Sets:

Directed network $G=(V,A)$; $V=\{0,1,2,...,v,v+1\}$ (nodes 0 and $v+1$ represent depot)

V: set of nodes in network G;

A: set of arcs in network G;

C_1: set of customers who can only be served by truck

C_2: set of customers who can only be served by drone

C: set of customers; $C=\{1,2,3,...,v\}$; $C\setminus(C_1\cup C_2)$ represents the set of customers who can be served by either truck or drone

V_1: set of nodes can be visited by trucks， $V_1=(C\setminus C_2)\cup\{0,v+1\}$

V_2: set of nodes can be visited by drones individually or with trucks (note that drones by attaching to trucks can visit all nodes that can be visited by trucks; thus, $V_2=V$)

A_1: set of arcs that can be traveled by truck, $A_1=\{(i,j)\in A \mid i\notin C_2, j\notin C_2, i\neq v+1\}$

A_2: set of arcs that can be traveled by drone (note that drones by attaching to trucks can travel all arcs that can be traveled by trucks; thus, $A_2=\{(i,j)\in A \mid i\neq v+1\}$)

$\Delta_m^+(i)$: forward star of node i corresponding to transportation mode m (m = 1 truck; m = 2 drone), determined by A_1 (when m = 1) and A_2 (when m = 2)

$\Delta_m^-(i)$: backward star of node i corresponding to transportation mode m (m = 1 truck; m = 2 drone), determined by A_1 (when m = 1) and A_2 (when m = 2)

Parameters:

d_{ij}: travel time on arc $(i,j)\in A_1$ corresponding to trucks

r_{ij}: travel time on arc $(i,j)\in A_2$ corresponding to drones

c_{ij}: travel cost on arc $(i,j)\in A_1$ corresponding to trucks

e_{ij}: travel cost on arc $(i,j)\in A_2$ corresponding to drones

s_i^m: service time at node $i\in V$ corresponding to transportation mode m (m = 1 truck; m = 2 drone)

$[a_i,b_i]$: time window associated with node $i\in V$

T: flying duration of drones

M: a large constant

Variables:

x_{ij} : a binary variable that equals to 1 if truck travels on arc $(i,j)\in A_1$ and 0 otherwise

y_{ij} : a binary variable that equals to 1 if drone travels on arc $(i,j)\in A_2$ and 0 otherwise

t_i : start of service time at node $i\in V_1$ by truck

p_i : start of service time at node $i\in V_2$ by drone

$z_{ij}\in\{0,1\} \quad \forall (i,j)\in A_1 \cap A_2$

Model formulation:

$$\min \sum_{(i,j)\in A_1} c_{ij}x_{ij} + \sum_{(i,j)\in A_2} e_{ij}y_{ij} - \sum_{(i,j)\in A_1\cap A_2} e_{ij}z_{ij} \tag{1}$$

Subject to:

$$\sum_{j\in\Delta_1^+(i)} x_{ij} + \sum_{j\in\Delta_2^+(i)} y_{ij} \ge 1 \quad \forall i\in C \tag{2}$$

$$\sum_{j\in\Delta_1^+(0)} x_{0j} = 1 \tag{3}$$

$$\sum_{j\in\Delta_2^+(0)} y_{0j} = 1 \tag{4}$$

$$\sum_{i\in\Delta_1^-(j)} x_{ij} - \sum_{i\in\Delta_1^+(j)} x_{ji} = 0 \quad \forall j\in V_1\setminus\{0,v+1\} \tag{5}$$

$$\sum_{i\in\Delta_2^-(j)} y_{ij} - \sum_{i\in\Delta_2^+(j)} y_{ji} = 0 \quad \forall j\in V_2\setminus\{0,v+1\} \tag{6}$$

$$\sum_{i\in\Delta_1^-(v+1)} x_{i(v+1)} = 1 \tag{7}$$

$$\sum_{i\in\Delta_2^-(v+1)} y_{i(v+1)} = 1 \tag{8}$$

$$x_{ij} + y_{ij} - 1 \le z_{ij} \le x_{ij} \quad \forall (i,j)\in A_1\cap A_2 \tag{9}$$

$$x_{ij} + y_{ij} - 1 \le z_{ij} \le y_{ij} \quad \forall (i,j)\in A_1\cap A_2 \tag{10}$$

$$t_i + s_i^1 + d_{ij} - t_j \le (1-x_{ij})M \quad \forall (i,j)\in A_1 \tag{11}$$

$$p_i + s_i^2 + r_{ij} - p_j \le (1-y_{ij})M \quad \forall (i,j)\in A_2 \tag{12}$$

$$a_i \le t_i \le b_i \quad \forall i\in V_1\cup\{0,v+1\} \tag{13}$$

$$a_i \le p_i \le b_i \quad \forall i\in V_2\cup\{0,v+1\} \tag{14}$$

$$p_i + \left(\sum_{j\in\Delta_1^-(i)} x_{ji} + \sum_{j\in\Delta_2^-(i)} y_{ji} - 2\right)M \le t_i \le p_i - \left(\sum_{j\in\Delta_1^-(i)} x_{ji} + \sum_{j\in\Delta_2^-(i)} y_{ji} - 2\right)M \tag{15}$$

$\forall i\in (V_1\cap V_2)\setminus\{0,v+1\}$

$$\sum_{m\in\Delta_1^-(i)} x_{mi} + \sum_{m\in\Delta_1^-(l)} x_{ml} \geq 2 - \left(\left(1-y_{ij}\right)M + \left(x_{ij}\right)M + \left(1-y_{jl}\right)M + \left(x_{jl}\right)M\right)$$
$$\forall(i,j),(j,l)\in A_1 \tag{16}$$
$$\sum_{m\in\Delta_1^-(i)} x_{mi} + \sum_{m\in\Delta_1^-(l)} x_{ml} \geq 2 - \left(\left(1-y_{ij}\right)M + \left(1-y_{jl}\right)M\right) \forall(i,j),(j,l)\in A_2 \setminus A_1 \tag{17}$$
$$\sum_{m\in\Delta_1^-(i)} x_{mi} + \sum_{m\in\Delta_1^-(l)} x_{ml} \geq 2 - \left(\left(1-y_{ij}\right)M + \left(1-y_{jl}\right)M + \left(x_{jl}\right)M\right)$$
$$\forall(i,j)\in A_2 \setminus A_1,(j,l)\in A_1 \tag{18}$$
$$\sum_{m\in\Delta_1^-(i)} x_{mi} + \sum_{m\in\Delta_1^-(l)} x_{ml} \geq 2 - \left(\left(1-y_{jl}\right)M + \left(1-y_{ij}\right)M + \left(x_{ij}\right)M\right)$$
$$\forall(i,j)\in A_1,(j,l)\in A_2 \setminus A_1 \tag{19}$$
$$r_{ij} + s_j^2 + r_{jl} \leq T + \left(\left(1-y_{ij}\right)M + \left(x_{ij}\right)M + \left(1-y_{jl}\right)M + \left(x_{jl}\right)M\right)$$
$$\forall(i,j),(j,l)\in A_1 \tag{20}$$
$$r_{ij} + s_j^2 + r_{jl} \leq T + \left(\left(1-y_{ij}\right)M + \left(1-y_{jl}\right)M\right) \quad \forall(i,j),(j,l)\in A_2 \setminus A_1 \tag{21}$$
$$r_{ij} + s_j^2 + r_{jl} \leq T + \left(\left(1-y_{ij}\right)M + \left(1-y_{jl}\right)M + \left(x_{jl}\right)M\right)$$
$$\forall(i,j)\in A_2 \setminus A_1,(j,l)\in A_1 \tag{22}$$
$$r_{ij} + s_j^2 + r_{jl} \leq T + \left(\left(1-y_{ij}\right)M + \left(x_{ij}\right)M + \left(1-y_{jl}\right)M\right)$$
$$\forall(i,j)\in A_1,(j,l)\in A_2 \setminus A_1 \tag{23}$$
$$\sum_{j\in\Delta_1^-(i)} x_{ji} = 1 \quad \forall i\in C_1 \tag{24}$$
$$\sum_{j\in\Delta_1^-(i)} z_{ji} = \sum_{j\in\Delta_1^+(i)} z_{ij} \quad \forall i\in C_1 \tag{25}$$
$$x_{ij}\in\{0,1\} \quad \forall(i,j)\in A_1 \tag{26}$$
$$y_{ij}\in\{0,1\} \quad \forall(i,j)\in A_2 \tag{27}$$
$$z_{ij}\in\{0,1\} \quad \forall(i,j)\in A_1\cap A_2 \tag{28}$$
$$t_i\in\mathbb{R} \quad \forall i\in V_1\cup\{0,v+1\} \tag{29}$$
$$p_i\in\mathbb{R} \quad \forall i\in V_2\cup\{0,v+1\} \tag{30}$$

Objective function (1) minimizes the total cost. The third term in (1) is the cost that is doubly counted in the first two terms. The value of z_{ij} is confined by constraints (9) and (10). Constraints (2) ensure that every customer is served by either a truck or a drone. Note that if a truck equipped with a drone visits a customer, then the customer is considered as visited twice. Constraints (3)-(8) define source-to-sink paths in G for all trucks and drones. Constraints (11) through (14) guarantee schedule feasibility with respect to both trucks and drones. Constraints (15) synchronize trucks with drones if they visit the same nodes. Constraints (16) guarantee capacity feasibility with respect to drones. On the other hand, constraints (16) through (19) ensure that a drone serves exactly one customer during a time period of launching and landing on its corresponding truck. Constraints (20) through (23) ensure duration feasibility of drones. Constraints (24) and (25) ensure that the customers who can only be served by trucks because of capacity and/or geographical constraints are served by trucks. It is noted that a truck is possibly equipped with a drone at the time of serving such a customer. In addition, drones cannot fly in and/or out of the customers. Note that we do not need to impose constraints to ensure that the customers who can only be served by drones are served by drones because the customers can only be reached by drones through the arcs in $A_2 \setminus A_1$.