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Exercises 4
CS409 Algorithmic Game Theory
Term 2, 2018/2019

1. (a) Find a maximin-strategy for player I in the following game of Chicken.

I \ II	swerve	straight
swerve	0 0	1 -1
straight	-1 1	-10 -25

   (b) Find a maximin-strategy for player II in the above game. Here, a maximin-strategy y of player II is defined as

$$\boldsymbol{y} \in \arg \max_{\boldsymbol{y}' \in \Delta^{|S_2|}} \min_{j \in [|S_1|]} (\boldsymbol{B}\boldsymbol{y}')_j \ ,$$

   where $\boldsymbol{B}$ is the payoff matrix of player II and S_1 and S_2 are the pure strategy sets of player I and II respectively.

   (c) Check whether these two strategies form a Nash equilibrium.

2. One version of Brouwer's fixed point theorem states that any continuous function $f \colon S \to S$, from a convex compact set S to itself, must have a fixed point. A compact set is a set that is both bounded and closed. Show that removing any single one of the four stated conditions ("continuous", "convex", "bounded", and "closed") makes the statement of the theorem false in general. In other words, show the following:

   - There is a (non-continuous) function $f \colon S \to S$ from a convex compact set S to itself that does *not* have a fixed point.
   - There is a continuous function $f \colon S \to S$ from a compact set S to itself that does *not* have a fixed point.
   - There is a continuous function $f \colon S \to S$ from a convex bounded set S to itself that does *not* have a fixed point.
   - There is a continuous function $f \colon S \to S$ from a convex closed set S to itself that does *not* have a fixed point.

   If you do not recall what the terms continuous, convex, bounded, or closed mean, look them up and re-familiarize yourself with them.

3. Recall the poisoned drink game.

I \ II	own	other
own	-1, 1	1, -1
other	1, -1	-1, 1

Consider the function f defined in the proof of Nash's theorem and calculate the values of $f(x, y)$ for the following strategy profiles of the game.

(a) $\sigma_1 := ((1, 0), (1, 0))$

(b) $\sigma_2 := f(\sigma_1)$

(c) $\sigma_3 := f(\sigma_2)$

(d) $\sigma_4 := ((1/2, 1/2), (1/2, 1/2))$

4. In the lecture we used Sperner's lemma to prove Brouwer's fixed point theorem. Try to do it the other way around, i.e., use Brouwer's fixed point theorem to prove Sperner's lemma. For this, for any Sperner coloring of a triangle T that is subdivided into smaller triangles, construct a continuous function from T to T such that the function only has a fixed point if there exists a tri-chromatic triangle.