ALGORITHMIC GAME THEORY #1

PAOLO TURRINI

Normal form games and equilibria

1) Each one of two bars [1] charges its own price for a beer, choosing between £2, £4, or £5. The cost of obtaining and serving the beer can be neglected. It is expected that 6000 beers per month are drunk in a bar by tourists, who choose one of the two bars uniformly at random, and 4000 beers per month are drunk by natives, who go to the bar with the lowest price, and split evenly in case both bars offer the same price.

1.1) Model this scenario as a normal form game.

1.2) Calculate all the equilibria in strictly dominant strategies.

1.3) Calculate all the pure strategy Nash Equilibria.

1.4) Show the procedure of iterated elimination of strictly dominated strategies, explaining each step, until convergence.

2) Consider the following game:

	L	R
T	5, 1	0, 10
B	−19, 1	−20, −2

2.1) Calculate all the Pareto Optimal outcomes.

2.2) Calculate all the mixed strategy Nash equilibria. [2]

2.3) Show the procedure of iterated elimination of strictly dominated strategies, explaining each step, until convergence.

[1] This question appeared in a previous exam paper of mine.

[2] Reflect on what happens with the procedure provided in class. Try out different payoffs.

3) An important result in game theory, known as the Oddness Theorem (Wilson 1971), says that almost all normal form games have a finite number of Nash Equilibria (either pure or mixed), and that number is odd.
Find a 2x2 game that has an even number of Nash equilibria (either pure or mixed) [3].

4) Prove that each dominant strategy equilibrium is also a pure Nash equilibrium.[4]

[3]This question requires a certain degree of stubbornness. Don't give up immediately!
[4]We have done it together, make sure you can do it on your own.