

## ALGORITHMIC GAME THEORY #2

PAOLO TURRINI

### EQUILIBRIA

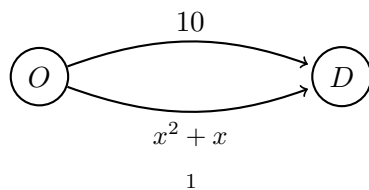
1) Consider the following game:

	L	R
T	$\begin{array}{c} -5 \\ 3 \end{array}$	$\begin{array}{c} 0 \\ 0 \end{array}$
B	$\begin{array}{c} -1 \\ 2 \end{array}$	$\begin{array}{c} 0 \\ -2 \end{array}$

- 1.1) Is it a potential game? If so, provide a potential function.
  - 1.2) Does it admit correlated equilibria that are not Nash equilibria? If so, provide one.
  - 1.3) Give maximin and minimax strategies, for both players. Are they unique?
- 2) Every normal form game with a finite set of actions per player has a mixed strategy Nash equilibrium. Find a game with an infinite set of actions that has no mixed strategy Nash equilibrium.

### CONGESTION GAMES

1) 10 individuals want to reach node  $D$  from node  $O$  in the network below, where  $x$  is the amount of individuals using the route.



1.1) Calculate all Nash Equilibria.

1.2) Calculate the optimal flow.

1.3) Calculate the POA.<sup>1</sup>

2) Take a non-atomic congestion game played on a Pigou network and suppose that the cost function is affine, i.e., of the form  $c(x) = ax + b$ , for some  $a, b > 0$ , and  $x \leq r \in \mathbb{R}_+$ . What is the worst POA you can get in this type of networks? Show your reasoning.

---

<sup>1</sup>What if there are multiple equilibria? how does this affect the definition of POA?