DEMOGRAPHIC, TRADE, AND GROWTH *

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ABSTRACT

Motivated by China's recent economic slowdown, the relocation of labor-intensive industries, and an aging population, this study examines how demographic forces shape China's economic growth and trade patterns. Country-level panel regressions and a VARX model indicate that countries with a larger working-age population share experience higher productivity growth and greater investment as a share of GDP. Building on these findings, I develop and calibrate an overlapping generations (OLG) trade model with three key features: age-varying abilities to generate ideas that drive knowledge accumulation, age-varying saving behaviors affecting capital accumulation, and a multi-sector structure that captures both Heckscher-Ohlin and Ricardian comparative advantage within an Eaton-Kortum trade framework. In a counterfactual analysis, I compare China's baseline case to a hypothetical scenario where China's fertility and survival rates align with those of the rest of the world. Results indicate a trade-off in China's unique demographics: short-term gains in capital and income per worker due to a saving-favorable age distribution, along with a stronger comparative advantage in capital-intensive sectors, but a long-term outcome of a lower productivity growth path and income per worker, as a smaller working-age population generates fewer new ideas post-2060.

Keywords: Demographics; Idea generation; Capital accumulation; International trade; Comparative advantage; Trade patterns change; Economic growth

JEL Codes: E21, F11, J11, O47, O11.

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1. Introduction

In recent decades, the world has faced significant demographic shifts, notably characterized by population aging and decline. By 2021, approximately 31% of global GDP was generated in countries experiencing population decline, many of which are also grappling with a shrinking working-age population. Among these countries, China stands out as one of the largest, and as its population declining and aging, its economic growth also slow down.

Historically, China's growth is mainly relied on a robust labor supply and significant productivity improvements. On the one hand, by opening up to trade, China leveraged its comparative advantage in labor-intensive goods, gaining added value from both domestic and international demand. On the other hand, productivity growth during this period also contributed significantly to the growth. However, the forces that fueled China's past growth are now shifting. The nation faces an aging and shrinking population alongside a slowdown in productivity growth. These demographic changes threaten to undermine China's traditional strengths in labor-intensive production, as a dwindling workforce and raised wages gradually erode its comparative advantage in labor-intensive production. Additionally, an aging population will dampen savings, hindering capital accumulation and further complicating the transition toward specialization in capital-intensive goods.

This transformation raises a critical question: How have demographic changes influenced China's trade patterns and economic growth in the past, and how will these forces continue to shape its future performance? As we know from economic theory, productivity and capital are widely recognized as fundamental drivers of economic growth. Furthermore, differences in productivity and capital-labor endowments play a crucial role in explaining shifts in trade patterns. This raises two important questions: Is there a relation between demographics and productivity? And is there a relation between demographics and capital accumulation? If such connections exist, demographics may impact economic growth and shape trade patterns through these channels.

In fact, the existing literature on economic growth reveals a strong relationship between age structure and productivity, observable at both macro and micro levels (Feyrer, 2008; Werding, 2008; Sevilla et al., 2007; Werding et al., 2007; Kögel, 2005). This literature shows that individuals in middle age tend to be more productive and creative compared to their younger and older counterparts. In parallel, the life-cycle hypothesis suggests that individual saving behavior follows a hump-shaped pattern: people tend to consume more and save less during youth and retirement, while saving primarily during their working years. Since savings are a key source of capital supply, this pattern indicates that demographic shifts have the potential to influence capital accumulation.

Hence, I aim to answer this question from a parsimonious perspective centers around these two mechanisms, utilizing both empirical and quantitative approaches. The empirical part focuses on examining the overall connections between demographic structure, productivity, and investment. Furthermore, I estimate the impulse response functions (IRFs) of demographic shocks to assess their connections over time. Based on these empirical findings, I develop an overlapping generations (OLG) open economy model. Basically, I embed two mechanisms to explain how demographic change affects shifts in trade patterns and economic growth (age-dependent ability to generate new ideas and age-dependent saving behavior) in the model featuring the essential mechanisms of relative price effects, comparative advantage, and investment. I then calibrate the model with five country groups and five sectors, spanning from 1970 to 2100, and conduct counterfactual analyses.

I find that China's particular demographic process (lower fertility rate and higher survival rate compared to that of the RoW) generates a short-run and long-run trade-off for real income per worker. In the short run, China's lower fertility rate and higher survival rate result in a demographic structure that favors saving, leading to both a higher level of capital per worker and real income per worker. In parallel, It also leads to higher degree of its comparative advantage in the capital-intensive sector compared to the counterfactual case. Specifically, compared with the counterfactual case where China's fertility rate and survival rate are replaced by those of the rest of the world, by 2020, China's real income per worker is about 5.08% higher than the counterfactual level, and China's revealed comparative advantage index in the capital-intensive sector is about 6.79% higher than its counterfactual level. These short-run trends would persist until 2060. In the long run, after 2060, China's particular demographic process implies a less working-age population. Fewer workers imply fewer new ideas, thereby lowering the productivity growth path. As a result, it leads to a lower level of real income per worker than the counterfactual level. By 2070, compared with the counterfactual in which China's fertility rate and survival rate are substituted with those of the RoW, China's projected real income per worker is estimated to be approximately 4.08% less than the counterfactual level.

The empirical analysis uses a balanced panel of 74 countries covering the period from 1971 to 2019. The empirical part is divided into two sub-parts. In the first part, I employ panel regression to estimate the relationships between demographic structure and various indicators such as TFP growth, consumption share of GDP, and investment share of GDP using country-level panel regression. I find that countries with a higher share of the working-age population (or a lower dependency ratio) exhibit a higher TFP growth rate, while a larger share of the elderly does not have significant effects on the TFP growth rate. Additionally, I find an inverse U-shaped relationship between the share of the population in different age groups and the productivity growth rate. A similar inverse U-shaped relationship is also found between age and the investment share of GDP, as well as the capital-labor ratio. In the second part, I employ a panel Vector Autoregressive with Exogenous Variables (VARX) model to investigate the dynamic effects of demographic shocks on productivity, income per capita, and the capital-labor ratio. The impulse response function (IRF) stemming from a 1 percentage point shock to the share of the young cohort exhibits an inverse U-shaped response, indicating that the shock will pass down as the cohort ages.

In the theoretical part, I develope and calibrate an OLG-trade model that incorporates the empirical features identified in the empirical sections. The model incorporate three key features. (key model features, features three key driving forces:) Firstly, the demographic structure will be one of those elements driving TFP growth as Schlenker and Roberts (2009); Rudik, Lyn, Tan, and Ortiz-Bobea (2023), named as demographic-induced TFP growth here after. Secondly, the model will have both dynamic and OLG features to capture the impact of demographic structure on capital accumulation, as Ravikumar, Santacreu, and Sposi (2019); Sposi (2022). Lastly, the sectoral production function will incorporate labor, capital, and TFP, making it a multi-sector trade model that integrates both Heckscher-Ohlin and Ricardian forces, following the framework proposed by Sposi, Yi, and Zhang (2021b). By utilizing this model, I aim to investigate China's past growth and conduct model-based projections for future growth from the perspective of demographics.

In the model, the main driving forces—age-time varying fertility rates and survival rates—mediated through the model's mechanisms, affect both sectoral productivities and capital accumulations. These forces, together with the exogenous trade cost changes, affect the sectoral prices, which, in turn, affects the allocation of production across sectors and locations, and ultimately affects both trade patterns and economic performance.

For example, a higher survival rate leads to greater knowledge stock and capital stock accumulation, raising the balanced growth path by enhancing both productivity and capital. Free trade induce specialization, which encourages higher productivity and lowers prices, ultimately leading to even greater capital accumulation. In addition, A lower fertility rate impacts both knowledge and capital stocks. In the short run, a reduced young population raises capital per person, temporarily boosting economic output. However, over time, this benefit is offset by slower productivity growth due to the demographic shift, ultimately causing capital per person to fall below the previous growth path. Trade liberalization can mitigate this long-term drawback by maintaining capital per person above the old growth path for a longer period, which extends the overall economic benefits in comparison to a closed economy scenario.

I calibrate my model to match the real data, such as sectoral trade flows, total sectoral output, value added, capital stock, wage rates, rental rates, and sectoral prices. Additionally, fertility rates and survival rates can be derived from the UN demographic database. The main time-varying forces are the sectoral knowledge stock and trade costs. The calibration results show that China's productivity exhibits an upward trend across all sectors, with growth occurring at a faster pace than in other regions, despite starting from a lower initial level. Overall, trade costs are higher in the services sector, while trade barriers decline across all sectors, with a faster rate of decline in the manufacturing sectors.

To assess the role of each driving force affecting economic growth and trade patterns over time, I conduct three counterfactual exercises. First, I remove both the demographic-induced-saving effects and demographic-induced-productivity changes. Specifically, I replace the age-varying fertility and survival rates in China with those of the rest of the world (RoW) and allow productivity to change in response to changes in demographics. I refer to this as the *without demographic scenario*. Second, I remove only the demographic-induced-saving effects. In this scenario, I again replace China's age-varying fertility and survival rates with those of the RoW but retain the usual productivity changes. I call this the *demographic-capital channel scenario*. Third, I focus solely on the demographic-induced-productivity changes. In this case, I maintain the original age-varying fertility and survival rates of China and its implied demographic process. However, I allow productivity to change as if China's demographic structure were aligned with that of the rest of the world (RoW), where China's age-varying fertility and survival rates mirror those of the RoW. I refer to this as the *demographic-idea channel scenario*. For each counterfactual scenario, I calculate the dynamic equilibrium as it transitions from one balanced growth equilibrium to another, guided by the corresponding exogenous processes under perfect foresight. All three scenarios start from the same initial equilibrium but converge to different final equilibria, each determined by the projected demographic processes at the final year.

Counterfactual analysis reveals that China's unique demographic characteristics—specifically, a lower fertility rate and higher survival rate relative to the rest of the world (RoW)—generate a short-run and long-run trade-off for real income per worker.

In the short run, the *demographic-capital channel* exerts a stronger influence than the *demographic-idea channel*. Under a lower fertility rate and higher survival rate compared to the RoW, China's demographic structure in the short run is characterized by a higher share of working-age and senior working-age populations. Consequently, this age structure, which is more conducive to savings, through the *demographic-capital channel*, increases the level of capital stock per worker, leading to higher real income per worker. In parallel, this structure also enhances China's comparative advantage in capital-intensive sectors compared to the counterfactual scenario. Specifically, compared to the counterfactual scenario in which China's fertility rate and survival rate are replaced by those of the RoW, by 2020, China's real income per worker is approximately 5.08% higher, and China's revealed comparative advantage index in capital-intensive sectors is around 6.79% higher than the counterfactual level. These short-run trends are projected to persist until 2060.

In the long run, however, the *demographic-idea channel* becomes more influential than the *demographic-capital channel*. After 2060, China's demographic process (characterized by a lower fertility rate and higher survival rate compared to the RoW) implies a reduced number of working-age population. Fewer workers suggest fewer new ideas, which in turn lower the productivity growth path. Consequently, the *demographic-idea channel* leads to a lower level of real income per worker than the counterfactual scenario. By 2070, compared with the counterfactual in which China's fertility rate and survival rate are substituted with those of the RoW, China's projected real income per worker is estimated to be approximately 4.08% less than the counterfactual level.

This paper contributes to three strands of literature. First, it connects to the literature on demographic structure and productivity, both empirically and theoretically. On the empirical side, several studies have examined the relationship between age structure and productivity at both the macro and micro levels. At the macro level, studies have shown that the age composition of populations has significant effects on a country's productivity Feyrer (2007); Maestas, Mullen, and Powell (2023). At the micro level, based on data from patents and innovation, Jones (2010); Azoulay, Graff Zivin, and Wang (2010) have shown that people's ability to generate ideas varies with age. These findings indicate that shifts in the age composition within a country may influence productivity through the quality of innovation and the generation of new ideas. In my paper, due to data availability, I replicate these results at the macro level, using more countries and more recent years.

On the theoretical side, this paper relates to literature exploring the mechanisms through which demographic changes influence productivity from three perspectives. First, some models incorporate age-dependent productivity, where the effectiveness of labor varies by age cohort, as seen in the work of Lindh and Malmberg (1999). Second, models of endogenous growth, such as those by Becker, Murphy, and Tamura (1990), show that demographic changes can alter incentives for investment in human capital, which in turn affects productivity growth. Third, recent work by Aksoy, Basso, Smith, and Grasl (2019) develops a framework where age structure affects a country's innovation rate by influencing the distribution of skills and experience across the population. In this paper, I model the demographic-productivity relationship through the assumption of age-varying ability in generating new ideas. Specifically, Buera and Oberfield (2020) builds the dynamics of knowledge stock through the process of exogenous idea arrival and learning from the external ideas distribution. I further assume that people of varying ages differ in their ability to find new ideas, and I introduce more sectors.

Second, this paper relates to the literature on multi-country trade models with capital accumulation. Among the most relevant is Sposi (2022), which examines how demographic transitions impact global trade imbalances through a multi-country Ricardian trade model.

Other papers in this field include Eaton, Kortum, Neiman, and Romalis (2016), Alvarez (2017), Ravikumar, Santacreu and Sposi (2019), Anderson, Larch, and Yotov (2020), and Sposi, Yi, and Zhang (2021a), with specific focuses other than demographics. However, my paper differs by linking capital accumulation with demographics, offering a novel perspective on the interplay between demographics, trade-induced relocation, and economic growth.

Third, this paper is related to two strands of literature on trade and the Chinese economy. The first strand involves research on explaining and quantifying the forces driving China's growth through a trade perspective, either at the aggregate or distributional level. This research often incorporates internal migration and internal trade across China's regions into trade models, emphasizing the effects of internal migration or internal trade to varying degrees, depending on the paper's focus, such as in Liu and Ma (2018); Tombe and Zhu (2019); Fan (2019); Hao, Sun, Tombe, and Zhu (2020); Ma and Tang (2020). In addition, there is a wealth of research focused on explaining changes in China's trade patterns per se. This strand is represented by papers such as those by Alessandria, Khan, Khederlarian, Ruhl, and Steinberg (2021); Hanwei, Jiandong, and Yue (2024). My paper focuses on quantifying both trade pattern changes and economic growth from a demographic perspective, which has yet to be explored in previous papers.

The rest of the paper is organized as follows. Section 2 presents some motivating facts. Section 3 lays out the model and discusses the mechanisms. Section 4 describes the data, calibrates the main parameters and shocks, and then briefly discusses the shocks I backed out. Section 5 presents counterfactual structural decomposition experiments, and Section 6 concludes.

The remaining of the paper is organized as follows. Section 2 provides the empirical analysis. Section 3 describes the model and conducts some numerical experiments to demonstrate how the model works. Section 4 describes the data, calibrates the main parameters and shocks, and then briefly discusses the calibrated shocks and model fitness. Section 5 conduct counterfactual analysis, and Section 6 concludes.

2. Empirical Evidence

In this section, I present the data, empirical model, empirical results. The empirical part is divided into two sub-parts. In the first part, I estimate the relationships between demographic structure and various macroeconomic outcomes such as TFP growth, consumption share of GDP, and investment share of GDP using country-level panel regression. In the second part, I employ panel regression and a panel VARX model to investigate the impact of trade costs and demographics on capital-labor ratios. By employing these methods, I aim to assess the effects of demographics and/or trade liberalization on various macroeconomic outcomes and analyze the dynamic effects of these shocks.

2.1. Data

I use panel data encompassing 76 countries across different income levels, spanning the period from 1975 to 2019. The selection of country groups and time periods for analysis was based on data availability, resulting in 34 high-income countries, 21 upper middle-income countries, 16 lower middle-income countries, and 5 lower-income countries according to the United Nations classifications. For detailed information about the country list, please refer to Table C.4.

The empirical analysis incorporated four types of variables. Firstly, there were 7 types of demographic structure indices, including the dependency ratio, young dependency ratio, old dependency ratio, working age share, young population share, old population share, and population distribution across different age cohorts. These variables were calculated or directly obtained from the United Nations World Population Prospects report. Secondly, the total factor productivity (TFP) growth was calculated based on TFP data from the Penn World Table 10.01. The capital-labor ratio was also derived from this dataset. Thirdly, various other macroeconomic outcomes, including investment, consumption share of GDP, and real GDP per capita, were acquired from the World Development Indicators database. Lastly, the trade cost index was calculated based on data from the CEPII database. For more information regarding the data sources and variable construction, please refer to Appendix C.

2.2. Demographics, technology change, and other macroeconomic outcomes

This section presents the empirical panel regression model that examines the relationship between demographics and Total Factor Productivity (TFP) change, along with macro variables such as capital formation, and consumption.

2.2.1. The effect of demographic structure on technology change

Regression model

In order to examine the relationship between demographic variables and technology change, I consider a panel regression with both country and time fixed effects.

$$GRTFP_{it,t+4} = Constant + \beta_1 Demographic_{it} + \beta_2 Control_{it} + f_i + f_t + \varepsilon_{it} \tag{1}$$

where i means country, t means year. The dependent variable $GRTFP_{it,t+4}$ means average TFP growth rate (%) for country i during the period from t to $t+4$, and calculated as follows:

$$GRTFP_{it,t+4} = \left(\frac{TFP_{i,s+4}}{TFP_{i,s}}\right)^{\frac{1}{4}} - 1$$

The variable $Demographic_{it}$ represents demographic-relevant variables for country i at time t, such as the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The old dependency ratio is defined as the ratio of people aged 65 and above to people aged 15-64. The working age share is defined as the share of people aged 15-64. The young population share is defined as the share of people aged 0-14, and the old population share is defined as the share of people aged 65 and above, or the population share at different age cohorts. The variable $Control_{it}$ represents a control variable for country i at time t, specifically the initial log real GDP per person. f_i and f_t are country and time fixed effects.

As is common in the literature, I reduce the influence of business cycle fluctuations by calculating 5-year growth rates and dividing the entire period of 1975–2019 into nine non-overlapping 5-year sub-periods period 1 (1975–1979), period 2 (1980–1984), period 3 (1985–1989), period 4 (1990–1994),..., and period 9 (2015–2019). Since I treat all demographic variables measured at the start of the sample period, before the growth has occurred, the regression analysis carries a stronger sense of causality. Similarly, in the following other types of regression, the time lag between the independent variables and the dependent variable reduces the likelihood of endogeneity issues caused by reverse causality. For the purpose of conducting robust checks, I performed the same regression analysis for five non-overlapping 8-year periods: period 1 (1980-1987), period 2 (1988-1995), period 3 (1996-2003), period 4 (2004-2011), and period 5 (2012-2019). The results of these robust regressions can be found in Appendix C.

I would expect that countries with a higher proportion of their population in the working age group would exhibit a higher TFP growth rate, indicating a positive relationship. Thus, I expect a positive sign for the coefficient $\hat{\beta}_1$ when using the working age share as an index for demographics. Conversely, for the share of young people and elderly, as well as the dependence ratios (including both young and old dependence ratios), I anticipate a negative sign for the coefficient $\hat{\beta}_1$. I would also expect negative sign for $\hat{\beta}_2$, which indicates that the developed countries usually showing a slowdown TFP growth.

Results

Table 1 and Table 2 report the main results of the regression analysis. In Table C.10, columns (1) to (4) use different indices to capture changes in age structure, ensuring the robustness of the empirical estimates. In Table 3, indices such as patent applications per 1000 people and industrial design applications per 1000 people are also used to represent TFP changes as robust checks. Table 1 and Table 2 reports the main results of regression. In Table C.10, In columns (1) to (4), I use different index to capture the changes of age structure to ensure robustness of empirical estimates. .

Overall, the estimation results are consistent with the expected signs expect for the estimators of old dependency ratio and share of elderly. It shows that increasing young people share or decreasing working age share related to TFP growth rate decline, while increasing elderly share has positive but no significant effects on TFP growth rate. This results is similar with Kögel (2005).

Specifically, 1 percentage point (p.p) increase, or 1 s.d. increase, in the working age share is associated with a corresponding increase of 0.11 percentage points (p.p), or increase of 0.81 s.d., in the average TFP growth rate over the following 4-year period. Additionally, an increase of 1 percentage point (p.p), or 1 s.d. change, in the child (age below 15) share is associated with a corresponding decrease of 0.14 percentage points (p.p), or decrease of 1.72 s.d., in the average TFP growth rate over the following 4-year period. However, changes in the elderly (age 65 and above) share do not have a significant effect on TFP growth.

In Table 2, to account for different age cohorts and their effects on the TFP growth, the population is further decomposed into 3, 4, or 5 distinct age cohorts. To ensure the inclusion of all types of age cohort dummy variables, the regression model is specified without a constant term. The results of the regression analysis reveal interesting patterns. In the 3-cohort regression, it is found that the share of the population aged 15 to 64 (15-64) has the most beneficial impact on the TFP growth rate. In the 4-cohort and 5-cohort regressions, the most beneficial cohorts are observed to be those aged 50 to 74 and 60 to 79, respectively. This implies that a large share of extreme young or old individuals is not as beneficial as a share of individuals at the median age.

In addition to the regression analysis, polynomial curves of different orders are employed to fit the estimated coefficients obtained from the cohort regressions, as shown in Figure 1, Figure 2, and Figure 3. This finding further reinforces the notion that a substantial proportion of extremely young or old individuals is not as advantageous as a share of individuals at the median age. The graphical representation provides visual evidence of the inverse U shape for the relationship between demographic structure and its impact on the TFP growth, providing valuable insights for the modeling.

TABLE 1

THE EFFECT OF DEMOGRAPHIC STRUCTURE CHANGE

	Average value in the future 4 years			
VARIABLES	Δ TFP/TFP	Cap.F.(% GDP)	Cons.(% GDP)	Δ k / k
Work.Share (15-64)/ToT	11.43***	28.80**	-33.75**	13.34**
	(3.33)	(2.17)	(-2.00)	(2.49)
Trade Cost				-0.87**
				(-2.27)
Initial.Log.Dependent	-3.09***			-2.24***
	(-4.82)			(-4.12)
PoP.Growth				-28.93
				(-1.55)
Constant	20.96***	4.09	98.56***	22.19***
	(3.65)	(0.53)	(9.84)	(3.63)
Observations	732	724	725	758
R-squared	0.259	0.575	0.753	0.586
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable $Child.Share$ represents the young population share, which is defined as the share of people aged 0-14, and the variable $Old.Share$ represents old population share, which is defined as the share of people aged 65 and above.

TABLE 2

THE EFFECT OF DEMOGRAPHIC STRUCTURE CHANGE

	Average value in the future 4 years			
VARIABLES	Δ TFP/TFP	Cap.F.(% GDP)	Cons.(% GDP)	Δ k / k
Child.Share (0-14)/ToT	-13.98***	-24.76*	33.75*	-11.22*
	(-3.68)	(-1.74)	(1.99)	(-1.82)
Old.Share (65+)/ToT	2.79	-65.97***	33.77	-24.65**
	(0.39)	(-2.65)	(0.90)	(-2.38)
Trade Cost				-0.83**
				(-2.13)
Initial.Log.Dependent	-3.46***			-1.99***
	(-4.77)			(-3.45)
PoP.Growth				-33.14*
				(-1.84)
Constant	35.46***	34.10***	64.81***	32.98***
	(5.19)	(6.74)	(9.15)	(5.32)
Observations	732	724	725	758
R-squared	0.266	0.581	0.753	0.589
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable $Child.Share$ represents the young population share, which is defined as the share of people aged 0-14, and the variable $Old.Share$ represents old population share, which is defined as the share of people aged 65 and above.

TABLE 3

THE EFFECT OF DEMOGRAPHIC STRUCTURE ON TECHNOLOGY CHANGE

VARIABLES	TFP growth rate	Patent.Applications (per 1000 people)	Industrial.Design.Applications (per 1000 people)
		Average value in the future 4 years	
Work.Share	11.43***	1.53***	1.36***
(15-64)/ToT	(3.33)	(3.12)	(4.78)
Initial.Log	-3.09***		
.Dependent	(-4.82)		
Constant	20.96***	-0.92***	-0.76***
	(3.65)	(-3.06)	(-4.39)
Observations	732	395	215
R-squared	0.259	0.826	0.913
Time FE	YES	YES	YES
Country FE	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable $Child.Dep.R$ represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable $Old.Dep.R$ represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable $Child.Share$ represents the young population share, which is defined as the share of people aged 0-14, and the variable $Old.Share$ represents old population share, which is defined as the share of people aged 65 and above.

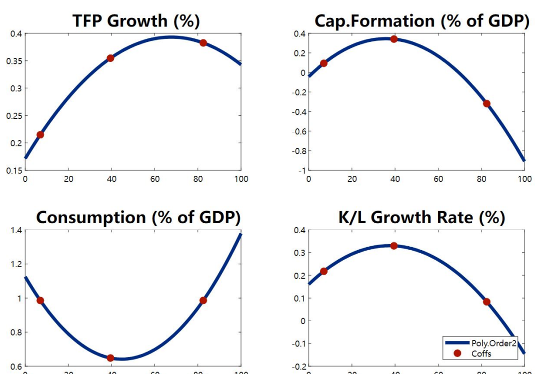


FIGURE 1

3 COHORTS

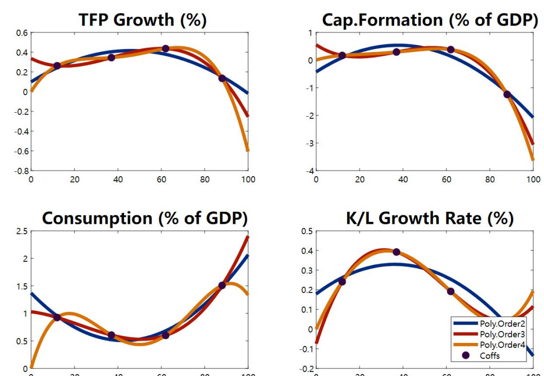


FIGURE 2

4 COHORTS

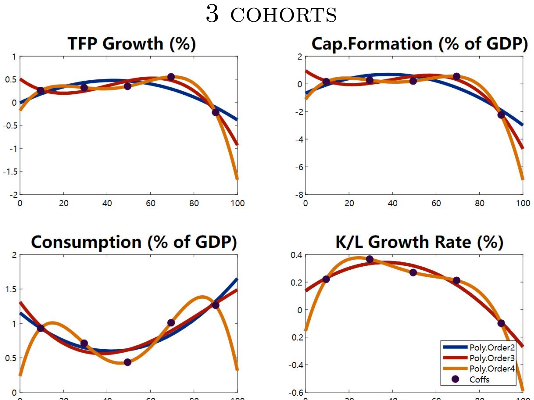


FIGURE 3

5 COHORTS

2.2.2. The effect of demographic structure on other macroeconomic outcomes

Regression model

The relationship between demographic variables and macroeconomic outcomes (capital formation, and consumption) is examined by estimating the following model

$$Ave.Y_{it,t+4} = Constant + \beta_1 Demographic_{it} + f_i + f_t + \varepsilon_{it} \tag{2}$$

The variable $Ave.Y_{it,t+4}$ means average investment, or consumption share of GDP (%) during the period t to $t+4$:

$$Ave.Y_{it,t+4} = \sum_{s=t+0}^{t+4} \frac{Y_{i,s}}{5}$$

The remaining variables used in the analysis are consistent with those mentioned in subsubsection 2.2.1. Since young and old people tend to save less and consume more compared to the working-age people, I would expect that countries with a larger share of working-age people are associated with a higher investment share of GDP, and a lower consumption share of GDP. Consequently, when the demographic index used is the working age share, I anticipate a positive estimated coefficient, $\hat{\beta}_1$, for investment share of GDP. On the other hand, if the demographic index is the share of young people and elderly, as well as the dependence ratios (including both young and old dependence ratios), I expect the estimated coefficient for investment to be negative. For the consumption share of GDP, the expectations are reversed. Specifically, when the demographic index used is the working age share, I anticipate a negative estimated coefficient for consumption. However, if the demographic index is the share of young people and elderly, as well as the dependence ratios (including both young and old dependence ratios), I expect the estimated coefficient for consumption to be positive.

Results

Table 1 and Table 2 reports the main regression results, and Table C.12 and Table C.13 using different indices to capture changes in age structure to ensure robustness of empirical estimates. Overall, the estimation results are consistent with the expected signs. The findings indicate that countries with a larger proportion of working-age individuals , or a lower share of elder or young people, or a lower dependency ratio tend to exhibit higher shares of investment in GDP, along with a lower share of consumption in GDP. While the increasing share of elderly population (age 65 and above) does not appear to have a significant impact on consumption.

Specifically, a 1 percentage point (p.p) increase, or 1 s.d. increase, in the working age share leads to a corresponding increase of 0.29 percentage points (p.p), or an increase of 0.33 s.d., in the average capital formation share of GDP over the next four years. On the other hand, a 1 percentage point (p.p) increase, or 1 s.d. increase, in the working age share is associated with a subsequent decrease of 0.34 percentage points (p.p), or an increase of 0.21 s.d., in the average consumption share of GDP over the following four years. The rise in the share of young population (age below 15) has an inverse impact on these two shares compared to the increase in working age share. However, increase in the elderly share have positive but not significant effect on consumption shares of GDP. For the capital formation share of GDP, It would be 0.66 percentage points (p.p) lower, or 0.65 s.d. lower if the elderly share is 1 percentage point (p.p) higher, or 1 s.d. higher.

Furthermore, Table 2 provides additional regression results that take into account the effects of the share of different age cohorts on investment, and consumption. The population is further divided into 3, 4, or 5 distinct age cohorts to capture the heterogeneity within the demographic structure. Figure 1, Figure 2, and Figure 3 depict the polynomial curves of different orders fitted to the estimated coefficients derived from the cohort regressions. These figures illustrate the relationship between age and the shares of consumption, and investment in GDP. The curves reveal an inverse U-shaped relationship between age and the investment shares of GDP. This suggests that countries tend to have higher investment shares when the population is composed of individuals at intermediate ages, while extreme youth or old age groups have a relatively smaller influence on these shares. In contrast, the relationship between age and the consumption share of GDP follows a U-shaped pattern. This implies that countries with a larger proportion of individuals at younger or older ages tend to have higher consumption shares in GDP. These findings highlight the importance of considering the age structure's impact on consumption, and investment behaviors.

2.3. Demographics, trade cost, endowments change and economic growth

In this section, I employ panel regression and VARX model to explore the relationship between demographics, globalization, capital endowments, and economic growth. The capital labor ratio is utilized as indicators of the relative size of capital and labor endowment. The analysis aims to examine how changes in demographics and trade costs impact the accumulation of capital endowments and economic growth.

2.3.1. The effects of demographic structure and trade cost change on capital-labor ratio

Regression model

The effect of demographic variables and trade cost change on capital labor ratio is examined by estimating the following model

$$GR.K/L_{it,t+4} = Constant + \beta_1 Demographic_{it} + \beta_2 TradeCost_{it} + \beta_3 Control_{it} + f_i + f_t + \varepsilon_{it} \tag{3}$$

The variable $GR.K/L_{it,t+4}$ means average capital per person (k) growth rate (%) for country i during the period from t to $t+4$, and calculated as follows:

$$GR.K/L_{it,t+4} = \left(\frac{k_{i,s+4}}{k_{i,s}}\right)^{\frac{1}{4}} - 1$$

The trade cost for country i at time t $TradeCost_{it}$ are constructed as the Head-Ries (HR) index (Head and Ries, 1997). I calculated it as follows:

$$TradeCost_{it} = \left(\frac{\pi_{i,row}}{\pi_{row,row}}\frac{\pi_{row,i}}{\pi_{ii}}\right)^{-\frac{1}{2\theta}}$$

Where $\pi_{i,row}$ is the share of country i's total expenditure on goods from Rest of the World (ROW) at t and $\theta = 4$ is index which capture the trade elasticity. To calculate the $\pi_{i,row}$, I still need the total output X_i of country i and total output X_{row} for Rest of the world with respected to country i. Due to data availability constraints, I made simplifications in the analysis. Specifically, I assumed a fixed labor share of $\alpha = 0.7$ in the production function. The total output, denoted as X_i, was calculated $X_i = \dfrac{GDP_i}{\alpha}$. I also calculated the trade cost variable for two cases: one with $\alpha = 1$ and another with $\alpha = 0.5$. The computed trade costs exhibited similar trends across all three cases. I utilized the trade cost calculated with $\alpha = 1$. The variable *Control* means control variable, and it includes the initial log capital per person, and population growth rate during period t to $t+4$. The remaining variables used in the analysis are consistent with those mentioned in subsubsection 2.2.1.

I would expect a country with a larger share of working-age people is associated with a higher growth rate of capital per person. So when the demographic index used is the working age share, $\hat{\beta}_1 > 0$, and if demographic index is the share of young people and elderly, as well as the dependence ratios (including both young and old dependence ratios), $\hat{\beta}_1 < 0$. Furthermore, Sposi, Yi and Zhang (2021b) shows that trade integration stimulates capital accumulation by reducing investment prices. Therefore, I would expect that a country with lower trade costs is associated with a higher growth rate of capital per person ($\hat{\beta}_2 < 0$).

Results

Table 1 and Table 2 presents the main regression results analyzing the relationship between age structure and capital per person growth rate, while Table C.15 considering different indices to capture changes in age structure. Overall, the estimation results are consistent with the expected signs. The results indicate that countries with a larger proportion of working-age individuals or lower dependence ratios tend to experience higher capital per person growth rates. Additionally, the findings suggest that a decrease in trade costs leads to higher capital per person growth rates, highlighting the positive impact of trade globalization on capital accumulation.

Specifically, a 1 percentage point (p.p) increase, or 1 standard deviation (s.d.) increase, in the working age share yields a subsequent increase of 0.13 percentage points (p.p), or an increase of 0.41 s.d., in the average capital-labor ratio growth over the next four years. For the trade cost, It would be around 0.93 percentage points (p.p) lower, or 0.38 s.d. lower if the trade cost is 1 unit higher, or 1 s.d. higher.

To further investigate the effects of different age cohorts on capital per person growth rate, Table 2 decomposes the population into 3, 4, or 5 distinct age cohorts. Figure 1, Figure 2, and Figure 3 depict the polynomial curves of different orders fitted to the estimated coefficients obtained from these cohort regressions. The curves visually confirm the inverse U-shaped relationship between age and capital per person growth rate. This observation implies that age structure can also affect comparative advantage forces. By influencing the accumulation or relative size of capital endowments, age structure indirectly impacts a country's growth through comparative advantage in endowments and trade.

2.3.2. VARX: Dynamic effects of demographics shock and trade cost shock

To further analyze the dynamic effects of demographic shocks and trade cost shocks on macroeconomic outcomes, a VARX model is employed. Here I focus on how changes in demographics and trade costs impact the accumulation of capital endowments, ultimately influencing economic growth.

VARX model

The dynamic effects of demographics shock and trade cost shock is examined by estimating the following VARX model:

$$Y_{n,t} = C + AY_{n,t-1} + BX_{n,t-1} + \varepsilon_{n,t}$$

Endogenous variable list:

$$Y_{n,t} = \begin{bmatrix} \text{the 5 year growth rate of } TFP \ (\%) \\ \text{the 5 year growth rate of the real GDP per capita } (\%) \\ \text{the 5 year growth rate of capital per person } (\%) \end{bmatrix}_{Country\ n, time\ t}$$

Exogenous variables: Demographic Structure (age shares):

$$X_{n,t-1} = \begin{bmatrix} young\ people\ share\ (\%),\ (0-14) \\ old\ people\ share\ (\%),\ (65+) \\ trade\ cost\ change\ (\%) \\ the\ 5\ year\ growth\ rate\ of\ population (\%) \end{bmatrix}_{Country\ n, time\ t-1}$$

Since demographics variables changes slowly over time, I calculate above index at every five year level and dividing the entire period of 1975–2019 into nine 5-year sub-periods: period 1 (1975–1979), period 2 (1980–1984), period 3 (1985–1989), period 4 (1990–1994),..., and period 9 (2015–2019). The growth rate variable is calculated as 5-year growth rates and the level variable are calculated as 5-year average values. The unit time lag here is 5 years. (e.g. t = 1 means first 5 years.)

Results

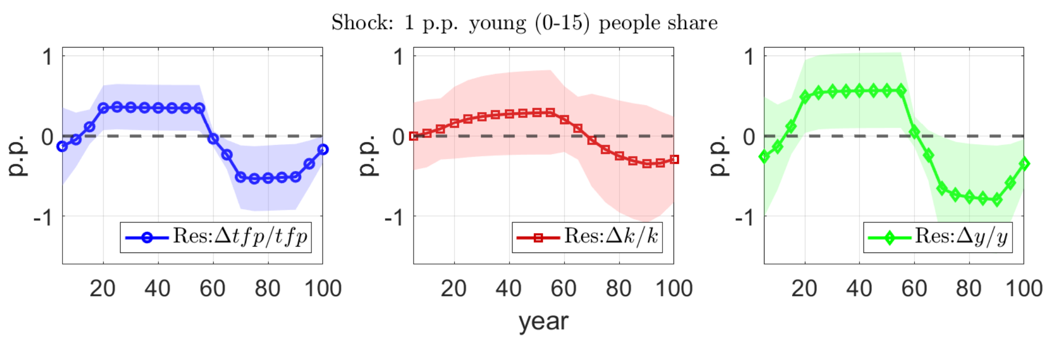


FIGURE 4
IRF OF EXOGENOUS DEMOGRAPHIC SHOCK

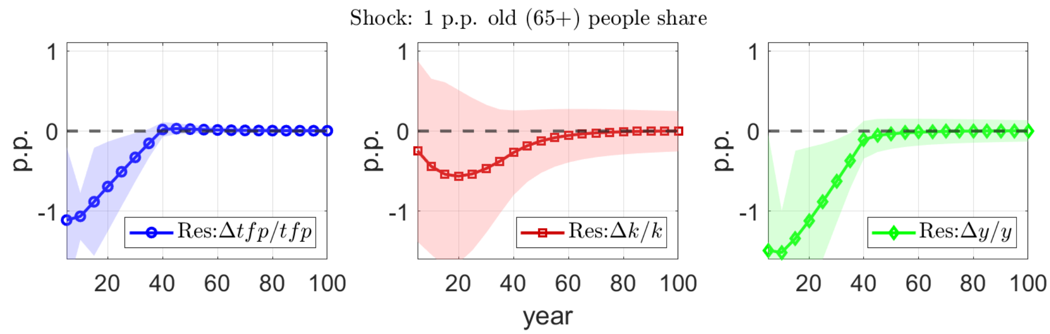


Figure 5
IRF of exogenous demographic shock

In Figure 4 and Figure 5, I plot the impulse response function illustrating the impact of demographic shocks on variables such as TFP growth, real GDP per capita growth, and capital per person growth. The graph visually represents the dynamic effects of these shocks on the specified variables over time. For detailed information on all the impulse response functions, please refer to Table C.10.

In general, one unit (1 percentage point (p.p)) positive shock to the share of younger or older people leads to an initial decrease in the growth rates of capital per person, TFP, and real GDP per person. For younger people, as they age, the effects show a hump-shaped pattern. For older people, as they pass away, the effects converge and eventually recover back to zero.

A one-unit positive shock to trade costs (higher trade costs) negatively impacts the growth rates of capital per person, real GDP per person, and TFP. The effects on capital per person and real GDP per person align with the literature, which shows that trade liberalization increases a country's real GDP and stimulates capital accumulation through lower capital prices. Regarding TFP growth, this suggests that changes in trade costs may affect TFP growth through trade-induced technology diffusion.

2.4. Summary

In this section, I first investigate empirically the effects of demographics and/or trade liberalization on TFP growth and other macroeconomic outcomes, employing panel data regression techniques. Second, I examined the dynamic effects of these shocks utilizing a panel Vector Autoregressive with Exogenous Variables (VARX) model.

The findings highlight the significant impact of demographic structure, particularly the share of working-age individuals, on TFP growth, investment, and consumption. Countries with a higher proportion of working-age people tend to exhibit higher TFP growth rates and greater shares of investment in GDP, while having a smaller consumption share. Additionally, the study explores the effects of demographics structure and trade costs on capital-labor ratios. This ratio is utilized as indicators of the relative size of capital and labor endowment. It shows that higher trade costs and a larger share of working-age population are associated with higher growth rates of the capital-labor ratio. To capture the dynamic effects of demographic shocks and trade cost shocks, a VARX model is employed. The results show that a positive shock to the share of younger or older people leads to an initial decrease in the growth rates of capital per person, TFP, and real GDP per person. For younger people, as they age, the effects exhibit a hump-shaped pattern. For older people, as they pass away, the effects converge and eventually return to zero.

The empirical analysis in this study underscores the pivotal role of demographic structure, particularly age distribution, in influencing TFP growth, investment, and consumption. It also sheds light on how trade costs affect capital accumulation and economic growth. These findings offer valuable insights into the complex relationships between demographics, capital accumulation, TFP changes, and trade liberalization. In the next section, I will develop a life-cycle model of demographics and trade, aligned with the key results presented here.

3. Model: multi-sector open economy

In this section, I first develop an overlapping generations (OLG) trade model that integrates the empirical features identified earlier. I then characterize both the stationary balanced growth equilibrium and the dynamic transitional equilibrium. Finally, I conduct several numerical experiments to demonstrate how the model operates. Using this framework, I aim to investigate China's past growth and provide model-based projections to assess the impacts of demographic forces on China's future growth.

The theoretical framework builds on Ravikumar, Santacreu and Sposi (2019) and Sposi, Yi and Zhang (2021b), with three key extensions. First, I integrate age-productivity links into the model, following the approach of Lucas Jr (2009), Alvarez, Buera, Lucas, et al. (2013), Alvarez, Buera, Lucas, et al. (2008), and Buera and Oberfield (2020). Age-varying abilities to generate ideas become key mechanisms behind the knowledge stock accumulation, which I refer to as "Demographic-induced knowledge stock change." Second, I incorporate overlapping generations (OLG) features similar to those in Sposi (2022), where age-varying saving behavior influences capital accumulation. These forces interact with demographic changes, which I refer to as "Demographic-induced capital stock change." Third, I distinguish between capital-intensive and labor-intensive sectors. By considering multiple sectors and incorporating both capital and labor into the production function, the model implicitly captures the forces of both Heckscher-Ohlin and Ricardian comparative advantage.

There is no uncertainty, and households have perfect foresight. The economy has N regions or countries and J sectors. Time is discrete and denoted by year t. In the notation below, country-specific parameters and variables have subscript n, cohort-specific variables have subscript g, and variables that vary over time have subscript t.

3.1. Households

In each region, households follow an overlapping generations model. Economic activity begins when individuals reach age $g = 1$ and continues until either death or they reach the maximum age $g = G$. Households supply labor inelastically during their working years, specifically when g falls within the interval $[G_0 + 1, G_1]$, with $l_g = 1$. After retirement, which occurs when $g \in [G_1 + 1, G]$, their labor supply drops to $l_g = 0$.

$$l_g = \begin{cases} 0 \; if \;\; g \in [1, G_0] \\ 1 \; if \;\; g \in [G_0 + 1, G_1] (4) \\ 0 \; if \;\; g \in [G_1{+}1, G] \end{cases}$$

Let $N_{n,g,t}$ represent the number of households at age g that are alive at time t. We define $N_{n,t}$ as the total population in the economy at any given period t, which can be expressed as:

$$N_{n,t} \equiv \sum_{g=1}^{G} N_{n,g,t} \qquad \forall t. \tag{5}$$

The labor endowment for age group g, denoted $l_{n,g}$, is adjusted for human capital (schooling), $E_{n,t}$, which is derived from the human capital measure available in the PWT10.01 dataset.[1] Among the working-age population, a fraction $\tau^L_{n,t}$ are excluded from employment due to factors unrelated to age, such as business cycle conditions, female labor force participation rates, or distortions in the labor market. Hereafter, $\tau^L_{n,t}$ will be referred to as a labor market distortion. Let $L_{n,t}$ denote the total labor supply, and $L^e_{n,t}$ the total labor supply adjusted for human capital. These can be defined as:

$$L_{n,t} = \left(1 - \tau^L_{n,t}\right) \sum_{g=G_0+1}^{G_1} N_{n,g,t} l_g = \left(1 - \tau^L_{n,t}\right) \sum_{g=1}^{G} N_{n,g,t} l_g \qquad \forall (n, t), \tag{6}$$

and

$$L^e_{n,t} = E_{n,t} L_{n,t}. \tag{7}$$

To simplify the notation, I omitted country subscript n here. Population dynamics in the model are driven by exogenous fertility and survival rates. The variable $N_{g,t}$ represents the number of individuals aged g alive at time t. Fertility rates for households of age g at time t are denoted by $f_{g,t}$, while $s_{g,t}$ represents the survival probability from age $g - 1$ to g.

1. This adjustment means that the wage rate, $W_{n,t}$, reflects the price of an efficiency unit of labor.

Notably, $s_{G+1,t} = 0$, indicating that individuals live no longer than age G.

The implied unconditional probability of surviving g periods up to time t is given by:

$$S_{g,t} = \prod_{k=1}^{g} s_{k,t+k-g}$$

The unconditional probability of people dying at age g at time t is:

$$D_{g,t} = S_{g-1,t-1}(1 - s_{g,t}) = S_{g-1,t-1} - S_{g,t}$$

The expected life expectancy for people born at t is:

$$\bar{g}_t = \sum_{k=1}^{G} k \cdot D_{k,t+k-1} \quad (1)$$

$$N_{1,t+1} = s_{1,t} \sum_{g=1}^{G} f_{g,t} N_{g,t}$$

$$N_{g+1,t+1} = s_{g+1,t+1} N_{g,t}$$

The demographic process can be represented as:

$$\begin{bmatrix} N_{1,t+1} \\ \vdots \\ N_{g,t+1} \\ \vdots \\ N_{G,t+1} \end{bmatrix} = \begin{bmatrix} s_{1,t+1}f_{1,t} & \cdots & s_{1,t+1}f_{g,t} & \cdots & s_{1,t+1}f_{G,t} \\ s_{2,t+1} & 0 & 0 & \cdots & 0 \\ 0 & s_{g+1,t+1} & 0 & \cdots & 0 \\ 0 & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & s_{G-1,t+1} & \cdots & 0 \\ 0 & 0 & 0 & s_{G,t+1} & 0 \end{bmatrix} \cdot \begin{bmatrix} N_{1,t} \\ \vdots \\ N_{g,t} \\ \vdots \\ N_{G,t} \end{bmatrix},$$

where $N_t = \Omega_t N_t$. Matrix Ω_t depends on fertility rates $f_{g,t}$ and survival rates $s_{g,t+1}$.

At time t, if the demographic process reaches a steady state or a balance growth path, the proportion of individuals aged g remains constant, denoted by $\tilde{N}_{n,g}$. Therefore, the population at age g in period t can be expressed as:

$$N_{n,g,t} = \tilde{N}_{n,g} N_{n,t} \tag{8}$$

3.1.1. Individual lifetime consumption and saving

For each country n, households of age g, born in period t, make decisions regarding their lifetime consumption $\{c_{n,g,t+g-1}\}_{g=1}^{G}$ and savings $\{b_{n,g+1,t+g}\}_{g=1}^{G-1}$ to maximize their expected lifetime utility, subject to the budget constraints and non-negativity constraints.

The utility derived from consumption in each period is modeled using a constant relative risk aversion (CRRA) function $u(c) = \dfrac{c^{1-1/\sigma}}{1-1/\sigma}$, where $u'(c) > 0$, $u''(c) < 0$, and as $c \to 0$, $\lim_{c\to 0} u(c) = -\infty$. The lifetime utility for a cohort born in country n at time t is defined as:

$$\max_{\{c_{g,t+g-1}\}_{g=1}^{G}\{a_{g+1,t+g}\}_{g=1}^{G-1}} \sum_{g=1}^{G} \psi_{n,t+g-1}\beta^{g-1}S_{g,t+g-1}u(c_{g,t+g-1})$$

The variable $\psi_{n,t}$ represents an external shock to the discount factor or saving wedges, accounting for influences on saving that are not explained by demographic factors in the model. These influences include factors such as capital taxes and other country-specific distortions. Meanwhile, β stands for the discount factor. The term $S_{g,t+g-1}$ denotes the unconditional probability that a household, born in period t, survives for g periods.

The budget and non-negativity constraints are given by:

$$\begin{aligned} P_{n,C,t+g-1}c_{n,g,t+g-1} + P_{n,I,t+g-1}a_{n,g+1,t+g} = {} & P_{n,I,t+g-1}\left(1 + r_{n,t+g-1}\right)a_{n,g,t+g-1} \\ & + W_{n,t+g-1}\left(1 - \tau^{L}_{n,t+g-1}\right)E_{n,t+g-1}l_g \\ & + ts^{D}_{n,t+g-1} + ts^{T}_{n,t+g-1} \quad \forall g \in [1, G] \end{aligned} \tag{9}$$

$$a_{1,t} = a_{G+1,t} = 0 \;\; and \;\; c_{n,g,t} > 0 \tag{10}$$

In each period t, if a household is of working age, it earns a wage $W_{n,t}$ by supplying 1 unit of labor inelastically to domestic firms. Households also receive returns on their savings and earn interest based on the savings from the previous period or previous age.

Households face a consumption-saving trade-off, deciding whether to save or borrow in financial markets, with the interest rate $r_{n,t}$ at time t. The amount of savings held by a household of age g, born in period t, and chosen in the prior period, is denoted as $a_{n,g,t+g-1}$. Similarly, households may save an amount $a_{g+1,t+g}$, which will accumulate interest for the next period. Households start with zero assets at age 1 and end with zero assets at age G: $a_{n,1,t} = a_{n,G+1,t} = 0$.

The term $tr^{D}_{n,t}$ denotes the accidental bequests relevant transfer. The accidental bequests left by households who passed away before age G accidentally between period $t-1$ to t are distributed equally across the total population as a lump-sum transfer, denoted as: [2]

$$tr^{D}_{n,t} \equiv P_{n,I,t}\left(1 + r_{n,t}\right)\frac{\sum_{g=2}^{G}\left(N_{n,g-1,t-1} - N_{n,g,t}\right)a_{n,g,t}}{N_{n,t}}, \qquad \forall t. \tag{11}$$

2. tr^{D}_{t} represents total accidental bequests available in period t from households who died in period $t-1$. Here, for the size of Individual at cohort $(s-1, t-1)$ and (s, t), the change of the size of individual can either counted as net death $(\eta_{n,g-1,t-1} - \eta_{n,g,t} > 0)$ or net immigrant $(\eta_{n,g-1,t-1} - \eta_{n,g,t} < 0)$ for country n. For the net death case, I treated it as positive bequests. For the net immigrant case, I assume that the net immigrant (g, t) enter country n with zero assets, and treated it as negative bequests. This assumption greatly simplifies the state vector of the model.

The term $tr^T_{n,t}$ refers to trade-related transfers, which are defined as the trade deficit divided by the total population Nn, t. This implies that the total net transfers are evenly distributed among the population.

$$tr^T_{n,t} \equiv \frac{D_{n,t}}{N_{n,t}}, \qquad \forall t. \tag{12}$$

Further details on the construction and definition of $D_{n,t}$ will be explained later.

3.1.2. Euler equation

For households at age $g \in [1, G]$ born at time t, Given the sequences of prices, transfers, households optimize on the intertemporal decisions of consumption and saving $\{c_{n,g,t+g-1},\ a_{n,g+1,t+g}\}$. The intertemporal Euler equation determines consumption and saving choices:

$$u'\left(c_{n,g,t+g-1}\right) = \left(\beta s_{n,g+1,t+g}\right)\left(\frac{\psi_{n,t+g}}{\psi_{n,t+g-1}}\right)\frac{\frac{P_{n,I,t+g}}{P_{n,C,t+g}}}{\frac{P_{n,I,t+g-1}}{P_{n,C,t+g-1}}}\left(1 + r_{n,t+g}\right)u'\left(c_{n,g+1,t+g}\right) \quad \forall\, g \in [1, G-1] \tag{13}$$

3.1.3. Structure of Consumption and investment

Define $C_{n,t}$ as aggregate consumption in country n and time t, which is a CES aggregate of sectoral consumption:

$$C_{n,t} \equiv \prod_{j=1}^{J} {C^j_{n,t}}^{\alpha^j_{n,C,t}} \tag{14}$$

where $\sum_{j=1}^{J} \alpha^j_{n,C,t} = 1$.

Similar relation for the aggregate investment and sectoral investment:

$$I_{n,t} \equiv \prod_{j=1}^{J} {I^j_{n,t}}^{\alpha^j_{n,I,t}} \tag{15}$$

where $\sum_{j=1}^{J} \alpha^j_{n,I,t} = 1$.

The overall aggregate price level is given by:

$$P_{I,n,t} = \prod_{j=1}^{J}\left[\frac{P^j_{n,t}}{\alpha^j_{I,n}}\right]^{\alpha^j_{I,n}}; P_{C,n,t} = \prod_{j=1}^{J}\left[\frac{P^j_{n,t}}{\alpha^j_{C,n}}\right]^{\alpha^j_{C,n}} \tag{16}$$

The structure of aggregate consumption and aggregate investment implies that:

$$P_{n,t}^{j} I_{n}^{j} = \alpha_{I,n}^{j} P_{I,n,t} I_{n,t}; P_{n,t}^{j} C_{n}^{j} = \alpha_{C,n}^{j} P_{C,n,t} C_{n,t} \tag{17}$$

or

$$\sum_{j=1}^{J} P_{n,t}^{j} C_{n,t}^{j} = P_{C,n,t} C_{n,t}; \sum_{j=1}^{J} P_{n,t}^{j} I_{n,t}^{j} = P_{I,n,t} I_{n,t} \tag{18}$$

3.2. Firms

The model consists of N countries, each with J sectors. These countries and sectors are interconnected through input-output linkages. Firms in country n and sector j produce a continuum of goods. Every variety within each sector is tradable and indexed by $\omega \in [0,1]$, using a constant returns to scale (CRS) technology:

$$y_{n,t}^{j}(\omega) \equiv q_{n,t}^{j}(\omega) \left[L_{n,t}^{j}(\omega)^{\beta_{n}^{j}} K_{n,t}^{j}(\omega)^{1-\beta_{n}^{j}} \right]^{\gamma_{n}^{j}} \prod_{k=1}^{J} m_{n,t}^{k,j}(\omega)^{\gamma_{n}^{k,j}} \tag{19}$$

Here, $m_{n,t}^{k,j}(\omega)$ represents the quantity of intermediate composite goods from sector k used as inputs to produce $y_{n,t}^{j}(\omega)$ units of sector j variety ω, while $K_{n,t}^{j}$ and $L_{n,t}^{j}$ denote the quantities of capital and efficient labor used, respectively. The CES production function implies that $\sum_{k=1}^{J} \gamma_{n}^{k,j} + \gamma_{n}^{j} = 1$.

The productivity of variety ω in country n and sector j, denoted $q_{n,t}^{j}(\omega)$, is a random variable that follows a Fréchet distribution: $F_{n,t}^{j}(q) = \exp\left(-\lambda_{n,t}^{j} q^{-\theta}\right)$. The mean of this distribution, $\lambda_{n,t}^{j}$, is commonly referred to as the knowledge stock.

I assume a perfectly competitive economy where firms require both capital and labor to operate. Households supply labor directly to firms, earning a wage rate denoted by W_t, and allocate their savings to financial markets, where they receive an interest rate r_t. Firms, on the other hand, borrow capital from financial markets at a rental rate R_t. Under the assumptions of zero frictions, zero profits in financial markets, and a financially autarkic economy, one can get:

$$r_{n,t} = \frac{R_{n,t}}{P_{n,I,t}} - \delta$$

3.2.1. Stock of knowledge

For the remaining paragraphs of this section, sector subscript j is omitted for simplicity.

For each economy n in which there is a continuum of intermediate varieties produced in the unit interval. For each variety, there is a large set of potential producers who have different technologies to produce the good. Each potential producer is characterized by the productivity of her idea, which we denote by q, to produce an intermediate variety.

Individuals of varying ages exhibit differences in their ability to generate new ideas. Between time t and time $t+1$, individuals of diverse ages within the economy can generate their new ideas to produce a variety, and producers are exposed to these new ideas to produce a variety. Both the number of new ideas and the productivity of them are stochastic, which generates randomness in the usage of the new ideas. The producer only adopts a new idea if the new ideas' productivity is greater than q.

Specifically, The number of new ideas per unit of time to which the producer is exposed follows a Poisson distribution. For each external new idea that arrives to the producer corresponds to a new productivity for producing a variety, given by

$$q = zq\prime^{\rho}.$$

Producers combine random "original components", z, with random "external insights" or "external idea", q', to generate new productivity. The "original component" originates from their internal source of ideas, drawn from their own distribution of original ideas $H(z)$. The "external insights" are insights obtained from the arrived new idea, drawn from the current productivity distribution $F_t(q')$. ρ measures the diffuse rate of new ideas, also known as learning intensity. The current productivity distribution describes the set of insights that producers might access.

Between time t and time $t+1$, individual varies at ages generate new ideas and local producers are exposed to these new ideas about producing the variety. Among the arrived new ideas during t and $t+1$, the probability that the best new idea has productivity no greater than q for each age g producer is defined as $F_{t,g}^{best\ new}$.

Under Assumption 1, One can get Proposition 1 which characterizes the C.D.F of the productivity of the best new idea.

Assumption 1

a) The number of new ideas arriving between t and $t+1$ follows a Poisson distribution with mean $\alpha_t \bar{z}^{-\theta}$. The term α_t controls the mean number of new ideas arriving between t and $t+1$.

b) Total mean number of new ideas arriaved between t and $t+1$ is defined as

$$\alpha_t \equiv \left[\sum_g \eta_g N_{n,g,t} \right]^{\varphi}$$

i) η_g controls the number of arrived new idea per unit of time per age g people

ii) $\varphi < 1$ reflects some kind of crowding effects, or duplication of idea. .

c) The Internal original idea z draw from Pareto distribution with C.D.F: $H(z) = 1 - (\frac{z}{\bar{z}})^{-\theta}$,

where $\bar{z}$ is lower bound of support and shape parameter $\theta > 1$.
d) $\beta \in [0, 1)$ measures the strength of ideas diffusion .
e) The productivity distribution F_t has sufficiently thin tail; i.e $\lim_{\bar{z}\to 0} \bar{z}^{-\theta} \left[1 - F_t\left(\left(\frac{q}{\bar{z}}\right)^{\frac{1}{\rho}}\right)\right] = 0$.

***Proposition 1**: under Assumption 1, between t and $t+1$, the C.D.F of the productivity of the best new idea is given by*

$$F_t^{best\ new}(q) = exp\left(-\alpha_t q^{-\theta} \int_0^\infty x^{\rho\theta} dF_t(x)\right), \text{ where } \alpha_t \equiv \left(\sum_g \eta_g N_{n,g,t}\right)^{\varphi}$$

in the limiting case when $\bar{z} \to 0$.

The distribution of the productivity of best new idea during t and $t+1$ and local productivity distribution at t together determine the next period's local productivity distribution:

$$F_{t+1}(q) = F_t^{best\ new}(q)\, F_t(q)$$

Assume that the initial frontier of knowledge at time t follows a Fréchet distribution given by $F_t(q) = exp(-\lambda_t q^{-\theta})$, one can get the frontier of knowledge in the economy and the evolution of the stock of knowledge over time.

$$F_{t+1}(q) = exp\left[-q^{-\theta}\left(\lambda_t + \alpha_t \int_0^\infty x^{\rho\theta} dF_t(x)\right)\right], \text{ where } \alpha_t \equiv \left(\sum_g \eta_g N_{n,g,t}\right)^{\varphi}$$

and the law of motion for the knowledge stock is given by

$$\lambda_{n,t+1} - \lambda_{n,t} = \alpha_t (\lambda_{n,t})^{\rho}\, \Gamma(1-\rho), \text{ where } \alpha_t \equiv \left(\sum_g \eta_g N_{n,g,t}\right)^{\varphi} \tag{20}$$

Equation 20 implies that, on the one hand, a higher the number of arrived new idea per unit of time per age g people. (higher η_g) leads to a higher knowledge stock for the following period, which in turn implies a higher overall productivity level. On the other hand, at the steady state, λ_t grows at the constant rate g_λ, and $1 + g_\lambda = (1 + g_n)^{\varphi/(1-\rho)}$.

With sector subscripts,

$$\lambda_{n,t+1}^j - \lambda_{n,t}^j = N_n^{\varphi^j}\left(\lambda_{n,t}^j\right)^{\rho}\left[\sum_g \eta_g^j \bar{N}_{n,g,t}\right]^{\varphi^j} \Gamma(1-\rho)$$

3.2.2. Composite goods

The non-tradable sectoral composite intermediate good $Q_{n,t}^{j}$ in country n sector j is consistent with tradable intermediate good $q_{n,t}^{j}(\omega)$:

$$Q_{n,t}^{j} \equiv \left[\int_{0}^{1} q_{n,t}^{j}(\omega)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}} \tag{21}$$

$$Q_{n,t}^{j} = I_{n,t}^{j} + C_{n,t}^{j} + \sum_{k=1}^{J} \int_{0}^{1} m_{n,t}^{j,k}(\omega)\, d\omega \tag{22}$$

where σ is the elasticity of substitution between varieties. The sectoral composite intermediate good $Q_{n,t}^{j}$ has three uses. It can be used for final consumption in country n: $C_{n,t} \equiv \prod_{j=1}^{J} C_{n,t}^{j}{}^{\alpha_{n,C}^{j}}$. It can be used for investment: $I_{n,t} \equiv \prod_{j=1}^{J} I_{n,t}^{j}{}^{\alpha_{n,I}^{j}}$. Finally, it can be used as an intermediate in the production of individual goods ω in each sector k: $\int_{0}^{1} m_{n,t}^{j,k}(\omega)\, d\omega$.

The price of sectoral composite intermediate good $Q_{n,t}^{j}$ is $P_{n,t}^{j}$. Define total expenditures on good j by region n as $X_{n,t}^{j}$, and I have $P_{n,t}^{j} Q_{n,t}^{j} = X_{n,t}^{j}$.

Cost minimization under constant returns to scale implies that, within each sector, expenditure on factors and intermediate inputs exhaust the value of output:

$$W_{n,t} L_{n,t}^{e} = \sum_{j=1}^{J} \beta_{n}^{j} \gamma_{n}^{j} \sum_{i=1}^{N} \pi_{in,t}^{j} X_{i,t}^{j} \tag{23}$$

$$R_{n,t} K_{n,t} = \sum_{j=1}^{J} \left(1-\beta_{n}^{j}\right) \gamma_{n}^{j} \sum_{i=1}^{N} \pi_{in,t}^{j} X_{i,t}^{j} \tag{24}$$

$$W_{n,t} L_{n,t}^{e} + R_{n,t} K_{n,t} = \sum_{j=1}^{J} \gamma_{n}^{j} \sum_{i=1}^{N} \pi_{in,t}^{j} X_{i,t}^{j} \tag{25}$$

3.3. International Trade

Trade is subject to "iceberg" trade costs. One unit of a tradable good in sector j shipped from region i to region n require $\kappa_{ni,t}^{j} \geq 1$ units in i, and the trade cost within region equal to 1, $\kappa_{nn,t}^{j} = 1$. The unit price of an input bundle is given by:

$$c_{n,t}^{j} \equiv \Upsilon_{n}^{j} \left[\left(W_{n,t}\right)^{\beta_{n}^{j}} \left(R_{n,t}\right)^{1-\beta_{n}^{j}}\right]^{\gamma_{n}^{j}} \prod_{k=1}^{J} P_{n,t}^{k}{}^{\gamma_{n}^{k,j}} \tag{26}$$

where $\Upsilon_n^j \equiv \gamma_n^j \beta_n^{j-\gamma_n^j\beta_n^j}\gamma_n^j\left(1-\beta_n^j\right)^{-\gamma_n^j\left(1-\beta_n^j\right)}\prod_{k=1}^{J}\gamma_n^{k,j-\gamma_n^{k,j}}$.

As in Eaton and Kortum (2002), the fraction of country n's expenditures in sector j goods source from country i is given by:

$$\pi_{ni,t}^j = \frac{\lambda_{i,t}^j\left(c_{i,t}^j\kappa_{ni,t}^j\right)^{-\theta}}{\sum_{m=1}^N \lambda_{m,t}^j\left(c_{m,t}^j\kappa_{nm,t}^j\right)^{-\theta}} \tag{27}$$

and

$$P_{n,t}^j=A^j\cdot\left[\sum_{i=1}^N\lambda_{i,t}^j\left(\kappa_{ni,t}^j c_{i,t}^j\right)^{-\theta}\right]^{-\frac{1}{\theta}} \tag{28}$$

$$\pi_{ni,t}^j=\lambda_{i,t}^j\left(\frac{A^j c_{i,t}^j\kappa_{ni,t}^j}{P_{n,t}^j}\right)^{-\theta} \tag{29}$$

where $A^j \equiv \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$. The expenditure by region n of sector j goods from region i is defined as $X_{ni,t}^j$, with $X_{n,t}^j=\sum_{i=1}^N X_{ni,t}^j$. Total revenue of country n on sector j goods is defined as $Y_{n,t}^j=\sum_{i=1}^N X_{in,t}^j$. The expenditure share is defined as $\pi_{ni,t}^j=\frac{X_{ni,t}^j}{\sum_{i=1}^N X_{ni,t}^j}$.

I abstract international borrowing and lending and model trade imbalances as transfers between countries, following Caliendo, Parro, Rossi-Hansberg, and Sarte (2018). A predetermined share of GDP, $\phi_{n,t}$ is sent to a global portfolio, which in turn disperses a per-capita lump-sum transfer, T_t^P,to every country.[3] The net transfer, also recognized as trade deficit, are calculated as:

$$D_{n,t}=-\phi_{n,t}\left(R_{n,t}K_{n,t}+W_{n,t}E_{n,t}N_{n,t}\right)+N_{n,t}T_t^P \qquad For\ \forall t \tag{30}$$

Dividing by the total population $N_{n,t}$ implies that total bequests are equally distributed across the population.

Under the trade deficit D_n, trade balance condition is:

$$\sum_{j=1}^J\sum_{i=1}^N X_{in,t}^j-\sum_{j=1}^J\sum_{i=1}^N X_{ni,t}^j=NX_n=-D_{n,t} \tag{31}$$

3. While the share of GDP allocated to the global portfolio is exogenous, the proceeds are endogenous to clear the global market. This feature is particularly useful in the counterfactual analysis.

3.4. Aggregation and equilibrium dynamics

3.4.1. Aggregation

Three markets must clear in this model: the labor market, the capital market, and the goods market. Each of these equations amounts to a statement of supply equals demand.

Total demand of capital and labor:

$$\sum_{j=1}^{J}\int_{0}^{1} l_{n,t}^{j}(\omega)d\omega = N_{n,t} \tag{32}$$

$$\sum_{j=1}^{J}\int_{0}^{1} k_{n,t}^{j}(\omega)d\omega = K_{n,t} \tag{33}$$

Now, aggregate individual variables across cohorts, I have:

$$\begin{aligned} C_{n,t} &\equiv \sum_{g=1}^{G} N_{n,g,t}c_{n,g,t} \\ K_{n,t} &\equiv \sum_{g=2}^{G} N_{n,g-1,t-1}a_{n,g,t} \end{aligned} \tag{34}$$

Aggregating individual budget constraints across ages. the budget constraint at the aggregate level is

$$\begin{aligned} P_{n,C,t}\sum_{g=1}^{G} N_{n,g,t}c_{n,g,t} + P_{n,I,t}\sum_{g=1}^{G} N_{n,g,t}a_{n,g+1,t+1} &= P_{n,I,t}\left(1+r_{n,t}\right)\sum_{g=1}^{G} N_{n,g,t}a_{n,g,t} \\ &+ \sum_{g=1}^{G} N_{n,g,t}W_{n,t}\left(1-\tau_{n,t}^{L}\right)E_{n,t}l_{g} + \sum_{g=1}^{G} N_{n,g,t}ts_{n,t}^{D} + \sum_{g=1}^{G} N_{n,g,t}ts_{n,t}^{T} \end{aligned} \tag{35}$$

Aggregate investment is defined as

$$I_t \equiv K_{t+1} - (1-\delta)K_t \tag{36}$$

Equation 35 and Equation 36 implies:

$$P_{n,C,t}C_{n,t} + P_{n,I,t}I_{n,t} = \left(r_{n,t}+\delta\right)P_{n,I,t}K_{n,t} + L_{n,t}^{e}W_{n,t} + D_{n,t} \tag{37}$$

$$= R_{n,t}K_{n,t} + L_{n,t}^{e}W_{n,t} + D_{n,t}, \tag{38}$$

and

$$P_{n,C,t}C_{n,t} + P_{n,I,t}I_{n,t} = R_{n,t}K_{n,t} + L_{n,t}^{e}W_{n,t} + D_{n,t} \tag{39}$$

Finally, each composite good is used as an intermediate and as final consumption, total expenditure on a composite good in sector j, region n is:

$$X_{n,t}^{j} = \alpha_{C,n}^{j} P_{C,n,t} C_{n,t} + \alpha_{I,n}^{j} P_{I,n,t} I_{n,t} + \sum_{k=1}^{J} \gamma_{n}^{j,k} \left(\sum_{i=1}^{N} X_{in,t}^{k} \right) \tag{40}$$

where $\alpha_{C,n}^{j} P_{C,n,t} C_{n,t} + \alpha_{I,n}^{j} P_{I,n,t} I_{n,t}$ is final demand for good j by workers in region n.

3.4.2. Stationary balanced growth equilibrium

The world consists of $n = 1, \cdots, N$ countries. Each country is populated by G period lived overlapping generations. There is no uncertainty and agents have perfect foresight.

Definition 1: Stationary balanced growth equilibrium: A stationary balanced growth competitive equilibrium in the perfect foresight overlapping generations model with G period lived agents, and exogenous population dynamics, is defined as constant allocations of stationary consumption, capital and prices: $\left\{ \{c_{n,g}\}_{g=1,\ n=1}^{G,\ N},\ \{b_{n,g+1}\}_{g=1,\ n=1}^{G-1,N},\ \{W_n,\ R_n\}_{n=1}^{N} \right\}$, such that:

i. The households taking prices transfer and deficit as given, optimize lifetime utility.

ii. Firms taking prices as given, minimize production cost.

iii. Each country purchases intermediate varieties from the least costly supplier/country subject to the trade cost.

iv. All markets are clear.

v. The population distribution reaches a stationary steady-state distribution before the economy reaches a steady state.

I take world GDP as the numeraire. This means all prices are expressed in units of current world GDP. Table E.1 provides a list of equilibrium conditions that these objects must satisfy.

3.4.3. Dynamic transitional equilibrium

Definition 2: Dynamics equilibrium

Given a set of initial capital distributions and exogenous forces across countries and over time, the transitional dynamics equilibrium (equilibrium transition path) in the perfect foresight overlapping generations trade model with G-period lived agents is defined as allocations of consumption, capital and prices: $\left\{ \{c_{n,g}\}_{g=1,\ n=1}^{G,\ N},\ \{b_{n,g+1}\}_{g=1,\ n=1}^{G-1,N},\ \{W_n,\ R_n\}_{n=1}^{N} \right\}_{t=1,\cdots,T+1}$ satisfies the following conditions:

i. The households at different ages taking prices, transfer and deficit as given, optimize lifetime utility.

iii. Firms taking prices as given, minimize production cost.

iv. Each country purchases intermediate varieties from the least costly supplier/country subject to the trade cost.

v. All markets are clear.

At each point in time, I take world GDP as the numeraire $\sum_n R_{n,t}K_{n,t} + W_{n,t}L^e_{n,t} = c, \forall t$. That means all prices are expressed in units of current world GDP. Table E.2 provides a list of equilibrium conditions that these objects must satisfy.

3.5. Discussion

In this section, through numerical experiments on both the stationary balanced growth equilibrium and the dynamic transitional equilibrium, I discuss the key mechanisms of the model.

The differences in demographic structure arise from variations in survival and fertility rates, denoted as $s_{g,t}$, the probability of surviving to age g at time t, given that one was alive at age $g-1$, and $f_{g,t}$, the number of newborns from each age g cohort at time t. Figure 6 shows the age-varying survival rate and fertility rate for two steady states, one with a high average level and one with a low average level. The main difference in the left-side figure between the two survival rate series is that after age 35, the red line has a higher survival rate on average for each age above 35. The implied average lifespan for the red line is thus longer than for the blue. The main difference in the right-side figure is that, on average, the red line shows a higher level of fertility.

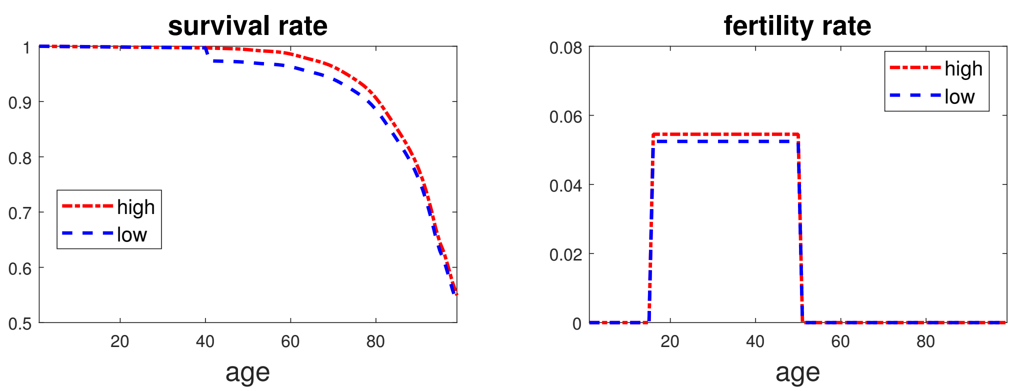


Figure 6
IRF of exogenous demographic shock

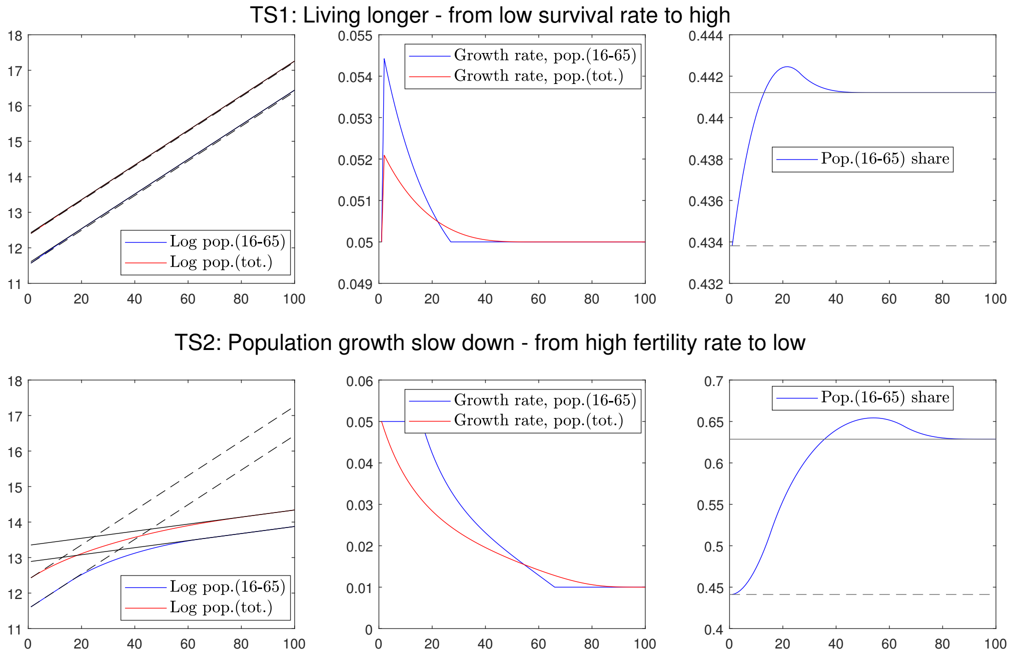


Figure 7
IRF of exogenous demographic shock

3.5.1. Demographics and knowledge stocks

$$\frac{\lambda_{t+1} - \lambda_t}{\lambda_t} = \Gamma\,(1-\rho)\,\alpha_t\,(\lambda_t)^{\rho-1}\,; \qquad \alpha_t \equiv (\sum_g \eta_g N_{g,t})^{\varphi} \tag{41}$$

To show how demographic structure affects knowledge stocks over time, I present a simple application. Assuming that the economy is on a balanced growth path in the initial period and that only working-age people contribute to new idea generation, we have:

$$\eta_g = c > 0 \quad \text{if} \quad g \in (16,65), \quad \text{and} \quad \eta_g = 0 \quad \text{if} \quad g \notin (16,65).$$

I conduct two counterfactual experiments to show how productivity responds to demographic shocks. Figure 8 and Figure 7 illustrate the effects of two types of shocks.

The left side of Figure 8 and Figure 7 shows the impact when Country 1 transitions from a low survival rate to a high survival rate, all else equal. The working-age population initially increases, leading to a rise in the knowledge stock. As the growth of the working-age population slows, the growth of the knowledge stock also decelerates, eventually matching the initial growth rate.

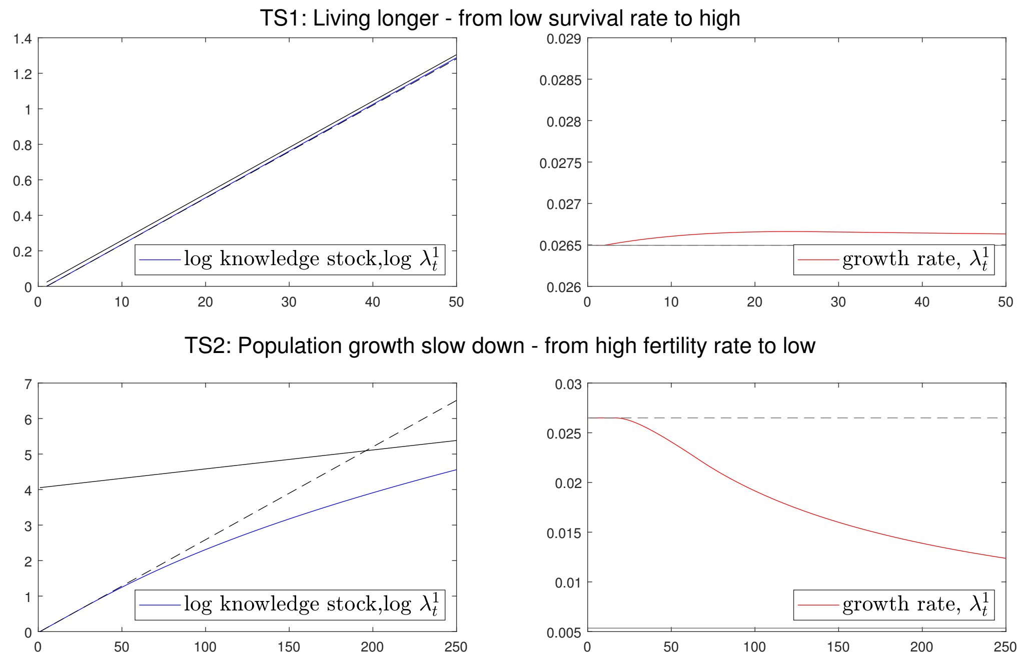


Figure 8
Demographics and knowledge stocks

The right side shows the effect of transitioning from a high fertility rate to a low fertility rate in Country 1. All else equal, the growth of both the total population and the working-age population slows, eventually stabilizing at a lower level compared to the previous growth path, with a growth rate that remains lower than before. As the working-age population growth slows, the growth of the knowledge stock also decelerates. Once the working-age population growth rate stabilizes at a lower level, the growth of the knowledge stock will stabilize at this lower rate.

Implications An increase in the level of the working-age population leads to a higher level of knowledge stock. On the balanced growth path, higher population growth implies higher knowledge stock growth.

3.5.2. Demographics and capital stocks

Besides the effects on knowledge stocks, in this paragraph, I further explore the effects of demographics on capital stocks. Specifically, I first compare balanced growth equilibria with different structures, then I show how capital and other variables change over time during the transition process from one balanced growth equilibrium to another.

Consider a 2x2 (two-country, two-sector) economy, where Sector 1 is labor-intensive and Sector 2 is capital-intensive. Table 4 presents the stationary balanced equilibrium under various conditions. The four columns are derived from a closed economy model. Columns (1S)

TABLE 4
STATIONARY BALANCE GROWTH EQUILIBRIUM COMPARISON. CLOSE ECONOMY

	(1S)	(2S)	(3S)	(1A)	
Country	sym.	sym.	sym.	cty1	cty2
Survival rate	low	high	high	high	low
Fertility rate	high	high	low	high	high
Average lifespan	60.1	70.8	70.8	70.8	60.1
Population growth	1.050	1.050	1.010	1.050	1.050
Implied TFP growth	0.025	0.025	1.003	1.025	1.025
Working age share	0.44	0.46	0.72	0.46	0.44
Trade cost	Autarky	Autarky	Autarky	Autarky	Autarky
Capital share of VA	0.5000	0.5000	0.5000	0.5000	0.5000
Per efficient person					
Capital stock	0.007	0.009	0.022	0.009	0.007
Output	0.0026	0.0029	0.0054	0.0029	0.0026
Consumption	0.0016	0.0017	0.0038	0.0017	0.0016
Investment	0.0010	0.0012	0.0016	0.0012	0.0010
Capital - efficient labor ratio	0.016	0.019	0.030	0.019	0.016
Price ratio					
Real wage rate	0.003	0.003	0.004	0.003	0.003
Real rental rate	0.179	0.165	0.125	0.165	0.179

to (3S) depict symmetric countries, while the countries in (1A) are asymmetric. Although the model operates under a closed economy framework, the symmetry or asymmetry does not significantly affect the results. This distinction becomes relevant when analyzing the effects of trade liberalization, as the closed economy counterparts serve as a control group for evaluating trade impacts.

Comparing columns $(2S)$ and $(1S)$, a higher survival rate allows individuals to live longer, which in turn enhances their ability to save. This increase in savings contributes to a larger supply of capital, resulting in a higher capital per efficient person. Meanwhile, when we compare column $(3S)$ with column $(2S)$, a lower fertility rate leads to a slowdown in population growth, which also implies a deceleration in productivity growth. Consequently, less capital is distributed among efficient individuals, thereby leading to a higher capital per efficient person. Finally, the capital-labor ratio reflects a relative abundance of capital compared to labor, highlighting the significant effects of these demographic changes on the economy.

Figure 9 illustrates how capital and other variables change over time following a positive shock to the survival rate. Basically, a higher survival rate stimulates capital accumulation and elevates the balanced growth path. In the short run, as the population of Country 1 lives longer, savings increase, resulting in higher capital per person until it reaches a new

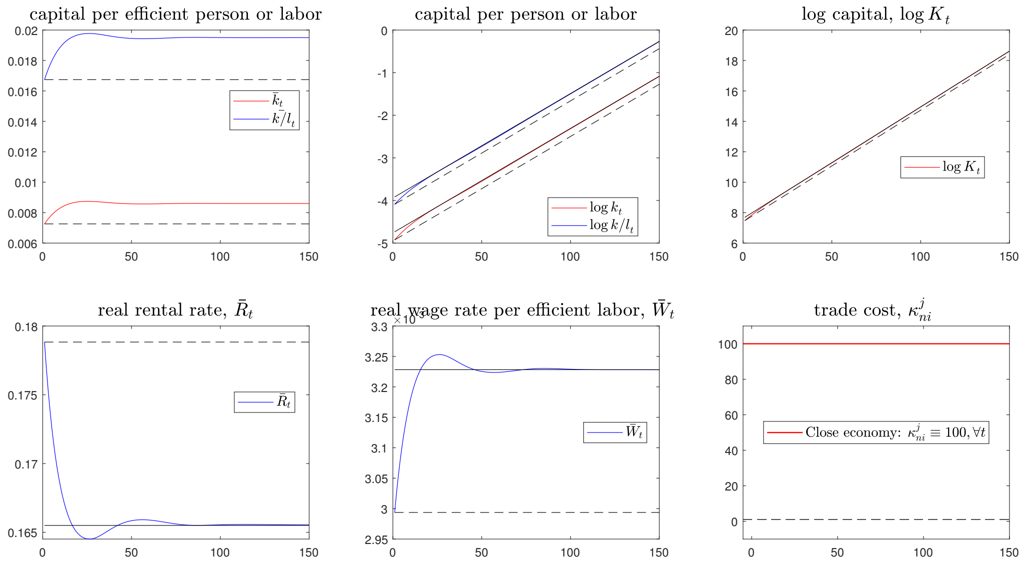


Figure 9
LIVING LONGER. CLOSE ECONOMY

growth path.

The first figure in Figure 9 depicts both capital per labor and capital per person. The difference between these two indices reflects changes in the working-age share. The trends for capital per labor and capital per person are driven by the growth rate of the knowledge stock: the initial increase in the working-age population leads to a corresponding rise in the knowledge stock. However, as the growth of the working-age population slows, the growth of the knowledge stock also decelerates, eventually aligning with the initial growth rate.

Figure 10 illustrates how capital and other variables change over time following a negative shock to the fertility rate. In the short run, as the population growth rate slows down, this can be beneficial, resulting in a higher level of capital per person and capital per labor due to a reduced young population. However, in the long run, this trend is harmful, as it leads to a slowdown in productivity growth.

Implications A higher survival rate stimulates capital accumulation, elevating the balanced growth path. The effects of a fertility rate shock are twofold: in the short run, a lower population growth rate raises capital per person above the previous growth path due to a reduced young population. However, in the long run, this benefit is counterbalanced by a slowdown in productivity growth, ultimately resulting in capital per person falling below the old growth path.

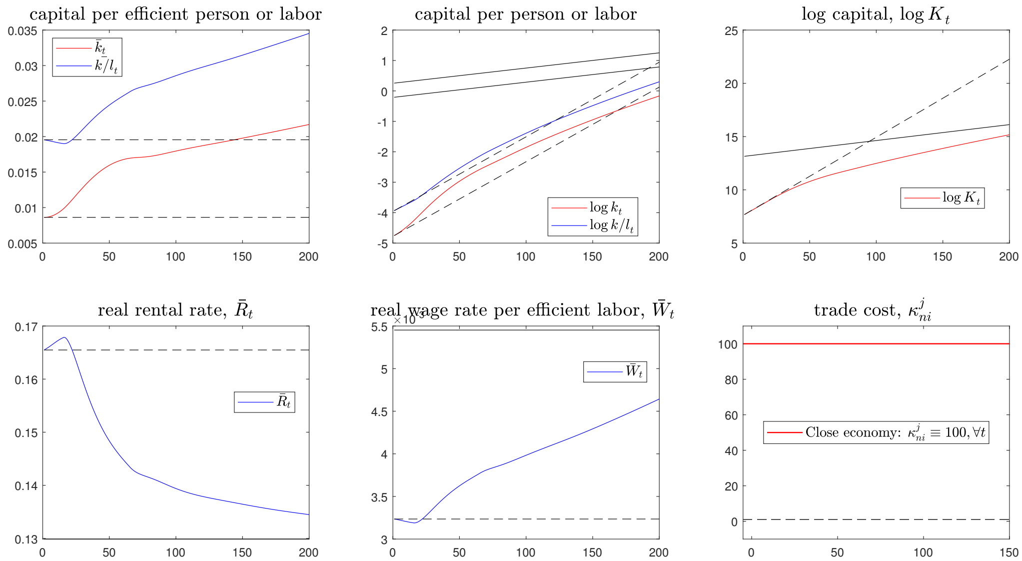


Figure 10
Population growth slows down. Close economy

3.5.3. Demographics and trade

In this section, I explore the interactive effects of demographics and trade liberalization. Specifically, I first compare balanced growth equilibrium under different structural conditions, both with and without free trade. Then, I analyze how capital and other variables evolve over time during the transition process in both scenarios.

Table 5 presents a comparison between stationary balanced growth equilibrium under free trade and autarky (no trade) conditions. The table shows different scenarios represented by columns (1S), (2S), (3S), and (1A), where "sym." indicates symmetrical countries and "cty1" and "cty2" represent two asymmetrical countries. The rows below the "Free Trade vs. Autarky (=1)" line show various economic indicators as ratios of their values under trade to those under autarky. For instance, the capital stock, output, consumption, and investment per efficient person are all equal to 2.83 in the symmetrical trade scenarios, indicating that free trade has a stimulating effect on capital accumulation across these cases. Moreover, real wage rates and real rental rates also adjust under trade conditions, reflecting shifts in productivity and capital distribution.

For the asymmetrical scenario (1A), the table indicates that "cty1" has a higher survival rate, leading to higher capital stock and investment levels compared to "cty2." This difference implies that "cty1" has a comparative advantage in producing capital-intensive goods. Consequently, the capital share of value added (VA) is higher in "cty1," indicating better allocation efficiency due to trade. Across all columns (1S, 2S, 3S, 1A), the larger supply of

TABLE 5
STATIONARY BALANCE GROWTH EQUILIBRIUM COMPARISON. FREE TRADE V.S. CLOSE ECONOMY

	(1S)	(2S)	(3S)	(1A)	
Country	sym.	sym.	sym.	cty1	cty2
Survival rate	low	high	high	high	low
Fertility rate	high	high	low	high	high
Average lifespan	60.1	70.8	70.8	70.8	60.1
Population growth	1.050	1.050	1.010	1.050	1.050
Implied TFP growth	0.025	0.025	1.003	1.025	1.025
Working age share	0.44	0.46	0.72	0.46	0.44
Free trade v.s. Autarky(=1)					
Capital share of VA	1.000	1.000	1.000	1.002	0.998
Per efficient person					
Capital stock	2.83	2.83	2.83	2.75	2.90
Output	2.83	2.83	2.83	2.74	2.93
Consumption	2.83	2.83	2.83	2.72	2.94
Investment	2.83	2.83	2.83	2.75	2.90
Capital - efffcient labor ratio	2.83	2.83	2.83	2.75	2.90
Price ratio					
Real wage rate	2.83	2.83	2.83	2.71	2.95
Real rental rate	1.000	1.000	1.000	1.001	0.999

capital under free trade is evident, resulting from trade's selection effect. Trade encourages higher productivity and lowers prices, which in turn stimulates capital accumulation and enhances income. This improved capital accumulation, driven by trade, enables countries to reach higher levels of income.

As depicted in Figure 11, the economy initially operates under a closed economy setup, but from period 2 onwards, trade costs are reduced to zero, indicating the onset of free trade.

When a positive shock to the survival rate occurs, it results in higher life expectancy, which increases the incentive for saving and investing, leading to an accumulation of capital. Both in closed and open economies, this higher survival rate elevates the balanced growth path.

In addition, free trade amplifies these effects by reallocating resources more efficiently and stimulating higher productivity from selection effects, thereby boosting capital accumulation even further.

In Figure 12, the economy undergoes a negative shock to the fertility rate. Initially, the reduced population growth due to lower fertility leads to an increase in capital per person and capital per labor, enhancing productivity in the short run. However, over time, as the young population declines, there is a risk of lower labor force growth, which can ultimately harm productivity and economic growth.

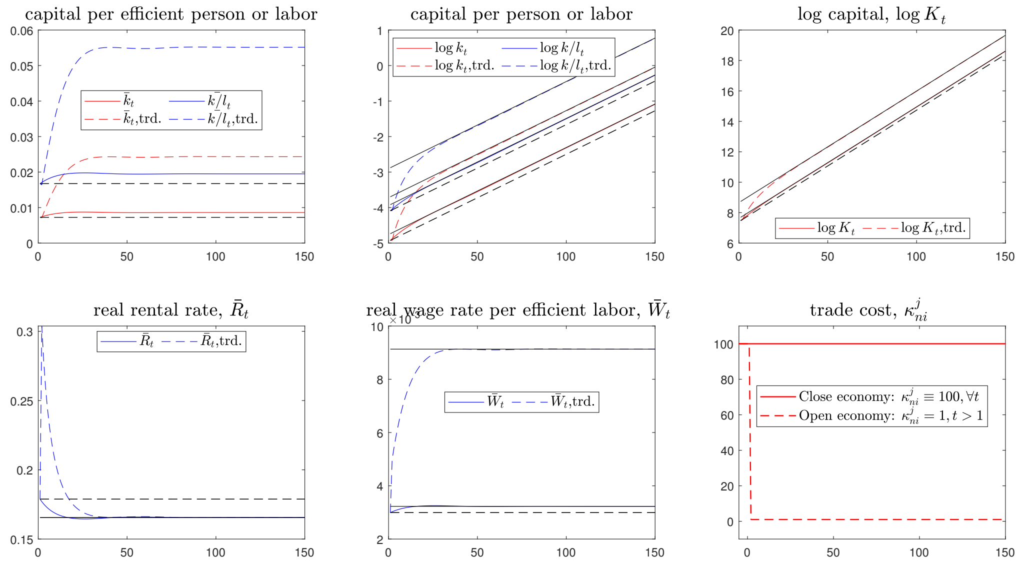


Figure 11

living longer. Symmetric Open economy v.s. Close

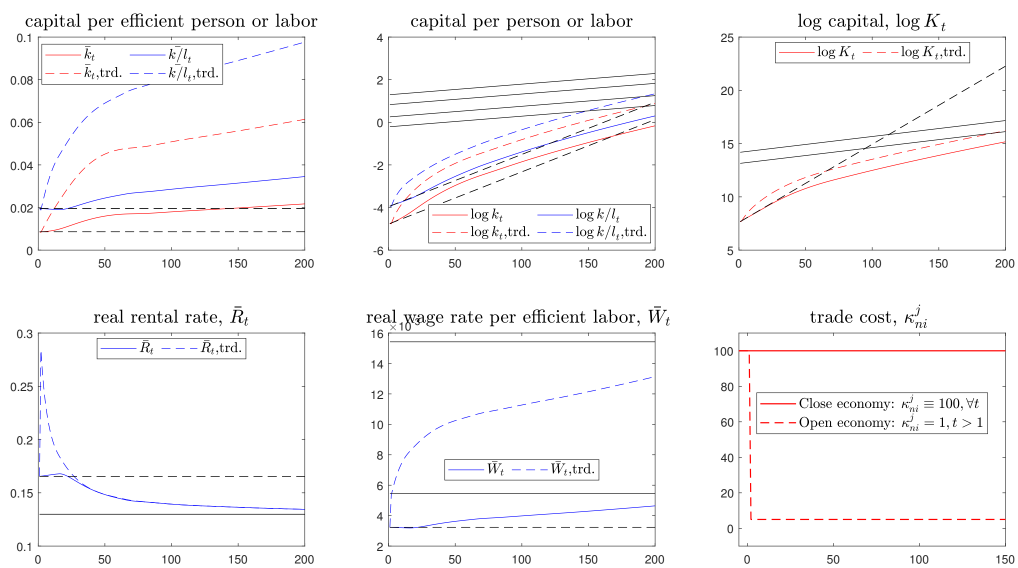


Figure 12

Population growth slows down. Symmetric Open economy v.s. Close

Trade liberalization mitigates this long-term downside by prolonging the period in which capital per person remains higher than in the closed economy scenario, thereby extending the period of economic benefits. Free trade, by selection effects and enhancing resource allocation, allows the economy to sustain a higher level of capital per person for a longer duration.

Implications A higher survival rate stimulates capital accumulation, elevating the balanced growth path. Free trade amplifies these effects by selection effects and enhancing resource allocation.

The effects of a fertility rate shock are beneficial in the short run, but adverse in the long run. Trade liberalization mitigates this long-term downside by prolonging the period in which capital per person remains higher than in the closed economy scenario, thereby extending the period of economic benefits.

Summary The main driving forces—age-time varying fertility rates and survival rates—mediated through the model's mechanisms, affect sectoral productivities and capital accumulations. These forces, together with the exogenous trade cost changes, affect the sectoral prices, which, in turn, affects the allocation of production across sectors and locations, and ultimately affects both trade patterns and economic performance.

For example, a higher survival rate leads to greater knowledge stock and capital stock accumulation, raising the balanced growth path by enhancing both productivity and capital. Free trade induce specialization, which encourages higher productivity and lowers prices, ultimately leading to even greater capital accumulation.

Similarly, a lower fertility rate impacts both knowledge and capital stocks. In the short run, a reduced young population raises capital per person, temporarily boosting economic output. However, over time, this benefit is offset by slower productivity growth due to the demographic shift, ultimately causing capital per person to fall below the previous growth path. Trade liberalization can mitigate this long-term drawback by maintaining capital per person above the old growth path for a longer period, which extends the overall economic benefits in comparison to a closed economy scenario.

4. Calibration

For forward-looking households, it is necessary to define the time-varying processes that extend beyond the sample period of 1970–2020. The extra 80 years (2020-2100) is intentionally designed to build solid expectations for individuals born in 2020, which is the last year of interest. Thus, the model is calibrated and solved from the assumed initial steady state in 1970 to the final steady state in 2100, as showed in Table 6. From 1970 to 2100, I calibrate time-varying productivity and trade costs using data on trade flow, input-output tables, capital stock, demographic variables, and sectoral prices. Data used in the calibration from 1970-2020 are historical data, while data or shocks used in the calibration from 2021-2200 are based on projections and model imputes.

For the initial steady state, the population growth rate, real wage growth rate and demographic distribution is calculated as the average value across both years from 1965–1975 and regions. Since demographic variables change slowly over time, I assume that each country is in a steady state characterized by these calculated average demographic distributions and an average constant growth rate. With these calculated average growth rates for real wages and population, plus the calibrated productivity levels in 1970, I can, on the one hand, calibrate the parameters $\eta_g, g \in [16, 65]$ in the demographic structure and knowledge stocks equation[4]. On the other hand, I calculate the model-implied saving distributions. In the steady state, the saving wedge $\psi_1^{c,\star}$ governs the saving tendency and determines both the steady-state capital stock and saving distribution, which I adjust to ensure that the model-generated capital stock matches the actual data, as I will show in the following paragraph.

For final steady state and the period after 2100, I assume that each region will maintain both its total population and population distribution, which implies that demographic parameters (fertility and survival rates) stabilize by 2100. The new steady state implies zero population growth, which also suggests a zero productivity growth rate. However, after 2100, the productivity growth rate is still approaching zero but has not yet reached it. Therefore, I calculate an average growth rate as an approximation. Specifically, given the calibrated productivity values for 2100, I impute the productivity levels for each region over the next 85 years. Then, I calculate the average productivity growth rate between 2100 and 2185. I take this average growth rate as an approximation for the growth rate on the balanced growth path. The results yield a steady-state saving distribution across different ages. These variables act as terminal conditions for the model and are key to solving the transition dynamics between the two steady states. While the assumed approximate steady state in 2100 is not a true steady state, it is sufficiently far from 2020, which is the last period of interest, so it exerts minimal influence on the period of interest.

During the period 2020–2100, data such as trade flow, input-output tables, and capital stock are not directly observable. However, I can impute these data using appropriate estimations for productivity and demographic variables. The demographic variables are obtained from UN imputations. Based on the demographic data, I can impute time-varying total factor productivity (TFP) through the demographic-knowledge stock link. I also maintain trade costs at 2020 levels for the entire period from 2020 to 2100. By solving the CP model year by year with the imputed investment rate, I can generate time series data for trade flows, sectoral prices, and other key variables[5].

For the rest of this section, I first calibrate exogenous, time-invariant parameters, and then I derive the time-varying shocks. For the time-varying historical shocks during sample period 1970-2020, I further present these shocks over time and provide an interpretation

4. For calibration details for the knowledge stock process, please refer to Appendix F.

5. The investment rate is imputed by estimating the relationship between the investment rate, a country-fixed effect, the lagged investment rate, and the contemporaneous demographic index for the years 1965–2020. Check Appendix F for details.

TABLE 6
STATIONARY BALANCE GROWTH EQUILIBRIUM, CHINA

Stationary balance growth equilibrium, China		
	Initial, at 1970	**Final, at 2100**
Survival rate to age 65	0.54	0.94
Average Fertility rate	0.10	0.02
Expected lifespan	60.20	80.45
Average Population growth	0.017	0.000
Implied Real wage growth	0.021	0.000
Per person		
Capital stock	1.248	43180.8
Consumption	0.060	2937.7
Investment	0.123	3037.2
Capital - efffcient labor ratio	1.89	35.72
Price ratio		
Real wage rate	176.07	2507.77
Real rental rate	0.050	0.042

for them. Finally, I feed these shocks into the model to demonstrate the accuracy of my calibration. Table 7 displays an overview of the calibration. For information on data used in the calibration, see Appendix C.

4.1. Time invariant parameters

As shown in Table 7, households are born with age 1, join the labor market after age $G_0 = 15$, retire after age $G_1 = 65$, and die after age $G = 85$. The annual discount factor is estimated to be $beta = 0.96$, and the annual depreciation rate of capital is $\delta = 0.06$. The coefficient of relative risk aversion is $\sigma = 1$. Following Simonovska and Waugh (2014), I set the trade elasticity $\theta = 4$. The elasticity of substitution between varieties within the composite good plays no quantitative role in the model, I set $\rho = 2$, similar to most trade literature. In the remaining part of this section, I omit the time subscripts t for convenience.

The sectoral expenditure data X_{ni}^j and sectoral value-added data V_n^j are derived from the IO Table. The value of region i's exports of sector j goods to region n is denoted as X_{ni}^j. Similarly, the value of region i's imports of sector j goods from region n is denoted as X_{in}^j. The trade deficit of region n in sector j is defined as $D_n^j = \sum_{i=1}^{N}(X_{ni}^j - X_{ni}^j)$. The aggregate trade deficit of region n is represented as D_n, and it is calculated as $D_n = \sum_{j=1}^{J} D_n^j$. The gross production value of sector j goods in region n is denoted as Y_n^j, and it is computed as

TABLE 7
CALIBRATION SUMMARY

Parameter	Description	Value or source
Time Invariant Parameters		
N	# of countries	5
J	# of sectors	5
G_0+1	Age join labor market	16
G_1+1	Retried age	66
G	Lifespan for households	85
σ	Risk aversion	1
$\rho_{knowledge}$	Existing knowledge stock coefficient	0.7 (Buera and Oberfield, 2020)
φ^j	Idea duplication coefficient	$[0.67, 0.28, 0.19, 0.69, 0.41]$
β	Annual discount factor	0.96
δ	Capital depreciation rate	0.06
θ	Trade elasticity	4
ρ	Elasticity of substitution between varieties	2
$\gamma^{k,j}$	Sectoral composite goods shares in output	IO table (average across t)
γ^j	Value added shares in output	IO table (average across t)
β^j	Labor's share in value added	IO table (average across t)
α_C^j	Preference parameters	IO table (average across t)
α_I^j	Investment parameters	IO table (average across t)
Time Varing Shocks		
N_{n,t_0}	Initial labor supply	PWT 10.01
$\overline{N}_{n,g,t_0}$	Initial age distribution	United Nations
$s_{n,g,t}$	Conditional survival rate	United Nations
$f_{n,g,t}$	Fertility rate	United Nations
$\tau_{n,t}^L$	Labor supply wedges	PWT
$\phi_{n,t}$	Trade imbalance wedges	IO table
$\lambda_{n,t}^j$	Knowledge stock	Gravity Regression
$\kappa_{ni,t}^j$	Trade cost	Gravity Regression
$\psi_{n,t}$	Saving wedges	Calculation
$\psi_{n,g}$	Steady state saving wedges	Calculation
$\eta_{n,t}^j$	Idea coefficient	Calculation
Time Varing Endogenous Variables		
$N_{n,t}$	Total labor supply	PWT 10.01
$\overline{N}_{n,g,t}$	Age distribution	United Nations

$Y_n^j = \sum_{i=1}^{N} X_{in}^j$. The gross expenditure value of sector j goods in region n is denoted as X_n^j, and it is computed as $X_n^j = \sum_{i=1}^{N} X_{ni}^j$. The value added by region n from sector j is V_n^j, and the total value added by region n is denoted as V_n, where $V_n \equiv W_n L_n^e + R_n K_n = \sum_{j=1}^{J} V_n^j$. Furthermore, the value of demand for sector j goods in region n's production of sector k goods is represented as $V_n^{j,k}$, and the production share parameters can be backed out through:

$$\gamma_n^j = \frac{V_n^j}{Y_n^j}, \qquad \gamma_n^{j,k} = (1-\gamma_n^j)\frac{V_n^{j,k}}{\sum_{j=1}^{J} V_n^{j,k}} \tag{42}$$

The preference parameters $\alpha^j_{C,n}$ and investment parameters $\alpha^j_{I,n}$ can be backed out through:

$$\alpha^j_{C,n} = \frac{P^j_n C^j_n}{P_{n,C} C_n}, \quad \alpha^j_{I,n} = \frac{P^j_n I^j_n}{P_{n,I} I_n} = \frac{X^j_n - \sum_{k=1}^{J} \gamma^{j,k}_n Y^k_n - P^j_n C^j_n}{IN_n - P_{n,C} C_n}, \quad IN_n \equiv R_n K_n + W_n L^e_n + D_n \tag{43}$$

where value of country n's sectoral goods consumption $P^j_n C^j_n$ and value of country n's total consumption $P_{n,C} C_n$ can be got directly from IO table. I then take the average of those parameters which generated from IO table across the years and regions.

4.2. Time varying shocks

In this section, I demonstrate how I calibrate the time-varying exogenous shocks, including conditional survival rate, fertility rate , labor market distortion, trade imbalance wedges, saving wedges, and trade cost. Additionally, I will explain the sources or the way to back out the endogenous variables, such as labor supply, age distribution, aggregate capital stocks and knowledge stocks.

4.2.1. Demographics, Labor input, capital and investment

Given the number of age g population for country n at time t, the conditional survival rate, which represents the probability of continuing survival from age g to $g+1$, is defined as $s_{n,g+1,t+1} = \frac{N_{n,g+1,t+1}}{N_{n,g,t}}$. The model does not distinguish between women and men. Therefore, a parsimonious way to define the survival rate is to consider the number of newborn cohorts (age $g = 1$) at time t divided by the population within the fertility age range. The fertility age is defined as individuals between age $g = 21$ and age $g = 49$. Consequently, the fertility rate is calculated as $f_{n,g,t} = \frac{N_{n,g=1,t}}{\sum_{g=21,\cdots,49} N_{n,g,t-1}}, g \in [21, 49]$ and $f_{n,g,t} = 0, g \notin [21, 49]$.

The labor supply denoted as $L_{n,t}$, is derived directly from the "number of people engaged (in millions)" data in the Penn World Table 10.01 Feenstra, Inklaar, and Timmer (2015). The human capital (schooling) index is also available in PWT 10.01. The same dataset provides data on capital stock, specifically the "Capital stock at constant 2017 national prices (in millions of 2017 US\$)," which I use to calibrate the initial aggregate capital stocks, K_{n,t_1}, in line with my model. Information on aggregate and sectoral investment values ($P_{I,n,t} I_{n,t}$ and $P^j_{n,t} I^j_{n,t}$) is sourced from the World Input-Output Database Timmer, Dietzenbacher, Los, Stehrer, and De Vries (2015); Woltjer, Ggouma, and Timmer (2021). Using sectoral intermediate prices, I calculate the quantities of investment, $I_{n,t}$, and capital stock, $K_{n,t}$.

The total population $N_{n,t}$ and age distribution data $N_{n,g,t}$ are sourced from the United Nations World Population Prospects 2022. Labor market distortions are computed using the formula $\tau^L_{n,t} = 1 - \frac{N_{n,t}}{\bar{L}_{n,t}}$. The trade imbalance shifters $\phi_{n,t}$ are defined as the ratio of net exports to GDP, given by $\phi_{n,t} = -\dfrac{D_{n,t}}{V_{n,t}}$.

4.2.2. Calibrate saving (or investment) wedges

One caveat is that, while I calibrate productivities and trade costs using only contemporaneous data, the saving wedges from 1970 to 2200 are calibrated using the entire period's data (1970–2100).

These wedges are used to ensure that the model-generated time-varying aggregate saving (or investment) matches the real data counterpart.

Since Saving wedges $\psi_{n,t}$ are adjusted to ensure that the ratio $\dfrac{\psi_{n,t+1}}{\psi_{n,t}}$ adheres to the cohort level Euler equation, with an initial condition of $\psi_{n,1} = c$. However, cohort-level consumption cannot be canceled or replaced by aggregate consumption through simple aggregation. Moreover, future shocks can influence the Euler equation via the budget constraint across cohorts, which must be accounted for when calibrating the wedges.

$$\left(\frac{c_{n,g+1,t+g}}{c_{n,g,t+g-1}}\right)^{1/\sigma} = \beta\left(\frac{\psi_{n,t+g}}{\psi_{n,t+g-1}}\right)\left(1 + \frac{R_{n,t+g}}{P_{n,I,t+g}} - \delta\right)\frac{\frac{P_{n,I,t+g}}{P_{n,C,t+g}}}{\frac{P_{n,I,t+g-1}}{P_{n,C,t+g-1}}} \quad \forall\ g \in [1, G-1] \tag{44}$$

The model suggests that higher saving wedges, $\psi_{n,t}$, indicate a stronger incentive to save for period t. Since savings provide the supply of investment, this leads to a higher capital stock in that period. Therefore, I use the aggregate capital stock, $K_{n,t}$, as targets to calibrate the evolution of saving wedges, $\psi_{n,t}$, over time. For calibration details, please refer to Appendix F.

4.2.3. Knowledge stock and trade cost

In this section, I demonstrate how to calibrate the knowledge stock (or productivity parameters) $\lambda^j_{n,t}$ and trade costs $\kappa^j_{ni,t}$. The expenditure data $X^j_{ni,t}$ (the value of sector j goods imported by region n from region i) and value-added data $V^j_{n,t}$ used in the calibration are sourced from the Input-Output (IO) Table. Using Equation Equation 29, I derive the following structural gravity equation:

$$\ln\left(\frac{X^j_{ni,t}}{X^j_{nn,t}}\right) = \ln\left(\lambda^j_{i,t}\left(c^j_{i,t}\right)^{-\theta}\right) - \ln\left(\lambda^j_{n,t}\left(c^j_{n,t}\right)^{-\theta}\right) - \theta\ln\left(\kappa^j_{ni,t}\right) \tag{45}$$

I assume that, for each sector at each time, the unobserved trade cost terms $\kappa^j_{ni,t}$ can be described as follows (suppressing time subscripts for convenience):

$$\begin{aligned}\ln\left(\kappa_{ni}^{j}\right) &= EX_{i}^{j} + X_{ni}^{j}\beta^{j} + \epsilon_{ni}^{j} \\ &= EX_{i}^{j} + \sum_{k}\gamma_{k}^{j}Dist_{ni} + \alpha_{1}^{j}Border_{ni} + \alpha_{2}^{j}Currency_{ni} + \alpha_{3}^{j}RTA_{ni} + \epsilon_{ni}^{j}\end{aligned} \tag{46}$$

In this equation, $\gamma_k^j Dist_{ni}$ represents the contribution of the distance between n and i (classified into different intervals indexed by k) to sector j trade costs. Following Eaton and Kortum (2002), I define the distance intervals (in miles) as $[0, 350]$, $[350, 750]$, $[750, 1500]$, $[1500, 3000]$, $[3000, 6000]$, and $[6000, \max)$. Additional variables include whether the two regions share a common border, belong to a currency union, or participate in a regional trade agreement. The term EXi^j represents exporter i's fixed effects for sector j. The notation $Distni$ refers to the geographic distance between the capital cities of regions n and i. The currency union indicator comes from Rose (2004), and was updated for the post-2000 period using publicly available information (such as the membership in the Euro area, and the dollarization of Ecuador and El Salvador). The geographic distance data and other control variables are obtained from the CEPII database (see Conte, Cotterlaz, Mayer, et al. (2022)).

If region n or region i represents an economic group, then $Dist_{ni}$ refers to the population-weighted geographic distance between regions n and i, defined as follows:

$$Dist_{ni} = \sum_{r\in\{r_1^n,\ldots,r_p^n\}}\left(\frac{pop_r}{pop_n}\sum_{s\in\{r_1^i,\ldots,r_q^i\}}\left(\frac{pop_s}{pop_i}\right)Dist_{rs}\right), \tag{47}$$

where pop_n represents the population of economic group n. Each economic group n comprises a set of economies denoted as $r_1^n, \ldots, r_p^n$. Here pop_n represents the population of economic group n. Each economic group n comprises a set of economies denoted as $r_1^n, \ldots, r_p^n$. The population figures are taken as of the base year 1970. I also use the average population over the sample period, and the resulting distance measures are not sensitive to the choice of year. The population data are taken as of the base year 1970. I also use the average population over the sample period, and the resulting distance measures are not sensitive to the choice of year.

Combining Equation 46 and Equation 45, I get the following structural equation:

$$\begin{aligned}\ln\left(\frac{X_{ni,t}^{j}}{X_{nn,t}^{j}}\right) &= \left\{\ln\left(\lambda_{i,t}^{j}\left(c_{i,t}^{j}\right)^{-\theta}\right) - \theta \mathrm{EX}_{i,t}^{j}\right\} + \left\{-\ln\left(\lambda_{n,t}^{j}\left(c_{n,t}^{j}\right)^{-\theta}\right)\right\} - X_{ni,t}\theta\beta_{t}^{j} - \theta\epsilon_{ni,t}^{j} \\ &= E_{i,t}^{j} + M_{n,t}^{j} + X_{ni,t}\Theta_{t}^{j} + \nu_{ni,t}^{j}\end{aligned} \tag{48}$$

where $E_{i,t}^j \equiv S_{i,t}^j - \theta\ \mathrm{EX}_{i,t}^j$, $M_{n,t}^j \equiv -S_{n,t}^j$, $\Theta_t^j \equiv -\theta\beta_t^j$, and $S_{n,t}^j \equiv \ln\left(\lambda_{n,t}^j\left(c_{n,t}^j\right)^{-\theta}\right)$.

Estimate $\tilde{M}^j_{n,t}$ I estimate the fixed effects $\tilde{M}^j_{n,t}$ by running the regression specified in Equation 48 separately for each year and sector. For detailed regression results, refer to appendix.

Calibrate Trade Costs Based on the estimated values of $\tilde{M}^j_{n,t}$, the sector-location-specific technology states, $S^j_{n,t}$, are calculated as $S^j_{n,t} = -M^j_{n,t}$. Given the available degrees of freedom, $M^j_{n,t}$ is identified up to a normalization for each sector. I use the United States (USA) as the reference location, setting $\tilde{M}^j_{USA,t} = 0$. Consequently, the estimated fixed effects ($\tilde{M}^j_{n,t}$) are expressed relative to the reference location's $\tilde{M}^j_{USA,t}$ for each year and sector, and thus, $S^j_{n,t} - S^j_{USA,t} = -\tilde{M}^j_{n,t}$.

From Equation 45, the trade cost can be derived as follows:

$$\kappa^j_{ni,t} = \{\left(\frac{X^j_{ni,t}}{X^j_{nn,t}}\right) exp(S^j_{n,t} - S^j_{i,t})\}^{-\frac{1}{\theta}} = \{\left(\frac{X^j_{ni,t}}{X^j_{nn,t}}\right) exp(\tilde{M}^j_{i,t} - \tilde{M}^j_{n,t})\}^{-\frac{1}{\theta}} \tag{49}$$

Calibrate knowledge stock Equation 29 indicate that

$$P^j_{n,t} = A^j \left[\sum_{i=1}^{N} exp(S^j_{i,t}) \left(\kappa^j_{ni,t}\right)^{-\theta}\right]^{-\frac{1}{\theta}} = A^j \left(\frac{exp(S^j_{n,t})}{\pi^j_{nn,t}}\right)^{-\frac{1}{\theta}}. \tag{50}$$

So, given the sectorial price of U.S.A, I can back out:

$$S^j_{USA,t} = ln\left[\pi^j_{USA\ USA,t} \cdot \left(\frac{P^j_{USA,t}}{A^j}\right)^{-\theta},\right] \tag{51}$$

and the technology term for other regions can be calculated as:

$$S^j_{n,t} = S^j_{USA,t} - \tilde{M}^j_{n,t} \quad \forall n \neq \text{USA} \tag{52}$$

Given the value of the term $S^j_{n,t}$, the sectoral price $P^j_{n,t}$ for regions other than base region U.S.A. can be imputed using Equation 50. From $S^j_{n,t} \equiv \ln\left(\lambda^j_{n,t}\left(c^j_{n,t}\right)^{-\theta}\right)$, the knowledge stock $\lambda^j_{n,t}$ can be derived as follows:

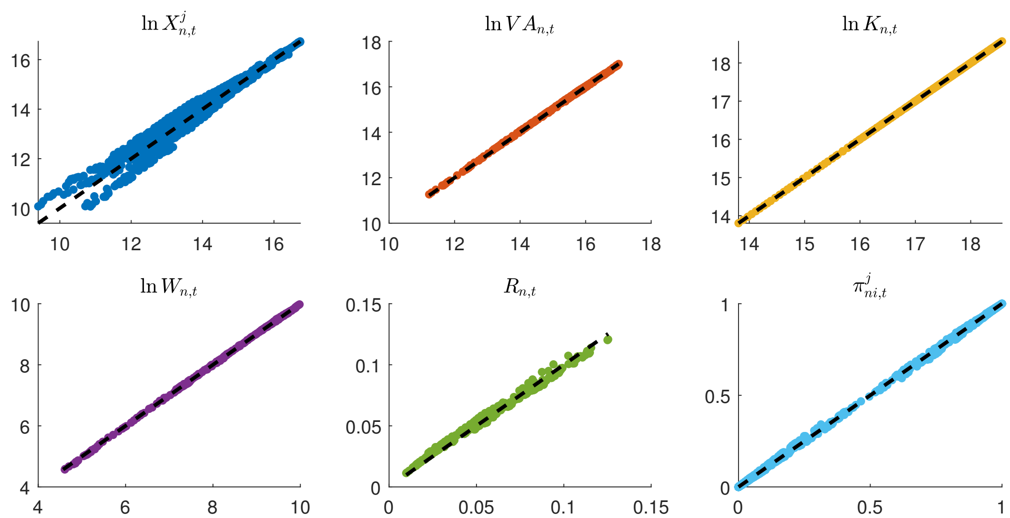
$$\lambda^j_{n,t} = \frac{exp\left(S^j_{n,t}\right)}{\left(c^j_{n,t}\right)^{-\theta}} \tag{53}$$

where $c^j_{n,t} = \Upsilon^j_n W_{n,t}{}^{\beta^j_n\gamma^j_n} R_{n,t}{}^{\left(1-\beta^j_n\right)\gamma^j_n} \prod_{k=1}^{J} P^k_{n,t}{}^{\gamma^{k,j}_n}$, and $\Upsilon^j_n \equiv \gamma^j_n \beta^j_n{}^{-\gamma^j_n\beta^j_n} \gamma^j_n \left(1-\beta^j_n\right)^{-\gamma^j_n\left(1-\beta^j_n\right)} \prod_{k=1}^{J} \gamma^{k,j}_n{}^{-\gamma^{k,j}_n}$.

The wage rate of region n can be obtained from: ($W_{n,t} = \{\sum_{j=1}^{J} \beta^j_n \gamma^j_n \sum_{i=1}^{N} \pi^j_{in,t} X^j_{i,t}\}/L^e_{n,t}$).

4.3. Model fit and discussion

In this section, I first reintroduce both the calibrated time-invariant parameters and the time-varying shocks into the model and solve it. I compare the model generated results with the real world counterpart to assess the accuracy of the calibration. Then, I presents and discusses each of these calibrated time-varying shocks.



Note: The scatter plots display real data on the x-axis and model-generated values on the y-axis, with a 45-degree reference line on the diagonal.

Figure 13
Calibration Efficiency

Figure 13 presents a comparison between model-generated values (y-axis) and real-world values (x-axis). The data points include the log of total sectoral expenditure, value added, capital, wage rate, and the level of rental rate, and sectoral expenditure share. The correlation between the model-generated data and the real-world data aligns closely with the 45-degree line, demonstrating that the calibrated shocks accurately replicate real-world observations.

Demographics Figure 14 presents the calibrated average fertility rate and unconditional survival rate up to age 65, alongside China's total population and working-age share over time. China's total population peaks around 2010, followed by a decline in the working age share starting approximately five years later, in 2015. Specifically, the working age share rose more rapidly in China than in any other region in the sample. The fertility rate, as shown in Figure 14, continues its steady decline after the 1990s, reflecting long-term demographic changes. Meanwhile, the survival rate demonstrates a consistent upward trend across all regions, indicating improvements in life expectancy over time.

Labor supply wedges ($(1-\tau^L_{n,t})$) and saving (or investment) wedges ($\psi_{n,t}$) Figure 15

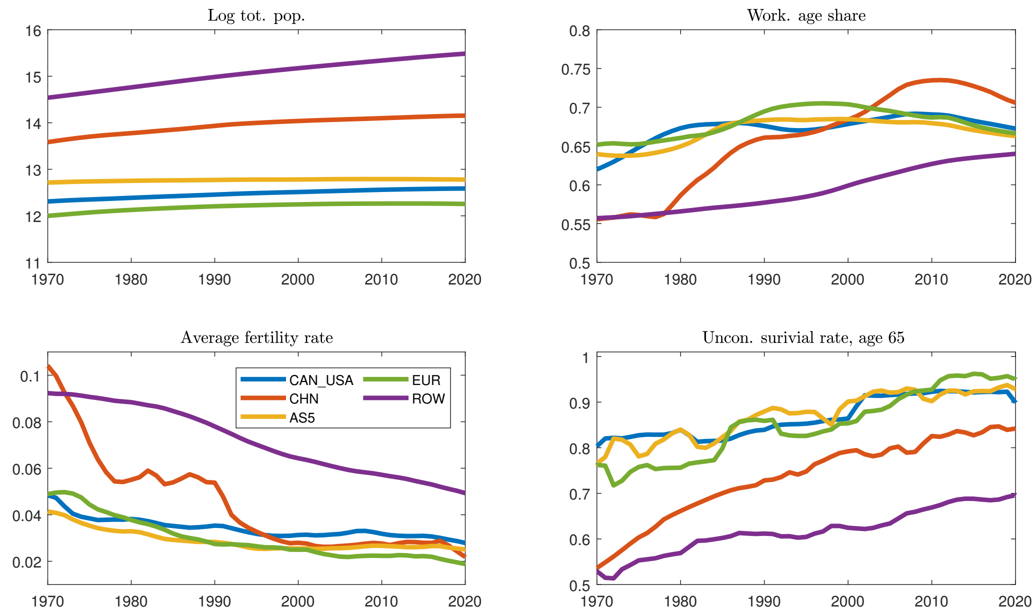


FIGURE 14
DEMOGRAPHICS

plots labor supply wedges and saving wedges over time. The labor supply wedges reflect various frictions such as female labor force participation, business cycles, and other labor market distortions. Compared to other regions, China's labor supply is less distorted in terms of levels, although it exhibits a declining trend in labor supply after the 1990s, while most other regions show an upward trend. The saving wedges capture forces influencing individuals' saving behavior, aside from demographic factors. Generally, these saving wedges display similar patterns of change across countries, with fluctuations centered around a value of 1.

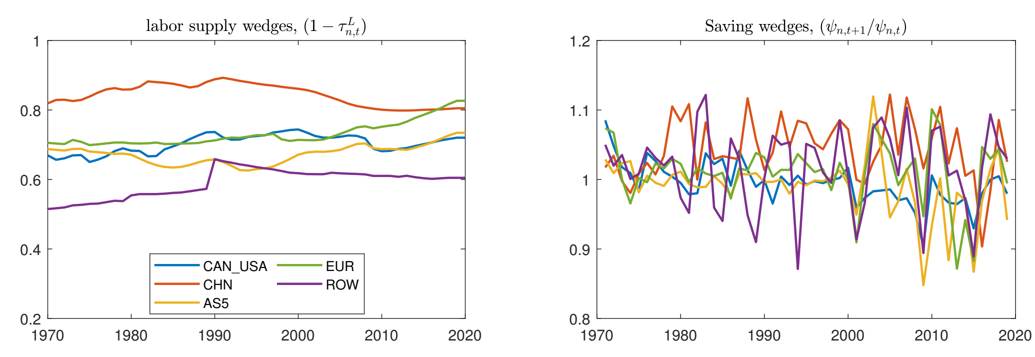


FIGURE 15
LABOR SUPPLY WEDGES AND SAVING (OR INVESTMENT) WEDGES

Knowledge stocks ($\lambda^j_{n,t}$) and Trade costs ($\kappa^j_{ni,t}$) Figure 16 presents the calibrated

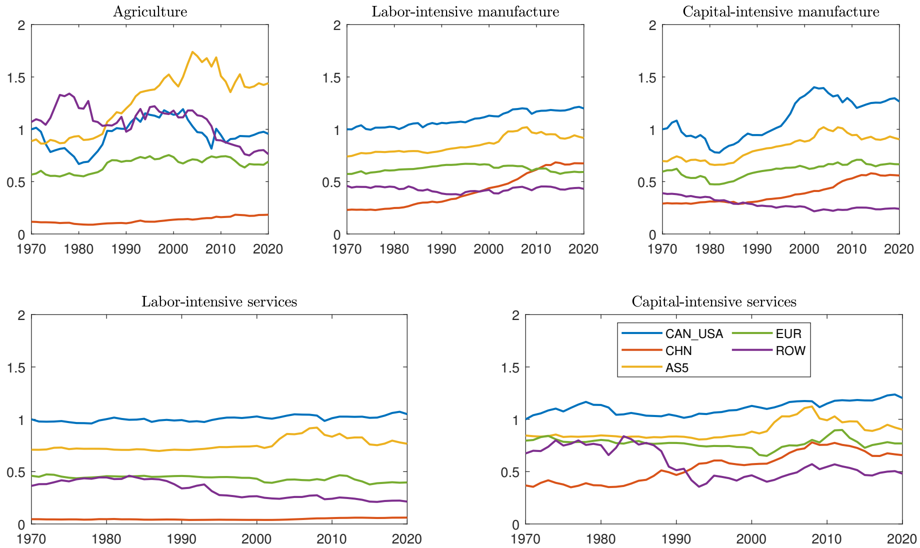


FIGURE 16
KNOWLEDGE STOCKS

productivity ${\lambda_{n,t}^{j}}^{1/\theta}$ across sectors over time. The productivity levels for the U.S. and Canada region in 1970 are normalized to 1. Overall, China's productivity exhibits an upward trend across all sectors, characterized by a steeper trend compared to other regions, although starting from lower initial levels compared to other regions.

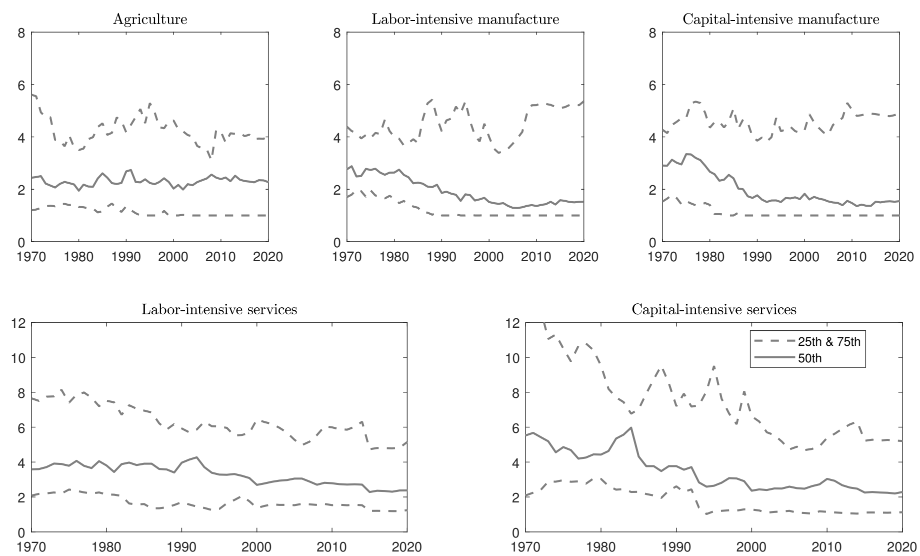


FIGURE 17
TRADE COSTS

Figure 17 depicts the cross-region distribution of estimated trade costs for each sector over time. It is evident that trade costs are generally higher in the services sector. In addition. although trade barriers decrease across all sectors, the rate of decline is faster in the manufacturing sectors.

To summarize, the behavior of these wedges and shocks differs across periods, regions, and sectors, with most changes aligning with intuition. A complete story requires further counterfactual analysis.

5. Quantitative Analysis

In this section, I conduct a counterfactual analysis to quantify the effects of demographic structures on China's economic growth and trade patterns, as well as to conduct projections for China's future form preservative of demographics.

I explore three counterfactual scenarios. First, I remove both the demographic-induced-saving effects and demographic-induced-productivity changes. Specifically, I replace the age-varying fertility and survival rates in China with those of the rest of the world (RoW) and allow productivity to change in response to changes in demographics. I refer to this as the *without demographic scenario*.

Second, I remove only the demographic-induced-saving effects. In this scenario, I again replace China's age-varying fertility and survival rates with those of the RoW but retain the usual productivity changes. I call this the *demographic-capital channel scenario*.

Third, I focus solely on the demographic-induced-productivity changes. In this case, I maintain the original age-varying fertility and survival rates of China and its implied demographic process. However, I allow productivity to change as if China's demographic structure were aligned with that of the rest of the world (RoW), where China's age-varying fertility and survival rates mirror those of the RoW. I refer to this as the *demographic-idea channel scenario*.

For each counterfactual scenario, I calculate the dynamic equilibrium as it transitions from one balanced growth equilibrium to another, guided by the corresponding exogenous processes under perfect foresight. All three scenarios start from the same initial equilibrium but converge to different final equilibria, each determined by the projected demographic processes at the final year.

The remainder of this section is organized as follows: subsection 5.1 provides an overview of the time-varying processes in China's demographic structures and their implications for capital stock and knowledge stock, comparing these to scenarios in which China's age-varying fertility and survival rates align with those of the RoW. subsection 5.2 assesses the effects of China's demographic structures on its past growth, separately analyzing the impacts of the demographic-capital channel and the demographic-idea channel. Finally, subsection 5.3 examines the projected effects of demographic changes on China's future growth.

5.1. Demographics, knowledge stock, and capital stock

I first examine the roles of the fertility rate and survival rate in shaping demographic structures over time. Figure 18 presents time series plots of China's total population, working-age population, and working-age share, represented by solid lines. For each of these variables, I also plot their counterparts with dotted lines, which represent scenarios where China's fertility and survival rates are replaced by those of the "Rest of the World" (Row). Comparing the solid lines with the dotted lines provides insights into the effects of China's unique fertility and survival rates on its demographic evolution.

As illustrated in Figure 18, all else being equal and assuming the same initial population distribution, if China's fertility and survival rates had aligned with those of the rest of the world[6], China would have experienced a larger total population than that which has been observed since the mid-1970s. This trend also applies to the working-age population after the mid-1990s, while the opposite is true for the share of working-age individuals before the mid-1990s.

Under the scenario in which China's fertility and survival rates are replaced by those of the rest of the world (Row), the working-age population initially falls below the observed (actual) trajectory. However, after the mid-1990s, the working-age population surpasses the observed path and continues to grow, diverging further from the actual demographic trend. This demographic shift implies similar patterns for knowledge stock, as a larger working-age population generates, on average, more ideas, thereby contributing to knowledge stock growth. Additionally, this shift suggests comparable trends for capital stock, as working-age individuals tend to save, while younger and older individuals generally rely on borrowing (either from future or past income).

Specifically, as illustrated in Figure 19, which presents the time series of knowledge stock across five sectors in China, and in Figure 20, which displays the time series of China's aggregate capital stock and capital stock per person.

In both figures, solid lines represent the actual trends, while each variable depicted with a solid line has a corresponding dotted counterpart that represents scenarios in which China's fertility and survival rates are replaced by those of the 'Rest of the World' (Row). To facilitate comparison, I normalize each year's actual data to 1, allowing the dotted counterparts to be shown relative to this baseline (1). I also plot figures present the same indices in levels, as detailed in Figure H.1 and Figure H.2 in subsection H.2.

6. Overall, this scenario would result in a decrease in China's survival rate and an increase in China's fertility rate

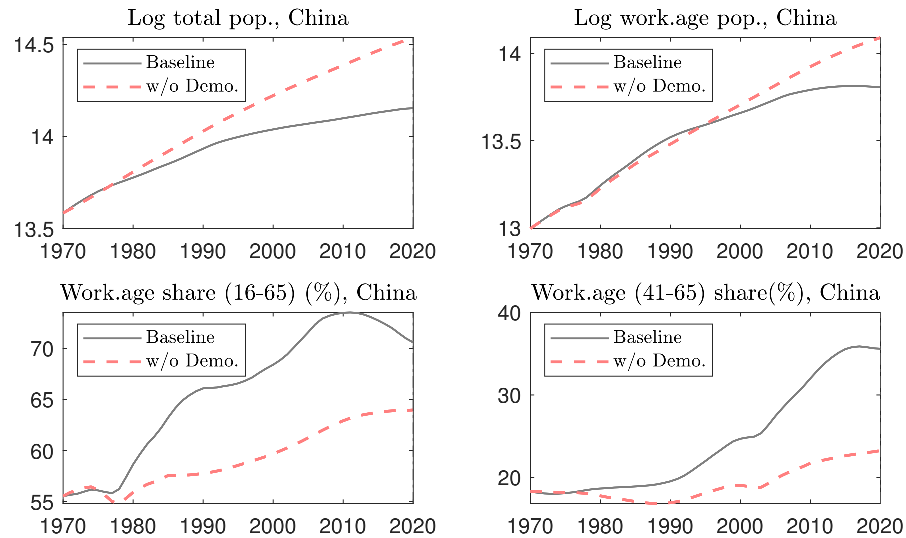


FIGURE 18
DEMOGRAPHIC PROCESS

Overall, had China's fertility and survival rates aligned with those of the rest of the world, the knowledge stock in each sector would exhibit similar patterns: initially increasing slowly, with levels remaining below the old path, followed by a upward trend after the 2000s, ultimately surpassing the old path. Furthermore, given that sectors vary in their idea duplication coefficient, φ_j, labor-intensive and capital-intensive manufacturing sectors, characterized by lower φ_j values, display smaller variations in knowledge stock, even when faced with similar changes in the working-age population.

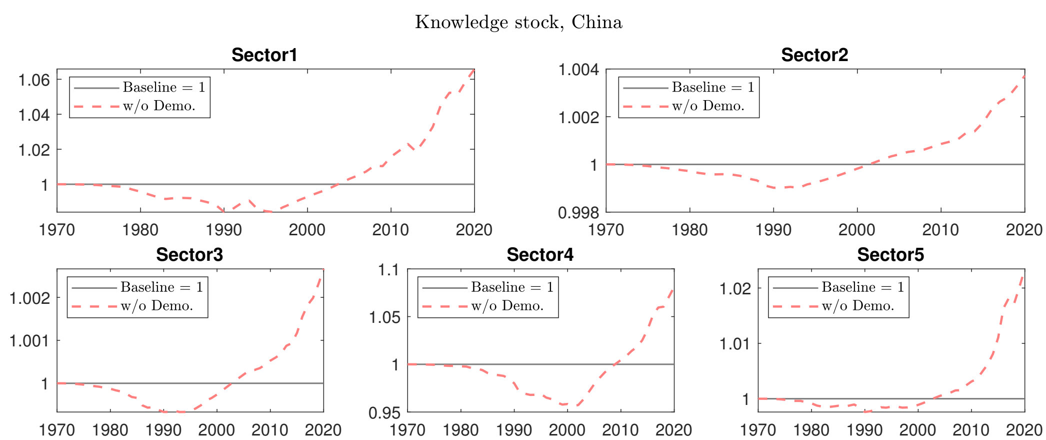


FIGURE 19
KNOWLEDGE STOCKS

For aggregate capital stock, if China's fertility and survival rates had matched those of the rest of the world, aggregate capital would initially increase slowly and remain below the old path, then rise quickly above the old path after the 2000s. The aggregate capital stock is fundamentally related to the total population and the age distribution within that population. A vague measure of these two factors combined is the level of the working-age population. One can observe that as the working-age population increased and surpassed the old path in the mid-1990s, aggregate capital also began to rise, albeit still remaining significantly below the old path. Subsequently, it began to approximate the old path and then exceed it. The working-age population contributes to savings, thus fueling capital accumulation.

Regarding capital stock per person, if China's fertility and survival rates had matched those of the rest of the world, it would have declined and remained below the old path. This phenomenon is driven by changes in the population distribution across age groups, as individuals of varying ages save differently. Consequently, an age structure that favors savings will increase the level of capital stock per person, and vice versa. One can also observe the declining share of the working-age population relative to the baseline. All else being equal, this decline will lead to less saving per person, which in turn drives capital stock per person down.

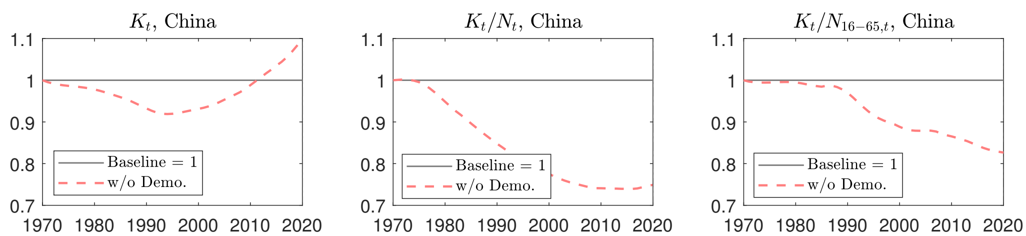


FIGURE 20
CAPITAL STOCKS

In sum, if China's fertility and survival rates had aligned with those of the rest of the world, its demographic evolution would change differently. The aggregate population would have grown more quickly and exceeded the previous path several years after the 1970s. Initially, the working-age population would fall below the old path but would surpass it after the mid-1990s. The working-age share will continue to increase at a slow pace; thus, it will remain below the old path.

The shift in the working-age population would impact knowledge dynamics. The knowledge stock would initially increase slowly, with levels below the old path, and then rise rapidly after the 1990s, ultimately exceeding the old path.

Similarly, the aggregate capital stock would initially grow more slowly; it would then cease to deviate from the old path, and finally surpass the old path due to higher savings stemming from a larger working-age population. Regarding capital stock per person, the slow increase in the working-age share would keep the working-age share below the old path, which would, in turn, maintain capital stock per person below the old path.

5.2. Demographics, economic growth, and trade patterns change

This subsection highlights the effects of China's demographic processes on its economic growth and changing trade patterns, as well as the two channels through which demographics exert their influence.

As previously discussed, I conduct three counterfactual scenarios. First, in the *no-demographics scenario*, to eliminate both the demographic-induced savings effects and demographic-induced productivity changes, I replace China's age-specific fertility and survival rates with those of the rest of the world (RoW) and allow productivity to adjust in response to demographic changes.

Second, in the *demographic-capital channel scenario*, I isolate only the demographic-induced savings effects by again replacing China's age-time-specific fertility and survival rates with those of the RoW while retaining the usual productivity changes.

Third, in the *demographic-idea channel scenario*, I focus on isolating only the demographic-induced productivity effects. In this case, I maintain China's original age-time-specific fertility and survival rates and their implied demographic processes. However, I allow productivity to adjust as if China's demographic structure (fertility, survival rates, and their implied demographic processes) were aligned with that of the rest of the world (RoW).

Figure 21 presents time series plots of China's real income per working-age person, represented by solid lines in each subplot. For each variable, I also include its counterparts from each counterfactual scenario, depicted with dotted lines, which represent scenarios where specific forces are muted. This same approach is applied in Figure 22, where I similarly plot the Revealed Comparative Advantage (RCA) index for the capital-intensive sector. To facilitate comparison, I normalize each year's actual data to 1, allowing the dotted counterparts to be interpreted relative to this baseline. I also display these indices in levels, as shown in Figure H.3, and Figure H.4.

Comparing the solid and dotted lines provides insights into the effects of China's unique demographic processes on economic growth, and changes in trade patterns.

Demographics and economic growth As shown in Figure 21, if China's fertility and survival rates were equal to those of the rest of the world, the real income per working-age person in China would be lower relative to the old path, with an increasing gap over time. By 2020, had China's demographic trends mirrored those of the rest of the world, the real income per working-age person in China would have been about 5.08% less than its actual level.

Two main forces drive these deviations from the historical path: the *Demographic-capital channel* and the *Demographic-idea channel*. As illustrated in Figure 21, the *Demographic-*

capital channel pulls real income per working-age individual in China below the historical path. When China's demographic trends align with those of the rest of the world, its population distribution skews towards higher consumption rather than saving (e.g., a smaller proportion of older working-age individuals or a larger share of younger individuals). This demographic shift reduces capital per working-age person, thereby driving real income per working-age individual below the historical trajectory. By 2020, through the *Demographic-capital channel*, China's real income per working-age person would be about 6.99% lower than observed.

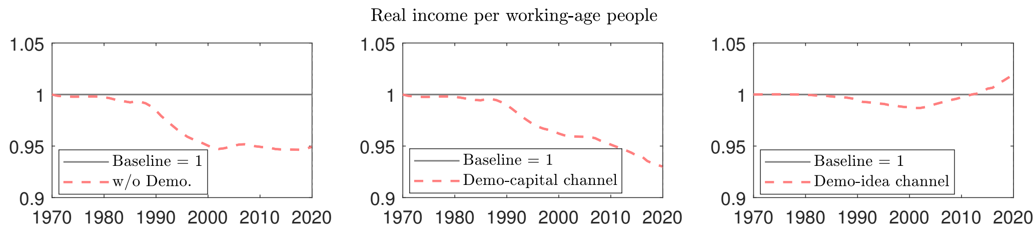


Figure 21
Real income per working-age people

Additionally, the *Demographic-idea channel* initially pushes real income per working-age individual below the historical path before pulling it above that path post-2000. When China's demographic trends follow those of the rest of the world, the working-age population initially falls below the historical path but eventually rises above it. Since the size of the working-age population correlates positively with productivity—given that a larger working-age population generates more ideas—this demographic structure ultimately raises real income per person, or per working-age individual, above the historical path. By 2020, through the *Demographic-idea channel*, real income per working-age individual in China would be about 1.95% higher than the observed level.

In sum, the *Demographic-capital channel* exerts a stronger influence than the *Demographic-idea channel*. Thus, if China's fertility and survival rates had matched those of the rest of the world, China's transitional path would reflect a lower real income per working-age individual than in its real-world counterpart. As of 2020, this income difference would amount to about 5.08% decrease from observed levels.

Demographics and Trade Pattern Changes Building on the counterfactual exercises previously discussed, I now focus on changes in trade patterns and specialization. I utilize the Revealed Comparative Advantage (RCA) index for the capital-intensive sector to quantify shifts in comparative advantage across sectors.

The Revealed Comparative Advantage (RCA) index, as developed by Balassa (1965), is used here to identify implied comparative advantage forces within each sector. The RCA for the capital-intensive sector is defined as:

$$RCA_{nj} = \left(\frac{Export_{n,j}}{\sum_n Export_{n,j}}\right) / \left(\frac{\sum_j Export_{n,j}}{\sum_{j,n} Export_{n,j}}\right),$$

where n refers to the country, j refers to the sector, and $Export_{n,j}$ denotes the value of country n's sector j exports. A higher RCA_{nj} indicates a greater degree of specialization for country n in the products of sector j.

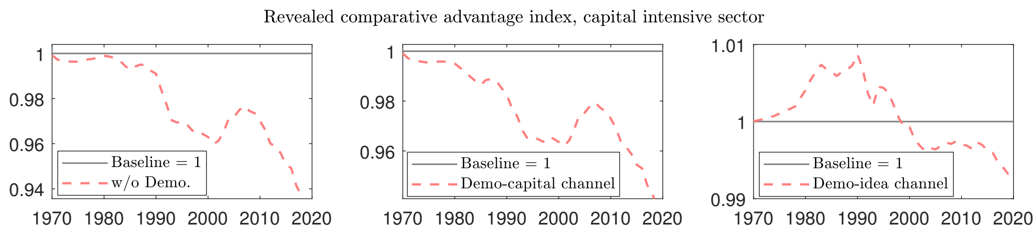


FIGURE 22
REVEALED COMPARATIVE ADVANTAGE INDEX

As shown in Figure 22, if China's fertility and survival rates were aligned with those of the rest of the world, the RCA index for China's capital-intensive sector would decrease relative to its previous trajectory, with the gap increasing over time. This downward trend implies that, under RoW similar demographic trends, China's comparative advantage in the capital-intensive sector would decline. Conversely, this shift would indicate a rising comparative advantage in the labor-intensive sector. By 2020, had China's demographics matched those of RoW, its degree of comparative advantage in the capital-intensive sector would be approximately 6.79% lower than the reality level, implying a stronger comparative advantage in the labor-intensive sector.

Two primary forces drive these deviations from the previous trajectory: the *Demographic-Capital Channel* and the *Demographic-Idea Channel*.

As illustrated in Figure 22, the *Demographic-Capital Channel* reduces China's comparative advantage in the capital-intensive sector relative to its previous path. Under demographic trends similar to the rest of the world (RoW), China's capital per working-age person would fall below its actual level, thereby implying a lower comparative advantage in the capital-intensive sector.

By 2020, the *Demographic-Capital Channel* would have reduced China's comparative advantage in the capital-intensive sector about 6.15% below its actual level, indicating a corresponding increase in the labor-intensive sector's comparative advantage.

From Figure 21, the *Demographic-idea channel* initially drove the RCA index for China's capital-intensive sector above the old path, before pushing it below the old path after the 1990s.

As China's demographic trends began to align with those of the rest of the world (RoW), China's working-age population initially fell below the old path, subsequently rising above it. Since the size of the working-age population is positively related to productivity (as a larger working-age cohort generates more ideas), the knowledge stock first trailed below the old path before later exceeding it.

Furthermore, given that sectors differ in their idea duplication coefficient, "φ_j," capital-intensive manufacturing and services sectors, which exhibit relatively lower "φ_j" values, imply a lower elasticity of knowledge stock (or the number of new ideas) with respect to the working-age population. Consequently, despite similar shifts in the working-age population, the variation in the capital-intensive sector will be smaller. Specifically, for the capital-intensive sector, the knowledge stock (productivity) growth rate declines less initially and rises less subsequently, suggesting a higher comparative advantage at the beginning, but a lower advantage thereafter.

By 2020, if China's demographic trends had continued to mirror those of the RoW, then through the *Demographic-idea channel*, China's comparative advantage in the capital-intensive sector would be approximately 0.068% lower than in reality.

Overall, the *Demographic-Capital Channel* exerts a greater influence than the *Demographic-Idea Channel*. Thus, if China's fertility and survival rates equaled those of the rest of the world, China would experience a reduction in comparative advantage in the capital-intensive sector compared to its real-world counterpart along the transition path. By 2020, China's comparative advantage in the capital-intensive sector would be about 6.79% lower than its actual level.

5.3. Model based projection

In this subsection, I analyze how China's demographic trends influence its future economic growth, drawing on the counterfactual exercises presented in the previous section.

Figure 23 displays the time series for the total population, working-age population, and working-age share over an extended period from 1970 to 2070. Assuming an identical initial population distribution, if China's fertility and survival rates were aligned with those of the rest of the world,[7] China's total population would have been larger than the actual observed levels, with this gap expected to widen over time.

A similar trend can be observed within the working-age population. This demographic shift implies similar patterns for knowledge stock, as a larger working-age population , on average, generates more ideas, thereby contributing to knowledge stock growth.

The working-age share, however, presents a contrasting pattern. Initially, this counterfactual share falls below the old path, with the gap gradually widening up to the 2010s.

7. Overall, this scenario would result in a lower trajectory for China's increasing survival rate and a higher trajectory for China's decreasing fertility rate.

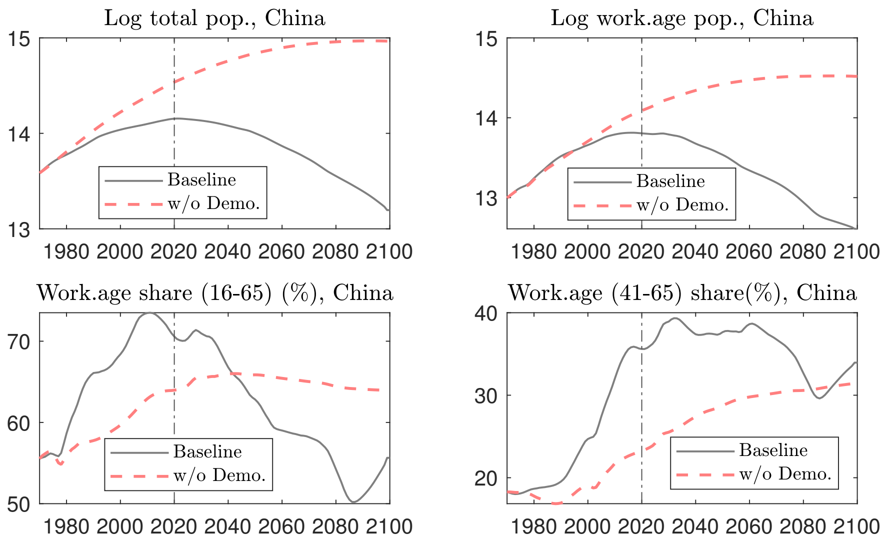


FIGURE 23
DEMOGRAPHIC PROCESS

Afterward, however, this gap begins to narrow, with the counterfactual working-age share converging toward the old path around 2040. Beyond the 2040s, the counterfactual share continues to rise, diverging further from the old path.

These demographic shifts suggest analogous trends for the capital stock process. As depicted in Figure 24, regarding capital stock per person, had China's fertility and survival rates aligned with RoW, the capital stock per person would have experienced a decline and remained below the old path for the entire period from 1970 to 2070. This trend is influenced by variations in population distribution across age groups, as individuals at different life stages tend to save at differing rates. Consequently, an age structure favoring savings leads to an increase in capital stock per working-age person, and vice versa.

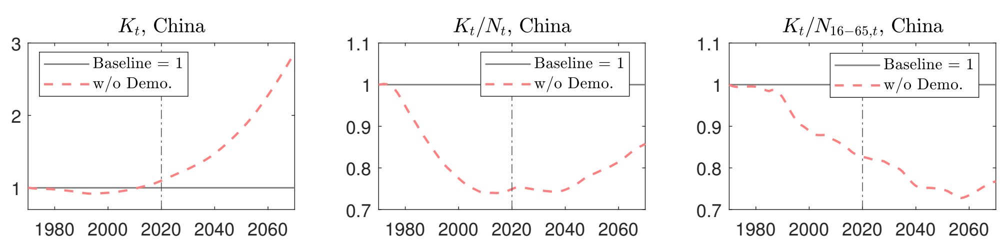


FIGURE 24
CAPITAL STOCKS

Figure 25 presents the time-varying process of China's real income per working-age person, along with its counterfactual counterparts under three aforementioned scenarios. If China's fertility and survival rates were to align with those of the rest of the world, the real income per working-age person in China would be lower relative to the baseline path, with the gap increasing over time. This gap begins to narrow after 2020, and by around 2060, the counterfactual path of real income per worker surpasses the old path.

Two primary forces drive the U-shaped deviations of the counterfactual real income per working-age person from the historical path: the *Demographic-capital channel* and the *Demographic-idea channel*. As illustrated in Figure 24 and Figure 25, the *Demographic-capital channel* pulls the counterfactual real income per working-age person in China below the historical path due to the declining capital stock per working-age person relative to the baseline. The *Demographic-idea channel* has driven the counterfactual real income per working-age person in China above its historical path. If China's fertility rate and survival rate were equal to those of the rest of the world (RoW), it would imply a larger working-age population than in reality. Since the size of the working-age population correlates positively with productivity, the *Demographic-idea channel* leads to higher productivity levels, which in turn raises real income per person above the actual path.

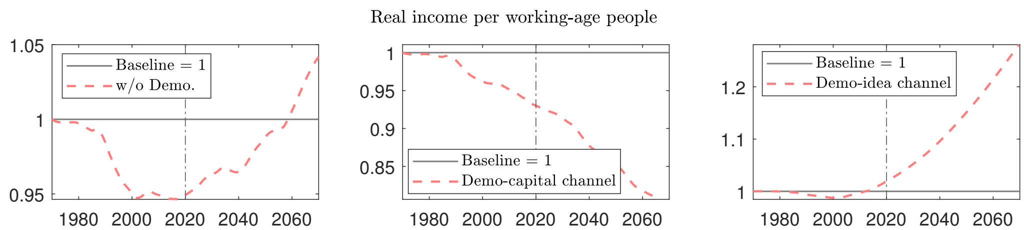


FIGURE 25
REAL INCOME PER WORKING-AGE PEOPLE

These two forces operate in opposite directions, with their relative strengths reversing around the year 2060. This implies that, before 2060, assuming an identical initial population distribution, if China's fertility rate and survival rate were equal to that of the rest of the world (RoW), the *Demographic-capital channel* effects would dominate. The counterfactual demographic structure favors consumption more than the actual demographic structure, which results in a lower capital per worker compared to the baseline, thus leading to a lower real income per worker relative to the baseline.

After 2060, the *Demographic-idea channel* effects become larger. The counterfactual demographic structure implies a higher working-age population relative to reality. Since the size of the working-age population is positively correlated with productivity, this results in higher productivity, leading to a higher real income per worker compared to the baseline.

These facts reveal a short-run and long-run trade-off for China's paiticular demographics process compared to that the rest of the world (RoW). If China's fertility rate and survival rate were equal to that of the rest of the world (RoW), in the short run until 2060, the counterfactual demographic structure would imply lower capital stocks per worker, resulting in a lower level of real income per worker. In the long run, after 2060, the counterfactual demographic structure features a higher proportion of the working-age population. This shift would bring more ideas, thereby boosting productivity. As a result, the long-run effect leads to a higher level of real income per worker than in the reality scenario.

6. Conclusion

Motivated by China's recent economic slowdown, the relocation of labor-intensive industries, and its aging population, this paper investigates how demographic forces have shaped China's economic growth and trade patterns, and provides model-based projections for its future growth..

I first estimate the relationship between demographics and trade liberalization on various macroeconomic outcomes using country-level panel regressions and a VARX model. The findings suggest that countries with a higher share of the working-age population experience higher productivity growth rates and a larger share of savings or investment in GDP. Moreover, a smaller working-age population share is associated with a lower growth rate in the capital-labor ratio. Additionally, the impulse response function (IRF) resulting from a 1 percentage point shock to the share of the young cohort exhibits a hump-shaped response, indicating that the effects of the shock diminish as the cohort ages.

I then develop and calibrate an OLG-trade model that incorporates the empirical features identified in the previous sections. The model includes three key features: First, given that people vary by age and ability to generate new ideas, the demographic structure will be a crucial element driving TFP growth. Second, the model incorporates both dynamic and OLG features to capture the impact of demographic structure on capital accumulation. Finally, the sectoral production function integrates labor, capital, and TFP, making it a multi-sector trade model that combines both Heckscher-Ohlin and Ricardian forces.

In this model, the primary driving forces—age-time varying fertility rates and survival rates—affect both sectoral productivity and capital accumulation through the model's mechanisms. These forces, together with exogenous trade cost changes, influence sectoral prices, which, in turn, affect the allocation of production across sectors and locations, ultimately impacting both trade patterns and economic performance. For example, a higher survival rate leads to greater knowledge and capital stock accumulation, raising the balanced growth path by enhancing both productivity and capital. Free trade induces specialization, which encourages higher productivity and lowers prices, ultimately leading to greater capital accumulation. In addition, a lower fertility rate impacts both knowledge and capital stocks. In the short run, a reduced young population raises capital per person, temporarily boosting economic output. However, over time, this benefit is offset by slower productivity growth due to the demographic shift, ultimately causing capital per person to fall below the previous growth path. Trade liberalization can mitigate this long-term drawback by maintaining capital per person above the old growth path for a longer period, thereby extending the overall economic benefits compared to a closed economy scenario.

The calibrated model quantitatively replicates China's capital stock process and income changes over time. It also replicates many other endogenous variables in the model, such as trade flow, the value of total production, and expenditure. To assess the role of each driving force affecting trade patterns and economic growth over time, I conduct three counterfactual exercises. First, I eliminate both the demographic-induced saving effects and demographic-induced productivity changes. Specifically, I replace the age-varying fertility and survival rates in China with those of the rest of the world (RoW), while allowing productivity to change in response to demographic shifts. I refer to this as the *without demographic scenario*. Second, I remove only the demographic-induced saving effects. In this scenario, I again replace China's age-varying fertility and survival rates with those of the RoW, but retain the usual productivity changes. I refer to this as the *demographic-capital channel scenario*. Third, I focus solely on demographic-induced productivity changes. In this case, I maintain the original age-varying fertility and survival rates of China, and its implied demographic process. However, I allow productivity to change as if China's demographic structure were aligned with that of the rest of the world (RoW), where China's age-varying fertility and survival rates mirror those of the RoW. I refer to this as the *demographic-idea channel scenario*.

I find that, in the short run, the *demographic-capital channel* exerts a stronger influence than the *demographic-idea channel*. Thus, if China's fertility and survival rates had matched those of the rest of the world, China's counterfactual transitional path would reflect a lower real income per worker than its real-world counterpart. In parallel, China would experience a lower degree of its comparative advantage in the capital-intensive sector compared to its real-world counterpart. By 2020, China's real income per worker would be about 5.08% lower than its actual level, and China's revealed comparative advantage index in the capital-intensive sector would be about 6.79% lower than its actual level. These short-run trends would persist until 2060. In the long run, the *demographic-idea channel* exerts a stronger influence than the *demographic-capital channel*. After 2060, the counterfactual demographic structure implies a higher working-age population relative to reality. Since the size of the working-age population is positively correlated with productivity, this results in higher productivity, leading to a higher real income per worker compared to the baseline. By 2070, China's real income per worker would be about 4.08% higher than its projected baseline level.

In the framework, demographic structure affects productivity through the proposed idea generation process. However, another potential link between demographic structure and productivity involves incorporating age-dependent productivity would also deserved to try, where the effectiveness of labor varies by age cohort. Additionally, the model currently assumes a financial autarky, with the trade balance being exogenous. Allowing for cross-border capital flows (or cross-border borrowing) and endogenous trade deficits would provide further insights into whether demographic structure affects economic growth and trade patterns through cross-border capital flows. Finally, including countries that have not yet experienced population aging as single country, such as India and Vietnam, would provide additional insights into how global demographic transitions shape global production reallocation and their effects on income. I leave these and other exercises for future research.

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Appendix A: Illustrative figures and/or tables

TABLE A.1
Country Groups

couty	countrycode	country_nam	couty	countrycode	country_nam
1	AUS	Australia	14	IND	India
2	AUT	Austria	15	IRL	Ireland
3	BEL	Belgium	16	ITA	Italy
4	BRA	Brazil	17	JPN	Japan
5	CAN	Canada	18	KOR	Korea, Republic of
6	CHN	China	19	MEX	Mexico
7	DEU	Germany	20	NLD	Netherlands
8	DNK	Denmark	21	PRT	Portugal
9	ESP	Spain	22	SWE	Sweden
10	FIN	Finland	23	TWN	Taiwan
11	FRA	France	24	USA	United States of America
12	GBR	United Kingdom	25	ROW	Rest of the World
13	GRC	Greece			

TABLE A.2
Sector classifications

ISIV 3	Sector Description	ISIV 3.1	ISIV 4
AtB	Agriculture, Hunting, Forestry and Fishing	AtB	At1t3
C	Mining and Quarrying	C	B
D15t16	Food, Beverages and Tobacco	15t16	C10t12
D17t19	Textiles, Textile, Leather and Footwear	17t19	C13t15
D20	Wood and Products of Wood and Cork	20	C16
D21t22	Pulp, Paper, Paper, Printing and Publishing	21t22	C17, C18, J58tJ60
D23	**Coke, Refined Petroleum and Nuclear Fuel**	23	C19
D24	**Chemicals and Chemical Products**	24	C20, C21
D25	Rubber and Plastics	25	C22
D26	Other NonMetallic Mineral	26	C23
D27t28	Basic Metals and Fabricated Metal	27t28	C24, C25
D29	Machinery, Nec	29	C28
D30t33	**Electrical and Optical Equipment**	30t33	C26, C27
D34t35	Transport Equipment	34t35	C29, C30
Dnec	Manufacturing, Nec; Recycling	36t37	C31t33
E	**Electricity, Gas and Water Supply**	E	D35, E36t39
F	Construction	F	F
G	Wholesale and Retail Trade	50t52	G45t47
H	Hotels and Restaurants	H	I
I60t63	Transport and Storage	60t63	H49t52
I64	**Post and Telecommunications**	64	H53, J61
J	**Financial Intermediation**	J	K64t66
K	**Real Estate, Renting and Business Activities**	70t74	J62,J63, L68tM70, M71t75, N
LtQ	Community Social and Personal Services	LtQ	O84, P85, Q, R,S, T, U

TABLE A.3
SECTOR CLASSIFICATIONS

#1.	5 Sector Classification	Index 1	Index 2	#2.	Sector Description
1	Agriculture, Mining and Quarrying	0.76	0.87	1	Agriculture, Hunting, Forestry and Fishing
1	Agriculture, Mining and Quarrying	0.40	0.34	2	Mining and Quarrying
2	Manufacture-labor intensive	0.59	0.72	3	Food, Beverages and Tobacco
2	Manufacture-labor intensive	0.64	0.72	4	Textiles, Textile, Leather and Footwear
2	Manufacture-labor intensive	0.63	0.78	5	Wood and Products of Wood and Cork
2	Manufacture-labor intensive	0.60	0.68	6	Pulp, Paper, Paper, Printing and Publishing
3	**Manufacture-capital intensive**	**0.47**	**0.44**	7	**Coke, Refined Petroleum and Nuclear Fuel**
3	**Manufacture-capital intensive**	**0.44**	**0.41**	8	**Chemicals and Chemical Products**
2	Manufacture-labor intensive	0.56	0.60	9	Rubber and Plastics
2	Manufacture-labor intensive	0.52	0.52	10	Other NonMetallic Mineral
2	Manufacture-labor intensive	0.51	0.51	11	Basic Metals and Fabricated Metal
2	Manufacture-labor intensive	0.57	0.62	12	Machinery, Nec
3	**Manufacture-capital intensive**	**0.49**	**0.44**	13	**Electrical and Optical Equipment**
2	Manufacture-labor intensive	0.55	0.56	14	Transport Equipment
2	Manufacture-labor intensive	0.66	0.81	15	Manufacturing, Nec; Recycling
3	**Manufacture-capital intensive**	**0.41**	**0.33**	16	**Electricity, Gas and Water Supply**
4	Services-labor intensive	0.72	0.93	17	Construction
4	Services-labor intensive	0.61	0.95	18	Wholesale and Retail Trade
4	Services-labor intensive	0.76	0.91	19	Hotels and Restaurants
4	Services-labor intensive	0.68	0.89	20	Transport and Storage
5	**Services-capital intensive**	**0.42**	**0.50**	21	**Post and Telecommunications**
5	**Services-capital intensive**	**0.50**	**0.51**	**22**	**Financial Intermediation**
5	**Services-capital intensive**	**0.44**	**0.40**	**23**	**Real Estate, Renting and Business Activities**
4	Services-labor intensive	0.75	0.86	24	Community Social and Personal Services

APPENDIX B: MECHANISM: ONE-SECTOR CLOSE ECONOMY

To better understand the mechanisms by which demographic structure affects productivity changes and capital accumulation over time and their effects on GDP, this section presents a one-sector closed economy model and characterizes its general equilibrium.

The one-sector closed economy model serves as a basis for understanding the multi-sector open economy model discussed in the following section. Both models share similar features, but the one-sector closed economy model places greater emphasis on the direct effects of demographic structure on GDP changes over time through its effects on productivity changes and capital accumulation. Furthermore, by comparing the closed economy with the open economy introduced in the following section, we can better understand the indirect effects of demographic structure on GDP over time through trade based on changes in productivity and capital accumulation.

B.1. Households

The households join the economy at age $g = 1$ and continue their participation until they either experience untimely demise or reach age $g = G$. The households supply labor $l_g = 1$ inelastically during working age $g \in [G_0 + 1, G_1]$ and are retired with $l_g = 0$ during $g \in [G_1 + 1, G]$. Population dynamics are exogenous in the model.

The measure $N_{g,t}$ is the number of households of age g alive at time t. Denote N_t as the number of total population in the economy at any period t, $\bar{L}_t$ as the number of total working-age households plus the retired households, and L_t as the aggregate labor supply. Thus,

$$N_t \equiv \sum_{g=1}^{G} N_{g,t} \quad \forall t \tag{B.1}$$

$$\bar{L}_t \equiv \sum_{g=G_0+1}^{G} N_{g,t} \quad \forall t \tag{B.2}$$

$$L_t = \sum_{g=1}^{G} N_{g,t} l_g = \sum_{g=G_0+1}^{G_1} N_{g,t} \quad \forall t \tag{B.3}$$

$$l_g = \begin{cases} 0 \ if \ g \in [1, G_0] \\ 1 \ if \ g \in [G_0+1, G_1] \text{(B.4)} \\ 0 \ if \ g \in [G_1{+}1, G] \end{cases}$$

Define the new cohort growth rate is n. The number of households at age g is $(1+n)$ times as large as of its younger cohort, So

$$N_{g,t} = (1+n)N_{g-1,t}$$

and thus

$$N_{t+1} = (1+n)N_t$$

$$\bar{L}_{t+1} = (1+n)\bar{L}_t$$

$$L_{t+1} = (1+n)L_t$$

At each time t, share of people at age g is constant and defined as $\tilde{N}_g$: Thus

$$\tilde{N}_g = \tilde{N}_{g,t} = \frac{(1+n)^{g-1}}{\sum_{g=1,2,3}(1+n)^{g-1}} \tag{B.5}$$

$$N_{g,t} = \tilde{N}_g N_t \tag{B.6}$$

B.1.I. Demographic process

The term $N_{g,t}$ is the number of households of age g alive at time t. The term $f_{g,t}$ is the fertility rate of age g households at time t. $s_{g,t}$ is the probability of surviving to age g at time t, given that they were alive at $g-1$. Here, define $s_{G+1,t} = 0$ which implies people live until age G.

The implied unconditional probability of surviving g periods up to time t is given by:

$$S_{g,t} = \prod_{k=1}^{g} s_{k,t+k-g}$$

The unconditional probability of people dying at age g at time t is:

$$D_{g,t} = S_{g-1,t-1}(1 - s_{g,t}) = S_{g-1,t-1} - S_{g,t}$$

The expected life expectancy for people born at t is:

$$\bar{g}_t = \sum_{k=1}^{G} k \cdot D_{k,t+k-1} \quad (1)$$

The demographic process can be represented as:

$$N_{1,t+1} = s_{1,t} \sum_{g=1}^{G} f_{g,t} N_{g,t}$$

$$N_{g+1,t+1} = s_{g+1,t+1} N_{g,t}$$

which implies

$$\begin{bmatrix} N_{1,t+1} \\ \vdots \\ N_{g,t+1} \\ \vdots \\ N_{G,t+1} \end{bmatrix} = \begin{bmatrix} s_{1,t+1}f_{1,t} & \cdots & s_{1,t+1}f_{g,t} & \cdots & s_{1,t+1}f_{G,t} \\ s_{2,t+1} & 0 & 0 & \cdots & 0 \\ 0 & s_{g+1,t+1} & 0 & \cdots & 0 \\ 0 & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & s_{G-1,t+1} & \cdots & 0 \\ 0 & 0 & 0 & s_{G,t+1} & 0 \end{bmatrix} \cdot \begin{bmatrix} N_{1,t} \\ \vdots \\ N_{g,t} \\ \vdots \\ N_{G,t} \end{bmatrix}$$

where $N_t = \Omega_t N_t$. The matrix Ω_t is a function of fertility rates $f_{g,t}$ and survival rates $s_{g,t+1}$.

B.1.II. Individual lifetime consumption and saving

An age g households that were born in period t choose lifetime consumption $\{c_{g,t+g-1}\}_{g=1}^{G}$ and savings $\{a_{g+1,t+g}\}_{g=1}^{G-1}$ to maximize expected lifetime utility, subject to the budget constraints and non-negativity constraints. The utility function is $u(c) = \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$.

$$\max_{\{c_{g,t+g-1}\}_{g=1}^{G}\{a_{g+1,t+g}\}_{g=1}^{G-1}} \sum_{g=1}^{G} \beta^{g-1} S_{g,t+g-1} u(c_{g,t+g-1})$$

$$c_{g,t+g-1} + a_{g+1,t+g} = (1 + r_{t+g-1})\, a_{g,t+g-1} \quad + \omega_{t+g-1} l_g + ts_{t+g-1} \qquad \forall g \in [1, G], t \tag{B.7}$$

$$a_{1,t} = a_{G+1,t} = 0 \quad and \quad c_{g,t} > 0 \tag{B.8}$$

The accidental bequests left by households who passed away accidentally between period

$t-1$ to t are distributed equally across the total population as a lump-sum transfer, denoted as ts_t.

$$ts_t \equiv \frac{(1+r_t)\sum_{g=2}^{G}\left(N_{g-1,t-1}-N_{g,t}\right)a_{g,t}}{N_t} \qquad \forall t \tag{B.9}$$

At each age, households earn income from inelastically supply labor forces to domestic firms and earn interest from last period's or age's saving. Households can save or borrow in the financial markets with interest rate r_t at each period t. The quantity of savings owned by an households of age g, born at time t, that was chosen in the previous period is denoted by $a_{g,t+g-1}$. They face a consumption-saving decision. Age g households born at time t can save with quantity $a_{g+1,t+g}$ to be returned to them with interest in the next period. Households born with zero assets position at age 1 and dies with zeros asset position at G: $a_{1,t} = a_{G+1,t} = 0$.

B.1.III. Euler equation

For households at age $g \in [1, G]$ born at time t, Given the sequences of prices, transfers, households optimize on the intertemporal decisions of consumption and investment $\{c_{g,t+g-1},\ a_{g+1,t+g}\}$. The intertemporal Euler equation determines consumption and investment choices:

$$u'\left(c_{g,t+g-1}\right) = \left(\beta s_{g+1,t+g}\right)\left(1+r_{t+g}\right)u'\left(c_{g+1,t+g}\right) \qquad \forall\ g \in [1, G-1] \tag{B.10}$$

which implies:

$$\frac{c_{g+1,t+1}}{c_{g,t}} = \left[\left(\beta s_{g+1,t+1}\right)\left(1+r_{t+1}\right)\right]^{\sigma}, \forall\ g \in [1, G-1] \tag{B.11}$$

Now, aggregate individual variables across cohorts, I have:

$$\begin{aligned} C_t &\equiv \sum_{g=1}^{G} N_{g,t}c_{g,t} \\ K_t &\equiv \sum_{g=2}^{G} N_{g-1,t-1}a_{g,t} \end{aligned} \tag{B.12}$$

$$\begin{aligned} c_{g,t} + a_{g+1,t+1} &= (1+r_t)\,a_{g,t} + \omega_t l_g + ts_t \\ ts_t &\equiv \frac{(1+r_t)\sum_{g=2}^{G}\left(N_{g-1,t-1}-N_{g,t}\right)a_{g,t}}{N_t} \end{aligned} \tag{B.13}$$

Aggregating individual budget constraints across ages. the budget constraint at the aggregate level is

$$\sum_{g=1}^{G} N_{g,t}c_{g,t} + \sum_{g=1}^{G} N_{g,t}a_{g+1,t+1} \tag{B.14}$$

$$= (1+r_t)\sum_{g=1}^{G} N_{g,t}a_{g,t} + \sum_{g=1}^{G} N_{g,t}\omega_t l_g + \sum_{g=1}^{G} N_{g,t}ts_t \tag{B.15}$$

which implies

$$C_t + K_{t+1} = (1+r_t)\,K_t + L_t\omega_t \tag{B.16}$$

Aggregate investment can be calculated from

$$I_t \equiv K_{t+1} - (1-\delta)\,K_t \tag{B.17}$$

Which implies:

$$C_t + I_t = (r_t+\delta)\,K_t + \omega_t L_t \tag{B.18}$$

B.1.IV. Variables under steady state

At each time t, for each household at age g, the number of consumption and saving is growth at constant rate $\Gamma = (1+\gamma)^{(1-\alpha)} - 1$ and defined as $\{c_{g,t}\}_{g=1}^{G} = \left\{(1+\Gamma)^t \bar{c}_g\right\}_{g=1}^{G}$ and savings $\{a_{g+1,t}\}_{g=1}^{G} = \left\{(1+\Gamma)^t \bar{a}_{g+1}\right\}_{g=1}^{G}$, and real wage $\omega_t = (1+\Gamma)^t\bar{\omega}$.

In the main text, I prove that if the number of consumption and saving $\{c_{g,t}, a_{g,t}, \omega_t\}$ is growth at constant for any t, then C_t and K_t is growth at the constant rate $(1+n)(1+\Gamma)-1$; In the **??**, I prove above inversely: if C_t and K_t is growth at the constant rate, then the number of consumption and saving $\{c_{g,t}, a_{g,t}\}$ is growth at constant rate too.

$$TS_t \equiv ts_t N_t = (1+r_t)\sum_{g=2}^{G}\left(N_{g-1,t-1} - N_{g,t}\right)a_{g,t} \tag{B.19}$$

$$ts_t = \frac{1}{N_t}TS_t = \frac{N_t}{N_t}\left(1+\bar{r}\right)\sum_{g=2}^{G}\frac{\left(N_{g-1,t-1} - N_{g,t}\right)}{N_t}a_{g,t} \tag{B.20}$$

$$= (1+\bar{r})\sum_{g=2}^{G}\left(\frac{\bar{N}_{g-1}}{(1+n)} - \bar{N}_g\right)a_{g,t} \tag{B.21}$$

Thus, $ts_t = \dfrac{TS_t}{N_t}$ growth at same speed with $a_{g,t}$. Define $\overline{ts} = (1+\bar{r})\sum_{g=2}^{G}\left(\dfrac{\bar{N}_{g-1}}{(1+n)} - \bar{N}_g\right)\bar{a}_g$.

The budget constraint

$$c_{g,t} + a_{g+1,t+1} = (1 + r_t)\, a_{g,t} + \omega_t l_g + ts_t,\ \forall g \geq 1, t \tag{B.22}$$
$$c_{g+1,t+1} = c_{g,t} \left[(\beta s_{g+1,t+1}) (1 + \bar{r}) \right]^{\sigma} \tag{B.23}$$

implies:

$$(1+\Gamma)^t \bar{c}_g + (1+\Gamma)^{t+1} \bar{a}_{g+1} = (1+\bar{r})(1+\Gamma)^t \bar{a}_g + (1+\Gamma)^t \bar{\omega} l_g + (1+\Gamma)^t \overline{ts},\ \forall g \geq 1, t \tag{B.24}$$
$$(1+\Gamma)^{t+1} \bar{c}_{g+1} = (1+\Gamma)^t \bar{c}_g \left[(\beta s_{g+1,t+1}) (1+\bar{r}) \right]^{\sigma} \tag{B.25}$$

so

$$\bar{c}_g + (1+\Gamma)\bar{a}_{g+1} = (1+\bar{r})\, \bar{a}_g + \bar{\omega} l_g + \overline{ts},\ \forall g \geq 1, t \tag{B.26}$$
$$(1+\Gamma)\bar{c}_{g+1} = \bar{c}_g \left[(\beta s_{g+1,t+1}) (1+\bar{r}) \right]^{\sigma} \tag{B.27}$$

.

Here $\bar{c}_g$ and $\bar{b}_g$ is per efficient for age g cohorts and defined as:

$$\bar{c}_g = \frac{c_{g,t}}{(1+\Gamma)^t}; \qquad \bar{a}_g = \frac{a_{g,t}}{(1+\Gamma)^t} \tag{B.28}$$

and

$$\overline{ts} = \frac{ts_t}{(1+\Gamma)^t}. \tag{B.29}$$

Here $\bar{r}$ and $\bar{\omega}$ is real interest rate and real wage per efficient labor.

The aggregate consumption and capital stock is thus growth at a constant rate n:

$$\begin{aligned} C_{t+1} &\equiv \sum_{g=1}^{G} N_{g,t+1} c_{g,t+1} = \sum_{g=1}^{G} (1+n)(1+\Gamma) N_{g,t} c_{g,t} = (1+n)(1+\Gamma) C_t \\ K_{t+1} &\equiv \sum_{g=2}^{G} N_{g-1,t} a_{g,t} = \sum_{g=2}^{G} (1+n)(1+\Gamma) N_{g-1,t-1} a_{g,t-1} = (1+n)(1+\Gamma) K_t \\ TS_{t+1} &\equiv ts_{t+1} N_{t+1} = (1+\Gamma)(1+n) ts_t N_t = (1+n)(1+\Gamma) TS_t \end{aligned} \tag{B.30}$$

.

So, the aggregate investment is proportional to K_t:

$$I_t = K_{t+1} - (1-\delta)\, K_t = \left((1+n)(1+\Gamma) + \delta - 1 \right) K_t \tag{B.31}$$

The investment per person, i_t, in the steady state:

$$i_t = \left((1+n)(1+\Gamma) + \delta - 1 \right) k_t \tag{B.32}$$

and

$$I_{t+1} = (1+n)(1+\Gamma)I_t \tag{B.33}$$

.

Define $\bar{x}_g = \dfrac{x_{g,t}}{(1+\Gamma)^t}$ and $x_t = \dfrac{X_t}{N_t}$, then $\bar{x} = \dfrac{x_t}{(1+\Gamma)^t} = \dfrac{X_t}{N_t(1+\Gamma)^t}$;

$$\bar{c} = \frac{C_t}{N_t(1+\Gamma)^t} = \frac{\sum_{g=1}^{G} N_{g,t}c_{g,t}}{N_t(1+\Gamma)^t} = \sum_{g=1}^{G} \tilde{N}_g \bar{c}_g$$

$$\bar{k} = \frac{K_t}{N_t(1+\Gamma)^t} = \frac{\sum_{g=1}^{G} N_{g-1,t-1}a_{g,t}}{N_{t-1}(1+n)(1+\Gamma)^t} = \frac{1}{(1+n)}\sum_{g=1}^{G} \bar{N}_{g-1}\bar{a}_g = \frac{1}{(1+n)}\sum_{g=2}^{G} \bar{N}_{g-1}\bar{a}_g$$

; and

$$\bar{i} = ((1+n)(1+\Gamma)+\delta-1)\,\bar{k}; \qquad \bar{i} = \frac{i_t}{(1+\Gamma)^t}; \qquad \bar{k} = \frac{k_t}{(1+\Gamma)^t} \tag{B.34}$$

.

B.2. Firms

Firms produce a continuous range of goods, with each variety indexed by $\omega \in [0,1]$ and utilizing constant returns to scale (CRS) technology:

$$Y_t(\omega) \equiv z_t(\omega)\, L_t(\omega)^{1-\alpha} K_t(\omega)^{\alpha} \tag{B.35}$$

The terms $K_t(\omega)$ and $L_t(\omega)$ are the quantities of capital and labor used in the production of Y_t units of good ω. The productivity of good ω is $z_t(\omega)$, which is the random variable follows a Fréchet distribution $F_t(A) = exp\left(-\lambda_t z^{-\theta}\right)$.

B.2.I. Stock of knowledge

The productivity distribution at time t is defined as the technology frontier, represented by the CDF of the distribution. The scale parameter of this distribution is defined as the stock of knowledge. Based on (Buera and Oberfield, 2020), which model the evolution of the knowledge stock driven by the process of new idea arrival.

I introduce 'age-varying idea arrive rate' to the model. This is a scale variable that captures people's age-varying ability in finding new ideas per unit of time.

The new idea is drawn from the technology frontier itself. one can get the evolution of scale parameters as λ_t:

$$\lambda_{n,t+1} - \lambda_{n,t} = \alpha_t\,(\lambda_{n,t})^{\rho}\,\Gamma(1-\rho)\,, \text{ where } \alpha_t \equiv \left[\sum_g \eta_g N_{n,g,t}\right]^{\varphi}. \tag{B.36}$$

The term α_t is defined as the average number of arrived new ideas in Oberfield and Buera, 2019. It controls the mean number of new ideas arriving between t and $t+1$.

The term η_g is the number of arrived new idea per unit of time per age g people. With the number of age a people at country n time t $N_{n,g,t}$, the total number of arrived idea per unit of time is $\sum_g \eta_g N_{n,g,t}$. The term $\varphi < 1$ reflects some kind of crowding effects, or duplication of idea. Thus, total number of efficient idea arriaved between t and $t+1$ is

$$\alpha_t \equiv \left[\sum_g \eta_g N_{n,g,t} \right]^{\varphi}$$

.

Equation B.36 implies that, on the one hand, a higher the number of arrived new idea per unit of time per age g people. (higher η_g) leads to a higher knowledge stock for the following period, which in turn implies a higher overall productivity level. On the other hand, at the steady state, λ_t grows at the constant rate g_λ, and $1 + g_\lambda = (1 + g_n)^{\varphi/(1-\rho)}$.

B.2.II. Composite goods

The composite intermediate good Y_t is consistent with intermediate good $y_t(\omega)$:

$$Y_t \equiv \left[\int_0^1 Y_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} \tag{B.37}$$

which implies:

$$Y_t(\omega) = \left[\frac{P_t(\omega)}{P_t} \right]^{-\sigma} Y_t; \quad P_t = \left[\int_0^1 P_t(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}};$$

$$\mathrm{P}_t = \Gamma \left(\frac{1+\theta-\sigma}{\theta} \right)^{\frac{1}{1-\sigma}} \frac{c_t}{{\lambda_t}^{1/\theta}} = \frac{\Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}(W_t)^{(1-\alpha)}(R_t)^{\alpha}}{{\lambda_t}^{1/\theta}}$$

$$A_t \equiv \left[\int_0^1 A_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} = \lambda_t^{1/\theta}(B)^{-1}$$

.

where $B = \Gamma\left(1 + \frac{1-\sigma}{\theta}\right)^{\frac{\theta}{1-\sigma}}$, and σ is the elasticity of substitution between varieties. The composite intermediate good Y_t has two uses. It can be used for consumption (C_t) and investment(I_t):

$$Y_t = I_t + C_t. \tag{B.38}$$

The price of composite intermediate good Y_t is P_t. Define value of total production as X_t, thus $P_t Y_t = X_t$, and $P_t Y_t = P_t C_t + P_t I_t = R_t K_t + W_t L_t$.

Given λ_t growth at rate g_λ, and $A_t \equiv \lambda_t^{1/\theta}(B)^{-1}$ growth at rate g_A, then $1 + g_A =$

$(1+g_\lambda)^{1/\theta}$. Define $1+\Gamma=(1+g_A)^{1/(1-\alpha)}$. Balance growth path implies ${A_t}^{1/(1-\alpha)}L_t$, Y_t and, K_t growth at the same speed:

$$\begin{aligned}\Delta Y_{t+1}/Y_t=\Delta K_{t+1}/K_t&=\Delta\left({A_{t+1}}^{1/(1-\alpha)}L_{t+1}\right)/\left({A_t}^{1/(1-\alpha)}L_t\right)\\&=(1+g_n)\,(1+\Gamma)-1.\end{aligned}$$

The firm hires household and rents capital to maximize profits implies:

$$W_tL_t=(1-\alpha)P_tY_t;\qquad R_tK_t=\alpha P_tY_t$$

Define $\bar{y}_t$ as output per efficient unit of labor,

$$\bar{y}_t=\frac{Y_t}{\left({A_t}^{1/(1-\alpha)}N_t\right)}=\frac{\left({A_t}^{1/(1-\alpha)}L_t\right)^{1-\alpha}{K_t}^{\alpha}}{\left({A_t}^{1/(1-\alpha)}N_t\right)}=l_t^{1-\alpha}\bar{k}_t^{\alpha}\tag{B.39}$$

where $l_t\equiv\dfrac{L_t}{N_t}$ is labor share at time t. Also, one can get

$$\bar{r}_t=\frac{R_t}{P_t}-\delta=(\alpha)\left(\frac{K_t}{{A_t}^{1/(1-\alpha)}L_t}\right)^{-(1-\alpha)}-\delta=(\alpha)\left(\frac{\bar{k}_t}{l_t}\right)^{-(1-\alpha)}-\delta\tag{B.40}$$

$$\omega_t=\frac{W_t}{P_t}=(1-\alpha)\left({A_t}^{1/(1-\alpha)}\right)\left(\frac{K_t}{{A_t}^{1/(1-\alpha)}L_t}\right)^{\alpha}=(1-\alpha)\left({\lambda_t}^{\frac{1}{\theta(1-\alpha)}}\right)\left(\frac{\bar{k}_t}{l_t}\right)^{\alpha}.\tag{B.41}$$

$$\bar{\omega}_t=\frac{W_t}{P_t}=(1-\alpha)\left(\frac{K_t}{{A_t}^{1/(1-\alpha)}L_t}\right)^{\alpha}=(1-\alpha)\left(\frac{\bar{k}_t}{l_t}\right)^{\alpha}.\tag{B.42}$$

Thus, real wage growth at Γ and real interest rate is constant, and

$$\bar{y}=\bar{c}+\bar{i}=\bar{r}\bar{k}+\frac{\omega_t}{{A_t}^{1/(1-\alpha)}}\frac{L_t}{N_t}=\bar{r}\bar{k}+\frac{\omega_t}{{A_t}^{1/(1-\alpha)}}l_t\tag{B.43}$$

where

$$\bar{c}=\frac{C_t}{\left({A_t}^{1/(1-\alpha)}N_t\right)};\qquad\bar{i}=\frac{I_t}{\left({A_t}^{1/(1-\alpha)}N_t\right)};\qquad\bar{k}=\frac{K_t}{\left({A_t}^{1/(1-\alpha)}N_t\right)}\tag{B.44}$$

B.2.III. Financial intermediate

The financial market received deposits of $a_{g,t}P_t$ from individuals, and must repay those individuals an amount $a_{g,t}\,(1+r_t)\,P_t$. The financial market loaned an amount $K_t=\sum_{g=2}^{G}N_{g-1,t-1}a_{g,t}$

to firms to use in production, and from them it receives an amount $\left(1 + \frac{R_t}{P_t} - \delta\right) P_t K_t$, where note that we have incorporated the fact that some of the capital depreciates in use, and so the total amount returned to the financial market is smaller by this amount. To clear the market it must be that these amounts are equal

$$\sum_{g=2}^{G} N_{g-1,t-1} a_{g,t} \left(1 + r_t\right) P_t = \left(1 + \frac{R_t}{P_t} - \delta\right) P_t K_t$$

and as this is a financial autarky economy, the total assets in the economy must be equal to the total capital stock $K_t = \sum_{g=2}^{G} N_{g-1,t-1} a_{g,t}$, What this implies is that

$$r_t = \frac{R_t}{P_t} - \delta$$

Under steady state, at each time t, the real interest rate is constant and defined as

$$\bar{r} = r_t = \frac{R_t}{P_t} - \delta = R_r - \delta \tag{B.45}$$

B.3. Equilibrium

B.3.I. Dynamic equilibrium

TABLE B.1
Dynamic equilibrium conditions

I1	$\lambda_{n,t+1} - \lambda_{n,t} = N_{n,t}{}^{\varphi} \left(\lambda_{n,t}\right)^{\rho} \left[\sum_g \eta_g \bar{N}_{n,g,t}\right]^{\varphi} \Gamma\left(1-\rho\right);\ 1+g_\lambda = \left(1+g_n\right)^{\varphi/(1-\rho)}$	$\forall(t)$
I2	$A_t \equiv \lambda_t^{1/\theta}\left(B\right)^{-1},B = \Gamma\left(1+\dfrac{1-\sigma}{\theta}\right)^{\frac{\theta}{1-\sigma}};\ 1+g_A = 1+\gamma = \left(1+g_\lambda\right)^{1/\theta};\ 1+\Gamma = \left(1+g_A\right)^{1/(1-\alpha)}$	$\forall(t)$
H1	$N_t \equiv \sum_{g=1}^{G} N_{g,t};\ \bar{L}_t \equiv \sum_{g=G_0+1}^{G} N_{g,t};\ L_t = \sum_{g=G_0+1}^{G_1} N_{g,t} l_g = \sum_{g=1}^{G} N_{g,t} l_g;$	$\forall(t)$
H2	$c_{g,t} + a_{g+1,t+1} = \left(1+r_t\right) a_{g,t} + \omega_t l_g + tr_t,\ \forall g \in [1,G]$	$\forall(t)$
H3	$a_{1,t} = a_{G+1,t} = 0;\ c_{g,t} > 0,\ \{c_{g,t+g-1}\}_{g=1}^{G};\ \{a_{g+1,t+g}\}_{g=1}^{G-1}$	$\forall(t)$
H4	$tr_t \equiv \dfrac{TR_t}{N_t} = \left(1+r_t\right)\sum_{g=2}^{G}\dfrac{\left(N_{g-1,t-1} - N_{g,t}\right)}{N_t} a_{g,t}$	$\forall(t)$
H5	$c_{g+1,t+1} = c_{g,t}\left[\beta s_{g+1,t+1}\left(1+r_{t+1}\right)\right]^{\sigma};\quad \forall\ g \in [1,G-1]$	$\forall(t)$
H6	$c_t = \sum_{g=1}^{G} \tilde{N}_{g,t} c_{g,t};\quad k_t = \dfrac{K_t}{N_t} = \dfrac{\sum_{g=1}^{G} N_{g-1,t-1} a_{g,t}}{N_t} = \sum_{g=1}^{G}\dfrac{N_{g-1,t-1}}{N_t} a_{g,t} = \sum_{g=2}^{G}\dfrac{N_{g-1,t-1}}{N_t} a_{g,t}$	$\forall(t)$
F1	$\omega_t = \dfrac{W_t}{P_t} = (1-\alpha)\left(A_t{}^{1/(1-\alpha)}\right)\left(\dfrac{K_t}{A_t{}^{1/(1-\alpha)} L_t}\right)^{\alpha} = (1-\alpha)\left(A_t{}^{1/(1-\alpha)}\right)\left(\dfrac{k_t}{A_t{}^{1/(1-\alpha)} l_t}\right)^{\alpha}$	$\forall(t)$
F2	$r_t = \dfrac{R_t}{P_t} - \delta = (\alpha)\left(\dfrac{K_t}{A_t{}^{1/(1-\alpha)} L_t}\right)^{-(1-\alpha)} - \delta = (\alpha)\left(\dfrac{k_t}{A_t{}^{1/(1-\alpha)} l_t}\right)^{-(1-\alpha)} - \delta$	$\forall(t)$
T1	$y_t = c_t + i_t = r_t k_t + \omega_t \dfrac{L_t}{N_t} = r_t k_t + \omega_t l_t$	$\forall(t)$
T2	$y_t = \dfrac{Y_t}{N_t} = A_t \dfrac{\left(L_t\right)^{1-\alpha} K_t{}^{\alpha}}{N_t} = A_t l_t^{1-\alpha} k_t^{\alpha}$	$\forall(t)$
T3	$C_t + I_t = (r_t + \delta) K_t + \omega_t L_t = Y_t$	$\forall(t)$
T3'	$\mathrm{C}_t + K_{t+1} = \left(1 + r_t - \delta\right) K_t + \omega_t L_t = Y_t$	$\forall(t)$
T4	$K_{t+1} = K_t(1-\delta) + I_t$	$\forall(t)$
S1	$x_t = \bar{x}\left(A_t{}^{1/(1-\alpha)}\right);\ x_{g,t} = \bar{x}_g\left(A_t{}^{1/(1-\alpha)}\right)$	$\forall(t)$
S2	$x_t \equiv \dfrac{X_t}{N_t};\bar{x} \equiv \dfrac{X_t}{\left(A_t{}^{1/(1-\alpha)} N_t\right)} \equiv \dfrac{x_t}{\left(A_t{}^{1/(1-\alpha)}\right)};x_t = \bar{x}\left(A_t{}^{1/(1-\alpha)}\right);$	$\forall(t)$
S3	$x_{g,t} \equiv \dfrac{X_{g,t}}{N_{g,t}};\bar{x}_g \equiv \dfrac{X_{g,t}}{\left(A_t{}^{1/(1-\alpha)} N_{g,t}\right)} \equiv \dfrac{x_{g,t}}{\left(A_t{}^{1/(1-\alpha)}\right)};x_{g,t} = \bar{x}_g\left(A_t{}^{1/(1-\alpha)}\right)$	$\forall(t)$

B.3.II. Steady-state

TABLE B.2
Steady-state conditions

g_n	$N_{t+1} = N_t(1+g_n)$	$\forall t \in [T-1,\infty)$
g_λ	$\lambda_{t+1} = (1+g_\lambda)\,\lambda_t;\ 1+g_A \equiv (1+g_\lambda)^{1/\theta};\ 1+g_\lambda = (1+g_n)^{\varphi/(1-\rho)}$	$\forall t \in [T,\infty)$
Γ	$X \in [\omega_t, tr_t, a_{g,t}, c_{g,t}]\ ;\ X_{t+1} = (1+\Gamma)\,X_t;\ 1+\Gamma = (1+g_A)^{1/(1-\alpha)}$	$\forall t \in [T,\infty)$
g_K	$X \in [C_t, I_t, K_t, Y_t]\ ;\ X_{t+1} = (1+g_K)\,X_t;\ 1+g_K = (1+\Gamma)\,(1+g_n)$	$\forall j, t \in [T,\infty)$
I1	$\lambda_{n,T+1} - \lambda_{n,T} = {N_{n,t}}^{\varphi}\,(\lambda_{n,T})^{\rho}\left[\sum_g \eta_g \bar{N}_{n,g,T}\right]^{\varphi}\Gamma\,(1-\rho)$	$\forall t \in [T,\infty)$
I2	$A_T \equiv \lambda_T^{1/\theta}\,(B)^{-1}\ , B = \Gamma\left(1+\dfrac{1-\sigma}{\theta}\right)^{\frac{\theta}{1-\sigma}};\ A_{t+1} = A_t(1+g_A);\ 1+g_A = (1+g_\lambda)^{\frac{1}{\theta}}$	$\forall t \in [T,\infty)$
H1	$1 \equiv \sum_{g=1}^{G} \bar{N}_g;\ \bar{L} \equiv \sum_{g=G_0+1}^{G} \bar{N}_g;\ l = \overline{labor} = \sum_{g=G_0+1}^{G_1} N_g l_g = \sum_{g=1}^{G} N_g l_g;\ l = \dfrac{L_t}{N_t}$	$\forall t \in [T,\infty)$
H2	$\bar{c}_g + (1+\Gamma)\bar{a}_{g+1} = (1+\bar{r})\,\bar{a}_g + \bar{\omega} l_g + \overline{tr};\forall g \in [1,G]$	$\forall t \in [T,\infty)$
H3	$\bar{a}_1 = \bar{a}_{G+1} = 0;\ \bar{c}_g > 0,\ \{\bar{c}_g\}_{g=1}^{G};\ \{\bar{a}_{g+1}\}_{g=1}^{G-1}$	$\forall t \in [T,\infty)$
H4	$\overline{tr} = (1+\bar{r})\sum_{g=2}^{G}\left(\dfrac{\bar{N}_{g-1}}{(1+n)} - \bar{N}_g\right)\bar{a}_g$	$\forall t \in [T,\infty)$
H5	$(1+\Gamma)\bar{c}_{g+1} = \bar{c}_g\,[\beta s_{g+1}\,(1+\bar{r})]^{\sigma};\quad \forall\ g \in [1, G-1]$	$\forall t \in [T,\infty)$
H6	$\bar{c} = \sum_{g=1}^{G} \tilde{N}_g \bar{c}_g\ ;\ \bar{k} = \dfrac{1}{(1+n)}\sum_{g=2}^{G} \bar{N}_{g-1}\bar{a}_g$	$\forall t \in [T,\infty)$
F1	$\omega_t = (1-\alpha)\left({A_t}^{1/(1-\alpha)}\right)\left(\dfrac{\bar{k}}{l}\right)^{\alpha};\ \bar{\omega} = (1-\alpha)\left(\dfrac{\bar{k}}{l}\right)^{\alpha}$	$\forall t \in [T,\infty)$
F2	$\bar{r} = (\alpha)\left(\dfrac{\bar{k}}{l}\right)^{-(1-\alpha)} - \delta$	$\forall t \in [T,\infty)$
T1	$\bar{y} = \bar{c} + \bar{i} = \bar{r}\bar{k} + \dfrac{\omega_t}{{A_t}^{1/(1-\alpha)}}\dfrac{L_t}{N_t} = \bar{r}\bar{k} + \dfrac{\omega_t}{{A_t}^{1/(1-\alpha)}} l$	$\forall t \in [T,\infty)$
T2	$\bar{y} = \bar{k}^{\alpha} l^{1-\alpha};\ y_t = \dfrac{Y_t}{N_t} = A_t\dfrac{(L_t)^{1-\alpha}\,{K_t}^{\alpha}}{N_t} = A_t l_t^{1-\alpha} k_t^{\alpha}$	$\forall t \in [T,\infty)$
T3	$\bar{i} = ((1+g_n)(1+\Gamma)+\delta-1)\,\bar{k};\ I_t = ((1+g_n)(1+\Gamma)+\delta-1)\,K_t$	$\forall t \in [T,\infty)$
T3'	$C_t + (1+g_n)(1+\Gamma)K_t = (1+r_t-\delta)\,K_t + \omega_t L_t = Y_t;$	$\forall t \in [T,\infty)$
T4	$(1+g_n)(1+\Gamma)K_t = K_{t+1} = K_t(1-\delta) + I_t$	$\forall t \in [T,\infty)$
S1	$x_t = \bar{x}\left({A_t}^{1/(1-\alpha)}\right);\ x_{g,t} = \bar{x}_g\left({A_t}^{1/(1-\alpha)}\right)$	$\forall t \in [T,\infty)$
S2	$x_t \equiv \dfrac{X_t}{N_t};\bar{x} \equiv \dfrac{X_t}{\left({A_t}^{1/(1-\alpha)}N_t\right)} \equiv \dfrac{x_t}{\left({A_t}^{1/(1-\alpha)}\right)};x_t = \bar{x}\left({A_t}^{1/(1-\alpha)}\right);$	$\forall t \in [T,\infty)$
S3	$x_{g,t} \equiv \dfrac{X_{g,t}}{N_{g,t}};\bar{x}_g \equiv \dfrac{X_{g,t}}{\left({A_t}^{1/(1-\alpha)}N_{g,t}\right)} \equiv \dfrac{x_{g,t}}{\left({A_t}^{1/(1-\alpha)}\right)};x_{g,t} = \bar{x}_g\left({A_t}^{1/(1-\alpha)}\right)$	$\forall t \in [T,\infty)$

Tilde "$\bar{x}$" or "$\bar{x}_g$" means x per efficiency unit of household, or x per efficiency unit of age g household, or real interest rate which is constant over time. Define $\bar{x}_g = \dfrac{x_{g,t}}{A_t^{1/(1-\alpha)}}$ and $x_t = \dfrac{X_t}{N_t}$,then $\bar{x} = \dfrac{x_t}{A_t^{1/(1-\alpha)}} = \dfrac{X_t}{N_t A_t^{1/(1-\alpha)}}$

Appendix C: Empirical analysis

Summary statistics

The specifics of summary statistics are listed in Table C.1.

TABLE C.1
DECRIPTIVE STATISTICS: NON-OVERLAPPING 5 YEARS BETWEEN 1975 - 2019, 76 COUNTRIES

VARIABLES	N	mean	sd	between.sd	within.sd	min	max	skewness	kurtosis
Age [0-9] share	684	0.212	0.0828	0.07694	0.03166	0.0723	0.383	0.229	1.689
Age [10-19] share	684	0.188	0.0463	0.04075	0.02246	0.0871	0.283	-0.306	1.828
Age [20-29] share	684	0.163	0.0207	0.01460	0.01479	0.0992	0.235	-0.106	3.298
Age [30-39] share	684	0.134	0.0217	0.01526	0.01547	0.0838	0.216	0.216	2.966
Age [40-49] share	684	0.108	0.0299	0.02501	0.01665	0.0547	0.192	0.252	2.050
Age [50-59] share	684	0.0834	0.0317	0.02830	0.01462	0.0300	0.170	0.398	1.977
Age [60-69] share	684	0.0602	0.0294	0.02784	0.00984	0.0169	0.144	0.523	2.020
Age [70-79] share	684	0.0368	0.0236	0.02267	0.00707	0.00726	0.113	0.679	2.221
Age [80-89] share	684	0.0143	0.0124	0.01123	0.00540	0.000645	0.0666	1.077	3.352
Age [90+] share	684	0.00197	0.00227	0.00183	0.00135	2.72e-06	0.0161	1.784	6.735
Child share	684	0.308	0.109	0.10071	0.04256	0.116	0.522	0.107	1.639
Working age share	684	0.611	0.0667	0.05812	0.03333	0.457	0.785	-0.311	2.051
Elderly share	684	0.0803	0.0518	0.04929	0.01674	0.0167	0.273	0.728	2.391
Dependence ratio	684	0.656	0.190	0.16679	0.09339	0.273	1.187	0.616	2.283
Young dependence ratio	684	0.530	0.244	0.22360	0.10056	0.159	1.142	0.433	1.899
Old dependence ratio	684	0.126	0.0748	0.07111	0.02452	0.0317	0.455	0.849	2.824
TFP growth (%)	684	-0.113	2.268	0.81878	2.11648	-19.36	10.03	-1.440	15.61
Final consumption (% of GDP)	653	77.19	10.93	9.51946	5.61180	35.71	115.5	-0.360	4.567
Gross capital formation (% of GDP)	653	24.25	6.912	5.02894	4.88331	1.525	62.67	0.918	5.233
Trade cost	657	3.182	0.928	0.86823	0.39593	1.174	8.334	1.364	6.684
(K/L) growth (%)	684	2.578	2.769	1.87536	2.04689	-5.274	12.78	0.681	4.394
log(K/L)	684	10.53	1.423	1.37333	0.40142	5.859	12.84	-0.541	2.638
GDP per capita at constant 2015 price	650	15,177	18,299	17,583.80	6,095.80	172.9	106,544	1.738	6.394

Pairwise correlation

TABLE C.2
PAIRWISE CORRELATION

Variables	ChildDep	OldDep	Dep	ChildSre	OldSre	WorkingSre
ChildDep	1					
OldDep	-0.787***	1				
Dep	0.967***	-0.604***	1			
ChildSre	0.990***	-0.850***	0.928***	1		
OldSre	-0.832***	0.995***	-0.664***	-0.891***	1	
WorkingSre	-0.961***	0.599***	-0.994***	-0.931***	0.663***	1

TABLE C.3
PAIRWISE CORRELATION

Variables	WorkingSre	TFP_GR	Capital/GDP	Cons/GDP	TradeCost	K/L_GR	POP_GR
WorkingSre	1						
TFP_GR	0.129***	1					
Capital/GDP	0.143***	-0.149***	1				
Cons/GDP	-0.380***	0.091***	-0.526***	1			
TradeCost	-0.640***	-0.017	-0.256***	0.409***	1		
K/L_GR	0.242***	0.011	0.374***	-0.165***	-0.151***	1	
POP_GR	-0.728***	-0.114***	0.023	0.146***	0.499***	-0.157***	1

C.1. Data description

I used panel data for 76 countries at different income levels, covering the period from 1975 to 2019. The selection of country groups and periods for analysis was based on the availability of data. The data were constructed from various sources, including the United Nations, World Population Prospects report, World Development Indicators database, Penn World Table 10.01, and CEPII database. The specifics of the data sources are listed in Table C.5.

TABLE C.4
Country Group

Income Group	Countries	Num.
High income	Australia, Austria, Belgium, Barbados, Canada, Switzerland, Chile, Cyprus, Germany, Denmark, Spain, Finland, France, United Kingdom, China (Hong Kong SAR), Greece, Ireland, Iceland, Israel, Italy, Japan, Republic of Korea, Luxembourg, Malta, Netherlands, Norway, New Zealand, Panama, Portugal, Romania, Singapore, Sweden, Uruguay, United States of America	34
Upper middle income	Brazil, Botswana, China, Colombia, Costa Rica, Dominican Republic, Jamaica, Jordan, Mexico, Mauritius, Malaysia, Namibia, Peru, Paraguay, Thailand, Ecuador, Gabon, Guatemala, Türkiye, South Africa, Venezuela (Bolivarian Republic of)	21
Lower middle income	Bolivia (Plurinational State of), Côte d'Ivoire, Cameroon, Egypt, Indonesia, India, Iran (Islamic Republic of), Kenya, Sri Lanka, Morocco, Nigeria, Philippines, Senegal, Tunisia, Tanzania, Zimbabwe	16
Lower income	Burkina Faso, Mozambique, Niger, Rwanda, Zambia	5

Notes: According to United Nations list of countries classified by income level

TABLE C.5
Definitions of Variables and Data Sources.

Variables	Description and Construction	Source
Population at every 5 year cohorts	Population at every 5 year cohorts: $[0,4],[5,9],\ldots,[90,94],[95,99]$ and $[100,+]$, 1975-2019	UN, World Population Prospects, 2022 version.
Real GDP per capita	Real GDP per capita at constant 2015 prices (in US \$), 1975-2019	World Development Indicators
Gross capital formation (% of GDP)	Gross capital formation (% of GDP), 1975-2019	World Development Indicators
Final consumption expenditure (% of GDP)	Final consumption expenditure (% of GDP), 1975-2019	World Development Indicators
Capital Stock	Capital Stock at constant 2017 prices (in millions US \$), 1975-2019	Penn World Table 10.01
Population	Total population (in millions US \$), 1975-2019	Penn World Table 10.01
Total factor productivity (TFP)	Total factor productivity (TFP), 1975-2019	Penn World Table 10.01
Gross Domestic Products	Destination and origin country GDP at current prices (in thousands US \$), 1975-2019	CEPII
Trade flow	Trade flows as reported by the destination at current prices (in thousands US \$), 1975-2019	CEPII

TABLE C.6

Decriptive statistics: 1975 - 2019, 76 countries

VARIABLES	N	mean	sd	between.sd	within.sd	min	max	skewness	kurtosis
Age [0-9] share	3,420	0.208	0.0826	0.07698	0.03123	0.0718	0.383	0.276	1.720
Age [10-19] share	3,420	0.185	0.0470	0.04144	0.02263	0.0746	0.287	-0.263	1.797
Age [20-29] share	3,420	0.162	0.0212	0.01525	0.01490	0.0966	0.236	-0.174	3.267
Age [30-39] share	3,420	0.135	0.0214	0.01521	0.01511	0.0765	0.217	0.176	3.030
Age [40-49] share	3,420	0.109	0.0300	0.02513	0.01665	0.0542	0.193	0.184	2.008
Age [50-59] share	3,420	0.0849	0.0324	0.02879	0.01531	0.0300	0.170	0.360	1.940
Age [60-69] share	3,420	0.0612	0.0299	0.02807	0.01083	0.0169	0.145	0.500	1.997
Age [70-79] share	3,420	0.0376	0.0241	0.02312	0.00742	0.00726	0.128	0.675	2.265
Age [80-89] share	3,420	0.0149	0.0129	0.01169	0.00560	0.000645	0.0740	1.076	3.435
Age [90+] share	3,420	0.00212	0.00246	0.00198	0.00148	2.07e-06	0.0209	1.881	7.724
Child share	3,420	0.303	0.109	0.10113	0.04205	0.116	0.523	0.155	1.655
Working age share	3,420	0.615	0.0661	0.05784	0.03260	0.457	0.785	-0.366	2.103
Elderly share	3,420	0.0824	0.0532	0.05046	0.01787	0.0167	0.293	0.727	2.447
Dependence ratio	3,420	0.647	0.188	0.16511	0.09093	0.273	1.189	0.672	2.363
Young dependence ratio	3,420	0.518	0.242	0.22284	0.09830	0.158	1.144	0.485	1.947
Old dependence ratio	3,420	0.129	0.0773	0.07303	0.02675	0.0314	0.499	0.880	3.010
TFP growth (%)	3,116	-0.0566	2.211	0.73687	2.08662	-19.36	20.61	-0.620	14.92
Final consumption (% of GDP)	3,285	77.31	11.08	9.53507	5.83873	11.61	148.5	-0.350	5.486
Gross capital formation (% of GDP)	3,284	24.14	7.148	5.03478	5.23456	1.525	89.38	1.437	9.718
Trade cost	3,286	3.181	0.940	0.86371	0.42245	1.174	10.49	1.550	8.612
(K/L) growth (%)	3,116	2.478	2.744	1.93121	1.96096	-5.326	14.27	0.716	4.643
log(K/L)	3,420	10.58	1.417	1.36937	0.39737	5.859	12.87	-0.546	2.613
GDP per capita at constant 2015 price	3,266	15,711	18,828	18,136.57	6,168.70	165.9	112,418	1.715	6.261

C.2. Detail results

TABLE C.7

The effect of demographic structure change

	Average value in the future 4 years							
VARIABLES	TFP growth rate		Cap.Formation(% GDP)		Gross.Consumption(% GDP)		K/L growth rate	
Work.Share	11.43***		28.80**		-33.75**		13.34**	
	(3.33)		(2.17)		(-2.00)		(2.49)	
Child.Share		-13.98***		-24.76*		33.75*		-11.22*
		(-3.68)		(-1.74)		(1.99)		(-1.82)
Old.Share		2.79		-65.97***		33.77		-24.65**
		(0.39)		(-2.65)		(0.90)		(-2.38)
Trade Cost							-0.87**	-0.83**
							(-2.27)	(-2.13)
Initial.Log.Dependent							-2.24***	-1.99***
							(-4.12)	(-3.45)
Initial.Log.GDP.pc	-3.09***	-3.46***						
	(-4.82)	(-4.77)						
PoP.Growth							-28.93	-33.14*
							(-1.55)	(-1.84)
Constant	20.96***	35.46***	4.09	34.10***	98.56***	64.81***	22.19***	32.98***
	(3.65)	(5.19)	(0.53)	(6.74)	(9.84)	(9.15)	(3.63)	(5.32)
Observations	732	732	724	724	725	725	758	758
R-squared	0.259	0.266	0.575	0.581	0.753	0.753	0.586	0.589
Time FE	YES	YES	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable $Child.Share$ represents the young population share, which is defined as the share of people aged 0-14, and the variable $Old.Share$ represents old population share, which is defined as the share of people aged 65 and above.

TABLE C.8
The effect of demographic cohort structure change

	Average value in the future 4 years											
VARIABLES	TFP growth rate			Cap.Formation(% GDP)			Gross.Consumption(% GDP)			K/L growth rate		
(0-14)/ToT.	21.48***			9.34			98.55***			21.77***		
	(3.61)			(0.98)			(9.21)			(3.69)		
(15-64)/ToT.	35.46***			34.10***			64.81***			32.98***		
	(5.19)			(6.74)			(9.15)			(5.32)		
(65+)/ToT.	38.25***			-31.87			98.58***			8.34		
	(3.42)			(-1.30)			(2.95)			(0.61)		
(0-24)/ToT.		26.22***			16.69***			92.44***			24.08***	
		(4.24)			(2.64)			(14.84)			(4.50)	
(25-49)/ToT.		34.48***			29.11***			60.55***			39.21***	
		(4.28)			(4.14)			(5.58)			(5.36)	
(50-74)/ToT.		43.60***			37.83**			59.95***			19.18	
		(4.41)			(2.05)			(3.23)			(1.66)	
(75+)/ToT.		13.47			-124.60***			150.74***			4.22	
		(0.90)			(-2.77)			(3.21)			(0.24)	
(0-19)/ToT.			25.36***			15.40**			92.98***			22.11***
			(4.08)			(2.32)			(12.62)			(4.02)
(20-39)/ToT.			31.80***			26.71**			71.18***			36.72***
			(4.35)			(2.52)			(5.39)			(5.30)
(40-59)/ToT.			34.74***			20.39			43.58*			27.00***
			(3.46)			(1.13)			(1.85)			(2.98)
(60-79)/ToT.			55.17***			53.93**			100.97**			21.25
			(5.35)			(2.37)			(2.47)			(1.41)
(80+)/ToT.			-21.89			-224.74***			126.47*			-9.87
			(-1.08)			(-3.07)			(1.75)			(-0.33)
Trade Cost										-0.83**	-0.83**	-0.79**
										(-2.13)	(-2.11)	(-2.00)
Initial.Log.Dependent	-3.46***	-3.51***	-3.51***							-1.99***	-1.98***	-1.93***
	(-4.77)	(-4.49)	(-4.55)							(-3.45)	(-3.21)	(-3.14)
PoP.Growth										-33.14*	-35.31**	-30.58
										(-1.84)	(-2.08)	(-1.64)
Observations	732	732	732	724	724	724	725	725	725	758	758	758
R-squared	0.266	0.263	0.272	0.971	0.972	0.972	0.996	0.996	0.996	0.785	0.787	0.787
Time FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

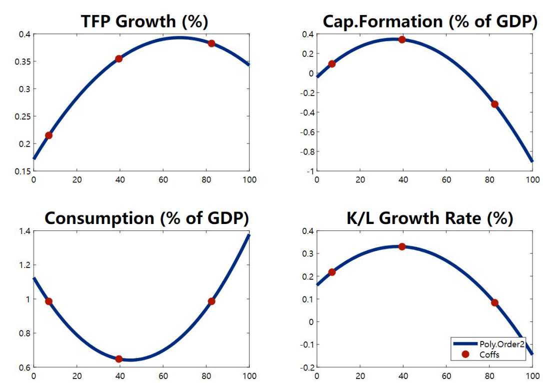

Figure C.1
3 cohorts

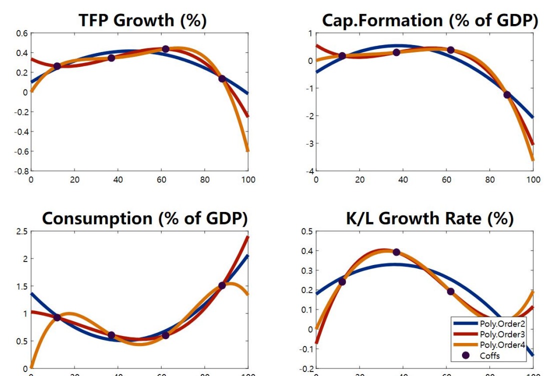

Figure C.2
4 cohorts

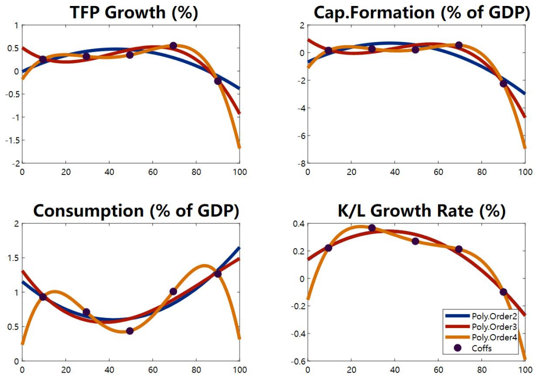

FIGURE C.3
5 COHORTS

TABLE C.9
THE EFFECT OF DEMOGRAPHIC STRUCTURE ON TECHNOLOGY CHANGE

	Average value in the future 4 years								
VARIABLES	TFP growth rate			Patent.Applications (per 1000 people)			Industrial.Design.Applications (per 1000 people)		
(0-14)/ToT.	21.48***			-1.60***			-0.89***		
	(3.61)			(-4.60)			(-3.84)		
(15-64)/ToT.	35.46***			0.58***			0.63***		
	(5.19)			(2.73)			(4.98)		
(65+)/ToT.	38.25***			2.29**			-0.42		
	(3.42)			(2.50)			(-0.98)		
(0-24)/ToT.		26.22***			-1.56***			-0.55***	
		(4.24)			(-7.06)			(-3.87)	
(25-49)/ToT.		34.48***			0.18			0.71***	
		(4.28)			(0.46)			(2.87)	
(50-74)/ToT.		43.60***			4.90***			1.08***	
		(4.41)			(7.40)			(2.93)	
(75+)/ToT.		13.47			-2.59			-1.85**	
		(0.90)			(-1.59)			(-1.99)	
(0-19)/ToT.			25.36***			-1.11***			-0.53***
			(4.08)			(-4.09)			(-2.87)
(20-39)/ToT.			31.80***			-1.72***			0.08
			(4.35)			(-4.06)			(0.31)
(40-59)/ToT.			34.74***			3.59***			1.75***
			(3.46)			(6.47)			(5.20)
(60-79)/ToT.			55.17***			4.23***			-0.31
			(5.35)			(3.99)			(-0.46)
(80+)/ToT.			-21.89			-7.67***			-1.09
			(-1.08)			(-2.62)			(-0.57)
Initial.Log.Dependent	-3.46***	-3.51***	-3.51***						
	(-4.77)	(-4.49)	(-4.55)						
Observations	732	732	732	395	395	395	215	215	215
R-squared	0.266	0.263	0.272	0.859	0.880	0.886	0.935	0.939	0.942
Time FE	YES	YES	YES	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable $Child.Dep.R$ represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable $Old.Dep.R$ represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable $Child.Share$ represents the young population share, which is defined as the share of people aged 0-14, and the variable $Old.Share$ represents old population share, which is defined as the share of people aged 65 and above.

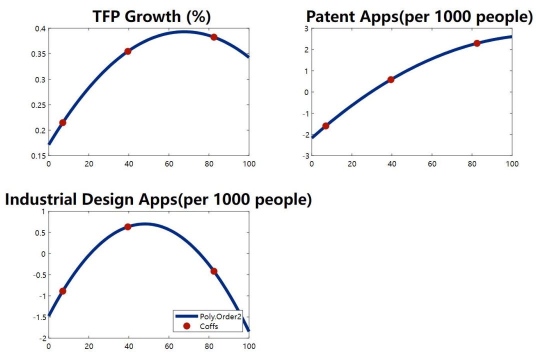

Figure C.4
3 cohorts

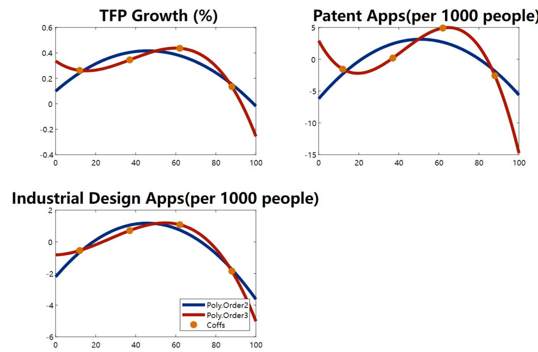

Figure C.5
4 cohorts

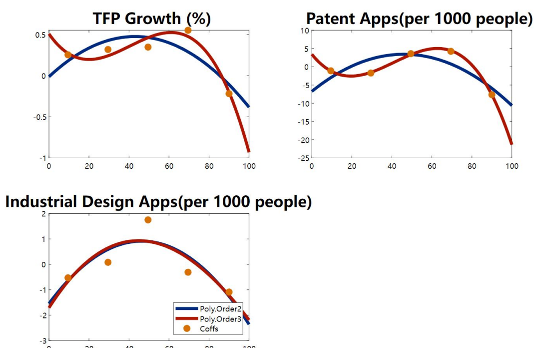

Figure C.6
5 cohorts

TABLE C.10

The effect of demographic structure on technology change

VARIABLES	Average TFP (%) growth rate in the future 4 years			
Initial.ln.RGDP.p.c	-2.93***	-3.22***	-3.09***	-3.46***
	(-4.61)	(-4.62)	(-4.82)	(-4.77)
Dep.Ratio	-3.31***			
	(-2.86)			
Child.Dep.R		-4.64***		
		(-3.40)		
Old.Dep.R		5.83		
		(1.30)		
Work.Share			11.43***	
			(3.33)	
Child.Share				-13.98***
				(-3.68)
Old.Share				2.79
				(0.39)
Constant	28.56***	30.82***	20.96***	35.46***
	(4.94)	(4.91)	(3.65)	(5.19)
Observations	732	732	732	732
R-squared	0.254	0.262	0.259	0.266
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Dep.Ratio* represents the dependency ratio, which is defined as the ratio of people aged (0-14) and (65, +) to people aged 15-64. The variable *Child.Dep.R* represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable *Old.Dep.R* represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable *Child.Share* represents the young population share, which is defined as the share of people aged 0-14, and the variable *Old.Share* represents old population share, which is defined as the share of people aged 65 and above.

TABLE C.11

The effect of demographic cohort structure on technology change

VARIABLES	Average TFP (%) growth rate in the future 4 years		
Initial.ln.RGDP.p.c	-3.46***	-3.51***	-3.51***
	(-4.77)	(-4.49)	(-4.55)
(0-14)/ToT.	21.48***		
	(3.61)		
(15-64)/ToT.	35.46***		
	(5.19)		
(65+)/ToT.	38.25***		
	(3.42)		
(0-24)/ToT.		26.22***	
		(4.24)	
(25-49)/ToT.		34.48***	
		(4.28)	
(50-74)/ToT.		43.60***	
		(4.41)	
(75+)/ToT.		13.47	
		(0.90)	
(0-19)/ToT.			25.36***
			(4.08)
(20-39)/ToT.			31.80***
			(4.35)
(40-59)/ToT.			34.74***
			(3.46)
(60-79)/ToT.			55.17***
			(5.35)
(80+)/ToT.			-21.89
			(-1.08)
Observations	732	732	732
R-squared	0.266	0.263	0.272
Time FE	YES	YES	YES
Country FE	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

TABLE C.12

The effect of demographic structure on capital formation, and consumption

VARIABLES	Average value (% GDP) in the future 4 years			
	Gross.Cap.Formation		Gross.Consumption	
Dep.Ratio	-10.20**		9.93*	
	(-2.13)		(1.68)	
Child.Dep.R		-7.93		9.88
		(-1.37)		(1.61)
Old.Dep.R		-28.54*		10.38
		(-1.73)		(0.45)
Constant	28.24***	29.06***	71.74***	71.72***
	(7.87)	(9.10)	(17.11)	(16.16)
Observations	724	724	725	725
R-squared	0.575	0.579	0.751	0.751
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Dep.Ratio* represents the dependency ratio, which is defined as the ratio of people aged (0-14) and (65, +) to people aged 15-64. The variable *Child.Dep.R* represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable *Old.Dep.R* represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64.

TABLE C.13

The effect of demographic structure on capital formation, and consumption

VARIABLES	Average value (% GDP) in the future 4 years			
	Gross.Cap.Formation		Gross.Consumption	
Work.Share	28.80**		-33.75**	
	(2.17)		(-2.00)	
Child.Share		-24.76*		33.75*
		(-1.74)		(1.99)
Old.Share		-65.97***		33.77
		(-2.65)		(0.90)
Constant	4.09	34.10***	98.56***	64.81***
	(0.53)	(6.74)	(9.84)	(9.15)
Observations	724	724	725	725
R-squared	0.575	0.581	0.753	0.753
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Work.Share* represents the working age share, which is defined as the share of people aged 15-64. The variable *Child.Share* represents the young population share, which is defined as the share of people aged 0-14, and the variable *Old.Share* represents old population share, which is defined as the share of people aged 65 and above.

TABLE C.14

THE EFFECT OF DEMOGRAPHIC COHORT STRUCTURE ON CAPITAL FORMATION, AND CONSUMPTION

	Average value (% GDP) in the future 4 years					
VARIABLES	Gross.Cap.Formation			Gross.Consumption		
(0-14)/ToT.	9.34			98.55***		
	(0.98)			(9.21)		
(15-64)/ToT.	34.10***			64.81***		
	(6.74)			(9.15)		
(65+)/ToT.	-31.87			98.58***		
	(-1.30)			(2.95)		
(0-24)/ToT.		16.69***			92.44***	
		(2.64)			(14.84)	
(25-49)/ToT.		29.11***			60.55***	
		(4.14)			(5.58)	
(50-74)/ToT.		37.83**			59.95***	
		(2.05)			(3.23)	
(75+)/ToT.		-124.60***			150.74***	
		(-2.77)			(3.21)	
(0-19)/ToT.			15.40**			92.98***
			(2.32)			(12.62)
(20-39)/ToT.			26.71**			71.18***
			(2.52)			(5.39)
(40-59)/ToT.			20.39			43.58*
			(1.13)			(1.85)
(60-79)/ToT.			53.93**			100.97**
			(2.37)			(2.47)
(80+)/ToT.			-224.74***			126.47*
			(-3.07)			(1.75)
Observations	724	724	724	725	725	725
R-squared	0.971	0.972	0.972	0.996	0.996	0.996
Time FE	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

TABLE C.15

THE EFFECT OF DEMOGRAPHIC STRUCTURE AND TRADE COST CHANGE ON CAPITAL-LABOR RATIO CHANGE

VARIABLES	Average K/L (%) growth rate in the future 4 years			
Trade Cost	-0.83**	-0.82**	-0.87**	-0.83**
	(-2.13)	(-2.06)	(-2.27)	(-2.13)
Dep.Ratio	-4.39**			
	(-2.22)			
Child.Dep.R		-3.25		
		(-1.27)		
Old.Dep.R		-11.37		
		(-1.66)		
Work.Share			13.34**	
			(2.49)	
Child.Share				-11.22*
				(-1.82)
Old.Share				-24.65**
				(-2.38)
Initial.ln.K/L	-2.13***	-1.93***	-2.24***	-1.99***
	(-3.90)	(-3.41)	(-4.12)	(-3.45)
PoP.Growth	-28.85	-34.06*	-28.93	-33.14*
	(-1.53)	(-1.90)	(-1.55)	(-1.84)
Constant	31.86***	29.96***	22.19***	32.98***
	(5.77)	(5.31)	(3.63)	(5.32)
Observations	758	758	758	758
R-squared	0.585	0.588	0.586	0.589
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

"

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Dep.Ratio* represents the dependency ratio, which is defined as the ratio of people aged (0-14) and (65, +) to people aged 15-64. The variable *Child.Dep.R* represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable *Old.Dep.R* represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable *Child.Share* represents the young population share, which is defined as the share of people aged 0-14, and the variable *Old.Share* represents old population share, which is defined as the share of people aged 65 and above.

"

TABLE C.16

The effects of demographic cohort structure and trade cost change on capital-labor ratio change

VARIABLES	Average K/L (%) growth rate in the future 4 years		
Trade Cost	-0.83**	-0.83**	-0.79**
	(-2.13)	(-2.11)	(-2.00)
(0-14)/ToT.	21.77***		
	(3.69)		
(15-64)/ToT.	32.98***		
	(5.32)		
(65+)/ToT.	8.34		
	(0.61)		
(0-24)/ToT.		24.08***	
		(4.50)	
(25-49)/ToT.		39.21***	
		(5.36)	
(50-74)/ToT.		19.18	
		(1.66)	
(75+)/ToT.		4.22	
		(0.24)	
(0-19)/ToT.			22.11***
			(4.02)
(20-39)/ToT.			36.72***
			(5.30)
(40-59)/ToT.			27.00***
			(2.98)
(60-79)/ToT.			21.25
			(1.41)
(80+)/ToT.			-9.87
			(-0.33)
Initial.ln.K/L	-1.99***	-1.98***	-1.93***
	(-3.45)	(-3.21)	(-3.14)
PoP.Growth	-33.14*	-35.31**	-30.58
	(-1.84)	(-2.08)	(-1.64)
Observations	758	758	758
R-squared	0.785	0.787	0.787
Time FE	YES	YES	YES
Country FE	YES	YES	YES

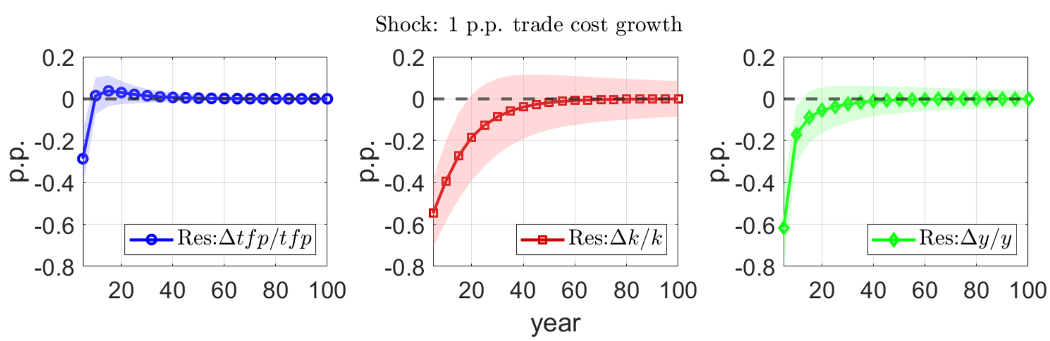

Figure C.7

IRF of exogenous trade cost shock

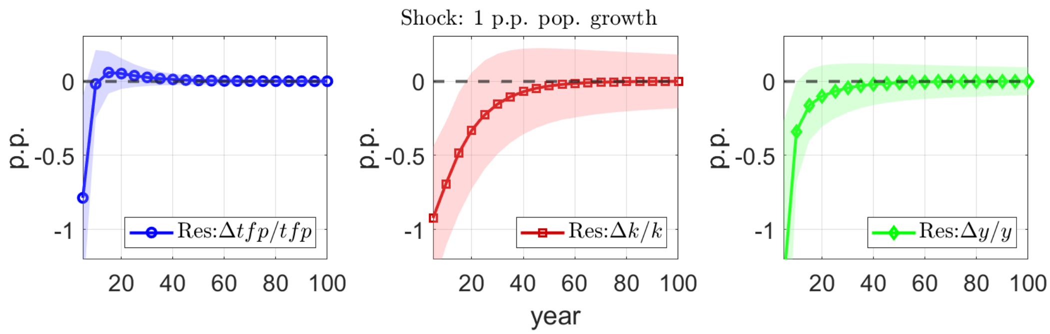

FIGURE C.8
IRF OF EXOGENOUS POPULATION SHOCK

C.3. Robust Checks

For the purpose of conducting robust checks, I performed the same regression analysis by dividing the entire period of 1980–2019 into five 8-year sub-periods: period 1 (1980–1987), period 2 (1988–1995), period 3 (1996–2003), period 4 (2004–2011), and period 5 (2012–2019).

C.3.I. Demographics, technology change, and macroeconomic outcomes

TABLE C.17
The effect of demographic structure on technology change

VARIABLES	Average TFP (%) growth rate in the future 7 years			
Initial.ln.RGDP.p.c	-2.78***	-2.92***	-2.93***	-3.10***
	(-4.32)	(-4.17)	(-4.55)	(-4.28)
Dep.Ratio	-2.11*			
	(-1.88)			
Child.Dep.R		-2.70**		
		(-2.08)		
Old.Dep.R		2.45		
		(0.55)		
Work.Share			8.31***	
			(2.76)	
Child.Share				-9.41***
				(-2.80)
Old.Share				-1.02
				(-0.14)
Constant	25.75***	26.77***	20.69***	30.42***
	(4.44)	(4.30)	(3.53)	(4.56)
Observations	439	439	439	439
R-squared	0.361	0.364	0.367	0.370
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Dep.Ratio* represents the dependency ratio, which is defined as the ratio of people aged (0-14) and (65, +) to people aged 15-64. The variable *Child.Dep.R* represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable *Old.Dep.R* represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64. The variable *Work.Share* represents the working age share, which is defined as the share of people aged 15-64. The variable *Child.Share* represents the young population share, which is defined as the share of people aged 0-14, and the variable *Old.Share* represents old population share, which is defined as the share of people aged 65 and above.

TABLE C.18

THE EFFECT OF DEMOGRAPHIC STRUCTURE ON TECHNOLOGY CHANGE

VARIABLES	Average TFP (%) growth rate in the future 7 years		
Initial.ln.RGDP.p.c	-3.10***	-3.15***	-3.19***
	(-4.28)	(-3.98)	(-4.05)
(0-14)/ToT.	21.01***		
	(3.48)		
(15-64)/ToT.	30.42***		
	(4.56)		
(65+)/ToT.	29.40**		
	(2.56)		
(0-24)/ToT.		24.19***	
		(3.82)	
(25-49)/ToT.		30.46***	
		(3.94)	
(50-74)/ToT.		34.35***	
		(3.32)	
(75+)/ToT.		14.89	
		(1.05)	
(0-19)/ToT.			24.13***
			(3.75)
(20-39)/ToT.			27.20***
			(3.80)
(40-59)/ToT.			31.92***
			(3.27)
(60-79)/ToT.			43.14***
			(3.86)
(80+)/ToT.			-18.13
			(-0.84)
Observations	439	439	439
R-squared	0.372	0.369	0.378
Time FE	YES	YES	YES
Country FE	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

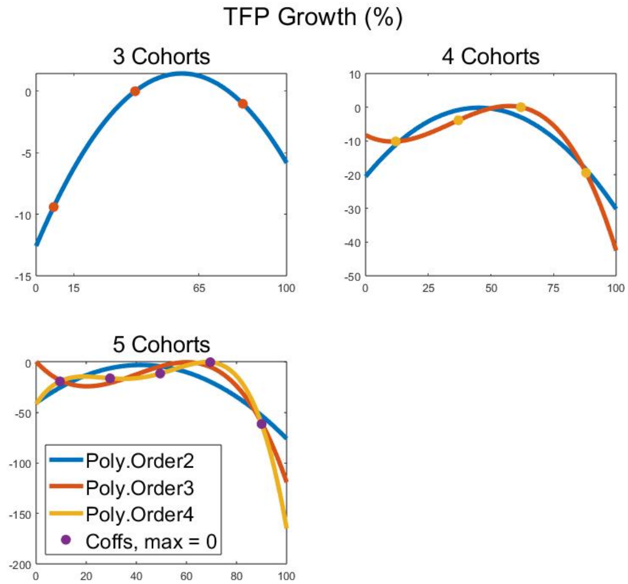

FIGURE C.9
DEMOGRAPHIC COHORT STRUCTURE AND TFP GROWTH (%)

TABLE C.19
THE EFFECT OF DEMOGRAPHIC STRUCTURE ON CAPITAL FORMATION, AND CONSUMPTION

VARIABLES	Average value (% GDP) in the future 7 years			
	Gross.Cap.Formation		Gross.Consumption	
Dep.Ratio	-9.79*		7.63	
	(-1.91)		(1.26)	
Child.Dep.R		-7.61		7.66
		(-1.22)		(1.26)
Old.Dep.R		-29.93		7.33
		(-1.62)		(0.28)
Constant	28.48***	29.65***	73.35***	73.37***
	(7.44)	(8.83)	(17.00)	(15.32)
Observations	431	431	432	432
R-squared	0.627	0.631	0.792	0.792
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Dep.Ratio* represents the dependency ratio, which is defined as the ratio of people aged (0-14) and (65, +) to people aged 15-64. The variable *Child.Dep.R* represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable *Old.Dep.R* represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64.

TABLE C.20

THE EFFECT OF DEMOGRAPHIC STRUCTURE ON CAPITAL FORMATION, AND CONSUMPTION

VARIABLES	Average value (% GDP) in the future 7 years			
	Gross.Cap.Formation		Gross.Consumption	
Work.Share	27.58*		-27.48	
	(1.91)		(-1.61)	
Child.Share		-24.21		27.63
		(-1.57)		(1.63)
Old.Share		-66.88**		25.77
		(-2.47)		(0.62)
Constant	5.36	34.61***	94.86***	67.45***
	(0.64)	(6.32)	(9.42)	(9.08)
Observations	431	431	432	432
R-squared	0.627	0.633	0.794	0.794
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable $Child.Share$ represents the young population share, which is defined as the share of people aged 0-14, and the variable $Old.Share$ represents old population share, which is defined as the share of people aged 65 and above.

TABLE C.21

The effect of demographic cohort structure on capital formation, and consumption

	Average value (% GDP) in the future 7 years					
VARIABLES	Gross.Cap.Formation			Gross.Consumption		
(0-14)/ToT.	10.40			95.08***		
	(1.00)			(9.09)		
(15-64)/ToT.	34.61***			67.45***		
	(6.32)			(9.08)		
(65+)/ToT.	-32.27			93.22**		
	(-1.21)			(2.51)		
(0-24)/ToT.		17.81**			90.93***	
		(2.55)			(14.34)	
(25-49)/ToT.		32.06***			62.28***	
		(4.08)			(4.83)	
(50-74)/ToT.		31.34			63.14***	
		(1.65)			(2.91)	
(75+)/ToT.		-107.89**			133.38**	
		(-2.31)			(2.60)	
(0-19)/ToT.			15.33**			90.29***
			(2.01)			(12.74)
(20-39)/ToT.			33.13***			74.63***
			(2.99)			(5.53)
(40-59)/ToT.			18.45			46.92*
			(0.97)			(1.90)
(60-79)/ToT.			41.73*			97.07**
			(1.67)			(2.00)
(80+)/ToT.			-172.04**			112.79
			(-2.18)			(1.36)
Observations	431	431	431	432	432	432
R-squared	0.977	0.977	0.978	0.997	0.997	0.997
Time FE	YES	YES	YES	YES	YES	YES
Country FE	YES	YES	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

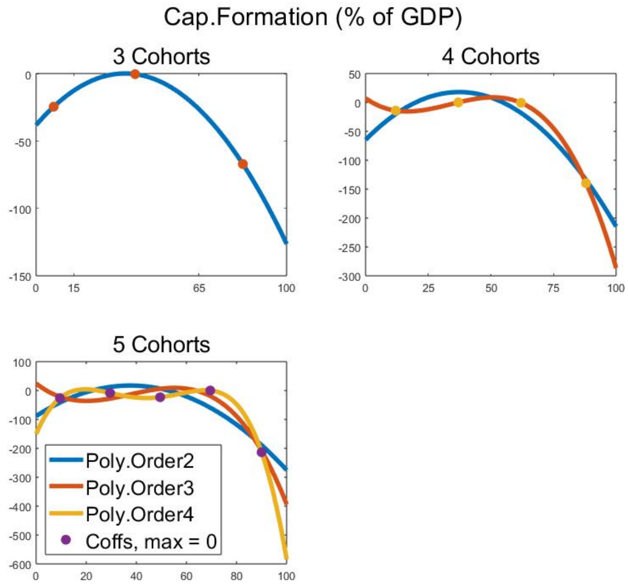

Figure C.10
Demographic cohort structure and capital formation (% of GDP)

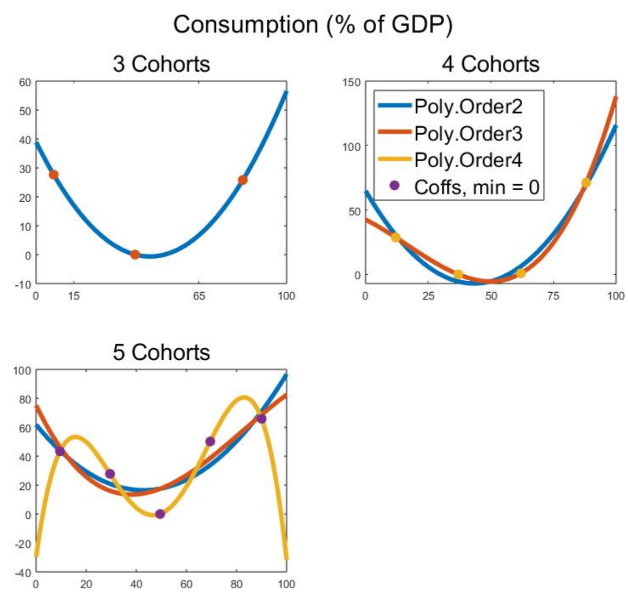

Figure C.11
Demographic cohort structure and consumption (% of GDP)

C.4. Demographics, trade cost, endowments change and economic growth

C.4.I. The effects of demographic structure and trade cost change on capital-labor ratio

TABLE C.22

THE EFFECT OF DEMOGRAPHIC STRUCTURE AND TRADE COST CHANGE ON CAPITAL-LABOR RATIO CHANGE

VARIABLES	Average K/L (%) growth rate in the future 7 years			
Trade Cost	-0.97***	-0.96***	-1.01***	-1.00***
	(-3.13)	(-3.05)	(-3.26)	(-3.10)
Dep.Ratio	-4.73**			
	(-2.42)			
Child.Dep.R		-4.45		
		(-1.62)		
Old.Dep.R		-6.65		
		(-0.79)		
Work.Share			13.85**	
			(2.60)	
Child.Share				-13.21**
				(-2.04)
Old.Share				-17.93
				(-1.49)
Initial K/L	-2.50***	-2.44***	-2.61***	-2.52***
	(-4.33)	(-3.85)	(-4.57)	(-3.95)
PoP.Growth	-5.08	-6.58	-5.89	-7.46
	(-0.29)	(-0.39)	(-0.34)	(-0.45)
Constant	35.87***	35.37***	25.69***	38.69***
	(5.79)	(5.11)	(3.88)	(5.17)
Observations	454	454	454	454
R-squared	0.659	0.659	0.660	0.660
Time FE	YES	YES	YES	YES
Country FE	YES	YES	YES	YES

Notes: Robust t-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The variable *Dep.Ratio* represents the dependency ratio, which is defined as the ratio of people aged (0-14) and (65, +) to people aged 15-64. The variable *Child.Dep.R* represents the young dependency ratio, which is defined as the ratio of people aged (0-14) to people aged 15-64. The variable *Old.Dep.R* represents the old dependency ratio, which is defined as the ratio of people aged 65 and above to people aged 15-64. The variable $Work.Share$ represents the working age share, which is defined as the share of people aged 15-64. The variable *Child.Share* represents the young population share, which is defined as the share of people aged 0-14, and the variable *Old.Share* represents old population share, which is defined as the share of people aged 65 and above.

TABLE C.23

THE EFFECTS OF DEMOGRAPHIC COHORT STRUCTURE AND TRADE COST CHANGE ON CAPITAL-LABOR RATIO CHANGE

VARIABLES	Average K/L (%) growth rate in the future 7 years		
Trade Cost	-1.00***	-1.00***	-0.94***
	(-3.10)	(-2.95)	(-2.84)
(0-14)/ToT.	25.49***		
	(3.90)		
(15-64)/ToT.	38.69***		
	(5.17)		
(65+)/ToT.	20.76		
	(1.25)		
(0-24)/ToT.		28.69***	
		(4.59)	
(25-49)/ToT.		45.13***	
		(5.19)	
(50-74)/ToT.		26.85*	
		(1.92)	
(75+)/ToT.		14.79	
		(0.72)	
(0-19)/ToT.			25.80***
			(4.15)
(20-39)/ToT.			43.05***
			(5.15)
(40-59)/ToT.			30.92***
			(2.88)
(60-79)/ToT.			33.28**
			(2.07)
(80+)/ToT.			-1.10
			(-0.03)
Initial.ln.K/L	-2.52***	-2.51***	-2.45***
	(-3.95)	(-3.72)	(-3.68)
PoP.Growth	-7.46	-11.56	-3.34
	(-0.45)	(-0.72)	(-0.18)
Observations	454	454	454
R-squared	0.827	0.830	0.830
Time FE	YES	YES	YES
Country FE	YES	YES	YES

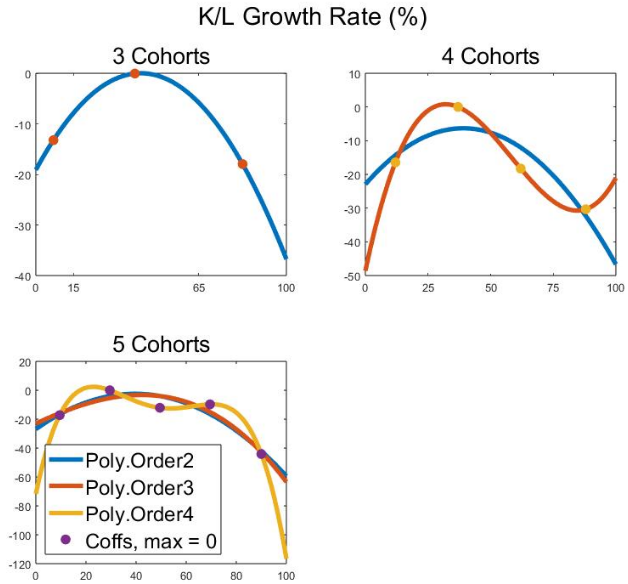

Figure C.12
Demographic cohort structure and Capital-labor ratio

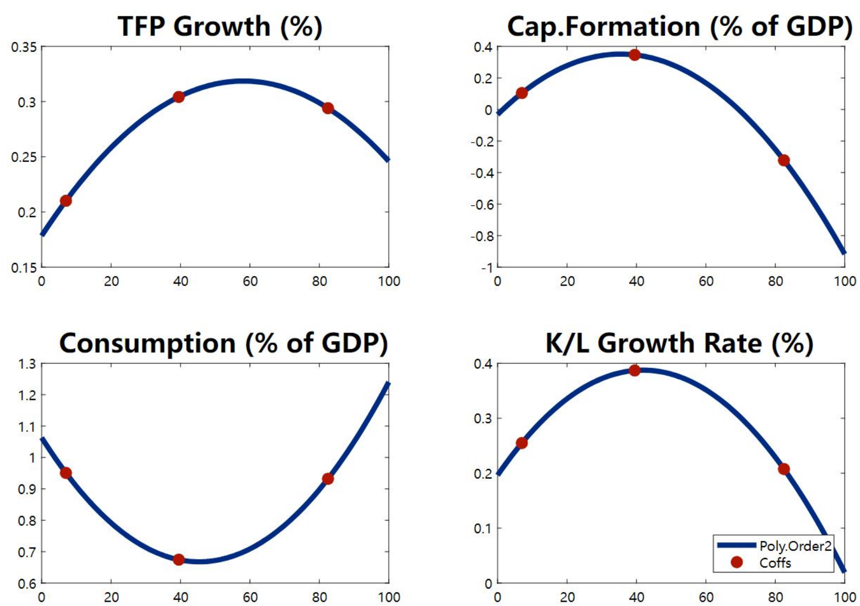

Figure C.13
Effects of demographic structure

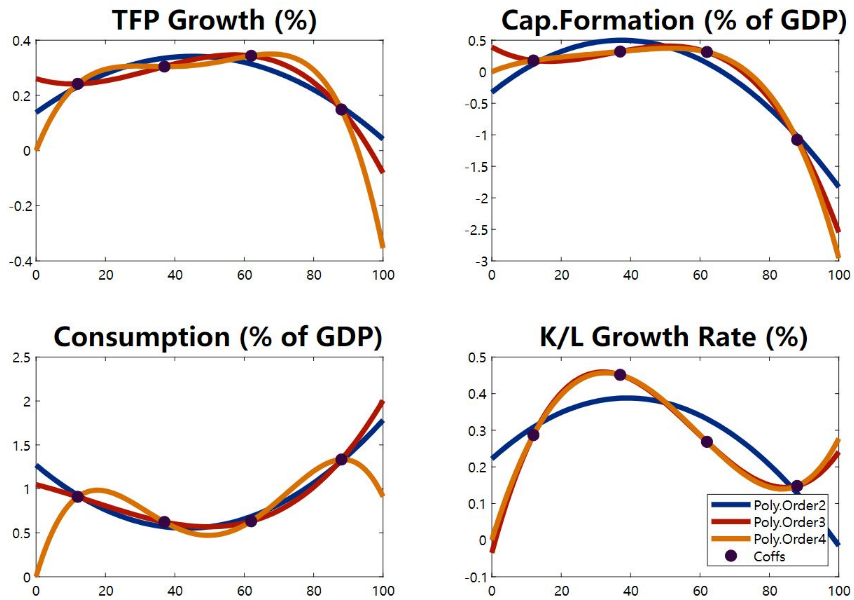

FIGURE C.14
EFFECTS OF DEMOGRAPHIC STRUCTURE

APPENDIX D: ONE-SECTOR CLOSE ECONOMY DETAILS

D.1. Equilibrium and/or equation derivations

D.1.I. Steady state prove

In the main text, I prove that if the number of consumption and saving per efficient unit of age cohort g, $c_{g,t}, a_{g,t}$ is constant for any t, then C_t and K_t grow at a constant rate.

Following, I will prove that if K_t, C_t grow at the constant rate, then $c_{g,t}, a_{g,t}, \dfrac{W_t}{P_t}$ also grow at the constant rate.

Real Wage and Real Rental Rate

$$\bar{r} = r_t = \frac{R_t}{P_t} - \delta = (\alpha)\left(\frac{K_t}{A_t^{\frac{1}{1-\alpha}} L_t}\right)^{-(1-\alpha)} - \delta \tag{C.1}$$

$$\omega_t = \frac{W_t}{P_t} = (1-\alpha)\left(\frac{A_t^{\frac{1}{1-\alpha}}}{K_t^{\frac{1}{1-\alpha}} L_t}\right)^{\alpha} \tag{C.2}$$

Real wage growth at $\Gamma = (1+\gamma)^{\frac{1}{1-\alpha}} - 1$, and $\gamma = g_A = (1+g_\lambda)^{\frac{1}{\theta}} - 1$.

Consumption

As for the growth rate of households' consumption $c_{g,t}$, from the budget constraint:

$$c_{g,t+g-1} + a_{g+1,t+g} = (1+\bar{r})a_{g,t+g-1} + \frac{W_{t+g-1}}{P_{t+g-1}} l_g$$

The household born at time t, his total wealth is:

$$W_t = \sum_{g=1}^{2} \frac{W_{t+g-1}}{P_{t+g-1}}$$

Thus $W_{t+1} = (1+\Gamma)W_t$.

The Euler equation implies:

$$\frac{c_{g+1,t+1}}{c_{g,t}} = [(1-\alpha)(1+r_{t+1})]^{\sigma}$$

This wealth is used for lifetime consumption:

$$W_t = \sum_{g=1}^{2} \frac{W_{t+g-1}}{P_{t+g-1}} = \sum_{g=1}^{G} c_{g,t+g-1} = c_{1,t} \sum_{g=1}^{G} [[(1-\alpha)(1+r^*)]^{\sigma}]^{g-1}$$

Thus at the household level, for the same cohort g, the consumption growth rate is constant:

$$c_{1,t+1} = (1+\Gamma)c_{1,t}$$

$$c_{g,t+g} = (1+\Gamma)c_{g,t+g-1}$$

We know the growth rate for cohort population $N_{g,t+1}$ and consumption $c_{g,t+1}$, thus the growth for aggregate consumption is:

$$C_{t+1} = \sum_{g=1}^{G} N_{g,t+1}c_{g,t+1} = (1+n)(1+\Gamma)\sum_{g=1}^{G} N_{g,t}c_{g,t} = (1+n)(1+\Gamma)C_t \tag{C.3}$$

Saving

As for the growth of households' saving stock $b_{g,t}$:

Since the growth rate of $c_{g,t}$ and $\frac{W_t}{P_t}$ is the same, $(1+\Gamma)$, thus:

$$c_{g,t+1} - \frac{W_{t+1}}{P_{t+1}}l_g = (1+\Gamma)\left(c_{g,t} - \frac{W_t}{P_t}l_g\right)$$

The budget constraint implies:

$$(1+\Gamma)\left(c_{g,t} - \frac{W_t}{P_t}l_g\right) = (1+\Gamma)[(1+r^*)a_{g,t} - a_{g+1,t+1}]$$

$$(1+\Gamma)\left(c_{g,t} - \frac{W_t}{P_t}l_g\right) = c_{g,t+1} - \frac{W_{t+1}}{P_{t+1}}l_g = (1+r^*)a_{g,t+1} - a_{g+1,t+2}$$

Define the change of saving as:

$$\delta_{g+1,t+1} = a_{g+1,t+1} - (1+r^*)a_{g,t}$$

Thus, from about 3 equations:

$$\delta_{g+1,t+1}(1+\Gamma) = \delta_{g+1,t+2}$$

So:

$$a_{g,t+1} = \delta_{g,t+1} + (1+r^*)a_{g-1,t}$$

$$a_{g-1,t} = \delta_{g-1,t} + (1+r^*)a_{g-2,t-2}$$

$$a_{2,t+3-g} = \delta_{2,t+3-g} + (1+r^*)a_{1,t-g+1}$$

So:

$$a_{g,t+1} = \sum_{j=0}^{g-2}(1+r^*)^j \delta_{g-j,t+1-j}$$

The growth of households' saving stock is also constant:

$$a_{g,t} = \frac{1}{(1+\Gamma)} \sum_{j=0}^{g-2}(1+r^*)^j \delta_{g-j,t-j}$$

$$a_{g,t+1} = \frac{1}{(1+\Gamma)} a_{g,t+1}$$

Thus, the growth for aggregate capital is:

$$K_{t+1} = \sum_{g=2}^{G} N_{g-1,t} a_{g,t+1} = (1+n)(1+\Gamma) \sum_{g=2}^{G} N_{g-1,t-1} a_{g,t} = (1+n)(1+\Gamma)K_t \qquad \text{(C.4)}$$

D.2. Computational algorithm

D.2.I. Steady-state

TABLE D.1
STEADY-STATE CONDITIONS

g_n	$N_{t+1} = N_t(1+g_n)$	$\forall t \in [T-1, \infty)$
g_λ	$\lambda_{t+1} = (1+g_\lambda)\,\lambda_t;\ 1+g_A \equiv (1+g_\lambda)^{1/\theta};\ 1+g_\lambda = (1+g_n)^{\varphi/(1-\rho)}$	$\forall t \in [T, \infty)$
Γ	$X \in [\omega_t, tr_t, a_{g,t}, c_{g,t}]\ ;\ X_{t+1} = (1+\Gamma)\,X_t;\ 1+\Gamma = (1+g_A)^{1/(1-\alpha)}$	$\forall t \in [T, \infty)$
g_K	$X \in [C_t, I_t, K_t, Y_t]\ ;\ X_{t+1} = (1+g_K)\,X_t;\ 1+g_K = (1+\Gamma)\,(1+g_n)$	$\forall j, t \in [T, \infty)$
I1	$\lambda_{n,T+1} - \lambda_{n,T} = {N_{n,t}}^{\varphi}\,(\lambda_{n,T})^{\rho} \left[\sum_g \eta_g \bar{N}_{n,g,T}\right]^{\varphi} \Gamma\,(1-\rho)$	$\forall t \in [T, \infty)$
I2	$A_T \equiv \lambda_T^{1/\theta}\,(B)^{-1}\ , B = \Gamma \left(1 + \frac{1-\sigma}{\theta}\right)^{\frac{\theta}{1-\sigma}};\ A_{t+1} = A_t(1+g_A);\ 1+g_A = (1+g_\lambda)^{\frac{1}{\theta}}$	$\forall t \in [T, \infty)$
H1	$1 \equiv \sum_{g=1}^{G} \bar{N}_g;\ \bar{L} \equiv \sum_{g=G_0+1}^{G} \bar{N}_g;\ l = \overline{labor} = \sum_{g=G_0+1}^{G_1} N_g l_g = \sum_{g=1}^{G} N_g l_g;\ l = \frac{L_t}{N_t}$	$\forall t \in [T, \infty)$
H2	$\bar{c}_g + (1+\Gamma)\bar{a}_{g+1} = (1+\bar{r})\,\bar{a}_g + \bar{\omega} l_g + \overline{tr}; \forall g \in [1, G]$	$\forall t \in [T, \infty)$
H3	$\bar{a}_1 = \bar{a}_{G+1} = 0;\ \bar{c}_g > 0,\ \{\bar{c}_g\}_{g=1}^{G};\ \{\bar{a}_{g+1}\}_{g=1}^{G-1}$	$\forall t \in [T, \infty)$
H4	$\overline{tr} = (1+\bar{r}) \sum_{g=2}^{G} \left(\frac{\bar{N}_{g-1}}{(1+n)} - \bar{N}_g\right) \bar{a}_g$	$\forall t \in [T, \infty)$
H5	$(1+\Gamma)\bar{c}_{g+1} = \bar{c}_g \left[\beta s_{g+1}\,(1+\bar{r})\right]^{\sigma};\quad \forall\ g \in [1, G-1]$	$\forall t \in [T, \infty)$
H6	$\bar{c} = \sum_{g=1}^{G} \tilde{N}_g \bar{c}_g\ ;\ \bar{k} = \frac{1}{(1+n)} \sum_{g=2}^{G} \bar{N}_{g-1} \bar{a}_g$	$\forall t \in [T, \infty)$
F1	$\omega_t = (1-\alpha)\left({A_t}^{1/(1-\alpha)}\right)\left(\frac{\bar{k}}{l}\right)^{\alpha};\ \bar{\omega} = (1-\alpha)\left(\frac{\bar{k}}{l}\right)^{\alpha}$	$\forall t \in [T, \infty)$
F2	$\bar{r} = (\alpha)\left(\frac{\bar{k}}{l}\right)^{-(1-\alpha)} - \delta$	$\forall t \in [T, \infty)$
T1	$\bar{y} = \bar{c} + \bar{i} = \bar{r}\bar{k} + \frac{\omega_t}{{A_t}^{1/(1-\alpha)}} \frac{L_t}{N_t} = \bar{r}\bar{k} + \frac{\omega_t}{{A_t}^{1/(1-\alpha)}} l$	$\forall t \in [T, \infty)$
T2	$\bar{y} = \bar{k}^{\alpha} l^{1-\alpha};\ y_t = \frac{Y_t}{N_t} = A_t \frac{(L_t)^{1-\alpha}\,{K_t}^{\alpha}}{N_t} = A_t l_t^{1-\alpha} k_t^{\alpha}$	$\forall t \in [T, \infty)$
T3	$\bar{i} = ((1+g_n)(1+\Gamma) + \delta - 1)\,\bar{k};\ I_t = ((1+g_n)(1+\Gamma) + \delta - 1)\,K_t$	$\forall t \in [T, \infty)$
T3'	$C_t + (1+g_n)(1+\Gamma)K_t = (1 + r_t - \delta)\,K_t + \omega_t L_t = Y_t;$	$\forall t \in [T, \infty)$
T4	$(1+g_n)(1+\Gamma)K_t = K_{t+1} = K_t(1-\delta) + I_t$	$\forall t \in [T, \infty)$
S1	$x_t = \bar{x}\left({A_t}^{1/(1-\alpha)}\right);\ x_{g,t} = \bar{x}_g \left({A_t}^{1/(1-\alpha)}\right)$	$\forall t \in [T, \infty)$
S2	$x_t \equiv \frac{X_t}{N_t}; \bar{x} \equiv \frac{X_t}{\left({A_t}^{1/(1-\alpha)} N_t\right)} \equiv \frac{x_t}{\left({A_t}^{1/(1-\alpha)}\right)}; x_t = \bar{x}\left({A_t}^{1/(1-\alpha)}\right);$	$\forall t \in [T, \infty)$
S3	$x_{g,t} \equiv \frac{X_{g,t}}{N_{g,t}}; \bar{x}_g \equiv \frac{X_{g,t}}{\left({A_t}^{1/(1-\alpha)} N_{g,t}\right)} \equiv \frac{x_{g,t}}{\left({A_t}^{1/(1-\alpha)}\right)}; x_{g,t} = \bar{x}_g \left({A_t}^{1/(1-\alpha)}\right)$	$\forall t \in [T, \infty)$

Tilde "$\bar{x}_t$" or "$\bar{x}_{g,t}$" means x per efficiency unit of household, or x per efficiency unit of age g household, or real interest rate which is constant over time. Define $\bar{x}_g = \frac{x_{g,t}}{A_t^{1/(1-\alpha)}}$ and $x_t = \frac{X_t}{N_t}$; then $\bar{x}_t = \frac{x_t}{A_t^{1/(1-\alpha)}}$.

Algorithm 1 Close Economy Steady State

1: Guess $\bar{k} = \bar{k}^i_{\text{guess}}$, and i record the order of guess.

2: Given $\bar{k} = \bar{k}_{\text{guess}}$, calculate $\bar{\omega} = (1-\alpha)\left(\frac{\bar{k}}{l}\right)^{\alpha}$, $\bar{R}_r = \alpha\left(\frac{\bar{k}}{l}\right)^{1-\alpha}$, and $\bar{r} = \bar{R}_r - \delta$.

3: Solve $(G-1)$ Euler equations and get $\bar{a}_g = \{2, \ldots, G\}$; $\bar{c}_g = \{1, \ldots, G\}$:

$$\begin{aligned}
\bar{c}_g + (1+\Gamma)\bar{a}_{g+1} &= (1+\bar{r})\bar{a}_g + \bar{\omega} l_g + ts, \quad \forall g \geq 1, t \\
(1+\Gamma)\bar{c}_{g+1} &= \bar{c}_g\left[\beta s_{g+1,t+1}(1+\bar{r})\right]^{\sigma}
\end{aligned}$$

$$\begin{aligned}
\bar{t}s &= (1+\bar{r})\sum_{g=2}^{G}\left(\frac{\bar{N}_{g-1}}{1+n} - \bar{N}_g\right)\bar{a}_g \\
\bar{a}_1 &= \bar{a}_{G+1} = 0
\end{aligned}$$

Specifically, solving an unknown vector that satisfies the above-implied matrix.

4: Calculate $\bar{k}_1^{\prime\text{guess}}$:

$$\bar{k} = \frac{1}{1+n}\sum_{g=2}^{G}\bar{N}_{g-1}\bar{a}_g$$

5: For $i = 1$, check if:

$$\|\bar{k}_1^{\prime\text{guess}} - \bar{k}^1_{\text{guess}}\| < \epsilon$$

If the above holds, then $\bar{k} = \bar{k}_1^{\prime\text{guess}}$, model solved.
If not, update new guess for k:

$$\bar{k}^{i+1}_{\text{guess}} = \zeta\bar{k}^i_{\text{guess}} + (1-\zeta)\bar{k}_1^{\prime\text{guess}}, \quad \zeta \in (0,1).$$

Then, repeat from step 2.

D.2.II. Dynamic equilibrium

TABLE D.2
Dynamic equilibrium conditions

I1	$\lambda_{n,t+1} - \lambda_{n,t} = {N_{n,t}}^{\varphi} \left(\lambda_{n,t}\right)^{\rho} \left[\sum_{g} \eta_g \bar{N}_{n,g,t}\right]^{\varphi} \Gamma\left(1-\rho\right);\ 1+g_{\lambda} = \left(1+g_n\right)^{\varphi/(1-\rho)}$	$\forall(t)$
I2	$A_t \equiv \lambda_t^{1/\theta} \left(B\right)^{-1}, B = \Gamma\left(1+\frac{1-\sigma}{\theta}\right)^{\frac{\theta}{1-\sigma}};\ 1+g_A = 1+\gamma = \left(1+g_{\lambda}\right)^{1/\theta};\ 1+\Gamma = \left(1+g_A\right)^{1/(1-\alpha)}$	$\forall(t)$
H1	$N_t \equiv \sum_{g=1}^{G} N_{g,t};\ \bar{L}_t \equiv \sum_{g=G_0+1}^{G} N_{g,t};\ L_t = \sum_{g=G_0+1}^{G_1} N_{g,t} l_g = \sum_{g=1}^{G} N_{g,t} l_g;$	$\forall(t)$
H2	$c_{g,t} + a_{g+1,t+1} = \left(1+r_t\right) a_{g,t} + \omega_t l_g + tr_t,\ \forall g \in [1,G]$	$\forall(t)$
H3	$a_{1,t} = a_{G+1,t} = 0;\ c_{g,t} > 0,\ \{c_{g,t+g-1}\}_{g=1}^{G};\ \{a_{g+1,t+g}\}_{g=1}^{G-1}$	$\forall(t)$
H4	$tr_t \equiv \frac{TR_t}{N_t} = \left(1+r_t\right) \sum_{g=2}^{G} \frac{\left(N_{g-1,t-1} - N_{g,t}\right)}{N_t} a_{g,t}$	$\forall(t)$
H5	$c_{g+1,t+1} = c_{g,t} \left[\beta s_{g+1,t+1} \left(1+r_{t+1}\right)\right]^{\sigma};\quad \forall\ g \in [1, G-1]$	$\forall(t)$
H6	$c_t = \sum_{g=1}^{G} \tilde{N}_{g,t} c_{g,t};\qquad k_t = \frac{K_t}{N_t} = \frac{\sum_{g=1}^{G} N_{g-1,t-1} a_{g,t}}{N_t} = \sum_{g=1}^{G} \frac{N_{g-1,t-1}}{N_t} a_{g,t} = \sum_{g=2}^{G} \frac{N_{g-1,t-1}}{N_t} a_{g,t}$	$\forall(t)$
F1	$\omega_t = \frac{W_t}{P_t} = (1-\alpha)\left({A_t}^{1/(1-\alpha)}\right)\left(\frac{K_t}{{A_t}^{1/(1-\alpha)} L_t}\right)^{\alpha} = (1-\alpha)\left({A_t}^{1/(1-\alpha)}\right)\left(\frac{k_t}{{A_t}^{1/(1-\alpha)} l_t}\right)^{\alpha}$	$\forall(t)$
F2	$r_t = \frac{R_t}{P_t} - \delta = (\alpha)\left(\frac{K_t}{{A_t}^{1/(1-\alpha)} L_t}\right)^{-(1-\alpha)} - \delta = (\alpha)\left(\frac{k_t}{{A_t}^{1/(1-\alpha)} l_t}\right)^{-(1-\alpha)} - \delta$	$\forall(t)$
T1	$y_t = c_t + i_t = r_t k_t + \omega_t \frac{L_t}{N_t} = r_t k_t + \omega_t l_t$	$\forall(t)$
T2	$y_t = \frac{Y_t}{N_t} = A_t \frac{\left(L_t\right)^{1-\alpha} {K_t}^{\alpha}}{N_t} = A_t l_t^{1-\alpha} k_t^{\alpha}$	$\forall(t)$
T3	$C_t + I_t = (r_t + \delta) K_t + \omega_t L_t = Y_t$	$\forall(t)$
T3'	$\mathrm{C}_t + K_{t+1} = \left(1 + r_t - \delta\right) K_t + \omega_t L_t = Y_t$	$\forall(t)$
T4	$K_{t+1} = K_t(1-\delta) + I_t$	$\forall(t)$
S1	$x_t = \bar{x}\left({A_t}^{1/(1-\alpha)}\right);\ x_{g,t} = \bar{x}_g\left({A_t}^{1/(1-\alpha)}\right)$	$\forall(t)$
S2	$x_t \equiv \frac{X_t}{N_t}; \bar{x} \equiv \frac{X_t}{\left({A_t}^{1/(1-\alpha)} N_t\right)} \equiv \frac{x_t}{\left({A_t}^{1/(1-\alpha)}\right)}; x_t = \bar{x}\left({A_t}^{1/(1-\alpha)}\right);$	$\forall(t)$
S3	$x_{g,t} \equiv \frac{X_{g,t}}{N_{g,t}}; \bar{x}_g \equiv \frac{X_{g,t}}{\left({A_t}^{1/(1-\alpha)} N_{g,t}\right)} \equiv \frac{x_{g,t}}{\left({A_t}^{1/(1-\alpha)}\right)}; x_{g,t} = \bar{x}_g\left({A_t}^{1/(1-\alpha)}\right)$	$\forall(t)$

Algorithm 2 Close Economy Dynamic Equilibrium

1: The economy starts at t_1 with $\bar{a}_{t_1 g=1,\dots,G}$ and reaches a new steady state at t_{T+1} with $\bar{a}_{t_{T+1} g=1,\dots,G}$.

2: With

$$c_t = \sum_{g=1}^{G} \bar{N}_{g,t} c_{g,t}, \quad k_t = \sum_{g=2}^{G} \frac{N_{g-1,t-1}}{N_t} a_{g,t}, \quad a_{g,t} = \bar{a}_{g,t} \left(A_t^{\frac{1}{1-\alpha}} \right)$$

Calculate k_t, k_{t+1}, guess $k^i = \{k_{t2}^{i,\text{guess}}, \dots, k_{tT}^{i,\text{guess}}\}$, and set $i = 1$.

3: Given k^i, $\forall t \in [t_1, t_{T+1}]$, calculate:

$$r_t = \frac{R_t}{P_t} - \delta = (\alpha) \left(\frac{K_t}{A_t^{\frac{1}{1-\alpha}} L_t} \right)^{-(1-\alpha)} - \delta = (\alpha) \left(\frac{k_t}{A_t^{\frac{1}{1-\alpha}} l_t} \right)^{-(1-\alpha)} - \delta \tag{C.5}$$

$$\omega_t = \frac{W_t}{P_t} = (1-\alpha) A_t^{\frac{1}{1-\alpha}} \left(\frac{K_t}{A_t^{\frac{1}{1-\alpha}} L_t} \right)^{\alpha} = (1-\alpha) A_t^{\frac{1}{1-\alpha}} \left(\frac{k_t}{A_t^{\frac{1}{1-\alpha}}{}_t} \right)^{\alpha} \tag{C.6}$$

4: Guess $tr_t^{i,j}$, repeat following until $tr_t^{i,j}$ convergence, and set $tr_t^i = tr_t^{i,j0}$ for some j_0.

(a) Solve Euler equations and get $\forall t \in [t_2, t_T]$: $a_{g,t}^i = \{2, \dots, G\}$; $c_{g,t}^i = \{1, \dots, G\}$, and $tr_t^{i,j'}$:

$$c_{g,t} + a_{g+1,t+1} = (1+r_t) a_{g,t} + \omega_t l_g + tr_t, \quad \forall g \geq 1, t$$

$$c_{g+1,t+1} = c_{g,t} \left[\beta s_{g+1,t+1} (1 + \bar{r}_{t+1}) \right]^{\sigma}$$

$$tr_t = \frac{TR_t}{N_t} = (1+r_t) \sum_{g=2}^{G} \left(\frac{N_{g-1,t-1} - N_{g,t}}{N_t} \right) a_{g,t}$$

$$a_{1,t} = a_{G+1,t} = 0$$

(b) For j, check:

$$\| tr_t^{i,j} - tr_t^{i,j'} \| < \epsilon \cdot 1E - 2$$

If the above holds, then $tr_t^i = tr_t^{i,j}$, model solved. If not, update new guess $tr_t^{i,j+1}$:

$$tr_t^{i,j+1} = \zeta' tr_t^{i,j} + (1-\zeta') tr_t^{i,j'}, \quad \zeta' \in (0,1)$$

Then, repeat from Step 3(a).

5: $\forall t \in [t_2, t_T]$, calculate $k^i = \{k_{t_1}^{i,\prime}, k_{T-1}^{i,\prime}\}$:

$$k_t = \sum_{g=2}^{G} \frac{N_{g-1,t-1}}{N_t} a_{g,t}, \quad \bar{k}_t = \frac{k_t}{A_t^{\frac{1}{1-\alpha}}}, \quad \bar{k}_t = \sum_{g=2}^{G} \frac{N_{g-1,t-1}}{N_t} \bar{a}_{g,t}$$

6: For i, check:

$$\| k_t^{i,\prime\text{guess}} - k_t^{i,\text{guess}} \| < \epsilon$$

If the above holds, then $k = k_t^{i,\prime\text{guess}}$, model solved. If not, update new guess for k:

$$k_t^{i+1,\text{guess}} = \zeta k_t^{i,\prime\text{guess}} + (1-\zeta) k_t^{i,\text{guess}}, \quad \zeta \in (0,1)$$

Then, repeat from Step 2.

Appendix E: Multi-sector open economy model details

E.1. Equilibrium and/or equation derivations

E.1.I. equilibrium conditions

TABLE E.1

Stationary balanced growth equilibrium

g_n	$N_{n,g,t+1} = (1+g_n)\, N_{n,g,t}$	$\forall n, t \in [T-1, \infty)$
g_{λ^j}	$\lambda^j_{n,t+1} = (1+g_{\lambda^j})\, \lambda^j_{n,t}$; $(1+g_{\lambda^j}) = (1+g_n)^{\frac{\varphi^j}{(1-\rho)}}$; $1+g_{A^j} \equiv (1+g_{\lambda^j})^{1/\theta}$	$\forall n, j, t \in [T, \infty)$
g_ω	$X \in [\frac{W_{n,t}}{P_{n,C,t}}, \frac{ts^T_{n,t}}{P_{C,n,t}}, \frac{ts^D_{n,t}}{P_{n,C,t}}, a_{n,g,t}, c_{n,g,t}]$; $X_{t+1} = (1+g_\omega)\, X_t$; $1+g_\omega = (1+g_{A^j})^{\frac{1}{\beta^j\gamma^j}} = (1+g_{\lambda^j})^{\frac{1}{\beta^j\gamma^j\theta}}$	$\forall n, t \in [T, \infty)$
$g_{rc^j_n}$	$X \in [\frac{c^j_{n,t}}{P^j_{n,t}}]$; $X_{t+1} = \left(1+g_{rc^j_n}\right) X_t$; $1+g_{rc^j_n} = (1+g_\omega)^{\beta^j\gamma^j} = (1+g_{\lambda^j})^{1/\theta}$	$\forall n, t \in [T, \infty)$
g_K	$X \in [C_{n,t}, C^j_{n,t}, I_{n,t}, I^j_{n,t}, K_{n,t}, Y^j_{n,t}, \frac{X^j_{n,t}}{P^j_{n,t}}, \frac{D_{n,t}}{P^j_{n,t}}, \frac{D_{n,t}}{P_{n,Ct}}, \frac{D_{n,t}}{P_{n,I,t}}]$; $X_{t+1} = (1+g_K)\, X_t$; $1+g_K = (1+g_\omega)(1+g_n)$	$\forall n, j, t \in [T, \infty)$
	$1+g_\omega = (1+g_n)^{\frac{\varphi^j}{\theta\beta^j\gamma^j(1-\rho)}}$; $\varphi^j/\varphi^k = \beta^j\gamma^j/\beta^k\gamma^k$; $\varphi^j = \theta(1-\rho)\,\beta^j\gamma^j \frac{\log(1+g_\omega)}{\log(1+g_n)}$;	$\forall n, j$
F0	$\lambda^j_{n,T+1} - \lambda^j_{n,T} = {N_{n,T}}^{\varphi^j} \left(\lambda^j_{n,T}\right)^\rho \left[\sum_g \eta^j_g \bar{N}_{n,g,T}\right]^{\varphi^j} \Gamma(1-\rho)$	$\forall(n)$
H1	$N_{n,T} \equiv \sum_{g=1}^{G} N_{n,g,T}$; $\bar{L}_{n,T} \equiv \sum_{g=G_0+1}^{G} N_{n,g,T}$; $L_{n,T} = \left(1-\tau^L_{n,T}\right) \sum_{g=G_0+1}^{G_1} N_{n,g,T} l_g$; $L^e_{n,T} = E_{n,T} L_{n,T}$	$\forall(n)$
H2	$P_{n,C,T} c_{n,g,T} + P_{n,I,T}(1+g_\omega)\, a_{n,g+1,T} = P_{n,I,T}(1+r_{n,T})\, a_{n,g,T} + W_{n,T}\left(1-\tau^L_{n,T}\right) E_{n,T} l_g + tr^D_{n,T} + tr^T_{n,T}$; $g \in [1, G]$	$\forall(n)$
H3	$a_{1,T} = a_{G+1,T} = 0$; $c_{n,g,T} > 0$, $\{c_{n,g,T}\}_{g=1}^{G}$; $\{a_{n,g+1,T}\}_{g=1}^{G-1}$	$\forall(n)$
H4	$tr^T_{n,T} \equiv \frac{D_{n,T}}{N_{n,T}}$; $tr^D_{n,T} = P_{n,I,T}(1+r_{n,T}) \sum_{g=2}^{G} \left(\frac{\overline{N}_{n,g-1,T}}{1+g_n} - \overline{N}_{n,g,T}\right) a_{n,g,T}$	$\forall(n)$
H4'	$tr^D_{n,T} = tr^{D,1}_{n,T} + tr^{D,2}_{n,T} = P_{n,I,T}(1-\delta) \sum_{g=2}^{G} \left(\frac{\overline{N}_{n,g-1,T}}{1+g_n} - \overline{N}_{n,g,T}\right) a_{n,g,T} + P_{n,I,T}\left(\frac{R_{n,T}}{P_{n,I,T}}\right) \sum_{g=2}^{G} \left(\frac{\overline{N}_{n,g-1,T}}{1+g_n} - \overline{N}_{n,g,T}\right) a_{n,g,T}$	$\forall(n)$
H4"	$P_{n,C,T} c_{n,g,T} + P_{n,I,T} i_{n,g,T} = R_{n,T} a_{n,g,T} + W_{n,T}\left(1-\tau^L_{n,T}\right) E_{n,T} l_g + tr^{D,2}_{n,T} + tr^T_{n,T}$	$\forall(n)$
H4"'	$P_{n,I,T} i_{n,g,T} = P_{n,I,T}(1+g_\omega)\, a_{n,g+1,T} - \left[P_{n,I,T}(1-\delta)\, a_{n,g,T} + tr^{D,1}_{n,T}\right]$	$\forall(n)$
H5	$(1+g_\omega)\, c_{n,g+1,T} = \left[(\beta s_{n,g+1,T})\left(\frac{\psi_{n,g+1,T+1}}{\psi_{n,g,T}}\right)(1+r_{n,T})\right]^\sigma c_{n,g,T}$; $\forall\ g \in [1, G-1]$	$\forall(n)$
H6	$C_{n,T} \equiv \sum_{g=1}^{G} N_{n,g,T} c_{n,g,T}$; $K_{n,T} \equiv \sum_{g=2}^{G} \frac{N_{n,g-1,T}}{1+g_n} a_{n,g,T}$	$\forall(n)$
H7	$C_{n,T} \equiv \prod_{j=1}^{J} {C^j_{n,T}}^{\alpha^j_C}$; $I_{n,T} \equiv \prod_{j=1}^{J} {I^j_{n,T}}^{\alpha^j_I}$; $P_{n,I,T} = \prod_{j=1}^{J} \left[\frac{P^j_{n,T}}{\alpha^j_I}\right]^{\alpha^j_I}$; $P_{n,C,T} = \prod_{j=1}^{J} \left[\frac{P^j_{n,T}}{\alpha^j_C}\right]^{\alpha^j_C}$	$\forall(n)$
H8	$P^j_{n,T} I^j_{n,T} = \alpha^j_{I,n} P_{n,I,T} I_{n,T}$; $P^j_{n,T} C^j_{n,T} = \alpha^j_{C,n} P_{n,C,T} C_{n,T}$	$\forall(n, j)$
F1	$W_{n,T} L^e_{n,T} = \sum_{j=1}^{J} \beta^j\gamma^j \sum_{i=1}^{N} \pi^j_{in,T} X^j_{i,T}$; $R_{n,T} K_{n,T} = \sum_{j=1}^{J} \left(1-\beta^j\right)\gamma^j \sum_{i=1}^{N} \pi^j_{in,T} X^j_{i,T}$	$\forall(n)$
F2	$r_{n,T} = \frac{R_{n,T}}{P_{n,I,T}} - \delta$	$\forall(n)$
T1	$c^j_{n,T} \equiv \Upsilon^j \left[(W_{n,T})^{\beta^j} (R_{n,T})^{1-\beta^j}\right]^{\gamma^j} \prod_{k=1}^{J} {P^k_{n,T}}^{\gamma^{k,j}}$; $\Upsilon^j \equiv \gamma^j \beta^{j^{-\gamma^j\beta^j}} \gamma^j \left(1-\beta^j\right)^{-\gamma^j\left(1-\beta^j\right)} \prod_{k=1}^{J} \gamma^{k,j^{-\gamma^{k,j}}}$	$\forall(n, j)$
T2	$P^j_{n,T} = A \cdot \left[\sum_{i=1}^{N} \lambda^j_{i,T}\left(\kappa^j_{ni,T} c^j_{i,T}\right)^{-\theta}\right]^{-\frac{1}{\theta}}$; $A \equiv \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$	$\forall(n, j)$
T3	$\pi^j_{ni,T} \equiv \frac{X^j_{ni,T}}{\sum_{i=1}^{N} X^j_{ni,T}} = \frac{\lambda^j_{i,t}\left(c^j_{i,t}\kappa^j_{ni,T}\right)^{-\theta}}{\sum_{m=1}^{N} \lambda^j_{m,T}\left(c^j_{m,T}\kappa^j_{nm,T}\right)^{-\theta}} = \lambda^j_{i,T}\left(\frac{A c^j_{i,T}\kappa^j_{ni,T}}{P^j_{n,T}}\right)^{-\theta}$	$\forall(n, i, j)$
T4	$P_{C,n,T} C_{n,T} + P_{I,n,T} I_{n,T} = R_{n,T} K_{n,T} + W_{n,T} E_{n,T} L_{n,T} + D_{n,T} = R_{n,T} K_{n,T} + W_{n,T} L^e_{n,T} + D_{n,T} \equiv IN_{n,T}$	$\forall(n)$
T4'	$P_{n,C,T} C_{n,T} + P_{n,I,T}(1+g_K)\, K_{n,T} = \left(1 + \frac{R_{n,T}}{P_{n,I,T}} - \delta\right) P_{n,I,T} K_{n,T} + W_{n,T} L^e_{n,T} + D_{n,T}$	$\forall(n)$
T5	$(1+g_K)\, K_{n,T} = K_{n,T+1} = I_{n,T} + (1-\delta)\, K_{n,T}$; $(g_K+\delta)\, K_{n,T} = I_{n,T}$	$\forall(n)$
T6	$\sum_{j=1}^{J}\sum_{i=1}^{N} X^j_{in,T} - \sum_{j=1}^{J}\sum_{i=1}^{N} X^j_{ni,T} = NX_{n,T} = -D_{n,T}$	$\forall(n, j)$
T6'	$X^j_{n,T} = \alpha^j_C P_{C,n,T} C_{n,T} + \alpha^j_I P_{I,n,T} I_{n,T} + \sum_{k=1}^{J} \gamma^{j,k} \left(\sum_{i=1}^{N} X^k_{in,T}\right)$	$\forall(n, j)$
T7	$D_{n,T} = -\phi_{n,T}\left(R_{n,T} K_{n,T} + W_{n,T} L^e_{n,T}\right) + N_{n,T} T^P_T$; $T^P_T = \frac{\sum_{n=1}^{N} \phi_{n,T}\left(R_{n,T} K_{n,T} + W_{n,T} L^e_{n,T}\right)}{\sum_{n=1}^{N} N_{n,T}}$	$\forall(n)$
T7'	$D_{n,T} = -\phi_{n,T}\left(R_{n,T} K_{n,T} + W_{n,T} L^e_{n,T}\right) + \frac{N_{n,T}}{\sum_{n=1}^{N} N_{n,T}} \sum_{n=1}^{N} \phi_{n,T}\left(R_{n,T} K_{n,T} + W_{n,T} L^e_{n,T}\right)$	$\forall(n)$

TABLE E.2

DYNAMIC TRANSITIONAL EQUILIBRIUM

I1	$\lambda^j_{n,t+1} - \lambda^j_{n,t} = \left(\lambda^j_{n,t}\right)^{\rho} \left(\sum_g \eta^j_g N_{n,g,t}\right)^{\varphi^j} \Gamma\left(1-\rho\right) = {N_{n,t}}^{\varphi^j} \left(\lambda^j_{n,t}\right)^{\rho} \left(\sum_g \eta^j_g \bar{N}_{n,g,t}\right)^{\varphi^j} \Gamma\left(1-\rho\right)$	$\forall(n,t)$
H1	$N_{n,t} \equiv \sum_{g=1}^{G} N_{n,g,t};\ \bar{L}_{n,t} \equiv \sum_{g=G_0+1}^{G} N_{n,g,t};\ L_{n,t} = \left(1-\tau^L_{n,t}\right) \sum_{g=G_0+1}^{G_1} N_{n,g,t} l_g = \left(1-\tau^L_{n,t}\right) \sum_{g=1}^{G} N_{n,g,t} l_g;\ L^e_{n,t} = E_{n,t} L_{n,t}$	$\forall(n,t)$
H2	$P_{n,C,t} c_{n,g,t} + P_{n,I,t} a_{n,g+1,t+1} = P_{n,I,t}\left(1+r_{n,t}\right) a_{n,g,t} + W_{n,t}\left(1-\tau^L_{n,t}\right) E_{n,t} l_g + tr^D_{n,t} + tr^T_{n,t}\ ;\ g \in [1,G]$	$\forall(n,t)$
H3	$a_{1,t} = a_{G+1,t} = 0;\ c_{n,g,t} > 0,\ \left\{c_{n,g,t+g-1}\right\}_{g=1}^{G};\ \left\{a_{n,g+1,t+g}\right\}_{g=1}^{G-1}$	$\forall(n,t)$
H4	$tr^T_{n,t} \equiv \frac{D_{n,t}}{N_{n,t}};\ tr^D_{n,t} \equiv P_{n,I,t}\left(1+r_{n,t}\right) \frac{\sum_{g=2}^{G}\left(N_{n,g-1,t-1} - N_{n,g,t}\right) a_{n,g,t}}{N_{n,t}}$	$\forall(n,t)$
H4’	$tr^D_{n,t} = tr^{D,1}_{n,t} + tr^{D,2}_{n,t} = P_{n,I,t}\left(1-\delta\right) \sum_{g=2}^{G} \left(\frac{N_{n,g-1,t-1} - N_{n,g,t}}{N_{n,t}}\right) a_{n,g,t} + P_{n,I,t}\left(\frac{R_{n,t}}{P_{n,I,t}}\right) \sum_{g=2}^{G} \left(\frac{N_{n,g-1,t-1} - N_{n,g,t}}{N_{n,t}}\right) a_{n,g,t}$	$\forall(n)$
H4”	$P_{n,C,t} c_{n,g,t} + P_{n,I,t} i_{n,g,t} = R_{n,t} a_{n,g,t} + W_{n,t}\left(1-\tau^L_{n,t}\right) E_{n,t} l_g + tr^{D,2}_{n,t} + tr^T_{n,t}$	$\forall(n)$
H4”’	$P_{n,I,t} i_{n,g,t} = P_{n,I,t} a_{n,g+1,t+1} - \left[P_{n,I,t}\left(1-\delta\right) a_{n,g,t} + tr^{D,1}_{n,t}\right]$	$\forall(n)$
H5	$\frac{c_{n,g+1,t+1}}{c_{n,g,t}} = \left[\left(\beta s_{n,g+1,t+1}\right) \left(\frac{\psi_{n,t+1}}{\psi_{n,t}}\right) \frac{\frac{P_{n,I,t+1}}{P_{n,C,t+1}}}{\frac{P_{n,I,t}}{P_{n,C,t}}} \left(1+r_{n,t+1}\right)\right]^{\sigma}\ ;\forall\ g \in [1, G-1]$	$\forall(n,t)$
H6	$C_{n,t} \equiv \sum_{g=1}^{G} N_{n,g,t} c_{n,g,t};\ K_{n,t} \equiv \sum_{g=2}^{G} N_{n,g-1,t-1} a_{n,g,t}$	$\forall(n,t)$
H7	$C_{n,t} \equiv \prod_{j=1}^{J} {C^j_{n,t}}^{\alpha^j_{n,C,t}};\ I_{n,t} \equiv \prod_{j=1}^{J} {I^j_{n,t}}^{\alpha^j_{n,I,t}};\ P_{n,I,t} = \prod_{j=1}^{J} \left[\frac{P^j_{n,t}}{\alpha^j_{I,n}}\right]^{\alpha^j_{I,n}};\ P_{n,C,t} = \prod_{j=1}^{J} \left[\frac{P^j_{n,t}}{\alpha^j_{C,n}}\right]^{\alpha^j_{C,n}}$	$\forall(n,t)$
H8	$P^j_{n,t} I^j_{n,t} = \alpha^j_{I,n} P_{n,I,t} I_{n,t};\ P^j_{n,t} C^j_{n,t} = \alpha^j_{C,n} P_{n,C,t} C_{n,t}$	$\forall(n,j,t)$
F1	$W_{n,t} L^e_{n,t} = \sum_{j=1}^{J} \beta^j_n \gamma^j_n \sum_{i=1}^{N} \pi^j_{in,t} X^j_{i,t};\ R_{n,t} K_{n,t} = \sum_{j=1}^{J} \left(1-\beta^j_n\right) \gamma^j_n \sum_{i=1}^{N} \pi^j_{in,t} X^j_{i,t}$	$\forall(n,t)$
F2	$r_{n,t} = \frac{R_{n,t}}{P_{n,I,t}} - \delta$	$\forall(n,t)$
T1	$c^j_{n,t} \equiv \Upsilon^j_n \left[\left(W_{n,t}\right)^{\beta^j_n} \left(R_{n,t}\right)^{1-\beta^j_n}\right]^{\gamma^j_n} \prod_{k=1}^{J} {P^k_{n,t}}^{\gamma^{k,j}_n}$ where $\Upsilon^j_n \equiv \gamma^j_n {\beta^j_n}^{-\gamma^j_n \beta^j_n} \gamma^j_n \left(1-\beta^j_n\right)^{-\gamma^j_n\left(1-\beta^j_n\right)} \prod_{k=1}^{J} {\gamma^{k,j}_n}^{-\gamma^{k,j}_n}$	$\forall(n,j,t)$
T2	$P^j_{n,t} = A^j \cdot \left[\sum_{i=1}^{N} \lambda^j_{i,t} \left(\kappa^j_{ni,t} c^j_{i,t}\right)^{-\theta}\right]^{-\frac{1}{\theta}}$ where $A^j \equiv \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$	$\forall(n,j,t)$
T3	$\pi^j_{ni,t} \equiv \frac{X^j_{ni,t}}{\sum_{i=1}^{N} X^j_{ni,t}} = \frac{\lambda^j_{i,t}\left(c^j_{i,t}\kappa^j_{ni,t}\right)^{-\theta}}{\sum_{m=1}^{N} \lambda^j_{m,t}\left(c^j_{m,t}\kappa^j_{nm,t}\right)^{-\theta}} = \lambda^j_{i,t}\left(\frac{A^j c^j_{i,t} \kappa^j_{ni,t}}{P^j_{n,t}}\right)^{-\theta}$	$\forall(n,i,j,t)$
T4	$P_{n,C,t} C_{n,t} + P_{n,I,t} I_{n,t} = R_{n,t} K_{n,t} + W_{n,t} E_{n,t} L_{n,t} + D_{n,t} = R_{n,t} K_{n,t} + W_{n,t} L^e_{n,t} + D_{n,t} \equiv IN_{n,t}$	$\forall(n,t)$
T4’	$P_{n,C,t} C_{n,t} + P_{n,I,t} K_{n,t+1} = \left(1 + \frac{R_{n,t}}{P_{n,I,t}} - \delta\right) P_{n,I,t} K_{n,t} + W_{n,t} L^e_{n,t} + D_{n,t}$	$\forall(n,t)$
T5	$K_{n,t+1} = I_{n,t} + \left(1-\delta\right) K_{n,t}$	$\forall(n,t)$
T6	$\sum_{j=1}^{J}\sum_{i=1}^{N} X^j_{in,t} - \sum_{j=1}^{J}\sum_{i=1}^{N} X^j_{ni,t} = NX_{n,t} = -D_{n,t}$	$\forall(n,j,t)$
T6’	$X^j_{n,t} = \alpha^j_{C,n} P_{C,n,t} C_{n,t} + \alpha^j_{I,n} P_{I,n,t} I_{n,t} + \sum_{k=1}^{J} \gamma^{j,k}_n \left(\sum_{i=1}^{N} X^k_{in,t}\right)$	$\forall(n,j,t)$
T7	$D_{n,t} = -\phi_{n,t}\left(R_{n,t} K_{n,t} + W_{n,t} L^e_{n,t}\right) + N_{n,t} T^P_t\ ;\ T^P_t = \frac{\sum_{n=1}^{N} \phi_{n,t}\left(R_{n,t} K_{n,t} + W_{n,t} L^e_{n,t}\right)}{\sum_{n=1}^{N} N_{n,t}}$	$\forall(n,t)$
T7’	$D_{n,t} = -\phi_{n,t}\left(R_{n,t} K_{n,t} + W_{n,t} L^e_{n,t}\right) + \frac{N_{n,t}}{\sum_{n=1}^{N} N_{n,t}} \sum_{n=1}^{N} \phi_{n,t}\left(R_{n,t} K_{n,t} + W_{n,t} L^e_{n,t}\right)$	$\forall(n,t)$

E.1.II. Financial market

The financial market received deposits of $a_{n,g,t} P_{n,I,t}$ from individuals, and must repay those individuals an amount $a_{n,g,t}\left(1+r_{n,t}\right) P_{n,I,t}$. The financial market loaned an amount $K_{n,t} = \sum_{g=2}^{G} N_{n,g-1,t-1} a_{n,g,t}$ to firms to use in production, and from financial intermediate, households receives an amount $\left(1 + \frac{R_{n,t}}{P_{n,I,t}} - \delta\right) P_{n,I,t} K_{n,t}$, where note that we have incorporated the fact that some of the capital depreciates in use, and so the total amount returned to the financial market is smaller by this amount. To clear the market it must be that these amounts are equal

$$\sum_{g=2}^{G} N_{n,g-1,t-1}a_{n,g,t}\left(1+r_{n,t}\right)P_{n,I,t}=\left(1+\frac{R_{n,t}}{P_{n,I,t}}-\delta\right)P_{n,I,t}K_{n,t}$$

and as this is a financial autarky economy, the total assets in the economy must be equal to the total capital stock $K_{n,t}=\sum_{g=2}^{G} N_{n,g-1,t-1}a_{n,g,t}$, What this implies is that

$$r_{n,t}=\frac{R_{n,t}}{P_{n,I,t}}-\delta$$

E.1.III. Age structure

To simplify the notation, I omitted country subscript n here. The term $N_{g,t}$ is the number of households of age g alive at time t. $f_{g,t}$ is the fertility rate of age g households at time t. $s_{g,t}$ is the probability of surviving to age g at time t, given that they were alive at $g-1$. Here, define $s_{G+1,t}=0$ which implies maximum lifespan is G; also define $g\in[1,G]$ and$s_{1,t}=1$ which implies people born with age $g=1$ and with zero death probability.

The implied unconditional probability of surviving g periods up to time t is given by:

$$S_{g,t}=\prod_{k=1}^{g} s_{k,t+k-g}.$$

The unconditional probability of people dying at age g at time t is:

$$D_{g,t}=\begin{cases}0 & g=1\\ \mathrm{S}_{g-1,t-1}(1-s_{g,t})=S_{g-1,t-1}-S_{g,t} & g\in[2,G]\end{cases}.$$

The expected life expectancy for people born at t is:

$$LS_t=\sum_{k=1}^{G} k\cdot D_{k,t+k-1}\quad (1).$$

The demographic process can be describe as:

$$N_{1,t+1}=s_{1,t}\sum_{g=1}^{G} f_{g,t}N_{g,t},$$

$$N_{g+1,t+1}=s_{g+1,t+1}N_{g,t}.$$

or

$$\begin{bmatrix} N_{1,t+1} \\ \vdots \\ N_{g,t+1} \\ \vdots \\ N_{G,t+1} \end{bmatrix} = \begin{bmatrix} s_{1,t+1}f_{1,t} & \cdots & s_{1,t+1}f_{g,t} & \cdots & s_{1,t+1}f_{G,t} \\ s_{2,t+1} & 0 & 0 & \cdots & 0 \\ 0 & s_{g+1,t+1} & 0 & \cdots & 0 \\ 0 & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & s_{G-1,t+1} & \cdots & 0 \\ 0 & 0 & 0 & s_{G,t+1} & 0 \end{bmatrix} \cdot \begin{bmatrix} N_{1,t} \\ \vdots \\ N_{g,t} \\ \vdots \\ N_{G,t} \end{bmatrix}.$$

or $N_t = \Omega_t N_t$. The matrix Ω_t is a function of fertility rates $f_{g,t}$ and survival rates $s_{g,t+1}$. To decompose the effects of longer lifespan on the demographic process, use a different set of survival rates ($s^1_{g,t+1}$) corresponding to a longer lifespan, and define $\Omega^1_t = \Omega(f_{g,t}, s^1_{g,t+1})$ to generate the counterfactual demographic process N^1_t. Thus, N_t represents the original demographic process, N^1_t represents the counterfactual process with a longer lifespan, and $(N^1_t - N_t)$ is the difference between the real and counterfactual processes.

E.1.IV. Other algebra

Now, aggregate individual variables across cohorts, I have:

$$C_{n,t} \equiv \sum_{g=1}^{G} N_{n,g,t}c_{n,g,t} \qquad K_{n,t} \equiv \sum_{g=2}^{G} N_{n,g-1,t-1}a_{n,g,t} \tag{D.1}$$

$$P_{n,C,t}c_{n,g,t} + P_{n,I,t}a_{n,g+1,t+1} = P_{n,I,t}(1+r_{n,t})a_{n,g,t} + W_{n,t}\left(1-\tau^n_L\right)E_{n,t}l_g + ts^D_{n,t} + ts^T_{n,t}$$

$$ts^D_{n,t} \equiv P_{n,I,t}(1+r_{n,t})\frac{\sum_{g=2}^{G}(N_{n,g-1,t-1} - N_{n,g,t})a_{n,g,t}}{N_{n,t}} \tag{D.2}$$

Aggregating individual budget constraints across ages, the budget constraint at the aggregate level is

$$P_{n,C,t}\sum_{g=1}^{G} N_{n,g,t}c_{n,g,t} + P_{n,I,t}\sum_{g=1}^{G} N_{n,g,t}a_{n,g+1,t+1} = P_{n,I,t}(1+r_{n,t})\sum_{g=1}^{G} N_{n,g,t}a_{n,g,t} \tag{D.3}$$

$$+\sum_{g=1}^{G} N_{n,g,t}W_{n,t}\left(1-\tau^n_L\right)E_{n,t}l_g + \sum_{g=1}^{G} N_{n,g,t}ts^D_{n,t} + \sum_{g=1}^{G} N_{n,g,t}ts^T_{n,t} \tag{D.4}$$

$$P_{n,C,t}C_{n,t} + P_{n,I,t}K_{n,t+1} = L_{n,t}E_{n,t}W_{n,t} + (1+r_{n,t})P_{n,I,t}\sum_{g=1}^{G} N_{n,g,t}a_{n,g,t} \tag{D.5}$$

$$+(1+r_{n,t})P_{n,I,t}\sum_{g=2}^{G}\left(N_{n,g-1,t-1}-N_{n,g,t}\right)a_{n,g,t}+D_{n,t} \tag{D.6}$$

$$=(1+r_{n,t})P_{n,I,t}K_{n,t}+L_{n,t}E_{n,t}W_{n,t}+D_{n,t} \tag{D.7}$$

$$=\left(1+\frac{R_{n,t}}{P_{n,t}}-\delta\right)P_{n,I,t}K_{n,t}+L_{n,t}E_{n,t}W_{n,t}+D_{n,t} \tag{D.8}$$

$$=\left(1+\frac{R_{n,t}}{P_{n,t}}-\delta\right)P_{n,I,t}K_{n,t}+L^{e}_{n,t}W_{n,t}+D_{n,t} \tag{D.9}$$

Aggregate investment can be calculated from

$$I_t \equiv K_{t+1}-(1-\delta)K_t \tag{D.10}$$

Which implies:

$$P_{n,C,t}C_{n,t}+P_{n,I,t}I_{n,t}=(r_{n,t}+\delta)P_{n,I,t}K_{n,t}+L^{e}_{n,t}W_{n,t}+D_{n,t} \tag{D.11}$$

$$=R_{n,t}K_{n,t}+L^{E}_{n,t}W_{n,t}+D_{n,t} \tag{D.12}$$

$$P_{n,C,t}C_{n,t}+P_{n,I,t}I_{n,t}=R_{n,t}K_{n,t}+L^{e}_{n,t}W_{n,t}+D_{n,t} \tag{D.13}$$

E.2. Computational algorithm

E.2.I. Computing the Steady-state

TABLE E.3

STEADY-STATE CONDITIONS

g_n	$N_{n,g,t+1} = (1+g_n)\, N_{n,g,t}$	$\forall n, t \in [T-1, \infty)$
g_{λ^j}	$\lambda^j_{n,t+1} = (1+g_{\lambda^j})\, \lambda^j_{n,t};\ (1+g_{\lambda^j}) = (1+g_n)^{\frac{\varphi^j}{(1-\rho)}}\ ;\ 1+g_{A^j} \equiv (1+g_{\lambda^j})^{1/\theta}$	$\forall n, j, t \in [T, \infty)$
g_ω	$X \in [\frac{W_{n,t}}{P_{n,C,t}}, \frac{ts^T_{n,t}}{P_{C,n,t}}, \frac{ts^D_{n,t}}{P_{n,C,t}}, a_{n,g,t}, c_{n,g,t}]\ ;\ X_{t+1} = (1+g_\omega)\, X_t;\ 1+g_\omega = (1+g_{A^j})^{\frac{1}{\beta^j\gamma^j}} = (1+g_{\lambda^j})^{\frac{1}{\beta^j\gamma^j\theta}}$	$\forall n, t \in [T, \infty)$
$g_{rc^j_n}$	$X \in [\frac{c^j_{n,t}}{P^j_{n,t}}]\ ;\ X_{t+1} = \left(1+g_{rc^j_n}\right) X_t;\ 1+g_{rc^j_n} = (1+g_\omega)^{\beta^j\gamma^j} = (1+g_{\lambda^j})^{1/\theta}$	$\forall n, t \in [T, \infty)$
g_K	$X \in [C_{n,t}, C^j_{n,t}, I_{n,t}, I^j_{n,t}, K_{n,t}, Y^j_{n,t}, \frac{X^j_{n,t}}{P^j_{n,t}}, \frac{D_{n,t}}{P^j_{n,t}}, \frac{D_{n,t}}{P_{n,Ct}}, \frac{D_{n,t}}{P_{n,I,t}}]\ ;\ X_{t+1} = (1+g_K)\, X_t;\ 1+g_K = (1+g_\omega)(1+g_n)$	$\forall n, j, t \in [T, \infty)$
	$1+g_\omega = (1+g_n)^{\frac{\varphi^j}{\theta\beta^j\gamma^j(1-\rho)}};\ \varphi^j/\varphi^k = \beta^j\gamma^j/\beta^k\gamma^k;\ \varphi^j = \theta(1-\rho)\,\beta^j\gamma^j \frac{\log(1+g_\omega)}{\log(1+g_n)};$	$\forall n, j$
F0	$\lambda^j_{n,T+1} - \lambda^j_{n,T} = {N_{n,T}}^{\varphi^j} \left(\lambda^j_{n,T}\right)^\rho \left[\sum_g \eta^j_g \bar{N}_{n,g,T}\right]^{\varphi^j} \Gamma(1-\rho)$	$\forall(n)$
H1	$N_{n,T} \equiv \sum_{g=1}^{G} N_{n,g,T};\ \bar{L}_{n,T} \equiv \sum_{g=G_0+1}^{G} N_{n,g,T};\ L_{n,T} = \left(1-\tau^L_{n,T}\right) \sum_{g=G_0+1}^{G_1} N_{n,g,T} l_g;\ L^e_{n,T} = E_{n,T} L_{n,T}$	$\forall(n)$
H2	$P_{n,C,T} c_{n,g,T} + P_{n,I,T}(1+g_\omega)\, a_{n,g+1,T} = P_{n,I,T}(1+r_{n,T})\, a_{n,g,T} + W_{n,T}\left(1-\tau^L_{n,T}\right) E_{n,T} l_g + tr^D_{n,T} + tr^T_{n,T}\ ;\ g \in [1, G]$	$\forall(n)$
H3	$a_{1,T} = a_{G+1,T} = 0;\ c_{n,g,T} > 0, \{c_{n,g,T}\}_{g=1}^{G};\ \{a_{n,g+1,T}\}_{g=1}^{G-1}$	$\forall(n)$
H4	$tr^T_{n,T} \equiv \frac{D_{n,T}}{N_{n,T}};\ tr^D_{n,T} = P_{n,I,T}(1+r_{n,T}) \sum_{g=2}^{G} \left(\frac{\overline{N}_{n,g-1,T}}{1+g_n} - \overline{N}_{n,g,T}\right) a_{n,g,T}$	$\forall(n)$
H4'	$tr^D_{n,T} = tr^{D,1}_{n,T} + tr^{D,2}_{n,T} = P_{n,I,T}(1-\delta) \sum_{g=2}^{G} \left(\frac{\overline{N}_{n,g-1,T}}{1+g_n} - \overline{N}_{n,g,T}\right) a_{n,g,T} + P_{n,I,T} \left(\frac{R_{n,T}}{P_{n,I,T}}\right) \sum_{g=2}^{G} \left(\frac{\overline{N}_{n,g-1,T}}{1+g_n} - \overline{N}_{n,g,T}\right) a_{n,g,T}$	$\forall(n)$
H4"	$P_{n,C,T} c_{n,g,T} + P_{n,I,T} i_{n,g,T} = R_{n,T} a_{n,g,T} + W_{n,T}\left(1-\tau^L_{n,T}\right) E_{n,T} l_g + tr^{D,2}_{n,T} + tr^T_{n,T}$	$\forall(n)$
H4"'	$P_{n,I,T} i_{n,g,T} = P_{n,I,T}(1+g_\omega)\, a_{n,g+1,T} - \left[P_{n,I,T}(1-\delta)\, a_{n,g,T} + tr^{D,1}_{n,T}\right]$	$\forall(n)$
H5	$(1+g_\omega)\, c_{n,g+1,T} = \left[(\beta s_{n,g+1,T}) \left(\frac{\psi_{n,g+1,T+1}}{\psi_{n,g,T}}\right)(1+r_{n,T})\right]^\sigma c_{n,g,T};\ \forall\ g \in [1, G-1]$	$\forall(n)$
H6	$C_{n,T} \equiv \sum_{g=1}^{G} N_{n,g,T} c_{n,g,T};\ K_{n,T} \equiv \sum_{g=2}^{G} \frac{N_{n,g-1,T}}{1+g_n} a_{n,g,T}$	$\forall(n)$
H7	$C_{n,T} \equiv \prod_{j=1}^{J} {C^j_{n,T}}^{\alpha^j_C};\ I_{n,T} \equiv \prod_{j=1}^{J} {I^j_{n,T}}^{\alpha^j_I};\ P_{n,I,T} = \prod_{j=1}^{J} \left[\frac{P^j_{n,T}}{\alpha^j_I}\right]^{\alpha^j_I};\ P_{n,C,T} = \prod_{j=1}^{J} \left[\frac{P^j_{n,T}}{\alpha^j_C}\right]^{\alpha^j_C}$	$\forall(n)$
H8	$P^j_{n,T} I^j_{n,T} = \alpha^j_{I,n} P_{n,I,T} I_{n,T};\ P^j_{n,T} C^j_{n,T} = \alpha^j_{C,n} P_{n,C,T} C_{n,T}$	$\forall(n,j)$
F1	$W_{n,T} L^e_{n,T} = \sum_{j=1}^{J} \beta^j\gamma^j \sum_{i=1}^{N} \pi^j_{in,T} X^j_{i,T};\ R_{n,T} K_{n,T} = \sum_{j=1}^{J} \left(1-\beta^j\right) \gamma^j \sum_{i=1}^{N} \pi^j_{in,T} X^j_{i,T}$	$\forall(n)$
F2	$r_{n,T} = \frac{R_{n,T}}{P_{n,I,T}} - \delta$	$\forall(n)$
T1	$c^j_{n,T} \equiv \Upsilon^j \left[(W_{n,T})^{\beta^j} (R_{n,T})^{1-\beta^j}\right]^{\gamma^j} \prod_{k=1}^{J} {P^k_{n,T}}^{\gamma^{k,j}};\ \Upsilon^j \equiv \gamma^j {\beta^j}^{-\gamma^j\beta^j} \gamma^j \left(1-\beta^j\right)^{-\gamma^j\left(1-\beta^j\right)} \prod_{k=1}^{J} {\gamma^{k,j}}^{-\gamma^{k,j}}$	$\forall(n,j)$
T2	$P^j_{n,T} = A \cdot \left[\sum_{i=1}^{N} \lambda^j_{i,T} \left(\kappa^j_{ni,T} c^j_{i,T}\right)^{-\theta}\right]^{-\frac{1}{\theta}}\ ;\ A \equiv \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$	$\forall(n,j)$
T3	$\pi^j_{ni,T} \equiv \frac{X^j_{ni,T}}{\sum_{i=1}^{N} X^j_{ni,T}} = \frac{\lambda^j_{i,t}\left(c^j_{i,t}\kappa^j_{ni,T}\right)^{-\theta}}{\sum_{m=1}^{N} \lambda^j_{m,T}\left(c^j_{m,T}\kappa^j_{nm,T}\right)^{-\theta}} = \lambda^j_{i,T} \left(\frac{A c^j_{i,T}\kappa^j_{ni,T}}{P^j_{n,T}}\right)^{-\theta}$	$\forall(n,i,j)$
T4	$P_{C,n,T} C_{n,T} + P_{I,n,T} I_{n,T} = R_{n,T} K_{n,T} + W_{n,T} E_{n,T} L_{n,T} + D_{n,T} = R_{n,T} K_{n,T} + W_{n,T} L^e_{n,T} + D_{n,T} \equiv IN_{n,T}$	$\forall(n)$
T4'	$P_{n,C,T} C_{n,T} + P_{n,I,T}(1+g_K)\, K_{n,T} = \left(1 + \frac{R_{n,T}}{P_{n,I,T}} - \delta\right) P_{n,I,T} K_{n,T} + W_{n,T} L^e_{n,T} + D_{n,T}$	$\forall(n)$
T5	$(1+g_K)\, K_{n,T} = K_{n,T+1} = I_{n,T} + (1-\delta)\, K_{n,T};\ (g_K+\delta)\, K_{n,T} = I_{n,T}$	$\forall(n)$
T6	$\sum_{j=1}^{J}\sum_{i=1}^{N} X^j_{in,T} - \sum_{j=1}^{J}\sum_{i=1}^{N} X^j_{ni,T} = NX_{n,T} = -D_{n,T}$	$\forall(n,j)$
T6'	$X^j_{n,T} = \alpha^j_C P_{C,n,T} C_{n,T} + \alpha^j_I P_{I,n,T} I_{n,T} + \sum_{k=1}^{J} \gamma^{j,k} \left(\sum_{i=1}^{N} X^k_{in,T}\right)$	$\forall(n,j)$
T7	$D_{n,T} = -\phi_{n,T}\left(R_{n,T} K_{n,T} + W_{n,T} L^e_{n,T}\right) + N_{n,T} T^P_T;\ T^P_T = \frac{\sum_{n=1}^{N} \phi_{n,T}\left(R_{n,T} K_{n,T} + W_{n,T} L^e_{n,T}\right)}{\sum_{n=1}^{N} N_{n,T}}$	$\forall(n)$
T7'	$D_{n,T} = -\phi_{n,T}\left(R_{n,T} K_{n,T} + W_{n,T} L^e_{n,T}\right) + \frac{N_{n,T}}{\sum_{n=1}^{N} N_{n,T}} \sum_{n=1}^{N} \phi_{n,T}\left(R_{n,T} K_{n,T} + W_{n,T} L^e_{n,T}\right)$	$\forall(n)$

Algorithm 3 Open Economy Steady State

1: Guess a vector of capital stocks $K^i = \{K_1, \dots, K_N\}^i$. Then calculate investment $I^i = \{I_1, \dots, I_N\}^i$, through T5: $I_n = (g_K + \delta)K_n$, $\forall n$.
2: Solve the multi-sector (capital and labor as inputs with different share across sectors) trade model under fixed world GDP, through T1 to T7 and F1, F2, H1, H8, get $\{\pi_{ni}^j, P_n^j, P_n, I_n, P_{n,C}, R_n, W_n^j\}$, $\forall n, i$.
3: Solve $(G-1)$ Euler equations for each Country n, through H2, H3, H4, H5, get a vector of capital stocks
$$\{a_{n,1}^1 = a_{n,G+1}^1 = 0,\ \{a_{n,g+1}^1\}_{g=1}^{G-1}\}$$
4: Calculate $K_n^{1'} = \sum_{g=2}^{G} \frac{N_{n,g-1}}{1+g_n} a_{n,g}^1$, through H6 for each Country n.
5: Check error term, if $\|K^i - K^{i'}\| < \epsilon$, stop. Else, go to next step.
6: Go back to step 2 with updated new guess K^{i+1}:
$$K^{i+1} = \zeta K^{i'} + (1-\zeta)K^i \quad \text{where } \zeta \in (0,1)$$

E.2.I.1 Steady-state

E.2.I.2 Detail for Step 3 :

$$\frac{(1+g_\omega)c_{n,g+1,T}}{c_{n,g,T}} = \left[\beta \left(\frac{s_{n,g+1,T+1}\psi_{n,g+1,T+1}}{s_{n,g,T}\psi_{n,g,T}}\right)\left(1 + \frac{R_{n,T}}{P_{n,I,T}} - \delta\right)\right]^\sigma$$
$$\frac{(1+g_\omega)c_{n,g+1}}{c_{n,g}} = \left[\beta \left(\frac{s_{n,g+1}\psi_{n,g+1}}{s_{n,g}\psi_{n,g}}\right)(1+r_n)\right]^\sigma = [\beta S_{n,g+1}\Theta_n]^\sigma$$

Define $\Theta_n \equiv 1 + \frac{R_n}{P_{n,I}} - \delta = 1 + r_n$, $G_\omega \equiv 1 + g_\omega$, $S_{n,g+1} \equiv s_{n,g+1}\frac{\psi_{n,g+1}}{\psi_{n,g}}$.
Thus

$$P_{n,C}c_{n,g} = G_\omega \left(\beta S_{n,g+1}\Theta_n\right)^{-\sigma} P_{n,C}c_{n,g+1}$$

Define $\Lambda_{n,g+1} \equiv G_\omega \left(\beta S_{n,g+1}\Theta_n\right)^{-\sigma}$, thus

$$P_{n,C}c_{n,g} = G_\omega \left(\beta S_{n,g+1}\Theta_n\right)^{-\sigma} P_{n,C}c_{n,g+1} = \Lambda_{n,g+1}P_{n,C}c_{n,g+1}$$

From

$$P_{n,C}c_{n,g} = P_{n,I}(1+r_n)a_{n,g} - P_{n,I}(1+g_\omega)a_{n,g+1} + W_n\left(1-\tau_L^n\right)E_n l_g + ts_n^D + ts_n^T$$

and

$$P_{n,C}c_{n,g+1} = P_{n,I}(1+r_n)a_{n,g+1} - P_{n,I}(1+g_\omega)a_{n,g+2} + W_n\left(1-\tau_L^n\right)E_n l_{g+1} + ts_n^D + ts_n^T$$

I have

$$P_{n,I}\Theta_n a_{n,g} - P_{n,I}G_\omega a_{n,g+1} + W_n\left(1-\tau_L^n\right)E_n l_g + ts_n^D + ts_n^T$$
$$= \Lambda_{n,g+1}\left[P_{n,I}\Theta_n a_{n,g+1} - P_{n,I}G_\omega a_{n,g+2} + W_n\left(1-\tau_L^n\right)E_n l_{g+1} + ts_n^D + ts_n^T\right]$$

or

$$P_{n,I}\Theta_n a_{n,g} - \left[\Lambda_{n,g+1}\Theta_n + G_\omega\right]P_{n,I}a_{n,g+1} + \Lambda_{n,g+1}G_\omega P_{n,I}a_{n,g+2}$$
$$= W_n\left(1-\tau_L^n\right)E_n\left[\Lambda_{n,g+1}l_{g+1} - l_g\right] + ts_n^T\left[\Lambda_{n,g+1}-1\right] + ts_n^D\left[\Lambda_{n,g+1}-1\right] \tag{D.14}$$

where

$$ts_n^D \equiv P_{n,I,T}(1+r_{n,T})\sum_{g=2}^{G}\left(\frac{\bar{N}_{n,g-1,T}}{1+g_n} - \bar{N}_{n,g,T}\right)a_{n,g,T}$$
$$= P_{n,I}\Theta_n\sum_{g=2}^{G}\left(\frac{\bar{N}_{n,g-1}}{1+g_n} - \bar{N}_{n,g}\right)a_{n,g}$$
$$= P_{n,I}\Theta_n \odot \left\{\bar{N}_{n,g-1}(1:G-1,1)./(1+g_n) - \bar{N}_{n,g}(2:G,1)\right\}' \times \bar{a}_n$$

and

$$ts_n^T \equiv \frac{D_{n,T}}{N_{n,T}};\quad D_n = -\phi_n\left(R_nK_n + W_nL_n^e\right) + \frac{N_n}{\sum_{n=1}^{N}N_n}\sum_{n=1}^{N}\phi_n\left(R_nK_n + W_nL_n^e\right)$$

Define

$$\vec{a}_n = \begin{bmatrix} a_{n,2} \\ a_{n,3} \\ \vdots \\ a_{n,G-1} \\ a_{n,G}\end{bmatrix}_{(G-1)\times 1};\quad \vec{l} = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_{G-2} \\ l_{G-1}\end{bmatrix}_{(G-1)\times 1};\quad \vec{N}_n = \begin{bmatrix} \bar{N}_{n,1} \\ \bar{N}_{n,2} \\ \vdots \\ \bar{N}_{n,G-1} \\ \bar{N}_{n,G}\end{bmatrix}_{(G)\times 1};\quad \Lambda_n = \begin{bmatrix} \Lambda_{n,2} \\ \Lambda_{n,3} \\ \vdots \\ \Lambda_{n,G-1} \\ \Lambda_{n,G}\end{bmatrix}_{(G-1)\times 1}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}_{(G-1)} = \begin{bmatrix} \text{zeros}(1,G-2) & 0 \\ \text{eye}(G-2) & \text{zeros}(G-2,1)\end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{(G-1)} \quad ; \quad I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{(G-1)} = \text{eye}(G-1)$$

Equation (D.14)

$$P_{n,I}\Theta_n a_{n,g} - [\Lambda_{n,g+1}\Theta_n + G_\omega]P_{n,I}a_{n,g+1} + \Lambda_{n,g+1}G_\omega P_{n,I}a_{n,g+2} - ts_n^D[\Lambda_{n,g+1} - 1]$$

$$= W_n(1-\tau_n^L)E_n[\Lambda_{n,g+1}l_{g+1} - l_g] + ts_n^T[\Lambda_{n,g+1} - 1]; \quad \forall g \in [1, G-1]$$

can be written in matrix form as

$$\vec{a}_n = \text{LeftMatrix}^{-1} \cdot \text{RightVector}$$

where

$$\text{LeftMatrix} = [P_{n,I}\Theta_n \odot A] - [(G_\omega + \Theta_n \odot \Lambda_n)P_{n,I}] + [\Lambda_n \odot G_\omega P_{n,I} \odot A'] - [\Lambda_n - 1] \odot P_{n,I}\Theta_n \odot T$$

$$\text{RightVector} = W_n(1-\tau_n^L)E_n\left[\Lambda_n \odot A' - I\right] \times \vec{l} + [\Lambda_n - 1]ts_n^T \odot \text{ones}(G-1,1)$$

$$ts_n^D = P_{n,I}\Theta_n \sum_{g=2}^{G}\left(\frac{\bar{N}_{n,g-1}}{1+g_n} - \bar{N}_{n,g}\right)a_{n,g}$$

$$= P_{n,I}\Theta_n \odot \left\{\bar{N}_{n,g-1}(1:G-1,1)./(1+g_n) - \bar{N}_{n,g}(2:G,1)\right\}' \times \bar{a}_n$$

$$= P_{n,I}\Theta_n \odot \left[\text{ones}(G-1,1) \times \left(\frac{\bar{N}_{n,g-1}(1:G-1,1)}{1+g_n} - \bar{N}_{n,g}(2:G,1)\right)^T \times \bar{a}_n\right]$$

$$= P_{n,I}\Theta_n \odot T \times \bar{a}_n$$

and

$$T = \text{ones}(G-1,1) \times \left(\bar{N}_{n,g-1}(1:G-1,1)./(1+g_n) - \bar{N}_{n,g}(2:G,1)\right)^T$$

E.2.II. Computing the transition path

TABLE E.4
Dynamic equilibrium conditions

I1	$\lambda^j_{n,t+1} - \lambda^j_{n,t} = \left(\lambda^j_{n,t}\right)^{\rho} \left(\sum_g \eta^j_g N_{n,g,t}\right)^{\varphi^j} \Gamma\left(1-\rho\right) = N_{n,t}{}^{\varphi^j} \left(\lambda^j_{n,t}\right)^{\rho} \left(\sum_g \eta^j_g \bar{N}_{n,g,t}\right)^{\varphi^j} \Gamma\left(1-\rho\right)$	$\forall(n,t)$
H1	$N_{n,t} \equiv \sum_{g=1}^{G} N_{n,g,t};\ \bar{L}_{n,t} \equiv \sum_{g=G_0+1}^{G} N_{n,g,t};\ L_{n,t} = \left(1-\tau^L_{n,t}\right) \sum_{g=G_0+1}^{G_1} N_{n,g,t} l_g = \left(1-\tau^L_{n,t}\right) \sum_{g=1}^{G} N_{n,g,t} l_g;\ L^e_{n,t} = E_{n,t} L_{n,t}$	$\forall(n,t)$
H2	$P_{n,C,t} c_{n,g,t} + P_{n,I,t} a_{n,g+1,t+1} = P_{n,I,t}\left(1+r_{n,t}\right) a_{n,g,t} + W_{n,t}\left(1-\tau^L_{n,t}\right) E_{n,t} l_g + tr^D_{n,t} + tr^T_{n,t}\ ;\ g \in [1,G]$	$\forall(n,t)$
H3	$a_{1,t} = a_{G+1,t} = 0;\ c_{n,g,t} > 0,\ \{c_{n,g,t+g-1}\}_{g=1}^{G};\ \{a_{n,g+1,t+g}\}_{g=1}^{G-1}$	$\forall(n,t)$
H4	$tr^T_{n,t} \equiv \frac{D_{n,t}}{N_{n,t}};\ tr^D_{n,t} \equiv P_{n,I,t}\left(1+r_{n,t}\right) \frac{\sum_{g=2}^{G}\left(N_{n,g-1,t-1} - N_{n,g,t}\right) a_{n,g,t}}{N_{n,t}}$	$\forall(n,t)$
H4'	$tr^D_{n,t} = tr^{D,1}_{n,t} + tr^{D,2}_{n,t} = P_{n,I,t}\left(1-\delta\right) \sum_{g=2}^{G} \left(\frac{N_{n,g-1,t-1} - N_{n,g,t}}{N_{n,t}}\right) a_{n,g,t} + P_{n,I,t}\left(\frac{R_{n,t}}{P_{n,I,t}}\right) \sum_{g=2}^{G} \left(\frac{N_{n,g-1,t-1} - N_{n,g,t}}{N_{n,t}}\right) a_{n,g,t}$	$\forall(n)$
H4"	$P_{n,C,t} c_{n,g,t} + P_{n,I,t} i_{n,g,t} = R_{n,t} a_{n,g,t} + W_{n,t}\left(1-\tau^L_{n,t}\right) E_{n,t} l_g + tr^{D,2}_{n,t} + tr^T_{n,t}$	$\forall(n)$
H4"'	$P_{n,I,t} i_{n,g,t} = P_{n,I,t} a_{n,g+1,t+1} - \left[P_{n,I,t}\left(1-\delta\right) a_{n,g,t} + tr^{D,1}_{n,t}\right]$	$\forall(n)$
H5	$\frac{c_{n,g+1,t+1}}{c_{n,g,t}} = \left[\left(\beta s_{n,g+1,t+1}\right)\left(\frac{\psi_{n,t+1}}{\psi_{n,t}}\right) \frac{\frac{P_{n,I,t+1}}{P_{n,C,t+1}}}{\frac{P_{n,I,t}}{P_{n,C,t}}} \left(1+r_{n,t+1}\right)\right]^{\sigma};\ \forall\ g \in [1,G-1]$	$\forall(n,t)$
H6	$C_{n,t} \equiv \sum_{g=1}^{G} N_{n,g,t} c_{n,g,t};\ K_{n,t} \equiv \sum_{g=2}^{G} N_{n,g-1,t-1} a_{n,g,t}$	$\forall(n,t)$
H7	$C_{n,t} \equiv \prod_{j=1}^{J} C^j_{n,t}{}^{\alpha^j_{n,C,t}};\ I_{n,t} \equiv \prod_{j=1}^{J} I^j_{n,t}{}^{\alpha^j_{n,I,t}};\ P_{n,I,t} = \prod_{j=1}^{J}\left[\frac{P^j_{n,t}}{\alpha^j_{I,n}}\right]^{\alpha^j_{I,n}};\ P_{n,C,t} = \prod_{j=1}^{J}\left[\frac{P^j_{n,t}}{\alpha^j_{C,n}}\right]^{\alpha^j_{C,n}}$	$\forall(n,t)$
H8	$P^j_{n,t} I^j_{n,t} = \alpha^j_{I,n} P_{n,I,t} I_{n,t};\ P^j_{n,t} C^j_{n,t} = \alpha^j_{C,n} P_{n,C,t} C_{n,t}$	$\forall(n,j,t)$
F1	$W_{n,t} L^e_{n,t} = \sum_{j=1}^{J} \beta^j_n \gamma^j_n \sum_{i=1}^{N} \pi^j_{in,t} X^j_{i,t};\ R_{n,t} K_{n,t} = \sum_{j=1}^{J}\left(1-\beta^j_n\right) \gamma^j_n \sum_{i=1}^{N} \pi^j_{in,t} X^j_{i,t}$	$\forall(n,t)$
F2	$r_{n,t} = \frac{R_{n,t}}{P_{n,I,t}} - \delta$	$\forall(n,t)$
T1	$c^j_{n,t} \equiv \Upsilon^j_n \left[\left(W_{n,t}\right)^{\beta^j_n} \left(R_{n,t}\right)^{1-\beta^j_n}\right]^{\gamma^j_n} \prod_{k=1}^{J} P^k_{n,t}{}^{\gamma^{k,j}_n}$ where $\Upsilon^j_n \equiv \gamma^j_n \beta^j_n{}^{-\gamma^j_n \beta^j_n} \gamma^j_n \left(1-\beta^j_n\right)^{-\gamma^j_n\left(1-\beta^j_n\right)} \prod_{k=1}^{J} \gamma^{k,j}_n{}^{-\gamma^{k,j}_n}$	$\forall(n,j,t)$
T2	$P^j_{n,t} = A^j \cdot \left[\sum_{i=1}^{N} \lambda^j_{i,t} \left(\kappa^j_{ni,t} c^j_{i,t}\right)^{-\theta}\right]^{-\frac{1}{\theta}}$ where $A^j \equiv \Gamma\left(\frac{1+\theta-\sigma}{\theta}\right)^{\frac{1}{(1-\sigma)}}$	$\forall(n,j,t)$
T3	$\pi^j_{ni,t} \equiv \frac{X^j_{ni,t}}{\sum_{i=1}^{N} X^j_{ni,t}} = \frac{\lambda^j_{i,t}\left(c^j_{i,t}\kappa^j_{ni,t}\right)^{-\theta}}{\sum_{m=1}^{N} \lambda^j_{m,t}\left(c^j_{m,t}\kappa^j_{nm,t}\right)^{-\theta}} = \lambda^j_{i,t}\left(\frac{A^j c^j_{i,t} \kappa^j_{ni,t}}{P^j_{n,t}}\right)^{-\theta}$	$\forall(n,i,j,t)$
T4	$P_{n,C,t} C_{n,t} + P_{n,I,t} I_{n,t} = R_{n,t} K_{n,t} + W_{n,t} E_{n,t} L_{n,t} + D_{n,t} = R_{n,t} K_{n,t} + W_{n,t} L^e_{n,t} + D_{n,t} \equiv IN_{n,t}$	$\forall(n,t)$
T4'	$P_{n,C,t} C_{n,t} + P_{n,I,t} K_{n,t+1} = \left(1 + \frac{R_{n,t}}{P_{n,I,t}} - \delta\right) P_{n,I,t} K_{n,t} + W_{n,t} L^e_{n,t} + D_{n,t}$	$\forall(n,t)$
T5	$K_{n,t+1} = I_{n,t} + \left(1-\delta\right) K_{n,t}$	$\forall(n,t)$
T6	$\sum_{j=1}^{J}\sum_{i=1}^{N} X^j_{in,t} - \sum_{j=1}^{J}\sum_{i=1}^{N} X^j_{ni,t} = NX_{n,t} = -D_{n,t}$	$\forall(n,j,t)$
T6'	$X^j_{n,t} = \alpha^j_{C,n} P_{C,n,t} C_{n,t} + \alpha^j_{I,n} P_{I,n,t} I_{n,t} + \sum_{k=1}^{J} \gamma^{j,k}_n \left(\sum_{i=1}^{N} X^k_{in,t}\right)$	$\forall(n,j,t)$
T7	$D_{n,t} = -\phi_{n,t}\left(R_{n,t} K_{n,t} + W_{n,t} L^e_{n,t}\right) + N_{n,t} T^P_t\ ;\ T^P_t = \frac{\sum_{n=1}^{N} \phi_{n,t}\left(R_{n,t} K_{n,t} + W_{n,t} L^e_{n,t}\right)}{\sum_{n=1}^{N} N_{n,t}}$	$\forall(n,t)$
T7'	$D_{n,t} = -\phi_{n,t}\left(R_{n,t} K_{n,t} + W_{n,t} L^e_{n,t}\right) + \frac{N_{n,t}}{\sum_{n=1}^{N} N_{n,t}} \sum_{n=1}^{N} \phi_{n,t}\left(R_{n,t} K_{n,t} + W_{n,t} L^e_{n,t}\right)$	$\forall(n,t)$

E.2.II.1 Detail for Step 4 :

Algorithm 4 Open Economy Dynamic Equilibrium

1: Guess $\overline{K}^1_{t=2,\ldots,T} = \{K_1, \ldots, K_N\}^i$. The economy reaches a new steady state at $T+1$. The population dynamics reach the steady state at some time before $T+1$.

2: Get $\overline{I}^1_{t=1,\ldots,T} = \{I_1, \ldots, I_N\}^i_{t=1,\ldots,T}$, where $I^i_{n,t} = K^i_{n,t+1} - (1-\delta)K^i_{n,t}, \forall n \in \mathcal{N}$.

3: Solve the multi-sector (capital and labor as inputs with different shares across sectors) EK trade model under fixed world GDP, get $\{\pi^{ij}_n, P^j_n, I_n, R_n, C_n, W_n\}^1$.

4: Guess $\overrightarrow{tr}^{D,i,j}$, repeat the following until $\overrightarrow{tr}^{D,i,j}$ converges at some j_0, and set $\overrightarrow{tr}^{D,i}_{j_0} = \overrightarrow{tr}^{D,i,j}$:

(a) Solve $\forall G_0 \in \{1, \ldots, (G-1)\}$ Euler equations for each Country $n \in \mathcal{N}$, each beginning cohort $g_0 \in \{1, \ldots, (G-1)\}$, and begin year $t_0 \in \{1, \ldots, T\}$:

$$\{a_{n,g_0,t_0}, a_{n,g_0+G_0,t_0+G_0}, \{c_{n,g,t+g-1}\}_{g=g_0}^{g_0+G_0-1}, \{a_{n,g,t+1+g}\}_{g=g_0}^{g_0+G_0-1}\}$$

(b) For j, check:

$$\left\| \overrightarrow{tr}^{D,i,j} - \overrightarrow{tr}^{D,i,j'} \right\| < \varepsilon \cdot 1E-2$$

If it holds, return the value of $\overrightarrow{tr}^{D,i,j}$ and go to step 5; if not, update $\overrightarrow{tr}^{D,i,j+1}$ with:

$$\overrightarrow{tr}^{D,i,j+1} = \zeta' \overrightarrow{tr}^{D,i,j} + (1-\zeta')\overrightarrow{tr}^{D,i,j'}, \quad \zeta' \in (0,1)$$

Repeat from step 4.(a) with $j = j+1$ until it holds.

5: For i, calculate $\overline{K}^i_n$ for each Country $n \in \mathcal{N}$. Check $\left\| \overline{K}^i - \overline{K}^{i'} \right\| < \varepsilon$

6: **if** it holds **then**

7: Return the value of $\overline{K}^i$ and stop.

8: **else**

9: Update $\overline{K}^{i+1}$ with:

$$\overline{K}^{i+1} = \zeta \overline{K}^{i'} + (1-\zeta)\overline{K}^i, \quad \zeta \in (0,1)$$

10: Repeat from step 2 with $i = i+1$ until it holds.

11: **end if**

$$\frac{c_{n,g+1,t+1}}{c_{n,g,t}} = \left[\beta\left(s_{n,g+1,t+1}\frac{\psi_{n,t+1}}{\psi_{n,t}}\frac{P_{n,I,t+1}}{P_{n,C,t+1}}\left(1+\frac{R_{n,t+1}}{P_{n,I,t+1}}-\delta\right)\right)\right]^{\sigma} = \left[\beta S_{n,g+1,t+1}\Theta_{n,t+1}\frac{P_{n,IC,t+1}}{P_{n,IC,t}}\right]^{\sigma}$$

Define

$$S_{n,g+1,t+1} \equiv s_{n,g+1,t+1}\frac{\psi_{n,t+1}}{\psi_{n,t}}, \quad \Theta_{n,t+1} \equiv 1+\frac{R_{n,t+1}}{P_{n,I,t+1}}-\delta = 1+r_{n,t+1}, \quad P_{n,IC,t} \equiv \frac{P_{n,I,t}}{P_{n,C,t}}.$$

Thus

$$\frac{c_{n,g+1,t+1}}{c_{n,g,t}} = \left[\beta S_{n,g+1,t+1}\Theta_{n,t+1}\frac{P_{n,IC,t+1}}{P_{n,IC,t}}\right]^{\sigma},$$

and

$$\frac{P_{n,C,t+1}c_{n,g+1,t+1}}{P_{n,C,t}c_{n,g,t}} = \left[\beta S_{n,g+1,t+1}\Theta_{n,t+1}\frac{P^{1/\sigma-1}_{n,C,t+1}P_{n,I,t+1}}{P^{1/\sigma-1}_{n,C,t}P_{n,I,t}}\right]^{\sigma} = \left[\beta S_{n,g+1,t+1}\Theta_{n,t+1}\frac{P_{n,C\sigma I,t+1}}{P_{n,C\sigma I,t}}\right]^{\sigma}.$$

$$\frac{P_{n,C,t+1}c_{n,g+1,t+1}}{P_{n,C,t}c_{n,g,t}} = \left[\beta S_{n,g+1,t+1}\Theta_{n,t+1}P_{n,\sigma,t+1}\right]^{\sigma}.$$

Define $P_{n,C\sigma I,t} \equiv P^{1/\sigma-1}_{n,C,t}P_{n,I,t}$, $P_{n,\sigma,t+1} \equiv \frac{P_{n,C\sigma I,t+1}}{P_{n,C\sigma I,t}}$, $\Lambda_{n,g+1,t+1} \equiv \left[\beta S_{n,g+1,t+1}\Theta_{n,t+1}P_{n,\sigma,t+1}\right]^{-\sigma}$. Thus

$$P_{n,C,t}c_{n,g,t} = \left[\beta S_{n,g+1,t+1}\Theta_{n,t+1}P_{n,\sigma,t+1}\right]^{-\sigma}P_{n,C,t+1}c_{n,g+1,t+1} = \Lambda_{n,g+1,t+1}P_{n,C,t+1}c_{n,g+1,t+1}.$$

$$P_{n,C,t}c_{n,g,t} = \Lambda_{n,g+1,t+1}P_{n,C,t+1}c_{n,g+1,t+1}.$$

Substitute PC into the above formula, I have

$$P_{n,I,t}\Theta_{n,t}a_{n,g,t} - P_{n,I,t}a_{n,g+!,t+1} + W_{n,t}\left(1-\tau^L_{n,t}\right)E_{n,t}l_g + ts^D_{n,t} + ts^T_{n,t} =$$

$$\Lambda_{n,g+1,t+1}\left[P_{n,I,t+1}\Theta_{n,t+1}a_{n,g+1,t+1} - P_{n,I,t+1}a_{n,g+2,t+2} + W_{n,t+1}\left(1-\tau^L_{n,t+1}\right)E_{n,t+1}l_{g+1} + ts^D_{n,t+1} + ts^T_{n,t+1}\right].$$

If the above Euler equation is solved from (g,t) to $(g+G_0-1, t+G_0-1)$, which implies G_0 unknown variables under G_0 equations, with the given value of $a_{n,g,t}$ and $a_{n,g+G_0+1,t+G_0+1}$, the first and last equation are:

$$P_{n,I,t}\Theta_{n,t}a_{n,g,t} - \left[P_{n,I,t} + \Lambda_{n,g+1,t+1}P_{n,I,t+1}\Theta_{n,t+1}\right]a_{n,g+1,t+1} + \Lambda_{n,g+1,t+1}P_{n,I,t+1}a_{n,g+2,t+2}$$

$$= \left[\Lambda_{n,g+1,t+1}W_{n,t+1}\left(1-\tau^L_{n,t+1}\right)E_{n,t+1}l_{g+1} - W_{n,t}\left(1-\tau^L_{n,t}\right)E_{n,t}l_g\right] + \left[\Lambda_{n,g+1,t+1}ts^D_{n,t+1} - ts^D_{n,t}\right]$$

$$+\left[\Lambda_{n,g+1,t+1}ts^T_{n,t+1} - ts^T_{n,t}\right],$$

and

$$P_{n,I,t+G_0-1}\Theta_{n,t+G_0-1}a_{n,g+G_0-1,t+G_0-1}-\left[P_{n,I,t+G_0-1}+\Lambda_{n,g+G_0,t+G_0}P_{n,I,t+G_0}\Theta_{n,t+G_0}\right]a_{n,g+G_0,t+G_0}$$

$$+\Lambda_{n,g+G_0,t+G_0}P_{n,I,t+G_0}a_{n,g+G_0+1,t+G_0+1}=$$

$$\left[\Lambda_{n,g+G_0,t+G_0}W_{n,t+G_0}\left(1-\tau^L_{n,t+G_0}\right)E_{n,t+G_0}l_{g+G_0}-W_{n,t+G_0-1}\left(1-\tau^L_{n,t+G_0-1}\right)E_{n,t+G_0-1}l_{g+G_0-1}\right]$$

$$+\left[\Lambda_{n,g+G_0,t+G_0}ts^D_{n,t+G_0}-ts^D_{n,t+G_0-1}\right]+\left[\Lambda_{n,g+G_0,t+G_0}ts^T_{n,t+G_0}-ts^T_{n,t+G_0-1}\right].$$

Define

$$\overrightarrow{a}_n \equiv \begin{bmatrix} a_{n,g+1,t+1} \\ a_{n,g+2,t+2} \\ \vdots \\ a_{n,g+G_0-1,t+G_0-1} \\ a_{n,g+G_0,t+G_0} \end{bmatrix}_{(G_0)\times 1} ; \overrightarrow{a}^*_{n,g,t} \equiv \begin{bmatrix} a_{n,g,t} \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}_{(G_0)\times 1} ; \overrightarrow{a}^{**}_{n,g+G_0+1,t+G_0+1} \equiv \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ a_{n,g+G_0+1,t+G_0+1} \end{bmatrix}_{(G_0)\times 1} ;$$

$$A \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} = \left[\text{zeros}(1,G_0); eyes(G_0-1)zeros(G_0-1,1)\right]; E \equiv eyes(G_0).$$

$$A' \equiv \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} ; \quad \overrightarrow{\Theta P}_n \equiv \begin{bmatrix} \Theta_{n,t}P_{n,I,t} \\ \Theta_{n,t+1}P_{n,I,t+1} \\ \vdots \\ \Theta_{n,t+G_0-2}P_{n,I,t+G_0-2} \\ \Theta_{n,t+G_0-1}P_{n,I,t+G_0-1} \end{bmatrix}_{(G_0)\times 1} ;$$

$$\overrightarrow{\Lambda P}_n \equiv \begin{bmatrix} \Lambda_{n,g+1,t+1}P_{n,I,t+1} \\ \vdots \\ \vdots \\ \Lambda_{n,g+G_0,t+G_0}P_{n,I,t+G_0} \end{bmatrix}_{(G_0)\times 1} ;$$

$$P\Lambda\overrightarrow{\Theta P}_n \equiv \begin{bmatrix} P_{n,I,t} + \Lambda_{n,g+1,t+1}\Theta_{n,t+1}P_{n,I,t+1} \\ \vdots \\ \vdots \\ P_{n,I,t+G_0-1} + \Lambda_{n,g+G_0,t+G_0}\Theta_{n,t+G_0}P_{n,I,t+G_0} \end{bmatrix}_{(G_0)\times 1} \quad ;$$

$$\overrightarrow{l}(g+1:g+G_0)^T \equiv \begin{bmatrix} l_{g+1} \\ l_{g+2} \\ \vdots \\ l_{g+G_0} \end{bmatrix}_{(G_0)\times 1} \quad ; \quad \overrightarrow{l}(g:g+G_0-1)^T \equiv \begin{bmatrix} l_g \\ l_{g+2} \\ \vdots \\ l_{g+G_0-1} \end{bmatrix}_{(G_0)\times 1} \quad ;$$

$$A\overrightarrow{a}_n + E\overrightarrow{a}^*_{n,g,t} = \begin{bmatrix} a_{n,g,t} \\ a_{n,g+1,t+1} \\ \vdots \\ a_{n,g+G_0-2,t+G_0-2} \\ a_{n,g+G_0-1,t+G_0-1} \end{bmatrix}_{(G_0)\times 1} \quad ; \quad A'\overrightarrow{a}_n + E\overrightarrow{a}^*_{n,g+G_0+1,t+G_0+1} = \begin{bmatrix} a_{n,g+2,t+2} \\ a_{n,g+3,t+3} \\ \vdots \\ a_{n,g+G_0,t+G_0} \\ a_{n,g+G_0+1,t+G_0+1} \end{bmatrix}_{(G_0)\times 1} \quad ;$$

$$\Lambda\overrightarrow{W}El_n = \begin{bmatrix} \Lambda_{n,g+1,t+1}W_{n,t+1}\left(1-\tau^L_{n,t+1}\right)E_{n,t+1}l_{g+1} \\ \vdots \\ \vdots \\ \Lambda_{n,g+G_0,t+G_0}W_{n,t+G_0}\left(1-\tau^L_{n,t+G_0}\right)E_{n,t+G_0}l_{g+G_0} \end{bmatrix}_{(G_0)\times 1} \quad ;$$

$$W\overrightarrow{E}l_n = \begin{bmatrix} W_{n,t}\left(1-\tau^L_{n,t}\right)E_{n,t}l_g \\ \vdots \\ \vdots \\ W_{n,t+G_0-1}\left(1-\tau^L_{n,t+G_0-1}\right)E_{n,t+G_0-1}l_{g+G_0-1} \end{bmatrix}_{(G_0)\times 1} \quad ;$$

$$\Lambda ts\overrightarrow{D}_n = \begin{bmatrix} \Lambda_{n,g+1,t+1}ts^D_{n,t+1} - ts^D_{n,t} \\ \vdots \\ \vdots \\ \Lambda_{n,g+G_0,t+G_0}ts^D_{n,t+G_0} - ts^D_{n,t+G_0-1} \end{bmatrix}_{(G_0)\times 1} ; \Lambda ts\overrightarrow{T}_n = \begin{bmatrix} \Lambda_{n,g+1,t+1}ts^T_{n,t+1} - ts^T_{n,t} \\ \vdots \\ \vdots \\ \Lambda_{n,g+G_0,t+G_0}ts^T_{n,t+G_0} - ts^T_{n,t+G_0-1} \end{bmatrix}_{(G_0)\times 1} \quad ;$$

Here, $ts^T_{n,t} \equiv \frac{D_{n,t}}{N_{n,t}}$ and $D_{n,t} = -\phi_{n,t}\left(R_{n,t}K_{n,t} + W_{n,t}L^e_{n,t}\right) + \frac{N_{n,t}}{\sum_N N_{n,t}\sum_{n=1}^N}\phi_{n,t}\left(R_{n,t}K_{n,t} + W_{n,t}L^e_{n,t}\right)$.

$$ts^D_{n,t} \equiv P_{n,I,t}\left(1+\frac{R_{n,t}}{P_{n,I,t}}-\delta\right)\frac{\sum_{g=2}^{G}\left(N_{n,g-1,t-1}-N_{n,g,t}\right)a_{n,g,t}}{N_{n,t}}$$

$$=\frac{P_{n,I,t}}{N_{n,t}}\Theta_{n,t}\left[\left(\overrightarrow{N}_{n,t-1}(1:G-1,1)-\overrightarrow{\tilde{N}}_{n,t}(2:G,1)\right)^T \times \overrightarrow{a}_{n,g,t}(2:G,1)\right].$$

Thus,

$$Leftside \equiv \overrightarrow{\Theta P}_n \circ \left[A\overrightarrow{a}_n + E\overrightarrow{a}^*_{n,g,t}\right] - P\Lambda\overrightarrow{\Theta P}_n \circ \overrightarrow{a}_n + \Lambda\overrightarrow{P}_n \circ \left[A'\overrightarrow{a}_n + E\overrightarrow{a}^*_{n,g+G_0+1,t+G_0+1}\right]$$

$$=\left[\overrightarrow{\Theta P}_n \circ A - P\Lambda\overrightarrow{\Theta P}_n \circ E + \Lambda\overrightarrow{P}_n \circ A'\right]\overrightarrow{a}_n + \left[\overrightarrow{\Theta P}_n \circ E\overrightarrow{a}^*_{n,g,t} + \Lambda\overrightarrow{P}_n \circ E\overrightarrow{a}^*_{n,g+G_0+1,t+G_0+1}\right],$$

$$Matrix \equiv \left[\overrightarrow{\Theta P}_n \circ A - P\Lambda\overrightarrow{\Theta P}_n \circ E + \Lambda\overrightarrow{P}_n \circ A'\right], \quad Res \equiv \left[\overrightarrow{\Theta P}_n \circ E\overrightarrow{a}^*_{n,g,t} + \Lambda\overrightarrow{P}_n \circ E\overrightarrow{a}^*_{n,g+G_0+1,t+G_0+1}\right]$$

$$Rightside \equiv \Lambda\overrightarrow{W}El_n - W\overrightarrow{E}l_n + \Lambda ts\overrightarrow{D}_n + \Lambda ts\overrightarrow{T}_n,$$

$$\overrightarrow{a_n} = Matrix^{-1}(Rightside - Res).$$

One can solve for $\overrightarrow{c_n}$ from:

$$\begin{bmatrix} P_{n,C,t} \\ P_{n,C,t+1} \\ \vdots \\ P_{n,C,t+G_0-1} \\ P_{n,C,t+G_0} \end{bmatrix}_{(G_0+1)\times 1} \begin{bmatrix} c_{n,g,t} \\ c_{n,g+1,t+1} \\ \vdots \\ c_{n,g+G_0,t+G_0} \end{bmatrix}_{(G_0+1)\times 1} = \begin{bmatrix} \Theta_{n,t}P_{n,I,t}a_{n,g,t} \\ \vdots \\ \vdots \\ \Theta_{n,t+G_0}P_{n,I,t+G_0}a_{n,g+G_0,t+G_0} \end{bmatrix}_{(G_0+1)\times 1}$$

$$-\begin{bmatrix} P_{n,I,t} \\ \vdots \\ P_{n,I,t+G_0} \end{bmatrix}_{(G_0+1)\times 1} \begin{bmatrix} a_{n,g,t+1} \\ \vdots \\ a_{n,g+G_0+1,t+G_0+1} \end{bmatrix}_{(G_0+1)\times 1} + \begin{bmatrix} W_{n,t}\left(1-\tau^L_{n,t}\right)E_{n,t}l_g \\ \vdots \\ W_{n,t+G_0}\left(1-\tau^L_{n,t+G_0}\right)E_{n,t+G_0}l_{g+G_0} \end{bmatrix}_{(G_0+1)\times 1}$$

$$
+\begin{bmatrix} ts^D_{n,t}+ts^T_{n,t} \\ \vdots \\ ts^D_{n,t+G_0}+ts^T_{n,t+G_0} \end{bmatrix}_{(G_0+1)\times 1},
$$

and

$$
\begin{bmatrix} i_{n,g,t} \\ i_{n,g+1,t+1} \\ \vdots \\ i_{n,g+G_0-1,t+G_0-1} \\ i_{n,g+G_0,t+G_0} \end{bmatrix}_{(G_0+1)\times 1} = \begin{bmatrix} a_{n,g+1,t+1} \\ \vdots \\ \vdots \\ a_{n,g+G_0+1,t+G_0+1} \end{bmatrix}_{(G_0+1)\times 1} - (1-\delta)\begin{bmatrix} a_{n,g,t} \\ \vdots \\ a_{n,g+G_0,t+G_0} \end{bmatrix}_{(G_0+1)\times 1}
$$

$$
-\begin{bmatrix} ts^{D,1}_{n,t} \\ ts^{D,1}_{n,t+1} \\ \vdots \\ ts^{D,1}_{n,t+G_0-1} \\ ts^{D,1}_{n,t+G_0} \end{bmatrix}_{(G_0+1)\times 1} \Big/ \begin{bmatrix} P_{n,I,t} \\ P_{n,I,t+1} \\ \vdots \\ P_{n,I,t+G_0-1} \\ P_{n,I,t+G_0} \end{bmatrix}_{(G_0+1)\times 1}.
$$

E.3. Numerical experiment details

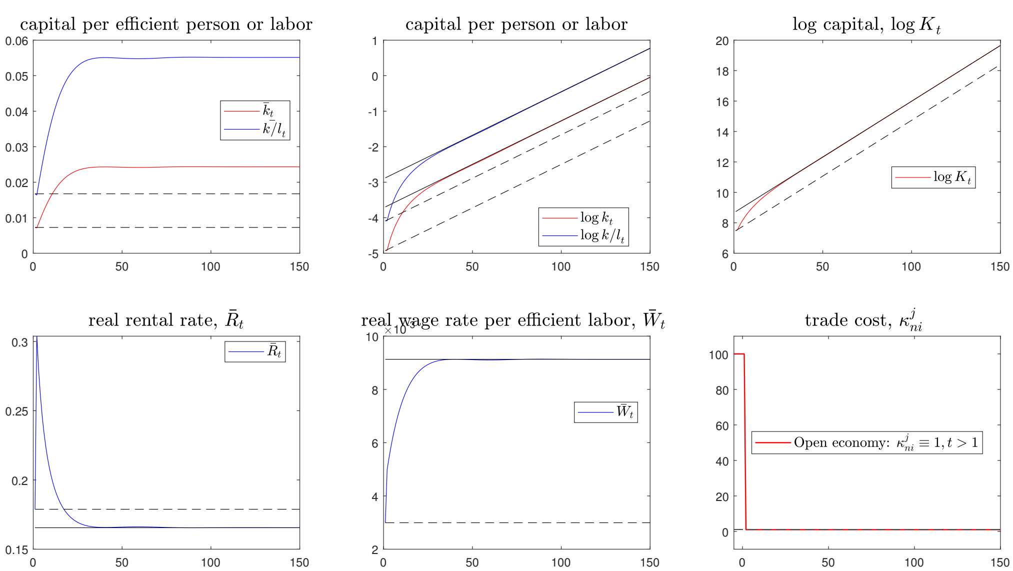

Figure E.1

IRF of exogenous demographic shock

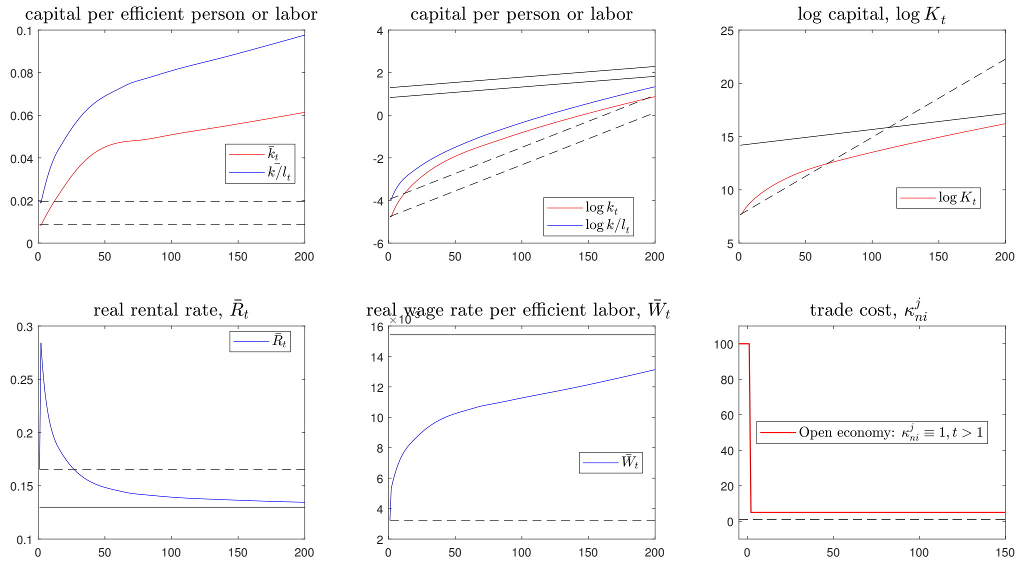

FIGURE E.2
IRF OF EXOGENOUS DEMOGRAPHIC SHOCK

APPENDIX F: CALIBRATION DETAILS

F.1. Data

F.1.I. Data sources

TABLE F.1
STEADY-STATE CONDITIONS

Variable description	Model counterpart	Data source (1971–2020)	Data source (2021–2100)
Age distribution	$\tilde{N}_{n,g,t}$	UN	UN, Imputed
Population	$N_{n,t}$	PWT	UN, Imputed
Employment	$L_{n,t}$	PWT	Imputed
Human capital index	$E_{n,t}$	PWT	Imputed
Value added	$W_{n,t}L_{n,t}E_{n,t} + R_{n,t}K_{n,t}$	WIOD & Long IO Table	Imputed
Gross output*	$P^j_{n,t}y_{n,t}$	WIOD & Long IO Table	Imputed
Gross expenditure*	$P^j_{n,t}Q^j_{n,t}$	WIOD & Long IO Table	Imputed
Trade flow*	$P^j_{n,t}Q^j_{n,t}T_{n,i,t}$	WIOD & Long IO Table	Imputed
Intermediate prices**	$P^j_{n,t}$	WIOD & Long IO Table	Imputed
Consumption***	$C_{n,t}$	WIOD & Long IO Table	Imputed
Investment***	$I_{n,t}$	WIOD & Long IO Table	Imputed
Initial capital stock***	$K_{n,t0}$	PWT	N/A

Notes: * Values are measured in current prices using market exchange rates. ** Prices are measured using PPP exchange rates. *** Quantities are measured as values deflated by prices. The sources of these data are Timmer et al. (2015); Feenstra, Inklaar and Timmer (2015); United Nations and Social Affairs (2024); Woltjer, Ggouma and Timmer (2021).

F.1.II. Constructing realized data from 1970-2020

Capital stock The initial capital stock is sourced directly from the Penn World Table 10.01 Feenstra, Inklaar and Timmer (2015), which provides "Capital stock at current PPPs (in mil. 2017US$)." This data allows me to align the initial aggregate capital stocks, $K_{n,t0}$, with my model.

Data on aggregate and sectoral investment values $(PI_{n,t}, PI^j_{n,t})$ is taken from the World Input-Output Database Timmer et al. (2015); Woltjer, Ggouma and Timmer (2021). The Penn World Table 10.01 also offers "Capital stock at constant 2017 national prices (in mil. 2017US$)," which is used to impute the initial investment price level for the first period:

$$I^{PWT}_{n,t1} = K^{PWT}_{n,t2} - (1-\delta)K^{PWT}_{n,t1},$$

and

$$P^{Implied}_{I,n,t1} = \frac{PI^{IO}_{n,t1}}{I^{PWT}_{n,I,t1}}, \quad \text{where} \quad P_{I,n,t} \equiv \prod_{j=1}^{J} \left[\frac{P^j_{n,t}}{\alpha^j_{I,n}}\right]^{\alpha^j_{I,n}}.$$

The sectoral intermediate prices for the base country are derived from the World Input-Output Database Social Economic Account, while for other regions, these prices are imputed using a gravity equation and model-implied relations. These sectoral intermediate prices $(P^{IO,j}_{n,t})$ are then used to compute the implied prices for investment $(P^{IO}_{I,n,t})$ and consumption $(P^{IO}_{C,n,t})$.

To correct the level of sectoral intermediate prices which is implied in the PWT capital stock variable, I apply the following scaling:

$$P^{scaled,j}_{n,t1} = P^{IO,j}_{n,t1}\frac{P^{Implied}_{I,n,t1}}{P^{IO}_{I,n,t1}}, \quad t = t_1$$

and for subsequent periods,

$$P^{scaled,j}_{n,tx} = P^{scaled,j}_{n,t1}\frac{P^{IO,j}_{n,tx}}{P^{IO,j}_{n,t1}}, \quad t_x > t_1.$$

These scaled sectoral prices, $P^{scaled,j}_{n,t}$, allow me to calculate the quantity of investment $I_{n,t}$ and capital stock $K_{n,t}$ over time.

Consumption or investment Final demand part of IO table includes: Final consumption expenditure by households, Final consumption expenditure by non-profit organisations serving households (NPISH), Final consumption expenditure by government, Gross fixed capital formation, Changes in inventories and valuables. I set

$$\text{Consumption} = \text{Government consumption} + \text{Household Consumption}$$

$$\text{Investment} = \text{Final demand} - \text{consumption}$$

F.1.III. Constructing realized data from 2020-2200

Investment Define investment rate as $sr_{n,t} = \dfrac{P_{n,I,t} I_{n,t}}{IN_{n,t}}$ where $IN_{n,t} \equiv R_{n,t} K_{n,t} + W_{n,t} E_{n,t} L_{n,t} + D_{n,t}$. I estimate the relationship between the investment rate against a country-fixed effect, the lagged investment rate, the contemporaneous and lagged demographic index, and the contemporaneous and lagged real GDP per capita for the years 1965-2020.

$$\log\left(\frac{sr_{n,t}}{1 - sr_{n,t}}\right) = \alpha_0 + \alpha_1 \log\left(\frac{sr_{n,t-1}}{1 - sr_{n,t-1}}\right) + \alpha_2 Young_{n,t} + \alpha_3 Old_{n,t} + f_n + \epsilon_{n,t}$$

Using $\log\left(\dfrac{sr}{1 - sr}\right)$ ensures that the imputed values of sr lie within the interval (0, 1).

TABLE F.2
Saving rate regression

	(1)	(2)	(3)
VARIABLES	SR	SR	SR
L1.SR		0.89***	
		(32.74)	
L5.SR	0.43***		
	(8.04)		
Young share	-1.06***	-0.19	-2.80***
	(-3.62)	(-1.34)	(-10.75)
Old share	-2.40***	-0.45*	-5.97***
	(-4.37)	(-1.66)	(-12.36)
Constant	-0.22**	-0.04	0.04
	(-2.24)	(-0.84)	(0.36)
Observations	255	275	280
R-squared	0.891	0.968	0.836
Region FE	YES	YES	YES

F.2. Calibrate Knowledge stock process

The process of knowledge stock is formulated as:

$$\frac{\lambda_{(n,t+1)} - \lambda_{(n,t)}}{\lambda_{(n,t)}} = \left(\lambda_{(n,t)}\right)^{\rho-1} \left[\sum_g \eta_g N_{(n,g,t)}\right]^{\varphi} \Gamma(1-\rho)$$

Here, following Buera and Oberfield (2020), I set $\rho = 0.7$ and calibrate φ.

On the balanced growth path, the knowledge stock formula can be written as:

$$1 + g_{\lambda,t+1} = 1 + g_\lambda = \left(\lambda_{(n,t)}\right)^{\rho-1} N_{n,t}^{\varphi} \left[\sum_g \eta_g \bar{N}_{(n,g)}\right]^{\varphi} \Gamma(1-\rho)$$

Thus, on the balanced growth path (BLG), population and knowledge stock must grow at a constant rate, with the relation:

$$(1+g_\lambda)^{1-\rho} = (1+g_n)^{\varphi}$$

where $1+g_n$ can be calculated from the population growth rate in 1970, and then averaged across regions. $1+g_\lambda$ can be backed out from the real wage growth rate with the relation:

$$1 + g_{\text{real wage}} = (1+g_\lambda)^{1/\theta\beta\gamma}$$

I then take the average across regions. Thus,

$$\varphi = \frac{(1-\rho)\log(1+g_\lambda)}{\log(1+g_n)}$$

To calibrate η_g, I assume that all working-age people have the same $\eta_g > 0$. In 1970, the world economy is assumed to be on the balanced growth path, which implies

$$1 + g_{\lambda,1970} = (\lambda_{n,1970})^{\rho-1} \left[\sum_{g\in[16,65]} \eta_g N_{n,g,1970}\right]^{\varphi} \Gamma(1-\rho)$$

Thus,

$$\eta_g = \frac{1+g_{\lambda,1970}}{(\lambda_{n,1970})^{\rho-1}\left(N_{n,g\in[16,65],1970}\right)^{\varphi}\Gamma(1-\rho)}$$

Derive $(1+g_\lambda)^{1-\rho} = (1+g_n)^{\varphi}$: Assume the economy is on the balanced growth path (BLG) in 1970, so $1+g_\lambda = 1+g_{\lambda,t+1} = \dfrac{\lambda_{n,t+1}-\lambda_{n,t}}{\lambda_{n,t}}$ for $\forall t \geq 1970$.

At $t = 1970$:

$$1 + g_\lambda = 1 + g_{\lambda,1971} = (\lambda_{n,1970})^{\rho-1} \left[\sum_g \eta_g N_{n,g,1970}\right]^{\varphi} \Gamma(1-\rho)$$

BLG implies that both $\lambda_{(n,t)}$ and $N_{n,g,t}$ grow at a constant rate. Thus, we can express:

$$\lambda_{n,t} \equiv \lambda_{n,1970}(1+g_\lambda)^{t-1970} \quad \text{and} \quad N_{n,g,t} \equiv N_{n,g,1970}(1+g_n)^{t-1970}$$

Therefore:

$$1 + g_{\lambda,t+1} = 1 + g_\lambda = (\lambda_{n,t})^{\rho-1} N_{n,t}^{\varphi} \left[\sum_g \eta_g \bar{N}_{n,g} \right]^{\varphi} \Gamma(1-\rho)$$

At $t = 1970$:

$$1 + g_{\lambda,1971} = (\lambda_{n,1970})^{\rho-1} \left[\sum_g \eta_g N_{n,g,1970} \right]^{\varphi} \Gamma(1-\rho)$$

This can be rewritten as:

$$1 + g_\lambda = (\lambda_{n,t})^{\rho-1} \left[\sum_g \eta_g N_{n,g,t} \right]^{\varphi} \Gamma(1-\rho)$$

Replacing $\lambda_{n,t} \equiv \lambda_{n,1970}(1+g_\lambda)^{t-1970}$ and $N_{n,g,t} \equiv N_{n,g,1970}(1+g_n)^{t-1970}$, we get:

$$1 + g_\lambda = \left(\lambda_{n,1970}(1+g_\lambda)^{t-1970}\right)^{\rho-1} \left[\sum_g \eta_g \left(N_{n,g,1970}(1+g_n)^{t-1970}\right) \right]^{\varphi} \Gamma(1-\rho)$$

$$= (\lambda_{n,1970})^{\rho-1} \left[\sum_g \eta_g N_{n,g,1970} \right]^{\varphi} \Gamma(1-\rho) \left((1+g_\lambda)^{t-1970}\right)^{\rho-1} \left((1+g_n)^{t-1970}\right)^{\varphi}$$

We already know that $1 + g_\lambda = 1 + g_{\lambda,1971} = (\lambda_{n,1970})^{\rho-1} \left[\sum_g \eta_g N_{n,g,1970} \right]^{\varphi} \Gamma(1-\rho)$, so substituting into the above equation yields:

$$1 + g_\lambda = (1+g_\lambda) \left((1+g_\lambda)^{t-1970}\right)^{\rho-1} \left((1+g_n)^{t-1970}\right)^{\varphi}$$

Thus,

$$1 = \left((1+g_\lambda)^{t-1970}\right)^{\rho-1} \left((1+g_n)^{t-1970}\right)^{\varphi}$$

Finally, we have:

$$(1+g_\lambda)^{1-\rho} = (1+g_n)^{\varphi}$$

F.3. Calibrate saving (or investment) wedges

The model suggests that higher saving wedges, $\psi_{n,t}$, indicate a stronger incentive to save for period t. Since savings provide the supply of investment, this leads to a higher capital stock in that period. Therefore, I use the aggregate capital stock, $K_{n,t}$, as targets to calibrate the evolution of saving wedges, $\psi_{n,t}$, over time.

F.3.I. Calibrate saving (or investment) wedges at steady state, ϕ_n

At the initial year, I assume the economy is in a steady state. I thus introduce the saving wedges, ψ_n, which serve as parameters to align the model's aggregate capital stock with the observed data. Mathematically, these parameter represent wedges to match the steady state capital stock, while intuitively, they capture the degree to which individuals discount next period's consumption. Even in the steady state, this variable can be included as it reflects the discount rate applied to next periods (or more precisely, to next ages), influencing saving behavior and ultimately affecting the capital stock at steady state.

Algorithm 5 Calibrate saving wedges, ψ_n, at Steady State

1: Guess a vector of saving wedges $\tilde{\psi}^i = \{\psi_1, ..., \psi_N\}^i$.

2: Given $\{P^j_{n,t}, P_{n,I,t}, P_{n,C,t}, R_{n,t}, W_{n,t}, I_{n,t}, K_{n,t}\}$ at $t = t_0$, solve $(G-1)$ Euler equations for each Country n, as described in Algorithm 3, to get a vector of capital stocks

$$\{a^1_{n,1}, a^1_{n,G+1} = 0, \{a^1_{n,g+1}\}^{G-1}_{g=1}\}.$$

3: Calculate K'_n as

$$K'_n = \sum_{g=2}^{G} \frac{N_{n,g-1} a^1_{n,g}}{1+g_n} \quad \text{for each Country } n.$$

4: Check error term, if $\left\|\tilde{K}^{data} - \tilde{K}^i\right\| < \epsilon$, stop. Else, go to next Step.

5: Go back to step 2 with updated new guess $\tilde{\psi}^{i+1}$:

$$\tilde{\psi}^{i+1} = \tilde{\psi}^i + \zeta \frac{\tilde{K}^i - \tilde{K}^{data}}{\tilde{K}^{data}} \quad \text{where } \zeta \in (0,1).$$

F.3.II. Calibrate saving wedges during transition dynamics

Algorithm 6 Calibrate saving wedges, $\psi_{n,t}$, during transition dynamics

---

1: Guess a vector of capital stocks $\tilde{\psi}_t^i = \{\psi_{1,t}, ..., \psi_{N,t}\}^i$, for $t = 2, \cdots, T+1$. $\tilde{\psi}_{t=1} = \tilde{\psi}_{t=1}$ is constant.

2: Given $\{P_{n,t}^j, P_{n,I,t}, P_{n,C,t}, R_{n,t}, W_{n,t}, I_{n,t}, K_{n,t}\}^1$ for $\forall t \in [1, T+1], n \in [1, ..., N]$, as described in *Algorithm 4*, solve $\forall G_0 \in \{1, ..., (G-1)\}$ Euler equations for each Country $n \in \mathcal{N}$, each begin cohort $g_0 \in \{1, ..., (G-1)\}$, and begin year $t_0 \in \{1, ..., T\}$:

$$a_{n,g_0,t_0}, a_{n,g_0+G_0,t_0+G_0}, \{c_{n,g,t+g-1}\}_{g=g_0}^{g_0+G_0-1}, \{a_{n,g,t+1+g}\}_{g=g_0}^{g_0+G_0-1}$$

and $\overrightarrow{tr}^{D^i}$.

3: Calculate $K_{n,t}^{\prime i} = \sum_{g=2}^{G} \frac{N_{n,g-1}}{1+g_n} a_{n,g}^1$ for each Country n and $t = 2, \cdots, T+1$.

Check error term, if $\left\| \tilde{K}_{t=2,\cdots,T+1}^{data} - \tilde{K}_{t=2,\cdots,T+1}^i \right\| < \epsilon$, stop. Else, go to next step.

4: Go back to step 2 with updated new guess $\tilde{\psi}_{t=2,\cdots,T+1}^{i+1}$:

$$\tilde{\psi}_{t=2,\cdots,T+1}^{i+1} = \zeta \tilde{\psi}_{t=2,\cdots,T+1}^i + \zeta \frac{\tilde{K}_{t=2,\cdots,T+1}^i - \tilde{K}_{t=2,\cdots,T+1}^{data}}{\tilde{K}_{t=2,\cdots,T+1}^{data}} \quad \text{where } \zeta \in (0,1).$$

Appendix G: Numerical analysis details

G.1. Revealed comparative advantage (RCA) index (Balassa, 1965)

$$RCA_{nj} = \frac{\frac{Export_{n,j}}{\sum_n Export_{n,j}}}{\frac{\sum_j Export_{n,j}}{\sum_{j,n} Export_{n,j}}} \tag{G.1}$$

where n means country, j means sector, $Export_{n,j}$ means the value of country n's sector j exports. The higher RCA_{nj}, the higher degree of specialization for country n in sector j products.

TABLE G.1

Stationary balance growth equilibrium comparison. Open economy

	(1S)	(2S)	(3S)	(1A)	
Country	sym.	sym.	sym.	cty1	cty2
Survival rate	low	high	high	high	low
Fertility rate	high	high	low	high	high
Average lifespan	60.1	70.8	70.8	70.8	60.1
Population growth	1.050	1.050	1.010	1.050	1.050
Implied TFP growth	1.025	1.025	1.005	0.025	0.025
Working age share	0.44	0.46	0.72	0.46	0.44
Trade cost	Free trade	Free trade	Free trade	Free trade	Free trade
Capital share of VA	0.5000	0.5000	0.5000	0.5009	0.4991
Per efffcient person					
Capital stock	0.021	0.024	0.061	0.024	0.021
Output	0.0073	0.0081	0.0152	0.0078	0.0076
Consumption	0.0046	0.0048	0.0107	0.0046	0.0047
Investment	0.0028	0.0033	0.0046	0.0032	0.0029
Capital - efffcient labor ratio	0.047	0.054	0.084	0.052	0.048
Price ratio					
Real wage rate	0.008	0.009	0.012	0.009	0.009
Real rental rate	0.179	0.165	0.125	0.166	0.179

Close economy, living longer, asymmetric two country

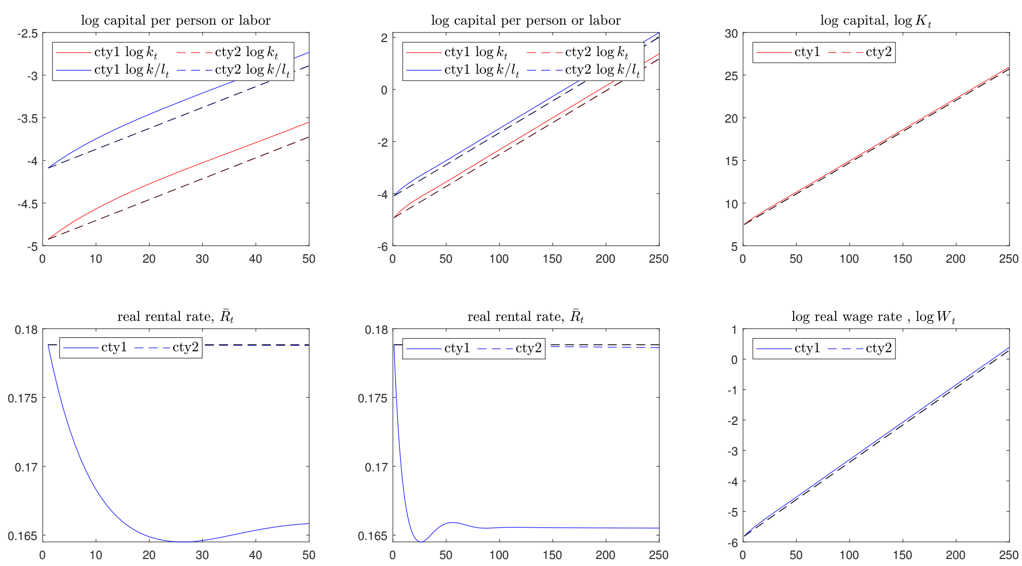

Figure G.1

Demographics and knowledge stocks

Open economy, living longer, asymmetric two country

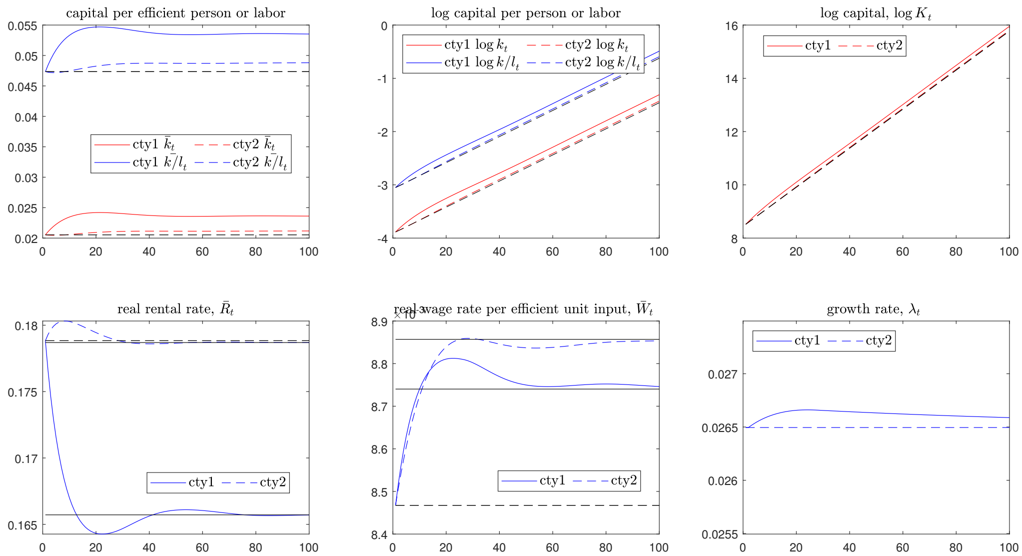

FIGURE G.2
DEMOGRAPHICS AND KNOWLEDGE STOCKS

Appendix H: Counterfactual analysis

H.1. Demographics, knowledge stocks, and capital stocks

H.2. Demographics, economic growth, and trade patterns change

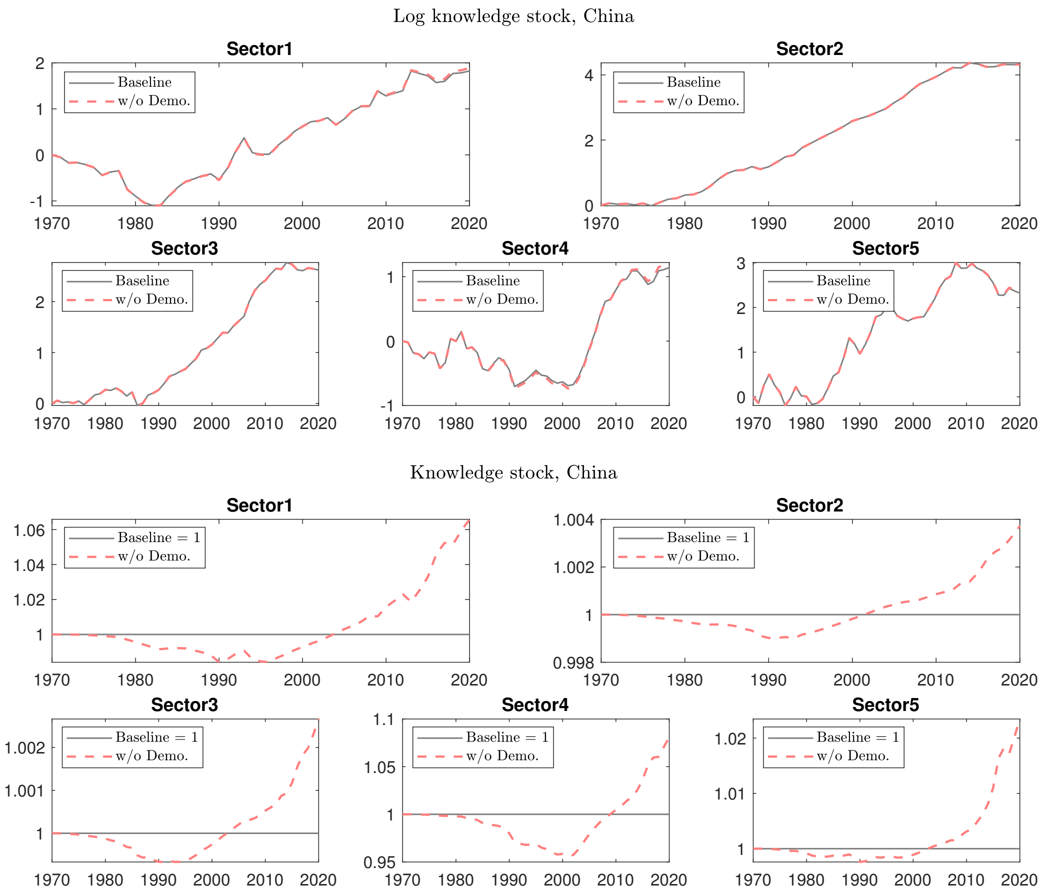

Figure H.1
Knowledge stocks

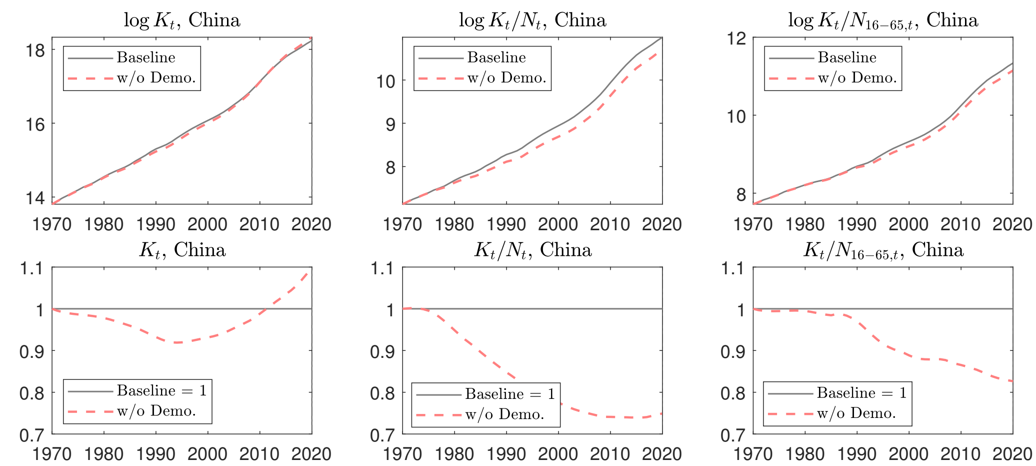

FIGURE H.2
CAPITAL STOCKS

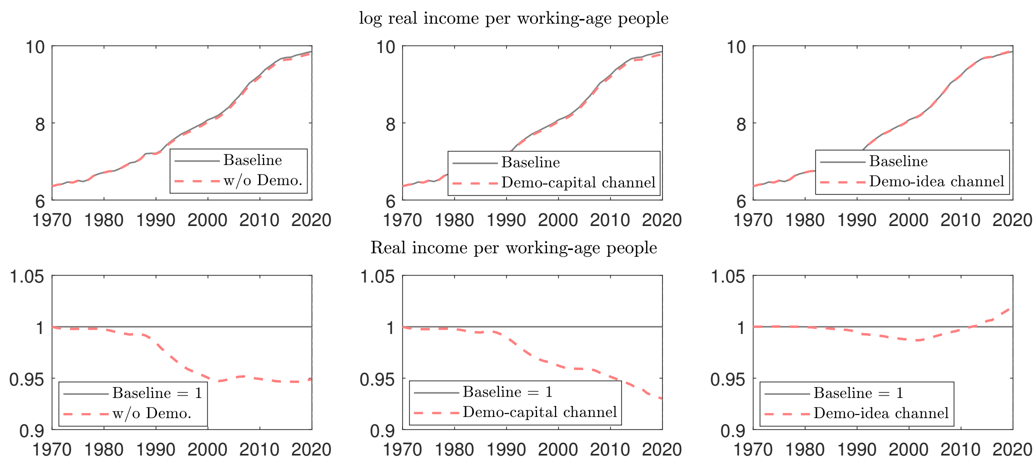

FIGURE H.3
REAL INCOME PER WORKING-AGE PEOPLE

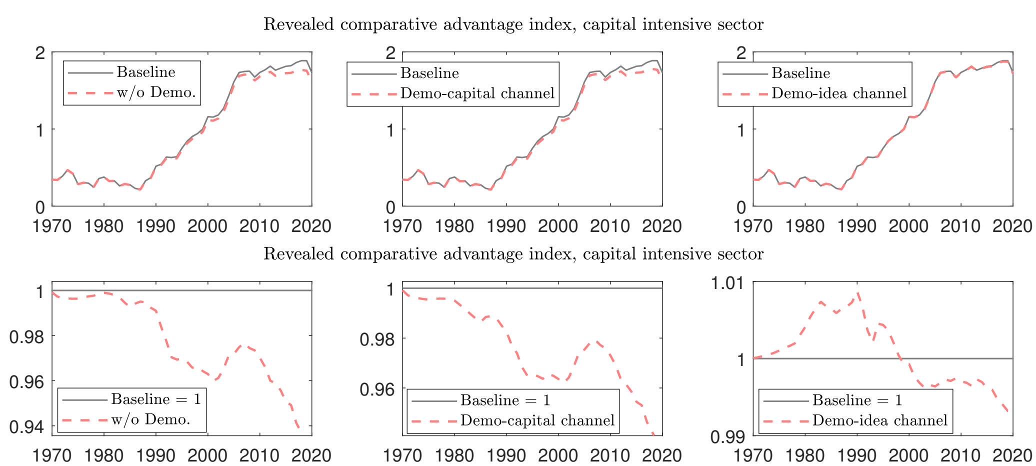

FIGURE H.4
REVEALED COMPARATIVE ADVANTAGE INDEX

H.3. Model based projection

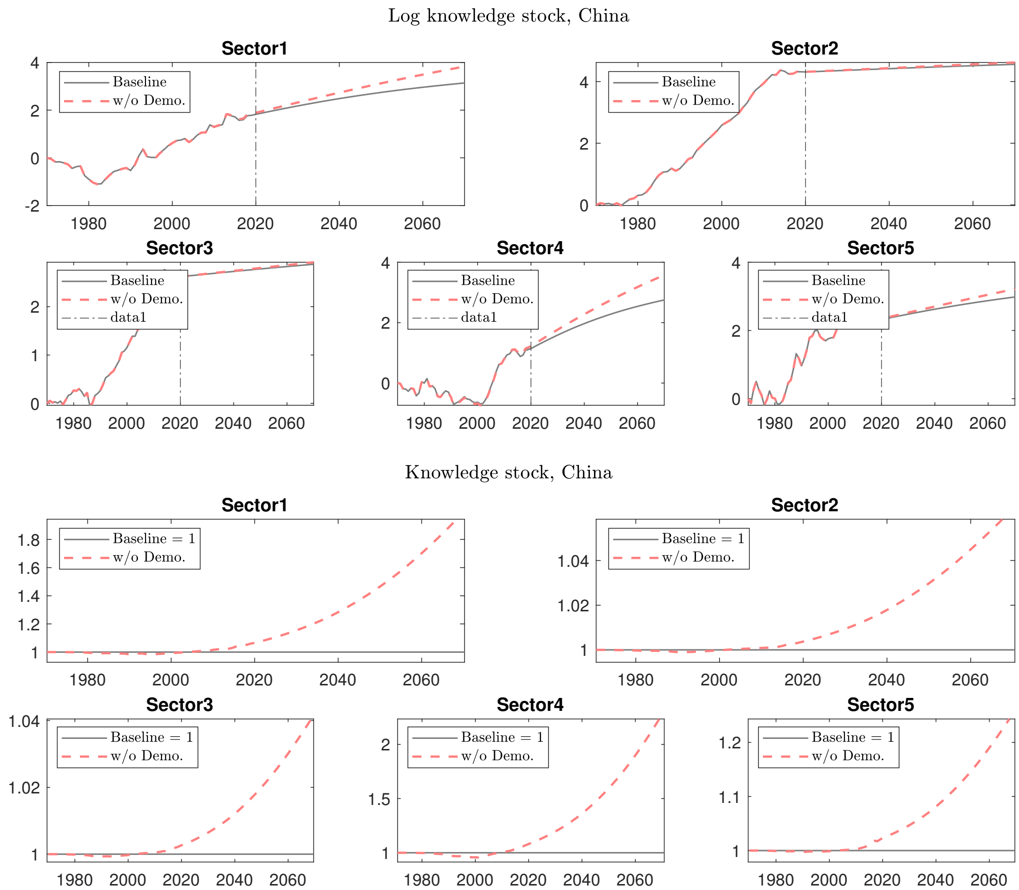

Figure H.5
Knowledge stocks

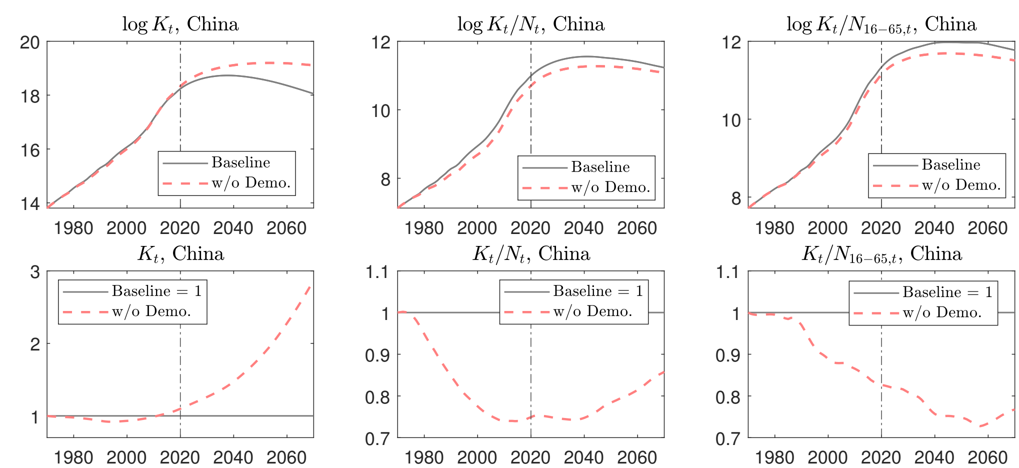

Figure H.6
Capital stocks

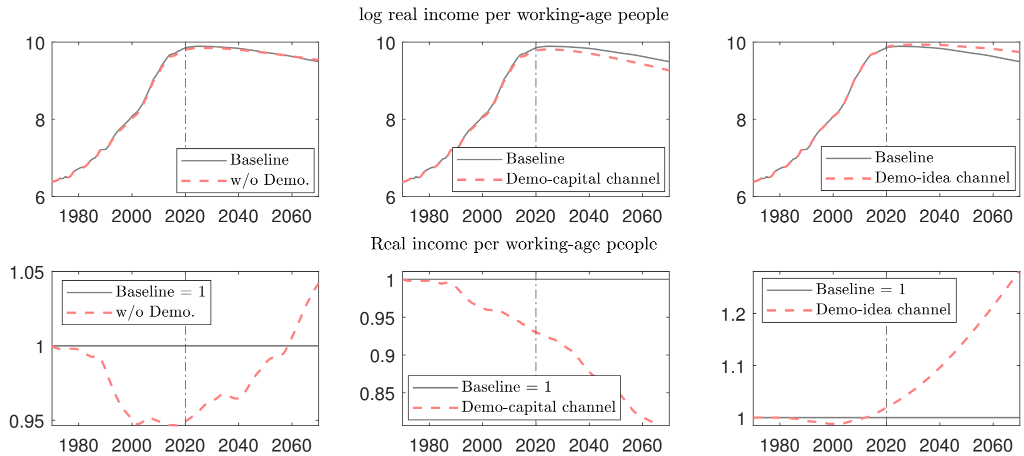

Figure H.7
Real income per Working-age people

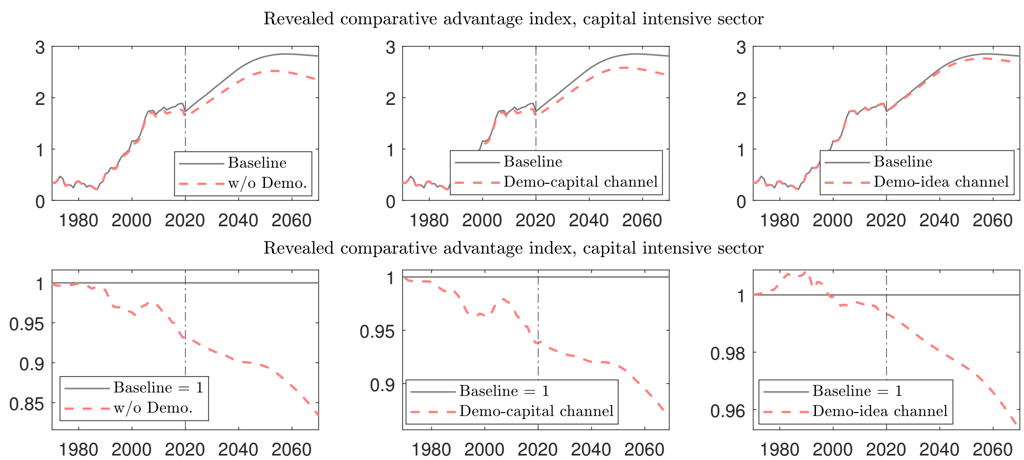

Figure H.8
Revealed comparative advantage index