

Below are the puzzles that lead to the meeting link. This is supposed to be *fun* (at least, in a nerd's perspective), so you're encouraged to attempt the hardest problem that you can. Both problems lead to the link, so you do not need to attempt both (but you're welcome to!) In case you do not have access to a *nix machine, I have set up an Ubuntu VM with everything you need:

IP address: 45.79.151.17

Username: guest

Password: password

For the second problem, a check program is set up in the home directory, which can be invoked as

```
$ ./check <answer>
```

This program will check if your answer is within 0.1 of the expected answer (the solution to either of the two parts of Problem 2 will work).

Of course, there are... *other* ways to get the meeting link; but if you have the technical expertise to use those methods, feel free to! By design, Problem 1 is not set up in a particularly secure manner—again, this is supposed to be *fun*!

Important: While this VM is set up so you can use it as you wish, please be mindful of other puzzle-solvers who might be using it. In particular:

1. Please do not leave your solutions in the home directory and ruin it for everyone else. Avoid writing to files. The problem is designed to be fully solvable using pipes.
2. You're not explicitly *disallowed* from trolling others (by, for instance, swapping out the message), but the SHA256 digest might make this a bit less fruitful.
3. You are, of course, free to `scp` the problem files over.

Problem Set

1. Use the `message.txt` file and any clues in the email to figure out the meeting link.

[*Hard mode:* Do this on Windows.]

2. Let $f(x) = \frac{1}{2}x^2$ with its domain restricted to \mathbb{R}_+ . Let there be four points initially located at $(0, 1)$, $(1, 0)$, $(0, -1)$, and $(-1, 0)$. A *windmill* process begins at time $t = 0$, where the points rotate clockwise at infinite speed around their center¹. As time progresses, the center of rotation moves along the curve of $f(t)$, at a speed proportional to its gradient. The four points maintain their relative distances and positions with respect to this moving center.

As they move, the rotating points sweep out an area in the Euclidean plane. At $t = \pi$, the rotation stops instantaneously. Calculate the total area covered by the points during their rotation from $t = 0$ to $t = \pi$.

[*Harder:* Now suppose the distance of each of the points from the center of rotation at time t is given by $f'(t)$. Calculate the new total area².]

[*Something to think about:* What if f was not Riemann-integrable? For example, consider the Dirichlet function $f(x) = \mathbb{1}_{\mathbb{Q}}(x)$, which is not Riemann-integrable, but has a Lebesgue integral of 0. Since the function is everywhere-discontinuous and therefore not differentiable, assume that the windmill process in this case “teleports” to each location instead. Is the original problem well-defined for this case, and if so, what is the new area? One could argue that since \mathbb{Q} is dense in \mathbb{R} , this area should be $2\pi^2$ (the area of the circle times the Lebesgue measure of the path, multiplied by 2 to account for both cases of the function); but the Lebesgue measure of (any subset of) \mathbb{Q} is 0, so one could also argue the area should be 0. What specific part(s) of this problem make this a challenge?]

¹More explicitly, the points cover the full circle instantaneously at every instant.

²Obviously, since f has a Lipschitz-continuous gradient, this problem is still well-defined. My fascination with the Hessian is widely-known at this point.