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Exercises 5
CS409 Algorithmic Game Theory
Term 2, 2018/2019

1. Find a Nash equilibrium of the following two-player zero-sum game. The values given in the matrix below are the payoffs for player I. Player I wants to maximize her payoff and player II wants to minimize player I's payoff.

I \ II	X	Y
A	12	6
B	8	9
C	5	10

2. Alice and you meet in a pub. Alice proposed to play the following game. Both of you simultaneously put down £1 or £5 on the table. If the two of you put down different amounts, then you get all the money on the table, and otherwise Alice gets all the money. Should you accept to play the game?

   (a) Model this formally as a two-player zero-sum game.

   (b) Find all Nash equilibria of the game. Argue why these are in fact all equilibria.

   (c) Derive your expected payoff and Alice's expected payoff for each of the Nash equilibria.

   (d) Do (a), (b), and (c) for general amounts of money £α and £β instead of £1 and £5.

3. Consider a finite n-player game in which S_i is the set of pure strategies for player i for each $i = 1, 2, \ldots, n$; we write $\Delta(S_i)$ for the set of mixed strategies of player i.

   (a) Prove that the set of mixed strategy profiles $\Delta(S_1) \times \Delta(S_2) \times \cdots \times \Delta(S_n)$ is convex.

   (b) Consider the set of mixed Nash equilibria (which is a subset of the set of mixed strategy profiles). Is it convex? If yes, give a proof; if not, give a counterexample.