Demographics, Trade, and Growth

Yang Pei

University of Houston

Presentation at The Hong Kong Polytechnic University

Februrary 26, 2025

Research Question

Motivation

- Nowadays, about one-third of global GDP is generated in countries with declining and aging populations
- Chief among them is China
  - As its population declines and ages, economic growth has also slowed down
- At the same time, the labor-intensive goods, that China used to specialize in, are now relocating their production
  - from China to other developing countries

Research Question: How much does demographic structure influence China's economic growth and trade patterns?

- Centering around two mechanisms
  - Age-dependent idea generation process that affects **productivity**
  - Age-dependent saving behavior that affects **capital accumulation**

What I do

- Conduct empirical analysis using Panel regression and Panel VARX model
  - I find a strong positive association between countries' working age share, and
    - Productivity growth; Investment or Saving share of GDP
  - I find an inverse U-shaped response from a 1-percentage point young cohort share shock on
    - Productivity growth; the growth rate of capital stock per person

- Develop and Calibrate a OLG trade model features
  - Demographic-induced TFP growth
  - Demographic-induced capital accumulation
  - Trade based on Ricardian and Heckscher-Ohlin comparative advantages (CA)

- By comparing a real-world scenario with a hypothetical one, in which China's fertility and survival rates are replaced by those of RoW, I find
  - Short run, a saving-favorable age structure leads to higher **capital**, and income per worker
    - along with a stronger comparative advantage in the capital-intensive sector
  - Long run, after 2060, a lower path of knowledge stocks leads to a lower **productivity**, and income per worker
  - Trade liberalization encourages specialization and selection, extends short-run benefit period

Related Literature

Demographic structure and productivity

- Empirical: Feyrer (2007); Maestas, Mullen, and Powell (2023); Jones (2010); Azoulay, Graff Zivin, and Wang (2010);
  - Replicate these results at the macro level using a larger set of countries and more recent years; further estimate the dynamic effects of demographic shocks
- Models: Becker, Murphy, and Tamura (1990); Lindh and Malmberg (1999); Aksoy, Basso, Smith, and Grasl (2019); Buera and Oberffeld (2020)
  - Model the relationship between demographics and productivity by assuming age-varying ability in generating new ideas

Multi-country trade models with capital accumulation

- Sposi (2022); Eaton, Kortum, Neiman, and Romalis (2016); Alvarez (2017); Ravikumar, Santacreu, and Sposi (2019); Anderson, Larch, and Yotov (2020); and Sposi, Yi, and Zhang (2021a)
  - Link capital accumulation to age-varying demographics and analyze its interaction with trade-induced relocation and economic growth

Changes in China's trade patterns and economic growth

- Liu and Ma (2018); Tombe and Zhu (2019); Fan (2019); Hao, Sun, Tombe, and Zhu (2020); Ma and Tang (2020), Alessandria, Khan, Khederlarian, Ruhl, and Steinberg (2021); Hanwei, Jiandong, and Yue (2024); Brandt, and Lim (2024)
  - Quantify trade pattern changes and economic growth from a demographic perspective

Empirical

Data source

The United Nations Statistics Division (UNSD)

- Age cohorts share for every 5 years, Dependence ratio, Old dependence ratio, Young dependence ratio, Total population

Penn World Table (PWT 10.01)

- Average annual hours worked by persons engaged, Number of persons engaged, Mean years of schooling, Capital stock, Real GDP, Average depreciation rate of the capital stock
- TFP calculated by PWT based on above variables

CEPII

- Imports and Exports between two countries

World Development Indicators (WDI)

- Share of household consumption, capital formation, government consumption (% share of GDP), residents new patents application, residents new industrial design application

Panel Regression

Effect of Demographic structure on TFP growth

$$Y_{it,t+4} = Constant + \alpha_1 Demographic_{it} + \alpha_2 Controls_{it} + f_i + f_t + \varepsilon_{it} \qquad (1)$$

- Dependent variables, $Y_{it,t+4}$:
  - Average yearly TFP growth rate; Average yearly Investment, or consumption share of GDP (during the period t to $t+4$)
- $Demographic_t$: Working age share [15-64/total]
- $Controls_{it}$: log real GDP per capita at t for country i; number of total population at t for country i
- f_i and f_t: country and year fixed effects
- Data sample: 74 countries. 10 non-overlapping 5 years from 1970 to 2019

Panel regression main results

	Average value in the future 4 years	
VARIABLES	TFP growth rate	Cap.Formation(% GDP)
Work.Share (15-64)/ToT	11.43***	28.80**
	(3.33)	(2.17)
Control	YES	YES
Observations	732	724
R-squared	0.259	0.575

1 p.p. (or 1 s.d.) increase, in the working age share, is related to a 0.11 p.p. (or 0.81 s.d.) increase, in the average TFP growth rate over the following 4-year period.

1 p.p. (or 1 s.d.) increase, in the working age share, is related to a 0.29 p.p. (or 0.33 s.d.) increase, in the average capital formation share of GDP over the next four years.

Robustness checks: Details

- Different age cohorts across total population: 3 cohorts: [0, 14], [15,64], [64,+); 4 cohorts: 0,24], [25,49], [50,74],[75, +); 5 cohorts: [0, 19], [20,39], [40,59], [60,79], [80,+)
- Age cohorts across labor force: [20, 29], [30, 39], [40, 49],
- Other variable: new patent applications (per 1000 people); new industrial design applications (per 1000 people)

Panel VARX model

Capital accumulation, TFP, and economic growth

VARX model:

$$Y_{n,t} = C + AY_{n,t-\mathbf{1}} + BX_{n,t-1} + \varepsilon_{n,t}$$

Endogenous variables:

$$Y_{nt} = \begin{bmatrix} the\ 5\ year\ growth\ rate\ of\ capital\ per\ person\ (\%) \\ the\ 5\ year\ growth\ rate\ of\ TFP\ (\%) \\ the\ 5\ year\ growth\ rate\ of\ the\ real\ GDP\ per\ capita\ (\%) \end{bmatrix}_{Country\ n, time\ t}$$

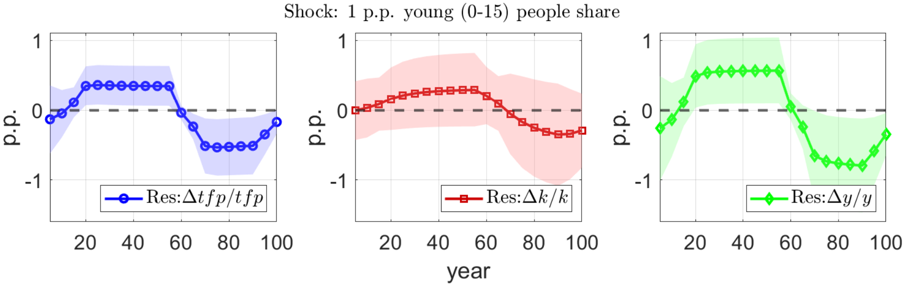
Exogenous variables:

$$X_{nt} = \begin{bmatrix} young\ people\ share\ (\%),\ (0-14) \\ old\ people\ share\ (\%),\ (65+) \\ trade\ cost\ change\ (\%) \\ the\ 5\ year\ growth\ rate\ of\ population (\%) \end{bmatrix}_{Country\ n, time\ t}$$

Time interval: 1 unit of time = 5 years. e.g. t = 1 means first 5 years

Panel VARX model main results

IRF of 1 p.p. young people share shock on
I.TFP growth; **II**. Growth rate of real capital stock per person **III**. Growth rate of real income stock per person



The IRF of +1 p.p. young people (0-15) share shock is hump shape

- Shock will pass down as people grow up

Other shock's IRFs

Empirical findings

- Panel regression: higher working age share is related to higher
  - Productivity growth
    - New patent applications (per 1000 people)
  - Investment share of GDP

- Panel VARX model: the hump shape for IRF of 1 p.p. young people share shock on
  - Productivity growth
  - Growth rate of capital stock per person

Model

Overview

- Producers produce tradable intermediate varieties given current knowledge stocks
- Heterogeneous households varying in ages
  - contribute to idea generation, which affects knowledge dynamics
  - face a consumption-investment trade-off under perfect foresight
- Comparative advantage regulates the allocation of production across locations
  - Ricardian
  - Heckscher-Ohlin

Production ▸ Back

Overview

- A continuum of intermediate good $\omega \in [0,1]$ from country n sector j, $y^j_{n,t}(\omega)$: are produced by labor, capital, and sectoral composite intermediate good

$$y^j_{n,t}(\omega) \equiv q^j_{n,t}(\omega)\left[N^j_{n,t}(\omega)^{\beta^j_n} K^j_{n,t}(\omega)^{1-\beta^j_n}\right]^{\gamma^j_n} \prod_{k=1}^{J} m^{k,j}_{n,t}(\omega)^{\gamma^{k,j}_n} \tag{2}$$

- Intermediate goods are aggregated to build sectoral composite good

- Sectoral composite good is used for consumption, Investment, and intermediate goods production

- The productivity of each variety ω, $q^j_{n,t}(\omega)$, is a r.v., drawn from Fréchet distribution
  - The CDF of the distribution, $F^j_{n,t}(q) = exp(-\lambda^j_{n,t} q^{-\theta})$: **Knowledge frontier**
  - The mean of the distribution, $\lambda_{n,t}$: **Knowledge stock**

Production

Knowledge stock dynamics (1/3)

(Omit the subscripts for sector j and country n for simplicity)

Between time t and $t+1$,

- The representative producer is characterized by its productivity level q, which is drawn from the current knowledge frontier
- Households generate some number of new ideas and share with producers
  - Both the number of new ideas and its productivity q_{new} are stochastic (Buera and Oberfield, Econometrica, 2019)
- Producers adopt the new idea if $q_{new} > q$

Model

Production: Knowledge stock dynamics (2/3)

Ideas arrive following a Poisson Process with mean parameter α_t

$$\alpha_t \equiv (\sum_g \eta_g N_{g,t})^{\varphi} \tag{3}$$

- η_g: mean of ideas arrived per age g people per period
- $N_{g,t}$: number of age g people at time t
- α_t: mean of ideas arrived per unit of time
- $\varphi < 1$: reflect some crowding effects, or duplication of idea

The productivity of a new idea q_{new} is a r.v., where $q_{new} = zq\prime^{\rho}$

- z is the original component; draw from distribution $H(z)$ (Buera and Oberfield, Econometrica, 2019)
- $q\prime$ is an insight drawn from current knowledge frontier
- ρ captures the contribution of the quality of insights from the current knowledge frontier to the productivity of new ideas

Model

Production: Knowledge stock dynamics (3/3)

- One can derive the **law of motion for stock of knowledge** (λ_t) :

$$\lambda_{t+1} - \lambda_t = \Gamma\,(1-\rho)\,\alpha_t\,(\lambda_t)^{\rho}\,; \qquad \alpha_t \equiv (\sum_g \eta_g N_{g,t})^{\varphi} \tag{4}$$

- An increase in the level of working-age population leads to higher knowledge stock
  - age-varing ability in generating ideas
- On the balanced growth path, higher population growth implies higher knowledge stock growth
  - more people generate more ideas, higher population growth rate implies higher idea growth

- w/o demographic: Chad Jones, 2022
- w/o demographic & insight drawn from external dist.: Oberfield and Buera, 2019

Households
Overview

(Omit country subscripts for simplicity)

- Three exogenous variables governing the demographic process ▸ Detail
  - The initial number of population across ages: N_{g,t_0}
  - $f_{g,t}$: number of the newborn from per age g cohort at time t
  - $s_{g,t}$: the probability of surviving to age g at time t, given that they were alive at $g-1$

- Households work at age 16, retired at age 65 and die at age $G = 85$

- The age g households that was born in period t choose lifetime consumption $\{c_{g,t+g-1}\}_{g=1}^{G}$ and savings $\{a_{g+1,t+g}\}_{g=1}^{G-1}$ to maximize expected lifetime utility

$$\sum_{g=1}^{G} \beta^{g-1} \psi_{t+g-1} S_{g,t+g-1} u\left(c_{g,t+g-1}\right), \text{ with } S_{g,t} \equiv \prod_{k=1}^{g} s_{k,t+k-g}$$

  - $u(c) = (c^{1-1/\sigma})/(1-1/\sigma)$
  - ψ_t: saving wedges, capture other forces (except demographics) impacting saving behavior

Households

Budget constraint

The budget constraint for households at age $g \in [1, G]$, time t is

$$P_{C,t}c_{g,t} + P_{I,t}a_{g+1,t+1} = P_{I,t}\left(1 + r_t\right)a_{g,t} + W_t\left(1 - \tau_t^L\right)E_t l_g + ts_t^D + ts_t^T$$
$$\forall\ t:\ a_{1,t} = a_{G+1,t} = 0$$

- $P_{C,t}$ and $P_{I,t}$: price level for consumption and investment
- W_t and R_t: wage and rental rate
- Household at age g own labor endowment $l_g = 1$, $\forall g \in [16, 65]$
- labor supply is adjusted for labor supply frictions τ_t^L and human capital index $E_{n,t}$
- Households save or borrow in the quantity of $a_{n,g+1,t+1}$ under interest rate ▸ Detail

$$r_{t+1} = \frac{R_{t+1}}{P_{I,t+1}} - \delta$$

- Transfers are equally distributed across the households
  - ts_t^D is the trade deficit induced transfer (Caliendo et.al, 2018) ▸ Detail
  - ts_t^T accidental death induced transfer: saving left by households who die before age G ▸ Detail

Trade

(I omit time t subscript to simplify notation)

- "Iceberg" trade costs: $\kappa_{ni}^{j} \geq 1$ for country n by sector j goods from country i

- Following Eaton and Kortum (2002), the fraction of country n's expenditures in sector j goods source from country i is:

$$\pi_{ni}^{j} = \frac{\lambda_i^j \left(c_i^j \kappa_{ni}^j\right)^{-\theta}}{\sum_{i=1}^{N} \lambda_i^j \left(c_i^j \kappa_{ni}^j\right)^{-\theta}} \tag{5}$$

  - c_n^j is the unit price of an input bundle in country n sector j

$$c_n^j \equiv \Upsilon_n^j \left[(W_n)^{\beta_n^j} (R_n)^{1-\beta_n^j}\right]^{\gamma_n^j} \prod_{k=1}^{J} P_n^{k\,\gamma_n^{k,j}} \tag{6}$$

    - P_n^j is the price of sectoral composite goods from country n sector j

Aggregation

Capital

$$\sum_{j=1}^{J}\int_{0}^{1}k_{n,t}^{j}\left(\omega\right)d\omega=K_{n,t}=\sum_{g=E+1}^{E+G}\eta_{n,g-1,t-1}a_{n,g,t} \tag{7}$$

Labor

$$\sum_{j=1}^{J}\int_{0}^{1}l_{n,t}^{j}\left(\omega\right)d\omega=N_{n,t}=\sum_{g=E+1}^{E+G}\eta_{n,g,t}l_{g} \tag{8}$$

Consumption

$$C_{n,t}=\sum_{g=E+1}^{E+G}\eta_{n,g,t}c_{n,g,t} \tag{9}$$

Investment

$$I_{n,t}\equiv K_{n,t+1}-\left(1-\delta\right)K_{n,t} \tag{10}$$

Steady State

Definition 1: Stationary balanced growth equilibrium: A stationary balanced growth competitive equilibrium in the perfect foresight overlapping generations model with G period lived agents, and exogenous population dynamics, is defined as constant allocations of stationary consumption, capital and prices: $\left\{ \{c_{n,g}\}_{g=1,\ n=1}^{G,\ N},\ \{b_{n,g+1}\}_{g=1,\ n=1}^{G-1,N},\ \{W_n,\ R_n\}_{n=1}^{N} \right\}$, such that:

i. The households taking prices transfer and deficit as given, optimize lifetime utility.

ii. Firms taking prices as given, minimize production cost.

iii. Each country purchases intermediate varieties from the least costly supplier/country subject to the trade cost.

iv. All markets are clear.

v. The population distribution reaches a stationary steady-state distribution before the economy reaches a steady state.

▸ Equations

Transitional Dynamics

Definition 2: Dynamics equilibrium
Given a set of initial capital distributions and exogenous forces across countries and over time, the transitional dynamics equilibrium (equilibrium transition path) in the perfect foresight overlapping generations trade model with G-period lived agents is defined as allocations of consumption, capital and prices: $\left\{\{c_{n,g}\}_{g=1,\ n=1}^{G,\ N},\ \{b_{n,g+1}\}_{g=1,\ n=1}^{G-1,N},\ \{W_n,\ R_n\}_{n=1}^{N}\right\}_{t=1,\cdots,T+1}$ satisfies the following conditions:

i. The households at different ages taking prices, transfer and deficit as given, optimize lifetime utility.

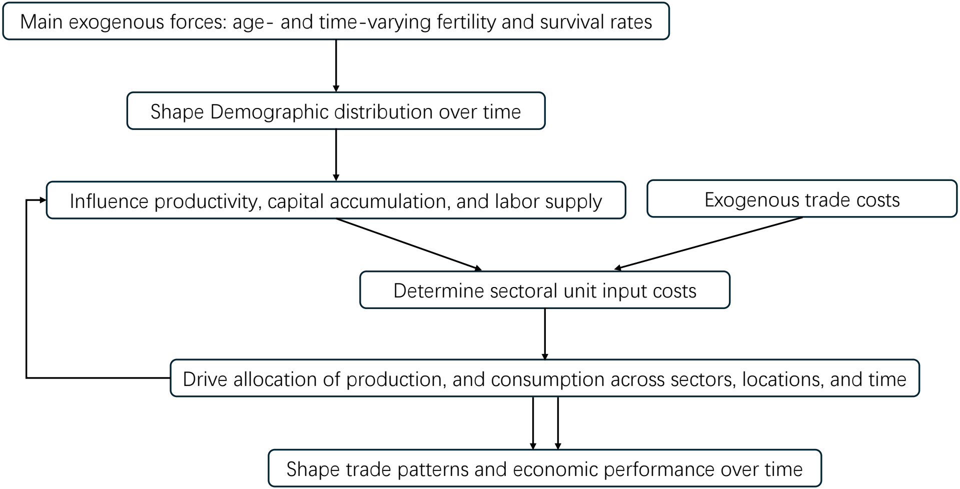
iii. Firms taking prices as given, minimize production cost.

iv. Each country purchases intermediate varieties from the least costly supplier/country subject to the trade cost.

v. All markets are clear.

Equations

How model works



Calibration

Overview

- 5 economy groups, 1970-2100 ▸ Detail
  - China; Asian 5(Japan, Taiwan, Korea, Australia, India); USA and Canada; Europe; ROW
- 5 sectors ▸ Detail
  - Agriculture; {Labor-intensive, Capital-intensive} $\otimes$ {Manufacturing, Services}
- Data sources ▸ Details ▸ Impute
  - WIOD; Long IO Table; Penn World Tables; UN Database

Calibration

Time Invariant Parameters

Index	Description	Value or source
N	# of countries	5
J	# of sectors	5
$G_0 + 1$	Age join labor market	16
$G_1 + 1$	Retried age	66
G	Lifespan for households	85
σ	Risk aversion	1
$\rho_{knowledge}$	Existing knowledge stock coefficient	0.7 (Burea and Oberfield, 2019)
φ^j	Idea duplication coefficient	$[0.67, 0.28, 0.19, 0.69, 0.41]$
β	Annual discount factor	0.96
δ	Capital depreciation rate	0.06
θ	Trade elasticity	4
ρ	Elasticity of substitution between varieties	2
$\gamma^{k,j}$	Sectoral composite goods shares in output	IO table (average across t)
γ^j	Value added shares in output	IO table (average across t)
β^j	Labor's share in value added	IO table (average across t)
α_C^j	Preference parameters	IO table (average across t)
α_I^j	Investment parameters	IO table (average across t)
$\eta_{n,t}^j$	Idea coefficient	Calculation

Calibration

Time Varying Driving Forces

Index	Description	Value or source
Time Varing Shocks		
N_{n,t_0}	Initial labor supply	PWT 10.01
$\overline{N}_{n,g,t_0}$	Initial age distribution	United Nations
$s_{n,g,t}$	Conditional survival rate	United Nations
$f_{n,g,t}$	Fertility rate	United Nations
$\tau^L_{n,t}$	Labor supply wedges	PWT
$\phi_{n,t}$	Trade imbalance wedges	IO table
$\lambda^j_{n,t}$	Knowledge stock	Match real data
$\kappa^j_{ni,t}$	Trade cost	Match real data
$\psi_{n,t}$	Saving wedges	Match real data
$\psi_{n,g}$	Steady state saving wedges	Match real data
Time Varing Endogenous Variables		
$N_{n,t}$	Total labor supply	PWT 10.01
$\overline{N}_{n,g,t}$	Age distribution	United Nations

Calibration

Key details

Several key time-varying shocks

$$\begin{pmatrix} \lambda^j_{n,t} \\ \kappa^j_{ni,t} \\ \psi_{n,t} \\ \phi_{n,t} \end{pmatrix} \equiv \begin{pmatrix} \text{knowledge stock} \\ \text{trade cost} \\ \text{saving wedges} \\ \text{trade imbalance wedges} \end{pmatrix} \leftrightarrow \begin{pmatrix} \text{sector prices} \\ \text{sector bilateral trade flows} \\ \text{aggregate saving rate} \\ \text{aggregate trade imbalance} \end{pmatrix}$$

Knowledge stock parameter, $\eta_{n,g}$ Details

- Assume that all working-age people have the same $\eta_g > 0, g \in [16, 65]$
- In 1970, the world economy is assumed to be on the balanced growth path, which implies

$$\eta_{n,g} = \frac{1 + g_{\lambda,1970}}{(\lambda_{n,1970})^{\rho-1} \left(N_{n,g\in[16,65],1970}\right)^{\varphi} \Gamma(1-\rho)}$$

- One can back out exogenous productivity shock, $\epsilon_{n,t}$, from

$$\lambda_{n,t+1} - \lambda_{n,t} = N_n{}^{\varphi} \left(\lambda_{n,t}\right)^{\rho} \left[\sum_g \eta_{n,g} \bar{N}_{n,g,t}\right]^{\varphi} \Gamma\left(1-\rho\right) + \epsilon_{n,t}$$

Calibration

Results ▸ Detail

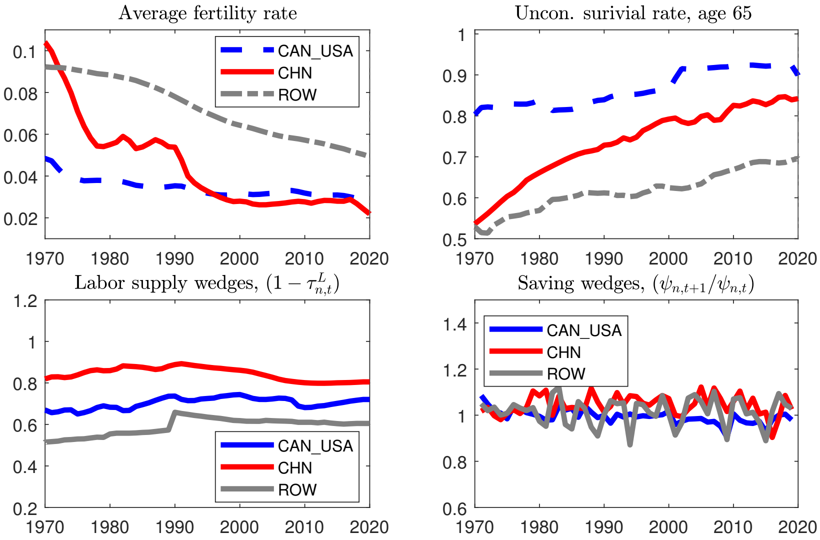


Figure: Demographic shocks and other wedges

Calibration

Results ▸ Detail

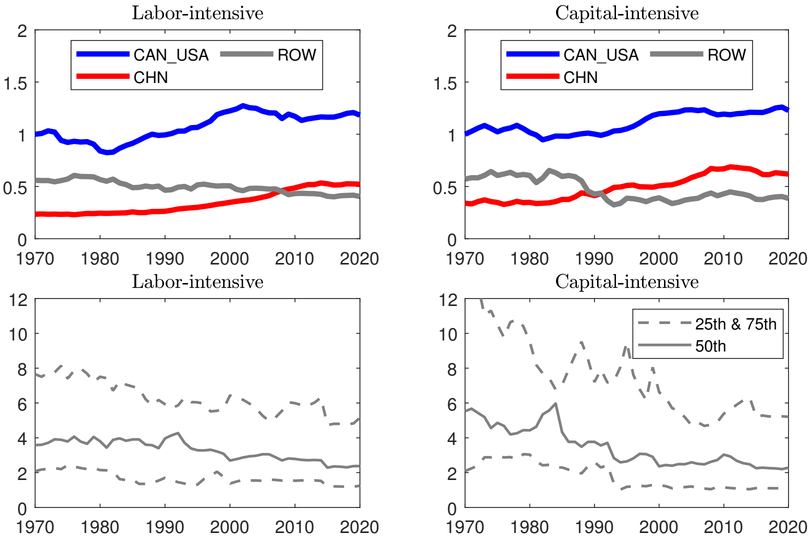
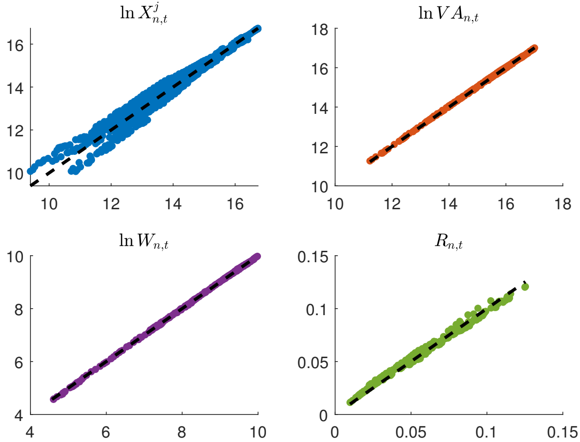


Figure: Knowledge stocks and trade costs

Calibration efficiency

Targeted Moments and other data



Note: vertical axis - model, horizontal axis - data.

Figure: Calibration Efficiency

Comparing baseline scenario with following three scenarios

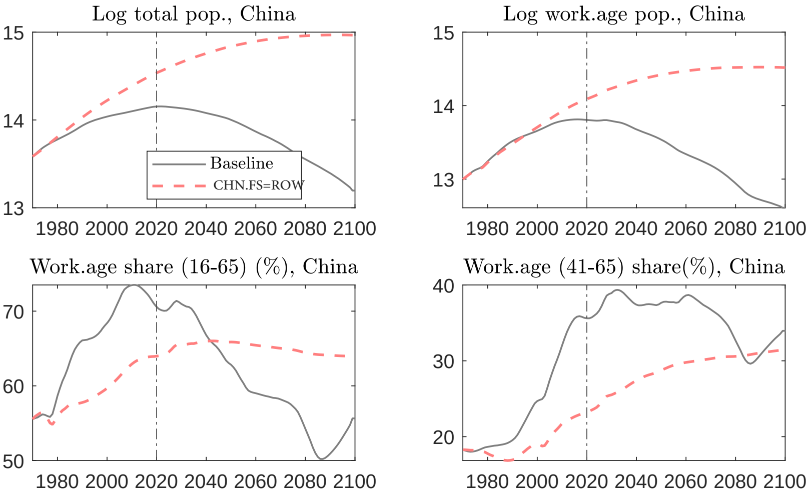
- *China's fertility and survival = ROW*, both channels work
  - Replace China's fertility and survival rates with those of the RoW
  - Open both channel: allow both productivity and saving change in response to demographic changes

- *China's fertility and survival = ROW, only demographic-capital channel works*
  - Replace China's fertility and survival rates with those of the RoW
  - Open capital channel: allow saving change in response to demographic changes
  - Mute productivity channel: but retain the baseline productivity changes

- *China's fertility and survival = ROW, only demographic-idea channel works*
  - Replace China's fertility and survival rates with those of the RoW
  - Open productivity channel: allow productivity to change as if China's demographic structure were replaced by that of RoW
  - Mute capital channel: maintain China's fertility and survival rates, and its implied demographic process

Counterfactual analysis

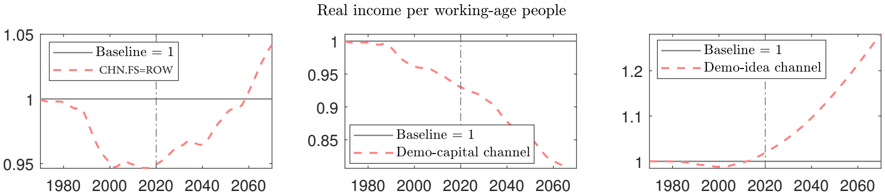
Demographic process



Turnpike Theorem

Counterfactual analysis

Implications for economic growth



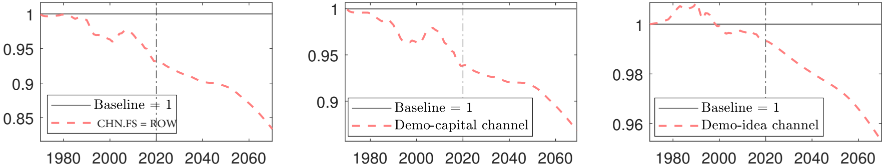
- China's low fertility and high survival rates compared to those of RoW, showing a short-run and long-run trade-off
  - Short run, a saving-favorable age structure leads to higher capital, and income per worker
    ▸ Capital process ▸ Saving-favorable age
  - Long run, after 2060, a lower path of knowledge stocks leads to a lower income per worker
    ▸ Knowledge process

▸ Details

Counterfactual analysis

Implications for trade pattern change

Revealed comparative advantage index, capital intensive sector



- Overall, China's low fertility and high survival rates compared to those of RoW, showing higher revealed CA on Capital-intensive production
  - Demo-capital channel: along the entire path, higher capital per worker—driven by a favorable age structure—enhances the comparative advantage in the capital-intensive sector
  - Demo-idea channel (Calibration showing that knowledge stock in the labor-intensive sector is more sensitive to the number of workers):
    - ★ Short run, more worker, leads to a greater increase in the knowledge stock for labor-intensive goods, thus showing lower RCA index for capital intensive sector
    - ★ Long run, less worker, leads to a greater slow down in the increase in the knowledge stock for labor-intensive goods, thus showing higher RCA index for capital intensive sector

▸ Revealed comparative advantage (RCA) index (Balassa, 1965) ▸ Details

How demographic structure affects China's growth and trade

Story from quantitative analysis

- China's low fertility and high survival rates compared to those of RoW, showing a short-run and long-run trade-off
  - Short run, a saving-favorable age structure leads to higher capital, and income per worker
    - along with a stronger comparative advantage in the capital-intensive sector
  - Long run, after 2060, a lower path of knowledge stocks leads to a lower income per worker
  - Trade liberalization encourages specialization and selection, extends short-run benefit period (numerical experiments)

Summary

How demographic forces shape China's economic growth and trade patterns?

- **Empirical Analysis**
  - A strong positive association between a country's working-age share and:
    - Productivity growth
    - Investment or saving share of GDP
  - An inverse U-shaped response of productivity growth and capital stock per person to a young cohort share shock.

- **Model and Counterfactual Analysis**
  - I build a OLG trade model feature aforementioned two mechanisms
  - I find a interesting trade-off in China's demographics compared to RoW's
    - **Short-run:** A saving-favorable age structure leads to higher capital, income per worker, and a stronger comparative advantage in capital-intensive sectors.
    - **Long-run (post-2060):** A lower knowledge stock trajectory results in lower productivity and income per worker.
    - Trade liberalization encourages specialization and selection, extending short-term benefits.

- **Future Work**
  - Exploring more direct ways to connect demographics and productivity:
    - Incorporating age-dependent productivity levels (i.e., the effectiveness of labor varies by age).
  - Designing a counterfactual to address:
    - To what extent demographic changes explain the recent slowdown in China's growth and the reallocation of labor-intensive production.

Thank You

For comments, please contact me through:
ypei1.work@gmail.com

Future work

- Simplify to 2 countries and 2 sectors: China and ROW; labor-intensive and capital-intensive sector
- Design a counterfactual try to answer
  - To what extent can demographic change explain the recent slowdown in Chinese growth and the reallocation of labor-intensive production?
- Other more direct way to connect demographics and productivity:
  - Incorporating age-dependent levels of productivity (i.e., the effectiveness of labor varies by age)

Panel Regression

Effects of demographic structure and trade cost change on capital/labor ratio

$$GR.K/L_{it,t+4} = Constant + \beta_1 Demographic_{it} + \beta_2 TradeCost_{it} + \beta_3 Control_{it} + f_i + f_t + \varepsilon_{it} \quad (11)$$

- $GR.K/L_{it,t+4}$: Average capital per person (k) growth rate (%) for country i during the period t to $t+4$:

$$GR.K/L_{it,t+4} = [\frac{k_{i,s+4}}{k_{i,s}}]^{\frac{1}{4}} - 1$$

- $TradeCost_{it}$: The trade cost for country i at time t, which is constructed as the Head-Ries (HR) index (Head and Mayer, 2004):

$$TradeCost_{it} = (\frac{\pi_{i,row}}{\pi_{row,row}}\frac{\pi_{row,i}}{\pi_{ii}})^{-\frac{1}{2\theta}}$$

Panel Regression Results

Under different cohort structure

	Average value in the future 4 years								
VARIABLES	TFP growth rate			Patent.Applications (per 1000 people)			Industrial.Design.Applications (per 1000 people)		
Different age intervals:	3 cohorts	4 cohorts	5 cohorts	3 cohorts	4 cohorts	5 cohorts	3 cohorts	4 cohorts	5 cohorts
3 cohorts:	21.48***	26.22***	25.36***	-1.60***	-1.56***	-1.11***	-0.89***	-0.55***	-0.53***
[0, 14], [15,64], [64,+)	(3.61)	(4.24)	(4.08)	(-4.60)	(-7.06)	(-4.09)	(-3.84)	(-3.87)	(-2.87)
	35.46***	34.48***	31.80***	0.58***	0.18	-1.72***	0.63***	0.71***	0.08
4 cohorts:	(5.19)	(4.28)	(4.35)	(2.73)	(0.46)	(-4.06)	(4.98)	(2.87)	(0.31)
[0,24], [25,49], [50,74],	38.25***	43.60***	34.74***	2.29**	4.90***	3.59***	-0.42	1.08***	1.75***
[75, +)	(3.42)	(4.41)	(3.46)	(2.50)	(7.40)	(6.47)	(-0.98)	(2.93)	(5.20)
		13.47	55.17***		-2.59	4.23***		-1.85**	-0.31
5 cohorts:		(0.90)	(5.35)		(-1.59)	(3.99)		(-1.99)	(-0.46)
[0, 19], [20,39], [40,59],			-21.89			-7.67***			-1.09
[60,79], [80,+)			(-1.08)			(-2.62)			(-0.57)
Initial.Log	-3.46***	-3.51***	-3.51***						
.Dependent	(-4.77)	(-4.49)	(-4.55)						
PoP.Growth									
Observations	732	732	732	395	395	395	215	215	215
R-squared	0.266	0.263	0.272	0.859	0.880	0.886	0.935	0.939	0.942
Time FE	YES	YES	YES	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES	YES	YES	YES

Back

Panel Regression Results

Under different cohort structure

	Average value in the future 4 years								
VARIABLES	Cap.Formation(% GDP)			Gross.Consumption(% GDP)			K/L growth rate		
Different age intervals:	3 cohorts	4 cohorts	5 cohorts	3 cohorts	4 cohorts	5 cohorts	3 cohorts	4 cohorts	5 cohorts
3 cohorts:	9.34	16.69***	15.40**	98.55***	92.44***	92.98***	21.77***	24.08***	22.11***
[0, 14], [15,64], [64,+)	(0.98)	(2.64)	(2.32)	(9.21)	(14.84)	(12.62)	(3.69)	(4.50)	(4.02)
	34.10***	29.11***	26.71**	64.81***	60.55***	71.18***	32.98***	39.21***	36.72***
4 cohorts:	(6.74)	(4.14)	(2.52)	(9.15)	(5.58)	(5.39)	(5.32)	(5.36)	(5.30)
[0,24], [25,49], [50,74],	-31.87	37.83**	20.39	98.58***	59.95***	43.58*	8.34	19.18	27.00***
[75, +)	(-1.30)	(2.05)	(1.13)	(2.95)	(3.23)	(1.85)	(0.61)	(1.66)	(2.98)
		-124.60***	53.93**		150.74***	100.97**		4.22	21.25
5 cohorts:		(-2.77)	(2.37)		(3.21)	(2.47)		(0.24)	(1.41)
[0, 19], [20,39], [40,59],			-224.74***			126.47*			-9.87
[60,79], [80,+)			(-3.07)			(1.75)			(-0.33)
Trade Cost							-0.83**	-0.83**	-0.79**
							(-2.13)	(-2.11)	(-2.00)
Initial.Log							-1.99***	-1.98***	-1.93***
.Dependent							(-3.45)	(-3.21)	(-3.14)
PoP.Growth							-33.14*	-35.31**	-30.58
							(-1.84)	(-2.08)	(-1.64)
Observations	724	724	724	725	725	725	758	758	758
R-squared	0.971	0.972	0.972	0.996	0.996	0.996	0.785	0.787	0.787
Time FE	YES	YES	YES	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES	YES	YES	YES

Back

Panel Regression Results

Regression Coefficients follows hump shape

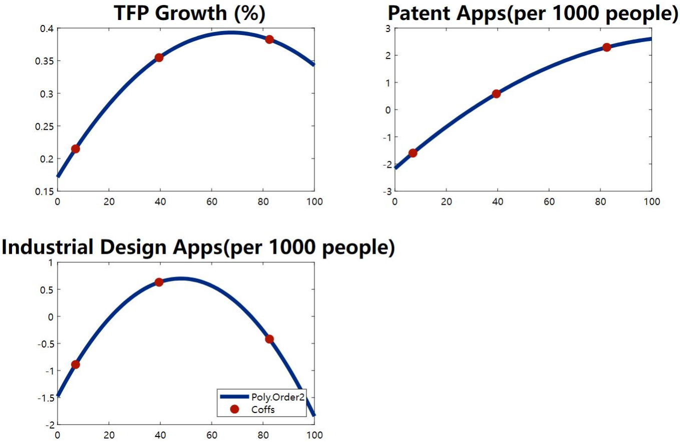


Figure: 3 cohorts: [0, 14], [15,64], [64,+)

Back

Panel Regression Results

Regression Coefficients follows hump shape

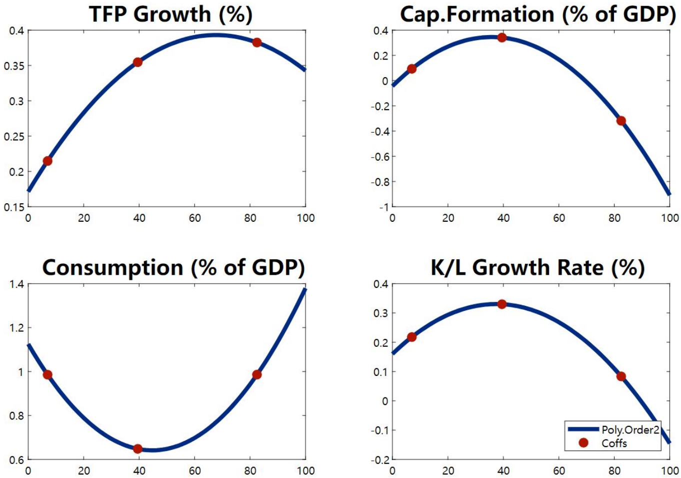


Figure: 3 cohorts: [0, 14], [15,64], [64,+)

Back

Panel Regression Results

Coefficients of different cohort

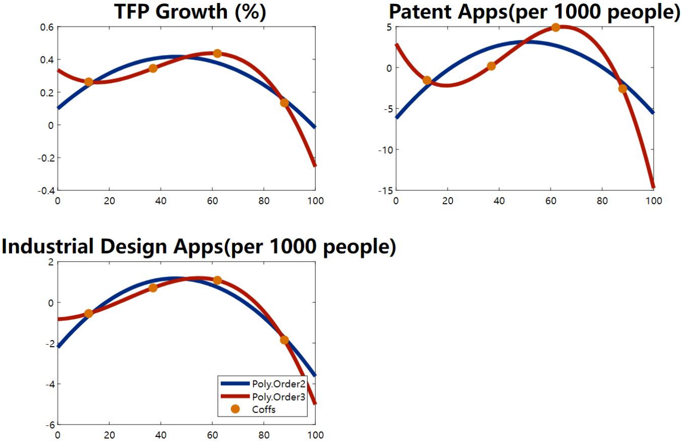


Figure: 4 cohorts

Back

Panel Regression Results

Coefficients of different cohort

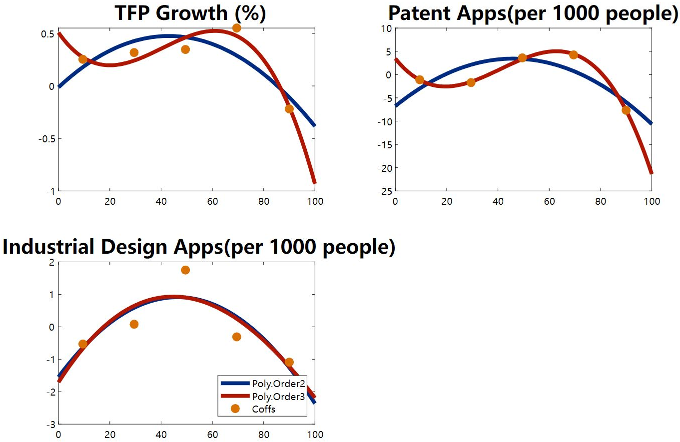


Figure: 5 cohorts

Back

Panel Regression Results

Coefficients of different cohort

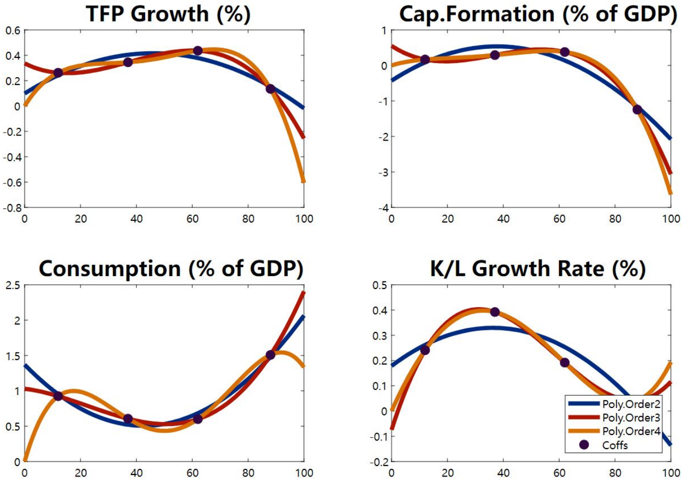


Figure: 4 cohorts

Back

Panel Regression Results

Coefficients of different cohort

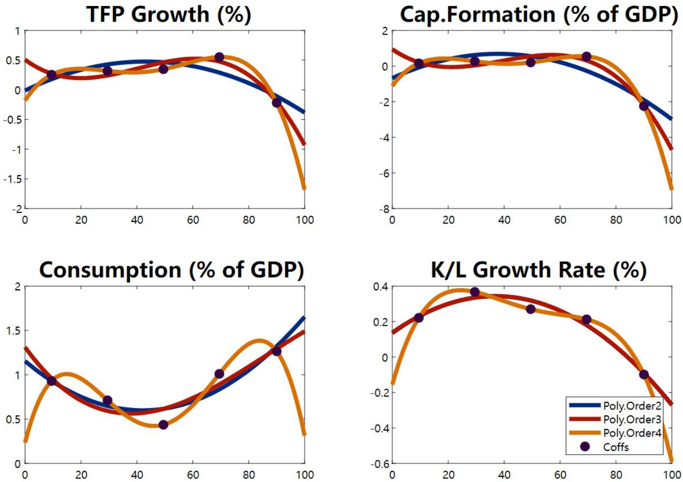
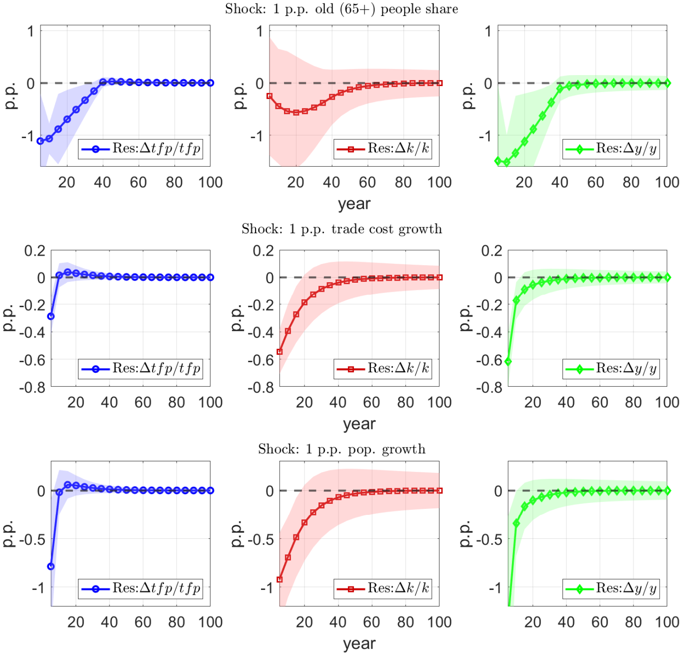


Figure: 5 cohorts

Back

Panel VARX model

▸ Back



Demographic structure

Back

$N_{g,t}$: the number of households of age g alive at time t
$f_{g,t}$: the fertility rate of age g households at time t
$s_{g,t}$: the probability of surviving to age g at time t, given that they were alive at $g-1$
The implied unconditional probability of surviving g periods up to time t is given by:

$$S_{g,t} = \prod_{k=1}^{g} s_{k,t+k-g}$$

The demographic process can be describe as:

$$N_{1,t+1} = s_{1,t} \sum_{g=1}^{G} f_{g,t} N_{g,t}, \, s_{1,t} \equiv 1$$

$$N_{g+1,t+1} = s_{g+1,t+1} N_{g,t}.$$

Demographic structure ▸ Back

$$
\begin{bmatrix} N_{1,t+1} \\ \vdots \\ N_{g,t+1} \\ \vdots \\ N_{G,t+1} \end{bmatrix} = \begin{bmatrix} f_{1,t} & \cdots & f_{g,t} & \cdots & f_{G,t} \\ s_{2,t+1} & 0 & 0 & \cdots & 0 \\ 0 & s_{g+1,t+1} & 0 & \cdots & 0 \\ 0 & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & s_{G-1,t+1} & \cdots & 0 \\ 0 & 0 & 0 & s_{G,t+1} & 0 \end{bmatrix} \cdot \begin{bmatrix} N_{1,t} \\ \vdots \\ N_{g,t} \\ \vdots \\ N_{G,t} \end{bmatrix}.
$$

or

$$N_{t+1} = \Omega_t N_t$$

At steady state

$$(1 + g_n) N_t = \Omega_t N_t$$

The financial market works with zero frictions

- Receive deposits of $P_{I,t} \sum a_{g,t} N_{g,t}$ from individuals
  - Repay those individuals an amount $(1 + r_t) P_{I,t} \sum a_{g,t} N_{g,t}$
- Loaned an amount $K_t = \sum a_{g,t} N_{g,t}$ to firms to use in production
  - Receives an amount $P_{I,t} \left(1 + \frac{R_t}{P_{I,t}} - \delta\right) K_t$ from firms
- Market clear implies

$$r_t = \frac{R_t}{P_{I,t}} - \delta \tag{12}$$

Back

Model

Trade deficit-induced transfers

Back

- A pre-determined share of GDP, $\phi_{n,t}$ is sent to a global portfolio, which in turn disperses a per-capita lump-sum transfer, T_t^P ,to every country

- The net transfer, also recognized as trade deficit, are calculated as:

$$D_{n,t} = -\phi_{n,t} \left(R_{n,t} K_{n,t} + W_{n,t} E_{n,t} N_{n,t} \right) + \bar{L}_{n,t} T_t^P \tag{13}$$

- Dividing by the total economically relevant population $\bar{L}_{n,t}$ implies that total bequests are equally distributed across the population

$$D_{n,t} = -\phi_{n,t} \left(R_{n,t} K_{n,t} + W_{n,t} E_{n,t} N_{n,t} \right) + \frac{\bar{L}_{n,t}}{\sum_{n=1}^{N} \bar{L}_{n,t}} \sum_{n=1}^{N} \phi_{n,t} \left(R_{n,t} K_{n,t} + W_{n,t} E_{n,t} N_{n,t} \right) \tag{14}$$

Model

Demographics-induced transfers

- $TRSV_{n,t}$ is defined as demographic structure change-induced transfer which is due to the number of population changes between cohort $(s-1,t-1)$ and (s,t)

$$TRSV_{n,t} = P_{n,I,t}\,(1 + r_{n,t}) \sum_{g=E+2}^{E+S} (\eta_{n,g-1,t-1} - \eta_{n,g,t})\, a_{n,g,t} \tag{15}$$

  - The number of population change can either counted as net death $(\eta_{n,g-1,t-1} - \eta_{n,g,t} > 0)$ or net immigrant $(\eta_{n,g-1,t-1} - \eta_{n,g,t} < 0)$
  - The asset change due to net death is treated as positive bequests
  - The net immigrant (g,t) enter country n with zero assets, and is treated as negative bequests

Back

Model

Steady State (1/2)

Table: Steady-state conditions (1/2)

g_n	$N_{n,g,t+1} = (1+g_n)\, N_{n,g,t}$	$\forall n, t \in [T-1, \infty)$
g_{λ^j}	$\lambda^j_{n,t+1} = (1+g_{\lambda^j})\, \lambda^j_{n,t};\ (1+g_{\lambda^j}) = (1+g_n)^{\frac{\varphi^j}{(1-\rho)}}\ ;\ 1+g_{A^j} \equiv (1+g_{\lambda^j})^{1/\theta}$	$\forall n, j, t \in [T, \infty)$
g_ω	$X \in [\frac{W_{n,t}}{P_{n,C,t}}, \frac{ts^T_{n,t}}{P_{C,n,t}}, \frac{ts^D_{n,t}}{P_{n,C,t}}, a_{n,g,t}, c_{n,g,t}]\ ;\ X_{t+1} = (1+g_\omega)\, X_t;\ 1+g_\omega = (1+g_{A^j})^{\frac{1}{\beta^j\gamma^j}} = (1+g_{\lambda^j})^{\frac{1}{\beta^j\gamma^j\theta}}$	$\forall n, t \in [T, \infty)$
$g_{rc^j_n}$	$X \in [\frac{c^j_{n,t}}{P^j_{n,t}}]\ ;\ X_{t+1} = \left(1+g_{rc^j_n}\right) X_t;\ 1+g_{rc^j_n} = (1+g_\omega)^{\beta^j\gamma^j} = (1+g_{\lambda^j})^{1/\theta}$	$\forall n, t \in [T, \infty)$
g_K	$X \in [C_{n,t}, C^j_{n,t}, I_{n,t}, I^j_{n,t}, K_{n,t}, Y^j_{n,t}, \frac{X^j_{n,t}}{P^j_{n,t}}, \frac{D_{n,t}}{P^j_{n,t}}, \frac{D_{n,t}}{P_{n,Ct}}, \frac{D_{n,t}}{P_{n,I,t}}]\ ;\ X_{t+1} = (1+g_K)\, X_t;\ 1+g_K = (1+g_\omega)(1+g_n)$	$\forall n, j, t \in [T, \infty)$
	$1+g_\omega = (1+g_n)^{\frac{\varphi^j}{\theta\beta^j\gamma^j(1-\rho)}};\ \varphi^j/\varphi^k = \beta^j\gamma^j/\beta^k\gamma^k;\ \varphi^j = \theta(1-\rho)\,\beta^j\gamma^j\frac{\log(1+g_\omega)}{\log(1+g_n)};$	$\forall n, j$
F0	$\lambda^j_{n,T+1} - \lambda^j_{n,T} = {N_{n,T}}^{\varphi^j}\left(\lambda^j_{n,T}\right)^\rho \left[\sum_g \eta^j_g \bar{N}_{n,g,T}\right]^{\varphi^j} \Gamma(1-\rho)$	$\forall (n)$
H1	$N_{n,T} \equiv \sum_{g=1}^G N_{n,g,T};\ \bar{L}_{n,T} \equiv \sum_{g=G_0+1}^G N_{n,g,T};\ L_{n,T} = \left(1-\tau^L_{n,T}\right)\sum_{g=G_0+1}^{G_1} N_{n,g,T}l_g;\ L^e_{n,T} = E_{n,T}L_{n,T}$	$\forall (n)$
H2	$P_{n,C,T}c_{n,g,T} + P_{n,I,T}(1+g_\omega)\, a_{n,g+1,T} = P_{n,I,T}(1+r_{n,T})\, a_{n,g,T} + W_{n,T}\left(1-\tau^L_{n,T}\right) E_{n,T}l_g + tr^D_{n,T} + tr^T_{n,T}\ ;\ g \in [1, G]$	$\forall (n)$
H3	$a_{1,T} = a_{G+1,T} = 0;\ c_{n,g,T} > 0,\ \{c_{n,g,T}\}_{g=1}^G\ ;\ \{a_{n,g+1,T}\}_{g=1}^{G-1}$	$\forall (n)$
H4	$tr^T_{n,T} \equiv \frac{D_{n,T}}{N_{n,T}};\ tr^D_{n,T} = P_{n,I,T}(1+r_{n,T})\sum_{g=2}^G\left(\frac{\overline{N}_{n,g-1,T}}{1+g_n} - \overline{N}_{n,g,T}\right) a_{n,g,T}$	$\forall (n)$
H4'	$tr^D_{n,T} = tr^{D,1}_{n,T} + tr^{D,2}_{n,T} = P_{n,I,T}(1-\delta)\sum_{g=2}^G\left(\frac{\overline{N}_{n,g-1,T}}{1+g_n} - \overline{N}_{n,g,T}\right) a_{n,g,T} + P_{n,I,T}\left(\frac{R_{n,T}}{P_{n,I,T}}\right)\sum_{g=2}^G\left(\frac{\overline{N}_{n,g-1,T}}{1+g_n} - \overline{N}_{n,g,T}\right) a_{n,g,T}$	$\forall (n)$
H4"	$P_{n,C,T}c_{n,g,T} + P_{n,I,T}i_{n,g,T} = R_{n,T}a_{n,g,T} + W_{n,T}\left(1-\tau^L_{n,T}\right) E_{n,T}l_g + tr^{D,2}_{n,T} + tr^T_{n,T}$	$\forall (n)$
H4"'	$P_{n,I,T}i_{n,g,T} = P_{n,I,T}(1+g_\omega)\, a_{n,g+1,T} - \left[P_{n,I,T}(1-\delta)\, a_{n,g,T} + tr^{D,1}_{n,T}\right]$	$\forall (n)$
H5	$(1+g_\omega)\, c_{n,g+1,T} = \left[(\beta s_{n,g+1,T})\left(\frac{\psi_{n,g+1,T+1}}{\psi_{n,g,T}}\right)(1+r_{n,T})\right]^\sigma c_{n,g,T};\forall\ g \in [1, G-1]$	$\forall (n)$
H6	$C_{n,T} \equiv \sum_{g=1}^G N_{n,g,T}c_{n,g,T};\ K_{n,T} \equiv \sum_{g=2}^G \frac{N_{n,g-1,T}}{1+g_n} a_{n,g,T}$	$\forall (n)$

Table: Steady-state conditions (2/2)

H7	$C_{n,T} \equiv \prod_{j=1}^{J} {C_{n,T}^{j}}^{\alpha_C^j}; I_{n,T} \equiv \prod_{j=1}^{J} {I_{n,T}^{j}}^{\alpha_I^j}; P_{n,I,T} = \prod_{j=1}^{J} \left[\frac{P_{n,T}^j}{\alpha_I^j}\right]^{\alpha_I^j}; P_{n,C,T} = \prod_{j=1}^{J} \left[\frac{P_{n,T}^j}{\alpha_C^j}\right]^{\alpha_C^j}$	$\forall(n)$
H8	$P_{n,T}^j I_{n,T}^j = \alpha_{I,n}^j P_{n,I,T} I_{n,T}; P_{n,T}^j C_{n,T}^j = \alpha_{C,n}^j P_{n,C,T} C_{n,T}$	$\forall(n,j)$
F1	$W_{n,T} L_{n,T}^e = \sum_{j=1}^{J} \beta^j \gamma^j \sum_{i=1}^{N} \pi_{in,T}^j X_{i,T}^j; R_{n,T} K_{n,T} = \sum_{j=1}^{J} \left(1-\beta^j\right) \gamma^j \sum_{i=1}^{N} \pi_{in,T}^j X_{i,T}^j$	$\forall(n)$
F2	$r_{n,T} = \frac{R_{n,T}}{P_{n,I,T}} - \delta$	$\forall(n)$
T1	$c_{n,T}^j \equiv \Upsilon^j \left[(W_{n,T})^{\beta^j} (R_{n,T})^{1-\beta^j}\right]^{\gamma^j} \prod_{k=1}^{J} {P_{n,T}^k}^{\gamma^{k,j}}; \Upsilon^j \equiv \gamma^j {\beta^j}^{-\gamma^j \beta^j} \gamma^j \left(1-\beta^j\right)^{-\gamma^j \left(1-\beta^j\right)} \prod_{k=1}^{J} {\gamma^{k,j}}^{-\gamma^{k,j}}$	$\forall(n,j)$
T2	$P_{n,T}^j = A \cdot \left[\sum_{i=1}^{N} \lambda_{i,T}^j \left(\kappa_{ni,T}^j c_{i,T}^j\right)^{-\theta}\right]^{-\frac{1}{\theta}}; A \equiv \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$	$\forall(n,j)$
T3	$\pi_{ni,T}^j \equiv \frac{X_{ni,T}^j}{\sum_{i=1}^{N} X_{ni,T}^j} = \frac{\lambda_{i,t}^j \left(c_{i,t}^j \kappa_{ni,T}^j\right)^{-\theta}}{\sum_{m=1}^{N} \lambda_{m,T}^j \left(c_{m,T}^j \kappa_{nm,T}^j\right)^{-\theta}} = \lambda_{i,T}^j \left(\frac{A c_{i,T}^j \kappa_{ni,T}^j}{P_{n,T}^j}\right)^{-\theta}$	$\forall(n,i,j)$
T4	$P_{C,n,T} C_{n,T} + P_{I,n,T} I_{n,T} = R_{n,T} K_{n,T} + W_{n,T} E_{n,T} L_{n,T} + D_{n,T} = R_{n,T} K_{n,T} + W_{n,T} L_{n,T}^e + D_{n,T} \equiv IN_{n,T}$	$\forall(n)$
T4'	$P_{n,C,T} C_{n,T} + P_{n,I,T} \left(1+g_K\right) K_{n,T} = \left(1 + \frac{R_{n,T}}{P_{n,I,T}} - \delta\right) P_{n,I,T} K_{n,T} + W_{n,T} L_{n,T}^e + D_{n,T}$	$\forall(n)$
T5	$\left(1+g_K\right) K_{n,T} = K_{n,T+1} = I_{n,T} + \left(1-\delta\right) K_{n,T}; \left(g_K+\delta\right) K_{n,T} = I_{n,T}$	$\forall(n)$
T6	$\sum_{j=1}^{J} \sum_{i=1}^{N} X_{in,T}^j - \sum_{j=1}^{J} \sum_{i=1}^{N} X_{ni,T}^j = NX_{n,T} = -D_{n,T}$	$\forall(n,j)$
T6'	$X_{n,T}^j = \alpha_C^j P_{C,n,T} C_{n,T} + \alpha_I^j P_{I,n,T} I_{n,T} + \sum_{k=1}^{J} \gamma^{j,k} \left(\sum_{i=1}^{N} X_{in,T}^k\right)$	$\forall(n,j)$
T7	$D_{n,T} = -\phi_{n,T} \left(R_{n,T} K_{n,T} + W_{n,T} L_{n,T}^e\right) + N_{n,T} T_T^P; T_T^P = \frac{\sum_{n=1}^{N} \phi_{n,T} \left(R_{n,T} K_{n,T} + W_{n,T} L_{n,T}^e\right)}{\sum_{n=1}^{N} N_{n,T}}$	$\forall(n)$
T7'	$D_{n,T} = -\phi_{n,T} \left(R_{n,T} K_{n,T} + W_{n,T} L_{n,T}^e\right) + \frac{N_{n,T}}{\sum_{n=1}^{N} N_{n,T}} \sum_{n=1}^{N} \phi_{n,T} \left(R_{n,T} K_{n,T} + W_{n,T} L_{n,T}^e\right)$	$\forall(n)$

Back

Model

Transitional Dynamics (1/2)

Table: Dynamic equilibrium conditions (1/2)

I1	$\lambda^j_{n,t+1}-\lambda^j_{n,t}=\left(\lambda^j_{n,t}\right)^{\rho}\left(\sum_g \eta^j_g N_{n,g,t}\right)^{\varphi^j}\Gamma\left(1-\rho\right)=N_{n,t}{}^{\varphi^j}\left(\lambda^j_{n,t}\right)^{\rho}\left(\sum_g \eta^j_g \bar{N}_{n,g,t}\right)^{\varphi^j}\Gamma\left(1-\rho\right)$	$\forall(n,t)$
H1	$N_{n,t}\equiv\sum_{g=1}^{G}N_{n,g,t}$; $\bar{L}_{n,t}\equiv\sum_{g=G_0+1}^{G}N_{n,g,t}$; $L_{n,t}=\left(1-\tau^L_{n,t}\right)\sum_{g=G_0+1}^{G_1}N_{n,g,t}l_g=\left(1-\tau^L_{n,t}\right)\sum_{g=1}^{G}N_{n,g,t}l_g$; $L^e_{n,t}=E_{n,t}L_{n,t}$	$\forall(n,t)$
H2	$P_{n,C,t}c_{n,g,t}+P_{n,I,t}a_{n,g+1,t+1}=P_{n,I,t}\left(1+r_{n,t}\right)a_{n,g,t}+W_{n,t}\left(1-\tau^L_{n,t}\right)E_{n,t}l_g+tr^D_{n,t}+tr^T_{n,t}$; $g\in[1,G]$	$\forall(n,t)$
H3	$a_{1,t}=a_{G+1,t}=0$; $c_{n,g,t}>0$, $\{c_{n,g,t+g-1}\}_{g=1}^{G}$; $\{a_{n,g+1,t+g}\}_{g=1}^{G-1}$	$\forall(n,t)$
H4	$tr^T_{n,t}\equiv\frac{D_{n,t}}{N_{n,t}}$; $tr^D_{n,t}\equiv P_{n,I,t}\left(1+r_{n,t}\right)\frac{\sum_{g=2}^{G}\left(N_{n,g-1,t-1}-N_{n,g,t}\right)a_{n,g,t}}{N_{n,t}}$	$\forall(n,t)$
H4'	$tr^D_{n,t}=tr^{D,1}_{n,t}+tr^{D,2}_{n,t}=P_{n,I,t}\left(1-\delta\right)\sum_{g=2}^{G}\left(\frac{N_{n,g-1,t-1}-N_{n,g,t}}{N_{n,t}}\right)a_{n,g,t}+P_{n,I,t}\left(\frac{R_{n,t}}{P_{n,I,t}}\right)\sum_{g=2}^{G}\left(\frac{N_{n,g-1,t-1}-N_{n,g,t}}{N_{n,t}}\right)a_{n,g,t}$	$\forall(n)$
H4"	$P_{n,C,t}c_{n,g,t}+P_{n,I,t}i_{n,g,t}=R_{n,t}a_{n,g,t}+W_{n,t}\left(1-\tau^L_{n,t}\right)E_{n,t}l_g+tr^{D,2}_{n,t}+tr^T_{n,t}$	$\forall(n)$
H4"'	$P_{n,I,t}i_{n,g,t}=P_{n,I,t}a_{n,g+1,t+1}-\left[P_{n,I,t}\left(1-\delta\right)a_{n,g,t}+tr^{D,1}_{n,t}\right]$	$\forall(n)$
H5	$\frac{c_{n,g+1,t+1}}{c_{n,g,t}}=\left[\left(\beta s_{n,g+1,t+1}\right)\left(\frac{\psi_{n,t+1}}{\psi_{n,t}}\right)\frac{\frac{P_{n,I,t+1}}{P_{n,C,t+1}}}{\frac{P_{n,I,t}}{P_{n,C,t}}}\left(1+r_{n,t+1}\right)\right]^{\sigma}$; $\forall\ g\in[1,G-1]$	$\forall(n,t)$
H6	$C_{n,t}\equiv\sum_{g=1}^{G}N_{n,g,t}c_{n,g,t}$; $K_{n,t}\equiv\sum_{g=2}^{G}N_{n,g-1,t-1}a_{n,g,t}$	$\forall(n,t)$
H7	$C_{n,t}\equiv\prod_{j=1}^{J}C^j_{n,t}{}^{\alpha^j_{n,C,t}}$; $I_{n,t}\equiv\prod_{j=1}^{J}I^j_{n,t}{}^{\alpha^j_{n,I,t}}$; $P_{n,I,t}=\prod_{j=1}^{J}\left[\frac{P^j_{n,t}}{\alpha^j_{I,n}}\right]^{\alpha^j_{I,n}}$; $P_{n,C,t}=\prod_{j=1}^{J}\left[\frac{P^j_{n,t}}{\alpha^j_{C,n}}\right]^{\alpha^j_{C,n}}$	$\forall(n,t)$
H8	$P^j_{n,t}I^j_{n,t}=\alpha^j_{I,n}P_{n,I,t}I_{n,t}$; $P^j_{n,t}C^j_{n,t}=\alpha^j_{C,n}P_{n,C,t}C_{n,t}$	$\forall(n,j,t)$

Model

Transitional Dynamics (2/2)

Table: Dynamic equilibrium conditions (2/2)

F1	$W_{n,t}L^e_{n,t} = \sum_{j=1}^{J} \beta^j_n \gamma^j_n \sum_{i=1}^{N} \pi^j_{in,t} X^j_{i,t}$; $R_{n,t}K_{n,t} = \sum_{j=1}^{J} \left(1-\beta^j_n\right) \gamma^j_n \sum_{i=1}^{N} \pi^j_{in,t} X^j_{i,t}$	$\forall (n,t)$
F2	$r_{n,t} = \frac{R_{n,t}}{P_{n,I,t}} - \delta$	$\forall (n,t)$
T1	$c^j_{n,t} \equiv \Upsilon^j_n \left[(W_{n,t})^{\beta^j_n} (R_{n,t})^{1-\beta^j_n} \right]^{\gamma^j_n} \prod_{k=1}^{J} P^k_{n,t}{}^{\gamma^{k,j}_n}$ where $\Upsilon^j_n \equiv \gamma^j_n \beta^j_n{}^{-\gamma^j_n \beta^j_n} \gamma^j_n \left(1-\beta^j_n\right)^{-\gamma^j_n (1-\beta^j_n)} \prod_{k=1}^{J} \gamma^{k,j}_n{}^{-\gamma^{k,j}_n}$	$\forall (n,j,t)$
T2	$P^j_{n,t} = A^j \cdot \left[\sum_{i=1}^{N} \lambda^j_{i,t} \left(\kappa^j_{ni,t} c^j_{i,t} \right)^{-\theta} \right]^{-\frac{1}{\theta}}$ where $A^j \equiv \Gamma \left(\frac{1+\theta-\sigma}{\theta} \right)^{\frac{1}{(1-\sigma)}}$	$\forall (n,j,t)$
T3	$\pi^j_{ni,t} \equiv \frac{X^j_{ni,t}}{\sum_{i=1}^{N} X^j_{ni,t}} = \frac{\lambda^j_{i,t} (c^j_{i,t} \kappa^j_{ni,t})^{-\theta}}{\sum_{m=1}^{N} \lambda^j_{m,t} (c^j_{m,t} \kappa^j_{nm,t})^{-\theta}} = \lambda^j_{i,t} \left(\frac{A^j c^j_{i,t} \kappa^j_{ni,t}}{P^j_{n,t}} \right)^{-\theta}$	$\forall (n,i,j,t)$
T4	$P_{n,C,t}C_{n,t} + P_{n,I,t}I_{n,t} = R_{n,t}K_{n,t} + W_{n,t}E_{n,t}L_{n,t} + D_{n,t} = R_{n,t}K_{n,t} + W_{n,t}L^e_{n,t} + D_{n,t} \equiv IN_{n,t}$	$\forall (n,t)$
T4'	$P_{n,C,t}C_{n,t} + P_{n,I,t}K_{n,t+1} = \left(1 + \frac{R_{n,t}}{P_{n,I,t}} - \delta\right) P_{n,I,t}K_{n,t} + W_{n,t}L^e_{n,t} + D_{n,t}$	$\forall (n,t)$
T5	$K_{n,t+1} = I_{n,t} + (1-\delta) K_{n,t}$	$\forall (n,t)$
T6	$\sum_{j=1}^{J} \sum_{i=1}^{N} X^j_{in,t} - \sum_{j=1}^{J} \sum_{i=1}^{N} X^j_{ni,t} = NX_{n,t} = -D_{n,t}$	$\forall (n,j,t)$
T6'	$X^j_{n,t} = \alpha^j_{C,n} P_{C,n,t} C_{n,t} + \alpha^j_{I,n} P_{I,n,t} I_{n,t} + \sum_{k=1}^{J} \gamma^{j,k}_n \left(\sum_{i=1}^{N} X^k_{in,t} \right)$	$\forall (n,j,t)$
T7	$D_{n,t} = -\phi_{n,t} \left(R_{n,t}K_{n,t} + W_{n,t}L^e_{n,t} \right) + N_{n,t} T^P_t$; $T^P_t = \frac{\sum_{n=1}^{N} \phi_{n,t} \left(R_{n,t}K_{n,t} + W_{n,t}L^e_{n,t} \right)}{\sum_{n=1}^{N} N_{n,t}}$	$\forall (n,t)$
T7'	$D_{n,t} = -\phi_{n,t} \left(R_{n,t}K_{n,t} + W_{n,t}L^e_{n,t} \right) + \frac{N_{n,t}}{\sum_{n=1}^{N} N_{n,t}} \sum_{n=1}^{N} \phi_{n,t} \left(R_{n,t}K_{n,t} + W_{n,t}L^e_{n,t} \right)$	$\forall (n,t)$

▸ Back

Compare Steady State ‣ Back

The role of demographics

	(1A)	(2A)	(3A)
Survival rate	low	**high**	high
Fertility rate	high	high	**low**
Trade cost	Autarky	Autarky	Autarky
Average lifespan	60.00	**71.00**	71.00
Population growth	0.05	0.05	**0.01**
Implied TFP growth	0.02	0.02	**0.01**
Working age share	0.43	0.44	0.63
Per efficient person			
Capital stock	0.0073	0.0086	0.0215
Output	0.0026	0.0029	0.0054
Consumption	0.0016	0.0017	0.0038
Investment	0.0010	0.0012	0.0016
capital - efficient labor ratio	0.0167	0.0195	0.0343
Price ratio			
Real wage rate	0.0030	0.0032	0.0043
Real rental rate	0.1788	0.1655	0.1250

- $(2A)$ v.s. $(1A)$: A higher average lifespan increases savings, which, acting as a supply of capital, leads to higher capital per efficient person
- $(3A)$ v.s. $(2A)$: With slower population and TFP growth, the number of effective persons grows more slowly. Less capital used to be spread across individuals, leads to higher capital per efficient person
- Capital-labor ratio implies a relative abundance of capital relative to labor

Compare Steady State ▸ Back

The role of trade

	(3A)	(3B)
Survival rate	high	high
Fertility rate	low	low
Trade cost	Autarky	**Free trade**
Average lifespan	71.00	71.00
Population growth	0.01	0.01
Implied TFP growth	0.01	0.01
Working age share	0.63	0.63
Per efficient person		
Capital stock	0.022	0.061
Output	0.005	0.015
Consumption	0.004	0.011
Investment	0.002	0.005
capital - efficient labor ratio	0.034	0.097
Price ratio		
Real wage rate	0.004	0.012
Real rental rate	0.125	0.125

- $(3B)$ v.s. $(3A)$: Trade stimulate capital accumulation

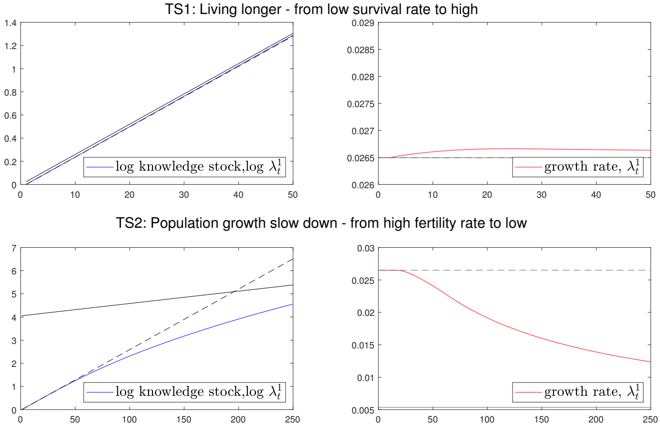
Compare Steady State ▸ Back

	(1A)	(1B)	(2A)	(2B)	(3A)	(3B)
Survival rate	low		**high**		high	
Fertility rate	high		high		**low**	
Trade cost	100	1	100	1	100	1
Average lifespan	60.00	60.00	**71.00**	**71.00**	71.00	71.00
Population growth	0.05	0.05	0.05	0.05	**0.01**	**0.01**
Implied TFP growth	0.02	0.02	0.02	0.02	**0.01**	**0.01**
working age share	0.43	0.43	0.44	0.44	0.63	0.63
Per efffcient person						
Capital stock	0.0073	0.0205	0.0086	0.0244	0.0215	0.0609
Output	0.0026	0.0073	0.0029	0.0081	0.0054	0.0152
Consumption	0.0016	0.0046	0.0017	0.0048	0.0038	0.0107
Investment	0.0010	0.0028	0.0012	0.0033	0.0016	0.0046
capital - efffcient labor ratio	0.0167	0.0473	0.0195	0.0553	0.0343	0.0970
Price ratio						
Real wage rate	0.0030	0.0085	0.0032	0.0092	0.0043	0.0121
Real rental rate	0.1788	0.1788	0.1655	0.1655	0.1250	0.1250

Transitional dynamics: Knowledge stock changes over time ▸ detail

Before $t = 1$, economy is on the old balance growth path

Shock at $t = 1$: survival Rate (or fertility Rate) changed forever



$$\frac{\lambda_{t+1} - \lambda_t}{\lambda_t} = (\lambda_t)^{\rho-1} \left(\sum_g \eta_g N_{g,t}\right)^{\varphi} \Gamma (1 - \rho)$$

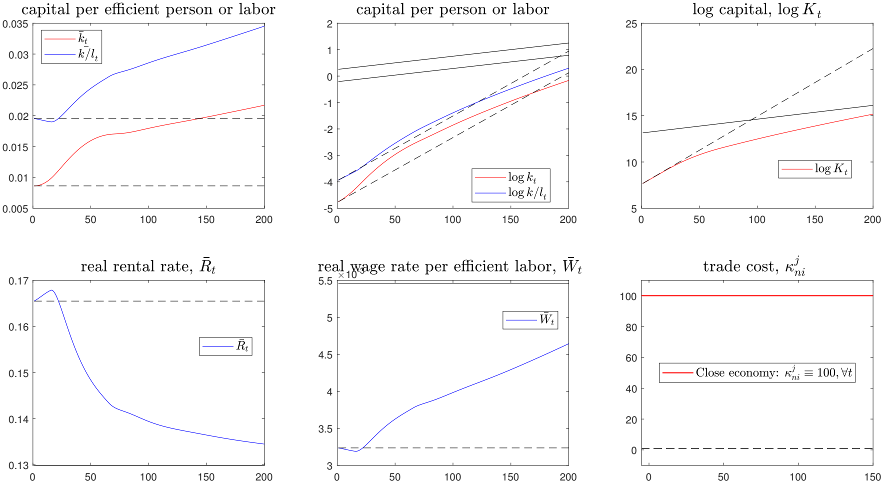
Simple application : assume only working-age people contribute to new idea generation

- $\eta_g = c > 0$ if $g \in (16, 65)$ and $\eta_g = 0$ if $g \notin (16, 65)$

◂ Back

Transitional dynamics - pop. growth slows down. Sym. Close economy

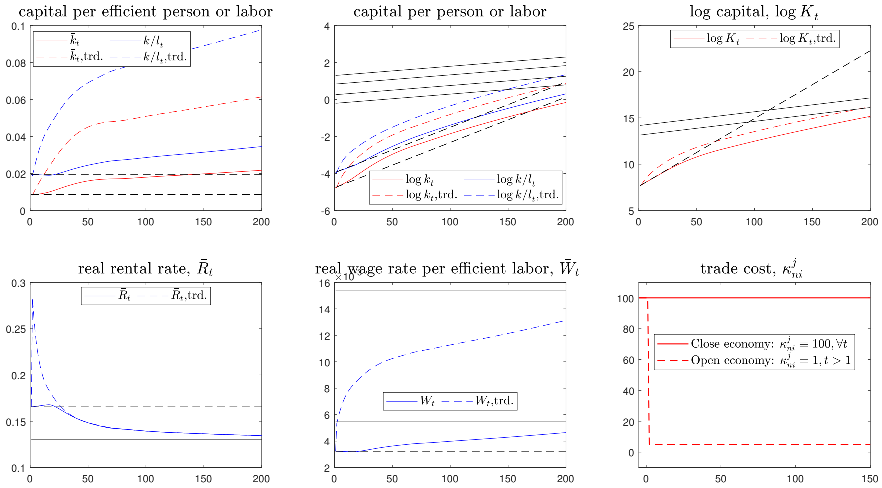
▸ Open economy deatil ▸ Living longer



Low fertility Rate: beneficial in the short run, but adverse in the long run.

- Short run, a lower population, raises capital per person above the old growth path
- Long run, productivity growth slows down, capital per person ultimately below old growth path

Transitional dynamics - pop. growth slows down. Close v.s. Open



Low fertility Rate Plus Trade liberalization

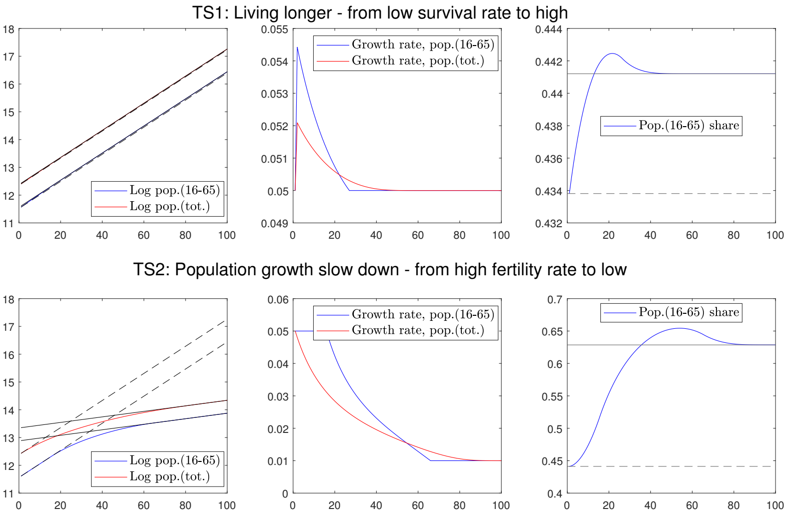
- Trade liberalization extends the beneficial period (during which capital per person remains above the old growth path).

Transitional dynamics: Population changes over time

Back

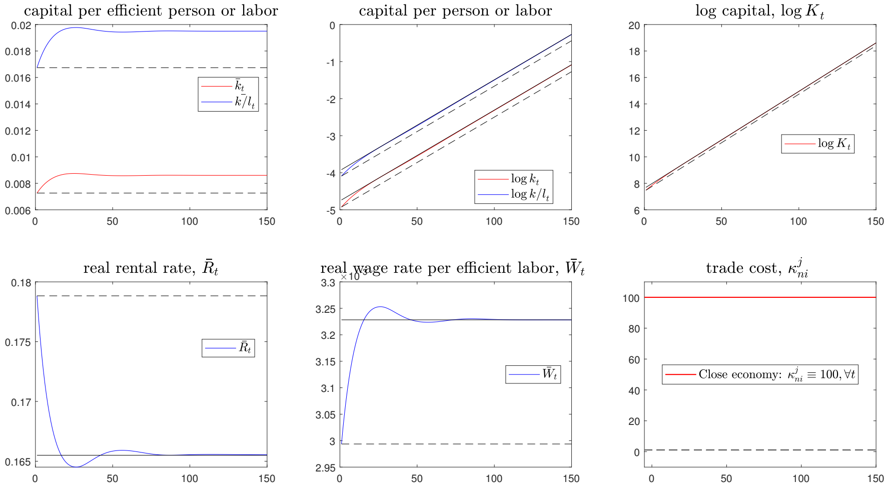
Before $t = 1$, economy is on the old balance growth path

Shock at $t = 1$: survival Rate (or fertility Rate) changed forever



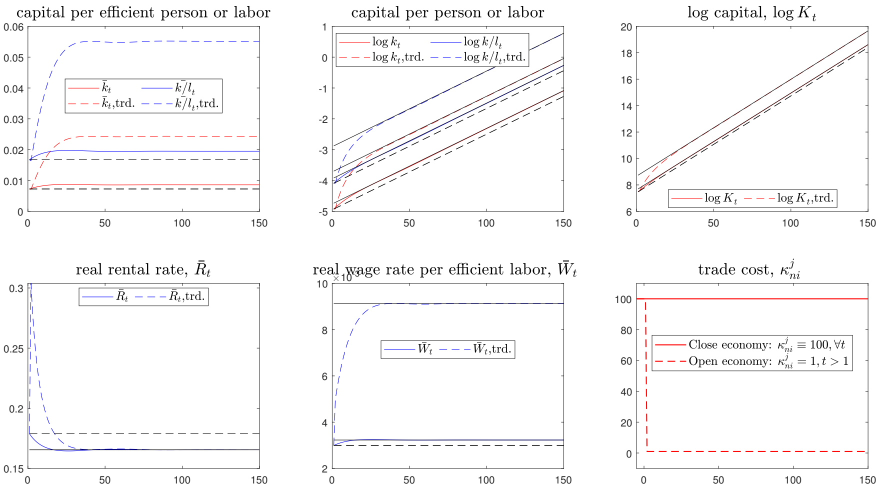
Transitional dynamics - living longer. Sym. Close economy

▸ Open economy ▸ Back



- A high survival rate stimulates capital accumulation and elevates the balanced growth path.

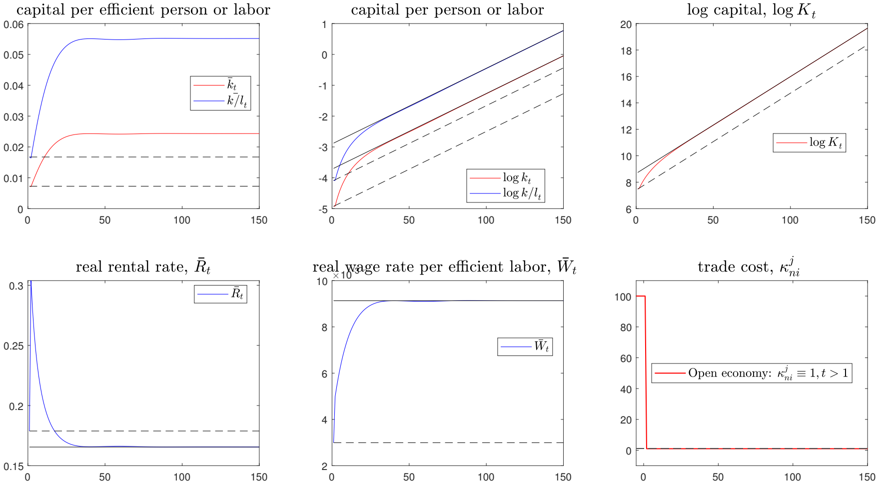
Transitional dynamics - living longer. Close v.s. Open ▸ Back



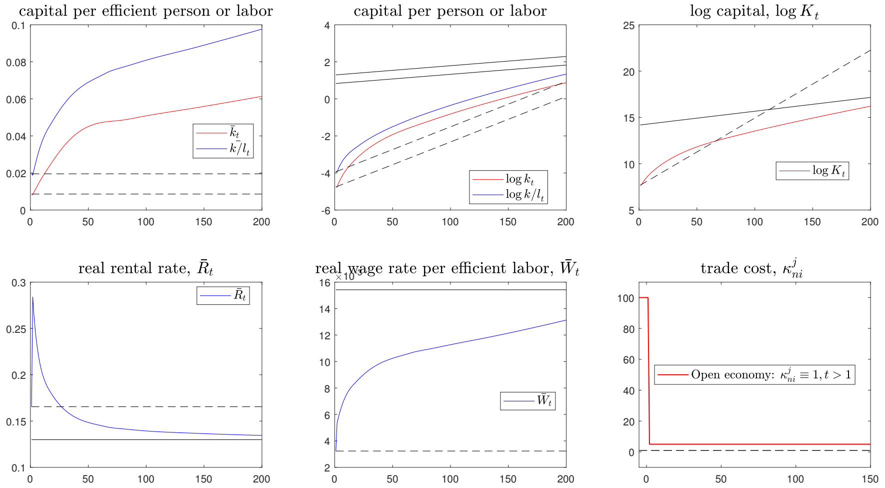
- Trade also stimulates capital accumulation and elevates the balanced growth path.

Transitional dynamics 1 - living longer. Sym. Open economy

back



Transitional dynamics 2 - pop. growth slow down. Sym. Open economy ▸ back



Country Groups

Table: Country Groups

couty	countrycode	country_nam	couty	countrycode	country_nam
1	AUS	Australia	14	IND	India
2	AUT	Austria	15	IRL	Ireland
3	BEL	Belgium	16	ITA	Italy
4	BRA	Brazil	17	JPN	Japan
5	CAN	Canada	18	KOR	Korea, Republic of
6	CHN	China	19	MEX	Mexico
7	DEU	Germany	20	NLD	Netherlands
8	DNK	Denmark	21	PRT	Portugal
9	ESP	Spain	22	SWE	Sweden
10	FIN	Finland	23	TWN	Taiwan
11	FRA	France	24	USA	United States of America
12	GBR	United Kingdom	25	ROW	Rest of the World
13	GRC	Greece			

Back

Sectors

Table: Sector classifications

#1.	5 Sector Classification	Index 1	Index 2	#2.	Sector Description
1	Agriculture, Mining and Quarrying	0.76	0.87	1	Agriculture, Hunting, Forestry and Fishing
1	Agriculture, Mining and Quarrying	0.40	0.34	2	Mining and Quarrying
2	Manufacture-labor intensive	0.59	0.72	3	Food, Beverages and Tobacco
2	Manufacture-labor intensive	0.64	0.72	4	Textiles, Textile, Leather and Footwear
2	Manufacture-labor intensive	0.63	0.78	5	Wood and Products of Wood and Cork
2	Manufacture-labor intensive	0.60	0.68	6	Pulp, Paper, Paper, Printing and Publishing
3	**Manufacture-capital intensive**	**0.47**	**0.44**	7	**Coke, Refined Petroleum and Nuclear Fuel**
3	**Manufacture-capital intensive**	**0.44**	**0.41**	8	**Chemicals and Chemical Products**
2	Manufacture-labor intensive	0.56	0.60	9	Rubber and Plastics
2	Manufacture-labor intensive	0.52	0.52	10	Other NonMetallic Mineral
2	Manufacture-labor intensive	0.51	0.51	11	Basic Metals and Fabricated Metal
2	Manufacture-labor intensive	0.57	0.62	12	Machinery, Nec
3	**Manufacture-capital intensive**	**0.49**	**0.44**	13	**Electrical and Optical Equipment**
2	Manufacture-labor intensive	0.55	0.56	14	Transport Equipment
2	Manufacture-labor intensive	0.66	0.81	15	Manufacturing, Nec; Recycling
3	**Manufacture-capital intensive**	**0.41**	**0.33**	16	**Electricity, Gas and Water Supply**
4	Services-labor intensive	0.72	0.93	17	Construction
4	Services-labor intensive	0.61	0.95	18	Wholesale and Retail Trade
4	Services-labor intensive	0.76	0.91	19	Hotels and Restaurants
4	Services-labor intensive	0.68	0.89	20	Transport and Storage
5	**Services-capital intensive**	**0.42**	**0.50**	21	**Post and Telecommunications**
5	**Services-capital intensive**	**0.50**	**0.51**	**22**	**Financial Intermediation**
5	**Services-capital intensive**	**0.44**	**0.40**	**23**	**Real Estate, Renting and Business Activities**
4	Services-labor intensive	0.75	0.86	24	Community Social and Personal Services

Back

Data sources

Table: Data sources

Variable description	Model counterpart	Data source (1971–2020)	Data source (2021–2100)
Age distribution	$\tilde{N}_{n,g,t}$	UN	UN, Imputed
Population	$N_{n,t}$	PWT	UN, Imputed
Employment	$L_{n,t}$	PWT	Imputed
Human capital index	$E_{n,t}$	PWT	Imputed
Value added	$W_{n,t}L_{n,t}E_{n,t} + R_{n,t}K_{n,t}$	WIOD & Long IO Table	Imputed
Gross output*	$P^j_{n,t}y_{n,t}$	WIOD & Long IO Table	Imputed
Gross expenditure*	$P^j_{n,t}Q^j_{n,t}$	WIOD & Long IO Table	Imputed
Trade flow*	$P^j_{n,t}Q^j_{n,t}T_{n,i,t}$	WIOD & Long IO Table	Imputed
Intermediate prices**	$P^j_{n,t}$	WIOD & Long IO Table	Imputed
Consumption***	$C_{n,t}$	WIOD & Long IO Table	Imputed
Investment***	$I_{n,t}$	WIOD & Long IO Table	Imputed
Initial capital stock***	$K_{n,t0}$	PWT	N/A

Notes: * Values are measured in current prices using market exchange rates. ** Prices are measured using PPP exchange rates. *** Quantities are measured as values deflated by prices.

Back

Constructing data from 2021-2200

Impute saving rate, then given total supply of labor and capital, along with the imputed productivity, solving the CP trade model under the fixed trade cost

$$\log\left(\frac{sr_{n,t}}{1-sr_{n,t}}\right) = \alpha_0 + \alpha_1 \log\left(\frac{sr_{n,t-1}}{1-sr_{n,t-1}}\right) + \alpha_2 Young_{n,t} + \alpha_3 Old_{n,t} + f_n + \epsilon_{n,t}$$

Table: SAVING RATE REGRESSION

	(1)	(2)	(3)
VARIABLES	SR	SR	SR
L1.SR		0.89***	
		(32.74)	
L5.SR	0.43***		
	(8.04)		
Young share	-1.06***	-0.19	-2.80***
	(-3.62)	(-1.34)	(-10.75)
Old share	-2.40***	-0.45*	-5.97***
	(-4.37)	(-1.66)	(-12.36)
Constant	-0.22**	-0.04	0.04
	(-2.24)	(-0.84)	(0.36)
Observations	255	275	280
R-squared	0.891	0.968	0.836
Region FE	YES	YES	YES

Back

Calibrate Knowledge stock process

On the balanced growth path (BLG), population and knowledge stock must grow at a constant rate, with the relation:

$$(1+g_\lambda)^{1-\rho} = (1+g_n)^{\varphi}$$

$1+g_n$ can be calculated from the population growth rate in 1970, and then averaged across regions. $1+g_\lambda$ can be backed out from the real wage growth rate with the relation:

$$1+g_{\text{real wage}} = (1+g_\lambda)^{1/\theta\beta\gamma}$$

Thus,

$$\varphi = \frac{(1-\rho)\log(1+g_\lambda)}{\log(1+g_n)} = \frac{(1-\rho)\theta\beta\gamma\log(1+g_{\text{real wage}})}{\log(1+g_n)}$$

Back

Calibrate Knowledge stock process

To calibrate η_g, I assume that all working-age people have the same $\eta_g > 0$. In 1970, the world economy is assumed to be on the balanced growth path, which implies

$$1 + g_{\lambda,1970} = (\lambda_{n,1970})^{\rho-1} \left[\sum_{g \in [16,65]} \eta_g N_{n,g,1970} \right]^{\varphi} \Gamma(1-\rho)$$

Thus,

$$\eta_g = \frac{1 + g_{\lambda,1970}}{(\lambda_{n,1970})^{\rho-1} \left(N_{n,g\in[16,65],1970} \right)^{\varphi} \Gamma(1-\rho)}$$

Back

Calibration results

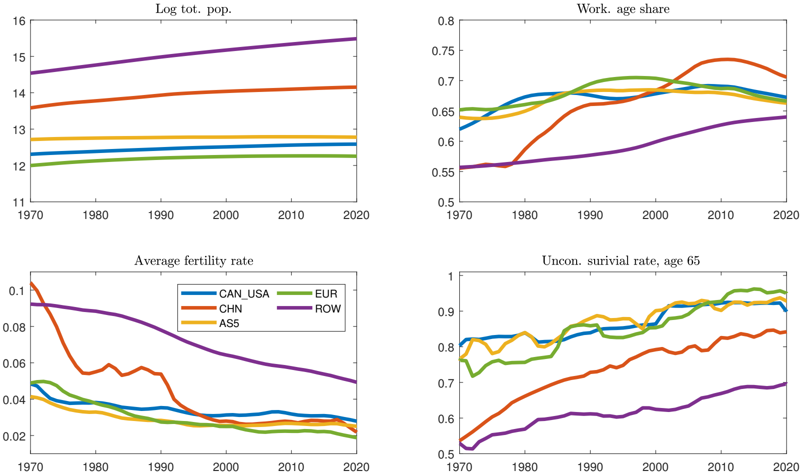


Figure: Demographics

Back

Calibration results

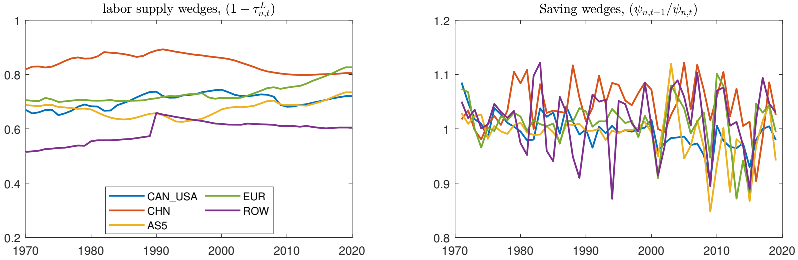


Figure: Demographics

▸ Back

Calibration results

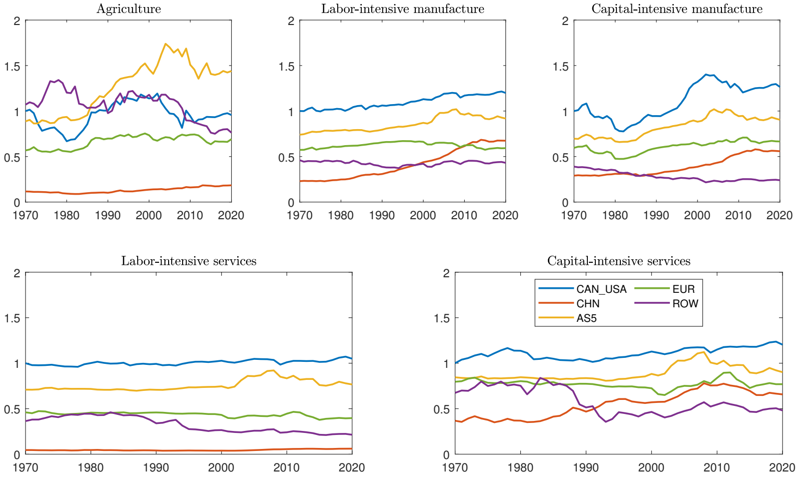


Figure: Demographics

Back

Calibration results

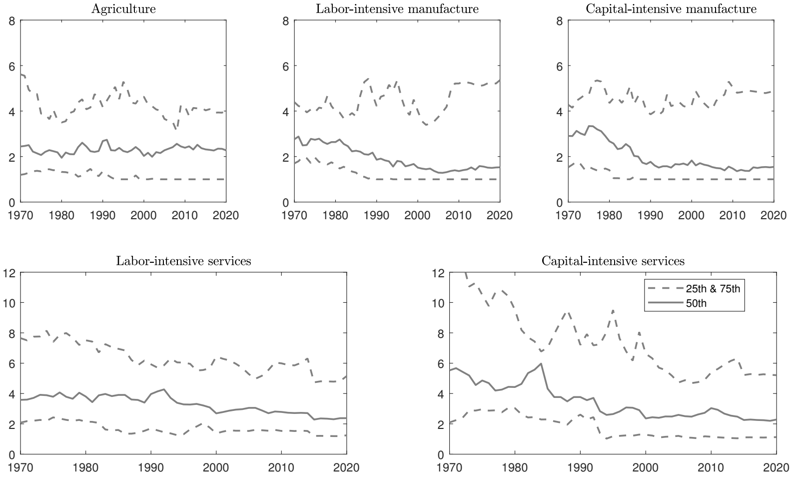
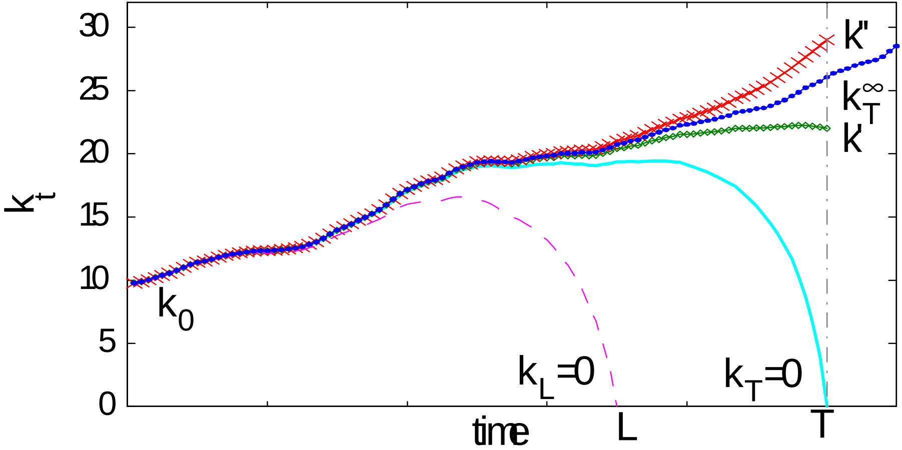


Figure: Demographics

Back

Illustration of turnpike theorem

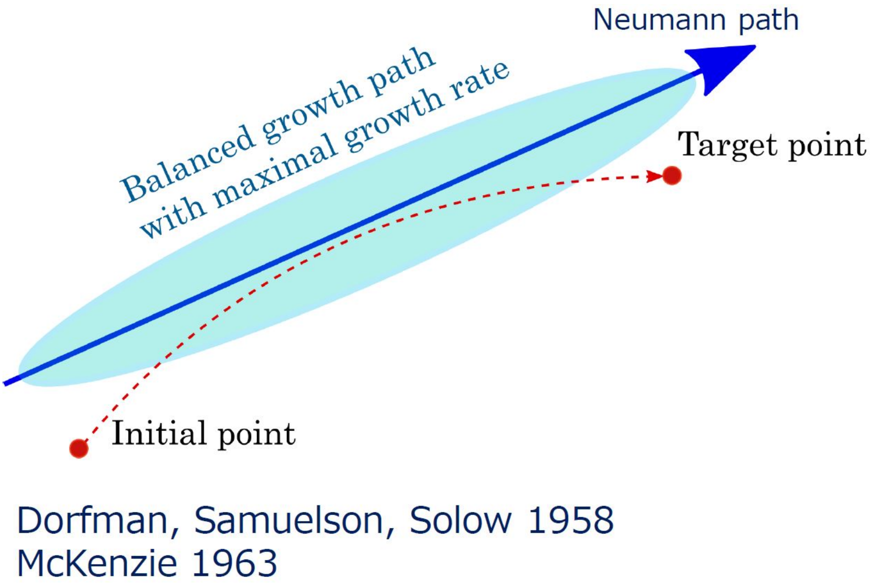
When you are young, you behave as if you will live forever...



Back *Sources*: Lilia Maliar and Serguei Maliar, 2017

Illustration of turnpike theorem

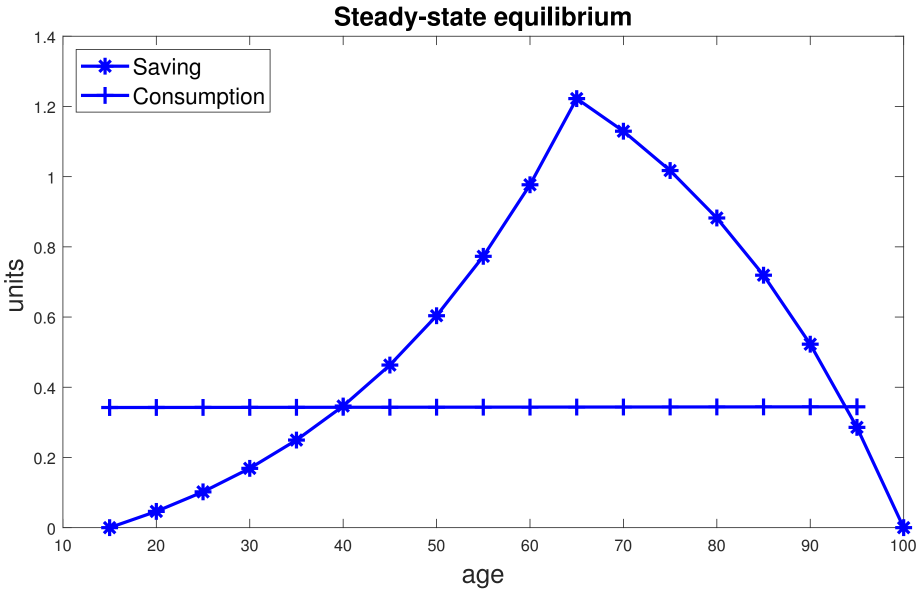
Put differently, terminal conditional has limited effects on the growth path



Back

Counterfactual analysis

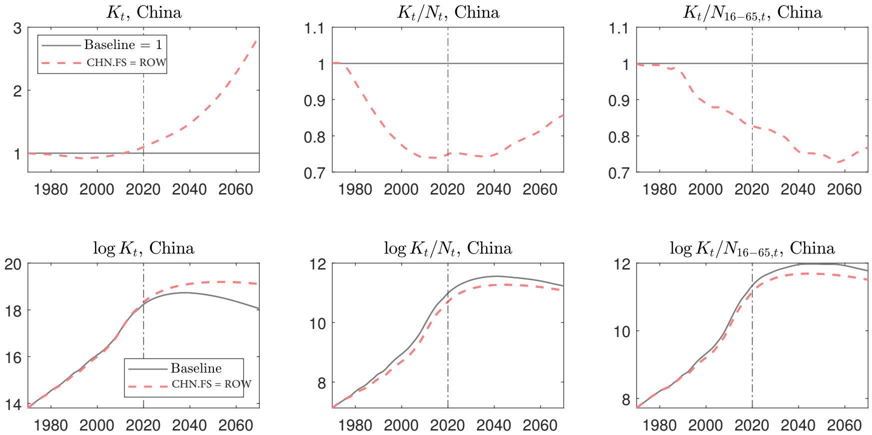
Age-varying savings stock



Back

Counterfactual analysis

Capital process

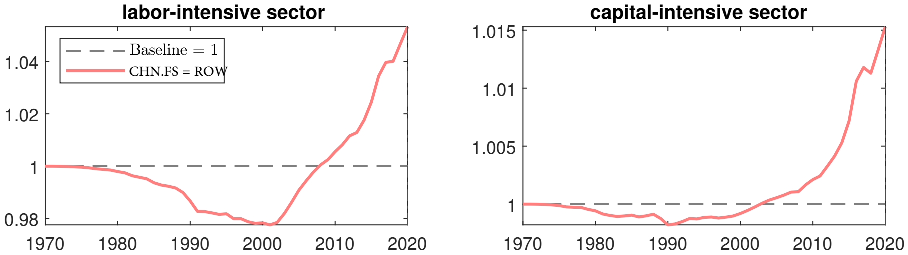


Back

Counterfactual analysis

Knowledge stock process

Knowledge stock, China

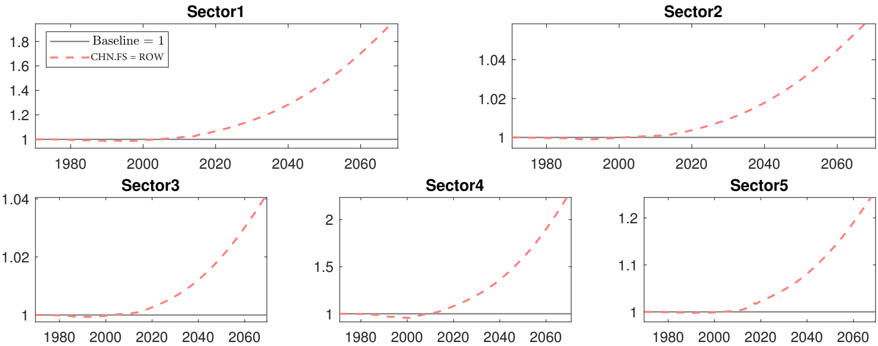


Back

Counterfactual analysis

Knowledge stock process

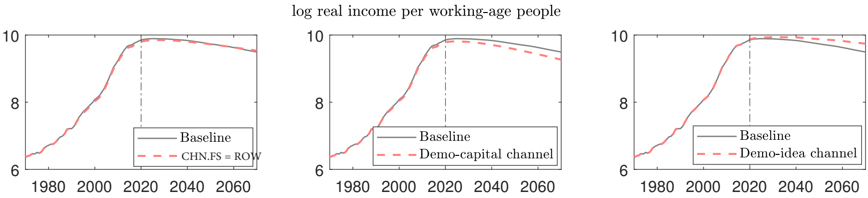
Knowledge stock, China



▸ Back

Counterfactual analysis

Income process

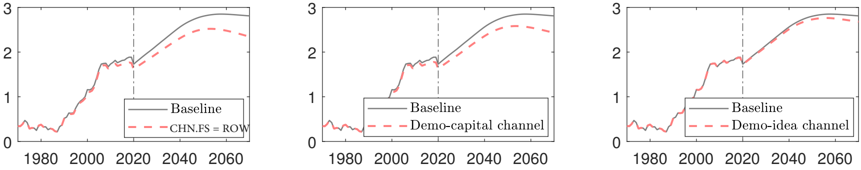


Back

Counterfactual analysis

Trade patterns

Revealed comparative advantage index, capital intensive sector



Back

Balassa (1965) Revealed comparative advantage (RCA) index

$$RCA_{nj} = \frac{\frac{Export_{n,j}}{\sum_n Export_{n,j}}}{\frac{\sum_j Export_{n,j}}{\sum_{j,n} Export_{n,j}}} \tag{16}$$

where n means country, j means sector, $Export_{n,j}$ means the value of country n's sector j exports.

- The higher RCA_{nj}, the higher degree of specialization for country n in sector j products.

Back