DEMOGRAPHIC, TRADE, AND GROWTH [*]

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November 28, 2023

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[*]I am deeply indebted to Kei-Mu Yi. I sincerely thank Bent Sorensen, German Cubas and all participants in UH Macro Workshop for very helpful discussions. All errors are my own.

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Abstract

Motivated by China's recent growth slowdown and aging population, this study investigates the role of demographic factors and trade liberalization in shaping China's past economic growth. It also examines how demographic forces and trade will continue to shape China's future growth.

In the empirical part, the study estimates the effects of demographics and/or trade liberalization on various macroeconomic outcomes through country-level panel regression and a VARX model. The findings indicate that countries with a higher share of working-age population exhibit a higher TFP growth rate. Additionally, these countries tend to save and invest more while consuming less. Moreover, higher trade cost growth rates and a larger share of working-age population are associated with a higher growth rate of the capital-labor ratio, which reflects Heckscher-Ohlin comparative advantage forces. Furthermore, shocks originating from changes in the share of the elderly population have a more substantial and persistent effect compared to shocks stemming from changes in the younger population.

In the theoretical part, an OLG-trade model is developed and calibrated to incorporate the empirical relations identified in the empirical sections. The model incorporates three key features. Firstly, it recognizes that the demographic structure is one of the elements driving TFP growth, known as demographic-induced TFP growth. Secondly, the model includes both dynamic and OLG features to capture the impact of the demographic structure on capital accumulation. Lastly, it is a multi-sector trade model that integrates both Heckscher-Ohlin and Ricardian forces. Counterfactual analysis reveals that China's past growth can largely be attributed to xxx (specific findings). Additionally, the model-based projections demonstrate that China's future demographic changes will lead to xxx (specific findings).

Keywords: Demographics; Dynamics; International trade; Comparative advantage; Economic Growth.

JEL Codes: E21, F11, J11, O47, O11.

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1. Introduction

China has experienced large economic growth over the past several decades. Literature have show that China's impressive growth can be largely attributed to its opening up to trade policy. By opening up to international trade, China has effectively utilized its abundant and expanding working-age population to leverage its comparative advantage in labor-intensive goods. This has allowed China to produce these goods domestically and meet the substantial external demand through exports. In parallel, the growth literature offers an alternative perspective by highlighting the substantial relationship between age structure and total factor productivity (TFP) growth (Demographic-induced TFP growth here after). Studies (e.g., (Feyrer, 2008; Werding, 2008; Sevilla et al., 2007; Werding et al., 2007; Kögel, 2005)) demonstrate that countries with a larger proportion of the working-age population tend to experience higher TFP growth. Specifically, Feyrer (2008) uncovers an inversely U-shaped relationship between the distribution of workers across different age groups and productivity levels. Therefore, an expanding share of the working-age population, accompanied by enhanced TFP growth, may also be one of those forces driven China's past economic expansion.

Despite China's impressive economic growth in the past, the country now faces challenges due to its aging population. As of 2021, China's median age of 37.9 was comparable to that of the United States (37.7), yet its GDP per capital was only 27.9% of that of the US. In brief, China now is too old to be rich. Moreover, projections[1] indicate that both China's total population and the share of its working-age population will continue to decline in the future. In light of these changes, the forces that drove past growth, such as demographic-induced TFP growth and comparative advantages in labor-intensive goods, may not have the same strong impact on China's future growth. Moreover, a large share of the aging population can lead to low domestic saving rates and a reduced supply of capital, thereby hampering capital accumulation. The declining and aging population combined with rising wage are gradually eroding China's old comparative advantage in labor-intensive goods, while the country may not have sufficient time to accumulate capital or develop productivity in capital-intensive goods. Taking these factors into account, this paper analyzes China's past economic growth and conduct model based projection for China's future growth from the perspective of demographics and trade.

The aim of this paper is to investigate the extent to which demographic structure and trade have interacted to drive China's past growth and how demographic forces and trade will continue to shape China's growth in the future. The paper consists of both empirical

1. Source: United Nations, World Population Prospects (2022), (Ritchie and Roser, 2019)

and theoretical parts. The empirical part is divided into two sub-parts. In the first part, I estimate the relationships between demographic structure and various indicators such as TFP growth, consumption share of GDP, and investment share of GDP using country-level panel regression. I find that countries with a higher share of working-age population (or a lower dependency ratio) exhibit a higher TFP growth rate, while larger share of elderly do not have significant effects on TFP growth rate. Additionally, I estimate an inverse U-shaped relationship between the share of population in different age groups and productivity growth rate. Moreover, countries with a higher proportion of working-age individuals tend to invest more while consuming less. When examining the investment and consumption shares of GDP, I also find an inverse U-shaped relationship between age and investment shares of GDP, and a U-shaped relationship between age and the consumption share of GDP. In the second part, I employ both panel regression and a panel Vector Autoregressive with Exogenous Variables (VARX) model to investigate the impact of trade costs and demographics on capital-labor ratios. This ratio is utilized as indicators of the relative size of capital and labor endowment, or Heckscher-Ohlin comparative advantage forces. By employing these methods, I aim to assess the effects of trade costs and demographic structure on the capital accumulation process. Panel regression result shows that lower trade costs or a larger share of working-age population are associated with a higher growth rate of the capital-labor ratio, which supports idea that trade integration stimulates capital accumulation (Sposi, Yi, and Zhang, 2021). Again, an inverse U-shaped relationship between age and the growth rate of the capital-labor ratio is evident. Furthermore, the impulse response function reveals that shocks stemming from changes in the elderly share have a larger and more lasting effect compared to shocks originating from changes in the younger population.

In the theoretical part, I develope and calibrate an OLG-trade model that incorporates the empirical features identified in the empirical sections. The model incorporate three key features. Firstly, the demographic structure will be one of those elements driving TFP growth as Schlenker and Roberts (2009); Rudik, Lyn, Tan, and Ortiz-Bobea (2023), named as demographic-induced TFP growth here after. Secondly, the model will have both dynamic and OLG features to capture the impact of demographic structure on capital accumulation, as Ravikumar, Santacreu, and Sposi (2019); Sposi (2022). Lastly, the sectoral production function will incorporate labor, capital, and TFP, making it a multi-sector trade model that integrates both Heckscher-Ohlin and Ricardian forces, following the framework proposed by Sposi, Yi and Zhang (2021). By utilizing this model, I aim to investigate China's past growth and conduct model-based projections to assess the impacts of demographic and trade forces on China's future growth.

The remaining of the paper is organized as follows. Section 2 provides the empirical regression and take empirical results as motivating facts. Section 3 describes the model and illustrates the mechanisms, where the equilibrium of the model is also defined.

2. Emprical Analysis

In this section, I present the data, empirical model, empirical results. The empirical part is divided into two sub-parts. In the first part, I estimate the relationships between demographic structure and various macroeconomic outcomes such as TFP growth, consumption share of GDP, and investment share of GDP using country-level panel regression. In the second part, I employ panel regression and a panel VARX model to investigate the impact of trade costs and demographics on capital-labor ratios. This ratio is utilized as indicators of the relative size of capital and labor endowment, or Heckscher-Ohlin comparative advantage forces. By employing these methods, I aim to assess the effects of demographics and/or trade liberalization on various macroeconomic outcomes and analyze the dynamic effects of these shocks.

2.1. Data

I use panel data encompassing 76 countries across different income levels, spanning the period from 1975 to 2019. The selection of country groups and time periods for analysis was based on data availability, resulting in 34 high-income countries, 21 upper middle-income countries, 16 lower middle-income countries, and 5 lower-income countries according to the United Nations classifications. For detailed information about the country list, please refer to Table A.1.

The empirical analysis incorporated four types of variables. Firstly, there were 7 types of demographic structure indices, including the dependency ratio, young dependency ratio, old dependency ratio, working age share, young population share, old population share, and population distribution across different age cohorts. These variables were calculated or directly obtained from the United Nations World Population Prospects report. Secondly, the total factor productivity (TFP) growth was calculated based on TFP data from the Penn World Table 10.01. The capital-labor ratio was also derived from this dataset. Thirdly, various other macroeconomic outcomes, including investment, consumption share of GDP, and real GDP per capita, were acquired from the World Development Indicators database. Lastly, the trade cost index was calculated based on data from the CEPII database. For more information regarding the data sources and variable construction, please refer to Appendix A.

Summary statistics

The specifics of summary statistics are listed in Table 1.

TABLE 1

Decriptive statistics: non-overlapping 5 years between 1975 - 2019, 76 countries

VARIABLES	N	mean	sd	between.sd	within.sd	min	max	skewness	kurtosis
Age [0-9] share	684	0.212	0.0828	0.07694	0.03166	0.0723	0.383	0.229	1.689
Age [10-19] share	684	0.188	0.0463	0.04075	0.02246	0.0871	0.283	-0.306	1.828
Age [20-29] share	684	0.163	0.0207	0.01460	0.01479	0.0992	0.235	-0.106	3.298
Age [30-39] share	684	0.134	0.0217	0.01526	0.01547	0.0838	0.216	0.216	2.966
Age [40-49] share	684	0.108	0.0299	0.02501	0.01665	0.0547	0.192	0.252	2.050
Age [50-59] share	684	0.0834	0.0317	0.02830	0.01462	0.0300	0.170	0.398	1.977
Age [60-69] share	684	0.0602	0.0294	0.02784	0.00984	0.0169	0.144	0.523	2.020
Age [70-79] share	684	0.0368	0.0236	0.02267	0.00707	0.00726	0.113	0.679	2.221
Age [80-89] share	684	0.0143	0.0124	0.01123	0.00540	0.000645	0.0666	1.077	3.352
Age [90+] share	684	0.00197	0.00227	0.00183	0.00135	2.72e-06	0.0161	1.784	6.735
Child share	684	0.308	0.109	0.10071	0.04256	0.116	0.522	0.107	1.639
Working age share	684	0.611	0.0667	0.05812	0.03333	0.457	0.785	-0.311	2.051
Elderly share	684	0.0803	0.0518	0.04929	0.01674	0.0167	0.273	0.728	2.391
Dependence ratio	684	0.656	0.190	0.16679	0.09339	0.273	1.187	0.616	2.283
Young dependence ratio	684	0.530	0.244	0.22360	0.10056	0.159	1.142	0.433	1.899
Old dependence ratio	684	0.126	0.0748	0.07111	0.02452	0.0317	0.455	0.849	2.824
TFP growth (%)	684	-0.113	2.268	0.81878	2.11648	-19.36	10.03	-1.440	15.61
Final consumption (% of GDP)	653	77.19	10.93	9.51946	5.61180	35.71	115.5	-0.360	4.567
Gross capital formation (% of GDP)	653	24.25	6.912	5.02894	4.88331	1.525	62.67	0.918	5.233
Trade cost	657	3.182	0.928	0.86823	0.39593	1.174	8.334	1.364	6.684
(K/L) growth (%)	684	2.578	2.769	1.87536	2.04689	-5.274	12.78	0.681	4.394
log(K/L)	684	10.53	1.423	1.37333	0.40142	5.859	12.84	-0.541	2.638
GDP per capita at constant 2015 price	650	15,177	18,299	17,583.80	6,095.80	172.9	106,544	1.738	6.394

Pairwise correlation

TABLE 2

Pairwise correlation

Variables	ChildDep	OldDep	Dep	ChildSre	OldSre	WorkingSre
ChildDep	1					
OldDep	-0.787***	1				
Dep	0.967***	-0.604***	1			
ChildSre	0.990***	-0.850***	0.928***	1		
OldSre	-0.832***	0.995***	-0.664***	-0.891***	1	
WorkingSre	-0.961***	0.599***	-0.994***	-0.931***	0.663***	1

TABLE 3

Pairwise correlation

Variables	WorkingSre	TFP_GR	Capital/GDP	Cons/GDP	TradeCost	K/L_GR	POP_GR
WorkingSre	1						
TFP_GR	0.129***	1					
Capital/GDP	0.143***	-0.149***	1				
Cons/GDP	-0.380***	0.091***	-0.526***	1			
TradeCost	-0.640***	-0.017	-0.256***	0.409***	1		
K/L_GR	0.242***	0.011	0.374***	-0.165***	-0.151***	1	
POP_GR	-0.728***	-0.114***	0.023	0.146***	0.499***	-0.157***	1

2.2. Demographics, technology change, and other macroeconomic outcomes

This section presents the empirical panel regression model that examines the relationship between demographics and Total Factor Productivity (TFP) change, along with macro variables such as capital formation, and consumption.

2.2.1. The effect of demographic structure on technology change

Regression model

In order to examine the relationship between demographic variables and technology change, I consider a panel regression with both country and time fixed effects.

$$GRTFP_{it,t+4} = Constant + \beta_1 Demographic_{it} + \beta_2 Control_{it} + f_i + f_t + \varepsilon_{it} \tag{1}$$

where i means country, t means year. The dependent variable $GRTFP_{it,t+4}$ means average TFP growth rate (%) for country i during the period from t to $t+4$, and calculated as follows:

$$GRTFP_{it,t+4} = [\frac{TFP_{i,s+4}}{TFP_{i,s}}]^{\frac{1}{4}} - 1$$

The variable $Demographic_{it}$ represents demographic-relevant variables for country i at time t, such as the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The old dependency ratio is defined as the ratio of people aged 65 and above to people aged 15-64. The working age share is defined as the share of people aged 15-64. The young population share is defined as the share of people aged 0-14, and the old population share is defined as the share of people aged 65 and above, or the population share at different age cohorts. The variable $Control_{it}$ represents a control variable for country i at time t, specifically the initial log real GDP per person. f_i and f_t are country and time fixed effects.

As is common in the literature, I reduce the influence of business cycle fluctuations by calculating 5-year growth rates and dividing the entire period of 1975–2019 into nine non-overlapping 5-year sub-periods period 1 (1975–1979), period 2 (1980–1984), period 3 (1985–1989), period 4 (1990–1994),..., and period 9 (2015–2019). Since I treat all demographic variables measured at the start of the sample period, before the growth has occurred, the regression analysis carries a stronger sense of causality. Similarly, in the following other types of regression, the time lag between the independent variables and the dependent variable reduces the likelihood of endogeneity issues caused by reverse causality. For the purpose of conducting robust checks, I performed the same regression analysis for five non-overlapping 8-year periods: period 1 (1980-1987), period 2 (1988-1995), period 3 (1996-2003), period 4 (2004-2011), and period 5 (2012-2019). The results of these robust regressions can be found in Appendix C.

I would expect that countries with a higher proportion of their population in the working age group would exhibit a higher TFP growth rate, indicating a positive relationship. Thus, I expect a positive sign for the coefficient $\hat{\beta}_1$ when using the working age share as an index for demographics. Conversely, for the share of young people and elderly, as well as the dependence ratios (including both young and old dependence ratios), I anticipate a negative sign for the coefficient $\hat{\beta}_1$. I would also expect negative sign for $\hat{\beta}_2$, which indicates that the developed countries usually showing a slowdown TFP growth.

Results

Table 5 and Table 7 reports the main results of regression. In Table B.1, In columns (1) to (4), I use different index to capture the changes of age structure to ensure robustness of empirical estimates. Overall, the estimation results are consistent with the expected signs expect for the estimators of old dependency ratio and share of elderly. It shows that increasing young people share or decreasing working age share related to TFP growth rate decline, while increasing elderly share has positive but no significant effects on TFP growth rate. This results is similar with Kögel (2005).

Specifically, 1 percentage point (p.p) increase, or 1 s.d. increase, in the working age share is associated with a corresponding increase of 0.11 percentage points (p.p), or increase of 0.81 s.d., in the average TFP growth rate over the following 4-year period. Additionally, an increase of 1 percentage point (p.p), or 1 s.d. change, in the child (age below 15) share is associated with a corresponding decrease of 0.14 percentage points (p.p), or decrease of 1.72 s.d., in the average TFP growth rate over the following 4-year period. However, changes in the elderly (age 65 and above) share do not have a significant effect on TFP growth.

In Table 7, to account for different age cohorts and their effects on the TFP growth, the population is further decomposed into 3, 4, or 5 distinct age cohorts. To ensure the inclusion of all types of age cohort dummy variables, the regression model is specified without a constant term. The results of the regression analysis reveal interesting patterns. In the 3-cohort regression, it is found that the share of the population aged 15 to 64 (15-64) has the most beneficial impact on the TFP growth rate. In the 4-cohort and 5-cohort regressions, the most beneficial cohorts are observed to be those aged 50 to 74 and 60 to 79, respectively. This implies that a large share of extreme young or old individuals is not as beneficial as a share of individuals at the median age.

In addition to the regression analysis, polynomial curves of different orders are employed to fit the estimated coefficients obtained from the cohort regressions, as shown in Figure **??** and **??**. This finding further reinforces the notion that a substantial proportion of extremely young or old individuals is not as advantageous as a share of individuals at the median age. The graphical representation provides visual evidence of the inverse U shape for the relationship between demographic structure and its impact on the TFP growth, providing valuable insights for the modeling.

2.2.2. The effect of demographic structure on other macroeconomic outcomes

Regression model

The relationship between demographic variables and macroeconomic outcomes (capital formation, and consumption) is examined by estimating the following model

$$Ave.Y_{it,t+4} = Constant + \beta_1 Demographic_{it} + f_i + f_t + \varepsilon_{it} \quad (2)$$

The variable $Ave.Y_{it,t+4}$ means average investment, or consumption share of GDP (%) during the period t to $t+4$:

$$Ave.Y_{it,t+4} = \sum_{s=t+0}^{t+4} \frac{Y_{i,s}}{5}$$

The remaining variables used in the analysis are consistent with those mentioned in subsubsection 2.2.1. Since young and old people tend to save less and consume more compared to the working-age people, I would expect that countries with a larger share of working-age people are associated with a higher investment share of GDP, and a lower consumption share of GDP. Consequently, when the demographic index used is the working age share, I anticipate a positive estimated coefficient, $\hat{\beta}_1$, for investment share of GDP. On the other hand, if the demographic index is the share of young people and elderly, as well as the dependence ratios (including both young and old dependence ratios), I expect the estimated coefficient for investment to be negative. For the consumption share of GDP, the expectations are reversed. Specifically, when the demographic index used is the working age share, I anticipate a negative estimated coefficient for consumption. However, if the demographic index is the share of young people and elderly, as well as the dependence ratios (including both young and old dependence ratios), I expect the estimated coefficient for consumption to be positive.

Results

Table 5 and Table 7 reports the main regression results, and Table B.3 and Table B.4 using different indices to capture changes in age structure to ensure robustness of empirical estimates. Overall, the estimation results are consistent with the expected signs. The findings indicate that countries with a larger proportion of working-age individuals , or a lower share of elder or young people, or a lower dependency ratio tend to exhibit higher shares of investment in GDP, along with a lower share of consumption in GDP. While the increasing share of elderly population (age 65 and above) does not appear to have a significant impact on consumption.

Specifically, a 1 percentage point (p.p) increase, or 1 s.d. increase, in the working age share leads to a corresponding increase of 0.29 percentage points (p.p), or an increase of 0.33 s.d., in the average capital formation share of GDP over the next four years. On the other hand, a 1 percentage point (p.p) increase, or 1 s.d. increase, in the working age share is associated with a subsequent decrease of 0.34 percentage points (p.p), or an increase of 0.21 s.d., in the average consumption share of GDP over the following four years. The rise in the share of young population (age below 15) has an inverse impact on these two shares compared to the increase in working age share. However, increase in the elderly share have positive but not significant effect on consumption shares of GDP. For the capital formation share of GDP, It would be 0.66 percentage points (p.p) lower, or 0.65 s.d. lower if the elderly share is 1 percentage point (p.p) higher, or 1 s.d. higher.

Furthermore, Table 7 provides additional regression results that take into account the effects of the share of different age cohorts on investment, and consumption. The population is further divided into 3, 4, or 5 distinct age cohorts to capture the heterogeneity within the demographic structure. **??**, and **??** depict the polynomial curves of different orders fitted to the estimated coefficients derived from the cohort regressions. These figures illustrate the relationship between age and the shares of consumption, and investment in GDP. The curves reveal an inverse U-shaped relationship between age and the investment shares of GDP. This suggests that countries tend to have higher investment shares when the population is composed of individuals at intermediate ages, while extreme youth or old age groups have a relatively smaller influence on these shares. In contrast, the relationship between age and the consumption share of GDP follows a U-shaped pattern. This implies that countries with a larger proportion of individuals at younger or older ages tend to have higher consumption shares in GDP. These findings highlight the importance of considering the age structure's impact on consumption, and investment behaviors.

2.3. Demographics, trade cost, endowments change and economic growth

In this section, I employ panel regression and VARX model to explore the relationship between demographics, globalization, capital endowments, and economic growth. The capital labor ratio is utilized as indicators of the relative size of capital and labor endowment, or Heckscher-Ohlin comparative advantage forces. The analysis aims to examine how changes in demographics and trade costs impact the accumulation of capital endowments and, ultimately, influence economic growth.

2.3.1. The effects of demographic structure and trade cost change on capital-labor ratio

Regression model

The effect of demographic variables and trade cost change on capital labor ratio is examined by estimating the following model

$$GR.K/L_{it,t+4} = Constant + \beta_1 Demographic_{it} + \beta_2 TradeCost_{it} + \beta_3 Control_{it} + f_i + f_t + \varepsilon_{it} \quad (3)$$

The variable $GR.K/L_{it,t+4}$ means average capital per person (k) growth rate (%) for country i during the period from t to $t+4$, and calculated as follows:

$$GR.K/L_{it,t+4} = [\frac{k_{i,s+4}}{k_{i,s}}]^{\frac{1}{4}} - 1$$

The trade cost for country i at time t $TradeCost_{it}$ are constructed as the Head-Ries (HR) index (Head and Ries, 1997). I calculated it as follows:

$$TradeCost_{it} = (\frac{\pi_{i,row}}{\pi_{row,row}} \frac{\pi_{row,i}}{\pi_{ii}})^{-\frac{1}{2\theta}}$$

Where $\pi_{i,row}$ is the share of country i's total expenditure on goods from Rest of the World (ROW) at t and $\theta = 4$ is index which capture the trade elasticity. To calculate the $\pi_{i,row}$, I still need the total output X_i of country i and total output X_{row} for Rest of the world with respected to country i. Due to data availability constraints, I made simplifications in the analysis. Specifically, I assumed a fixed labor share of $\alpha = 0.7$ in the production function. The total output, denoted as X_i, was calculated $X_i = \frac{GDP_i}{\alpha}$. I also calculated the trade cost variable for two cases: one with $\alpha = 1$ and another with $\alpha = 0.5$. The computed trade costs exhibited similar trends across all three cases. I utilized the trade cost calculated with $\alpha = 1$. The variable *Control* means control variable, and it includes the initial log capital per person, and population growth rate during period t to $t+4$. The remaining variables used in the analysis are consistent with those mentioned in subsubsection 2.2.1.

I would expect a country with a larger share of working-age people is associated with a higher growth rate of capital per person. So when the demographic index used is the working age share, $\hat{\beta}_1 > 0$, and if demographic index is the share of young people and elderly, as well as the dependence ratios (including both young and old dependence ratios), $\hat{\beta}_1 < 0$. Furthermore, Sposi, Yi and Zhang (2021) shows that trade integration stimulates capital accumulation by reducing investment prices. Therefore, I would expect that a country with lower trade costs is associated with a higher growth rate of capital per person ($\hat{\beta}_2 < 0$).

Results

Table 5 presents the main regression results analyzing the relationship between age structure and capital per person growth rate, while Table B.6 considering different indices to capture changes in age structure. Overall, the estimation results are consistent with the expected signs. The results indicate that countries with a larger proportion of working-age individuals or lower dependence ratios tend to experience higher capital per person growth rates. Additionally, the findings suggest that a decrease in trade costs leads to higher capital per person growth rates, highlighting the positive impact of trade globalization on capital accumulation.

Specifically, a 1 percentage point (p.p) increase, or 1 standard deviation (s.d.) increase, in the working age share yields a subsequent increase of 0.13 percentage points (p.p), or an increase of 0.41 s.d., in the average capital-labor ratio growth over the next four years. For the trade cost, It would be around 0.93 percentage points (p.p) lower, or 0.38 s.d. lower if the trade cost is 1 unit higher, or 1 s.d. higher.

To further investigate the effects of different age cohorts on capital per person growth rate, Table 7 decomposes the population into 3, 4, or 5 distinct age cohorts. **??** and **??** depicts the polynomial curves of different orders fitted to the estimated coefficients obtained from these cohort regressions. The curves visually confirm the inverse U-shaped relationship between age and capital per person growth rate. This observation implies that age structure can also affect comparative advantage forces, as capital per person serves as an indicator of Heckscher-Ohlin comparative advantage forces. By influencing the accumulation or relative size of capital endowments, age structure indirectly impacts a country's growth through comparative advantage in endowments and trade.

TABLE 4

THE EFFECT OF DEMOGRAPHIC STRUCTURE CHANGE

VARIABLES	Average value in the future 4 years		
	TFP growth rate	Patent.Applications (per 1000 people)	Industrial.Design.Applications (per 1000 people)
Work.Share	11.43***	1.53***	1.36***
(15-64)/ToT	(3.33)	(3.12)	(4.78)
Initial.Log	-3.09***		
.Dependent	(-4.82)		
Constant	20.96***	-0.92***	-0.76***
	(3.65)	(-3.06)	(-4.39)
Observations	732	395	215
R-squared	0.259	0.826	0.913
Time FE	YES	YES	YES
Country FE	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable *Child.Share* represents the young population share, which is defined as the share of people aged 0-14, and the variable *Old.Share* represents old population share, which is defined as the share of people aged 65 and above.

TABLE 5

THE EFFECT OF DEMOGRAPHIC STRUCTURE CHANGE

VARIABLES	Average value in the future 4 years							
	TFP growth rate		Cap.Formation(% GDP)		Gross.Consumption(% GDP)		K/L growth rate	
Work.Share	11.43***		28.80**		-33.75**		13.34**	
	(3.33)		(2.17)		(-2.00)		(2.49)	
Child.Share		-13.98***		-24.76*		33.75*		-11.22*
		(-3.68)		(-1.74)		(1.99)		(-1.82)
Old.Share		2.79		-65.97***		33.77		-24.65**
		(0.39)		(-2.65)		(0.90)		(-2.38)
Trade Cost							-0.87**	-0.83**
							(-2.27)	(-2.13)
Initial.Log.Dependent							-2.24***	-1.99***
							(-4.12)	(-3.45)
Initial.Log.GDP.pc	-3.09***	-3.46***						
	(-4.82)	(-4.77)						
PoP.Growth							-28.93	-33.14*
							(-1.55)	(-1.84)
Constant	20.96***	35.46***	4.09	34.10***	98.56***	64.81***	22.19***	32.98***
	(3.65)	(5.19)	(0.53)	(6.74)	(9.84)	(9.15)	(3.63)	(5.32)
Observations	732	732	724	724	725	725	758	758
R-squared	0.259	0.266	0.575	0.581	0.753	0.753	0.586	0.589
Time FE	YES	YES	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable *Child.Share* represents the young population share, which is defined as the share of people aged 0-14, and the variable *Old.Share* represents old population share, which is defined as the share of people aged 65 and above.

TABLE 6

The effect of demographic cohort structure change

VARIABLES	TFP growth rate			Patent.Applications (per 1000 people)			Industrial.Design.Applications (per 1000 people)		
	Average value in the future 4 years								
(0-14)/ToT.	21.48***			-1.60***			-0.89***		
	(3.61)			(-4.60)			(-3.84)		
(15-64)/ToT.	35.46***			0.58***			0.63***		
	(5.19)			(2.73)			(4.98)		
(65+)/ToT.	38.25***			2.29**			-0.42		
	(3.42)			(2.50)			(-0.98)		
(0-24)/ToT.		26.22***			-1.56***			-0.55***	
		(4.24)			(-7.06)			(-3.87)	
(25-49)/ToT.		34.48***			0.18			0.71***	
		(4.28)			(0.46)			(2.87)	
(50-74)/ToT.		43.60***			4.90***			1.08***	
		(4.41)			(7.40)			(2.93)	
(75+)/ToT.		13.47			-2.59			-1.85**	
		(0.90)			(-1.59)			(-1.99)	
(0-19)/ToT.			25.36***			-1.11***			-0.53***
			(4.08)			(-4.09)			(-2.87)
(20-39)/ToT.			31.80***			-1.72***			0.08
			(4.35)			(-4.06)			(0.31)
(40-59)/ToT.			34.74***			3.59***			1.75***
			(3.46)			(6.47)			(5.20)
(60-79)/ToT.			55.17***			4.23***			-0.31
			(5.35)			(3.99)			(-0.46)
(80+)/ToT.			-21.89			-7.67***			-1.09
			(-1.08)			(-2.62)			(-0.57)
Initial.Log.Dependent	-3.46***	-3.51***	-3.51***						
	(-4.77)	(-4.49)	(-4.55)						
Observations	732	732	732	395	395	395	215	215	215
R-squared	0.266	0.263	0.272	0.859	0.880	0.886	0.935	0.939	0.942
Time FE	YES	YES	YES	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

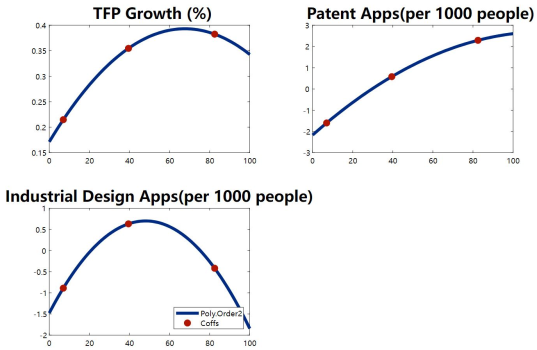

Figure 1
3 cohorts

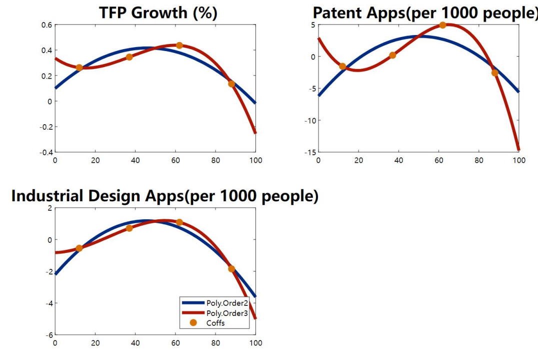

Figure 2
4 cohorts

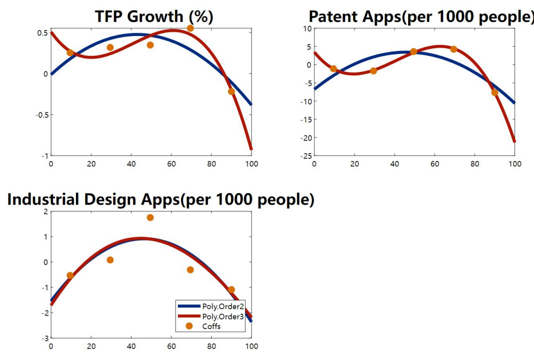

Figure 3
5 cohorts

TABLE 7

The effect of demographic cohort structure change

	Average value in the future 4 years											
VARIABLES	TFP growth rate			Cap.Formation(% GDP)			Gross.Consumption(% GDP)			K/L growth rate		
(0-14)/ToT.	21.48***			9.34			98.55***			21.77***		
	(3.61)			(0.98)			(9.21)			(3.69)		
(15-64)/ToT.	35.46***			34.10***			64.81***			32.98***		
	(5.19)			(6.74)			(9.15)			(5.32)		
(65+)/ToT.	38.25***			-31.87			98.58***			8.34		
	(3.42)			(-1.30)			(2.95)			(0.61)		
(0-24)/ToT.		26.22***			16.69***			92.44***			24.08***	
		(4.24)			(2.64)			(14.84)			(4.50)	
(25-49)/ToT.		34.48***			29.11***			60.55***			39.21***	
		(4.28)			(4.14)			(5.58)			(5.36)	
(50-74)/ToT.		43.60***			37.83**			59.95***			19.18	
		(4.41)			(2.05)			(3.23)			(1.66)	
(75+)/ToT.		13.47			-124.60***			150.74***			4.22	
		(0.90)			(-2.77)			(3.21)			(0.24)	
(0-19)/ToT.			25.36***			15.40**			92.98***			22.11***
			(4.08)			(2.32)			(12.62)			(4.02)
(20-39)/ToT.			31.80***			26.71**			71.18***			36.72***
			(4.35)			(2.52)			(5.39)			(5.30)
(40-59)/ToT.			34.74***			20.39			43.58*			27.00***
			(3.46)			(1.13)			(1.85)			(2.98)
(60-79)/ToT.			55.17***			53.93**			100.97**			21.25
			(5.35)			(2.37)			(2.47)			(1.41)
(80+)/ToT.			-21.89			-224.74***			126.47*			-9.87
			(-1.08)			(-3.07)			(1.75)			(-0.33)
Trade Cost										-0.83**	-0.83**	-0.79**
										(-2.13)	(-2.11)	(-2.00)
Initial.Log.Dependent	-3.46***	-3.51***	-3.51***							-1.99***	-1.98***	-1.93***
	(-4.77)	(-4.49)	(-4.55)							(-3.45)	(-3.21)	(-3.14)
PoP.Growth										-33.14*	-35.31**	-30.58
										(-1.84)	(-2.08)	(-1.64)
Observations	732	732	732	724	724	724	725	725	725	758	758	758
R-squared	0.266	0.263	0.272	0.971	0.972	0.972	0.996	0.996	0.996	0.785	0.787	0.787
Time FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

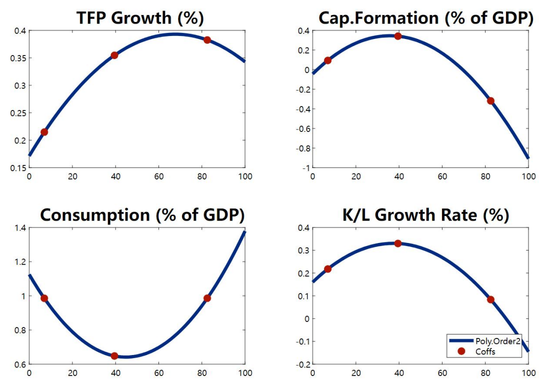

FIGURE 4
3 COHORTS

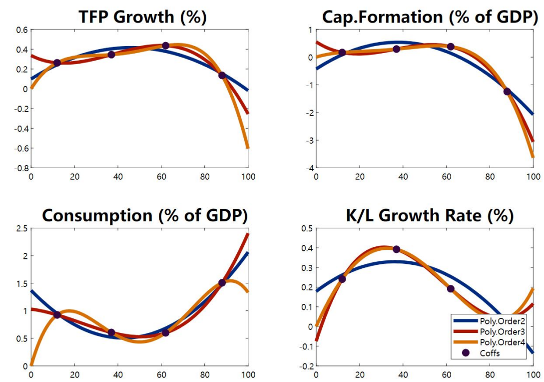

FIGURE 5
4 COHORTS

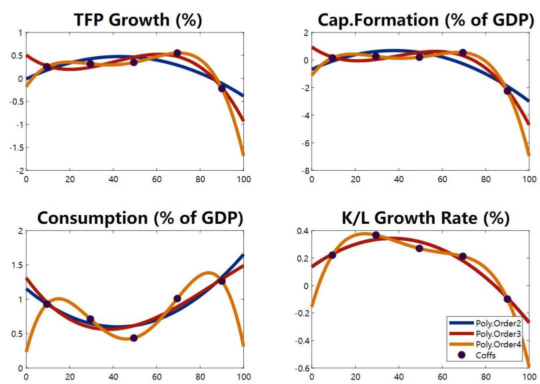

FIGURE 6
5 COHORTS

2.3.2. VARX: Dynamic effects of demographics shock and trade cost shock

To further analyze the dynamic effects of demographic shocks and trade cost shocks on macroeconomic outcomes, a VARX model is employed. Here we focus on how changes in demographics and trade costs impact the accumulation of capital endowments, ultimately influencing economic growth.

VARX model

The dynamic effects of demographics shock and trade cost shock is examined by estimating the following VARX model:

$$Y_{nt} = C + AY_{nt-1} + BX_{nt} + \varepsilon_{nt}$$

Endogenous variable list:

$$Y_{nt} = \begin{bmatrix} the\ 5\ year\ growth\ rate\ of\ TFP\ (\%) \\ the\ 5\ year\ growth\ rate\ of\ the\ real\ GDP\ per\ capita\ (\%) \\ the\ 5\ year\ growth\ rate\ of\ capital\ per\ person\ (\%) \end{bmatrix}_{Country\ n, time\ t}$$

Exogenous variables: Demographic Structure (age shares):

$$X_{nt} = \begin{bmatrix} young\ people\ share\ (\%),\ (0-14) \\ old\ people\ share\ (\%),\ (65+) \\ trade\ cost\ change\ (\%) \\ the\ 5\ year\ growth\ rate\ of\ population (\%) \end{bmatrix}_{Country\ n, time\ t}$$

Since demographics variables changes slowly over time, I calculate above index at every five year level and dividing the entire period of 1975–2019 into nine 5-year sub-periods: period 1 (1975–1979), period 2 (1980–1984), period 3 (1985–1989), period 4 (1990–1994),..., and period 9 (2015–2019). The growth rate variable is calculated as 5-year growth rates and the level variable are calculated as 5-year average values. The unit time lag here is 5 years. (e.g. t = 1 means first 5 years.)

Results

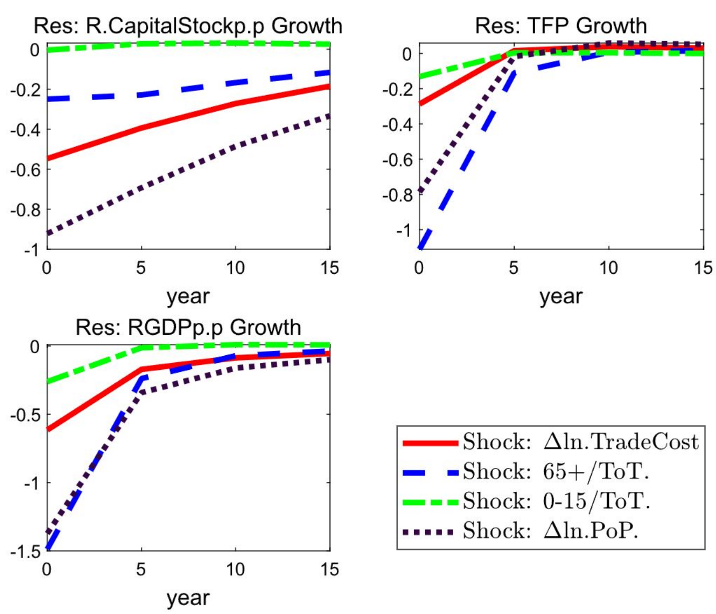

Figure 7

IRF of exogenous demographic shock and trade cost shock

In Figure 7, I plot the impulse response function illustrating the impact of demographic shocks and trade cost shocks on variables such as TFP growth, real GDP per capita growth, and capital per person growth. The graph visually represents the dynamic effects of these shocks on the specified variables over time. In general, one unit (1 percentage point (p.p)) positive shock to the share of younger people or older people leads to a decrease in the growth rates of capital per person, TFP, and real GDP per person. Similarly, one unit positive shock to trade costs also negatively impact the growth rates of capital per person, TFP, and real GDP per person. Notably, the impact of one unit shock to the share of elderly people is stronger and persists for a longer period compared to the impact of one unit shock to the share of young people.

2.4. Summary

In this section, I first investigate empirically the effects of demographics and/or trade liberalization on TFP growth and other macroeconomic outcomes, employing panel data regression techniques. Subsequently, I examined the dynamic effects of these shocks utilizing a panel Vector Autoregressive with Exogenous Variables (VARX) model.

The findings highlight the significant impact of demographic structure, particularly the share of working-age individuals, on TFP growth, investment, and consumption. Countries with a higher proportion of working-age people tend to exhibit higher TFP growth rates and greater shares of investment in GDP, while having a smaller consumption share. Additionally, the study explores the effects of demographics structure and trade costs on capital-labor ratios. This ratio is utilized as indicators of the relative size of capital and labor endowment, or Heckscher-Ohlin comparative advantage forces. It shows that higher trade costs and a larger share of working-age population are associated with higher growth rates of the capital-labor ratio. To capture the dynamic effects of demographic shocks and trade cost shocks, a VARX model is employed, showing that shocks stemming from changes in the share of elderly population have a more substantial and persistent effect compared to shocks from changes in the share of young population.

The empirical analysis in this study emphasizes the crucial role of demographic structure, particularly the share of working-age individuals, in shaping TFP growth, investment, and consumption. Furthermore, it highlights the impact of demographics and trade costs on capital-labor ratios and the dynamic effects of demographic shocks on capital accumulation and economic growth. These findings provide valuable insights into the complex interactions between demographics, trade liberalization, and macroeconomic outcomes. In the next section, I will develop a life-cycle model of trade, growth and demographic that is consistent with the main results in this section.

3. Model

In this section, I develope an OLG trade model that incorporates the empirical features identified in the empirical sections. The theoretical framework builds on Richard W. Evans (2022); Sposi (2022). I embody OLG features to the Eaton-Kortum trade model(Eaton and Kortum, 2002), which capture the impact of demographic structure on dynamic capital accumulation and interact effects with trade. I further augment model in two aspects: Firstly, I incorporate the identified empirical features into the model. Following the approach used by Lucas Jr (2009); Alvarez, Buera, Lucas, et al. (2013, 2008); Schlenker and Roberts (2009); Rudik, Lyn, Tan and Ortiz-Bobea (2023). the age distribution will be one of the elements changing TFP growth. I refer to this channel as "demographic-induced TFP growth" hereafter. Secondly, similar with the framework proposed by Sposi, Yi and Zhang (2021), in the proposed model, the production function incorporates labor, capital, and Total Factor Productivity (TFP), and allows for the existence of multiple sectors. By including both capital and labor in the production function and considering multiple sectors, it implicitly incorporates both the Heckscher-Ohlin and Ricardian comparative advantage forces into the model, and allows for these forces interact with demographics.

There is no uncertainty, and households have perfect foresight. The economy has N regions or countries, J sectors. Time is discrete and denoted by year t. In the notation below, country-specific parameters and variables have subscript n, cohort-specific variables have subscript g and the variables that vary over time have subscript t. By utilizing this model, I aim to investigate China's past growth and conduct model-based projections to assess the impacts of demographic and trade forces on China's future growth.

3.1. Households

Let the age of an individual be indexed by $g = \{1, 2, 3..., E + G\}$. Households or individuals are referred to as 'youth' and refrain from engaging in market activities during age $g \in [1, E]$. The individuals join the labor force and the economy in period $E + 1$ and continue their participation until they either experience untimely demise or reach the age $g = E + G$. Population dynamics are exogenous in the model. The measure $\eta_{g,t}$ is the number of households of age g alive at time t. The total population in the economy L_t at any period t is the sum of households in the economy. $\bar{L}_t$ is the working age population plus the retired population. I assume the individuals supply labor $l_g = 1$ inelastically during age $g \in [E + 1, E + G_0]$ and are retired with $l_g = 0.2$ during $g \in [E + G_0 + 1, E + G]$. $N_{n,t}$ are the total labor inputs.

$$L_{n,t} \equiv \sum_{g=1}^{E+G} \eta_{n,g,t} \quad \forall n,t \tag{4}$$

$$\bar{L}_{n,t} \equiv \sum_{g=E+1}^{E+G} \eta_{n,g,t} \quad \forall n,t \tag{5}$$

$$N_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g,t} l_g \quad \forall n,t \tag{6}$$

$$l_g = \begin{cases} 0 \ if \ g \in [1, E] \\ 1 \ if \ g \in [E+1, E+G_0] \\ 0 \ if \ g \in [E+G_0+1, E+G] \end{cases} \tag{7}$$

Individual lifetime consumption and saving

Define $c_{n,g,t+g-1}$ as the aggregate consumption of an age g individual at time $t+g-1$. β is the discount factor. Let the utility of consumption in each period be defined by the constant relative risk aversion function $\mathcal{U}(c) = \dfrac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$, such that $\mathcal{U}(\cdot)\prime > 0$, $\mathcal{U}(\cdot)\prime\prime < 0$, and $\lim_{c\to 0}\mathcal{U}(c) = -\infty$. Lifetime utility for cohort born at time t is defined as:

$$\max_{\{c_{n,g,t+g-1}\}_{g=E+1}^{E+G}\{b_{n,g+1,t+g}\}_{g=E+1}^{E+G-1}} \sum_{g=E+1}^{E+G} \beta^{g-E-1}\mathcal{U}\left(c_{n,g,t+g-1}\right) \tag{8}$$

Each period the individual inelastically supplies capital and labor to domestic firms. Individuals purchase consumption and investment goods from the domestic firms to maximize lifetime utility. Investment augments the stock of capital. An age g individual that was born in period t choose lifetime consumption $\{c_{n,g,t+g-1}\}_{g=E+1}^{E+G}$ and savings $\{b_{n,g+1,t+g}\}_{g=E+1}^{E+G-1}$ to maximize lifetime utility, subject to the budget constraints and non-negativity constraints:

$$P_{n,C,t}c_{n,g,t} + P_{n,I,t}i_{n,g,t} = R_{n,t}b_{n,g,t} + W_{n,t}l_g + \frac{TRS_{n,t}}{\bar{L}_{n,t}} + \frac{D_{n,t}}{\bar{L}_{n,t}} \quad For \ \forall g \geq E+1, t \tag{9}$$

$$b_{n,E+1,t} = b_{n,E+G+1,t} = 0 \quad For \ \forall \ t \tag{10}$$

$$c_{n,E+g,t} > 0 \quad For \ \forall g, t \tag{11}$$

where $b_{g+1,t+1} = i_{g,t} + (1-\delta)\, b_{g,t}$ So budget constraint Equation 9 can also write as:

$$P_{n,C,t}c_{n,g,t}+P_{n,I,t}b_{n,g+1,t+1} = \left(1+\frac{R_{n,t}}{P_{n,I,t}}-\delta\right)P_{n,I,t}b_{n,g,t}+W_{n,t}l_g+\frac{TRS_{n,t}}{\bar{L}_{n,t}}+\frac{D_{n,t}}{\bar{L}_{n,t}} \quad For\,\forall g \geq E+1, t \tag{12}$$

The right-hand side of equation accounts for income for individual at cohort g at time t, and is adjusted for the net population change and aggregate trade imbalances. Income accrues from capital and labor at the rates $R_{n,t}$ and $W_{n,t}$, respectively.

$TRS_{n,t}$represents total accidental bequests available in period t from households who died in period t-1. Here, for the size of Individual at cohort $(s-1,t-1)$ and (s,t), the change of the size of individual can either counted as net death $(\eta_{n,g-1,t-1} - \eta_{n,g,t} > 0)$ or net immigrant $(\eta_{n,g-1,t-1} - \eta_{n,g,t} < 0)$ for country n. For the net death case, I treated it as positive bequests. For the net immigrant case, I assume that the net immigrant (g,t) enter country n with zero assets, and treated it as negative bequests. Dividing by the total economically relevant population $\bar{L}_{n,t}$ implies that total bequests are equally distributed across the population. This assumption greatly simplifies the state vector of the model. Total bequests are characterized by the following equation:

$$TRS_{n,t} = P_{n,I,t}\left(1+\frac{R_{n,t}}{P_{n,I,t}}-\delta\right)\sum_{g=E+2}^{E+S}\left(\eta_{n,g-1,t-1}-\eta_{n,g,t}\right)b_{n,g,t} \qquad For\,\forall t \tag{13}$$

$D_{n,t}$ is defined as the net transfer, also recognized as trade deficit, Dividing by the total economically relevant population $\bar{L}_{n,t}$ implies that total net transfer are equally distributed across the population. Further illustration regarding the construction and definition of $D_{n,t}$ will be provided later.

Euler equation

For individual at age $g \in [E+1, E+G]$ born at time t, Given the sequences of prices, transfers and deficits, households optimize on the intertemporal decisions of aggregate consumption and investment $\{c_{n,g,t+g-1},\ b_{n,g+1,t+g}\}$. Aggregate consumption and investment choices are determined by an intertemporal Euler equation:

$$\mathcal{U}\prime\left(c_{n,g,t+g-1}\right) = \beta\left(1+\frac{R_{n,t+g}}{P_{n,I,t+g}}-\delta\right)\frac{\frac{P_{n,I,t+g}}{P_{n,C,t+g}}}{\frac{P_{n,I,t+g-1}}{P_{n,C,t+g-1}}}\mathcal{U}\prime\left(c_{n,g+1,t+g}\right) \quad For\,\forall\, g \in [E+1, E+G-1] \tag{14}$$

and constraint:

$$P_{n,C,t+g-1}c_{n,g,t+g-1} + P_{n,I,t+g-1}b_{n,g+1,t+g} = \left(1 + \frac{R_{n,t+g-1}}{P_{n,I,t+g-1}} - \delta\right) P_{n,I,t+g-1}b_{n,g,t+g-1} + W_{n,t+g-1}l_g + \frac{TRS_{n,t+g-1}}{\bar{L}_{n,t+g-1}} + \frac{D_{n,t+g-1}}{\bar{L}_{n,t+g-1}} \tag{15}$$

$$P_{n,C,t+g}c_{n,g+1,t+g} + P_{n,I,t+g}b_{n,g+2,t+g+1} = \left(1 + \frac{R_{n,t+g}}{P_{n,I,t+g}} - \delta\right) P_{n,I,t+g}b_{n,g+1,t+g} + W_{n,t+g}l_{g+1} + \frac{TRS_{n,t+g}}{\bar{L}_{n,t+g}} + \frac{D_{n,t+g}}{\bar{L}_{n,t+g}} \tag{16}$$

$$b_{E+1,t} = b_{E+G+1,t} = 0 \;\; and \;\; c_{n,E+g,t} > 0 \tag{17}$$

Specifically, For the second final cohort $g = E + G - 1$, the individual simply consumes all his resources during next period, and I have:

$$\mathcal{U}\prime(c_{n,E+G-1,t+E+G-2}) = \beta\left(1 + \frac{R_{n,t+E+G-1}}{P_{n,I,t+E+G-1}} - \delta\right)\frac{\frac{P_{n,I,t+E+G-1}}{P_{n,C,t+E+G-1}}}{\frac{P_{n,I,t+E+G-2}}{P_{n,C,t+E+G-2}}}\mathcal{U}\prime(c_{n,E+G,t+E+G-1}) \tag{18}$$

$$\begin{aligned} &P_{n,C,t+E+G-2}c_{n,E+G-1,t+E+G-2} + P_{n,I,t+E+G-2}b_{n,E+G,t+E+G-1} \\ &= \left(1 + \frac{R_{n,t+E+G-2}}{P_{n,I,t+E+G-2}} - \delta\right) P_{n,I,t+E+G-2}b_{n,E+G-1,t+E+G-2} \\ &+W_{n,t+E+G-2}l_{E+G-1} + \frac{TRS_{n,t+E+G-2}}{\bar{L}_{n,t+E+G-2}} + \frac{D_{n,t+E+G-2}}{\bar{L}_{n,t+E+G-2}} \end{aligned} \tag{19}$$

$$\begin{aligned} &P_{n,C,t+E+G-1}c_{n,E+G,t+E+G-1} + P_{n,I,t+E+G-1}b_{n,E+G+1,t+E+G} \\ &= \left(1 + \frac{R_{n,t+E+G-1}}{P_{n,I,t+E+G-1}} - \delta\right) P_{n,I,t+E+G-1}b_{n,E+G,t+E+G-1} \\ &+W_{n,t+E+G-1}l_{E+G} + \frac{TRS_{n,t+E+G-1}}{\bar{L}_{n,t+E+G-1}} + \frac{D_{n,t+E+G-1}}{\bar{L}_{n,t+E+G-1}} \end{aligned} \tag{20}$$

Since $b_{n,E+G+1,t+E+G} = 0$, I have 1 equation Equation 21 for 1 unknow $b_{n,E+G,t+E+G-1}$. So, I get get function $\psi_{n,E+G-1}$:

$$b_{n,E+G,t+E+G-1} = \psi_{n,E+G-1}\left(b_{n,E+G-1,t+E+G-2}, \{w_{t+u}, r_{t+u}\}_{u=E+G-2}^{E+G-1}, \{P_{n,I,t+u}, P_{n,C,t+u}\}_{u=E+G-2}^{E+G-1}\right) \tag{21}$$

Similarly, for $g = E + G - 2$, I have 2 equations Equation 21 and Equation 22 for 2 unknow

$b_{n,E+G,t+E+G-1}$ and $\mathrm{b}_{n,E+G-1,t+E+G-2}$:

$$
\begin{aligned}
& b_{n,E+G-1,t+E+G-2} \\
& = \varphi_{n,E+G-2} \left(b_{n,E+G-2,t+E+G-3}, b_{n,E+G,t+E+G-1}, \{w_{t+u}, r_{t+u}\}_{u=E+G-3}^{E+G-2}, \{P_{n,I,t+u}, P_{n,C,t+u}\}_{u=E+G-3}^{E+G-2} \right)
\end{aligned} \tag{22}
$$

So, I get get function $\psi_{n,E+G-2}$:

$$
b_{n,E+G-1,t+E+G-2} = \psi_{n,E+G-2} \left(b_{n,E+G-2,t+E+G-3}, \{w_{t+u}, r_{t+u}\}_{u=E+G-3}^{E+G-1}, \{P_{n,I,t+u}, P_{n,C,t+u}\}_{u=E+G-3}^{E+G-1} \right) \tag{23}
$$

So for $\forall t$ and $g \in [E+1, E+G-1]$, I have

$$
\begin{aligned}
& b_{n,g+1,t+g} \\
= & \varphi_{n,g} \left(b_{n,g,t+g-1}, \{b_{n,u,t+u-1}\}_{u=g-1}^{E+G-1}, \{w_{t+u}, r_{t+u}\}_{u=g-1}^{g}, \{P_{n,I,t+u}, P_{n,C,t+u}\}_{u=g-1}^{g} \right) \\
= & \psi_{n,g} \left(b_{n,g,t+g-1}, \{w_{t+u}, r_{t+u}\}_{u=g-1}^{E+G-1}, \{P_{n,I,t+u}, P_{n,C,t+u}\}_{u=g-1}^{E+G-1} \right)
\end{aligned} \tag{24}
$$

Structure of Consumption and investment

Define $C_{n,t}$ as aggregate consumption in country n and time t, which is a CES aggregate of sectoral consumption:

$$
C_{n,t} \equiv \prod_{j=1}^{J} {C_{n,t}^{j}}^{\alpha_{n,C,t}^{j}} \tag{25}
$$

where $\sum_{j=1}^{J} \alpha_{n,C,t}^{j} = 1$.

Similar relation for the aggregate investment and sectoral investment:

$$
I_{n,t} \equiv \prod_{j=1}^{J} {I_{n,t}^{j}}^{\alpha_{n,I,t}^{j}} \tag{26}
$$

where $\sum_{j=1}^{J} \alpha_{n,I,t}^{j} = 1$.

The overall aggregate price level is given by:

$$
P_{I,n,t} = \prod_{j=1}^{J} \left[\frac{P_{n,t}^{j}}{\alpha_{I,n}^{j}} \right]^{\alpha_{I,n}^{j}} \tag{27}
$$

$$
P_{C,n,t} = \prod_{j=1}^{J} \left[\frac{P_{n,t}^{j}}{\alpha_{C,n}^{j}} \right]^{\alpha_{C,n}^{j}} \tag{28}
$$

The structure of aggregate consumption and aggregate investment implies that:

$$P_{n,t}^{j} I_{n}^{j} = \alpha_{I,n}^{j} P_{I,n,t} I_{n,t} \tag{29}$$

$$P_{n,t}^{j} C_{n}^{j} = \alpha_{C,n}^{j} P_{C,n,t} C_{n,t} \tag{30}$$

or

$$\sum_{j=1}^{J} P_{n,t}^{j} C_{n,t}^{j} = P_{C,n,t} C_{n,t} \tag{31}$$

$$\sum_{j=1}^{J} P_{n,t}^{j} I_{n,t}^{j} = P_{I,n,t} I_{n,t} \tag{32}$$

Aggregation

Now, aggregate individual variable across cohorts, I have:

$$C_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g,t} c_{n,g,t} \quad For\ \forall\ t \tag{33}$$

$$I_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g,t} i_{n,g,t} \quad For\ \forall\ t \tag{34}$$

$$K_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g-1,t-1} b_{n,g,t} \quad For\ \forall\ t \tag{35}$$

Where $K_{n,t}$ is aggregate capital at time t.

The budget constraint at the aggregate level is:

$$P_{n,C,t} C_{n,t} + P_{n,I,t} I_{n,t} = R_{n,t} K_{n,t} + w_{n,t} N_{n,t} + D_{n,t} \quad For\ \forall\ t \tag{36}$$

$$K_{t+1} = I_t + (1-\delta) K_t \quad For\ \forall\ t \tag{37}$$

$$P_{n,C,t} C_{n,t} + P_{n,I,t} K_{n,t+1} = \left(1 + \frac{R_{n,t}}{P_{n,I,t}} - \delta\right) P_{n,I,t} K_{n,t} + W_{n,t} N_{n,t} + D_{n,t} \quad For\ \forall\ t \tag{38}$$

I abstract international borrowing and lending and model trade imbalances as transfers between countries, following Caliendo, Parro, Rossi-Hansberg, and Sarte (2018). A pre-determined share of GDP, $\phi_{n,t}$ is sent to a global portfolio, which in turn disperses a per-capita lump-sum transfer, T_t^P,to every country.[2] The net transfer, also recognized as trade

2. While the share of GDP allocated to the global portfolio is exogenous, the proceeds are endogenous to clear the global market. This feature is particularly useful in the counterfactual analysis.

deficit, are calculated as:

$$D_{n,t} = -\phi_{n,t}\left(R_{n,t}K_{n,t} + W_{n,t}N_{n,t}\right) + \bar{L}_{n,t}T_t^P \quad For\ \forall t \tag{39}$$

Dividing by the total economically relevant population $\bar{L}_{n,t}$ implies that total bequests are equally distributed across the population. The labor input in the country n is $N_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g,t} l_g$. The wage in the country n is $W_{n,t}$ and the consumers total income is IN_n, which is the sum of total value added of country n, population-induced transfer $TRS_{n,t}$ and trade deficit $D_{n,t}$:

$$IN_{n,t} \equiv R_{n,t}K_{n,t} + W_{n,t}N_{n,t} + D_{n,t} \tag{40}$$

3.2. Production

The model is consisted with N countries and each country has J sectors. The countries and sectors are linked by input-output linkages and firms in country N sector J produce a continuum of goods. Each variety within each sector is tradable and is indexed by $\omega \in [0,1]$, with CRS technology:

$$y_{n,t}^j(\omega) \equiv z_{n,t}^j(\omega)\left[l_{n,t}^j(\omega)^{\beta_n^j} k_{n,t}^j(\omega)^{1-\beta_n^j}\right]^{\gamma_n^j} \prod_{k=1}^{J} m_{n,t}^{k,j}(\omega)^{\gamma_n^{k,j}} \tag{41}$$

The CES production technology means $\sum_{k=1}^{N} \gamma_n^{k,j} + \gamma_n^j = 1$. The productivity of good ω in country n sector j is $z_{n,t}^j(\omega)$, which is the random variable follows a Fréchet distribution $F_{n,t}^j(z) = exp\left(-\lambda_{n,t}^j z^{-\theta^j}\right)$.

Productivity Growth

The distribution of productivity of market (n,j) at time t is the technology frontier, represented by C.D.F $G_n^j(z,t)$, where $G_n^j(z,t)$ follows a Fréchet distribution $G_n^j(z,t) = exp\left(-\lambda_{n,t}^j z^{-\theta}\right)$. The location parameter $\lambda_{n,t}^j > 0$ represents the level of time-varying fundamental productivity of market (n,j).

New ideas arrive at a deterministic rate $\alpha_{n,s,t}^j$ for people in country n at age cohort s, or the average rate (events per period) of new chance to draw an idea for people in country n at age cohort s is $\alpha_{n,s,t}^j$, and $\vec{\alpha}_{n,t}^j \equiv \{\alpha_{n,s,t}\}_s$. The age distribution for country n at time t is $\vec{\Omega}_{n,t} \equiv \{\Omega_{n,s,t}\}_s$, where $\Omega_{n,s,t}$ is share of cohort s people at time t in country n. For each country, there is a unit interval of population. Over the time $(t, t+h)$, people of different cohort s in country n draw ideas from the same distribution $G_n^j(z,t)$. Then, during the time t to $(t+h)$, total draw (aggregate deterministic arrival rate) would be $A_{n,t}^{j}=g\left(\vec{\Omega}_{n,t};\vec{\alpha}_{n,t}^{j}\right)$. Then, at time t+h the technology frontier will be

$$G_{n}^{j}\left(z,t+h\right)=G_{n}^{j}\left(z,t\right)\times Pr\left\{all\ \alpha h\ draws\leq z\right\}=G_{n}^{j}\left(z,t\right)\left(G_{n}^{j}\left(z,t\right)\right)^{A_{n,t}^{j}h} \tag{42}$$

Taking the limits as $h\to 0$ at time t, I have $\dfrac{\partial log(G_{n}^{j}\left(z,t\right))}{log\left(G_{n}^{j}\left(z,t\right)\right)\partial t}=\dfrac{d\lambda_{n,t}^{j}}{\lambda_{n,t}^{j}dt}=A_{n,t}^{j}$. So, aggregate deterministic arrival rate $A_{n,t}^{j}$ determines the growth rate of technology parameters at time t. I further assume that the fundamental productivity of market (n,j) grows at "demographic-induced" growth rate, but adjusted for errors $\vartheta_{n,t}^{j}$ which is irrelevant to demographic effects:

$$A_{n,t}^{j}=g\left(\vec{\Omega}_{n,t};\vec{\alpha}_{n,t}^{j}\right)+\vartheta_{n,t}^{j} \tag{43}$$

In the discrete case, I have

$$\frac{\lambda_{n,t+1}^{j}-\lambda_{n,t}^{j}}{\lambda_{n,t}^{j}}=A_{n,t}^{j}=g\left(\vec{\Omega}_{n,t};\vec{\alpha}_{n,t}^{j}\right)+\vartheta_{n,t}^{j} \tag{44}$$

or

$$\frac{\lambda_{n,t}^{j}}{\lambda_{n,t-1}^{j}}=1+A_{n,t-1}^{j}=1+g\left(\vec{\Omega}_{n,t-1};\vec{\alpha}_{n,t-1}^{j}\right)+\vartheta_{n,t-1}^{j} \tag{45}$$

where $\vec{\Omega}_{n,t}$ is a vector of age shares at different cohorts in region n in year t. $g\left(\vec{\Omega}_{n,t};\vec{\alpha}_{n,t}^{j}\right)$ is a flexible demographic response function with parameter vector $\vec{\alpha}_{n,t}^{j}$. Function g is linear, which I will estimate.

Composite goods

The non-tradable sectoral composite intermediate good $Q_{n,t}^{j}$ in country n sector j is consistent with tradable intermediate good $q_{n,t}^{j}\left(\omega\right)$:

$$Q_{n,t}^{j}\equiv\left[\int_{0}^{1}q_{n,t}^{j}\left(\omega\right)^{\frac{\sigma-1}{\sigma}}dj\right]^{\frac{\sigma}{\sigma-1}} \tag{46}$$

$$Q_{n}^{j}=I_{n}^{j}+C_{n}^{j}+\sum_{k=1}^{J}\int_{0}^{1}m_{n}^{j,k}\left(\omega\right)d\omega \tag{47}$$

where σ is the elasticity of substitution between varieties. The sectoral composite intermediate good Q_{n}^{j} has three uses. It can be used for final consumption in country n: $C_{n,t}\equiv\prod_{j=1}^{J}{C_{n,t}^{j}}^{\alpha_{n,C,t}^{j}}$. It can be used for investment: $I_{n,t}\equiv\prod_{j=1}^{J}{I_{n,t}^{j}}^{\alpha_{n,I,t}^{j}}$. Finally, it can be used as an intermediate in the production of individual goods ω in each sector k: $\int_0^1 m_n^{j,k}(\omega)\, d\omega$.

The price of sectoral composite intermediate good Q_n^j is P_n^j. Define total expenditures on good j by region n as $X_{n,t}^j$, and I have $P_{n,t}^j Q_{n,t}^j = X_{n,t}^j$.

Cost minimization under constant returns to scale implies that, within each sector, expenditure on factors and intermediate inputs exhaust the value of output:

$$W_{n,t}N_{n,t} = \sum_{j=1}^{J} \beta_n^j \gamma_n^j \sum_{i=1}^{N} \pi_{in,t}^j X_{i,t}^j \tag{48}$$

$$R_{n,t}K_{n,t} = \sum_{j=1}^{J} \left(1-\beta_n^j\right) \gamma_n^j \sum_{i=1}^{N} \pi_{in,t}^j X_{i,t}^j \tag{49}$$

$$W_{n,t}N_{n,t} + R_{n,t}K_{n,t} = \sum_{j=1}^{J} \gamma_n^j \sum_{i=1}^{N} \pi_{in,t}^j X_{i,t}^j \tag{50}$$

Total demand of capital and labor:

$$\sum_{j=1}^{J} \int_0^1 l_{n,t}^j(\omega) d\omega = N_{n,t} \tag{51}$$

$$\sum_{j=1}^{J} \int_0^1 k_{n,t}^j(\omega) d\omega = K_{n,t} \tag{52}$$

3.3. Trade

Trade is subject to "iceberg" trade costs. One unit of a tradable good in sector j shipped from region i to region n require $\kappa_{ni,t}^j \geq 1$ units in i, and the trade cost within region equal to 1, $\kappa_{nn,t}^j = 1$. The unit price of an input bundle is given by:

$$c_{n,t}^j \equiv \Upsilon_n^j \left[\left(W_{n,t}^j\right)^{\beta_n^j} \left(R_{n,t}^j\right)^{1-\beta_n^j} \right]^{\gamma_n^j} \prod_{k=1}^{J} {P_{n,t}^k}^{\gamma_n^{k,j}} \tag{53}$$

where $\Upsilon_n^j \equiv \gamma_n^j {\beta_n^j}^{-\gamma_n^j \beta_n^j} \gamma_n^j \left(1-\beta_n^j\right)^{-\gamma_n^j\left(1-\beta_n^j\right)} \prod_{k=1}^{J} {\gamma_n^{k,j}}^{-\gamma_n^{k,j}}$.

As in Eaton and Kortum (2002), the fraction of country n's expenditures in sector j goods source from country i is given by:

$$\pi_{ni,t}^j = \frac{\lambda_{i,t}^j \left(c_{i,t}^j \kappa_{ni,t}^j\right)^{-\theta}}{\sum_{m=1}^{N} \lambda_{m,t}^j \left(c_{m,t}^j \kappa_{nm,t}^j\right)^{-\theta}} \tag{54}$$

and

$$P_{n,t}^{j}=A^{j}\cdot\left[\sum_{i=1}^{N}\lambda_{i,t}^{j}\left(\kappa_{ni,t}^{j}c_{i,t}^{j}\right)^{-\theta}\right]^{-\frac{1}{\theta}} \tag{55}$$

$$\pi_{ni,t}^{j}=\lambda_{i,t}^{j}\left(\frac{A^{j}c_{i,t}^{j}\kappa_{ni,t}^{j}}{P_{n,t}^{j}}\right)^{-\theta} \tag{56}$$

where $A^{j}\equiv\Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$. The expenditure by region n of sector j goods from region i is defined as $X_{ni,t}^{j}$, with $X_{n,t}^{j}=\sum_{i=1}^{N}X_{ni,t}^{j}$. Total revenue of country n on sector j goods is defined as $Y_{n,t}^{j}=\sum_{i=1}^{N}X_{in,t}^{j}$. The expenditure share is defined as $\pi_{ni,t}^{j}=\frac{X_{ni,t}^{j}}{\sum_{i=1}^{N}X_{ni,t}^{j}}$. Each composite good is used as an intermediate and as final consumption, total expenditure on a composite good in sector j, region n is:

$$X_{n,t}^{j}=\alpha_{C,n}^{j}P_{C,n,t}C_{n,t}+\alpha_{I,n}^{j}P_{I,n,t}I_{n,t}+\sum_{k=1}^{J}\gamma_{n}^{j,k}\left(\sum_{i=1}^{N}X_{in,t}^{k}\right) \tag{57}$$

where $\alpha_{C,n}^{j}P_{C,n,t}C_{n,t}+\alpha_{I,n}^{j}P_{I,n,t}I_{n,t}$ is final demand for good j by workers in region n. Under the trade deficit D_n, trade balance condition is:

$$\sum_{j=1}^{J}\sum_{i=1}^{N}X_{in,t}^{j}-\sum_{j=1}^{J}\sum_{i=1}^{N}X_{ni,t}^{j}=NX_{n}=-D_{n,t} \tag{58}$$

and trade condition implies:

$$\sum_{n=1}^{N}\phi_{n,t}\left(R_{n,t}K_{n,t}+W_{n,t}N_{n,t}\right)=\sum_{n=1}^{N}\bar{L}_{n,t}T_{t}^{P} \tag{59}$$

$\Phi_{n,t}$ is a exogenous shock to the discount factor capturing the impact on investment dynamics of forces outside of the model: time varying demographics, capital taxes, and other distortions at the country level.

3.4. Equilibrium and equilibrium dynamics

3.4.1. Pseudo steady state equilibrium

Definition 1 (Pseudo Steady-state equilibrium): A non-autarkic steady-state competitive equilibrium in the perfect foresight overlapping generations model with $(E+G)$ period lived agents, and exogenous population dynamics, is defined as constant allocations of stationary consumption, capital and prices: $\left\{\{c^{\star}{}_{n,g}\}_{g=E+1,\ n=1}^{E+G,\ N},\ \{b^{\star}{}_{n,g+1}\}_{g=E+1,\ n=1}^{E+G-1,N},\ \{W^{\star}{}_{n},\ R^{\star}{}_{n}\}_{n=1}^{N}\right\}$, such that:

i. The households taking prices transfer and deficit as given, optimize lifetime utility.

ii. Firms taking prices as given, minimize production cost.

iii. Each country purchases intermediate varieties from the least costly supplier/country subject to the trade cost.

iv. All markets are clear.

v. The population distribution is not changed with time and defined as stationary steady-state distribution $\left\{\eta^{\star}{}_{n,g}\right\}_{g=1}^{E+G}$.

vi. In order to make this muti-sector OLG trade model exist steady state. Here, the growth rate of sectoral TFP in the steady state is assumed as zero, which implies that the demographic induced growth $g\left(\vec{\Omega}_{n,t};\vec{\alpha}_{n,t}^{j}\right)$ will be balanced out by orthogonal shocks $\vartheta_{n,t}^{j}$. I call this equilibrium as Pseudo steady state equilibrium.

I take world GDP as the numeraire $\sum_{n}\bar{R}_{n}K_{n}+\bar{W}_{n}N_{n}=1$. This means all prices are expressed in units of current world GDP. Table 8 provides a list of equilibrium conditions that these objects must satisfy.

$$\forall t \quad \frac{\lambda_{n,t+1}^{j}}{\lambda_{n,t}^{j}}=1+A_{n,t}^{j}=1+g\left(\vec{\Omega}_{n,t};\vec{\alpha}_{n,t}^{j}\right)+\vartheta_{n,t}^{j}=1$$

For $\forall g\in[E+2,\ E+G]$, and $\forall n\in[1,\ N]$:

$$\begin{aligned}&\mathcal{U}'\left[\left(1+\frac{R^{\star}{}_{n}\left(\Gamma^{\star}\right)}{P^{\star}{}_{n,I}}-\delta\right)\frac{P^{\star}{}_{n,I}}{P^{\star}{}_{n,C}}b^{\star}{}_{n,g}+\frac{W^{\star}{}_{n}\left(\Gamma^{\star}\right)l_{g}}{P^{\star}{}_{n,C}}+\frac{TRS^{\star}{}_{n}}{\bar{L}_{n}^{\star}P^{\star}{}_{n,C}}+\frac{D^{\star}{}_{n}}{\bar{L}_{n}^{\star}P^{\star}{}_{n,C}}-\frac{P^{\star}{}_{n,I}}{P^{\star}{}_{n,C}}b^{\star}{}_{n,g+1}\right]\\&=\beta\left(1+\frac{R^{\star}{}_{n}}{P^{\star}{}_{n,I}}-\delta\right)\mathcal{U}\prime\left(\left(1+\frac{R^{\star}{}_{n}\left(\Gamma^{\star}\right)}{P^{\star}{}_{n,I}}-\delta\right)\frac{P^{\star}{}_{n,I}}{P^{\star}{}_{n,C}}b^{\star}{}_{n,g+1}+\frac{W^{\star}{}_{n}\left(\Gamma^{\star}\right)l_{g+1}}{P^{\star}{}_{n,C}}+\frac{TRS^{\star}{}_{n}}{\bar{L}^{\star}{}_{n}P^{\star}{}_{n,C}}\right.\\&\left.+\frac{D^{\star}{}_{n}}{\bar{L}_{n}^{\star}P^{\star}{}_{n,C}}-\frac{P^{\star}{}_{n,I}}{P^{\star}{}_{n,C}}b^{\star}{}_{n,g+2}\right)\end{aligned} \tag{60}$$

These $N(G-1)$ equations are exactly identified in the steady state. That is, they are $(G-1)$ equations and $(G-1)$ unknowns for N countries. Define the distribution of capital for country n across ages as $\Gamma^{\star}=\{b^{\star}{}_{n,g+1}\}_{g=E+1,\ n=1}^{E+G-1,N}$. I can solve for steady-state by using an unconstrained optimization solver.

TABLE 8

STEADY-STATE CONDITIONS

H1	$L_n^\star \equiv \sum_{g=1}^{E+G} \eta_{n,g}^\star;\ \bar{L}_n^\star \equiv \sum_{g=E+1}^{E+G} \eta_{n,g}^\star;\ N_n^\star = \sum_{g=E+1}^{E+G} \eta_{n,g}^\star l_g$	$\forall(n)$
H2	$P_{n,C}^\star c_{n,g}^\star + P_{n,I}^\star i_{n,g}^\star = R_n^\star b_{n,g}^\star + W_n^\star l_g + \frac{TRS_n^\star}{\bar{L}_n^\star} + \frac{D_n^\star}{\bar{L}_n^\star}$ for $\forall g \geq E+1$	$\forall(n)$
H2'	$P_{n,C}^\star c_{n,g}^\star + P_{n,I}^\star b_{n,g+1}^\star = \left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right) P_{n,I}^\star b_{n,g}^\star + W_n^\star l_g + \frac{TRS_n^\star}{\bar{L}_n^\star} + \frac{D_n^\star}{\bar{L}_n^\star}$ for $\forall g \geq E+1$	$\forall(n)$
H3	$b_{g+1}^\star = i_g^\star + (1-\delta)\, b_g^\star,\ b_{n,E+1}^\star = b_{n,E+G+1}^\star = 0,\ c_{n,E+g}^\star > 0,\ \left\{c_{n,g}^\star\right\}_{g=E+1}^{E+G},\ \left\{b_{n,g+1}^\star\right\}_{g=E+1}^{E+G-1}$	$\forall(n)$
H4	$TRS_n^\star = P_{n,I}^\star \left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right) \sum_{g=E+2}^{E+S} \left(\eta_{n,g-1}^\star - \eta_{n,g}^\star\right) b_{n,g}^\star$	$\forall(n)$
H5	$\left(\frac{c_{n,g+1}^\star}{c_{n,g}^\star}\right)^{1/\sigma} = \beta \left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right) \frac{\frac{P_{n,I}^\star}{P_{n,C}^\star}}{\frac{P_{n,I}^\star}{P_{n,C}^\star}}$ for $\forall\ g \in [E+1, E+G-1]$	$\forall(n)$
H6	$C_n^\star = \sum_{g=E+1}^{E+G} \eta_{n,g}^\star c_{n,g}^\star;\ I_n^\star = \sum_{g=E+1}^{E+G} \eta_{n,g}^\star i_{n,g}^\star;\ K_n^\star = \sum_{g=E+1}^{E+G} \eta_{n-1,g-1}^\star b_{n,g}^\star$	$\forall(n)$
F1	$W_n^\star N_n^\star = \sum_{j=1}^{J} \beta_n^j \gamma_n^j \sum_{i=1}^{N} \pi_{in}^{\star\ j} X_i^{\star j}$	$\forall(n)$
F2	$R_n^\star K_n^\star = \sum_{j=1}^{J} \left(1 - \beta_n^j\right) \gamma_n^j \sum_{i=1}^{N} \pi_{in}^{\star\ j} X_i^{\star j}$	$\forall(n)$
F3	$X_n^{\star j} = \alpha_{C,n}^j P_{C,n}^\star C_n^\star + \alpha_{I,n}^j P_{I,n}^\star I_n^\star + \sum_{k=1}^{J} \gamma_n^{j,k} \left(\sum_{i=1}^{N} X_{in}^{\star\ k}\right)$	$\forall(n, j$
F4	$P_n^{\star j} I_n^{\star j} = \alpha_{I,n}^j P_{I,n}^\star I_n^\star;\ P_n^{\star j} C_n^{\star j} = \alpha_{C,n}^j P_{C,n}^\star C_n^\star$	$\forall(n, j)$
F5	$IN_n^\star \equiv R_n^\star K_n^\star + W_n^\star N_n^\star + D_n^\star = P_{C,n}^\star C_n^\star + P_{I,n}^\star I_n^\star$	$\forall(n)$
T1	$c_n^{\star j} \equiv \Upsilon_n^j \left[\left(W_n^{\star j}\right)^{\beta_n^j} \left(R_n^{\star j}\right)^{1-\beta_n^j}\right]^{\gamma_n^j} \prod_{k=1}^{J} P_n^{\star k \gamma_n^{k,j}}$ where $\Upsilon_n^j \equiv \gamma_n^j \beta_n^{j\ -\gamma_n^j \beta_n^j} \gamma_n^j \left(1 - \beta_n^j\right)^{-\gamma_n^j \left(1-\beta_n^j\right)} \prod_{k=1}^{J} \gamma_n^{k,j-\gamma_n^{k,j}}$	$\forall(n, j)$
T2	$P_n^{\star j} = A \cdot \left[\sum_{i=1}^{N} \lambda_i^{\star j} \left(\kappa_{ni}^{\star\ j} c_i^{\star j}\right)^{-\theta}\right]^{-\frac{1}{\theta}}$ where $A \equiv \Gamma \left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$	$\forall(n, j)$
T3	$\pi_{ni}^{\star\ j} = \frac{\lambda_i^{\star j} \left(c_i^{\star j} \kappa_{ni}^{\star\ j}\right)^{-\theta}}{\sum_{m=1}^{N} \lambda_m^{\star\ j} \left(c_m^{\star\ j} \kappa_{nm}^{\star\ j}\right)^{-\theta}} = \lambda_i^{\star j} \left(\frac{A^j c_i^{\star j} \kappa_{ni}^{\star\ j}}{P_n^{\star j}}\right)^{-\theta}$	$\forall(n, i, j)$
T4	$P_{n,C}^\star C_n^\star + P_{n,I}^\star I_n^\star = R_n^\star K_n^\star + W_n^\star N_n^\star + D_n^\star$	$\forall(n)$
T4'	$P_{n,C}^\star C_n^\star + P_{n,I}^\star K_n^\star = \left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right) P_{n,I}^\star K_n^\star + W_n^\star N_n^\star + D_n^\star$	$\forall(n)$
T5	$K_n^\star = I_n^\star + (1-\delta)\, K_n^\star$	$\forall(n)$
T6	$\sum_{j=1}^{J} \sum_{i=1}^{N} X_{in}^{\star\ j} - \sum_{j=1}^{J} \sum_{i=1}^{N} X_{ni}^{\star\ j} = NX_n^\star = -D_n^\star$	$\forall(n, j)$
T7	$D_n^\star = -\phi_n^\star \left(R_n^\star K_n^\star + W_n^\star N_n^\star\right) + \bar{L}_n^\star T^{\star P}$	$\forall(n)$
T8	$\sum_{n=1}^{N} \phi_n^\star \left(R_n^\star K_n^\star + W_n^\star N_n^\star\right) = \sum_{n=1}^{N} \bar{L}_n^\star T^{\star P}$	$\forall(n)$

3.4.2. Stationary Non-steady-state functional equilibrium

The world consists of n = 1,...N countries and time runs from t = t_0 to $t_1 = t_0 + T$. Each country is populated by $(E+G)$ period lived overlapping generations and individual join the economy in age $E+1$. There is no uncertainty and agents have perfect foresight.

$t_1 + G - 2$

The model economy is summarized by the time invariant parameters, time varying exogenous process of sectoral productivities and trade costs, the initial capital distribution, process of population dynamics and process controlling trade imbalances and discount factors.

Given a set of initial conditions (initial capital distribution) and exogenous forces across countries and over time, The transitional dynamics equilibrium (equilibrium transition path) consists of the following objects (prices and aggregate allocations):

Define Γ_t as the distribution of household savings across households at time t. $\forall t$, $\left\{R_{n,t}, W_{n,t}, P_{n,t}^j\right\}$ is a function of state variable $\Gamma_t \equiv \{b_{n,g+1}\}_{g=E+1,\ n=1}^{E+G-1,N}$ Let general beliefs about the future distribution of capital in period $t+u$ be characterized by the operator $\Omega(\cdot)$ such that: $\Gamma_{t+u}^E = \Omega^u(\Gamma_t)$, $\forall\ t, u \geq 1$, where the E superscript signifies that Γ_{t+u}^E is the expected distribution of wealth at time t+u based on general beliefs $\Omega(\cdot)$.

Definition 2 (Stationary Non-steady-state functional equilibrium): The transitional dynamics equilibrium in the perfect foresight overlapping generations trade model with E+G-period lived agents is defined as consistent sequences of capital distribution $\{b_{n,g+1,t+1} = \psi_{n,g}(\Gamma_t)\}_{g=E+1,n\in\mathrm{N}}^{E+G-1}$ and rental rates $R(\Gamma_t)$ and wage rates $W(\Gamma_t)$ satisfies the following conditions:

i.households have symmetric beliefs $\Omega(\cdot)$ about the evolution of the distribution of savings:

$$\Gamma_{t+u}^E = \Omega^u(\Gamma_t) \qquad \forall\ t, u \geq 1 \tag{61}$$

and those beliefs about the future distribution of savings equal the realized outcome (rational expectations):

$$\Gamma_{t+u} = \Gamma_{t+u}^E = \Omega^u(\Gamma_t) \qquad \forall\ t, u \geq 1 \tag{62}$$

ii. The households at different ages taking prices, transfer and deficit as given, optimize lifetime utility.

iii. Firms taking prices as given, minimize production cost.

iv. Each country purchases intermediate varieties from the least costly supplier/country subject to the trade cost.

v. All markets are clear.

vi. The population dynamics is exogenous.

At each point in time, I take world GDP as the numeraire $\sum_n R_{n,t}K_{n,t} + W_{n,t}N_{n,t} = 1, \forall t$. That means all prices are expressed in units of current world GDP. Table 9 provides a list of equilibrium conditions that these objects must satisfy.

TABLE 9

Dynamic equilibrium conditions

H1	$L_{n,t} \equiv \sum_{g=1}^{E+G} \eta_{n,g,t}$; $\bar{L}_{n,t} \equiv \sum_{g=E+1}^{E+G} \eta_{n,g,t}$; $N_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g,t} l_g$	$\forall (n,t)$
H2	$P_{n,C,t} c_{n,g,t} + P_{n,I,t} i_{n,g,t} = R_{n,t} b_{n,g,t} + W_{n,t} l_g + \frac{TRS_{n,t}}{\bar{L}_{n,t}} + \frac{D_{n,t}}{\bar{L}_{n,t}}$ for $\forall g \geq E+1$	$\forall (n,t)$
H2'	$P_{n,C,t} c_{n,g,t} + P_{n,I,t} b_{n,g+1,t+1} = \left(1 + \frac{R_{n,t}}{P_{n,I,t}} - \delta\right) P_{n,I,t} b_{n,g,t} + W_{n,t} l_g + \frac{TRS_{n,t}}{\bar{L}_{n,t}} + \frac{D_{n,t}}{\bar{L}_{n,t}}$ for $\forall g \geq E+1$	$\forall (n,t)$
H3	$b_{g+1,t+1} = i_{g,t} + (1-\delta) b_{g,t}$, $b_{n,E+1,t} = b_{n,E+G+1,t} = 0$, $c_{n,E+g,t} > 0$, $\{c_{n,g,t+g-1}\}_{g=E+1}^{E+G}$, $\{b_{n,g+1,t+g}\}_{g=E+1}^{E+G-1}$	$\forall (n,t)$
H4	$TRS_{n,t} = P_{n,I,t} \left(1 + \frac{R_{n,t}}{P_{n,I,t}} - \delta\right) \sum_{g=E+2}^{E+S} \left(\eta_{n,g-1,t-1} - \eta_{n,g,t}\right) b_{n,g,t}$	$\forall (n,t)$
H5	$\left(\frac{c_{n,g+1,t+g}}{c_{n,g,t+g-1}}\right)^{1/\sigma} = \beta \left(1 + \frac{R_{n,t+g}}{P_{n,I,t+g}} - \delta\right) \frac{\frac{P_{n,I,t+g}}{P_{n,C,t+g}}}{\frac{P_{n,I,t+g-1}}{P_{n,C,t+g-1}}}$ for $\forall\ g \in [E+1, E+G-1]$	$\forall (n,t)$
H6	$C_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g,t} c_{n,g,t}$; $I_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g,t} i_{n,g,t}$; $K_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g-1,t-1} b_{n,g,t}$	$\forall (n,t)$
F1	$W_{n,t} N_{n,t} = \sum_{j=1}^{J} \beta^j_n \gamma^j_n \sum_{i=1}^{N} \pi^j_{in,t} X^j_{i,t}$	$\forall (n,t)$
F2	$R_{n,t} K_{n,t} = \sum_{j=1}^{J} \left(1 - \beta^j_n\right) \gamma^j_n \sum_{i=1}^{N} \pi^j_{in,t} X^j_{i,t}$	$\forall (n,t)$
F3	$X^j_{n,t} = \alpha^j_{C,n} P_{C,n,t} C_{n,t} + \alpha^j_{I,n} P_{I,n,t} I_{n,t} + \sum_{k=1}^{J} \gamma^{j,k}_n \left(\sum_{i=1}^{N} X^k_{in,t}\right)$	$\forall (n,j,t)$
F4	$P^j_{n,t} I^j_n = \alpha^j_{I,n} P_{I,n,t} I_{n,t}$; $P^j_{n,t} C^j_n = \alpha^j_{C,n} P_{C,n,t} C_{n,t}$	$\forall (n,j,t)$
F5	$IN_{n,t} \equiv R_{n,t} K_{n,t} + W_{n,t} N_{n,t} + D_{n,t} = P_{C,n,t} C_{n,t} + P_{I,n,t} I_{n,t}$	$\forall (n,t)$
T1	$c^j_{n,t} \equiv \Upsilon^j_n \left[\left(W^j_{n,t}\right)^{\beta^j_n} \left(R^j_{n,t}\right)^{1-\beta^j_n}\right]^{\gamma^j_n} \prod_{k=1}^{J} P^k_{n,t}{}^{\gamma^{k,j}_n}$ where $\Upsilon^j_n \equiv \gamma^j_n \beta^j_n{}^{-\gamma^j_n \beta^j_n} \gamma^j_n \left(1 - \beta^j_n\right)^{-\gamma^j_n\left(1-\beta^j_n\right)} \prod_{k=1}^{J} \gamma^{k,j-\gamma^{k,j}_n}_n$	$\forall (n,j,t)$
T2	$P^j_{n,t} = A \cdot \left[\sum_{i=1}^{N} \lambda^j_{i,t} \left(\kappa^j_{ni,t} c^j_{i,t}\right)^{-\theta}\right]^{-\frac{1}{\theta}}$ where $A \equiv \Gamma \left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$	$\forall (n,j,t)$
T3	$\pi^j_{ni,t} = \frac{\lambda^j_{i,t} \left(c^j_{i,t} \kappa^j_{ni,t}\right)^{-\theta}}{\sum_{m=1}^{N} \lambda^j_{m,t} \left(c^j_{m,t} \kappa^j_{nm,t}\right)^{-\theta}} = \lambda^j_{i,t} \left(\frac{A^j c^j_{i,t} \kappa^j_{ni,t}}{P^j_{n,t}}\right)^{-\theta}$	$\forall (n,i,j,t)$
T4	$P_{n,C,t} C_{n,t} + P_{n,I,t} I_{n,t} = R_{n,t} K_{n,t} + W_{n,t} N_{n,t} + D_{n,t}$	$\forall (n,t)$
T4'	$P_{n,C,t} C_{n,t} + P_{n,I,t} K_{n,t+1} = \left(1 + \frac{R_{n,t}}{P_{n,I,t}} - \delta\right) P_{n,I,t} K_{n,t} + W_{n,t} N_{n,t} + D_{n,t}$	$\forall (n,t)$
T5	$K_{n,t+1} = I_{n,t} + (1-\delta) K_{n,t}$	$\forall (n,t)$
T6	$\sum_{j=1}^{J} \sum_{i=1}^{N} X^j_{in,t} - \sum_{j=1}^{J} \sum_{i=1}^{N} X^j_{ni,t} = NX_{n,t} = -D_{n,t}$	$\forall (n,j,t)$
T7	$D_{n,t} = -\phi_{n,t} \left(R_{n,t} K_{n,t} + W_{n,t} N_{n,t}\right) + \bar{L}_{n,t} T^P_t$	$\forall (n,t)$
T8	$\sum_{n=1}^{N} \phi_{n,t} \left(R_{n,t} K_{n,t} + W_{n,t} N_{n,t}\right) = \sum_{n=1}^{N} \bar{L}_{n,t} T^P_t$	$\forall (n,t)$

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Appendix A: Data Description

A.1. Data

I used panel data for 76 countries at different income levels, covering the period from 1975 to 2019. The selection of country groups and periods for analysis was based on the availability of data. The data were constructed from various sources, including the United Nations, World Population Prospects report, World Development Indicators database, Penn World Table 10.01, and CEPII database. The specifics of the data sources are listed in Table A.2.

TABLE A.1
Country Group

Income Group	Countries	Num.
High income	Australia, Austria, Belgium, Barbados, Canada, Switzerland, Chile, Cyprus, Germany, Denmark, Spain, Finland, France, United Kingdom, China (Hong Kong SAR), Greece, Ireland, Iceland, Israel, Italy, Japan, Republic of Korea, Luxembourg, Malta, Netherlands, Norway, New Zealand, Panama, Portugal, Romania, Singapore, Sweden, Uruguay, United States of America	34
Upper middle income	Brazil, Botswana, China, Colombia, Costa Rica, Dominican Republic, Jamaica, Jordan, Mexico, Mauritius, Malaysia, Namibia, Peru, Paraguay, Thailand, Ecuador, Gabon, Guatemala, Türkiye, South Africa, Venezuela (Bolivarian Republic of)	21
Lower middle income	Bolivia (Plurinational State of), Côte d'Ivoire, Cameroon, Egypt, Indonesia, India, Iran (Islamic Republic of), Kenya, Sri Lanka, Morocco, Nigeria, Philippines, Senegal, Tunisia, Tanzania, Zimbabwe	16
Lower income	Burkina Faso, Mozambique, Niger, Rwanda, Zambia	5

Notes: According to United Nations list of countries classified by income level

TABLE A.2

Definitions of Variables and Data Sources.

Variables	Description and Construction	Source
Population at every 5 year cohorts	Population at every 5 year cohorts: $[0,4],[5,9],\ldots,[90,94],[95,99]$ and $[100,+]$, 1975-2019	UN, World Population Prospects, 2022 version.
Real GDP per capita	Real GDP per capita at constant 2015 prices (in US \$), 1975-2019	World Development Indicators
Gross capital formation (% of GDP)	Gross capital formation (% of GDP), 1975-2019	World Development Indicators
Final consumption expenditure (% of GDP)	Final consumption expenditure (% of GDP), 1975-2019	World Development Indicators
Capital Stock	Capital Stock at constant 2017 prices (in millions US \$), 1975-2019	Penn World Table 10.01
Population	Total population (in millions US \$), 1975-2019	Penn World Table 10.01
Total factor productivity (TFP)	Total factor productivity (TFP), 1975-2019	Penn World Table 10.01
Gross Domestic Products	Destination and origin country GDP at current prices (in thousands US \$), 1975-2019	CEPII
Trade flow	Trade flows as reported by the destination at current prices (in thousands US \$), 1975-2019	CEPII

TABLE A.3

Decriptive statistics: 1975 - 2019, 76 countries

VARIABLES	N	mean	sd	between.sd	within.sd	min	max	skewness	kurtosis
Age [0-9] share	3,420	0.208	0.0826	0.07698	0.03123	0.0718	0.383	0.276	1.720
Age [10-19] share	3,420	0.185	0.0470	0.04144	0.02263	0.0746	0.287	-0.263	1.797
Age [20-29] share	3,420	0.162	0.0212	0.01525	0.01490	0.0966	0.236	-0.174	3.267
Age [30-39] share	3,420	0.135	0.0214	0.01521	0.01511	0.0765	0.217	0.176	3.030
Age [40-49] share	3,420	0.109	0.0300	0.02513	0.01665	0.0542	0.193	0.184	2.008
Age [50-59] share	3,420	0.0849	0.0324	0.02879	0.01531	0.0300	0.170	0.360	1.940
Age [60-69] share	3,420	0.0612	0.0299	0.02807	0.01083	0.0169	0.145	0.500	1.997
Age [70-79] share	3,420	0.0376	0.0241	0.02312	0.00742	0.00726	0.128	0.675	2.265
Age [80-89] share	3,420	0.0149	0.0129	0.01169	0.00560	0.000645	0.0740	1.076	3.435
Age [90+] share	3,420	0.00212	0.00246	0.00198	0.00148	2.07e-06	0.0209	1.881	7.724
Child share	3,420	0.303	0.109	0.10113	0.04205	0.116	0.523	0.155	1.655
Working age share	3,420	0.615	0.0661	0.05784	0.03260	0.457	0.785	-0.366	2.103
Elderly share	3,420	0.0824	0.0532	0.05046	0.01787	0.0167	0.293	0.727	2.447
Dependence ratio	3,420	0.647	0.188	0.16511	0.09093	0.273	1.189	0.672	2.363
Young dependence ratio	3,420	0.518	0.242	0.22284	0.09830	0.158	1.144	0.485	1.947
Old dependence ratio	3,420	0.129	0.0773	0.07303	0.02675	0.0314	0.499	0.880	3.010
TFP growth (%)	3,116	-0.0566	2.211	0.73687	2.08662	-19.36	20.61	-0.620	14.92
Final consumption (% of GDP)	3,285	77.31	11.08	9.53507	5.83873	11.61	148.5	-0.350	5.486
Gross capital formation (% of GDP)	3,284	24.14	7.148	5.03478	5.23456	1.525	89.38	1.437	9.718
Trade cost	3,286	3.181	0.940	0.86371	0.42245	1.174	10.49	1.550	8.612
(K/L) growth (%)	3,116	2.478	2.744	1.93121	1.96096	-5.326	14.27	0.716	4.643
log(K/L)	3,420	10.58	1.417	1.36937	0.39737	5.859	12.87	-0.546	2.613
GDP per capita at constant 2015 price	3,266	15,711	18,828	18,136.57	6,168.70	165.9	112,418	1.715	6.261

Appendix B: Figures and Tables

TABLE B.1

The effect of demographic structure on technology change

VARIABLES	Average TFP (%) growth rate in the future 4 years			
Initial.ln.RGDP.p.c	-2.93***	-3.22***	-3.09***	-3.46***
	(-4.61)	(-4.62)	(-4.82)	(-4.77)
Dep.Ratio	-3.31***			
	(-2.86)			
Child.Dep.R		-4.64***		
		(-3.40)		
Old.Dep.R		5.83		
		(1.30)		
Work.Share			11.43***	
			(3.33)	
Child.Share				-13.98***
				(-3.68)
Old.Share				2.79
				(0.39)
Constant	28.56***	30.82***	20.96***	35.46***
	(4.94)	(4.91)	(3.65)	(5.19)
Observations	732	732	732	732
R-squared	0.254	0.262	0.259	0.266
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Dep.Ratio* represents the dependency ratio, which is defined as the ratio of people aged (0-14) and (65, +) to people aged 15-64. The variable *Child.Dep.R* represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable *Old.Dep.R* represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable *Child.Share* represents the young population share, which is defined as the share of people aged 0-14, and the variable *Old.Share* represents old population share, which is defined as the share of people aged 65 and above.

TABLE B.2

The effect of demographic cohort structure on technology change

VARIABLES	Average TFP (%) growth rate in the future 4 years		
Initial.ln.RGDP.p.c	-3.46***	-3.51***	-3.51***
	(-4.77)	(-4.49)	(-4.55)
(0-14)/ToT.	21.48***		
	(3.61)		
(15-64)/ToT.	35.46***		
	(5.19)		
(65+)/ToT.	38.25***		
	(3.42)		
(0-24)/ToT.		26.22***	
		(4.24)	
(25-49)/ToT.		34.48***	
		(4.28)	
(50-74)/ToT.		43.60***	
		(4.41)	
(75+)/ToT.		13.47	
		(0.90)	
(0-19)/ToT.			25.36***
			(4.08)
(20-39)/ToT.			31.80***
			(4.35)
(40-59)/ToT.			34.74***
			(3.46)
(60-79)/ToT.			55.17***
			(5.35)
(80+)/ToT.			-21.89
			(-1.08)
Observations	732	732	732
R-squared	0.266	0.263	0.272
Time FE	YES	YES	YES
Country FE	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

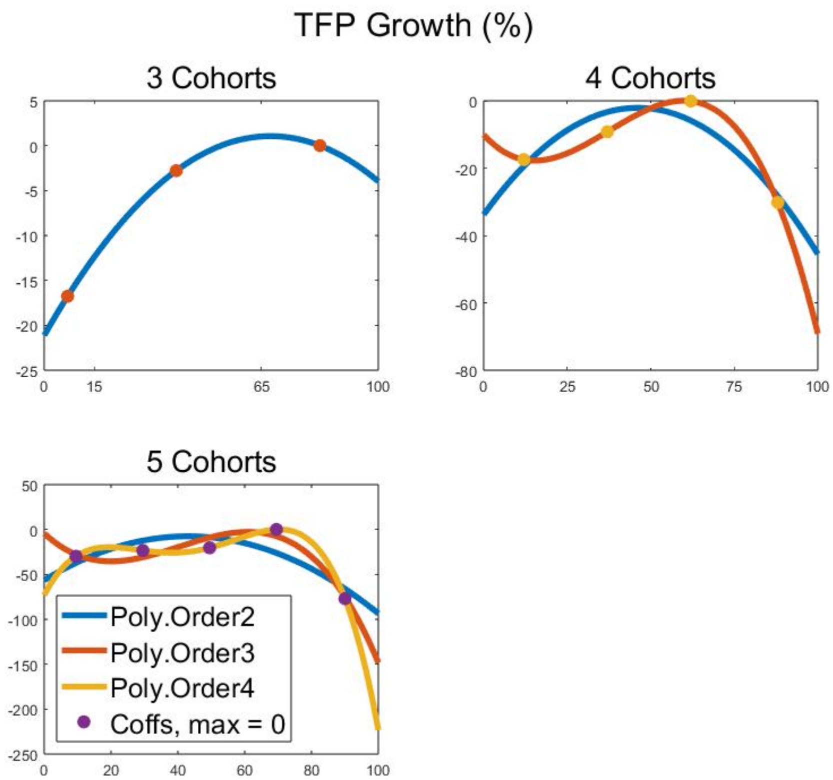


Figure B.1
Demographic cohort structure and TFP growth (%)

TABLE B.3

THE EFFECT OF DEMOGRAPHIC STRUCTURE ON CAPITAL FORMATION, AND CONSUMPTION

VARIABLES	Average value (% GDP) in the future 4 years			
	Gross.Cap.Formation		Gross.Consumption	
Dep.Ratio	-10.20**		9.93*	
	(-2.13)		(1.68)	
Child.Dep.R		-7.93		9.88
		(-1.37)		(1.61)
Old.Dep.R		-28.54*		10.38
		(-1.73)		(0.45)
Constant	28.24***	29.06***	71.74***	71.72***
	(7.87)	(9.10)	(17.11)	(16.16)
Observations	724	724	725	725
R-squared	0.575	0.579	0.751	0.751
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Dep.Ratio* represents the dependency ratio, which is defined as the ratio of people aged (0-14) and (65, +) to people aged 15-64. The variable *Child.Dep.R* represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable *Old.Dep.R* represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64.

TABLE B.4

THE EFFECT OF DEMOGRAPHIC STRUCTURE ON CAPITAL FORMATION, AND CONSUMPTION

VARIABLES	Average value (% GDP) in the future 4 years			
	Gross.Cap.Formation		Gross.Consumption	
Work.Share	28.80**		-33.75**	
	(2.17)		(-2.00)	
Child.Share		-24.76*		33.75*
		(-1.74)		(1.99)
Old.Share		-65.97***		33.77
		(-2.65)		(0.90)
Constant	4.09	34.10***	98.56***	64.81***
	(0.53)	(6.74)	(9.84)	(9.15)
Observations	724	724	725	725
R-squared	0.575	0.581	0.753	0.753
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Work.Share* represents the working age share, which is defined as the share of people aged 15-64. The variable *Child.Share* represents the young population share, which is defined as the share of people aged 0-14, and the variable *Old.Share* represents old population share, which is defined as the share of people aged 65 and above.

TABLE B.5

The effect of demographic cohort structure on capital formation, and consumption

	Average value (% GDP) in the future 4 years					
VARIABLES	Gross.Cap.Formation			Gross.Consumption		
(0-14)/ToT.	9.34			98.55***		
	(0.98)			(9.21)		
(15-64)/ToT.	34.10***			64.81***		
	(6.74)			(9.15)		
(65+)/ToT.	-31.87			98.58***		
	(-1.30)			(2.95)		
(0-24)/ToT.		16.69***			92.44***	
		(2.64)			(14.84)	
(25-49)/ToT.		29.11***			60.55***	
		(4.14)			(5.58)	
(50-74)/ToT.		37.83**			59.95***	
		(2.05)			(3.23)	
(75+)/ToT.		-124.60***			150.74***	
		(-2.77)			(3.21)	
(0-19)/ToT.			15.40**			92.98***
			(2.32)			(12.62)
(20-39)/ToT.			26.71**			71.18***
			(2.52)			(5.39)
(40-59)/ToT.			20.39			43.58*
			(1.13)			(1.85)
(60-79)/ToT.			53.93**			100.97**
			(2.37)			(2.47)
(80+)/ToT.			-224.74***			126.47*
			(-3.07)			(1.75)
Observations	724	724	724	725	725	725
R-squared	0.971	0.972	0.972	0.996	0.996	0.996
Time FE	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

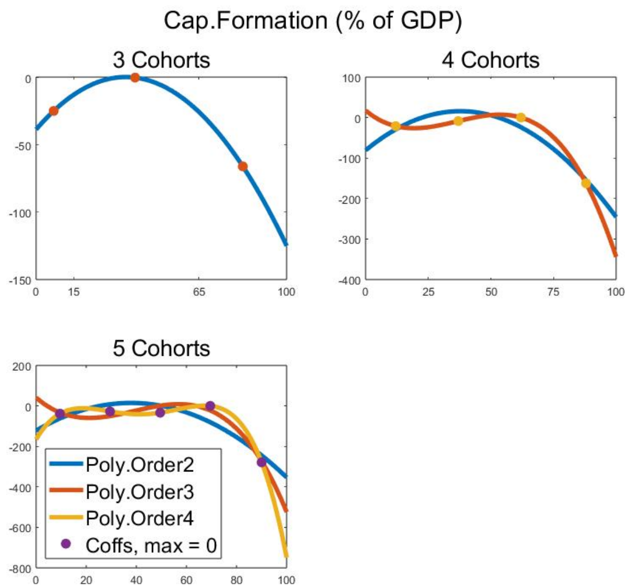


Figure B.2
Demographic cohort structure and capital formation (% of GDP)

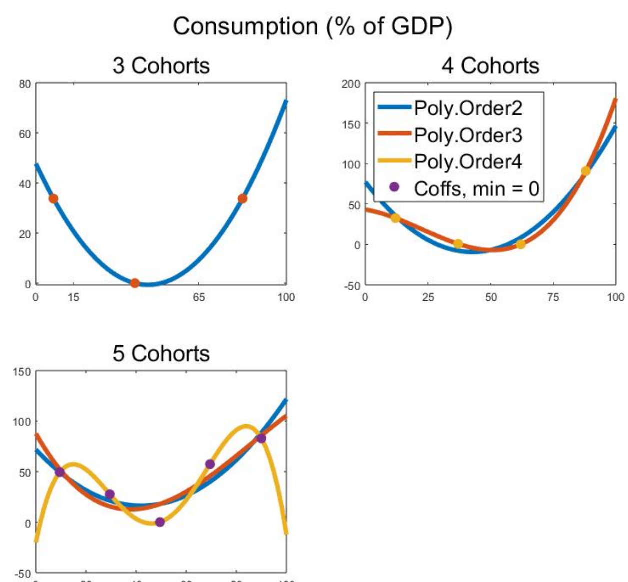


Figure B.3
Demographic cohort structure and consumption (% of GDP)

TABLE B.6

The effect of demographic structure and trade cost change on capital-labor ratio change

VARIABLES	Average K/L (%) growth rate in the future 4 years			
Trade Cost	-0.83**	-0.82**	-0.87**	-0.83**
	(-2.13)	(-2.06)	(-2.27)	(-2.13)
Dep.Ratio	-4.39**			
	(-2.22)			
Child.Dep.R		-3.25		
		(-1.27)		
Old.Dep.R		-11.37		
		(-1.66)		
Work.Share			13.34**	
			(2.49)	
Child.Share				-11.22*
				(-1.82)
Old.Share				-24.65**
				(-2.38)
Initial.ln.K/L	-2.13***	-1.93***	-2.24***	-1.99***
	(-3.90)	(-3.41)	(-4.12)	(-3.45)
PoP.Growth	-28.85	-34.06*	-28.93	-33.14*
	(-1.53)	(-1.90)	(-1.55)	(-1.84)
Constant	31.86***	29.96***	22.19***	32.98***
	(5.77)	(5.31)	(3.63)	(5.32)
Observations	758	758	758	758
R-squared	0.585	0.588	0.586	0.589
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

”

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Dep.Ratio* represents the dependency ratio, which is defined as the ratio of people aged (0-14) and (65, +) to people aged 15-64. The variable *Child.Dep.R* represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable *Old.Dep.R* represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable *Child.Share* represents the young population share, which is defined as the share of people aged 0-14, and the variable *Old.Share* represents old population share, which is defined as the share of people aged 65 and above.

”

TABLE B.7

The effects of demographic cohort structure and trade cost change on capital-labor ratio change

VARIABLES	Average K/L (%) growth rate in the future 4 years		
Trade Cost	-0.83**	-0.83**	-0.79**
	(-2.13)	(-2.11)	(-2.00)
(0-14)/ToT.	21.77***		
	(3.69)		
(15-64)/ToT.	32.98***		
	(5.32)		
(65+)/ToT.	8.34		
	(0.61)		
(0-24)/ToT.		24.08***	
		(4.50)	
(25-49)/ToT.		39.21***	
		(5.36)	
(50-74)/ToT.		19.18	
		(1.66)	
(75+)/ToT.		4.22	
		(0.24)	
(0-19)/ToT.			22.11***
			(4.02)
(20-39)/ToT.			36.72***
			(5.30)
(40-59)/ToT.			27.00***
			(2.98)
(60-79)/ToT.			21.25
			(1.41)
(80+)/ToT.			-9.87
			(-0.33)
Initial.ln.K/L	-1.99***	-1.98***	-1.93***
	(-3.45)	(-3.21)	(-3.14)
PoP.Growth	-33.14*	-35.31**	-30.58
	(-1.84)	(-2.08)	(-1.64)
Observations	758	758	758
R-squared	0.785	0.787	0.787
Time FE	YES	YES	YES
Country FE	YES	YES	YES

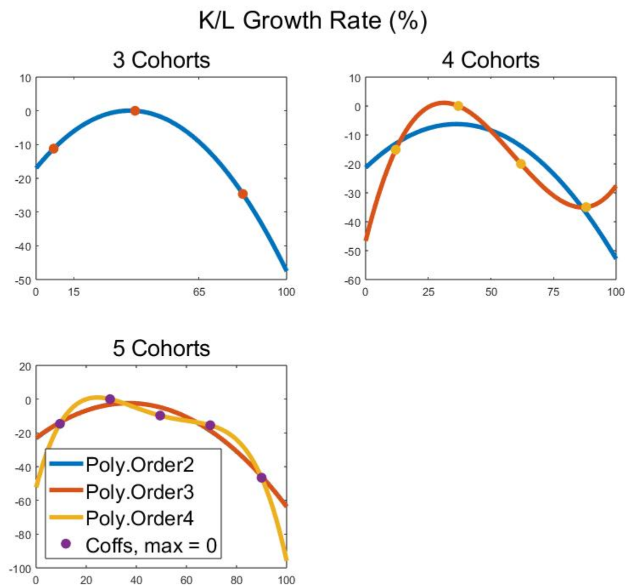


Figure B.4
Demographic cohort structure and Capital-labor ratio

Appendix C: Robust Checks

For the purpose of conducting robust checks, I performed the same regression analysis by dividing the entire period of 1980–2019 into five 8-year sub-periods: period 1 (1980–1987), period 2 (1988–1995), period 3 (1996–2003), period 4 (2004–2011), and period 5 (2012–2019).

C.1. Demographics, technology change, and macroeconomic outcomes

C.1.I. The effect of demographic structure on technology change

TABLE C.1

The effect of demographic structure on technology change

VARIABLES	Average TFP (%) growth rate in the future 7 years			
Initial.ln.RGDP.p.c	-2.78***	-2.92***	-2.93***	-3.10***
	(-4.32)	(-4.17)	(-4.55)	(-4.28)
Dep.Ratio	-2.11*			
	(-1.88)			
Child.Dep.R		-2.70**		
		(-2.08)		
Old.Dep.R		2.45		
		(0.55)		
Work.Share			8.31***	
			(2.76)	
Child.Share				-9.41***
				(-2.80)
Old.Share				-1.02
				(-0.14)
Constant	25.75***	26.77***	20.69***	30.42***
	(4.44)	(4.30)	(3.53)	(4.56)
Observations	439	439	439	439
R-squared	0.361	0.364	0.367	0.370
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Dep.Ratio* represents the dependency ratio, which is defined as the ratio of people aged (0-14) and (65, +) to people aged 15-64. The variable *Child.Dep.R* represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable *Old.Dep.R* represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable *Child.Share* represents the young population share, which is defined as the share of people aged 0-14, and the variable *Old.Share* represents old population share, which is defined as the share of people aged 65 and above.

TABLE C.2

The effect of demographic structure on technology change

VARIABLES	Average TFP (%) growth rate in the future 7 years		
Initial.ln.RGDP.p.c	-3.10***	-3.15***	-3.19***
	(-4.28)	(-3.98)	(-4.05)
(0-14)/ToT.	21.01***		
	(3.48)		
(15-64)/ToT.	30.42***		
	(4.56)		
(65+)/ToT.	29.40**		
	(2.56)		
(0-24)/ToT.		24.19***	
		(3.82)	
(25-49)/ToT.		30.46***	
		(3.94)	
(50-74)/ToT.		34.35***	
		(3.32)	
(75+)/ToT.		14.89	
		(1.05)	
(0-19)/ToT.			24.13***
			(3.75)
(20-39)/ToT.			27.20***
			(3.80)
(40-59)/ToT.			31.92***
			(3.27)
(60-79)/ToT.			43.14***
			(3.86)
(80+)/ToT.			-18.13
			(-0.84)
Observations	439	439	439
R-squared	0.372	0.369	0.378
Time FE	YES	YES	YES
Country FE	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

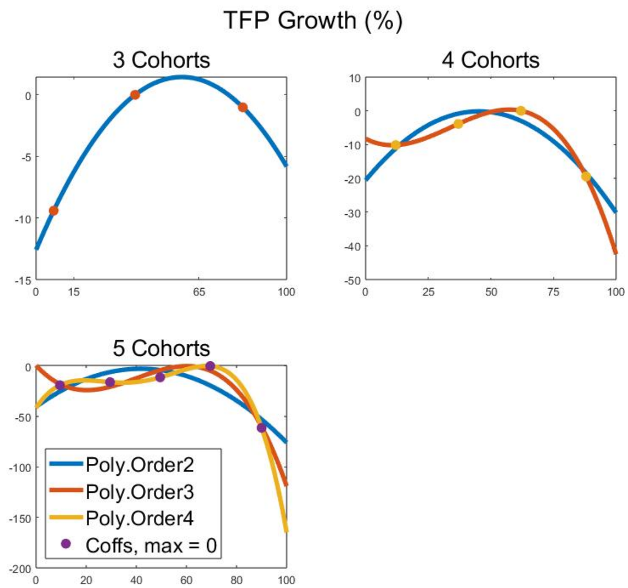


Figure C.1
Demographic cohort structure and TFP growth (%)

C.1.II. The effect of demographic structure on other macroeconomic variables

TABLE C.3
The effect of demographic structure on capital formation, and consumption

VARIABLES	Average value (% GDP) in the future 7 years			
	Gross.Cap.Formation		Gross.Consumption	
Dep.Ratio	-9.79*		7.63	
	(-1.91)		(1.26)	
Child.Dep.R		-7.61		7.66
		(-1.22)		(1.26)
Old.Dep.R		-29.93		7.33
		(-1.62)		(0.28)
Constant	28.48***	29.65***	73.35***	73.37***
	(7.44)	(8.83)	(17.00)	(15.32)
Observations	431	431	432	432
R-squared	0.627	0.631	0.792	0.792
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Dep.Ratio* represents the dependency ratio, which is defined as the ratio of people aged (0-14) and (65, +) to people aged 15-64. The variable *Child.Dep.R* represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable *Old.Dep.R* represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64.

TABLE C.4

THE EFFECT OF DEMOGRAPHIC STRUCTURE ON CAPITAL FORMATION, AND CONSUMPTION

VARIABLES	Average value (% GDP) in the future 7 years			
	Gross.Cap.Formation		Gross.Consumption	
Work.Share	27.58*		-27.48	
	(1.91)		(-1.61)	
Child.Share		-24.21		27.63
		(-1.57)		(1.63)
Old.Share		-66.88**		25.77
		(-2.47)		(0.62)
Constant	5.36	34.61***	94.86***	67.45***
	(0.64)	(6.32)	(9.42)	(9.08)
Observations	431	431	432	432
R-squared	0.627	0.633	0.794	0.794
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable $Child.Share$ represents the young population share, which is defined as the share of people aged 0-14, and the variable $Old.Share$ represents old population share, which is defined as the share of people aged 65 and above.

TABLE C.5

THE EFFECT OF DEMOGRAPHIC COHORT STRUCTURE ON CAPITAL FORMATION, AND CONSUMPTION

	Average value (% GDP) in the future 7 years					
VARIABLES	Gross.Cap.Formation			Gross.Consumption		
(0-14)/ToT.	10.40			95.08***		
	(1.00)			(9.09)		
(15-64)/ToT.	34.61***			67.45***		
	(6.32)			(9.08)		
(65+)/ToT.	-32.27			93.22**		
	(-1.21)			(2.51)		
(0-24)/ToT.		17.81**			90.93***	
		(2.55)			(14.34)	
(25-49)/ToT.		32.06***			62.28***	
		(4.08)			(4.83)	
(50-74)/ToT.		31.34			63.14***	
		(1.65)			(2.91)	
(75+)/ToT.		-107.89**			133.38**	
		(-2.31)			(2.60)	
(0-19)/ToT.			15.33**			90.29***
			(2.01)			(12.74)
(20-39)/ToT.			33.13***			74.63***
			(2.99)			(5.53)
(40-59)/ToT.			18.45			46.92*
			(0.97)			(1.90)
(60-79)/ToT.			41.73*			97.07**
			(1.67)			(2.00)
(80+)/ToT.			-172.04**			112.79
			(-2.18)			(1.36)
Observations	431	431	431	432	432	432
R-squared	0.977	0.977	0.978	0.997	0.997	0.997
Time FE	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

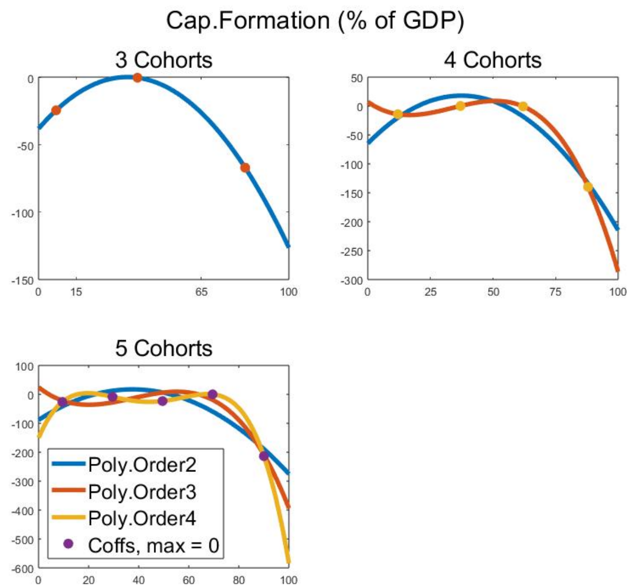


Figure C.2
Demographic cohort structure and capital formation (% of GDP)

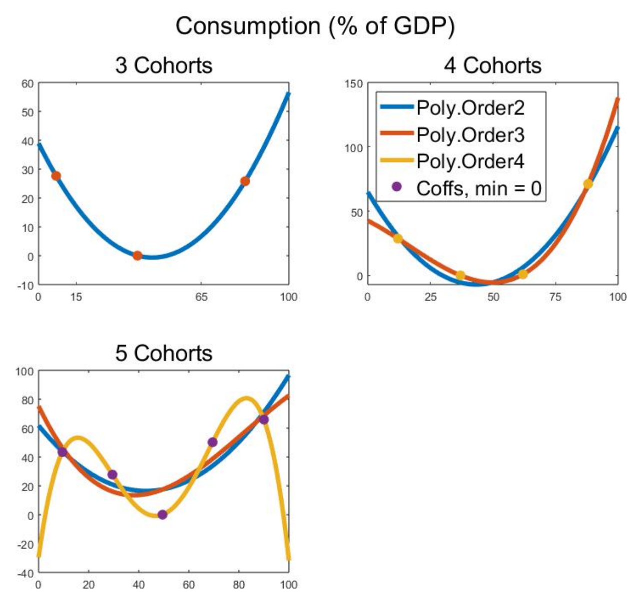


Figure C.3
Demographic cohort structure and consumption (% of GDP)

C.2. Demographics, trade cost, endowments change and economic growth

C.2.I. The effects of demographic structure and trade cost change on capital-labor ratio

TABLE C.6

The effect of demographic structure and trade cost change on capital-labor ratio change

VARIABLES	Average K/L (%) growth rate in the future 7 years			
Trade Cost	-0.97***	-0.96***	-1.01***	-1.00***
	(-3.13)	(-3.05)	(-3.26)	(-3.10)
Dep.Ratio	-4.73**			
	(-2.42)			
Child.Dep.R		-4.45		
		(-1.62)		
Old.Dep.R		-6.65		
		(-0.79)		
Work.Share			13.85**	
			(2.60)	
Child.Share				-13.21**
				(-2.04)
Old.Share				-17.93
				(-1.49)
Initial K/L	-2.50***	-2.44***	-2.61***	-2.52***
	(-4.33)	(-3.85)	(-4.57)	(-3.95)
PoP.Growth	-5.08	-6.58	-5.89	-7.46
	(-0.29)	(-0.39)	(-0.34)	(-0.45)
Constant	35.87***	35.37***	25.69***	38.69***
	(5.79)	(5.11)	(3.88)	(5.17)
Observations	454	454	454	454
R-squared	0.659	0.659	0.660	0.660
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Dep.Ratio* represents the dependency ratio, which is defined as the ratio of people aged (0-14) and (65, +) to people aged 15-64. The variable $Child.Dep.R$ represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable $Old.Dep.R$ represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable $Child.Share$ represents the young population share, which is defined as the share of people aged 0-14, and the variable $Old.Share$ represents old population share, which is defined as the share of people aged 65 and above.

TABLE C.7

The effects of demographic cohort structure and trade cost change on capital-labor ratio change

VARIABLES	Average K/L (%) growth rate in the future 7 years		
Trade Cost	-1.00***	-1.00***	-0.94***
	(-3.10)	(-2.95)	(-2.84)
(0-14)/ToT.	25.49***		
	(3.90)		
(15-64)/ToT.	38.69***		
	(5.17)		
(65+)/ToT.	20.76		
	(1.25)		
(0-24)/ToT.		28.69***	
		(4.59)	
(25-49)/ToT.		45.13***	
		(5.19)	
(50-74)/ToT.		26.85*	
		(1.92)	
(75+)/ToT.		14.79	
		(0.72)	
(0-19)/ToT.			25.80***
			(4.15)
(20-39)/ToT.			43.05***
			(5.15)
(40-59)/ToT.			30.92***
			(2.88)
(60-79)/ToT.			33.28**
			(2.07)
(80+)/ToT.			-1.10
			(-0.03)
Initial.ln.K/L	-2.52***	-2.51***	-2.45***
	(-3.95)	(-3.72)	(-3.68)
PoP.Growth	-7.46	-11.56	-3.34
	(-0.45)	(-0.72)	(-0.18)
Observations	454	454	454
R-squared	0.827	0.830	0.830
Time FE	YES	YES	YES
Country FE	YES	YES	YES

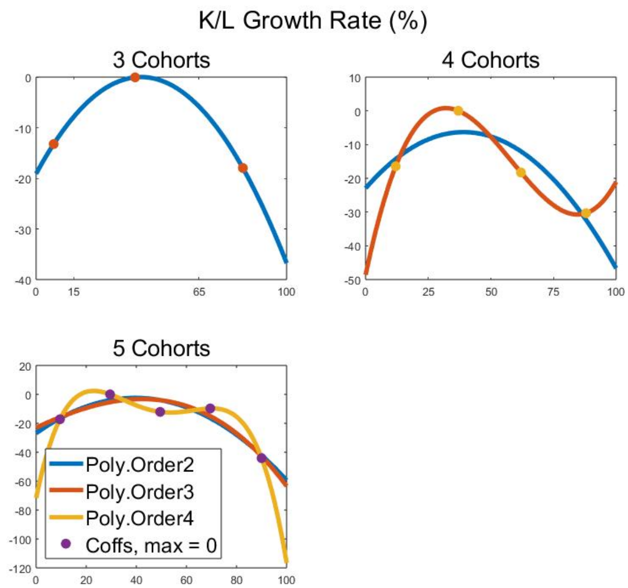


Figure C.4
Demographic cohort structure and Capital-labor ratio

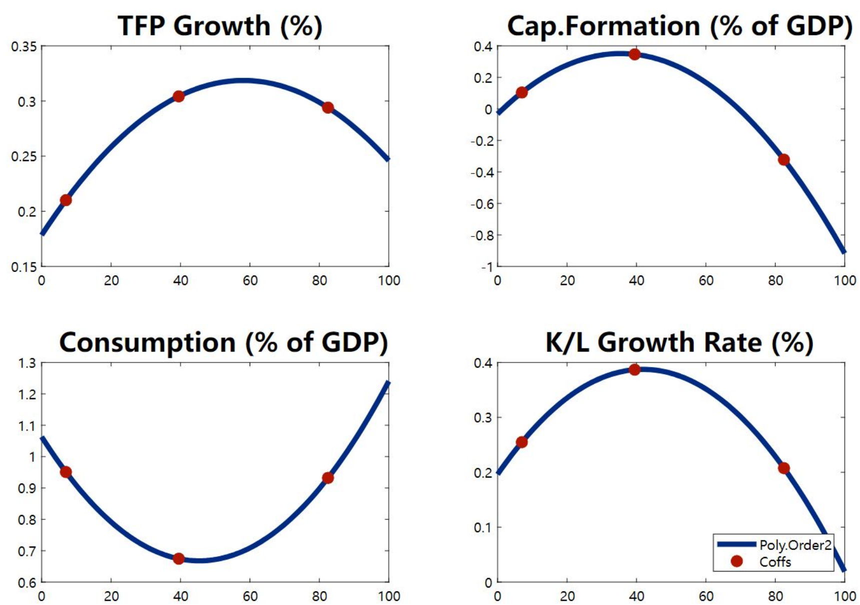


Figure C.5
Effects of demographic structure

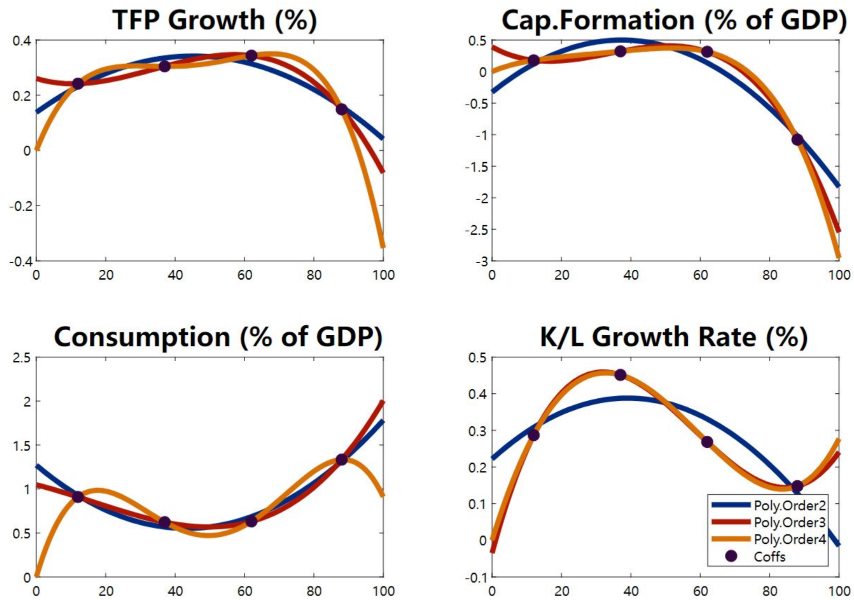


FIGURE C.6
EFFECTS OF DEMOGRAPHIC STRUCTURE

APPENDIX D: MODEL

D.1. Solution method: time path iteration (TPI)

D.2. Technology

Another relation can also be identified:

$$\frac{\lambda^j_{n,t}}{\lambda^j_{n,t-1}} = \left(1 + \vartheta^j_{n,t}\right) exp(g(D_{n,t}; \zeta^j)) \tag{D.1}$$

$$\begin{aligned} log\left(\frac{X^j_{ni,t}X^j_{nn,t}}{X^j_{ni,t-1}X^j_{nn,t-1}}\right) &= \left[g(D_{i,t}; \zeta^j) - g(D_{n,t}; \zeta^j)\right] + log\left(\frac{1+\vartheta^j_{i,t}}{1+\vartheta^j_{n,t}}\right) \\ &- \theta^j log\left(\frac{\kappa^j_{ni,t}}{\kappa^j_{ni,t-1}}\right) - \theta^j log\left(\frac{c^j_{i,t}c^j_{i,t-1}}{c^j_{n,t}c^j_{n,t-1}}\right) \\ &= \left[g(D_{i,t} - D_{n,t}; \zeta^j)\right] - \theta^j log\left(\frac{\kappa^j_{ni,t}}{\kappa^j_{ni,t-1}}\right) - \theta^j log\left(\frac{c^j_{i,t}c^j_{i,t-1}}{c^j_{n,t}c^j_{n,t-1}}\right) \\ &+ \mu^j_t + \mu^j_{ni} + \varepsilon^j_{ni,t} \end{aligned}$$

(D.2)

D.3. Calibration

$$X^j_{ni,t} = \pi^j_{ni,t} X^j_{n,t} = \left(A^j\right)^{-\theta^j} \lambda^j_{i,t} \left(\frac{c^j_{i,t}\kappa^j_{ni,t}}{P^j_{n,t}}\right)^{-\theta^j} X^j_{n,t} \tag{D.3}$$

$$X^j_{nn,t} = \pi^j_{nn,t} X^j_{n,t} = \left(A^j\right)^{-\theta^j} \lambda^j_{n,t} \left(\frac{c^j_{n,t}\kappa^j_{nn,t}}{P^j_{n,t}}\right)^{-\theta^j} X^j_{n,t} \tag{D.4}$$

$$\begin{aligned}
&log\left(\frac{X^j_{ni,t}X^j_{nn,t}}{X^j_{ni,t-1}X^j_{nn,t-1}}\right)\\
&= \left(\vartheta^j_{i,t} - \vartheta^j_{n,t}\right) + \left(g\left(D_{i,t};\zeta^j\right)\right) - \left(g\left(D_{n,t};\zeta^j\right)\right) - \theta^j log\left(\frac{\kappa^j_{ni,t}}{\kappa^j_{ni,t-1}}\right) - \theta^j log\left(\frac{c^j_{i,t}c^j_{i,t-1}}{c^j_{n,t}c^j_{n,t-1}}\right)\\
&= \left[g(D_{i,t} - D_{n,t};\zeta^j)\right] - \theta^j log\left(\frac{\kappa^j_{ni,t}}{\kappa^j_{ni,t-1}}\right) - \theta^j log\left(\frac{c^j_{i,t}c^j_{i,t-1}}{c^j_{n,t}c^j_{n,t-1}}\right)\\
&+ \mu^j_t + \mu^j_{ni} + \varepsilon^j_{ni,t}
\end{aligned} \tag{D.5}$$

$\kappa^j_{ni,t}$ and $c^j_{n,t}$ can be estimated through other gravity equation. μ^j_t,μ^j_{ni} to control for components of the unobserved fundamental growth rates that may be correlated with demographic. The error term $\varepsilon^j_{ni,t}$ thus captures within-industry components of fundamental base productivity growth that are changing differentially within an importer-exporter-industry triplet over time. The vector of ζ^j's is well-identified if $\varepsilon^j_{ni,t}$ are not correlated with demographic. I cluster our standard errors two ways at the importer and exporter level to account for autocorrelation and within-country correlation in errors across trading partners or industries. In our empirical application I estimate both the average response function across all industries, ζ , as well as industry-specific response functions, ζ^j. The sector-specific responses functions come from a single regression where we interact the response function g with a set of industry dummy variables.

TABLE D.1
Parameters & Shocks: Steady-state equilibrium

Parameters	σ, β, θ
	β^j_n, γ^j_n, $\gamma^{j,k}_n$, $\alpha^j_{C,n}$, $\alpha^j_{I,n}$
	$A \equiv \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$, $\Upsilon^j_n \equiv \gamma^j_n \beta^{j^{-\gamma^j_n\beta^j_n}}_n \gamma^j_n \left(1-\beta^j_n\right)^{-\gamma^j_n\left(1-\beta^j_n\right)} \prod_{k=1}^{J} \gamma^{k,j^{-\gamma^{k,j}_n}}_n$
Shocks	$\lambda^{\star j}_n$, $\kappa^{\star\, j}_{ni}$, $\{\eta^\star_{n,g}\}^{E+G}_{g=1}$
Solutions	$b^\star_{n,E+1} = b^\star_{n,E+G+1} = 0$, $c^\star_{n,E+g} > 0$, $\left\{c^\star_{n,g}\right\}^{E+G}_{g=E+1}$, $\left\{b^\star_{n,g+1}\right\}^{E+G-1}_{g=E+1}$

D.3..1 OLG trade model: steady state

TABLE D.2
Steady-state conditions

H1	$L_n^\star \equiv \sum_{g=1}^{E+G} \eta_{n,g}^\star;\ \bar{L}_n^\star \equiv \sum_{g=E+1}^{E+G} \eta_{n,g}^\star;\ N_n^\star = \sum_{g=E+1}^{E+G} \eta_{n,g}^\star l_g$	$\forall(n)$
H2	$P_{n,C}^\star c_{n,g}^\star + P_{n,I}^\star i_{n,g}^\star = R_n^\star b_{n,g}^\star + W_n^\star l_g + \frac{TRS_n^\star}{\bar{L}_n^\star} + \frac{D_n^\star}{\bar{L}_n^\star}$ for $\forall g \geq E+1$	$\forall(n)$
H2'	$P_{n,C}^\star c_{n,g}^\star + P_{n,I}^\star b_{n,g+1}^\star = \left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right) P_{n,I}^\star b_{n,g}^\star + W_n^\star l_g + \frac{TRS_n^\star}{\bar{L}_n^\star} + \frac{D_n^\star}{\bar{L}_n^\star}$ for $\forall g \geq E+1$	$\forall(n)$
H3	$b_{g+1}^\star = i_g^\star + (1-\delta)\, b_g^\star,\ b_{n,E+1}^\star = b_{n,E+G+1}^\star = 0,\ c_{n,E+g}^\star > 0,\ \left\{c_{n,g}^\star\right\}_{g=E+1}^{E+G},\ \left\{b_{n,g+1}^\star\right\}_{g=E+1}^{E+G-1}$	$\forall(n)$
H4	$TRS_n^\star = P_{n,I}^\star \left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right) \sum_{g=E+2}^{E+S} \left(\eta_{n,g-1}^\star - \eta_{n,g}^\star\right) b_{n,g}^\star$	$\forall(n)$
H5	$\left(\frac{c_{n,g+1}^\star}{c_{n,g}^\star}\right)^{1/\sigma} = \beta \left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right) \frac{\frac{P_{n,I}^\star}{P_{n,C}^\star}}{\frac{P_{n,I}^\star}{P_{n,C}^\star}}$ for $\forall\ g \in [E+1, E+G-1]$	$\forall(n)$
H6	$C_n^\star = \sum_{g=E+1}^{E+G} \eta_{n,g}^\star c_{n,g}^\star;\ I_n^\star = \sum_{g=E+1}^{E+G} \eta_{n,g}^\star i_{n,g}^\star;\ K_n^\star = \sum_{g=E+1}^{E+G} \eta_{n-1,g-1}^\star b_{n,g}^\star$	$\forall(n)$
F1	$W_n^\star N_n^\star = \sum_{j=1}^{J} \beta_n^j \gamma_n^j \sum_{i=1}^{N} \pi_{in}^{\star\ j} X_i^{\star j}$	$\forall(n)$
F2	$R_n^\star K_n^\star = \sum_{j=1}^{J} \left(1-\beta_n^j\right) \gamma_n^j \sum_{i=1}^{N} \pi_{in}^{\star\ j} X_i^{\star j}$	$\forall(n)$
F3	$X_n^{\star j} = \alpha_{C,n}^j P_{C,n}^\star C_n^\star + \alpha_{I,n}^j P_{I,n}^\star I_n^\star + \sum_{k=1}^{J} \gamma_n^{j,k} \left(\sum_{i=1}^{N} X_{in}^{\star\ k}\right)$	$\forall(n, j$
F4	$P_n^{\star j} I_n^{\star j} = \alpha_{I,n}^j P_{I,n}^\star I_n^\star;\ P_n^{\star j} C_n^{\star j} = \alpha_{C,n}^j P_{C,n}^\star C_n^\star$	$\forall(n, j)$
F5	$IN_n^\star \equiv R_n^\star K_n^\star + W_n^\star N_n^\star + D_n^\star = P_{C,n}^\star C_n^\star + P_{I,n}^\star I_n^\star$	$\forall(n)$
T1	$c_n^{\star j} \equiv \Upsilon_n^j \left[\left(W_n^{\star j}\right)^{\beta_n^j} \left(R_n^{\star j}\right)^{1-\beta_n^j}\right]^{\gamma_n^j} \prod_{k=1}^{J} P_n^{\star k^{\gamma_n^{k,j}}}$ where $\Upsilon_n^j \equiv \gamma_n^j \beta_n^{j^{-\gamma_n^j \beta_n^j}} \gamma_n^j \left(1-\beta_n^j\right)^{-\gamma_n^j\left(1-\beta_n^j\right)} \prod_{k=1}^{J} \gamma_n^{k,j^{-\gamma_n^{k,j}}}$	$\forall(n, j)$
T2	$P_n^{\star j} = A \cdot \left[\sum_{i=1}^{N} \lambda_i^{\star j} \left(\kappa_{ni}^{\star\ j} c_i^{\star j}\right)^{-\theta}\right]^{-\frac{1}{\theta}}$ where $A \equiv \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$	$\forall(n, j)$
T3	$\pi_{ni}^{\star\ j} = \frac{\lambda_i^{\star j} \left(c_i^{\star j} \kappa_{ni}^{\star\ j}\right)^{-\theta}}{\sum_{m=1}^{N} \lambda_m^{\star\ j} \left(c_m^{\star\ j} \kappa_{nm}^{\star\ j}\right)^{-\theta}} = \lambda_i^{\star j} \left(\frac{A^j c_i^{\star j} \kappa_{ni}^{\star\ j}}{P_n^{\star j}}\right)^{-\theta}$	$\forall(n, i, j)$
T4	$P_{n,C}^\star C_n^\star + P_{n,I}^\star I_n^\star = R_n^\star K_n^\star + W_n^\star N_n^\star + D_n^\star$	$\forall(n)$
T4'	$P_{n,C}^\star C_n^\star + P_{n,I}^\star K_n^\star = \left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right) P_{n,I}^\star K_n^\star + W_n^\star N_n^\star + D_n^\star$	$\forall(n)$
T5	$K_n^\star = I_n^\star + (1-\delta)\, K_n^\star$	$\forall(n)$
T6	$\sum_{j=1}^{J} \sum_{i=1}^{N} X_{in}^{\star\ j} - \sum_{j=1}^{J} \sum_{i=1}^{N} X_{ni}^{\star\ j} = NX_n^\star = -D_n^\star$	$\forall(n, j)$
T7	$D_n^\star = -\phi_n^\star \left(R_n^\star K_n^\star + W_n^\star N_n^\star\right) + \bar{L}_n^\star T^{\star P}$	$\forall(n)$
T8	$\sum_{n=1}^{N} \phi_n^\star \left(R_n^\star K_n^\star + W_n^\star N_n^\star\right) = \sum_{n=1}^{N} \bar{L}_n^\star T^{\star P}$	$\forall(n)$

TABLE D.3
Parameters & Shocks: Steady-state equilibrium

Parameters	$\sigma,\ \beta,\ \theta$
	$\beta_n^j,\ \gamma_n^j,\ \gamma_n^{j,k},\ \alpha_{C,n}^j,\ \alpha_{I,n}^j$
	$A \equiv \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$, $\Upsilon_n^j \equiv \gamma_n^j \beta_n^{j^{-\gamma_n^j \beta_n^j}} \gamma_n^j \left(1-\beta_n^j\right)^{-\gamma_n^j\left(1-\beta_n^j\right)} \prod_{k=1}^{J} \gamma_n^{k,j^{-\gamma_n^{k,j}}}$
Shocks	$\lambda_n^{\star j},\ \kappa_{ni}^{\star\ j},\ \left\{\eta_{n,g}^\star\right\}_{g=1}^{E+G}$
Solutions	$\left\{b_{n,g+1}^\star\right\}_{g=E+1}^{E+G-1}$
	$b_{n,E+1}^\star = b_{n,E+G+1}^\star = 0,\ c_{n,E+g}^\star > 0,\ \left\{c_{n,g}^\star\right\}_{g=E+1}^{E+G}$

Step 1: Guess $\tilde{\mathbf{K}}^1 = \{K_1, ..., K_N\}^1$. Get $\tilde{\mathbf{I}}^1 = \{I_1, ..., I_N\}^1$, where $I_n^1 = \delta K_n^1 \; \forall n \in \mathcal{N}$.
Step 2: Solve the multi-sector (capital and labor as inputs with different share across sectors) EK trade model under fixed world GDP, get $\{\pi_{ni}^j, P_n^j, P_{n,I}, P_{n,C}, R_n, W_n\}^1$.
Step 3: Solve $(G-1)$ Euler equations for each Country $n \in \mathcal{N}$:

$$\{b^\star_{n,E+1} = b^\star_{n,E+G+1} = 0, \{b^\star_{n,g+1}\}_{g=E+1}^{E+G-1}\}^1$$

Step 4: Calculate ${K^1}'_n = \sum_{g=E+1}^{E+G} \eta_{n,g} b_{n,g}^1$ for each Country $n \in \mathcal{N}$. for $i = 1$, Check

$$||\tilde{\mathbf{K}}^i - \tilde{\mathbf{K}}^{i\prime}|| < \epsilon$$

for $\zeta \in (0,1)$: $\tilde{\mathbf{K}}^{i+1} = \zeta \tilde{\mathbf{K}}^{i\prime} + (1-\zeta)\tilde{\mathbf{K}}^i$. This process is repeated until each country's aggregate capital stock is consistent with steady state of capital stock implied by household and firm optimization.

Detail:

Step 3: Solve $(G-1)$ Euler equations for each Country $n \in \mathcal{N}$:

$$TRS_n^\star = P_{n,I}^\star \left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right) \sum_{g=E+2}^{E+S} \left(\eta_{n,g-1}^\star - \eta_{n,g}^\star\right) b_{n,g}^\star$$

$$D_n^\star = -\phi_n^\star \left(R_n^\star K_n^\star + W_n^\star N_n^\star\right) + \bar{L}_n^\star T^{\star P}$$

$$\sum_{n=1}^{N} \phi_n^\star \left(R_n^\star K_n^\star + W_n^\star N_n^\star\right) = \sum_{n=1}^{N} \bar{L}_n^\star T^{\star P}$$

$$\{b_{n,E+1}^\star = b_{n,E+G+1}^\star = 0, \left\{b_{n,g+1}^\star\right\}_{g=E+1}^{E+G-1}\}^1$$

$\forall\ g \in [E+1, E+G-1]$,

$$\left(\frac{c_{n,g+1}^\star}{c_{n,g}^\star}\right)^{1/\sigma} = \beta\left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right)$$

$$\left(\frac{c_{n,g+1}^\star}{c_{n,g}^\star}\right)^{1/\sigma} = \left(\frac{P_{n,C}^\star c_{n,g+1}^\star}{P_{n,C}^\star c_{n,g}^\star}\right)^{1/\sigma} = \beta\left(1 + \frac{R_n^\star}{P_{n,I}^\star} - \delta\right) = \beta\Theta_n$$

$\forall g \geq E+1$,

$$P_{n,C}^\star c_{n,E+1}^\star + P_{n,I}^\star b_{n,E+2}^\star = \Theta_n P_{n,I}^\star b_{n,E+1}^\star + W_n^\star l_{E+1} + \frac{TRS_n^\star}{\bar{L}_n^\star} + \frac{D_n^\star}{\bar{L}_n^\star}$$

$$P_{n,C}^\star c_{n,E+G}^\star + P_{n,I}^\star b_{n,E+G+1}^\star = \Theta_n P_{n,I}^\star b_{n,E+G}^\star + W_n^\star l_{E+G} + \frac{TRS_n^\star}{\bar{L}_n^\star} + \frac{D_n^\star}{\bar{L}_n^\star}$$

Euler equation: $g \in [E+1, E+G-1]$,

$$P_{n,C}^\star c_{n,g}^\star = \left(\beta\Theta_n\right)^{-\sigma} P_{n,C}^\star c_{n,g+1}^\star$$

$$\begin{aligned}\Theta_n P_{n,I}^\star b_{n,E+1}^\star - \left(1 + \beta^{-\sigma}\Theta_n^{1-\sigma}\right) P_{n,I}^\star b_{n,E+2}^\star + \left(\beta\Theta_n\right)^{-\sigma} P_{n,I}^\star b_{n,E+3}^\star \\ = \left(\beta\Theta_n\right)^{-\sigma} W_n^\star l_{E+2} - W_n^\star l_{E+1} + \left(\left(\beta\Theta_n\right)^{-\sigma} - 1\right)\frac{TRS_n^\star}{\bar{L}_n^\star} + \left(\left(\beta\Theta_n\right)^{-\sigma} - 1\right)\frac{D_n^\star}{\bar{L}_n^\star}\end{aligned}$$

$$\ldots$$

$$\begin{aligned}\Theta P_{n,I}^\star b_{n,E+G-1}^\star - \left(1 + \beta^{-\sigma}\Theta^{1-\sigma}\right) P_{n,I}^\star b_{n,E+G}^\star + \left(\beta\Theta_n\right)^{-\sigma} P_{n,I}^\star b_{n,E+G+1}^\star \\ = \left(\beta\Theta_n\right)^{-\sigma} W_n^\star l_{E+G} - W_n^\star l_{E+G-1} + \left(\left(\beta\Theta_n\right)^{-\sigma} - 1\right)\frac{TRS_n^\star}{\bar{L}_n^\star} + \left(\left(\beta\Theta_n\right)^{-\sigma} - 1\right)\frac{D_n^\star}{\bar{L}_n^\star}\end{aligned}$$

Define the following:

$$\vec{B}^{\star} = \begin{bmatrix} \vec{b}_1^{\star} \\ \ldots \\ \vec{b}_N^{\star} \end{bmatrix}_{N\times 1} ; \quad \vec{b}_n^{\star} = \begin{bmatrix} b_{n,E+2}^{\star} \\ b_{n,E+3}^{\star} \\ \ldots \\ b_{n,E+G-1}^{\star} \\ b_{n,E+G}^{\star} \end{bmatrix}_{(G-1)\times 1} ; \quad \vec{l} = \begin{bmatrix} l_{E+1} \\ l_{E+2} \\ \ldots \\ l_{E+G-2} \\ l_{E+G-1} \end{bmatrix}_{(G-1)\times 1} ; \quad \vec{\eta_n^{\star}} = \begin{bmatrix} \eta_{n,1}^{\star} \\ \eta_{n,2}^{\star} \\ \ldots \\ \eta_{n,E+G-1}^{\star} \\ \eta_{n,E+G}^{\star} \end{bmatrix}_{(E+G)\times 1}$$

$$A = \begin{bmatrix} zeros\left(1, E+G-2\right) & 0 \\ eye\left(E+G-2\right) & zeros\left(E+G-2, 1\right) \end{bmatrix}; \quad I = \left[eye\left(E+G-1\right)\right]$$

$$\begin{bmatrix} 0 \\ b_{n,E+2}^{\star} \\ \ldots \\ b_{n,E+G-2}^{\star} \\ b_{n,E+G-1}^{\star} \end{bmatrix}_{(G-1)\times 1} = A \times \vec{b}_n^{\star}; \quad \begin{bmatrix} b_{n,E+2}^{\star} \\ b_{n,E+3}^{\star} \\ \ldots \\ b_{n,E+G-1}^{\star} \\ b_{n,E+G}^{\star} \end{bmatrix}_{(G-1)\times 1} = I \times \vec{b}_n^{\star}; \quad \begin{bmatrix} b_{n,E+3}^{\star} \\ b_{n,E+4}^{\star} \\ \ldots \\ b_{n,E+G}^{\star} \\ b_{n,E+G+1}^{\star} \end{bmatrix}_{(G-1)\times 1} = A' \times \vec{b}_n^{\star}$$

$$\begin{bmatrix} l_{E+2} \\ l_{E+3} \\ \ldots \\ l_{E+G-1} \\ l_{E+G} \end{bmatrix}_{(G-1)\times 1} = A' \times \vec{l}; \quad \begin{bmatrix} l_{E+1} \\ l_{E+2} \\ \ldots \\ l_{E+G-2} \\ l_{E+G-1} \end{bmatrix}_{(G-1)\times 1} = I \times \vec{l}$$

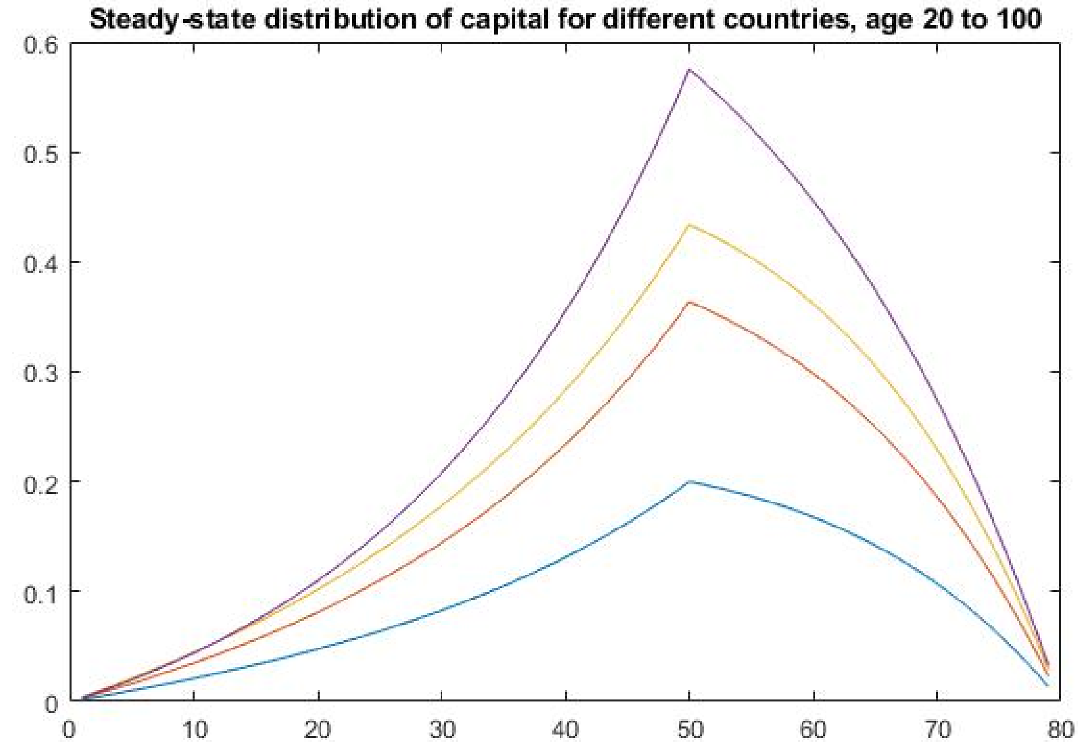
$$\bar{L}_n^{\star} \equiv \sum_{g=E+1}^{E+G} \eta_{n,g}^{\star} = ones\left(1, G\right) \times \vec{\eta_n^{\star}}\left(E+1 : E+G, 1\right)$$

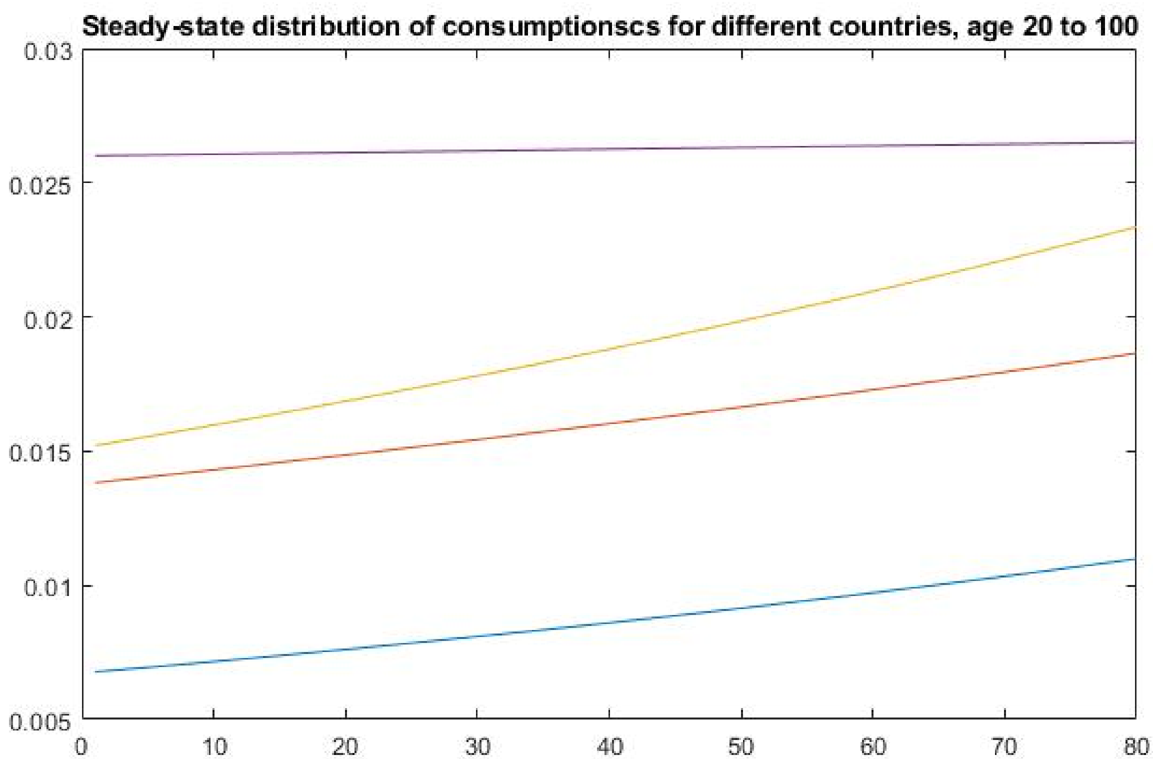
$$TRS_n^{\star} = P_{n,I}^{\star}\Theta_n \sum_{g=E+2}^{E+G} \left(\eta_{n,g-1}^{\star} - \eta_{n,g}^{\star}\right) b_{n,g}^{\star} = P_{n,I}^{\star}\Theta_n \odot \left\{\vec{\eta_n^{\star}}\left(E+1 : E+G-1, 1\right) - \vec{\eta_n^{\star}}\left(E+2 : E+G, 1\right)\right\}'$$

$$D_n^{\star} = -\phi_n^{\star}\left(R_n^{\star}K_n^{\star} + W_n^{\star}N_n^{\star}\right) + \bar{L}_n^{\star}T^{\star^P} = -\phi_n^{\star}\left(R_n^{\star}K_n^{\star} + W_n^{\star}N_n^{\star}\right) + \bar{L}_n^{\star}\frac{\sum_{n=1}^{N} \phi_n^{\star}\left(R_n^{\star}K_n^{\star} + W_n^{\star}N_n^{\star}\right)}{\sum_{n=1}^{N} \bar{L}_n^{\star}}$$

$$N_n^{\star} = \sum_{g=E+1}^{E+G} \eta_{n,g}^{\star} l_g = \sum_{g=E+1}^{E+G-1} \eta_{n,g}^{\star} l_g = \vec{l}' \times \vec{\eta_n^{\star}}\left(E+1 : E+G-1, 1\right)$$

$$K_n^{\star} = \sum_{g=E+1}^{E+G} \eta_{n,g}^{\star} b_{n,g}^{\star} = \sum_{g=E+2}^{E+G} \eta_{n,g}^{\star} b_{n,g}^{\star} = \vec{b}_n^{\star\prime} \times \vec{\eta_n^{\star}}\left(E+2 : E+G, 1\right)$$


Steady-state distribution of capital for different countries, age 20 to 100
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60
70
80


Steady-state distribution of consumptionscs for different countries, age 20 to 100
0.03
0.025
0.02
0.015
0.01
0.005
0
10
20
30
40
50
60
70
80

D.3..2 OLG trade model: Dynamic equilibrium conditions

TABLE D.4

Dynamic equilibrium conditions

H1	$L_{n,t} \equiv \sum_{g=1}^{E+G} \eta_{n,g,t};\ \bar{L}_{n,t} \equiv \sum_{g=E+1}^{E+G} \eta_{n,g,t};\ N_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g,t} l_g$	$\forall (n,t)$
H2	$P_{n,C,t} c_{n,g,t} + P_{n,I,t} i_{n,g,t} = R_{n,t} b_{n,g,t} + W_{n,t} l_g + \frac{TRS_{n,t}}{\bar{L}_{n,t}} + \frac{D_{n,t}}{\bar{L}_{n,t}}$ for $\forall g \geq E+1$	$\forall (n,t)$
H2'	$P_{n,C,t} c_{n,g,t} + P_{n,I,t} b_{n,g+1,t+1} = \left(1 + \frac{R_{n,t}}{P_{n,I,t}} - \delta\right) P_{n,I,t} b_{n,g,t} + W_{n,t} l_g + \frac{TRS_{n,t}}{\bar{L}_{n,t}} + \frac{D_{n,t}}{\bar{L}_{n,t}}$ for $\forall g \geq E+1$	$\forall (n,t)$
H3	$b_{g+1,t+1} = i_{g,t} + (1-\delta) b_{g,t},\ b_{n,E+1,t} = b_{n,E+G+1,t} = 0,\ c_{n,E+g,t} > 0,\ \{c_{n,g,t+g-1}\}_{g=E+1}^{E+G},\ \{b_{n,g+1,t+g}\}_{g=E+1}^{E+G-1}$	$\forall (n,t)$
H4	$TRS_{n,t} = P_{n,I,t}\left(1 + \frac{R_{n,t}}{P_{n,I,t}} - \delta\right) \sum_{g=E+2}^{E+S} \left(\eta_{n,g-1,t-1} - \eta_{n,g,t}\right) b_{n,g,t}$	$\forall (n,t)$
H5	$\left(\frac{c_{n,g+1,t+g}}{c_{n,g,t+g-1}}\right)^{1/\sigma} = \beta \left(1 + \frac{R_{n,t+g}}{P_{n,I,t+g}} - \delta\right) \frac{\frac{P_{n,I,t+g}}{P_{n,C,t+g}}}{\frac{P_{n,I,t+g-1}}{P_{n,C,t+g-1}}}$ for $\forall\ g \in [E+1, E+G-1]$	$\forall (n,t)$
H6	$C_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g,t} c_{n,g,t};\ I_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g,t} i_{n,g,t};\ K_{n,t} = \sum_{g=E+1}^{E+G} \eta_{n,g-1,t-1} b_{n,g,t}$	$\forall (n,t)$
F1	$W_{n,t} N_{n,t} = \sum_{j=1}^{J} \beta_n^j \gamma_n^j \sum_{i=1}^{N} \pi_{in,t}^j X_{i,t}^j$	$\forall (n,t)$
F2	$R_{n,t} K_{n,t} = \sum_{j=1}^{J} \left(1 - \beta_n^j\right) \gamma_n^j \sum_{i=1}^{N} \pi_{in,t}^j X_{i,t}^j$	$\forall (n,t)$
F3	$X_{n,t}^j = \alpha_{C,n}^j P_{C,n,t} C_{n,t} + \alpha_{I,n}^j P_{I,n,t} I_{n,t} + \sum_{k=1}^{J} \gamma_n^{j,k} \left(\sum_{i=1}^{N} X_{in,t}^k\right)$	$\forall (n,j,t)$
F4	$P_{n,t}^j I_n^j = \alpha_{I,n}^j P_{I,n,t} I_{n,t};\ P_{n,t}^j C_n^j = \alpha_{C,n}^j P_{C,n,t} C_{n,t}$	$\forall (n,j,t)$
F5	$IN_{n,t} \equiv R_{n,t} K_{n,t} + W_{n,t} N_{n,t} + D_{n,t} = P_{C,n,t} C_{n,t} + P_{I,n,t} I_{n,t}$	$\forall (n,t)$
T1	$c_{n,t}^j \equiv \Upsilon_n^j \left[\left(W_{n,t}^j\right)^{\beta_n^j} \left(R_{n,t}^j\right)^{1-\beta_n^j}\right]^{\gamma_n^j} \prod_{k=1}^{J} {P_{n,t}^k}^{\gamma_n^{k,j}}$ where $\Upsilon_n^j \equiv \gamma_n^j {\beta_n^j}^{-\gamma_n^j \beta_n^j} \gamma_n^j \left(1 - \beta_n^j\right)^{-\gamma_n^j\left(1-\beta_n^j\right)} \prod_{k=1}^{J} {\gamma_n^{k,j}}^{-\gamma_n^{k,j}}$	$\forall (n,j,t)$
T2	$P_{n,t}^j = A \cdot \left[\sum_{i=1}^{N} \lambda_{i,t}^j \left(\kappa_{ni,t}^j c_{i,t}^j\right)^{-\theta}\right]^{-\frac{1}{\theta}}$ where $A \equiv \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$	$\forall (n,j,t)$
T3	$\pi_{ni,t}^j = \frac{\lambda_{i,t}^j \left(c_{i,t}^j \kappa_{ni,t}^j\right)^{-\theta}}{\sum_{m=1}^{N} \lambda_{m,t}^j \left(c_{m,t}^j \kappa_{nm,t}^j\right)^{-\theta}} = \lambda_{i,t}^j \left(\frac{A^j c_{i,t}^j \kappa_{ni,t}^j}{P_{n,t}^j}\right)^{-\theta}$	$\forall (n,i,j,t)$
T4	$P_{n,C,t} C_{n,t} + P_{n,I,t} I_{n,t} = R_{n,t} K_{n,t} + W_{n,t} N_{n,t} + D_{n,t}$	$\forall (n,t)$
T4'	$P_{n,C,t} C_{n,t} + P_{n,I,t} K_{n,t+1} = \left(1 + \frac{R_{n,t}}{P_{n,I,t}} - \delta\right) P_{n,I,t} K_{n,t} + W_{n,t} N_{n,t} + D_{n,t}$	$\forall (n,t)$
T5	$K_{n,t+1} = I_{n,t} + (1-\delta) K_{n,t}$	$\forall (n,t)$
T6	$\sum_{j=1}^{J} \sum_{i=1}^{N} X_{in,t}^j - \sum_{j=1}^{J} \sum_{i=1}^{N} X_{ni,t}^j = NX_{n,t} = -D_{n,t}$	$\forall (n,j,t)$
T7	$D_{n,t} = -\phi_{n,t}\left(R_{n,t} K_{n,t} + W_{n,t} N_{n,t}\right) + \bar{L}_{n,t} T_t^P$	$\forall (n,t)$
T8	$\sum_{n=1}^{N} \phi_{n,t}\left(R_{n,t} K_{n,t} + W_{n,t} N_{n,t}\right) = \sum_{n=1}^{N} \bar{L}_{n,t} T_t^P$	$\forall (n,t)$

Step 1: Guess $\tilde{\mathbf{K}}_{t=1,...,T}^1 = \{K_1, ..., K_N\}^1$. T is large enough to reach The steady state before T. T also larger than population s.s T. Final T, capital distribution must be stationary. Initial capital distribution can at any level. Get $\tilde{\mathbf{I}}_{t=1,...,T}^1 = \{I_1, ..., I_N\}_{t=1,...,T}^1$, where $I_{n,t}^1 =$

$K^1_{n,t+1} - (1-\delta)K^1_{n,t}$ $\forall n \in \mathcal{N}$. Consumption rate can also backed out at the same time $\tilde{\phi}^{con,1}_{t=1,...,T-1}{}^1$.

Step 2: Solve the multi-sector (capital and labor as inputs with different share across sectors) EK trade model under fixed world GDP, get $\{\pi^j_{ni}, P^j_n, P_{n,I}, P_{n,C}, R_n, W_n\}^1$.

Step 3: Solve $\# = \{1, ..., (G-1)\}$ Euler equations for each Country $n \in \mathcal{N}$:

$$b_{n,E+1,t} = b_{n,E+G+1,t} = 0, c_{n,E+g,t} > 0, \{c_{n,g,t+g-1}\}^{E+G}_{g=E+1}$$
$$\{b_{n,g+1,t+g}\}^{E+G-1}_{g=E+1} \qquad \forall (n, t \in [1, ..., T], g)$$

Step 4: Calculate $K^{1\prime}_n = \sum_{g=E+1}^{E+G} \eta_{n,g} b^1_{n,g}$ for each Country $n \in \mathcal{N}$. for $i = 1$, Check

$$||\tilde{\mathbf{K}}^i - \tilde{\mathbf{K}}^{i\prime}|| < \epsilon$$

for $\zeta \in (0,1)$: $\tilde{\mathbf{K}}^{i+1} = \zeta\tilde{\mathbf{K}}^{i\prime} + (1-\zeta)\tilde{\mathbf{K}}^i$. This process is repeated until each country's aggregate capital stock is consistent with steady state of capital stock implied by household and firm optimization.

Step 4: Calculate $K^{1\prime}_n = \sum_{g=E+1}^{E+G} \eta_{n,g} b^1_{n,g}$ for each Country $n \in \mathcal{N}$. Calculate accumulated Euler equation and Check each single equation Repeat until Euler equation holds in every country/period.

D.3.I. Calibration