$M_n - E_n \equiv D_n \equiv 0 \tag{5}$$

where $E_n = \sum_{\ell \in \mathcal{C}} E_n^\ell$ and $M_n = \sum_{\ell \in \mathcal{C}} M_n^\ell$.

1.7.1 Country n's gross exports through chain l

$$E_n^\ell = \sum_{i \neq n} P_i Q_i \lambda_{\ell,i} \sum_{s=1}^{S} \widetilde{\gamma}^s 1_{\ell \in \mathcal{E}_n^s} + P_n Q_n \lambda_{\ell,n} \sum_{s=1}^{S-1} \widetilde{\gamma}^s 1_{\ell \in \mathcal{E}_n^s} \tag{6}$$

For any stage $s < S$, $\mathcal{E}_n^s = \left\{ \ell : \ell^s = n \text{ } and \text{ } \ell^{s+1} \neq n \right\}$. This is the set of Chains that generate exports of intermediate goods of stage s for country n from any country's demand on these chains including its own.
For any stage $s = S$, $\mathcal{E}_n^S = \left\{ \ell : \ell^S = n \right\} \equiv \Lambda_n^S$. This is the set of Chains that country n occupies the final stage. These chains generate exports of final goods for country n from foreign demand on this chain.

$$\mathcal{E}_{n=1}^{s=1} \equiv \{\mathcal{C}_2\} = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}; \qquad \mathcal{E}_{n=1}^{s=2} \equiv \Lambda_{n=2}^{s=S} = \{\mathcal{C}_1; \mathcal{C}_3\} = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$$

$$\mathcal{E}_{n=2}^{s=1} \equiv \{\mathcal{C}_3\} = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}; \qquad \mathcal{E}_{n=2}^{s=2} \equiv \Lambda_{n=2}^{s=S} = \{\mathcal{C}_2; \mathcal{C}_4\} = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}; \begin{pmatrix} 2 \\ 2 \end{pmatrix} \right\}$$

$$\begin{aligned}
\left\{E_n^\ell\right\}_{n=1,\ell=\mathcal{C}_1} &= X_{i=2}\lambda_{\ell=\mathcal{C}_1,i=2}\widetilde{\gamma}^{s=2} & \left\{E_n^\ell\right\}_{n=2,\ell=\mathcal{C}_1} &= 0 \\
\left\{E_n^\ell\right\}_{n=1,\ell=\mathcal{C}_2} &= X_{i=2}\lambda_{\ell=\mathcal{C}_2,i=2}\widetilde{\gamma}^{s=1} + X_{n=1}\lambda_{\ell=\mathcal{C}_2,n=1}\widetilde{\gamma}^{s=1} & \left\{E_n^\ell\right\}_{n=2,\ell=\mathcal{C}_2} &= X_{i=1}\lambda_{\ell=\mathcal{C}_2,i=1}\widetilde{\gamma}^{s=2} \\
\left\{E_n^\ell\right\}_{n=1,\ell=\mathcal{C}_3} &= X_{i=2}\lambda_{\ell=\mathcal{C}_3,i=2}\widetilde{\gamma}^{s=2} & \left\{E_n^\ell\right\}_{n=2,\ell=\mathcal{C}_3} &= X_{i=1}\lambda_{\ell=\mathcal{C}_3,i=1}\widetilde{\gamma}^{s=1} + X_2\lambda_{\mathcal{C}_3,2}\widetilde{\gamma}^1 \\
\left\{E_n^\ell\right\}_{n=1,\ell=\mathcal{C}_4} &= 0 & \left\{E_n^\ell\right\}_{n=2,\ell=\mathcal{C}_4} &= X_{i=1}\lambda_{\ell=\mathcal{C}_4,i=1}\widetilde{\gamma}^{s=2}
\end{aligned}$$

1.7.2 Country n's gross imports through chain l

$$M_n^\ell = P_n Q_n \lambda_{\ell,n} \sum_{s=1}^{S} \widetilde{\gamma}^s 1_{\ell\in\mathcal{M}_n^s} + \sum_{i\neq n} P_i Q_i \lambda_{\ell,i} \sum_{s=1}^{S-1} \widetilde{\gamma}^s 1_{\ell\in\mathcal{M}_n^s} \tag{7}$$

For any stage $s < S$, $\mathcal{M}_n^s = \left\{\ell : \ell^{s+1} = n \ and \ \ell^s \neq n\right\}$. This is the set of chains that country n incurs imports at stage s to use in production in stage s + 1

For any stage $s = S$, $\mathcal{M}_n^S = \left\{\ell : \ell^S \neq n\right\} \equiv \bigcup_{i\neq n} \Lambda_i^S$. This is the set of chains that country n imports final goods. These chains generate exports of final goods for country n from foreign demand on this chain.

$$\mathcal{M}_{n=1}^{s=1} \equiv \{\mathcal{C}_3\} = \left\{\begin{pmatrix}2\\1\end{pmatrix}\right\}; \qquad \mathcal{M}_{n=1}^{s=2} \equiv \{\mathcal{C}_2;\mathcal{C}_4\} = \left\{\begin{pmatrix}1\\2\end{pmatrix};\begin{pmatrix}2\\2\end{pmatrix}\right\}$$

$$\mathcal{M}_{n=2}^{s=1} \equiv \{\mathcal{C}_2\} = \left\{\begin{pmatrix}1\\2\end{pmatrix}\right\}; \qquad \mathcal{M}_{n=2}^{s=2} \equiv \{\mathcal{C}_1;\mathcal{C}_3\} = \left\{\begin{pmatrix}1\\1\end{pmatrix};\begin{pmatrix}2\\1\end{pmatrix}\right\}$$

$$\left\{M_n^\ell\right\}_{n=1,\ell=\mathcal{C}_1} = 0$$

$$\left\{M_n^\ell\right\}_{n=1,\ell=\mathcal{C}_2} = X_{n=1}\lambda_{\ell=\mathcal{C}_2,n=1}\widetilde{\gamma}^{s=2}$$

$$\left\{M_n^\ell\right\}_{n=1,\ell=\mathcal{C}_3} = X_{n=1}\lambda_{\ell=\mathcal{C}_3,n=1}\widetilde{\gamma}^{s=1} + X_{i=2}\lambda_{\ell=\mathcal{C}_3,i=2}\widetilde{\gamma}^{s=1}$$

$$\left\{M_n^\ell\right\}_{n=1,\ell=\mathcal{C}_4} = X_{n=1}\lambda_{\ell=\mathcal{C}_4,n=1}\widetilde{\gamma}^{s=2}$$

$$\left\{M_n^\ell\right\}_{n=2,\ell=\mathcal{C}_1} = X_{n=2}\lambda_{\ell=\mathcal{C}_1,n=2}\widetilde{\gamma}^{s=2}$$

$$\left\{M_n^\ell\right\}_{n=2,\ell=\mathcal{C}_2} = X_{n=2}\lambda_{\ell=\mathcal{C}_2,n=2}\widetilde{\gamma}^{s=1} + X_1\lambda_{\mathcal{C}_2,1}\widetilde{\gamma}^1$$

$$\left\{M_n^\ell\right\}_{n=2,\ell=\mathcal{C}_3} = X_{n=2}\lambda_{\ell=\mathcal{C}_3,n=2}\widetilde{\gamma}^{s=2}$$

$$\left\{M_n^\ell\right\}_{n=2,\ell=\mathcal{C}_4} = 0$$

1.8 Welfare

the composite retail good is not traded across countries, the supply of retail goods must equal domestic demand:

$$Q_n = C_n \tag{8}$$

$$P_n C_n = W_n L_n + D_n = I_n \tag{9}$$

where $Q_n = C_n$, $Welfare \equiv \frac{I_n}{P_n} = C_n$

2 Measure value added in exports

2.1 country n's value-added exports through chain l

$$V_n^\ell = \sum_{i\neq n} P_i Q_i \lambda_{\ell,i} \sum_{s=1}^{S} \widetilde{\gamma}^s \gamma^s 1_{\ell(s)=n} + P_n Q_n \lambda_{\ell,n} 1_{\ell\in\mathcal{E}_n^*} \sum_{s=1}^{S-1} \widetilde{\gamma}^s \gamma^s 1_{\ell(s)=n} \tag{10}$$

where $V_n = \sum_{\ell \in \mathcal{C}} V_n^\ell$, $\mathcal{E}_n^* = \left\{\ell : \ell \in \left(\bigcup_{j=1}^{S-1} \mathcal{E}_n^j\right)\right\}$.

$$\mathcal{E}_1^* = \left\{\left(\bigcup_{j=1}^{S-1} \mathcal{E}_1^j\right)\right\} = \mathcal{E}_1^1 = \mathcal{E}_{n=1}^{s=1} \equiv \{\mathcal{C}_2\} = \left\{\begin{pmatrix}1\\2\end{pmatrix}\right\}$$

$$\mathcal{E}_2^* = \left\{\left(\bigcup_{j=1}^{S-1} \mathcal{E}_2^j\right)\right\} = \mathcal{E}_2^1 = \mathcal{E}_{n=2}^{s=1} \equiv \{\mathcal{C}_3\} = \left\{\begin{pmatrix}2\\1\end{pmatrix}\right\}$$

$$\left\{V_n^\ell\right\}_{n=1,\ell=\mathcal{C}_1} = X_{i=2}\lambda_{\ell=\mathcal{C}_1,i=2}\left(\widetilde{\gamma}^{s=1}\gamma^{s=1} + \widetilde{\gamma}^{s=2}\gamma^{s=2}\right)$$

$$\left\{V_n^\ell\right\}_{n=1,\ell=\mathcal{C}_2} = X_{i=2}\lambda_{\ell=\mathcal{C}_2,i=2}\widetilde{\gamma}^{s=1}\gamma^{s=1} + X_{n=1}\lambda_{\ell=\mathcal{C}_2,n=1}\widetilde{\gamma}^{s=1}\gamma^{s=1}$$

$$\left\{V_n^\ell\right\}_{n=1,\ell=\mathcal{C}_3} = X_{i=2}\lambda_{\ell=\mathcal{C}_3,i=2}\widetilde{\gamma}^{s=2}\gamma^{s=2}$$

$$\left\{V_n^\ell\right\}_{n=1,\ell=\mathcal{C}_4} = 0$$

$$\left\{V_n^\ell\right\}_{n=2,\ell=\mathcal{C}_1} = 0$$

$$\left\{V_n^\ell\right\}_{n=2,\ell=\mathcal{C}_2} = X_{i=1}\lambda_{\ell=\mathcal{C}_2,i=1}\widetilde{\gamma}^{s=2}\gamma^{s=2}$$

$$\left\{V_n^\ell\right\}_{n=2,\ell=\mathcal{C}_3} = X_1\lambda_{\mathcal{C}_3,1}\widetilde{\gamma}^1\gamma^1 + X_2\lambda_{\mathcal{C}_3,2}\widetilde{\gamma}^1\gamma^1$$

$$\left\{V_n^\ell\right\}_{n=2,\ell=\mathcal{C}_4} = X_1\lambda_{\mathcal{C}_4,1}\left(\widetilde{\gamma}^1\gamma^1 + \widetilde{\gamma}^2\gamma^2\right)$$

2.2 Total value added in exports

$$DCE_n = \frac{V_n}{E_n} \tag{11}$$

3 Algorithm

Step 1: Guess $w^0 = \{w_1^0, ..., w_n^0\}$.
Step 2: Given TFP and trade cost: $\left\{A_n^s,\ d_{i,n}^s\right\}$, calculating $p^0 = \{p_1^0, ..., p_n^0\}$.
Step 3: Given TFP, trade cost and p^0, calculating spending share $\lambda_{\ell,n}^0$.
Step 4: calculating labor distribution and total spending: $\left\{L_n^{s\,0},\ X_n^0\right\}$.
Step 5: checking trade balance condition, if smaller than 10E-6, stop loops; otherwise, go back to Step 1 with new guess $w^0 = w^1$.
Update new wage rate w^1 under fixed World GDP, (Alvarez & Lucas Jr, 2007):

$$w_n^1 = w_n^0\left(1 + v\frac{M_n^0 - E_n^0}{L_n}\right)$$

Where $v \in (0,1)$ controls convergence speed.
For any fixed world GDP given by w_n^0

$$\sum_{n=1}^{N} w_n^0 L_n \equiv C$$

World GDP generate from w_n^1 still fixed:

$$\begin{aligned}\sum_{n=1}^{N} w_n^1 L_n &= \sum_{n=1}^{N} \left(w_n^0 \left(1 + v \frac{M_n^0 - E_n^0}{L_n w_n^0} \right) \right) L_n \\ &= \sum_{n=1}^{N} \left(w_n^0 L_n + v \left(M_n^0 - E_n^0 \right) \right) \\ &= \sum_{n=1}^{N} \left(w_n^0 L_n \right) + v \sum_{n=1}^{N} \left(M_n^0 - E_n^0 \right) \\ &= \sum_{n=1}^{N} \left(w_n^0 L_n \right) = C\end{aligned}$$

4 Trade Share of GDP

4.1 GDP

Define $\Lambda_n^s \equiv \{\ell \in \mathcal{C} : \ell^s = n\}$ as the set of chains that country n occupies position s, firms demand inputs so that marginal products of all inputs are equated:

$$w_n L_n^s = \gamma^s \widetilde{\gamma}^s \sum_{\ell \in \Lambda_n^s} \sum_{i=1}^{N} X_i \lambda_{\ell,i} \tag{12}$$

where $P_i Q_i \equiv X_i$, $\Lambda_n^s \equiv \{\ell \in \mathcal{C} : \ell^s = n\}$.

$$\sum_{s=1}^{S} L_n^s = L_n \tag{13}$$

and $L_1^1 + L_1^2 = L_1$; $L_2^1 + L_2^2 = L_2$.

$$VA_n = w_n L_n = \sum_{s=1,2} \gamma^s \widetilde{\gamma}^s \sum_{\ell \in \Lambda_n^s} \sum_{i=1}^{N} X_i \lambda_{\ell,i} \tag{14}$$

$$\Lambda_{n=1}^{s=1} \equiv \{\mathcal{C}_1; \mathcal{C}_2\} = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}; \qquad \Lambda_{n=1}^{s=2} \equiv \{\mathcal{C}_1; \mathcal{C}_3\} = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$$

$$\Lambda_{n=2}^{s=1} \equiv \{\mathcal{C}_3; \mathcal{C}_4\} = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}; \begin{pmatrix} 2 \\ 2 \end{pmatrix} \right\}; \qquad \Lambda_{n=2}^{s=2} \equiv \{\mathcal{C}_2; \mathcal{C}_4\} = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}; \begin{pmatrix} 2 \\ 2 \end{pmatrix} \right\}$$

$$w_1 L_1^1 = \gamma^1 \widetilde{\gamma}^1 \left\{ \sum_{i=1}^{N} X_i \left(\lambda_{\ell=\mathcal{C}_1,i} + \lambda_{\ell=\mathcal{C}_2,i} \right) \right\}$$

$$w_1 L_1^2 = \gamma^2 \widetilde{\gamma}^2 \left\{ \sum_{i=1}^{N} X_i \left(\lambda_{\ell=\mathcal{C}_1,i} + \lambda_{\ell=\mathcal{C}_3,i} \right) \right\}$$

$$w_2 L_2^1 = \gamma^1 \widetilde{\gamma}^1 \left\{ \sum_{i=1}^{N} X_i \left(\lambda_{\ell=\mathcal{C}_3,i} + \lambda_{\ell=\mathcal{C}_4,i} \right) \right\}$$

$$w_2 L_2^2 = \gamma^2 \widetilde{\gamma}^2 \left\{ \sum_{i=1}^{N} X_i \left(\lambda_{\ell=\mathcal{C}_2,i} + \lambda_{\ell=\mathcal{C}_4,i} \right) \right\}$$

$$\begin{aligned} VA_1 = w_1 \left(L_1^1 + L_1^2 \right) &= \gamma^1 \widetilde{\gamma}^1 \left\{ \sum_{i=1}^{N} X_i \left(\lambda_{\ell=\mathcal{C}_1,i} + \lambda_{\ell=\mathcal{C}_2,i} \right) \right\} + \gamma^2 \widetilde{\gamma}^2 \left\{ \sum_{i=1}^{N} X_i \left(\lambda_{\ell=\mathcal{C}_1,i} + \lambda_{\ell=\mathcal{C}_3,i} \right) \right\} \\ &= \gamma^1 \widetilde{\gamma}^1 \left\{ X_1 \left(\lambda_{\mathcal{C}_1,1} + \lambda_{\mathcal{C}_2,1} \right) + X_2 \left(\lambda_{\mathcal{C}_1,2} + \lambda_{\mathcal{C}_2,2} \right) \right\} + \gamma^2 \widetilde{\gamma}^2 \left\{ X_1 \left(\lambda_{\mathcal{C}_1,1} + \lambda_{\mathcal{C}_3,1} \right) + X_2 \left(\lambda_{\mathcal{C}_1,2} + \lambda_{\mathcal{C}_3,2} \right) \right\} \end{aligned}$$

$$\begin{aligned} VA_2 = w_2 \left(L_2^1 + L_2^2 \right) &= \gamma^1 \widetilde{\gamma}^1 \left\{ \sum_{i=1}^{N} X_i \left(\lambda_{\ell=\mathcal{C}_3,i} + \lambda_{\ell=\mathcal{C}_4,i} \right) \right\} + \gamma^2 \widetilde{\gamma}^2 \left\{ \sum_{i=1}^{N} X_i \left(\lambda_{\ell=\mathcal{C}_2,i} + \lambda_{\ell=\mathcal{C}_4,i} \right) \right\} \\ &= \gamma^1 \widetilde{\gamma}^1 \left\{ X_1 \left(\lambda_{\mathcal{C}_3,1} + \lambda_{\mathcal{C}_4,1} \right) + X_2 \left(\lambda_{\mathcal{C}_3,2} + \lambda_{\mathcal{C}_4,2} \right) \right\} + \gamma^2 \widetilde{\gamma}^2 \left\{ X_1 \left(\lambda_{\mathcal{C}_2,1} + \lambda_{\mathcal{C}_4,1} \right) + X_2 \left(\lambda_{\mathcal{C}_2,2} + \lambda_{\mathcal{C}_4,2} \right) \right\} \end{aligned}$$

4.2 Imports

$$M_n^\ell = P_n Q_n \lambda_{\ell,n} \sum_{s=1}^{S} \widetilde{\gamma}^s 1_{\ell \in \mathcal{M}_n^s} + \sum_{i \neq n} P_i Q_i \lambda_{\ell,i} \sum_{s=1}^{S-1} \widetilde{\gamma}^s 1_{\ell \in \mathcal{M}_n^s} \tag{15}$$

For any stage $s < S$, $\mathcal{M}_n^s = \left\{ \ell : \ell^{s+1} = n \textit{ and } \ell^s \neq n \right\}$. This is the set of chains that country n incurs imports at stage s to use in production in stage s + 1

For any stage $s = S$, $\mathcal{M}_n^S = \left\{ \ell : \ell^S \neq n \right\} \equiv \bigcup_{i \neq n} \Lambda_i^S$. This is the set of chains that country n imports final goods. These chains generate exports of final goods for country n from foreign demand on this chain.

The total imports of Country n, M_n is

$$M_n = \sum_{\ell = \ell_1, \ell_2, \ell_3, \ell_4} M_n^\ell \tag{16}$$

$\mathcal{M}_{n=1}^{s=1} \equiv \{\mathcal{C}_3\} = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$; $\qquad \mathcal{M}_{n=1}^{s=2} \equiv \{\mathcal{C}_2; \mathcal{C}_4\} = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}; \begin{pmatrix} 2 \\ 2 \end{pmatrix} \right\}$

$$\mathcal{M}_{n=2}^{s=1} \equiv \{\mathcal{C}_2\} = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}; \qquad \mathcal{M}_{n=2}^{s=2} \equiv \{\mathcal{C}_1; \mathcal{C}_3\} = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$$

$$\left\{M_n^\ell\right\}_{n=1,\ell=\mathcal{C}_1} = 0 \qquad \left\{M_n^\ell\right\}_{n=2,\ell=\mathcal{C}_1} = X_{n=2}\lambda_{\ell=\mathcal{C}_1,n=2}\widetilde{\gamma}^{s=2}$$

$$\left\{M_n^\ell\right\}_{n=1,\ell=\mathcal{C}_2} = X_{n=1}\lambda_{\ell=\mathcal{C}_2,n=1}\widetilde{\gamma}^{s=2} \qquad \left\{M_n^\ell\right\}_{n=2,\ell=\mathcal{C}_2} = X_{n=2}\lambda_{\ell=\mathcal{C}_2,n=2}\widetilde{\gamma}^{s=1} + X_1\lambda_{\mathcal{C}_2,1}\widetilde{\gamma}^1$$

$$\left\{M_n^\ell\right\}_{n=1,\ell=\mathcal{C}_3} = X_{n=1}\lambda_{\ell=\mathcal{C}_3,n=1}\widetilde{\gamma}^{s=1} + X_{i=2}\lambda_{\ell=\mathcal{C}_3,i=2}\widetilde{\gamma}^{s=1} \qquad \left\{M_n^\ell\right\}_{n=2,\ell=\mathcal{C}_3} = X_{n=2}\lambda_{\ell=\mathcal{C}_3,n=2}\widetilde{\gamma}^{s=2}$$

$$\left\{M_n^\ell\right\}_{n=1,\ell=\mathcal{C}_4} = X_{n=1}\lambda_{\ell=\mathcal{C}_4,n=1}\widetilde{\gamma}^{s=2} \qquad \left\{M_n^\ell\right\}_{n=2,\ell=\mathcal{C}_4} = 0$$

$$M_1 = \sum_{\ell=\ell_1,\ell_2,\ell_3,\ell_4} M_1^\ell = X_1\lambda_{\mathcal{C}_2,1}\widetilde{\gamma}^2 + X_1\lambda_{\mathcal{C}_3,1}\widetilde{\gamma}^1 + X_2\lambda_{\mathcal{C}_3,2}\widetilde{\gamma}^1 + X_1\lambda_{\mathcal{C}_4,1}\widetilde{\gamma}^2 \tag{17}$$

$$M_2 = \sum_{\ell=\ell_1,\ell_2,\ell_3,\ell_4} M_2^\ell = X_2\lambda_{\mathcal{C}_1,2}\widetilde{\gamma}^2 + X_2\lambda_{\mathcal{C}_2,2}\widetilde{\gamma}^1 + X_1\lambda_{\mathcal{C}_2,1}\widetilde{\gamma}^1 + X_2\lambda_{\mathcal{C}_3,2}\widetilde{\gamma}^2 \tag{18}$$

4.3 Trade share

$$ImporShareGDP_n = \frac{M_n}{VA_n} = \frac{\sum_{\ell=\ell_1,\ell_2,\ell_3,\ell_4} M_n^\ell}{\sum_{s=1,2}\gamma^s\widetilde{\gamma}^s\sum_{\ell\in\Lambda_n^s}\sum_{i=1}^N X_i\lambda_{\ell,i}} \tag{19}$$

$$M_1 = X_1\lambda_{\mathcal{C}_2,1}\widetilde{\gamma}^2 + X_1\lambda_{\mathcal{C}_3,1}\widetilde{\gamma}^1 + X_2\lambda_{\mathcal{C}_3,2}\widetilde{\gamma}^1 + X_1\lambda_{\mathcal{C}_4,1}\widetilde{\gamma}^2 \tag{20}$$

$$M_2 = X_2\lambda_{\mathcal{C}_1,2}\widetilde{\gamma}^2 + X_2\lambda_{\mathcal{C}_2,2}\widetilde{\gamma}^1 + X_1\lambda_{\mathcal{C}_2,1}\widetilde{\gamma}^1 + X_2\lambda_{\mathcal{C}_3,2}\widetilde{\gamma}^2 \tag{21}$$

$$VA_1 = \gamma^1\widetilde{\gamma}^1\left\{X_1\left(\lambda_{\mathcal{C}_1,1} + \lambda_{\mathcal{C}_2,1}\right) + X_2\left(\lambda_{\mathcal{C}_1,2} + \lambda_{\mathcal{C}_2,2}\right)\right\} + \gamma^2\widetilde{\gamma}^2\left\{X_1\left(\lambda_{\mathcal{C}_1,1} + \lambda_{\mathcal{C}_3,1}\right) + X_2\left(\lambda_{\mathcal{C}_1,2} + \lambda_{\mathcal{C}_3,2}\right)\right\}$$

$$VA_2 = \gamma^1\widetilde{\gamma}^1\left\{X_1\left(\lambda_{\mathcal{C}_3,1} + \lambda_{\mathcal{C}_4,1}\right) + X_2\left(\lambda_{\mathcal{C}_3,2} + \lambda_{\mathcal{C}_4,2}\right)\right\} + \gamma^2\widetilde{\gamma}^2\left\{X_1\left(\lambda_{\mathcal{C}_2,1} + \lambda_{\mathcal{C}_4,1}\right) + X_2\left(\lambda_{\mathcal{C}_2,2} + \lambda_{\mathcal{C}_4,2}\right)\right\}$$

$$ImporShareGDP_1 = \frac{M_1}{VA_1}$$

$$= \frac{X_1\lambda_{\mathcal{C}_2,1}\widetilde{\gamma}^2 + X_1\lambda_{\mathcal{C}_3,1}\widetilde{\gamma}^1 + X_2\lambda_{\mathcal{C}_3,2}\widetilde{\gamma}^1 + X_1\lambda_{\mathcal{C}_4,1}\widetilde{\gamma}^2}{\gamma^1\widetilde{\gamma}^1\left\{X_1\left(\lambda_{\mathcal{C}_1,1} + \lambda_{\mathcal{C}_2,1}\right) + X_2\left(\lambda_{\mathcal{C}_1,2} + \lambda_{\mathcal{C}_2,2}\right)\right\} + \gamma^2\widetilde{\gamma}^2\left\{X_1\left(\lambda_{\mathcal{C}_1,1} + \lambda_{\mathcal{C}_3,1}\right) + X_2\left(\lambda_{\mathcal{C}_1,2} + \lambda_{\mathcal{C}_3,2}\right)\right\}}$$

$$ImporShareGDP_2 = \frac{M_2}{VA_2}$$

$$= \frac{X_2\lambda_{\mathfrak{C}_1,2}\widetilde{\gamma}^2 + X_2\lambda_{\mathfrak{C}_2,2}\widetilde{\gamma}^1 + X_1\lambda_{\mathfrak{C}_2,1}\widetilde{\gamma}^1 + X_2\lambda_{\mathfrak{C}_3,2}\widetilde{\gamma}^2}{\gamma^1\widetilde{\gamma}^1\left\{X_1\left(\lambda_{\mathfrak{C}_3,1} + \lambda_{\mathfrak{C}_4,1}\right) + X_2\left(\lambda_{\mathfrak{C}_3,2} + \lambda_{\mathfrak{C}_4,2}\right)\right\} + \gamma^2\widetilde{\gamma}^2\left\{X_1\left(\lambda_{\mathfrak{C}_2,1} + \lambda_{\mathfrak{C}_4,1}\right) + X_2\left(\lambda_{\mathfrak{C}_2,2} + \lambda_{\mathfrak{C}_4,2}\right)\right\}}$$

5 Model implied sensitivity (welfare and import share of GDP) w.r.t trade cost change

I solve the above simplified SYZ (Sposi et al. (2020)) trade model basing on parameters in Table 1. I allows trade cost of both stage 1 and stage 2 changing from 3 (5.5 or 8) to 1 to simulate the globalization process and then calculate the model implied welfare change w.r.t trade cost change. These models with different trade cost values are solved under same fixed world GDP.

Table 1: Parameters Setup

Definition	Value
Number of Countries	Symmetric country, $N \equiv 2$
Number of stages	$S \equiv 2$
Trade elasiticity	$\theta \equiv 4$
Variety substitution coefficient	$\eta \equiv 2$
Stage 1 labor share	$\gamma^1 \equiv 1$, $\widetilde{\gamma}^1 \equiv 0.5$
Stage 2 labor share	$\gamma^2 \equiv 0.5$, $\widetilde{\gamma}^2 \equiv 1$
trade deficit	$D_1 = D_2 \equiv 0$
labor supply	$L_1 = L_2 \equiv 10$
World GDP	$W_1L_1 + W_2L_2 \equiv 1$

5.1 Welfare

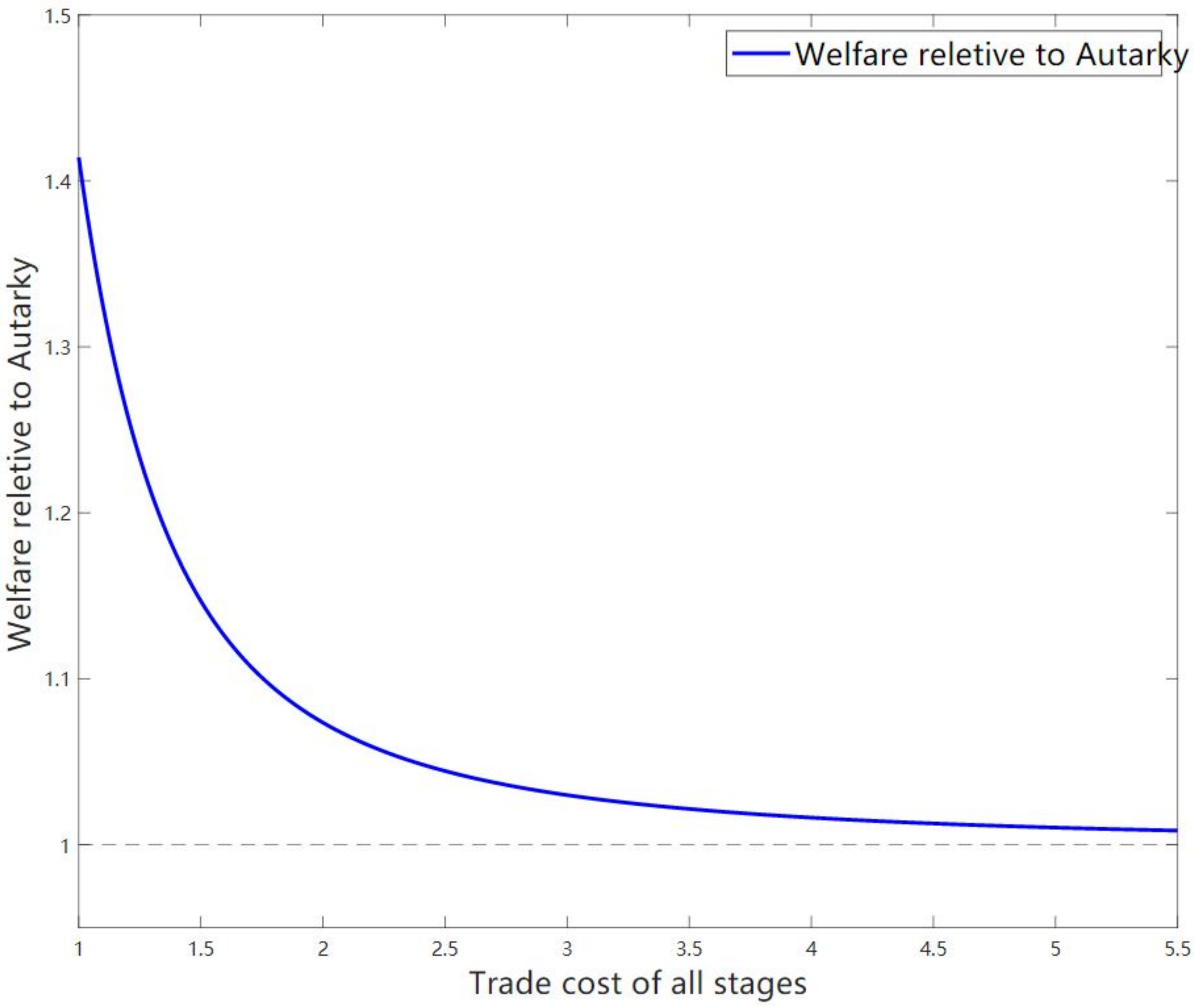

Figure 2: Welfare Change w.r.t Trade Cost Change

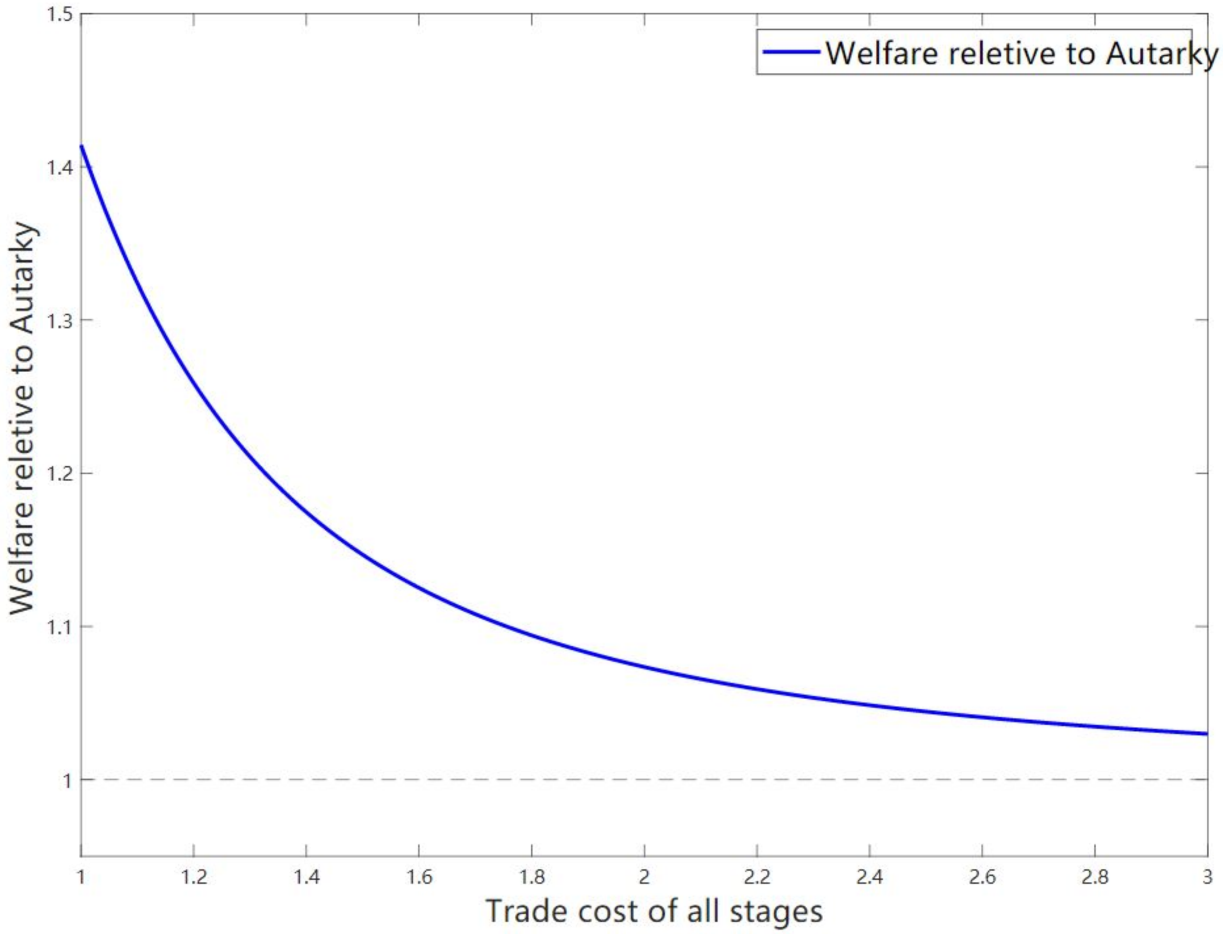

Figure 3: Welfare Change w.r.t Trade Cost Change

5.2 Import share of GDP

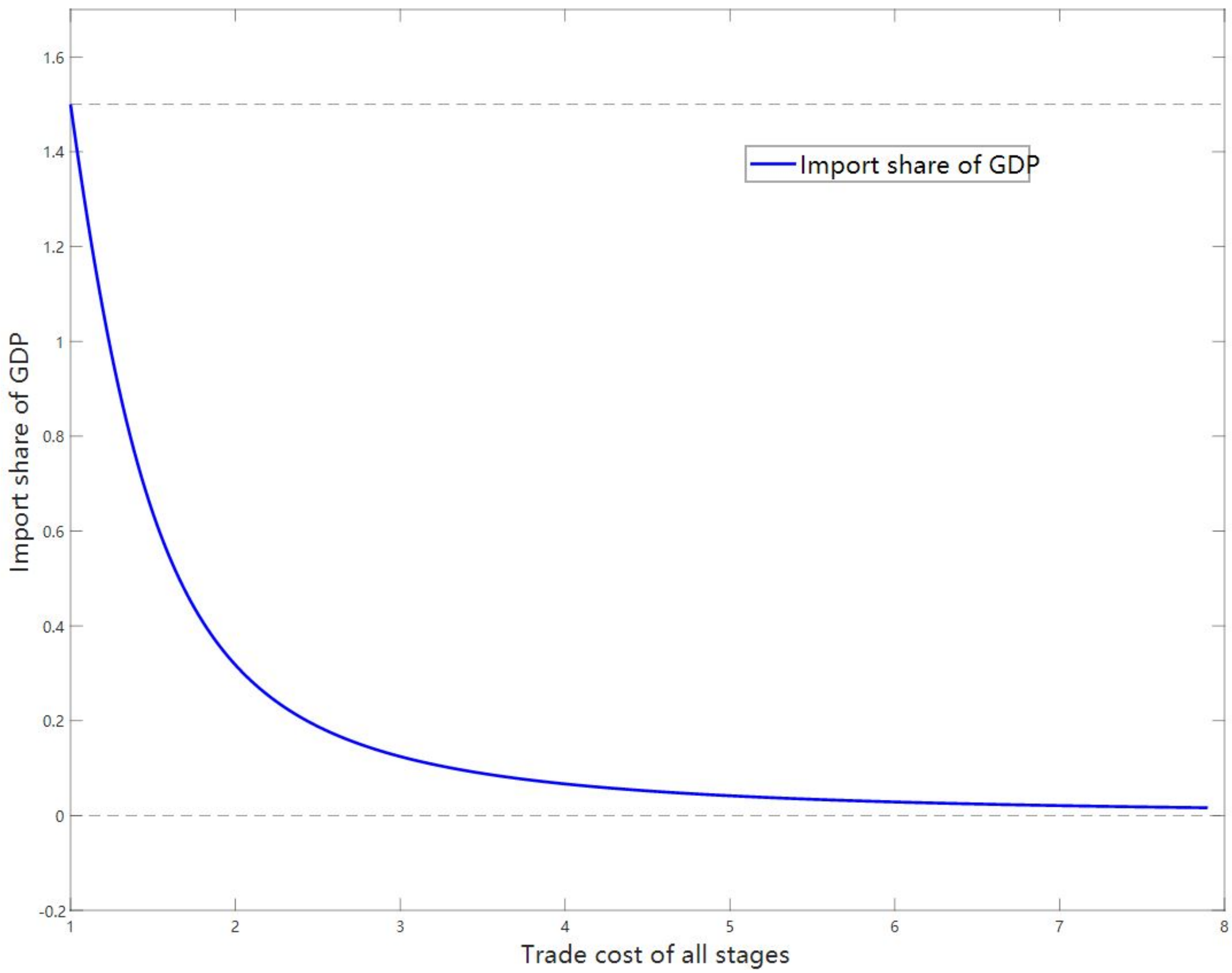

Figure 4: Import share of Change w.r.t Trade Cost Change

References

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Sposi, M., Yi, K.-M., & Zhang, J. (2020, November). Trade integration, global value chains, and capital accumulation [Working Paper]. (28087). Retrieved from `http://www.nber.org/papers/w28087` doi: 10.3386/w28087