AGENT-BASED SYSTEMS SEMINAR #1

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1. Strategies

1.1. **Chess.** Zermelo's theorem is formulated for chess. But what is really specific to chess in the theorem? Can you use the same result for other games?

Consider now a variant of the game of chess where either the game terminates with a win for White, or a win for Black or it goes on forever. Does Zermelo's theorem work for this variant? Prove it, or give a counterexample.

2. Knowledge

2.1. **Epistemic models.** Let $\{w_1, w_2, w_3\}$ be a set of worlds and $N = \{Ann, Bob\}$ be a set of agents [1].

(1) Let $E = \{w_1\}$ be a fact. Find an indistinguishability relation for each agent so that, at w_1, fact E is distributed knowledge at w_1.
(2) Let $E = \{w_1\}$ be a fact. Find an indistinguishability relation for each agent so that, at w_1, fact E is common knowledge at w_1.
(3) Let $E = \{w_1\}$ be a fact. Find an indistinguishability relation for each agent so that, at w_1, fact E is distributed knowledge at w_1 but not common knowledge at w_1.
(4) Is it possible to find a fact and an indistinguishability relation such that (1) w_3 is disconnected from all the other worlds by the indistinguishability relation of either agent and (2) at some world, that fact is general knowledge but not common knowledge? Explain.

2.2. **Facts in need of proofs.** The following statements are true. Prove it.

(1) $K_i E \subseteq E$

(2) $K_i E \subseteq K_i K_i E$

(3) $\neg K_i E \subseteq K_i \neg K_i E$ [2]

(4) If $E \subseteq F$ then $K_i E \subseteq K_i F$

[1] This question appeared in a previous exam paper of mine

[2] Reflect on the last three statements. Can you see a connection with the 'axioms' we imposed on the indistinguishability relation?

(5) $(K_iE \cap K_iF) \subseteq K_i(E \cap F)$

(6) $CKE \subseteq E$

(7) $CKE \subseteq K_iCKE$

2.3. **True of false?** Consider now the converse [3] of the statements above. Do they hold? Prove it, or give a counterexample.

[3]Either reverse the subset sign or the implication.