The velocity field

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I. CHOOSING THE BARYON VELOCITY

We consider a non-superfluid and non-superconducting neutron star. Let $\boldsymbol{u}_{\rm b}(r,\,\theta)$ be the baryon velocity in the stellar core. It can be shown, that, to a good approximation, it is this velocity that appears in the equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = {\rm rot}\,[\boldsymbol{u}_{\rm b} \times \boldsymbol{B}]. \tag{1}$$

Our aim is to specify $\boldsymbol{u}_{\rm b}$. Instead of $\boldsymbol{u}_{\rm b}$ it will be more convenient to work with the baryon current $\boldsymbol{j} = n_{\rm b}\boldsymbol{u}_{\rm b}$, where $n_{\rm b}(r)$ is the unperturbed baryon number density in the absence of the magnetic field. The baryon current is conserved, hence, in the quasistationary approximation (e.g., [1])

$$\mathrm{div}\,\boldsymbol{j} = 0. \tag{2}$$

Our problem is axisymmetric, hence we can present $\boldsymbol{j} \equiv (j_r,\, j_\theta,\, j_\phi)$ as

$$\boldsymbol{j} = \left(\sum_l a_l\, P_l,\ \sum_l b_l \frac{\partial P_l}{\partial \theta},\ j_\phi \right), \tag{3}$$

where $P_l(\cos\theta)$ is the Legendre polynomial and a_l, b_l are the coefficients that should be specified. These coefficients are not independent. Plugging (3) into (2) and taking into account that $\hat{\boldsymbol{L}}^2 P_l = l(l+1) P_l$ ($\hat{L}$ being the angular momentum operator), one finds

$$b_l = \frac{1}{l(l+1)}\,\frac{1}{r}\,\frac{\partial}{\partial r}\left(r^2 a_l\right). \tag{4}$$

Looking at this formula one sees that the component $l = 0$ in the expansion must be excluded. Moreover, the symmetry of the problem dictates $j_r \to 0$ at $r \to 0$. In addition, we shall require that $j_r = 0$ at the crust-core interface. Therefore, the solution should satisfy the following conditions [we assume that $\boldsymbol{j}(r,\,\theta)$ is an analytic function of r near the centre and crust-core boundary]

1. $a_0 = 0$.
2. $a_l \sim r^\alpha$ at $r \to 0$, where $\alpha \geq 1$.
3. $a_l \sim (r - R_{\rm cc})^\beta$ at $r \to R_{\rm cc}$, where $\beta \geq 1$ ($R_{\rm cc}$ is the radial coordinate of the crust-core interface).

Note that there are no conditions on the component j_ϕ.

II. NUMERICAL EXAMPLE

The simplest $\boldsymbol{j}$ satisfying these conditions is given by

$$\begin{aligned} & a_1 = r\,(r - R_{\rm cc}), \\ & a_l = 0, \quad \text{for} \quad l \neq 1, \\ & j_\phi = 0. \end{aligned} \tag{5}$$

Since we know $\boldsymbol{j}$, the baryon velocity $\boldsymbol{u}_{\mathrm{b}} = \boldsymbol{j}/n_{\mathrm{b}}$ is also determined provided that we know $n_b(r)$, which is easy to extract from the solution of TOV equations.

[1] P. Goldreich and A. Reisenegger, Astrophys. J. **395**, 250 (1992).