CS409

THE UNIVERSITY OF WARWICK

Coursework #1 (2018/2019)

Algorithmic Game Theory

Deadline: February 12th, 2019 (at noon).

Please see the instructions on Tabula and the module website.

Answer **FOUR** questions.

For all your answers, provide **ALL** the calculations.

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1. Alice and ten more friends are playing a game: each of them, independently and simultaneously, chooses an integer between 0 and 4, i.e., any $x \in [0, 4] \cap \mathbb{Z}$. The winner is the player whose choice is the closest to $\frac{2}{3}$ of the average of the choices of all players.

   (a) Model this scenario as a normal form game. [10]

   (b) Calculate all pure strategy Nash equilibria. [10]

   (c) What is the threshold on the fraction of the average that the game needs to have so that 0 is the only Nash equilibrium choice? [5]

Continued

2. Two food vendors are to decide where to park their truck for the street festival. There is only one road, which is a vector of positions $(p_0, p_1, \ldots, p_k)$. Vendors pick one position each. Once they do this, the leftmost vendor attracts all the customers on their left side, the rightmost vendor all the customers on their right side. They split in half all the customers in between their positions, comprising the ones in their positions as well.

   Assume there is only one customer per position, and that each customer is worth 1.

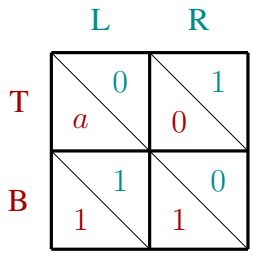
   (a) Model this scenario as a normal form game. [10]

   (b) Calculate all pure strategy Nash equilibria, for $k = 9$. [10]

   (c) Calculate the outcome of IESDS for k even. [5]

Continued

3. Let $a \in \mathbb{R}$. Consider the following game



(a) What are all the values of a that make (T, L) Pareto Optimal? [6]

(b) What are all the values of a that make (B, L) a pure strategy Nash equilibrium? [6]

(c) Let $a = 2$. Calculate all mixed strategies Nash equilibria. [7]

(d) Let $a = 1$. Is this a potential game? If so, provide a potential function. [6]

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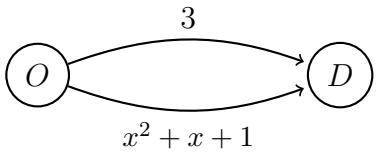
4. Consider the following non-atomic congestion game, where a divisible unit of traffic (i.e., traffic rate is 1) is going from O to D.



(a) Calculate the equilibrium flow. [6]

(b) Calculate the optimal flow. [9]

(c) Calculate the price of anarchy. [10]

End