

Introduction to Sampling and Hypothesis Testing

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Learning Outcomes

- ☐ **Random Variables and Distributions**

- ☐ **Sampling**

- Central Limit Theorem
- Sampling Distribution
- Sampling Variability

- ☐ **Statistical Inference**

- Confidence Intervals
- Hypothesis Testing

Review: Descriptive Statistics

Arithmetic Mean	$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$
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Median	Middle observation
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Mode	Value occurs most frequently
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Variance	$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$
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Standard Deviation	$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$
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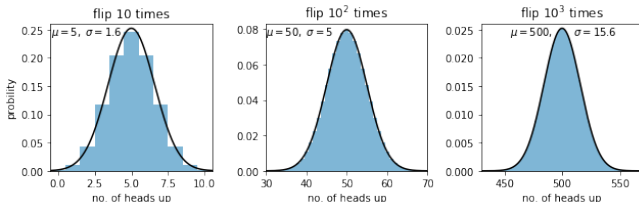
Random Variables

- **Random variable:** A variable whose possible values are outcomes of a random phenomenon
- Discrete random variable: Countable number of random variables in a given interval
 - Toss a coin, roll a dice
- **Continuous random variable:** Unaccountably infinite number of random variables in a given interval, could be any value
 - Daily rainfall, waiting time at a traffic light
 - **Probability density function $f(x)$:**

$$P(a < X < b) = \int_a^b f(x)dx$$

Distributions

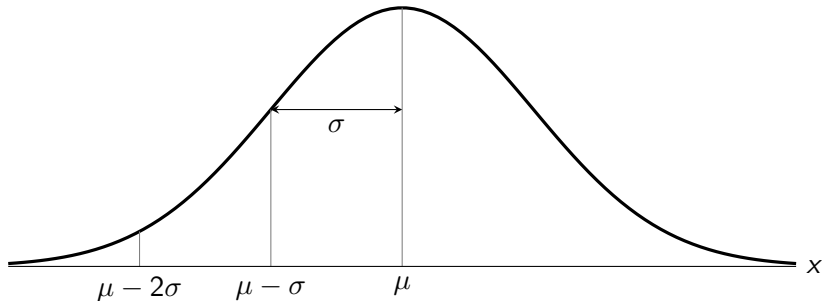
- **Probability Distribution:** mathematical function that provides the probabilities of occurrence of different possible outcomes in an experiment.
- The distribution of many variables can be approximated by **Normal (Gaussian) Distribution** with known properties.
- Example: flip a coin 10, 10^2 , 10^3 times, prob. of head up x times $p(x) = \binom{n}{x} \left(\frac{1}{2}\right)^n$



The Normal Distribution

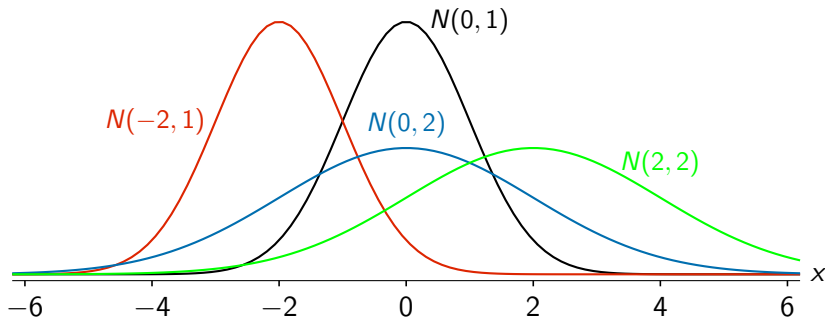
□ The Normal distribution has the shape of a "bell curve" with parameters μ and σ^2 that determine the center and spread.

□ Probability density function: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$



The Normal Distribution (cont'd)

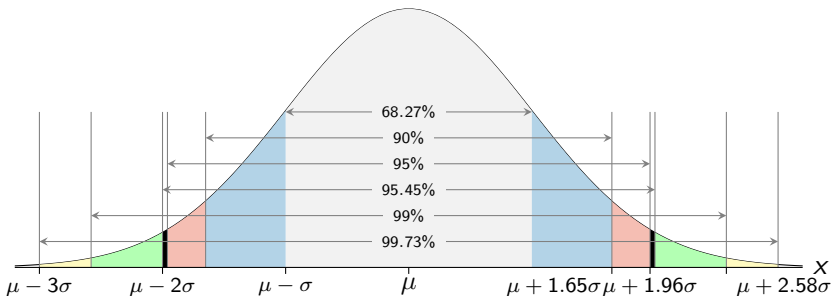
□ Each different value of μ and σ^2 gives a different Normal distribution, denoted by $N(\mu, \sigma^2)$.



Properties of the Normal Distribution

Intervals	$(\mu - \sigma, \mu + \sigma)$	$(\mu - 2\sigma, \mu + 2\sigma)$	$(\mu - 3\sigma, \mu + 3\sigma)$
Probability	68.27%	95.45%	99.73%

Intervals	$\mu \pm 1.65\sigma$	$\mu \pm 1.96\sigma$	$\mu \pm 2.58\sigma$
Probability	90%	95%	99%



Standardization

- If $N(0, 1)$, it is called **Standard Normal distribution**
- Any non-standard Normal distribution can be transformed into the standard one. This process is called **standardization**
- Why standardization:

$$P(x) = \int_{-\infty}^x \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(x-\mu)^2}{2\sigma^2}} dx \rightarrow P(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{\frac{x^2}{2}} dx$$

- **Linear transformation** is used to convert a non-standard Normal distribution into the standard one
- If $X \sim N(\mu, \sigma^2)$, we want $Z \sim N(0, 1)$, then we use the following equation:

$$Z = \frac{X - \mu}{\sigma}$$

Practice 1

☐ Fill in the blanks

$X \sim N(0, 1)$				
Intervals	$(-1, 1)$		$(-2, 2)$	$(-3, 3)$
Probability	90%	95%		99%

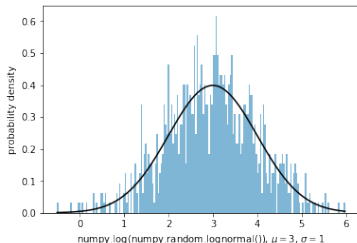
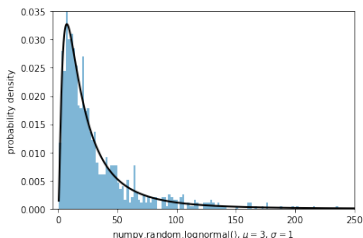
☐ An array $\mathbf{X} = [0, 1, 2, 3, 4, 5]$ follows the Normal distribution $N(7, 25)$. Please convert this array and let it follow the Standard Normal distribution.

Solution 1

- $X \sim N(7, 25)$ gives $\mu = 7$, $\sigma = 5$
- and $\mathbf{Z} = (\mathbf{X} - \mu)/\sigma$
- Hence, $\mathbf{Z} = [-1.4, -1.2, -1.0, -0.8, -0.6, -.4]$

Log Transformation for Skewed Distributions

- A distribution that is not symmetric but exhibit skewness with heavy right tail
- When appropriate the Log transformation has good statistical properties
- X is log-normal if $\log(X)$ is normally distributed and this property is key to transform your data back to its original scale



Sampling

- Statistical inference: using observed data to estimate characteristics of the whole population
 - Assumed that the observed data is sampled from population
 - Characteristics: mean, variance, correlation, etc ...
- Factor that influences our ability to make inference from a sample to a population:
 - **Bias**: sample is not a representative of the whole population
- Methods to avoid bias:
 - Random sampling: each element in the population has an equal probability of selection
 - Stratified sampling: random selection in each strata
 - Systematic sampling, cluster sampling, etc ...

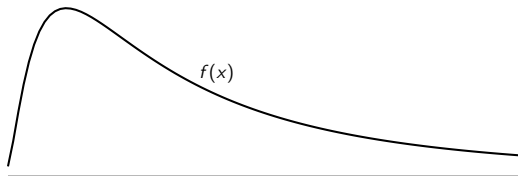
Sampling Variability

- Example: Random sampling from Lognormal distribution

$\ln X \sim N(0, 1)$, where $f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(\frac{-(\ln x - \mu)^2}{2\sigma^2}\right)$

- 5 trials, 50 samples per trial. Each trial gives a different $\overline{\ln X}$ and s

	$\overline{\ln X}$	s
1	-0.361	1.065
2	0.205	0.965
3	-0.138	0.914
4	-0.100	0.983
5	0.114	0.996

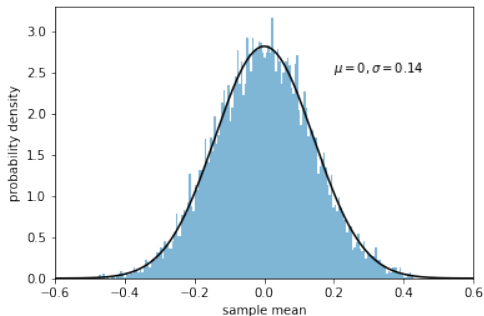


Distribution of the Sample Mean

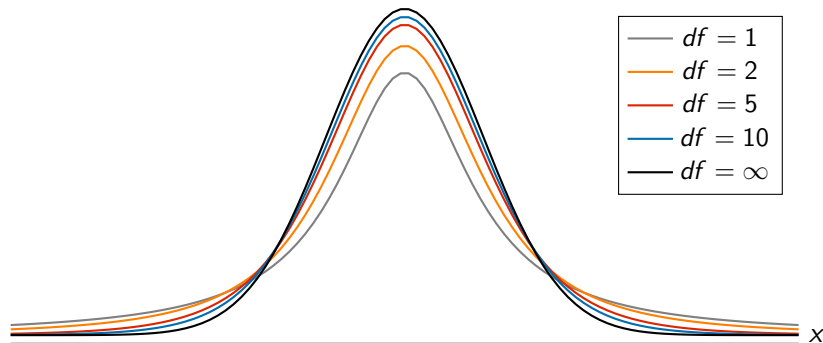
- What is the distribution of sample mean \bar{X} , or $\overline{\ln X}$ in the previous example?
- **Central Limit Theorem:** If a random sample X with n observations is drawn from a population with finite mean μ and variance σ^2 , then, when n is sufficiently large ($n > 30$), the sample mean \bar{X} can be approximated by $\bar{X} \sim N(\mu, \sigma^2/n)$
 - Sufficiently large: typically $n > 30$
 - Distribution of X is not necessarily Normal
 - Allow us use sample mean \bar{X} to make statistical inferences about the population mean μ

Distribution of the Sample Mean (cont'd)

- Example: $\ln X \sim N(0, 1)$, 10000 trials with 50 samples per trial. Plot the sample mean $\overline{\ln X}$
- According to Central Limit Theorem, we derive that the sample mean \overline{X} can be approximated by $\overline{X} \sim N(0, 1/50)$



What if $n \leq 30$? Student's t-Distribution



Student's t-Distribution

- ☐ When $n \leq 30$ and σ unknown, use **Student's t-distribution** instead of Normal distribution
- ☐ Closely related to standard Normal distribution
 - Continuous probability distribution
 - Heavier tails
 - 1 additional parameter: degree of freedom (df)
- ☐ For univariate t-distribution, $df = n - 1$

Sampling Variability and Standard Error

- **Standard Error** (SE) of the mean is the standard deviation (SD) of the sample mean, which quantifies the accuracy of the sample mean \bar{X} as an estimate of the population mean μ
- Standard error calculation
 - If σ is known, SE is calculated by $SE = \sigma/\sqrt{n}$
 - If σ is unknown, SE is estimated by $SE \approx s/\sqrt{n}$
 - Where σ is the population standard deviation, s is the sample standard deviation, and n is the sampling size

Standard Deviation vs. Standard Error

□ Standard error: quantifies the typical error or difference between the sample mean \bar{X} and the theoretical mean μ in the population from which the sample was drawn

$$SE = \frac{\sigma}{\sqrt{n}}$$

□ Standard deviation of a sample of observations measures how a typical observation in the sample deviates from the sample mean

$$SD = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \mu)^2}$$

Practice 2

□ You are investigating the Body Mass Index for people with different diet, calculate standard deviation and standard error for the two diet groups

Diet 1: low consumption of fruit/veg

24.1 23.5 18.5 16.7 26.3 28.5 25.2 23.4 22.5 29.9

Diet 2: high consumption of fruit/veg

22.1 20.5 18.5 16.9 24.1 26.0 22.2 28.4 21.5 31.9 23.0 17.2

Solution 2

- $n_{d1} = 10, n_{d2} = 12$
- $\mu_{d1} = 23.86, \mu_{d2} = 22.69$
- $SD_{d1} = 4.06, SD_{d2} = 4.47$
- $SE_{d1} = 1.28, SE_{d2} = 1.29$

Confidence Intervals

- **Confidence interval** (CI) quantifies the level of confidence that the parameter lies in the interval
 - e.g. level of confidence that CI contains the true value of the population mean μ

- Factors required to construct a confidence interval for the true value of the population mean
 - **Sample mean** \bar{X}
 - **Standard error** of the mean σ/\sqrt{n}
 - **Sample mean distribution**, e.g. Normal distribution
 - **Level of confidence**, e.g. 95%

Confidence Intervals (cont'd)

- Constructing a 95% confidence interval, when $n > 30$
 - If $X \sim N(\mu, \sigma^2)$, $\bar{X} \sim N(\mu, \sigma^2/n)$
 - Standardization \bar{X} gives:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1) \quad (1)$$

- 95% confidence interval refers to $P(-1.96 < Z < 1.96) = 0.95$
- Replace Z by equation (1) gives:

$$P\left(\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}\right) = 0.95$$

Confidence Intervals (cont'd)

- Constructing a 95% confidence interval, when $n > 30$
 - Therefore, 95% confidence interval for the true population mean μ is

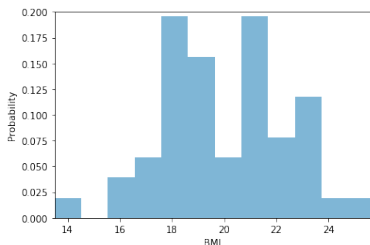
$$\left(\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

- Out of 100 samples, if we obtain a CI as above, 95 will contain the true but unknown population mean μ
 - Or, we are 95% sure that the true but unknown population mean μ lies on CI obtained from a sample mean \bar{X}

Confidence Intervals: Example

- Investigating BMI for women living in rural areas of Bangladesh

18.1	21.6	19.5	21.0	26.8	19.4	21.0	23.8	19.1	19.5	21.7	22.9	22.9
18.0	18.1	17.0	17.6	15.6	21.3	19.8	22.3	20.8	18.1	22.1	23.6	19.2
18.6	25.6	21.6	17.7	20.9	20.8	23.5	15.9	20.0	18.6	23.2	22.2	21.3
13.5	16.8	19.4	21.2	18.8	23.2	18.0	17.5	18.0	19.5	20.6		



Summary Statistics

Mean	20.14
Standard Deviation	2.60
Sample Size	50
Standard Error	0.37

Confidence Intervals: Example (cont'd)

- Sample mean $\bar{X} = 20.14$, however it is probably not exactly equal to the population mean μ
- We need to use sample mean to calculate a confidence interval of likely values for the population mean μ
- Assume:
 - Each observation is independently and randomly sampled from the population with unknown mean μ
 - Population standard deviation σ is equal to the sample standard deviation s , i.e. $\sigma = s$
- Then, $\bar{X} \sim N(\mu, \sigma^2/n)$

Confidence Intervals: Example (cont'd)

- A 95% confidence interval is given by

$$(\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}})$$

- Replacing σ and n from sample statistics, we have

$$(\square\square.\square\square, \square\square.\square\square)(\text{practice 3})$$

- We are 95% sure that the true population mean μ lies within this confidence interval

Solution 3

□ $(\bar{X} - 1.96SE, \bar{X} + 1.96SE)$

□ $\bar{X} = 20.14, SE = 0.37$

□ $(19.41, 20.87)$

Critical Value

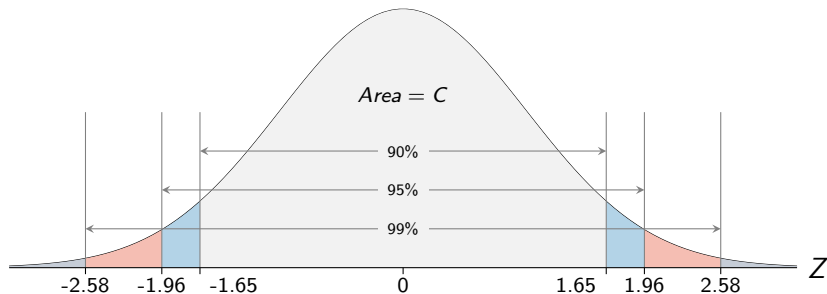
- A 95% confidence interval $(\bar{X} - 1.96\sigma/\sqrt{n}, \bar{X} + 1.96\sigma/\sqrt{n})$
 - 95%: **confidence level**
 - 1.96: **critical value** (Z)
- General formula for a $100C\%$ confidence interval for population mean μ when σ is known

$$(\bar{X} - Z \frac{\sigma}{\sqrt{n}}, \bar{X} + Z \frac{\sigma}{\sqrt{n}})$$

- where Z^* is the value from a standard Normal table that gives you a tail probability of $(1 - C)/2$
- A 95% confidence interval where $C = 0.95$ gives a tail probability of 0.025

Critical Value: Example

Confidence Level	90%	95%	99%
Critical Value Z	1.65	1.96	2.58



Confidence Intervals: What if $n \leq 30$?

	$n > 30$	$n \leq 30$
Known σ	$Z \sim N(0, 1)$	$Z \sim N(0, 1)$
Unknown σ	$Z \sim N(0, 1)$ (approx.)	$T \sim t_{df}$

□ For $Z \sim N(0, 1)$, we have

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

□ For $T \sim t_{df}$, where t_{df} is a Student t-distribution, we have

$$T = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

Confidence Intervals: Student's t-Distribution

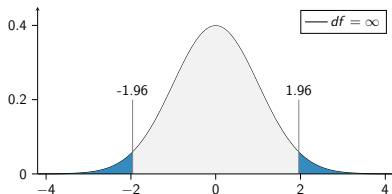
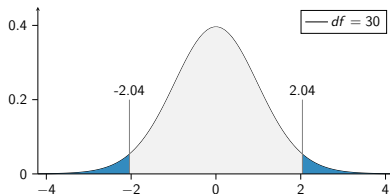
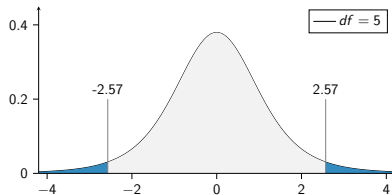
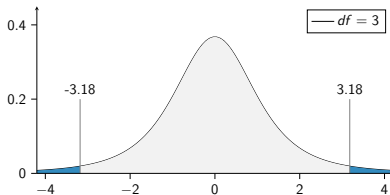
- If σ is unknown and sample size is large ($n > 30$), t_{df} where $df = n - 1$ approximates the standard Normal distribution
- General formula for a $100C\%$ confidence interval for population mean μ when σ is unknown and $n \leq 30$:

$$(\bar{X} - Ts/\sqrt{n}, \bar{X} + Ts/\sqrt{n})$$

- Example: 95% confidence interval for 9 observations ($n = 9$) with known and unknown σ

Known σ	Unknown σ
$(\bar{X} - 1.96\sigma/3, \bar{X} + 1.96\sigma/3)$	$(\bar{X} - 2.31s/3, \bar{X} + 2.31s/3)$

95% Confidence Interval Tail Areas



Confidence Intervals: Student's t-Distribution

- A confidence interval based on Student t-distribution will be **wider** than the corresponding interval based on the Normal distribution
- **Intuition**
 - Sample standard deviation is likely to be poor estimate of population standard deviation σ , when sample size is small
 - Confidence interval using sample standard deviation s should be wider than using σ
- Student t-distribution with large sample size ($n > 30$) is approximately the standard Normal distribution, so confidence intervals will be about the same under both distributions

Difference between Two Population Means: Example

- The birth weights of babies have been measured for a sample of mothers splitting into two categories: nonsmoking and heavy smoking
 - Two categories are measured independently from each other
 - Both come from Normal distributions
 - Same unknown variance σ
- We want to know is there a significant difference in mean birth weights between the two categories

Difference between Two Population Means: Example (cont'd)

□ Summary of Data

Category	Non-smoking				Heavy smoking			
Weight (kg)	3.99	3.79	3.60	3.73	3.18	2.84	2.90	3.27
	3.21	3.60	4.08	3.61	3.85	3.52	3.23	2.76
	3.83	3.31	4.13	3.26	3.60	3.75	3.59	3.63
	3.54	3.51	2.71		2.38	2.34		
Sample size	$n_1 = 15$				$n_2 = 14$			
Mean	$\bar{X}_1 = 3.59$				$\bar{X}_2 = 3.20$			
Sample SD	$s_1 = 0.37$				$s_2 = 0.49$			

Difference between Two Population Means: Example (cont'd)

- The difference between the sample means

$$d_{\bar{X}} = \bar{X}_1 - \bar{X}_2 = 0.39$$

- The difference $d_{\bar{X}}$ arises by chance, or it is significant?
- Build the confidence interval for the difference between population means $d_{\mu} = \mu_1 - \mu_2$
 - $(d_{\bar{X}} - t_{df} s_E, d_{\bar{X}} + t_{df} s_E)$
 - If CI contains 0, $d_{\bar{X}}$ could arise by chance
 - Otherwise, it could be significant

Practice 4

□ Build the confidence interval for μ at 95% confidence level, given following equations derived from the Student's t-distribution:

- Sample SE for difference of two sample means:

$$s_E^2 = s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)$$

- Where

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

- Critical value at $df = 27$: $t_{df} = 2.05$

□ What conclusion can we draw? [ans: (0.06, 0.72)]

Hypothesis Testing

- Alternative method: other than constructing a confidence interval, we assume that there is no difference in mean birth weights between the two categories, i.e. $d_\mu = 0$. Then we calculate under which condition the assumption does not hold
- The alternative method is called the **hypothesis testing**
- Hypothesis
 - **Null hypothesis** (H_0): A default position that there is no effect or change between two measured phenomena, any differences are due to chance. e.g. $d_\mu = 0$
 - **Alternative hypothesis** (H_1): the counterpart of H_0 . e.g. $d_\mu \neq 0$

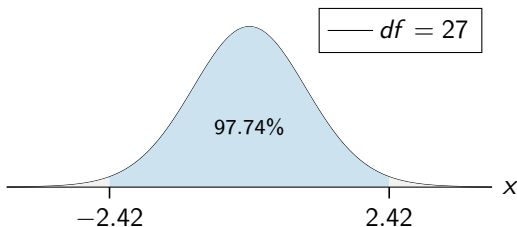
Hypothesis Testing: Difference between Two Population Means (cont'd)

- H_0 : there is no difference in mean birth weights between the two categories, $d_\mu = 0$
- H_1 : there is difference in mean birth weights between the two categories, $d_\mu \neq 0$
- Difference in sample means $d_{\bar{X}} = 0.39$ is a random variable of Student's t-distribution with $df = 27$, standardize $d_{\bar{X}}$ by

$$t_{df} = \frac{d_{\bar{X}} - d_\mu}{s_E}$$

Hypothesis Testing: Difference between Two Population Means (cont'd)

- For $X \sim t_{df=27}$, $t_{df} = 2.42$, $P(|X| < t_{df}) = 97.74\%$
- We are 97.74% sure to say that men birth weights between the two categories are not the same (reject H_0)
- Implies there is difference in mean birth weights between the two categories (H_1)

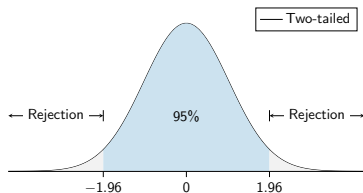
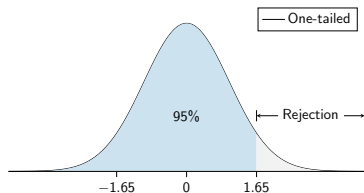


Hypothesis Testing Terminologies

- H_0 and H_1 are always two **rival** hypothesis, e.g. $H_0: \mu = 0$, $H_1: \mu \neq 0$
- **Test statistic**: A quantity derived from sample used in hypothesis testing, e.g. t_{df}
- **P-value**: The probability that the observed data is larger than test statistic, given the null hypothesis is true
 - e.g. $p = 0.03$
 - The smaller the p-value is, the more likely to reject H_0
- **Significance level** α is the probability that p-value under which H_0 will be rejected
 - e.g. If $\alpha = 0.05$, $p = 0.03 < \alpha$, reject H_0

One-Tailed Test vs Two-Tailed Test

- Two-tailed test: $H_0: \mu = 0, H_1: \mu \neq 0$
 - For z-test with a given significance level $\alpha = 0.05$, H_0 is rejected when $P(|X| > z_{\alpha/2}) < \alpha/2$, where $z_{\alpha/2} = 1.96$
- One-tailed test: $H_0: \mu < 0, H_1: \mu \geq 0$
 - For z-test with a given significance level $\alpha = 0.05$, H_0 is rejected when $P(X > z_{\alpha}) < \alpha$, where $z_{\alpha} = 1.65$



Test Statistic

- Measures the difference between the observed data and the null hypothesis
- One value, e.g. t_{df27} , that can be used to perform the hypothesis test
- Assumed underlying distributions have different test statistics

Student's t-distribution	t-test	$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$
Normal distribution	z-test	$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$
Chi-squared distribution	χ^2 test	$\chi^2 = \sum (f_o - f_e)^2 / f_e$
F-distribution	F-test	$F = s_1^2 / s_2^2$

Hypothesis Testing Process

1. Propose a research question regarding observed data set
2. Formulate null hypothesis H_0
3. Calculate the test statistic
4. Find the p-value
5. Compare p-value with required significance level α
6. Make a decision about whether to reject H_0

Practice 5

□ Given the amount of money invested (in dollars) in a plant divided by the delivered amount of energy (in quadrillion British thermal units) listed below, do these data provide sufficient evidence at $\alpha = 0.05$ to indicate a difference in the average investment/quad between gas plants and electric plants?

Electric				Gas			
204.15	0.57	62.76	0.78	0.78	16.66	74.94	0.01
85.46	0.35	89.72	0.65	0.54	23.59	88.79	0.64
44.38	9.28	78.60		0.82	91.84	66.64	7.20
				0.74	64.67	165.60	0.36

Solution 5

	Electric	Gas
Sample Size, n	$n_1 = 11$	$n_2 = 16$
Mean, \bar{x}	$\bar{x}_1 = 52.43$	$\bar{x}_2 = 37.74$
Sample SD, s	$s_1 = 62.43$	$s_2 = 49.05$
H_0	$D_0 = 0$, where $D_0 = \mu_1 - \mu_2$	
H_1	$D_0 \neq 0$, where $D_0 = \mu_1 - \mu_2$	
	$D_{\bar{x}} = \bar{x}_1 - \bar{x}_2 = 14.69$	
df	$df = n_1 + n_2 - 2$	
	$s_p^2 = 3002.05$, $s_E = 21.46$	
Test Statistic	$t = (D_{\bar{x}} - D_0)/s_E = .68$	
Results	$p = .13 \rightarrow p > \alpha$, H_0 is not rejected	

Type I & Type II Errors

Rejection Table

	H_0 True	H_0 False
Reject H_0	<i>Type I Error (α)</i>	<i>Correct Decision ($1 - \beta$)</i>
Fail to Reject H_0	<i>Correct Decision</i>	<i>Type II Error (β)</i>

□ Two possible errors can be made when using p-values to make a decision

- **Type I error:** reject the null hypothesis when it is true
- **Type II error:** not reject the null hypothesis when it is false

Type I & Type II Errors (cont'd)

- Probability of Type I and Type II errors:
 - α : the significance level α is the probability of Type I error.
 - β : the probability of Type II error relative to the H_1 is called β
- Power of a test $1 - \beta$: probability that H_0 is rejected when H_1 is true
- Methods to reduce errors:
 - $\alpha \downarrow \longrightarrow \beta \uparrow$
 - $\beta \downarrow \longrightarrow \alpha \uparrow$
 - Increase the **sample size** n

Example: Chi-Squared Test

- Chi-squared χ^2 test
 - Test for **association** between **two categorical variables**
 - Compare the observed frequencies with the frequencies that would be expected under H_0
- Observations: hospital patients with and without lung cancer were asked about previous smoking habits
- Question: whether there is association between lung cancer and smoking, given $\alpha = .05$?
- H_0 : There is no association between lung cancer and smoking
- H_1 : Lung cancer is associated to smoking

Observations

	Lung Cancer (L)	Other Diseases (O)	Sum
Smoker (S)	$f_{oSL} = 647$	$f_{oSO} = 621$	1268
Non-smoker (N)	$f_{oNL} = 2$	$f_{oNO} = 28$	30
Sum	649	649	1298

- ☐ H_0 assumes smoking and lung cancer are **independent**
- ☐ Then, $P(S, L) = P(S) * P(L) \dots$

Expected Observations under H_0

	Lung Cancer (L)	Other Diseases (O)	Sum
Smoker (S)	f_{eSL}	f_{eSO}	1268
Non-smoker (N)	f_{eNL}	f_{eNO}	30
Sum	649	649	1298

- $f_{eSL} = f_o P(S, L) = f_o P(S)P(L) = 1298 * \frac{1268}{1298} * \frac{649}{1298} = 634$
- $f_{eSO} = 634$
- $f_{eNL} = 15$
- $f_{eNO} = 15$

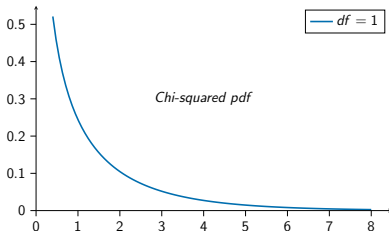
Chi-Squared Test

□ The Chi-square test compares the observed frequencies (f_{o*}) with the frequencies that would be expected under H_0 (f_{e*})

□ Test statistic

$$\chi^2 = \sum \frac{(f_{o*} - f_{e*})^2}{f_{e*}}$$

□ $df = (\text{no. of rows} - 1)(\text{no. of cols} - 1)$



Chi-Squared Test (cont'd)

□ Computation result

- $\chi^2 = 23.1$
- $df = 1$
- $p = 1.53e^{-6}$
- $p < \alpha$

□ Conclusions

- H_0 is rejected due to very small p-value
- Smoking and lung cancer are associated

Example: F - Test

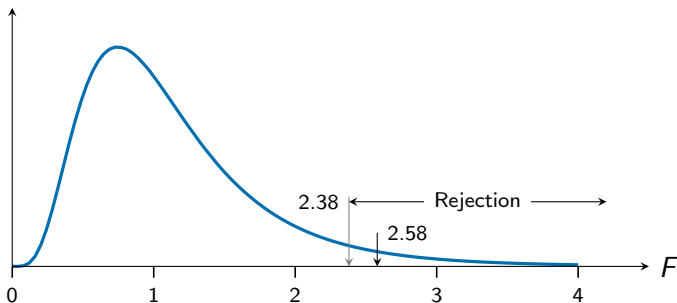
- ☐ F - test: Test for the ratio of **two population variances**
- ☐ Observations: A company imports products from two suppliers, the weight (in kg) slightly varies for each product
- ☐ Question: Whether there is a difference between the standard deviation of product weights from two suppliers, given $\alpha = .1$?
- ☐ H_0 : There is no difference in the standard deviation of product weights from two suppliers, i.e. $\sigma_1^2 = \sigma_2^2$
- ☐ H_1 : The standard deviation of product weights from two suppliers are different, i.e. $\sigma_1^2 \neq \sigma_2^2$

Summary of Observed Data

	Supplier 1	Supplier 2
Sample Size, n	$n_1 = 13$	$n_2 = 18$
Mean	5.60	5.90
Sample SD, s	$s_1 = 3.10$	$s_2 = 1.93$
H_0	$\sigma_1^2 = \sigma_2^2$	
H_1	$\sigma_1^2 \neq \sigma_2^2$	
Test Statistic	$F = s_1^2/s_2^2$	
df	$df_1 = 12$	$df_2 = 17$

F - Test

- $\square F = 2.58 \implies p = 0.04 < \alpha/2 \implies H_0$ is rejected
- \square The standard deviation of product weights from two suppliers are different



References



William Mendenhall, Terry Sincich. *Statistics for Engineering and the Sciences*. Pearson/Prentice Hall, Upper Saddle River, New Jersey, 2007.



Douglas G. Altman. *Practical Statistics for Medical Research*. CRC press, 1990.