CS4091

THE UNIVERSITY OF WARWICK

MSc Examinations: Summer 2016

Algorithmic Game Theory

- **Time allowed: 3 hours.**
- Answer exactly **FOUR** out of six questions. Make it clear on the cover of the answerbook which four questions you have selected.
- Read carefully the instructions on the answerbook and make sure that the particulars required are entered on each answerbook. Give yourself plenty of space, and start each question on a fresh page of the answerbook. Clearly mark any rough work.
- Calculators are not allowed.

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1. Consider the following game.

I \ II	l	c	r
T	0 2	2 2	3 1
B	5 0	4 4	2 3

(a) Find all Nash equilibria of the game. [8]

(b) In two-player zero-sum games, a pair of maximin- and minimax-strategies forms a Nash equilibrium. This is not necessarily true for games that are not two-player zero-sum games. However, even in games that are *not* zero-sum, maximin- and minimax-stratgies are well defined and we can compute them. Compute player I's maximin-strategy in the game above. [8]

(c) Find a maximin-strategy of player II. Here, a maximin-strategy y of player II is defined as

$$y \in \arg \max_{y' \in \Delta^{|S_2|}} \min_{j \in [|S_1|]} (By')_j \ ,$$

where B is the payoff matrix of player II and S_1 and S_2 are the pure strategy sets of player I and II respectively. [9]

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2. Consider the following two-player game. Each player chooses a number from $\{1,2,\ldots,m\}$ and writes it down on a piece of paper. Then, the players compare the two numbers. If the numbers differ by one, the player with the higher number wins £1 from the other player. If the two numbers differ by two or more, the player with the higher number pays £2 to the other player. If both players wrote down the same number, the game is a tie and no money changes hands.

   (a) Model this formally as a game for $m = 3$, that is, write down the payoff matrices for this case. [5]

   (b) Give a Nash equilibrium and the value of the game for $m = 3$. Explain your answer. [12]

   (c) Give a Nash equilibrium and the value of the game for arbitrary values of m. Explain your answer. [8]

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3. (a) Discuss the Pigou bound (for selfish routing games) and its use. [8]

   (b) Consider an auction in which a single item is sold to $n \geq 3$ bidders. The bidder with the highest bid receives the item and has to pay the amount of the third highest bid. Prove or disprove that this auction is incentive compatible. [7]

   (c) Consider a combinatorial auction over items {A, B, C, D, E, F} and with five single-minded bidders. The auction uses the VCG mechanism and the bidders declare the following bids:

   - $(\{A,C,D,E\},10)$,
   - $(\{B,C,E,F\},14)$,
   - $(\{A\},4)$,
   - $(\{E\},4)$, and
   - $(\{F\},6)$.

   i. Determine the winners of the auction. Explain your answer. [4]

   ii. Determine all payments and the revenue of the auctioneer. Explain your answer. [6]

4. Consider the following two-player game. The numbers in the lower left corners denote the payoff for player I and the numbers in the upper right corners denote the payoff for player II.

I \ II	A	B	C
1	0	2	3
	0	3	0
2	3	2	0
	2	2	0
3	0	0	1
	3	0	1

(a) For each player, draw the player's mixed strategy simplex and subdivide it into best response regions of the other player. Give the coordinates of all relevant points. [12]

(b) Label the facets of the mixed strategy simplices and use the resulting diagram to identify *all* the Nash equilibria of this game. Briefly explain your answer and give the resulting mixed strategies of the players as well as their expected payoffs. [7]

(c) Step by step, give the path which the Lemke-Howson algorithm follows if the first label dropped is label B. [6]

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5. (a) Consider a two-player game with mixed strategy sets Δ^{m_1} and Δ^{m_2} and payoff matrices A and B for player I and II, respectively. Give a continuous function from $\Delta^{m_1} \times \Delta^{m_2}$ to $\Delta^{m_1} \times \Delta^{m_2}$, whose fixed points correspond to Nash equilibria in the game. You do not have to prove that your function has the desired property. [7]

(b) Consider a load balancing game with four tasks and two machines. Both machines are identical with speed 1. The tasks have weights $2,2,4,4$, respectively.

(i) Find the best pure Nash equilibrium for this game. [4]

(ii) Find the worst pure Nash equilibrium for this game. [4]

(iii) Find the best mixed Nash equilibrium for this game. [4]

(iv) Find the worst mixed Nash equilibrium for this game. [6]

In each case, your task is to give the entire strategy profile and not only the values of the makespan. Explain your answers.

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6. (a) Consider the selfish routing game with an arbitrary number of nodes and edges, and with an arbitrary number of commodities, in which the possible latency functions are all constant functions. Give a tight bound for the price of anarchy for networks with such latency functions. [6]

   (b) Define the notion of a Wardrop equilibrium for selfish routing games. [5]

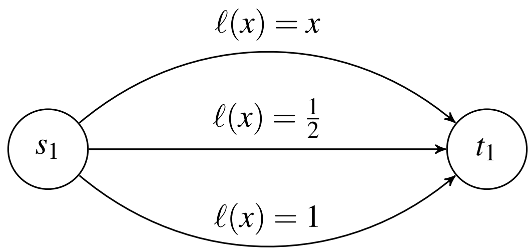

Figure 1: A selfish routing game.

   (c) Consider the instance of the selfish routing game from Figure 1 with single source s_1 and single destination t_1, unit traffic rate (i.e. $r_1 = 1$), and with latency functions marked at the top of each edge. Find the optimal flow and flow that is a Wardrop equilibrium and compare the cost of the two flows. Explain your answer. [9]

   (d) Give an example of a selfish routing game for which the price of anarchy is at least 1.5. [5]

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