THE UNIVERSITY OF WARWICK

MSc Examinations: Summer 2017

Algorithmic Game Theory

- **Time allowed: 3 hours.**
- Answer exactly **FOUR** out of six questions. Make it clear on the cover of the answerbook which four questions you have selected.
- Read carefully the instructions on the answerbook and make sure that the particulars required are entered on each answerbook. Give yourself plenty of space, and start each question on a fresh page of the answerbook. Clearly mark any rough work.
- Calculators are not allowed.

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1. (a) State Sperner's Lemma. [7]

   (b) State the definition of incentive compatibility for auction mechanisms. [4]

   (c) Consider a combinatorial auction over items $\{A, B, C, D\}$ and with seven single-minded bidders. The auction uses the VCG mechanism and the bidders declare the following bids:

   - ({C,D},7)
   - ({A},1)
   - ({A,B,C},8)
   - ({B},4)
   - ({B,C},5)
   - ({B,D},7)
   - ({C},3)

   i. Determine the winners of the auctions. [6]

   ii. Determine all payments and the revenue of the auctioneer. [8]

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2. (a) Solve the following two-player game using the iterated elimination of dominated strategies. Explain your answer step by step. [6]

I \ II	X	Y	Z
A	0 4	4 1	1 2
B	2 3	1 2	4 5
C	3 2	2 -1	1 3
D	1 7	2 6	3 4

(b) State the definition of a Nash equilibrium and the definition of an ε-Nash equilibrium. [6]

(c) Briefly discuss why, in general games, typically only algorithms to compute ε-Nash equilibrium are considered. [3]

(d) Give all Nash equilibria of the following two-player zero-sum game. Explain your answer. The values given are the payoffs for player I. Player I wants to maximize her payoff and player II wants to minimize player I's payoff. [10]

I \ II	X	Y	Z
A	0	2	5
B	9	4	1

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3. Consider the following two-player game:

I \ II	A	B
1	6 1	7 2
2	2 2	0 0

   (a) State the definition of a correlated equilibrium for a game. [6]

   (b) Find some Nash equilibrium for the two-player game above and explain why this is indeed a Nash equilibrium. [4]

   (c) Write down the constraints defining the set of correlated equilibria for this game. [10]

   (d) Write down the social utility function, and then derive a socially optimal correlated equilibrium (that is, one that maximises the social welfare) and the social utility value. [5]

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4. (a) Consider a load balancing game with six tasks and two machines. Both machines are identical with speed 1. The tasks have weights $1, 1, 3, 4, 4, 7$, respectively. Find the best (minimising the makespan) pure Nash equilibrium for this game and explain why this is a Nash equilibrium and why it is the best Nash equilibrium. [9]

   (b) Consider a load balancing game with six tasks and three machines. All three machines are identical with speed 1. The tasks have weights $1, 1, 3, 4, 4, 7$, respectively. Find the worst (maximising the makespan) pure Nash equilibrium for this game and explain why this is a Nash equilibrium and why it is the worst Nash equilibrium. [10]

   (c) Show that a mixed profile for the load balancing game in part (b), in which each task is assigned with probability $\frac{1}{3}$ to the first machine, $\frac{1}{3}$ to the second machine, and $\frac{1}{3}$ to the third machine, is a Nash equilibrium. [6]

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5. We want to use methods from the Lemke-Howson algorithm to find Nash equilibria in the following two-player game. The numbers in the lower left corners denote the payoff for player I and the numbers in the upper right corners denote the payoff for player II.

I \ II	3	4	5
1	2	6	7
	8	1	4
2	8	4	0
	6	4	3

   (a) Draw player I's mixed strategy simplex and subdivide it into best response regions of player II. Give the coordinates of all relevant points. [7]

   (b) Draw player II's mixed strategy simplex and subdivide it into best response regions of player I. Give the coordinates of all relevant points. [9]

   (c) Label the facets of the mixed strategy simplices in parts (a) and (b) and use the resulting diagrams to identify *all* Nash equilibria of the game. Explain your answer. [6]

   (d) Which of the equilibria does the Lemke-Howson algorithm find if the first label dropped is label 2? [3]

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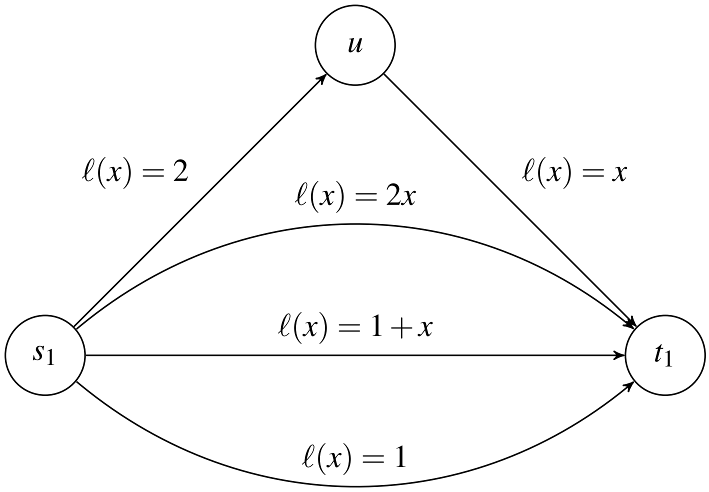

Figure 1: A selfish routing game.

6. (a) Define the price of anarchy for an instance of a nonatomic selfish routing game. [5]

   (b) Consider an instance of the network routing game in Figure 1 with single source s, single destination t, unit traffic rate, and with latency functions indicated above each edge. Find the optimal solution and a Wardrop equilibrium flow, and compute their costs. Explain your answer. [11]

   (c) State the Pigou bound and calculate the Pigou bound for the set of functions $\mathscr{L}$ of the form $\ell_e(x) = x + b_e$ with $b_e \geq 0$. [9]

End (of CS4091, pages 1–5)