(6/11/2022) - Yiren Qin & Arnaud Trouvé

We consider a stationary line fire that emits embers to flight distances (measured from the fireline location, i.e. from the center of cells where the level-set function *f* is equal to 0). The distribution of possible values of ember flight distances is characterized statistically by the probability density function (PDF) of , , and is assumed known. We also assume that the PDF takes non-zero values for .

We further assume a spatial resolution and discretize the PDF of into bins as follows:

We can therefore calculate the value of the PDF in bin number *k*, as:

We also define as the center of the bin where the value of the PDF takes an arbitrary low value:

or

And we have by construction:

Note that the -PDF is the modified (truncated) version of the PDF proposed by Sardoy *et al*., noted :

~~We therefore have:~~

OR (Yiren, 7/15/2022)

Define and based on Sardoy’s PDF:

Then normalize the new truncated PDF:

Let us now consider a particular line fire cell that emits embers at an average rate noted (measured in units of number of particles emitted per unit time) during a certain duration, noted . The total number of embers emitted during that period of time can be expressed as:

These embers land on the ground at different values of the flight distances . Let us call the total number of embers that have reached the bin after the particular line fire cell under consideration stops emitting. We can write:

and

We now turn to the time variations of the number of embers that reach the bin . These time variations are described through the introduction of a time delay, noted, associated with transport by the wind ( is the time required for transport of embers from the line fire cell to the bin *k*). We assume that the delay has the simple form with . More complex treatments of the time delay can be considered in the future.

The ember landing scheme is then:

With this scheme, the expected value of the rate of spread due to ember transport (and assuming for now that the probability of ignition is 1) is:

We now turn to the question of the value of the ember generation rate . There is a minimum value of that is required to guarantee that for each bin *k*, , we get:

Because is a decreasing function of , we see that the most demanding condition is found for the last bin :

This condition corresponds to a requirement for converged statistics. In the model, we call this critical value of required for converged statistics.

Note that with the choice , we have:

Consider the bin *k*. Note that the number of embers landing inside that bin during the time increment is: . If one requires that for each bin *k*, this number is greater than 1, then one gets a new set of condition for converged statistics:

(8/17/2022)

Considering now that multiple cells may emit embers, the ember landing scheme is modified as follows. We write:

where designates the total number of embers previously emitted from the leading edge cell, labelled as cell 0, located at , and that at time *t*, have reached cell , located at , and where is the time at which cell 0 starts emitting embers, the time at which cell 0 stops emitting embers, and is the transport delay between cell 0 and cell *k*, .

We more generally write:

where designates the total number of embers previously emitted from the upwind cell, labelled as cell , located at , and that at time *t*, have reached cell , located at , and where is the time at which cell starts emitting embers, the time at which cell stops emitting embers, and is the transport delay between cell and cell *k*, .

In this revised formulation, the total number of embers that are emitted from cell and that reach cell is:

and the total number of embers that reach cell is:

We have:

which leads to:

If we require that for each cell *k*, , we get:

This result suggests that is independent of and but depends on .

Note that one could argue that converged statistics require the following condition to be statisfied:

which leads to:

or

The most demanding condition is found for the last bin and with that argument, we get a more demanding criterion:

This modified result suggests that is independent of but depends on and .

[@Yiren: Can you plot the time variations of for different values of *k*?]

[@Yiren: Can you reconstruct the time variations of ?]