review

Sunday, June 2, 2019 3:38 PM

exam date 6/11, 3:55pm, room 5503, 2 hour long

range: chapter 1-8. 4 pieces of A4 - double sided formular sheet.

*: not in final exam

In class 60-min quiz(three in total) 30%; Final 30%; Homework 40%

☆: important, reviewing previous quiz is also helpful.

problem type: ① fill-in-the-blank, no need for derivation shown

② proving / solving - with derivation needed

Ch1

- Stern - Gerlach experiment
- concept of superposition
- Quantum state - kets, bras, operators

  $|\psi\rangle$, $\langle\psi|$, $\hat{A}|\psi\rangle = |\phi\rangle$

  $|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$, $|\phi\rangle = \begin{pmatrix} c \\ d \end{pmatrix}$, inner product $\langle\phi|\psi\rangle = c^* a + d^* b$

- Hermitian operator and basis

  eigenvalue, eigenstates ☆

  operator commutation ☆ mutual eigenstates.

  operator function.

  unitary operator $UU^\dagger = \mathbb{1} = U^\dagger U$ ☆

- operator / state expressed in a certain basis (representation theory)

  basis $\{|a_i\rangle\}$ — Hilbert space, normal, orthogonal state set

  $\langle a_i|a_j\rangle = \delta_{ij} = \begin{cases} 1, & i=j \\ 0, & \text{else.} \end{cases}$

  measurement and collapse of quantum state.

  $|\psi\rangle = \sum_i c_i |a_i\rangle$, c_i complex num.

  $|c_i|^2$ — probability of finding particle at $|a_i\rangle$ state.

  Pauli - operator and operation ☆²

  $\sigma_x, \sigma_y, \sigma_z$

  $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. $[\sigma_i, \sigma_j] = i\varepsilon_{ijk}\sigma_k$ ☆

  $\sigma_i^2 = 1$

- infinite dimension $\int \langle x_i|x_j\rangle = \delta(x_i - x_j)$

$\sigma_x = \begin{pmatrix} & \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} & \\ i & \end{pmatrix}$ $\sigma_i^2 = \mathbb{1}$

- infinite dimension

① position operator, $\hat{X}$, position eigenstate $\{|x\rangle\}$, $\begin{cases} \langle x_i|x_j\rangle = \delta(x_i - x_j) \\ \mathbb{1} = \int_{-\infty}^{+\infty} |x\rangle\langle x| \, dx \end{cases}$ ☆

wavefunction $\psi(x) = \langle x|\psi\rangle$, $|\psi(x)|^2 dx$ — probability of finding particle at $[x, x+dx]$

normalization $1 = \langle\psi|\psi\rangle = \int_{-\infty}^{+\infty} \psi^*(x)\psi(x)\, dx$, ☆

inner product $\langle\varphi|\psi\rangle = \int_{-\infty}^{+\infty} \varphi^*(x)\psi(x)\, dx$

requirement $\lim_{x\to\pm\infty} \psi(x) \to 0$ for a physical wavefunction, $\begin{cases} |\psi|^2 \text{ integrable.} \\ \text{single value} \\ \text{continuous} \end{cases}$

② momentum operator

translation operator $\xrightarrow{\text{reversible, Hermitian}}$ $\hat{p}$, $[\hat{x}, \hat{p}] = i\hbar\mathbb{1}$ ☆^{2}

position representation $\langle x|\hat{p}|\psi\rangle = -i\hbar \frac{\partial}{\partial x}\psi(x)$ ☆

$\begin{cases} xp - px = i\hbar\mathbb{1} \\ xp = px + i\hbar\mathbb{1} \\ px = xp - i\hbar\mathbb{1} \end{cases}$

momentum eigenvalue / eigenstate $\hat{p}|p\rangle = p|p\rangle$

$\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}$ ⟵ ☆

$\psi(p) = \langle p|\psi\rangle = \int_{-\infty}^{+\infty} dx \langle p|x\rangle\langle x|\psi\rangle = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} e^{-i\frac{px}{\hbar}} \psi(x)$

uncertainty principle $\langle\Delta x^2\rangle\langle\Delta p^2\rangle \geq \frac{1}{4}\hbar^2$, or $\Delta x \Delta p \geq \frac{\hbar}{2}$

expectation of an operator $\hat{A}$

$$\langle\hat{A}\rangle = \langle\psi|\hat{A}|\psi\rangle \overset{\text{position}}{=} \int_{-\infty}^{+\infty} dx\, dx' \langle\psi|x\rangle\langle x|A|x'\rangle\langle x'|\psi\rangle$$

(✱) postulates of QM

Ch 2 Q dynamics (non relativistic)

§1. time displacement op ⟶ Hermitian operator — Hamiltonian

Schrödinger's equation

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle \quad ☆$$

$H = T + V = \frac{p^2}{2m} + V$

$x \to i\hbar\frac{\partial}{\partial p}$

$\left(\frac{p^2}{2m} + V\right)\psi(p,t) = i\hbar\frac{\partial}{\partial t}\psi(p,t)$

Choice of representation $\begin{cases} \text{momentum} \\ \text{position} \end{cases}$

$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$

§2 static Schrödinger's equation

$H|\psi\rangle = E|\psi\rangle = i\hbar\frac{\partial}{\partial t}|\psi\rangle \Rightarrow |\psi(t)\rangle = e^{-iEt/\hbar}|\psi(0)\rangle$ ☆^{2}

a basis of eigenstates of H $\{|\psi_i\rangle\} \Rightarrow |\phi(t=0)\rangle = \sum_i a_i|\psi_i\rangle$

then $|\phi(t)\rangle = \sum_i a_i e^{-iE_it/\hbar}|\psi_i\rangle$ ☆^{3}

simple proof $|\phi(t)\rangle = \sum_i a_i(t)|\psi_i\rangle$

$\Rightarrow i\hbar\dot{a}_i(t) = a_i(t)E_i \Rightarrow a_i(t) = a_i(0)e^{-iE_it/\hbar}$

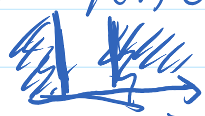
simple most $|\psi(t)\rangle = \sum_i a_i(t)|\psi_i\rangle$

$\Rightarrow i\hbar \dot{a}_i(t) = a_i(t) E$, $\Rightarrow a_i(t) = a_i(0) e^{-iEt/\hbar}$

$\psi(x,t) = \phi(x) e^{-iEt/\hbar}$ $\Rightarrow \begin{cases} -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\phi(x) + V(x)\phi(x) = E\phi(x) \\ \psi(x,t) = \phi(x) e^{-iEt/\hbar} \leftarrow \text{true S-eq sol.} \end{cases}$

time-independent

example $V(x) = \begin{cases} 0 & -\frac{a}{2} < x < \frac{a}{2} \\ +\infty & \text{other} \end{cases}$



$\phi(\pm\frac{a}{2}) = 0$ ☆

time-independent

eigen: $\eta(x) = \sqrt{\frac{2}{a}} \sin\left(k(x+\frac{a}{2})\right)$, $k = \frac{n\pi}{a}$, $E = \frac{n^2\pi^2\hbar^2}{2ma^2}$, n = integer $n = 1, 2, \dots$

eigen: $\boxed{\psi(x,t) = \sqrt{\frac{2}{a}} \sin\left(k(x+\frac{a}{2})\right) e^{-iEt/\hbar}}$ ☆ phase.

§2 potential barrier finite, $\eta(x)$, $\eta'(x)$ continuous

§3 1D Harmonic oscillator ☆

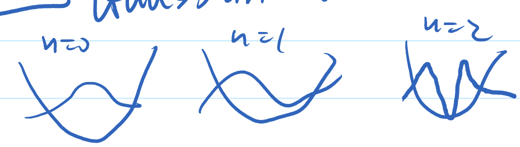
$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$, let $\begin{cases} a = \frac{1}{\sqrt{\hbar\omega}}\left(\sqrt{\frac{1}{2}m\omega^2}\,x + i\frac{p}{\sqrt{2m}}\right) \\ a^+ = \frac{1}{\sqrt{\hbar\omega}}\left(\sqrt{\frac{1}{2}m\omega^2}\,x - i\frac{p}{\sqrt{2m}}\right) \end{cases}$

$\begin{cases} [a, a^+] = \mathbb{1} \text{ ☆} \quad aa^+ = \mathbb{1} + a^+a \dots \\ H = \hbar\omega(a^+a + \frac{1}{2}\mathbb{1}) \end{cases}$, $[H, \hat{a}] = -\hbar\omega a$, $[H, a^+] = \hbar\omega a^+$

$H|n\rangle = \hbar\omega(n+\frac{1}{2})|n\rangle$, $a|n\rangle = \sqrt{n}|n-1\rangle$, $a^+|n\rangle = \sqrt{n+1}|n+1\rangle$

$\langle x|n=0\rangle = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{1}{2}m\omega x^2/\hbar} \leftarrow a|n=0\rangle = 0 \rightarrow$ Gaussian function

$\langle x|n\rangle = \frac{1}{\sqrt{n!}}\left(-\frac{\hbar}{\sqrt{2m}}\frac{\partial}{\partial x} + \sqrt{\frac{1}{2}m\omega}\,x\right)^n \langle x|0\rangle$



solve S-eq for q-harmonic by brute force ☆

☆ o spin resonance, ☆ rotating wave approximation

o ☆ density matrix

Ch3 angular momentum $\leftarrow$ orbital angular momentum

rotation operator $\xrightarrow{\text{unitary}}$ Hermitian operator $\begin{cases} \hat{L}_z = -i\hbar\frac{\partial}{\partial\varphi} \\ \hat{L}_z = xp_y - yp_x \end{cases}$ ☆

$L_y = zp_x - xp_z$, $L_x = yp_z - zp_y$

$[L_x, L_y] = i\hbar L_z$, $\begin{cases} [L_p, L_q] = i\hbar\,\varepsilon_{pqr} L_r \\ \varepsilon_{pqr} = 1, \text{ in order} \\ \quad\quad\ \ -1 \text{ out of order} \end{cases}$ for l, $m = -l, -l+1, \dots, l-1, l$

$L^2 = L_x^2 + L_y^2 + L_z^2$, $[L^2, L_i] = 0$ ☆ L_z eigenstate $\propto e^{im\varphi}$.

out of order

$\vec{L}^2 = L_x^2 + L_y^2 + L_z^2$, $[\vec{L}^2, L_i] = 0$ ☆, for l, $m = -l, -l+1, \dots, l-1, l$

eigenstate $\propto e^{im\varphi}$, $m = 0, \pm 1, \pm 2, \dots$ $\Rightarrow$ l also integer.

$\vec{L}^2 |l,m\rangle = \hbar^2 l(l+1) |l,m\rangle$

$L_z |l,m\rangle = \hbar m |l,m\rangle$ $\boxed{\langle\theta,\varphi|\rangle = Y_l^m}$ ☆

- generic angular momentum

$[\vec{J}^2, J_z] = 0$, $[J_p, J_q] = i\hbar \varepsilon_{pqr} J_r$,

$$\begin{cases} \vec{J}^2 |j,m\rangle = \hbar^2 j(j+1) |j,m\rangle \\ J_z |j,m\rangle = \hbar m |j,m\rangle \end{cases}$$

, $J_+ |j,m\rangle = \hbar\sqrt{j(j+1) - m(m+1)}\, |j,m+1\rangle$ ☆

$J_- |j,m\rangle = \hbar\sqrt{j(j+1) - m(m-1)}\, |j,m-1\rangle$

$J_+ = J_x + iJ_y$, $J_- = J_x - iJ_y$

$[J_z, J_+] = \hbar J_+$, $[J_z, J_-] = -\hbar J_-$

$[J_+, J_-] = 2\hbar J_z$

$J_- J_+ = J^2 - J_z^2 - \hbar J_z$, ...

- coupled 2-particle system

direct product $\otimes$

$|\psi_1\rangle = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$, $|\psi_2\rangle = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$,

$$|\psi_1\rangle \otimes |\psi_2\rangle = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} \otimes \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 a_2 \\ a_1 b_2 \\ b_1 a_2 \\ b_1 b_2 \end{pmatrix} = \begin{pmatrix} a_1 \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} \\ b_1 \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} \end{pmatrix}$$ ☆

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

$$A \otimes B = \begin{pmatrix} a_{11}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} & a_{12}(\ ) \\ a_{21}(\ ) & a_{22}(\ ) \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\ & \dots & & \end{pmatrix}_{4\times 4}$$

$$(A \otimes B) \cdot (|\psi_A\rangle \otimes |\psi_B\rangle) = (A|\psi_A\rangle) \otimes (B|\psi_B\rangle)$$

* entanglement

* quantum logic gate. partial trace and measurement of 2-partial system

- addition of angular momentum

$$\hat{\vec{J}} = \hat{\vec{S}} \otimes \hat{1}_L + \hat{1}_S \otimes \vec{L}$$

with $[\vec{J}^2, J_z] = 0$, $[J_p, J_q] = i\hbar \varepsilon_{pqr} J_r$

$|l - s| \le j \le s + l$ ☆

[illegible]

$|l-s| \le j \le s+l$

$-j \le m_j \le j$, $m_j = m_s + m_l$ ☆

* clebsch-Gordan coefficient

Chapter 4 3D dynamics ☆

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + V(x,y,z).$$

$V(x,y,z) = V(x) + V(y) + V(z)$

- 3D harmonic oscillator — separation of variable
  degeneracy ☆
- spherical coordinate (r,θ,φ), central potential

$$\begin{cases} H = \frac{p_r^2}{2m} + \frac{\vec{L}^2}{2mr^2} + V(r) \\ p_r^2 = -\hbar^2 \frac{1}{r^2}\left(\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}\right) \end{cases}$$ ☆

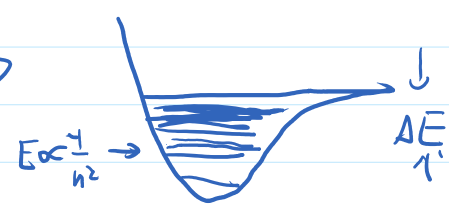
eigen $\begin{cases} \psi(r,\theta,\varphi) = R(r)\,\Phi(\theta,\varphi) \\ \Phi(\theta,\varphi) = Y_l^m(\theta,\varphi) \end{cases}$

eigen eq $R(r) = \frac{u(r)}{r} \Rightarrow \begin{cases} -\frac{\hbar^2}{2m}\frac{\partial^2 u}{\partial r^2} + \overbrace{\left[\frac{l(l+1)}{2mr^2}\hbar^2 + V(r)\right]}^{\text{effective potential}} u(r) = E u(r) \\ \text{boundary, physical: } u(r\to 0) = 0, \quad u(r\to\infty) = 0. \end{cases}$

example spherical square well ☆[3] derive a few times

example H-atom,

$\begin{cases} \text{concept of ionization energy ☆} \\ E \propto -\frac{1}{n^2} \\ \text{Bohr radius} \end{cases}$



* derivation for Hydrogen atom

example

* Linear combination of atomic orbital (LCAO).

Ch5 approximation methods.

§1. time-dependent p-th.

Taylor expansion of small parameter, discuss effects according to different orders.

$H = H_0 + H'$, $H_0 |n^{(0)}\rangle = E_n^{(0)} |n^{(0)}\rangle$

$H|n\rangle = E_n |n\rangle$,

→ derive the example a few times.

$$\begin{cases} |n\rangle = |n^{(0)}\rangle + \sum_{k\neq n} \frac{\langle k^{(0)}|H'|n^{(0)}\rangle}{E_n^{(0)} - E_k^{(0)}} |k^{(0)}\rangle + \cdots \\ E_n = E_n^{(0)} + \langle n^0 | H' | n^0\rangle + \sum_{k\neq n} \frac{|\langle k^{(0)}|H'|n^{(0)}\rangle|^2}{E_n^{(0)} - E_k^{(0)}} \end{cases}$$ ☆

example perturbation around 1-D harmonic oscillator, and in the HW ☆

* §2 time - dependent theory

* degenerate perturbation