Research Notes

Yang Pei[*]

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[*]Yang Pei, University of Houston, `ypei5@uh.edu`

CP model

Table 1: Equilibrium conditions

(F1)	$c_n^j = \Upsilon_n^j w_n{}^{\gamma_n^j} \prod_{k=1}^J P_n^k{}^{\gamma_n^{k,j}}$; $\Upsilon_n^j \equiv \prod_{k=1}^J \gamma_n^{k,j}{}^{-\gamma_n^{k,j}} \gamma_n^j{}^{-\gamma_n^j}$	$\forall(n,j)$
(F2)	$P_n^j = A^j (\sum_{i=1}^N \lambda_i^j \left(\kappa_{ni}^j c_i^j\right)^{-\theta})^{-\frac{1}{\theta}}$; $A^j = \Gamma\left(\frac{1+\theta-\sigma^j}{\theta}\right)^{\frac{1}{(1-\sigma^j)}}$	$\forall(n,j)$
(F3)	$\pi_{ni}^j = \frac{\lambda_i^j \left(c_i^j \kappa_{ni}^j\right)^{-\theta}}{\sum_{m=1}^N \lambda_m^j \left(c_m^j \kappa_{nm}^j\right)^{-\theta}} = \lambda_i^j \left(A^j \frac{c_i^j \kappa_{ni}^j}{P_n^j}\right)^{-\theta}$	$\forall(n,j)$
(H1)	$P_{n=} \prod_{j=1}^J \left(\frac{P_n^j}{\alpha_n^j}\right)^{\alpha_n^j}$	$\forall(n)$
(H2)	$w_n L_n + D_n = I_n$	$\forall(n)$
(M1)	$X_n^j = \alpha_n^j I_n + \sum_{k=1}^J \gamma_n^{j,k} \left(\sum_{i=1}^N X_{in}^k\right)$	$\forall(n,j)$
(M2)	$\sum_{j=1}^J \sum_{i=1}^N X_{ni}^j - D_n = \sum_{j=1}^J \sum_{i=1}^N X_{in}^j$	$\forall(n,j)$
(M2′)	$w_n L_n = \sum_{j=1}^J \gamma_n^j \sum_{i=1}^N \pi_{in}^j X_i^j$	$\forall(n)$

Step 1: guess $\vec{w^1}$, then get $\vec{P}_n^j(\vec{w}^1)$ and $\vec{c}_n^j(\vec{w}^1)$, $\vec{\pi}_{ni}^j(\vec{w}^1)$.

Step 2: get $\vec{X}_n^j(\vec{w}^1)$.

Step 3: check trade balance condition. until $\vec{w} = \vec{w}^*$.

Step 4: $w^2{}_i = T(w)_i = w_i(1 + v\frac{Z_i(\vec{w})}{L_i}), \forall i$, where

$$Z_n(\vec{w^1}) = \left[\sum_{k=1}^J \gamma_n^k \left(\sum_{i=1}^N X_{in}^k\right) - w_n L_n\right]$$

CP model with capital and labor

Table 2: Steady state equilibrium conditions

(F1)	$c_{n,t}^{j} \equiv \Upsilon_{n}^{j}\left[\left(W_{n,t}^{j}\right)^{\beta_{n}^{j}}\left(R_{n,t}^{j}\right)^{1-\beta_{n}^{j}}\right]^{\gamma_{n}^{j}} \prod_{k=1}^{J} {P_{n,t}^{k}}^{\gamma_{n}^{k,j}}$; $\Upsilon_{n}^{j} \equiv \left[\gamma_{n}^{j}\beta_{n}^{j}\right]^{-\gamma_{n}^{j}\beta_{n}^{j}}\left[\gamma_{n}^{j}\left(1-\beta_{n}^{j}\right)\right]^{-\gamma_{n}^{j}\left(1-\beta_{n}^{j}\right)} \prod_{k=1}^{J} {\gamma_{n}^{k,j}}^{-\gamma_{n}^{k,j}}$	$\forall(n,j)$
(F2)	$P_{n,t}^{j} = A^{j}\cdot\left[\sum_{i=1}^{N}\lambda_{i,t}^{j}\left(\kappa_{ni,t}^{j}c_{i,t}^{j}\right)^{-\theta}\right]^{-\frac{1}{\theta}}$; $A^{j} = \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$	$\forall(n,j)$
(F3)	$\pi_{ni,t}^{j} = \frac{\lambda_{i,t}^{j}\left(c_{i,t}^{j}\kappa_{ni,t}^{j}\right)^{-\theta}}{\sum_{m=1}^{N}\lambda_{m,t}^{j}\left(c_{m,t}^{j}\kappa_{nm,t}^{j}\right)^{-\theta}} = \lambda_{i,t}^{j}\left(\frac{A^{j}c_{i,t}^{j}\kappa_{ni,t}^{j}}{P_{n,t}^{j}}\right)^{-\theta}$	$\forall(n,j)$
(H1)	$P_{I,n,t} = \prod_{j=1}^{J}\left[\frac{P_{n,t}^{j}}{\alpha_{I,n}^{j}}\right]^{\alpha_{I,n}^{j}}$; $P_{C,n,t} = \prod_{j=1}^{J}\left[\frac{P_{n,t}^{j}}{\alpha_{C,n}^{j}}\right]^{\alpha_{C,n}^{j}}$	$\forall(n)$
(H2)	$P_{n,C,t}C_{n,t} + P_{n,I,t}I_{n,t} = W_{n,t}L_{n,t} + R_{n,t}K_{n,t} + D_{n,t} = IN_{n,t}$	$\forall(n)$
(S1)	S.S.: $I_{n,t} = \delta K_{n,t}$; $K_{n,t+1} = I_{n,t} + (1-\delta)K_{n,t}$	$\forall(n)$
(S2)	Exo.S: $I_{n,t} = \frac{s_{n,t}\cdot IN_{n,t}}{P_{I,n,t}}$	$\forall(n)$
(M1)	$X_{n,t}^{j} = P_{n,t}^{j}\left(C_{n,t}^{j} + I_{n,t}^{j}\right) + \sum_{k=1}^{J}\gamma_{n}^{j,k}\left(\sum_{i=1}^{N}X_{in,t}^{k}\right) = \alpha_{C,n}^{j}P_{C,n,t}C_{n,t} + \alpha_{I,n}^{j}P_{I,n,t}I_{n,t} + \sum_{k=1}^{J}\gamma_{n}^{j,k}\left(\sum_{i=1}^{N}X_{in,t}^{k}\right)$	$\forall(n,j)$
(M2)	$\sum_{j=1}^{J}\sum_{i=1}^{N}X_{ni,t}^{j} - D_{n,t} = \sum_{j=1}^{J}\sum_{i=1}^{N}X_{in,t}^{j}$	$\forall(n,j)$
(M2′)	$W_{n,t}L_{n,t} = \sum_{j=1}^{J}\beta_{n}^{j}\gamma_{n}^{j}\sum_{i=1}^{N}\pi_{in,t}^{j}X_{i,t}^{j}$; $R_{n,t}K_{n,t} = \sum_{j=1}^{J}\left(1-\beta_{n}^{j}\right)\gamma_{n}^{j}\sum_{i=1}^{N}\pi_{in,t}^{j}X_{i,t}^{j}$	$\forall(n)$

Table 3: Capital only as inputs, static equilibrium conditions

(F1)	$c_{n,t}^{j} \equiv \Upsilon_{n}^{j}\left[\left(W_{n,t}^{j}\right)^{\beta_{n}^{j}}\left(R_{n,t}^{j}\right)^{1-\beta_{n}^{j}}\right]^{\gamma_{n}^{j}} \prod_{k=1}^{J} {P_{n,t}^{k}}^{\gamma_{n}^{k,j}}$; $\Upsilon_{n}^{j} \equiv \left[\gamma_{n}^{j}\beta_{n}^{j}\right]^{-\gamma_{n}^{j}\beta_{n}^{j}}\left[\gamma_{n}^{j}\left(1-\beta_{n}^{j}\right)\right]^{-\gamma_{n}^{j}\left(1-\beta_{n}^{j}\right)} \prod_{k=1}^{J} {\gamma_{n}^{k,j}}^{-\gamma_{n}^{k,j}}$	$\forall(n,j)$
(F2)	$P_{n,t}^{j} = A^{j}\cdot\left[\sum_{i=1}^{N}\lambda_{i,t}^{j}\left(\kappa_{ni,t}^{j}c_{i,t}^{j}\right)^{-\theta}\right]^{-\frac{1}{\theta}}$; $A^{j} = \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$	$\forall(n,j)$
(F3)	$\pi_{ni,t}^{j} = \frac{\lambda_{i,t}^{j}\left(c_{i,t}^{j}\kappa_{ni,t}^{j}\right)^{-\theta}}{\sum_{m=1}^{N}\lambda_{m,t}^{j}\left(c_{m,t}^{j}\kappa_{nm,t}^{j}\right)^{-\theta}} = \lambda_{i,t}^{j}\left(\frac{A^{j}c_{i,t}^{j}\kappa_{ni,t}^{j}}{P_{n,t}^{j}}\right)^{-\theta}$	$\forall(n,j)$
(H1)	$P_{C,n,t} = \prod_{j=1}^{J}\left[\frac{P_{n,t}^{j}}{\alpha_{C,n}^{j}}\right]^{\alpha_{C,n}^{j}}$	$\forall(n)$
(H2)	$P_{n,C,t}C_{n,g,t} = W_{n,t}L_{n,t} + R_{n,t}K_{n,t} + D_{n,t} = IN_{n,t}$	$\forall(n)$
(M1)	$X_{n,t}^{j} = P_{n,t}^{j}\left(C_{n,t}^{j}\right) + \sum_{k=1}^{J}\gamma_{n}^{j,k}\left(\sum_{i=1}^{N}X_{in,t}^{k}\right) = \alpha_{C,n}^{j}IN_{n,t} + + \sum_{k=1}^{J}\gamma_{n}^{j,k}\left(\sum_{i=1}^{N}X_{in,t}^{k}\right)$	$\forall(n,j)$
(M2)	$\sum_{j=1}^{J}\sum_{i=1}^{N}X_{ni,t}^{j} - D_{n,t} = \sum_{j=1}^{J}\sum_{i=1}^{N}X_{in,t}^{j}$	$\forall(n,j)$
(M2′)	$W_{n,t}L_{n,t} = \sum_{j=1}^{J}\beta_{n}^{j}\gamma_{n}^{j}\sum_{i=1}^{N}\pi_{in,t}^{j}X_{i,t}^{j}$; $R_{n,t}K_{n,t} = \sum_{j=1}^{J}\left(1-\beta_{n}^{j}\right)\gamma_{n}^{j}\sum_{i=1}^{N}\pi_{in,t}^{j}X_{i,t}^{j}$	$\forall(n)$

Step 1: guess $\overrightarrow{W,R}^{1}$, then get $\vec{P}_{n}^{j}(\overrightarrow{W,R}^{1})$ and $\vec{c}_{n}^{j}(\overrightarrow{W,R}^{1})$, $\vec{\pi}_{ni}^{j}(\overrightarrow{W,R}^{1})$.

Step 2: get $\vec{X}_{n}^{j}(\overrightarrow{W,R}^{1})$.

Step 3: check trade balance condition. until $\overrightarrow{W,R} = \overrightarrow{W,R}^{*}$.

Step 4: $ZW(\overrightarrow{W,R})_{i} = \sum_{j=1}^{J}\beta_{n}^{j}\gamma_{n}^{j}\sum_{i=1}^{N}\pi_{in,t}^{j}X_{i,t}^{j} - W_{n,t}L_{n,t}, \forall i$, and $ZR(\overrightarrow{W,R})_{i} = \sum_{j=1}^{J}\left(1-\beta_{n}^{j}\right)\gamma_{n}^{j}\sum_{i=1}^{N}\pi_{in,t}^{j}X_{i,t}^{j} - R_{n,t}K_{n,t}, \forall i$

where

$$W^{2}{}_{i} = TW(\overrightarrow{W,R}) = W_{i}\left(1 + v\frac{ZW(\overrightarrow{W,R})_{i}}{L_{i,t}}\right)$$

$$R^{2}{}_{i} = TR(\overrightarrow{W,R}) = R_{i}\left(1 + v\frac{ZR(\overrightarrow{W,R})_{i}}{K_{i,t}}\right)$$