THE UNIVERSITY OF WARWICK

Coursework 2 (2018/2019)

Algorithmic Game Theory

Submission deadline: Wednesday 6 March 2019, 12:00 noon

Answer all **THREE** questions.

1. (a) **[12 marks]** Consider the following zero-sum game:

I \ II	A	B	C	D
1	3	1	4	2
2	4	4	5	3
3	2	6	3	5
4	4	5	2	2

Iteratively eliminate dominated strategies. For each elimination, indicate which strategy dominates the eliminated strategy.

(b) **[12 marks]** Solve the remaining game: find a Nash equilibrium and the value of the remaining smaller game.

(c) **[6 marks]** Consider the mixed strategy profile $(\mathbf{x}, \mathbf{y})$ in the original game 4×4 game in part (a) that extends the Nash equilibrium from part (b) by playing the strategies eliminated in part (a) with probability 0.

Argue that the strategies that were eliminated in part (a) are not best responses to the respective mixed strategies $\mathbf{x}$ or $\mathbf{y}$, and hence conclude that $(\mathbf{x}, \mathbf{y})$ is a Nash equilibrium in the original 4×4 game.

Continued

2. Consider the following two-player game:

I \ II	4	5	6
1	0, 1	1, 0	0, 2
2	1, 0	0, 3	0, 1
3	0, 3	0, 2	1, 0

(a) **[10 marks]** Draw the best response diagrams for the game.

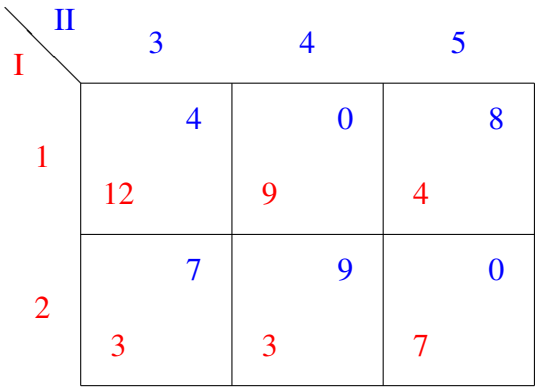
(b) **[10 marks]** Use the best reponse diagrams to find all the Nash equilibria of the game.

(c) **[10 marks]** Find all the Nash equilibria in the following game that differs from the one above only by the payoffs of Player I in strategies 2 and 3 being swapped.

I \ II	4	5	6
1	0, 1	1, 0	0, 2
2	1, 3	0, 2	0, 0
3	0, 0	0, 3	1, 1

Try to reuse your answer to part (a) as much as possible.

3. Consider the following two-player game:



(a) **[20 marks]** Use the Lemke-Howson algorithm to find the Nash equilibrium reached by dropping label 2 in the first step.

Do it graphically, by drawing the best response diagrams and then following the appropriate paths in them. For every step, explain which label is dropped and which label is gained, in which of the two best response diagrams.

(b) **[20 marks]** Use the Lemke-Howson algorithm to find the Nash equilibrium reached by dropping label 1 in the first step.

Do it algebraically, by setting up the appropriate systems of linear equations with variable non-negativity constraints, and by performing complementary pivoting steps to move from one dictionary (basic solution) to another. For every step, explain which label is dropped, which variables are swapped, and hence which label is gained.

End