Linear Algebra in R

Thursday, September 19

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Part 1

- 1 Let $u = \begin{bmatrix} 2 \\ -7 \\ 1 \end{bmatrix}$, $v = \begin{bmatrix} -3 \\ 0 \\ 4 \end{bmatrix}$, $w = \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}$. Find

a. $3u - 4v$
b. $2u + 3v - 5w$

```
u <- c(2,-7,1)
v <- c(-3,0,4)
w <- c(0,5,-8)
#a
3*u - 4*v
```

```
## [1]  18 -21 -13
```

```
#b
2*u + 3*v - 5*w
```

```
## [1]  -5 -39  54
```

- 2 Find the norm of the following vectors.

a. $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
b. $v = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

```
#a
u <- c(1, 2, 3)
unorm <- norm(as.matrix(u),"F")
unorm
```

```
## [1] 3.741657
```

```
#b
v <- c(3, 2, 1)
vnorm <- norm(as.matrix(v),"F")
vnorm
```

```
## [1] 3.741657
```

- 3 Find the unit vectors in the direction of the same vectors given below:

a. $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

b. $v = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

```
#a
u_unit <- u/unorm
u_unit
```

```
## [1] 0.2672612 0.5345225 0.8017837
```

```
#b
v_unit <- v/vnorm
v_unit
```

```
## [1] 0.8017837 0.5345225 0.2672612
```

- 4 Find the distance between the following two vectors.

a. $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

b. $v = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

```
norm(as.matrix(u-v),"F")
```

```
## [1] 2.828427
```

- 5 Find the dot product of the following vectors:

a. $u = \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

b. $v = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

```
#a
u <- c(4, 3, -1)
v <- c(1, -1, 1)
crossprod(u,v)
```

```
##      [,1]
## [1,]    0
```

```r
#b
u <- c(-1, 2)
v <- c(3, 1)
crossprod(u,v)
```

```
##      [,1]
## [1,]   -1
```

- 6 Find the angles between the following vectors:

a. $u = \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$
b. $v = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

```r
#a
u <- c(4, 3, -1)
v <- c(1, -1, 1)
dotuv <- crossprod(u,v)
unorm <- norm(as.matrix(u),"F")
vnorm <- norm(as.matrix(v),"F")
theta <- as.numeric(acos(dotuv/(unorm*vnorm)))
theta*180/pi
```

```
## [1] 90
```

```r
#b
u <- c(-1, 2)
v <- c(3, 1)
dotuv <- crossprod(u,v)
unorm <- norm(as.matrix(u),"F")
vnorm <- norm(as.matrix(v),"F")
theta <- as.numeric(acos(dotuv/(unorm*vnorm)))
theta*180/pi
```

```
## [1] 98.1301
```

- 7 If $u = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$, determine whether or not u and v are orthogonal

```r
u <- c(3/5, -4/5)
v <- c(1, 2)

crossprod(u,v)
```

```
##      [,1]
## [1,]   -1
```

```r
if(crossprod(u,v) == 0){
  cat("u and v are orthogonal")
} else {
  cat("u and v are not orthogonal")
}
```

```
## u and v are not orthogonal
```

Part 2

- 1 The EZ Life Company manufactures sofas and armchairs in three models, A, B, and C. The company has regional warehouses in New York, Chicago, and San Francisco.

In its August shipment, the company sends 10 model-A sofas, 12 model-B sofas, 5 model-C sofas, 15 model-A chairs, 20 model-B chairs, and 8 model-C chairs to each warehouse. a. Use a matrix to organize this information.

```r
#a
elements <- c(10, 15, 12, 20, 5, 8)
Shipment_Aug <- matrix(elements, nrow=2)

colnames(Shipment_Aug) <- c("Model A", "Model B", "Model C")
rownames(Shipment_Aug) <- c("Sofa", "Chair")

Shipment_Aug
```

```
##       Model A Model B Model C
## Sofa       10      12       5
## Chair      15      20       8
```

The September shipments from the EZ Life Company to the New York, San Francisco, and Chicago warehouses are given below

```
## Warning: package 'knitr' was built under R version 3.6.2
```

Table 1: New York

	Model A	Model B	Model C
Sofa	45	35	20
Chair	65	40	35

Table 2: San Francisco

	Model A	Model B	Model C
Sofa	30	32	28
Chair	43	47	30

Table 3: Chicago

	Model A	Model B	Model C
Sofa	22	25	38
Chair	31	34	35

b. Use a matrix to organize the September shipment information. What was the total amount shipped to the three warehouses in September?

```
#b
elements_NY <- c(45, 65, 35, 40, 20, 35)
Shipment_NY_Sep <- matrix(elements_NY, nrow=2)

colnames(Shipment_NY_Sep) <- c("Model A", "Model B", "Model C")
rownames(Shipment_NY_Sep) <- c("Sofa", "Chair")

elements_SF <- c(30, 43, 32, 47, 28, 30)
Shipment_SF_Sep <- matrix(elements_SF, nrow=2)

colnames(Shipment_SF_Sep) <- c("Model A", "Model B", "Model C")
rownames(Shipment_SF_Sep) <- c("Sofa", "Chair")

elements_CH <- c(22, 31,  25, 34, 38, 35)
Shipment_CH_Sep <- matrix(elements_CH, nrow=2)

colnames(Shipment_CH_Sep) <- c("Model A", "Model B", "Model C")
rownames(Shipment_CH_Sep) <- c("Sofa", "Chair")

Shipment_Total_Sep<- Shipment_CH_Sep+Shipment_NY_Sep+Shipment_SF_Sep
Shipment_Total_Sep
```

```
##       Model A Model B Model C
## Sofa       97      92      86
## Chair     139     121     100
```

The September sales for each model for the Chicago warehouse of the EZ Life Company is shown below.

Table 4: Chicago

	Model A	Model B	Model C
Sofa	5	10	8
Chair	11	14	15

c. Use a matrix to organize Chicago's September shipment information.

```
#c
elements_CH_sale <- c(5, 11, 10, 14, 8, 15)
Sale_CH_Sep <- matrix(elements_CH_sale, nrow=2)

colnames(Sale_CH_Sep) <- c("Model A", "Model B", "Model C")
rownames(Sale_CH_Sep) <- c("Sofa", "Chair")
```

d. What was the Chicago warehouse inventory on October 1, taking into account only the number of items received and sent out during the month?

```
#d
Inv_CH_Sep <- Shipment_Aug + Shipment_CH_Sep - Sale_CH_Sep
Inv_CH_Sep
```

```
##       Model A Model B Model C
## Sofa       27      27      35
## Chair      35      40      28
```

- 2 Microbucks Computer Company makes two computers, the Pomegranate II and the Pomegranate Classic, at two different factories. The Pom II requires 2 processor chips, 16 memory chips, and 20 vacuum tubes, while the Pom Classic requires 1 processor chip, 4 memory chips, and 40 vacuum tubes. Microbucks has in stock at the beginning of the year 500 processor chips, 5,000 memory chips, and 10,000 vacuum tubes at the Pom II factory and 200 processor chips, 2,000 memory chips, and 20,000 vacuum tubes at the Pom Classic factory. It manufactures 50 Pom IIs and 50 Pom Classics each month. Besides having the stock mentioned above, gets shipments of parts every month in the amounts of 100 processor chips, 1,000 memory chips, and 3,000 vacuum tubes at the Pom II factory and 50 processor chips, 1,000 memory chips, and 2,000 vacuum tubes at the Pom Classic factory. Find the company's inventory of parts after 6 months, using matrix operations. Use matrix algebra.

```
parts <- c(2, 1, 16, 4, 20, 40)
Parts <- matrix(parts, nrow = 2)

colnames(Parts) <- c("processor chips",
                     "memory chips",
                     "vacuum tubes")
rownames(Parts) <- c("Pom II",
                     "Pom Classic")

stock <- c(500, 200,  5000, 2000, 10000, 20000)
Stock <- matrix(stock, nrow = 2)

colnames(Stock) <- c("processor chips",
                     "memory chips",
                     "vacuum tubes")
rownames(Stock) <- c("Pom II",
                     "Pom Classic")

monthly_prod <- 50
months <- 6

shipment <- c(100, 50, 1000, 1000, 3000, 2000)
Shipment <- matrix(shipment, nrow = 2)

Inv_After_6_Month <- Stock + Shipment *  months -
  monthly_prod * Parts * months
Inv_After_6_Month
```

```
##              processor chips memory chips vacuum tubes
## Pom II                   500         6200        22000
## Pom Classic              200         6800        20000
```

- 3 An investment firm recommends that a client invest in AAA, A, and B rated bonds. The average yield on AAA bonds is 6%, on A bonds 6.5%, and on B bonds 8%. The client wants to invest twice as much in AAA bonds as in B bonds. How much should be invested in each type of bond if the total investment is $25,000, and the investor wants an annual return of $1,650 on the three investments? Use matrix algebra.

```r
# AAA +0 A - 2 B = 0
# AAA + A + B = 25000
# 0.06 AAA + 0.065 A + 0.08 B = 1650

first <- c(1, 1, 0.06, 0, 1, 0.065, -2, 1, 0.08)
First <- matrix(first, nrow = 3)
second <- c(0, 25000, 1650)
Second <- matrix(second, nrow = 3)
result <- solve(First) %*% Second

colnames(result) <- c("investment")
rownames(result) <- c("AAA", "A", "B")

result
```

```
##     investment
## AAA      10000
## A        10000
## B         5000
```

- 4 Two sectors of the U.S. economy are (1) lumber and wood products and (2) paper and allied products. In 1998 the input-output table involving these two sectors was as follows. (All figures are in millions of dollars.)

Table 5: Input_Output

	Wood	Paper
Wood	36000	7000
Paper	100	17000
Total Output	120000	120000

```r
demand <-  c(10000, 20000)
Ext_Demand <- matrix(demand, nrow = 2)
rownames(Ext_Demand) <- c("Wood", "Paper")
colnames(Ext_Demand) <- c("External Demand")

inpoutp <- c(36000/120000, 100/120000, 7000/120000, 17000/120000)
Sector_Matrix <- matrix(inpoutp, nrow = 2)
rownames(Sector_Matrix) <- c("Wood", "Paper")
colnames(Sector_Matrix) <- c("Wood", "Paper")
```

```r
# Create an identity matrix
I <- diag(x = 1, nrow = 2, ncol = 2)

## Inverse of I - Sector Matrix
Inv <- solve(I - Sector_Matrix)

Output <- Inv %*% Ext_Demand
rownames(Output) <- c("Wood", "Paper")
colnames(Output) <- c("Output")

Output
```

```
##         Output
## Wood  16228.77
## Paper 23316.73
```

- 5 Ever since Bob Misly was hired, things have been going awry at AllSmart CPAs Inc.. He has a tendency to lose important documents, especially around April, when tax returns of our business clients are due. Today Misly accidentally shredded Big Corp.'s investment records. We must therefore reconstruct them on the basis of the information he can gather. Bob recalls that the company earned an $8 million return on investments totaling $65 million last year. After a few frantic telephone calls to sources in Big Corp, he learned that Big Corp had made investments in four companies last year: A, B, C, and D. (For reasons of confidentiality their names are withheld.) Investments in company A earned 15% last year, investments in B depreciated by 20% last year, investments in C neither appreciated nor depreciated last year, while investments in D earned 20% last year. Misly was also told that Giant Corp invested twice as much in company X as in company Z, and three times as much in company W as in company Z. Does Misly have sufficient information to piece together Giant Corp's investment portfolio before its tax return is due next week? If so, what does the investment portfolio look like?

```r
# 0.15 A - 0.2 B + 0 C + 0.2 D = 8
# A + 0 B - 2 C + 0 D = 0
# 0 A + 0 B -3 C + 1 D = 0
# A + B + C + D = 65

left <- c(0.15, 1, 0, 1, -0.2, 0 , 0, 1,
           0, -2, -3, 1,
           0.2, 0, 1, 1)
Left <- matrix(left, nrow = 4)
right <- c(8, 0, 0, 65)
Right <- matrix(right, nrow = 4)
portfolio <- solve(Left) %*% Right

colnames(portfolio) <- c("investment")
rownames(portfolio) <- c("A", "B", "C", "D")

portfolio
```

```
##   investment
## A         20
## B          5
## C         10
## D         30
```

- 6 Aze Bookstore wants to ship books from its warehouses in Rancho Cordova and Fair Oaks to its stores, one in Folsom and another in Carmichael. Its warehouse in Rancho Cordova has 1,000 books, and its warehouse in Fair Oaks has 2,000. Each store orders 1,500 books. It costs \$5 to ship each book from Rancho Cordova to Folsom and \$1 to ship each book from Rancho Cordova to Carmichael. It costs \$4 to ship each book from Fair Oaks to Folsom and \$2 to ship each book from Fair Oaks to Carmichael. If Aze has a transportation budget of \$9,000 and it is willing to spend all of it, how many books should Aze ship from each warehouse to each store to fulfill the demand. Use matrix algebra even if you think that you can solve this question without a pencil and a paper. You will be graded based on your matrix algebra code.

```r
# a + b + 0 + 0 = 1000
# 0 + 0 + c + d = 2000
# 5a +1b + 4c + 2d = 9000
# a + 0 + c + 0 = 1500
# 0 + b + 0 + d = 1500

input <- c(1, 0, 5, 1, 0,
           1, 0, 1, 0, 1,
           0, 1, 4, 1, 0,
           0, 1, 2, 0, 1)
Input <- matrix(input, nrow = 5)
numbers <- c(1000, 2000, 9000, 1500, 1500)
Numbers <- matrix(numbers, nrow = 5)
Output <- qr.solve(Input, Numbers)

colnames(Output) <- c("number of books")
rownames(Output) <- c("RtF", "RtC", "FtF", "FtC")

Output
```

```
##     number of books
## RtF             500
## RtC             500
## FtF            1000
## FtC            1000
```

- 7 Consider the following information about stocks of Corporations A, B, and C.

Table 6: Stocks Information

	Price (\$)	Dividend Yield (%)
Corp A	16	7
Corp B	56	2
Corp C	80	2

You invested a total of \$8,400 in shares of the three stocks at the given prices and expected to earn \$248 in annual dividends. If you purchased a total of 200 shares, how many shares of each stock did you purchase? Use matrix algebra.

```r
# a + b + c = 200
# 16 a + 56 b + 80 c = 8400
# 0.07 * 16 a + 0.02 * 56 b + 0.02 * 80 c = 248

# yps stands for yield per share
yps_a <- 0.07*16
yps_b <- 0.02*56
yps_c <- 0.02*80

coeff_matrix <- matrix(c(1, 16, yps_a,
                         1, 56, yps_b,
                         1, 80, yps_c), nrow = 3)
numbers_matrix <- matrix(c(200, 8400, 248), nrow = 3)

answer <- solve(coeff_matrix)%*% numbers_matrix

colnames(answer) <- c("shares")
rownames(answer) <- c("A","B","C")

answer
```

```
##   shares
## A    100
## B     50
## C     50
```

Part 3

- 1 The human resource manager of a telemarketing firm is concerned about the rapid turnover of the firm's telemarketers. It appears that many telemarketers do not work very long before quitting. There may be a number of reasons, including relatively low pay, personal unsuitability for the work, and the low probability of advancement. Because of the high cost of hiring and training new workers, the manager decided to examine the factors that influence workers to quit. He reviewed the work history of a random sample of workers who have quit in the last year and recorded the number of weeks on the job before quitting and the age of each worker when originally hired. Use regression analysis to describe how the work period and age are related. Dataset is available under Part 3 Dataset 1 CSV file.

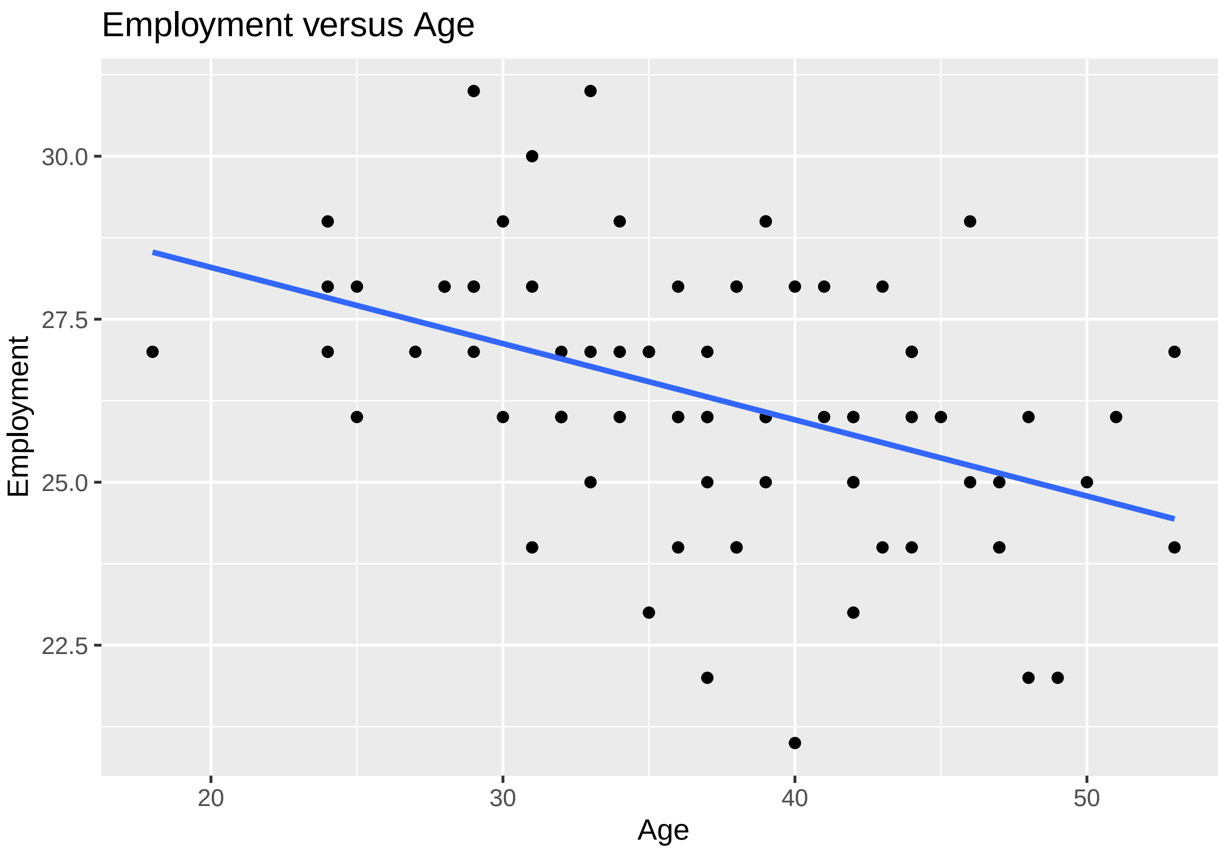
```r
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 3.6.2
```

```r
employees <- read.csv("Part 3 Question 1 Dataset.csv")

ggplot(employees, aes(x=Age, y=Employment)) + geom_point() +
  geom_smooth(method=lm, se=FALSE) + ggtitle("Employment versus Age")
```

```
## `geom_smooth()` using formula 'y ~ x'
```




```
## Create X and Y matrices

# Create matrix X containing all Age records
CoeffMatrix <- as.matrix(employees$Age, ncol = 1)
colnames(CoeffMatrix) <- c("Age")

# Create a matrix containing all 1's to hold the
# coefficients of the beta 0.
BetaZeroCoeff <- c(1)
OneMatrix <- matrix(BetaZeroCoeff, nrow = 80)
colnames(OneMatrix) <- c("BetaZeroCoeff")

# Final XMatrix Containing Odometer Readings and the
# BetaZeroCoeff
(XMatrix <- cbind(CoeffMatrix, OneMatrix))
```

```
##       Age BetaZeroCoeff
##  [1,]  37             1
##  [2,]  33             1
##  [3,]  53             1
##  [4,]  29             1
##  [5,]  24             1
##  [6,]  25             1
##  [7,]  36             1
##  [8,]  32             1
##  [9,]  32             1
```

```
## [10,]  37            1
## [11,]  36            1
## [12,]  47            1
## [13,]  38            1
## [14,]  30            1
## [15,]  40            1
## [16,]  36            1
## [17,]  38            1
## [18,]  33            1
## [19,]  42            1
## [20,]  34            1
## [21,]  46            1
## [22,]  38            1
## [23,]  39            1
## [24,]  31            1
## [25,]  45            1
## [26,]  39            1
## [27,]  41            1
## [28,]  44            1
## [29,]  50            1
## [30,]  24            1
## [31,]  33            1
## [32,]  44            1
## [33,]  42            1
## [34,]  39            1
## [35,]  47            1
## [36,]  34            1
## [37,]  37            1
## [38,]  39            1
## [39,]  35            1
## [40,]  35            1
## [41,]  25            1
## [42,]  36            1
## [43,]  44            1
## [44,]  44            1
## [45,]  28            1
## [46,]  37            1
## [47,]  35            1
## [48,]  34            1
## [49,]  49            1
## [50,]  39            1
## [51,]  41            1
## [52,]  42            1
## [53,]  46            1
## [54,]  37            1
## [55,]  42            1
## [56,]  43            1
## [57,]  24            1
## [58,]  29            1
## [59,]  48            1
## [60,]  47            1
## [61,]  31            1
## [62,]  32            1
## [63,]  30            1
```

```
## [64,]  29           1
## [65,]  27           1
## [66,]  29           1
## [67,]  40           1
## [68,]  38           1
## [69,]  31           1
## [70,]  28           1
## [71,]  42           1
## [72,]  43           1
## [73,]  18           1
## [74,]  35           1
## [75,]  53           1
## [76,]  32           1
## [77,]  48           1
## [78,]  51           1
## [79,]  42           1
## [80,]  39           1
```

```r
# Create a Matrix containing Y values
EmploymentMatrix <- as.matrix(employees$Employment, ncol = 1)
colnames(EmploymentMatrix) <- c("Employment")

# X-transpose %*% X
expr1 <- t(XMatrix) %*% XMatrix

# Inverse of it
expr2 <- solve(expr1)

# X-transpose %*% Matrix Y
expr3 <- t(XMatrix) %*% EmploymentMatrix

# Final Product
BetaMatrix <- expr2 %*% expr3

BetaMatrix
```

```
##               Employment
## Age           -0.1169168
## BetaZeroCoeff 30.6330746
```

```r
# Verify directly using Simple Linear Regression
Ex4_Model <- lm(formula = employees$Employment ~ employees$Age, data = employees)
summary(Ex4_Model)
```

```
##
## Call:
## lm(formula = employees$Employment ~ employees$Age, data = employees)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.9564 -0.9371  0.0175  0.9820  4.2252
##
## Coefficients:
```

```
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)   30.63307    1.04414  29.338  < 2e-16 ***
## employees$Age -0.11692    0.02748  -4.255 5.77e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.813 on 78 degrees of freedom
## Multiple R-squared:  0.1884, Adjusted R-squared:  0.178
## F-statistic:  18.1 on 1 and 78 DF,  p-value: 5.767e-05
```

```r
# interpretation
cat("According to the regression analysis, the work period is negatively
    related to the age. The model predicts that as age increases by 1, the
    duration of employment decreases by 0.117 week")
```

```
## According to the regression analysis, the work period is negatively
##     related to the age. The model predicts that as age increases by 1, the
##     duration of employment decreases by 0.117 week
```

- 2 The CSV file Part 3 Dataset 2 CSV file contains monthly returns for four stocks – Merck (MRK), Proctor & Gamble (PG), Johnson & Johnson (JNJ), and Pfizer (PFE). Calculate the mean and standard deviation of the portfolio. The proportions invested in each stock are 25%.

Also shown on the dataset are monthly returns for four stocks – Boeing (BA), Home Depot (HD), Chevron (CVX), and Exxon (XOM). Calculate the mean and standard deviation of this portfolio. The proportions invested in each stock are 25%.

Which portfolio would a risk-averse investor choose? Which portfolio would a risk-taking investor choose?

```r
returns <- read.csv("Part 3 Question 2 Dataset.csv")

# Portfolio 1
weights <- c(0.25)
matrix_weights <- matrix(weights, nrow = 4)
colnames(matrix_weights) <- c("Weights")

mean_MRK <- mean(returns$MRK)
mean_PG <- mean(returns$PG)
mean_JNJ <- mean(returns$JNJ)
mean_PFE <- mean(returns$PFE)

Mean_Matrix <- rbind(mean_MRK, mean_PG, mean_JNJ, mean_PFE)

Exp_Return <- t(matrix_weights) %*% Mean_Matrix
rownames(Exp_Return) <- c("Expected Return")
colnames(Exp_Return) <- c("")

Exp_Return
```

```
##
## Expected Return 0.01108921
```

```r
combined_matrix <- cbind(returns$MRK, returns$PG,
                         returns$JNJ, returns$PFE)
colnames(combined_matrix) <- c("MRK", "PG", "JNJ","PFE")

cov_matrix <- cov(combined_matrix)

port_var <- t(matrix_weights) %*% cov_matrix %*% matrix_weights
rownames(port_var) <- c("Portfolio Variance")
colnames(port_var) <- c("")

port_sd <- sqrt(port_var)
rownames(port_sd) <- c("Portfolio Std Dev")
colnames(port_sd) <- c("")

port_var
```

```
##
## Portfolio Variance 0.001096782
```

```r
port_sd
```

```
##
## Portfolio Std Dev 0.0331177
```

```r
Coeff_Var <- port_sd/Exp_Return * 100
rownames(Coeff_Var) <- c("Portfolio Coefficient of Variation")

Coeff_Var
```

```
##
## Portfolio Coefficient of Variation 298.648
```

```r
# Portfolio 2
weights2 <- c(0.25)
matrix_weights2 <- matrix(weights2, nrow = 4)
colnames(matrix_weights2) <- c("Weights2")

mean_BA <- mean(returns$BA)
mean_HD <- mean(returns$HD)
mean_CVX <- mean(returns$CVX)
mean_XOM <- mean(returns$XOM)

Mean_Matrix2 <- rbind(mean_BA, mean_HD, mean_CVX, mean_XOM)

Exp_Return2 <- t(matrix_weights2) %*% Mean_Matrix2
rownames(Exp_Return2) <- c("Expected Return2")
colnames(Exp_Return2) <- c("")

Exp_Return2
```

```
##
## Expected Return2 0.01050258
```

```r
combined_matrix2 <- cbind(returns$BA, returns$HD,
                          returns$CVX, returns$XOM)
colnames(combined_matrix2) <- c("BA", "HD", "CVX","XOM")

cov_matrix2 <- cov(combined_matrix2)

port_var2 <- t(matrix_weights2) %*% cov_matrix2 %*% matrix_weights2
rownames(port_var2) <- c("Portfolio Variance2")
colnames(port_var2) <- c("")

port_sd2 <- sqrt(port_var2)
rownames(port_sd2) <- c("Portfolio Std Dev2")
colnames(port_sd2) <- c("")

port_var2
```

```
##
## Portfolio Variance2 0.001586124
```

```r
port_sd2
```

```
##
## Portfolio Std Dev2 0.03982617
```

```r
Coeff_Var2 <- port_sd2/Exp_Return2 * 100
rownames(Coeff_Var2) <- c("Portfolio Coefficient of Variation2")

Coeff_Var2
```

```
##
## Portfolio Coefficient of Variation2 379.2036
```

```r
cat("Since the expected return of the first portfolio is larger, and
its relative dispersion is smaller at the same time, both risk-averse
and risk-taking investors should choose the first portfolio.")
```

```
## Since the expected return of the first portfolio is larger, and
## its relative dispersion is smaller at the same time, both risk-averse
## and risk-taking investors should choose the first portfolio.
```