Joint Equalization and Decoding Scheme using Spinal Codes

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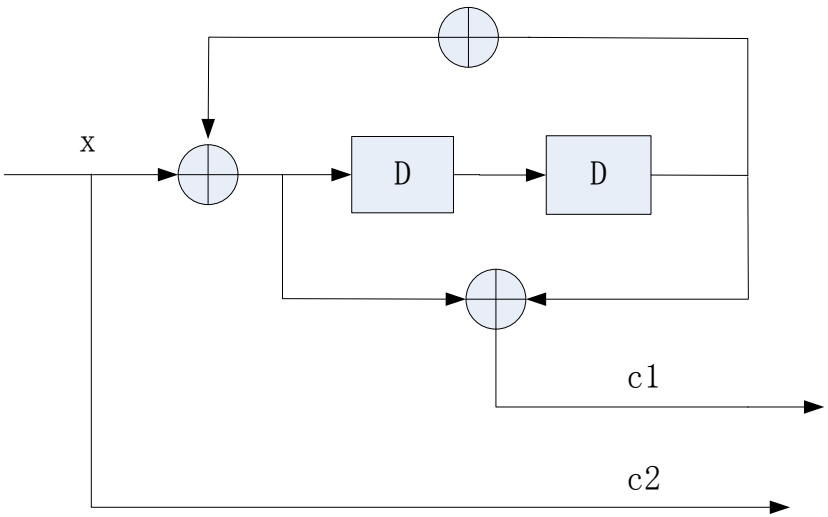
Overview

Hash Function

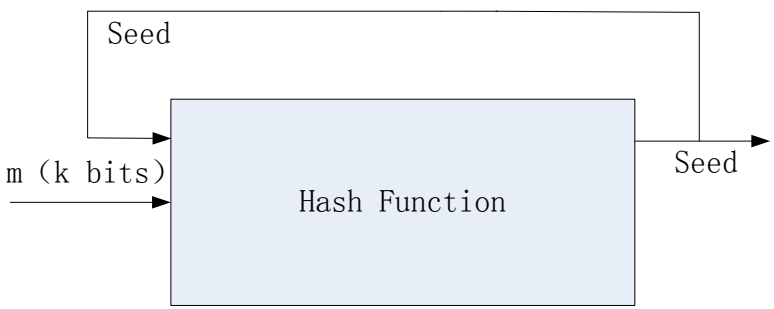
The Hash Function is a group of function that can be used to map data of arbitrary size to data of fixed size. In the Spinal code we could directly regard it as a **random number generator** with two inputs: the seed $S(32 - bits)$ and message segment $m(k - bits)$; One 32-bits output seed.

Recursive Systematic Convolutional Code



We firstly start from this (7,5) RSC code to better understand the Spinal code

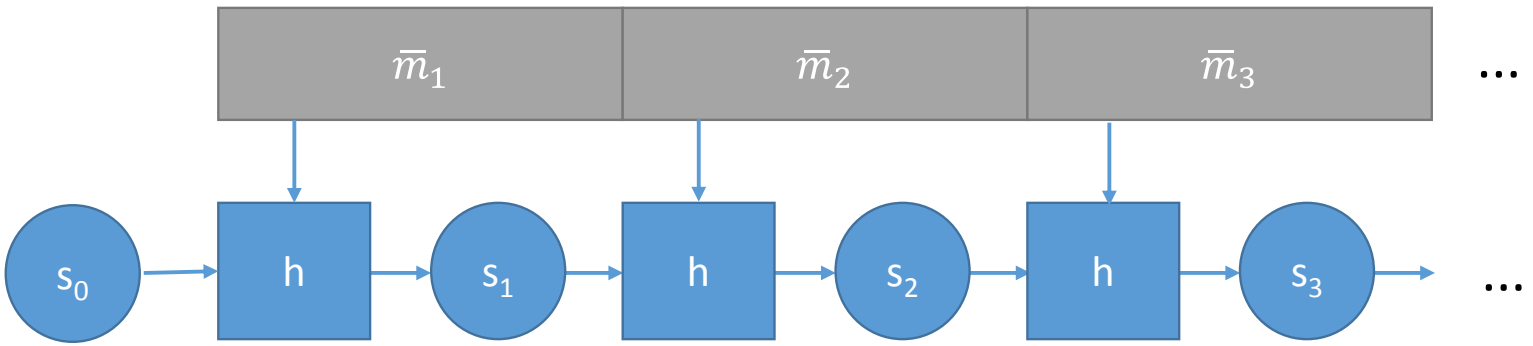
Core of Spinal Code



There are three modification from the encoder of RSC to the core of Spinal code:

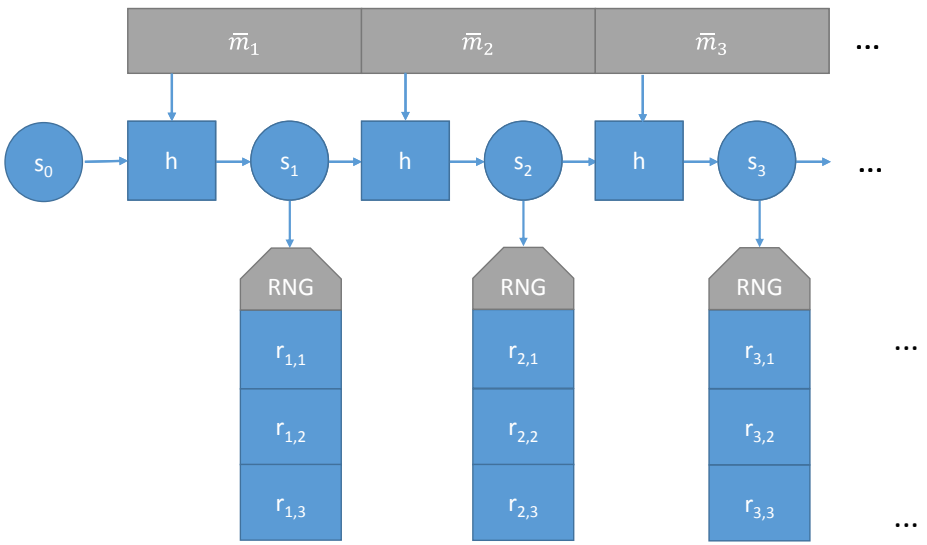
- Change the boolean polynomial function into a pseudo-random hash function.
- Change the input into segments of the message.
- The feedback and the output is a 32-bit seed.
- Don't transmit the message itself (not a system code).

Converse the recursive structure into sequence structure



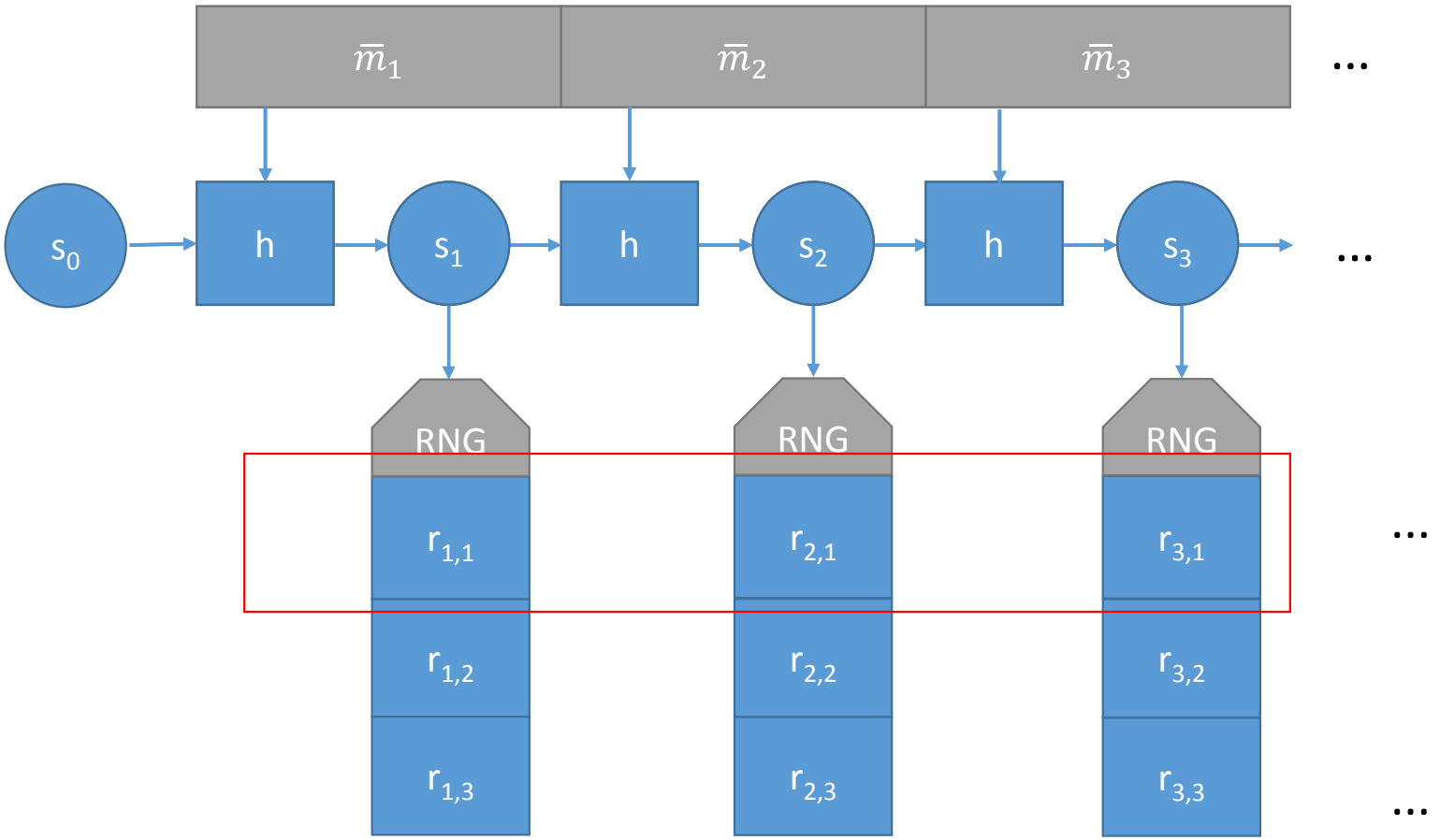
The message length is n, $\overline{m}_i$ is the k-bits segment of message. h represent the hash function. s_i is the seeds.

The whole encode scheme

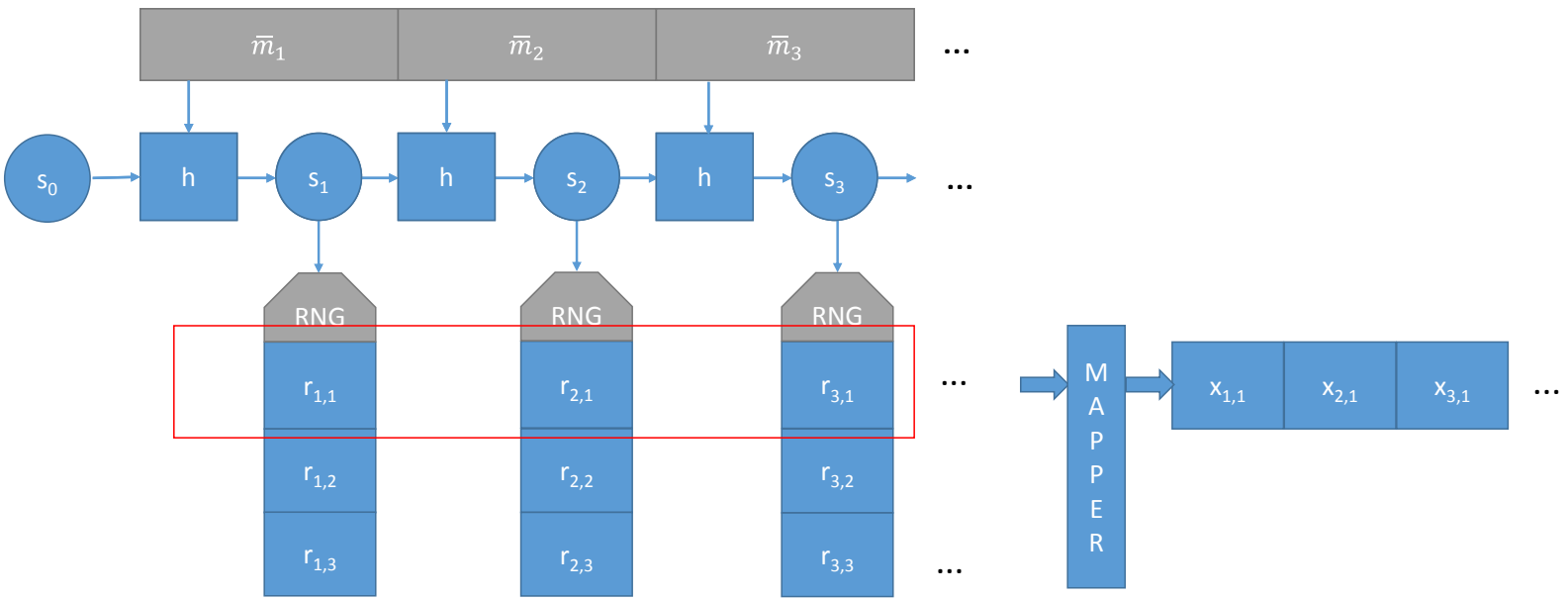


- $r_{i,j}$ is the j^{th} output of GNR seed by s_i. In fact the primal output generated by GNR is 32-bit. The final output is the lowest c-bits of primal ones.
- The size of c is depended on the mapper's degree. So the range of c is (1,32). Increasing the c would not increase the encoding complexity.

Rateless transmission scheme



Rateless transmission scheme



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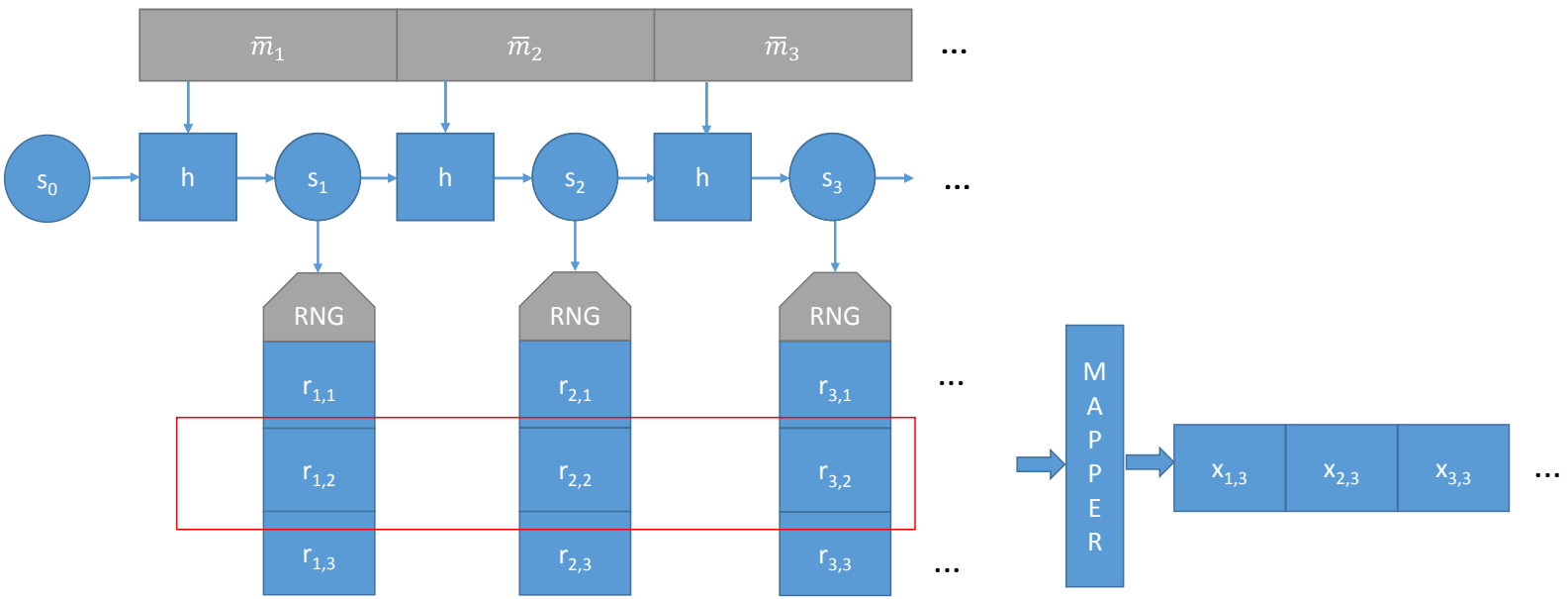

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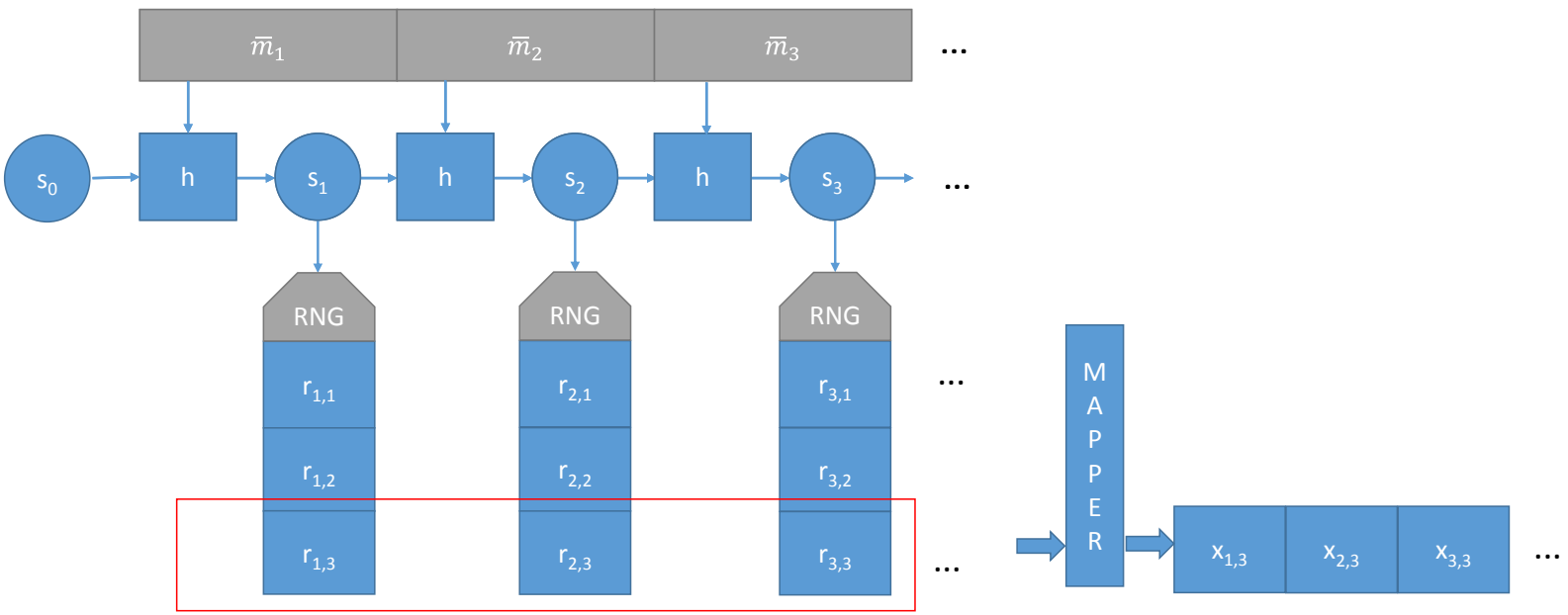
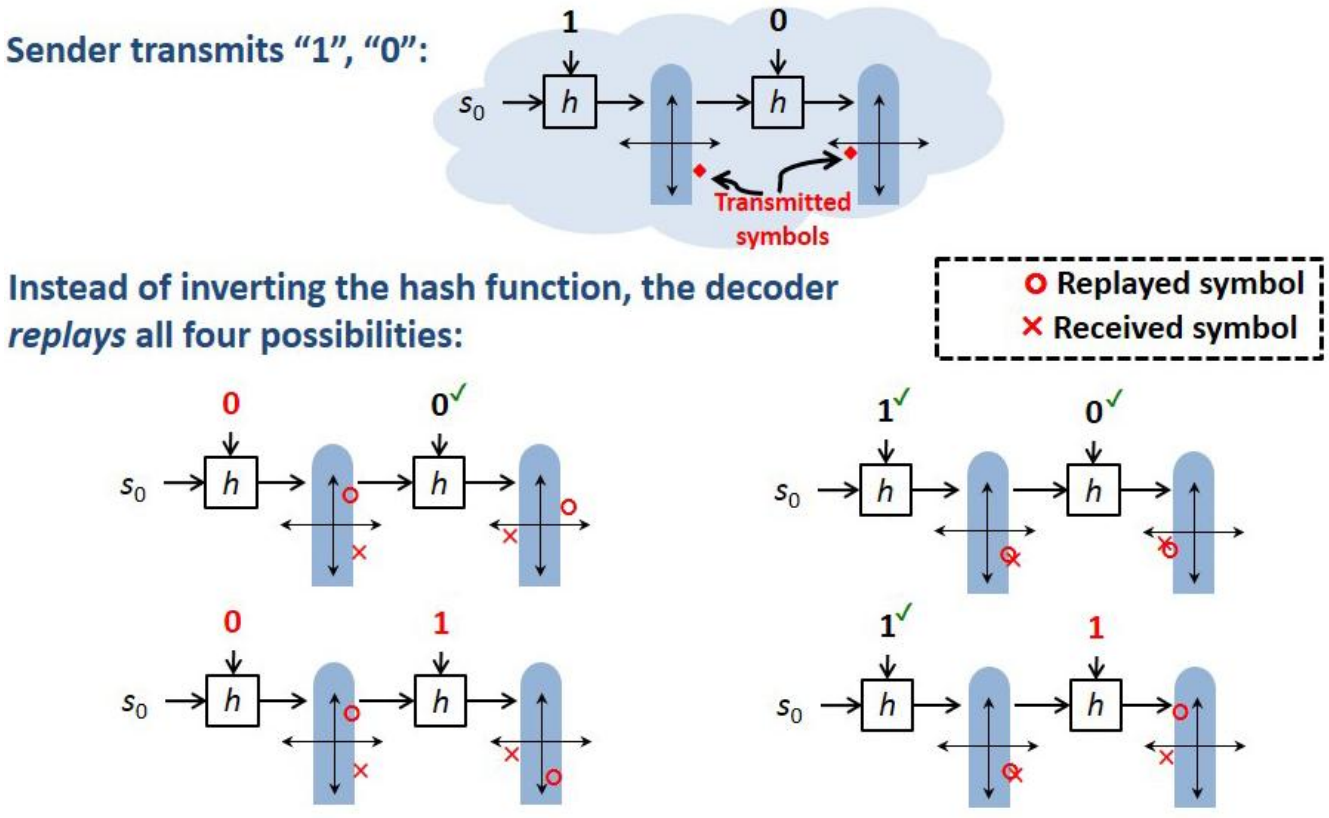


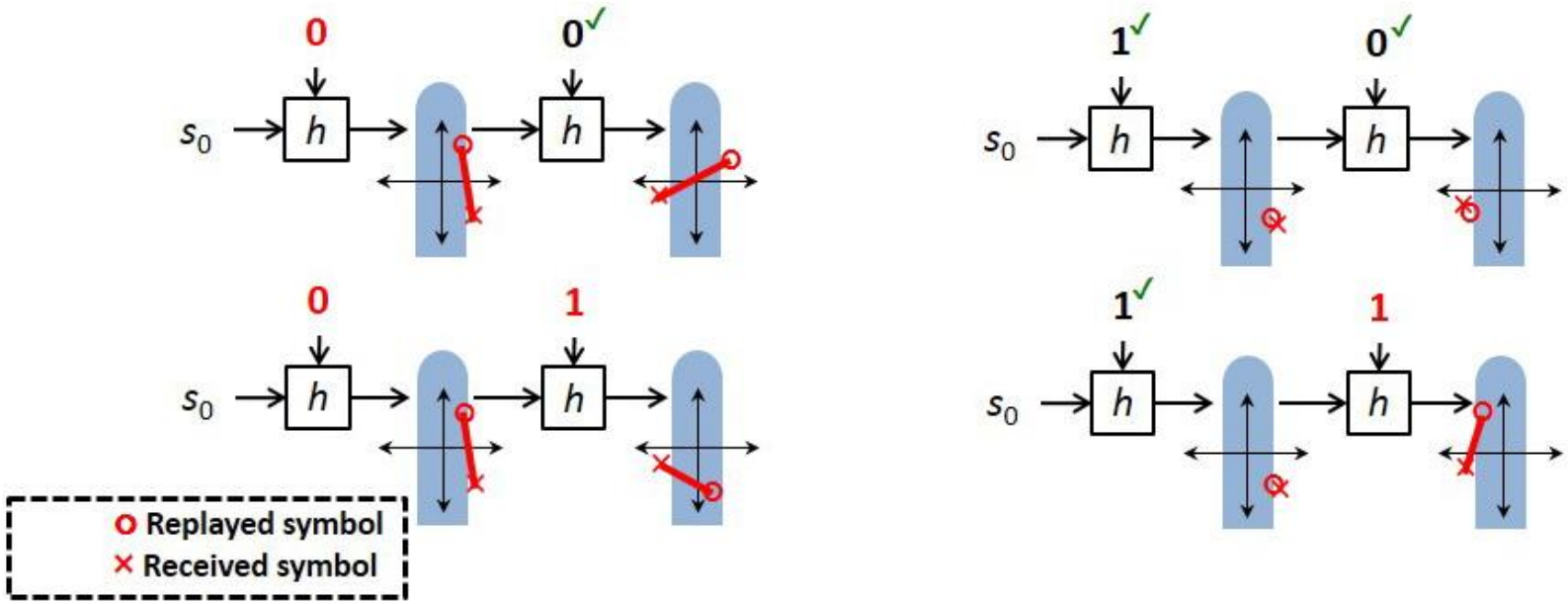
ML decoder

As the hash function could not be reverse. So the decode method is replay the encoder. A simply example: the k=1, transmitted message is '1' '0'.



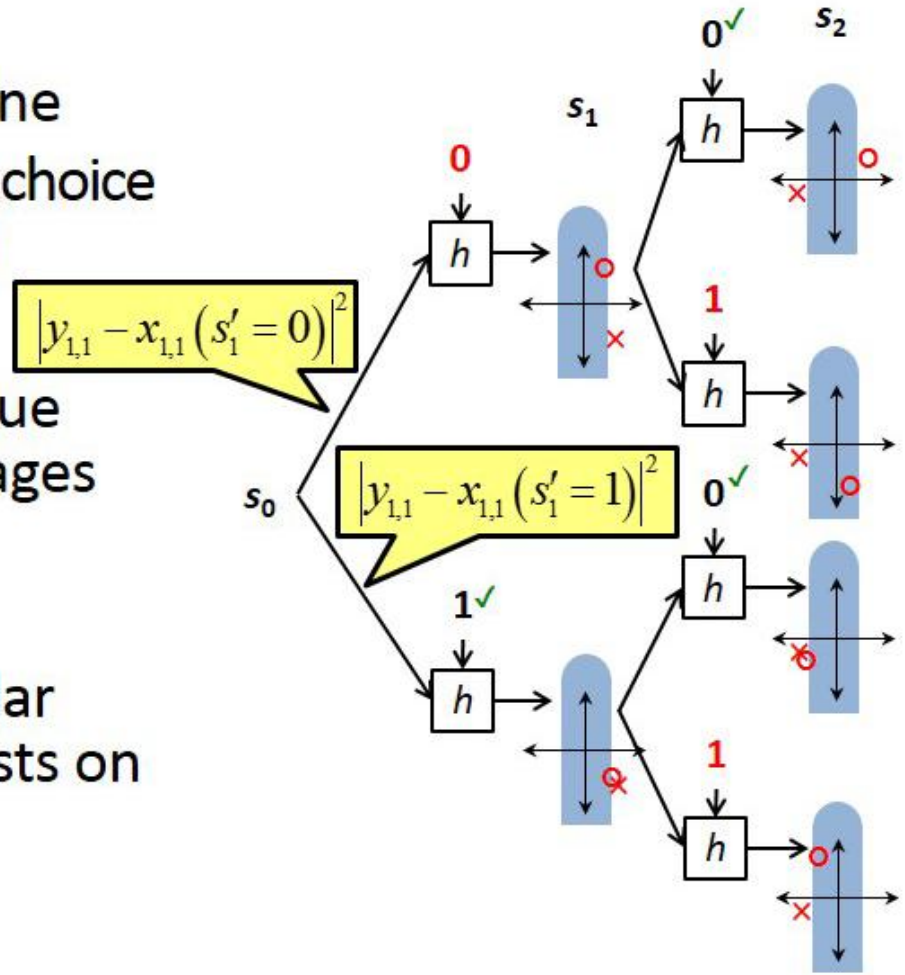
ML decoder

Measure total distance.



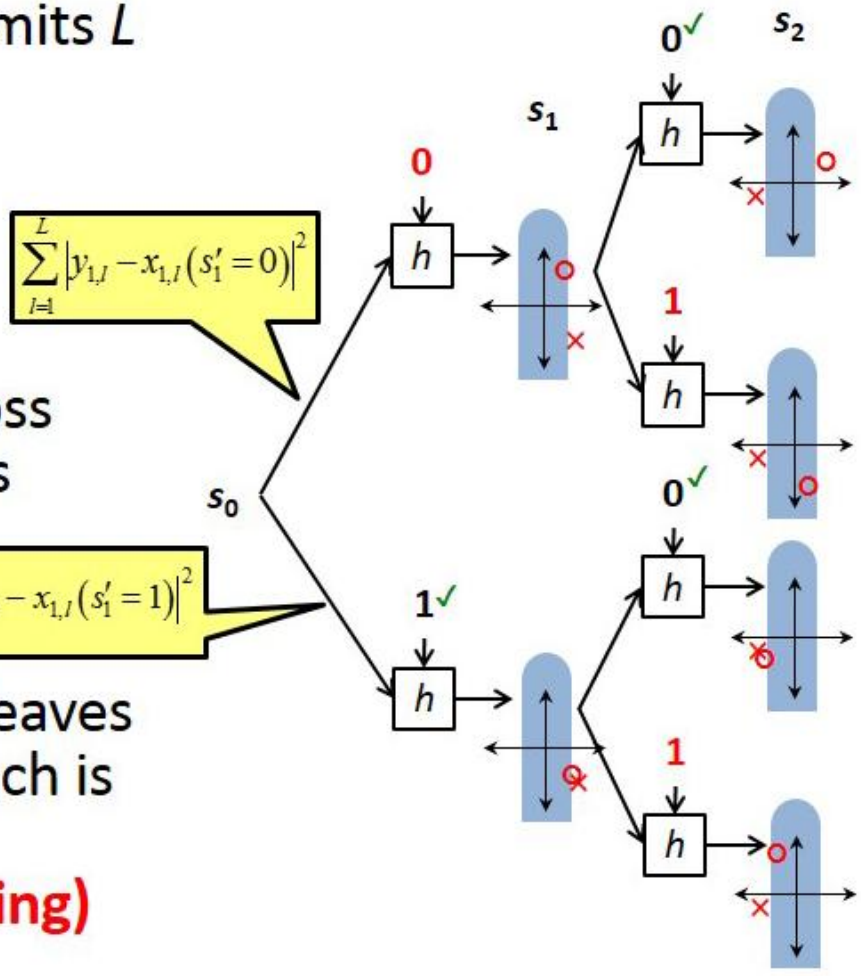
Tree decoder algorithm

- General tree properties:
  - n/k levels, one per spine
  - Branching factor 2^k (per choice of k-bit message chunk)
- Let s'_t be the t^{th} spine value associated with all messages that share s'_t
- We find cost of a particular message by summing costs on path from root to leaf

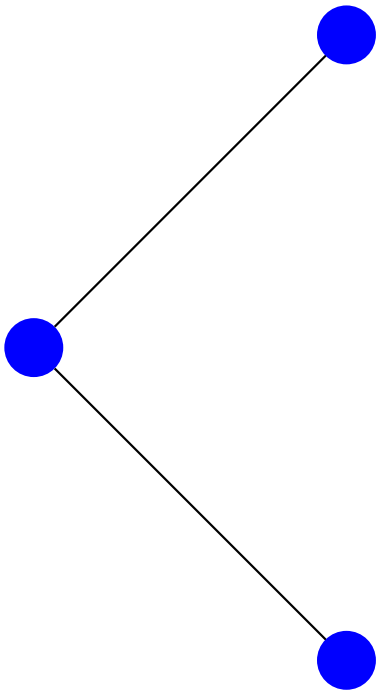


Tree decoder algorithm

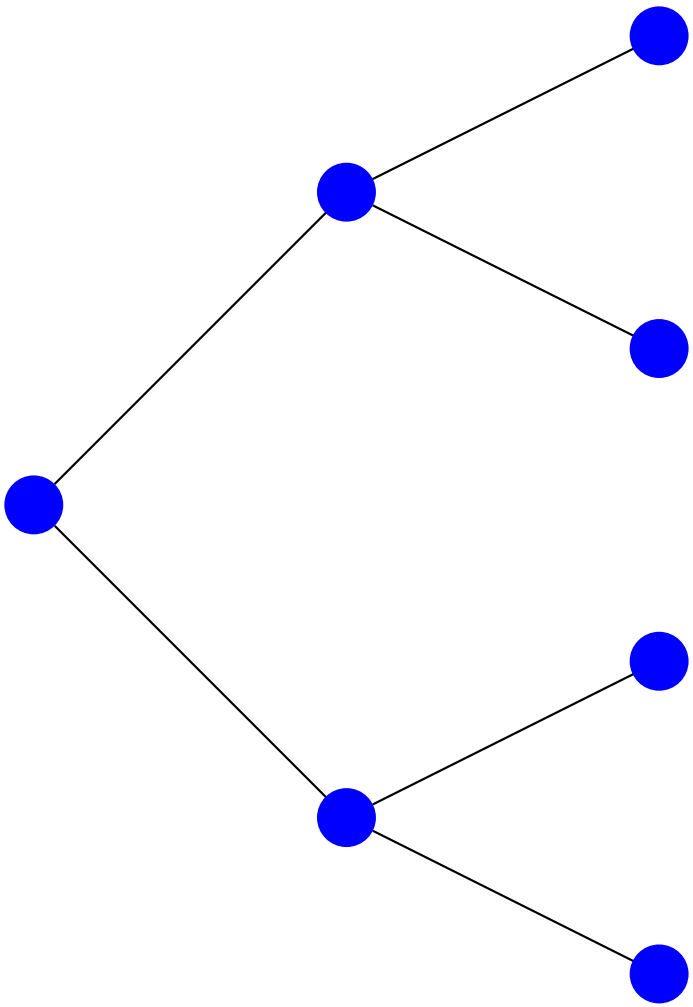
- Suppose the sender transmits L passes, in a poor channel
- Average (sum) metric across passes, and label branches
- However, the tree has 2^n leaves to compare so this approach is still **impracticable (too computationally demanding)**



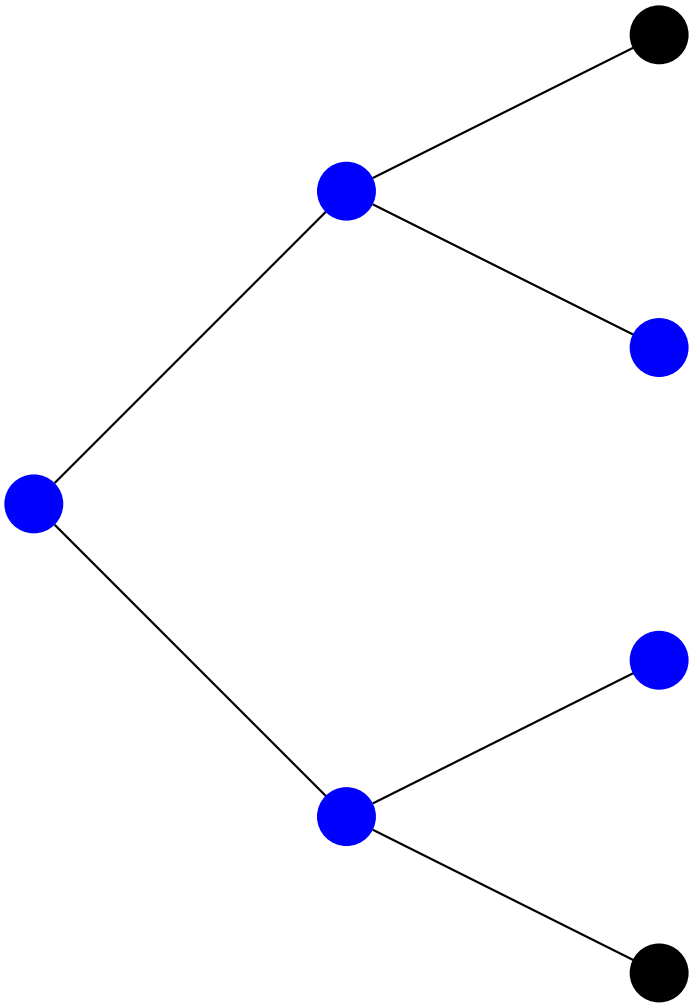
Approximate ML decoder: k=1,B=2 – Initialization



Approximate ML decoder: k=1,B=2 – Expand



Approximate ML decoder: k=1,B=2 – Score and Prune



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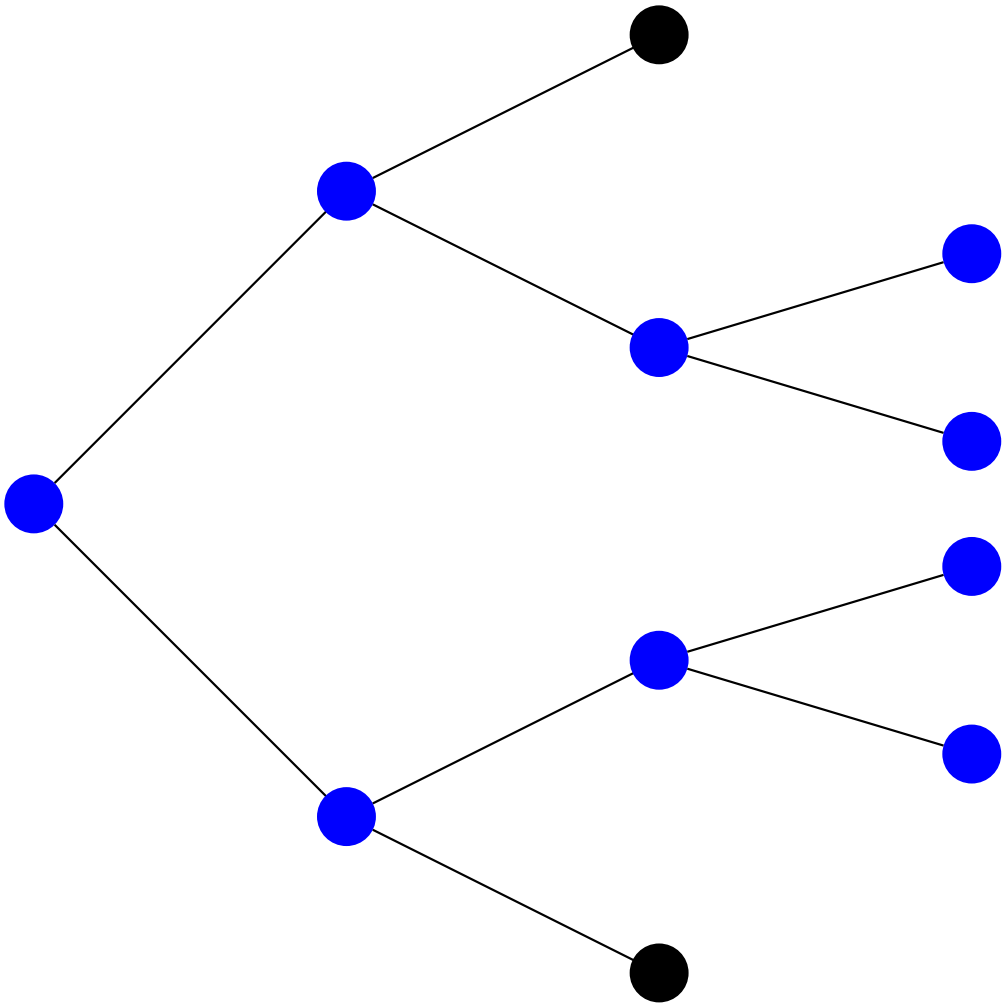

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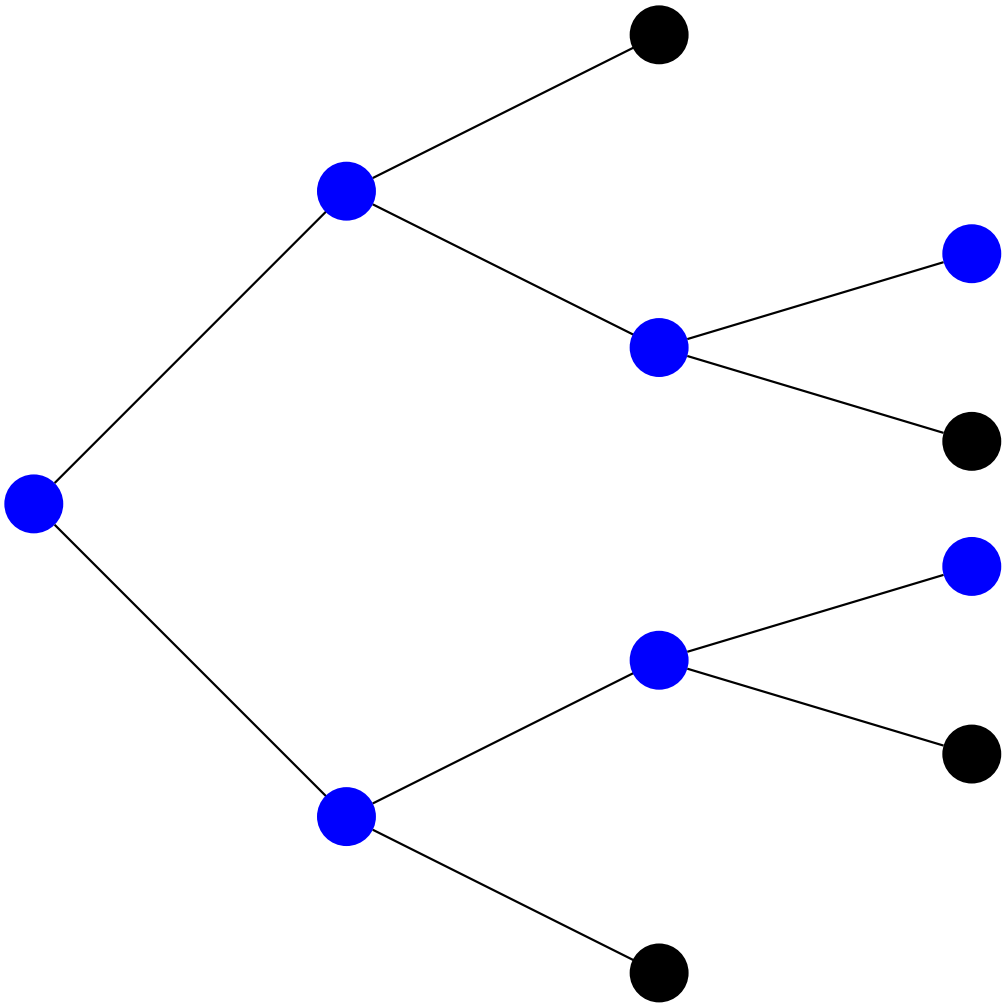
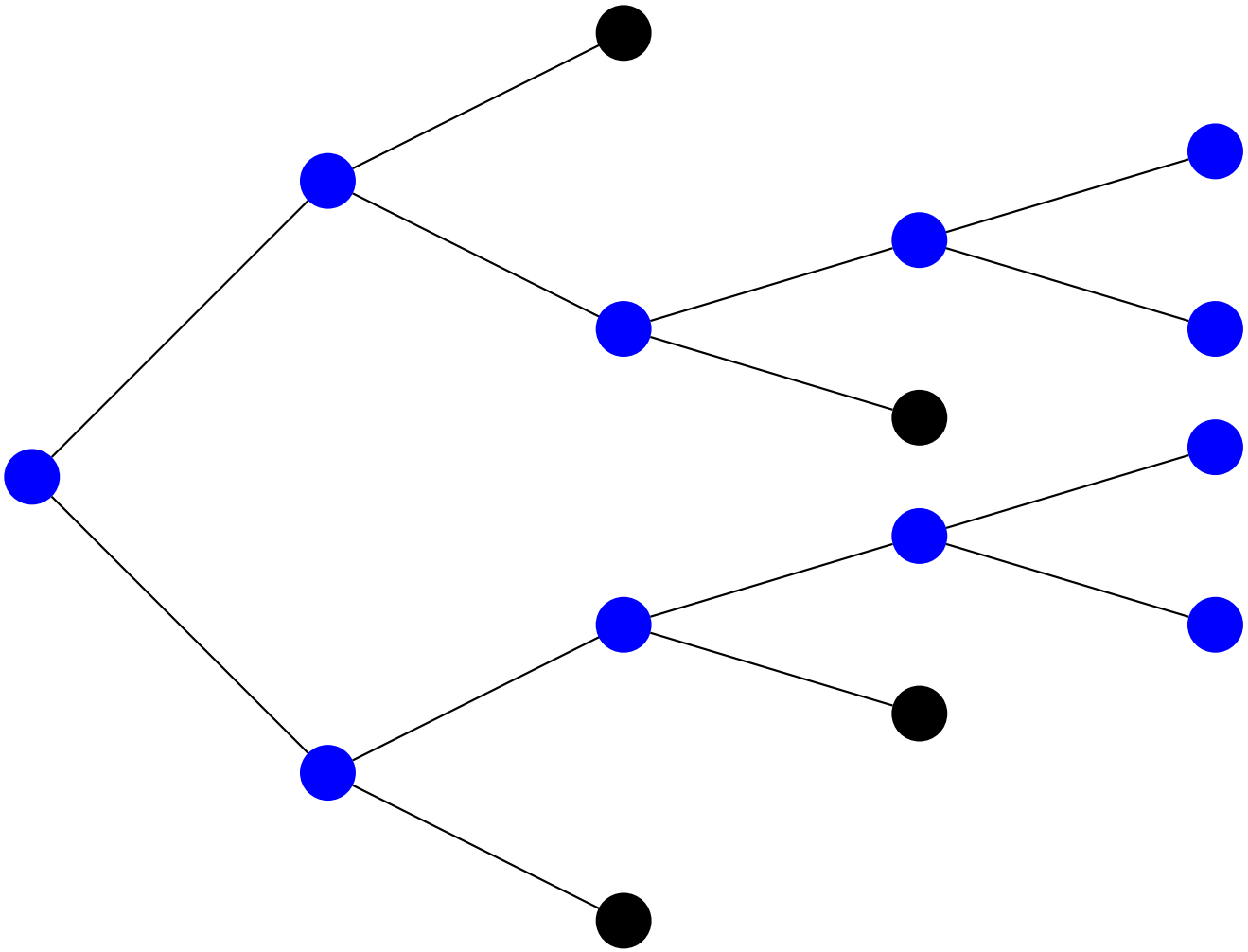
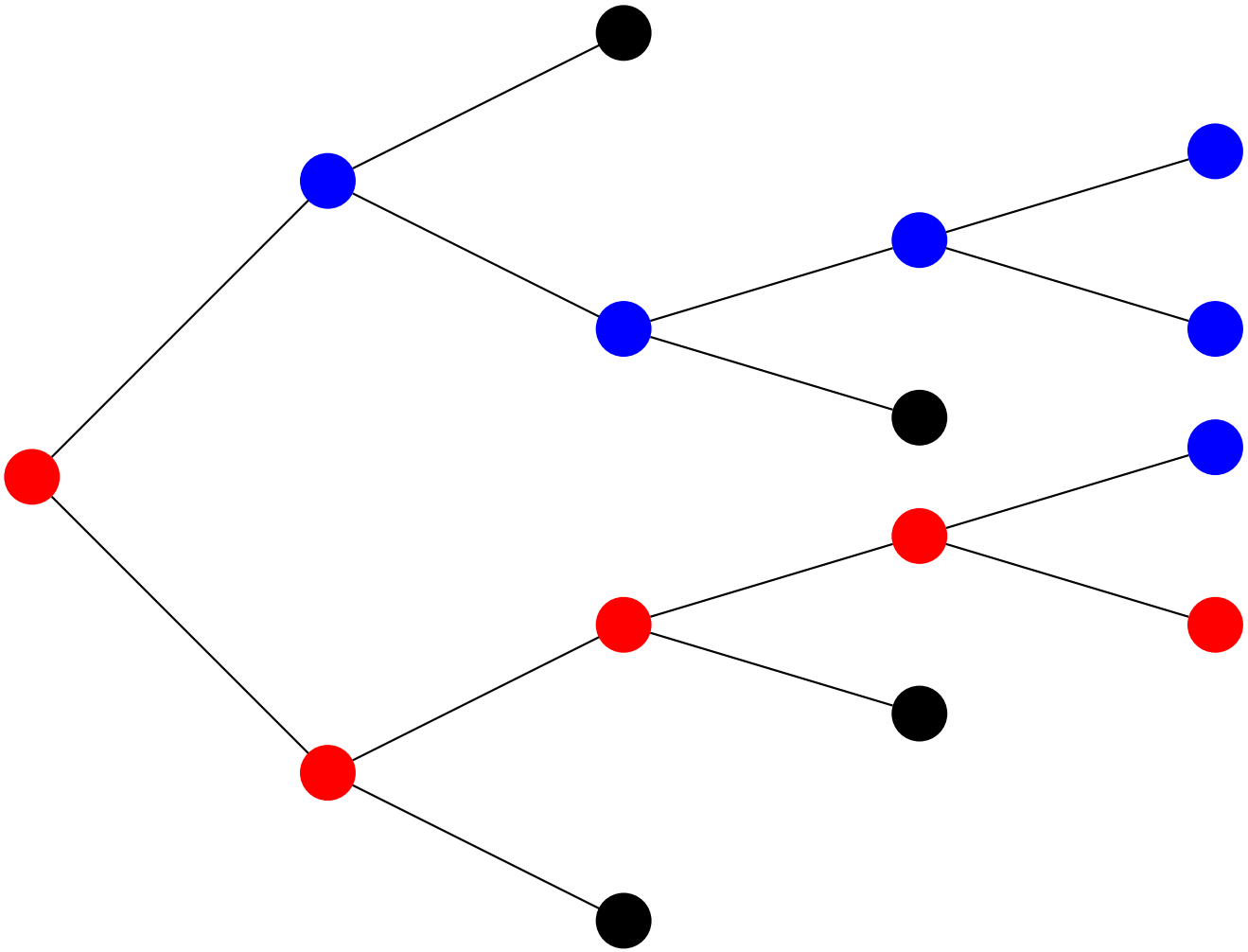


Approximate ML decoder: k=1,B=2 – Expand



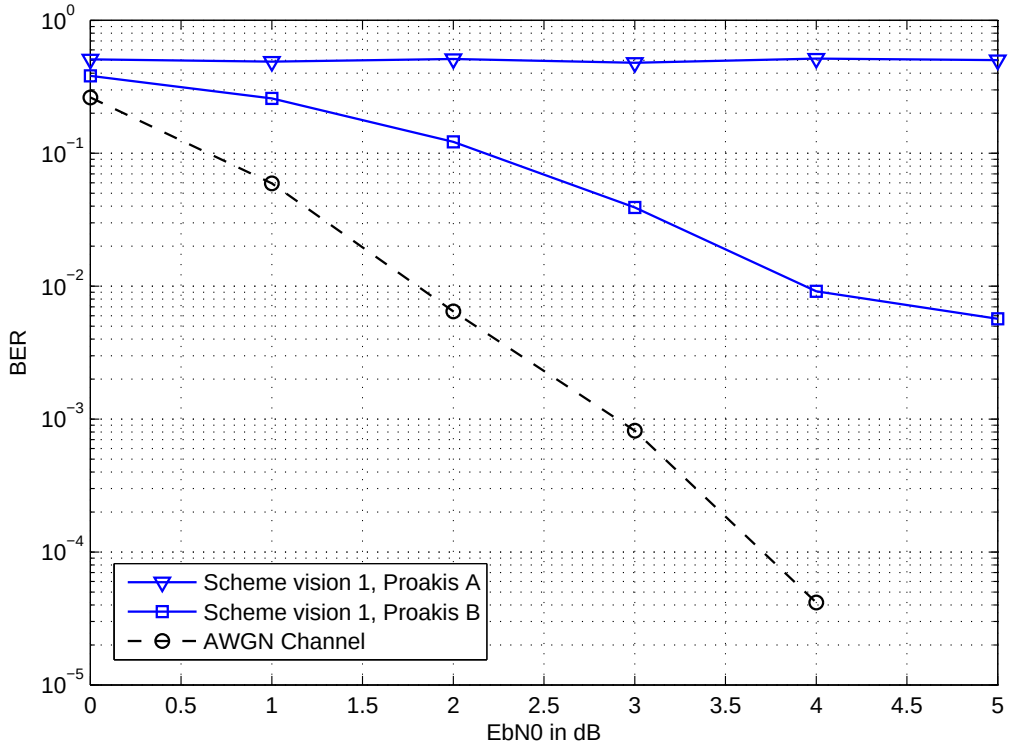
Approximate ML decoder: k=1,B=2 – Make Decision



ML equalization

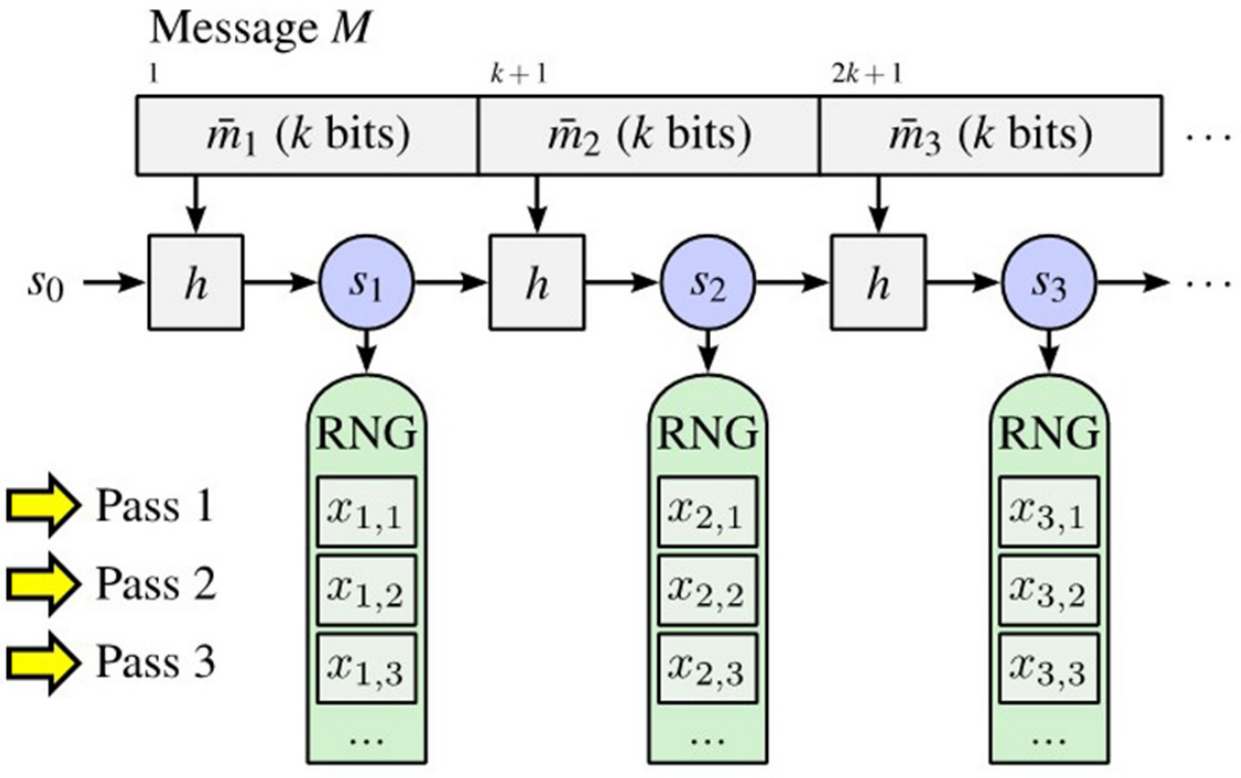
- A frequency-selective channel: $y_n = \sum_{l=0}^{L} h_l x_{n-l} + w_n$
- The ML equalization rule:
  $\widehat{M} = arg \min_{m_i \in \{0 \cdots 2^q\}} (\sum_{i=1}^{n} \| y_i - \sum_{l=1}^{L} h_l x_{i-l}(m_{i-l}) \|)$
- Considering the encoding process where $m_(i - l)$ are generated by RNG with seed s_j so
  $\widehat{M} = arg \min_{m_i \in \{0 \cdots 2^q\}} (\sum_{i=1}^{n} \| y_i - \sum_{l=1}^{L} h_l x_{i-l}(RNG(s_j)) \|)$

Simulation Results 1

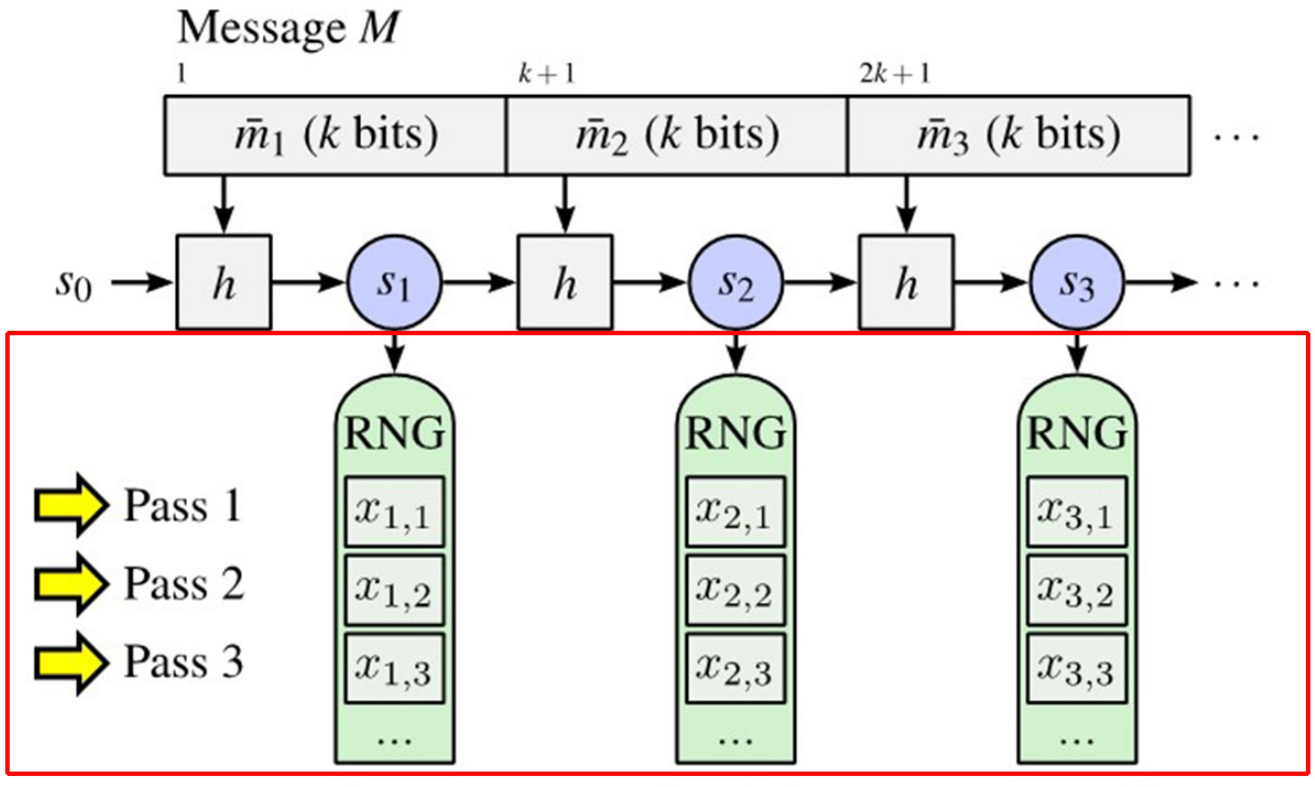


The results didn't desirable.
The mainly reason is the correlations between the spines are too strong and make the beam algorithm prune the wrong branch.

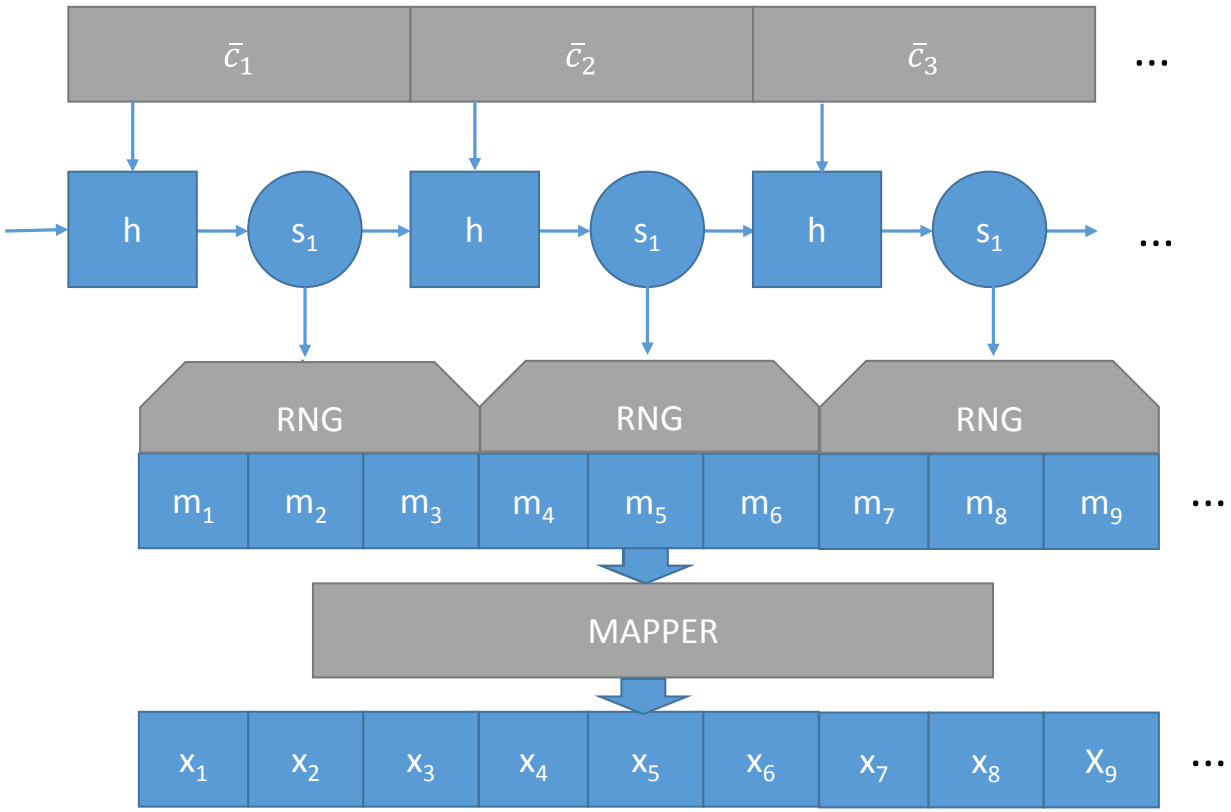
Original Diagram



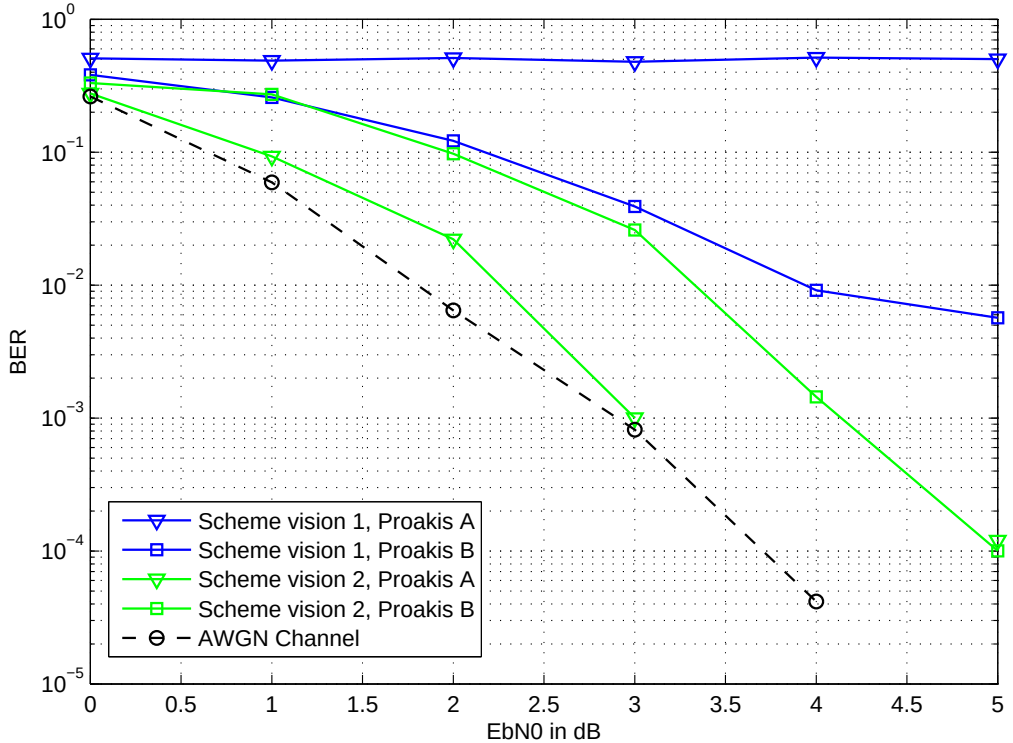
Original Diagram



Modified Diagram

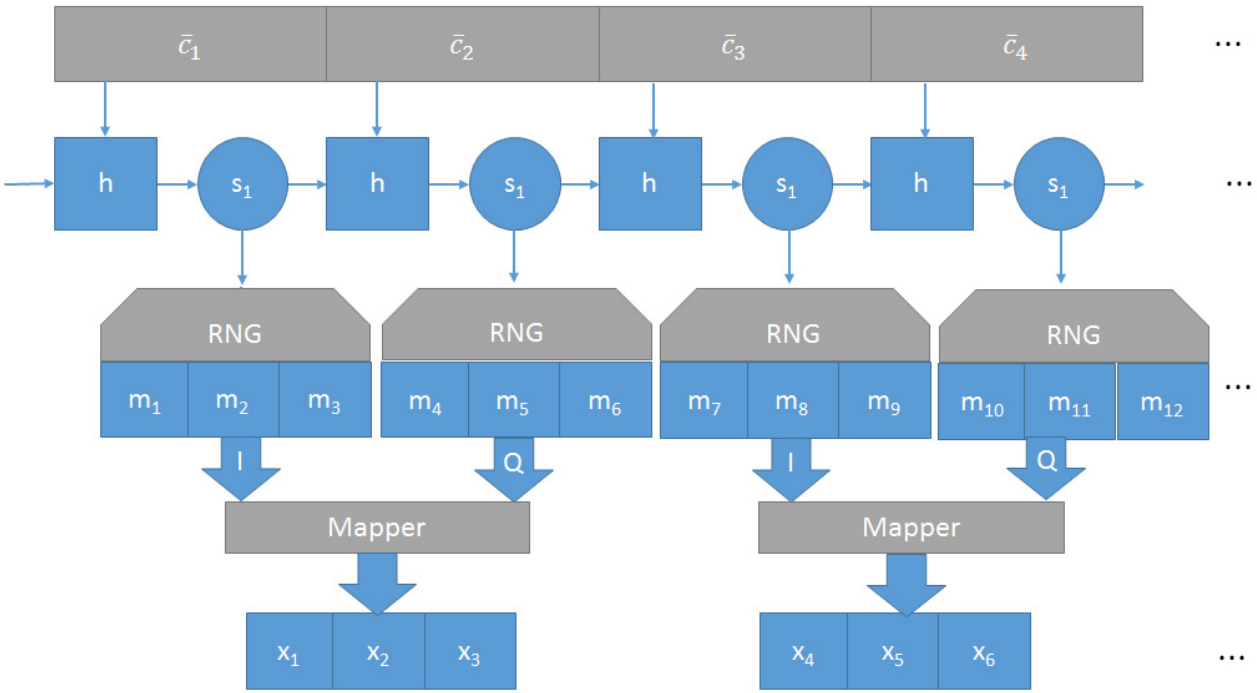


Simulation Results 2

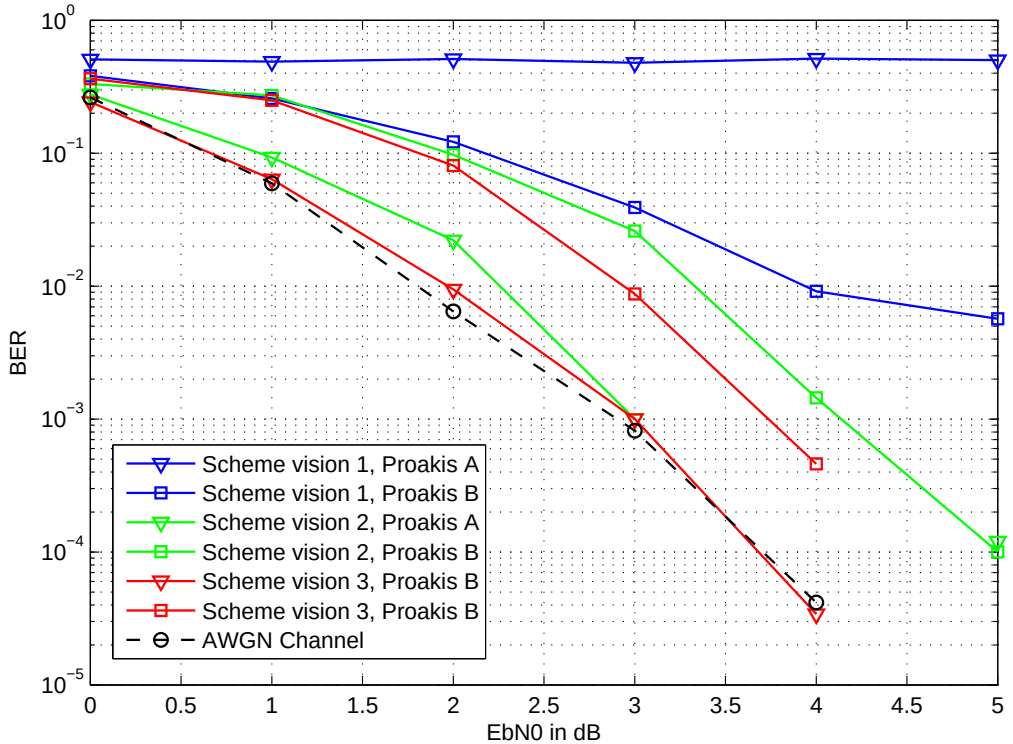


Modification of I/Q Paths

After the modification before. If the group length is long enough the only correlation just exits between adjacent spines. To break these correlations make the adjacent spines are sent respectively by I/Q paths.



Simulation Results 1



After the modification the results of channel Proakis A have been very closed to the AWGN performance. But the Proakis B one is still high.