Uncertainty

Experimental Research

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Estimating the ATE

- The sample mean is an unbiased estimator for the population mean
- The sample ATE is an unbiased estimator for the population ATE
  - Mean treated outcomes - mean control (untreated) outcomes
- How do we define bias in a statistical sense?
- We need some way of characterizing uncertainty because averages will vary from sample to sample due to random chance

Example

- A researcher wants to estimate the turnout rate for a small community (N = 10,000)
- The true population mean is .7
- They collect data on 100 residents and compute the average turnout rate for this sample (sample mean)
- On average, across many hypothetical repetitions, the sample mean will equal the population mean
- Sample means vary from sample to sample

Example

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1 0 1 1 1 1 1 0 1 1 1 1 0 0 1 1 1 1 1 1 1 1 0 1 1 0 1 1 0 1 1 1 1 1 1 1 1 1 0 1 0 0 0 0 1 1 1 1 1 0 0 1 1 1 1 0 1 0 1
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1 1 1 1 0 0 1 0 1 1 0 1 1 1 1 1 1 1 0 0 0 0 1 0 0 1 0 1 0 0 1 1 1 1 1 1 0 1 1 0 0 1 1 1 1 1 1 1 1 0 1 1 1 0 1 0
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Example

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 [1] 0 1 0 1 1 0 1 1 1 1 1 0 1 1 0 0 1 1 1 0 1 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 0 0 1 1 1
[61] 1 0 0 1 0 1 0 1 1 1 1 1 1 1 0 1 1 1 1 0 0 1 1 0 1 1 1 1 0 1 1 1 0 0 1 1 1 1 1 1
```

```
 [1] 0 0 1 1 1 1 1 0 1 1 1 1 1 1 0 1 1 0 1 1 0 1 1 1 0 1 0 1 1 0 1 0 1 1 1 0 1 1 0 1 1 0 0 1 1 0 0 1 1 1 0 1 1 1 1 1 1 1 1 1
[61] 1 1 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0 1 1 1 0 1 0 1 1
```

```
 [1] 0 1 1 1 0 1 0 1 0 1 0 0 1 1 1 1 1 0 1 1 1 1 1 0 1 1 1 0 0 1 0 1 0 1 1 0 1 1 1 1 1 0 1 1 1 0 1 0 1 1 1 1 1 1 1 0 1 1 1 1
[61] 1 1 0 0 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 1 0 1 1 0 1 1 0 1 1 1 1 1 1 1 0 1 0 1 1
```

```
 [1] 1 1 0 0 1 1 1 1 1 1 1 0 0 1 0 0 0 0 1 1 1 0 1 0 0 0 1 1 1 0 0 1 1 0 1 1 1 0 1 1 0 1 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1
[61] 1 0 0 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 0 0 0 1 1 1 0 1 0 1 0 1 0
```

Example

```
 [1] 0 1 0 1 1 0 1 1 1 1 1 0 1 1 0 0 1 1 1 0 1 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 0 0 1 1 1
[61] 1 0 0 1 0 1 0 1 1 1 1 1 1 1 0 1 1 1 1 0 0 1 1 0 1 1 1 1 0 1 1 1 0 0 1 1 1 1 1 1
```

Mean: .76

```
 [1] 0 0 1 1 1 1 1 0 1 1 1 1 1 1 0 1 1 0 1 1 0 1 1 1 0 1 0 1 1 0 1 0 1 1 1 0 1 1 0 1 1 0 0 1 1 0 0 1 1 1 0 1 1 1 1 1 1 1 1
[61] 1 1 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0 1 1 1 0 1 0 1 1
```

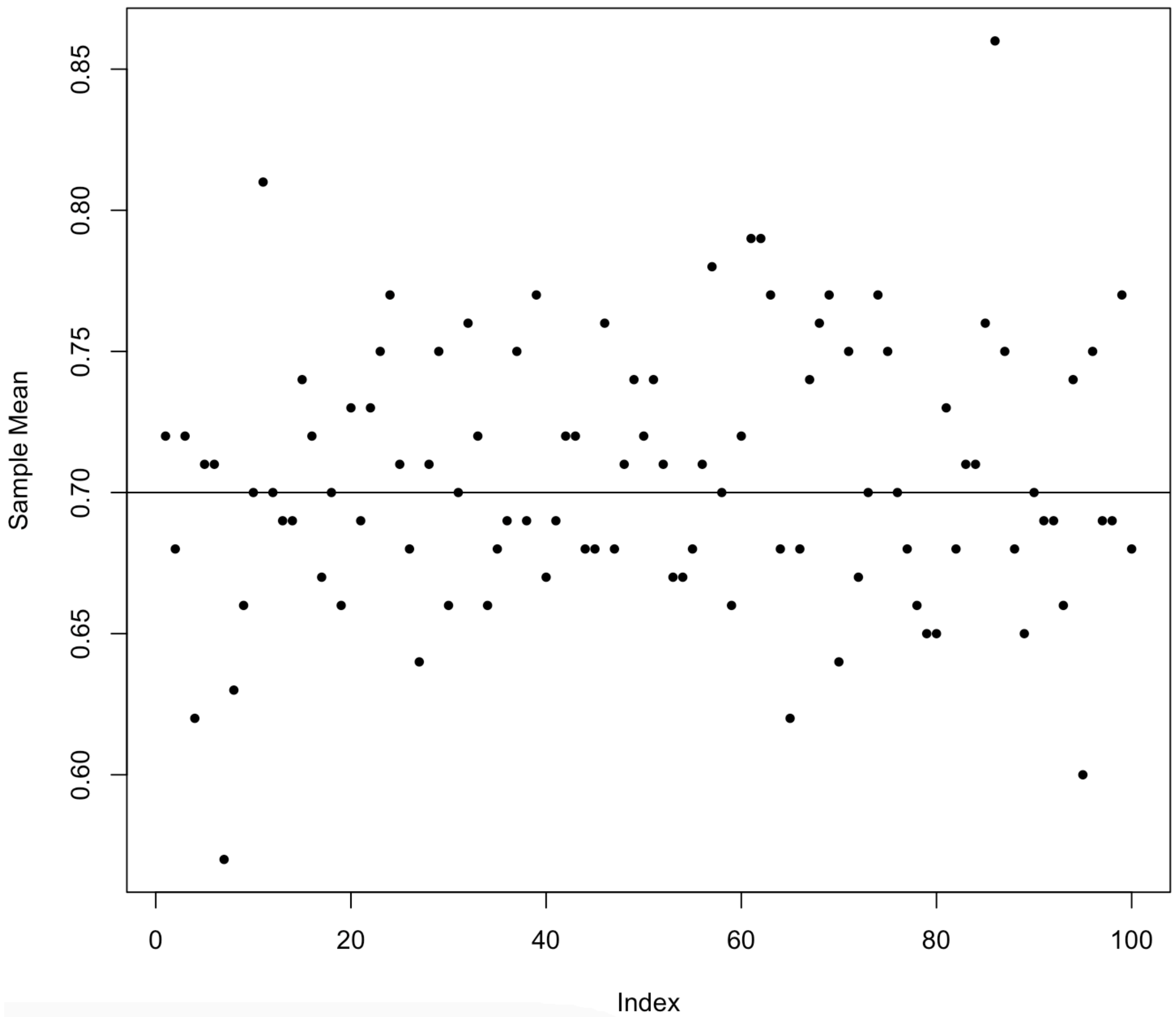
Mean: .76

```
 [1] 0 1 1 1 0 1 0 1 0 1 0 0 1 1 1 1 1 0 1 1 1 1 1 0 1 1 1 0 0 1 0 1 0 1 1 0 1 1 1 1 1 0 1 1 1 0 1 0 1 1 1 1 1 1 1 0 1 1 1 1
[61] 1 1 0 0 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 1 0 1 1 0 1 1 0 1 1 1 1 1 1 1 0 1 0 1 1
```

Mean: .74

```
 [1] 1 1 0 0 1 1 1 1 1 1 1 0 0 1 0 0 0 0 1 1 1 0 1 0 0 0 1 1 1 0 0 1 1 0 1 1 1 0 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1
[61] 1 0 0 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 0 0 0 1 1 1 0 1 0 1 0 1 0
```

Mean: .69


0.85
0.80
0.75
0.70
0.65
0.60
Sample Mean
0
20
40
60
80
100
Index

Standard Error

- We need a way of characterizing this uncertainty that we would hypothetically observe from sample to sample
- Standard error
  - Standard deviation of the sampling distribution
  - Variation in sample means across hypothetical samples
- $$SE = \frac{\sigma}{\sqrt{N}}$$
  - Sample standard deviation (outcome)
  - N = sample size

Standard Deviation

- Captures the variability in a random variable
- $\sigma = \sqrt{(\frac{\sum (x_i - \mu)^2}{N})}$
    - Population standard deviation
- $s = \sqrt{(\frac{\sum (x_i - \bar{x})^2}{n - 1})}$
    - Sample standard deviation
- Everything inside the parentheses is called the variance

Standard Deviation

n	x_n	$x_n - \overline{x}$	$(x_n - \overline{x})^2$
1	1.5	-1.38	1.90
2	2.0	-0.88	0.77
3	2.1	-0.78	0.61
4	2.5	-0.38	0.14
5	2.9	0.02	0.00
6	3.1	0.22	0.05
7	3.2	0.32	0.10
8	3.4	0.52	0.27
9	3.4	0.52	0.27
10	4.7	1.82	3.31

$$sqrt(\sum /(n-1))$$

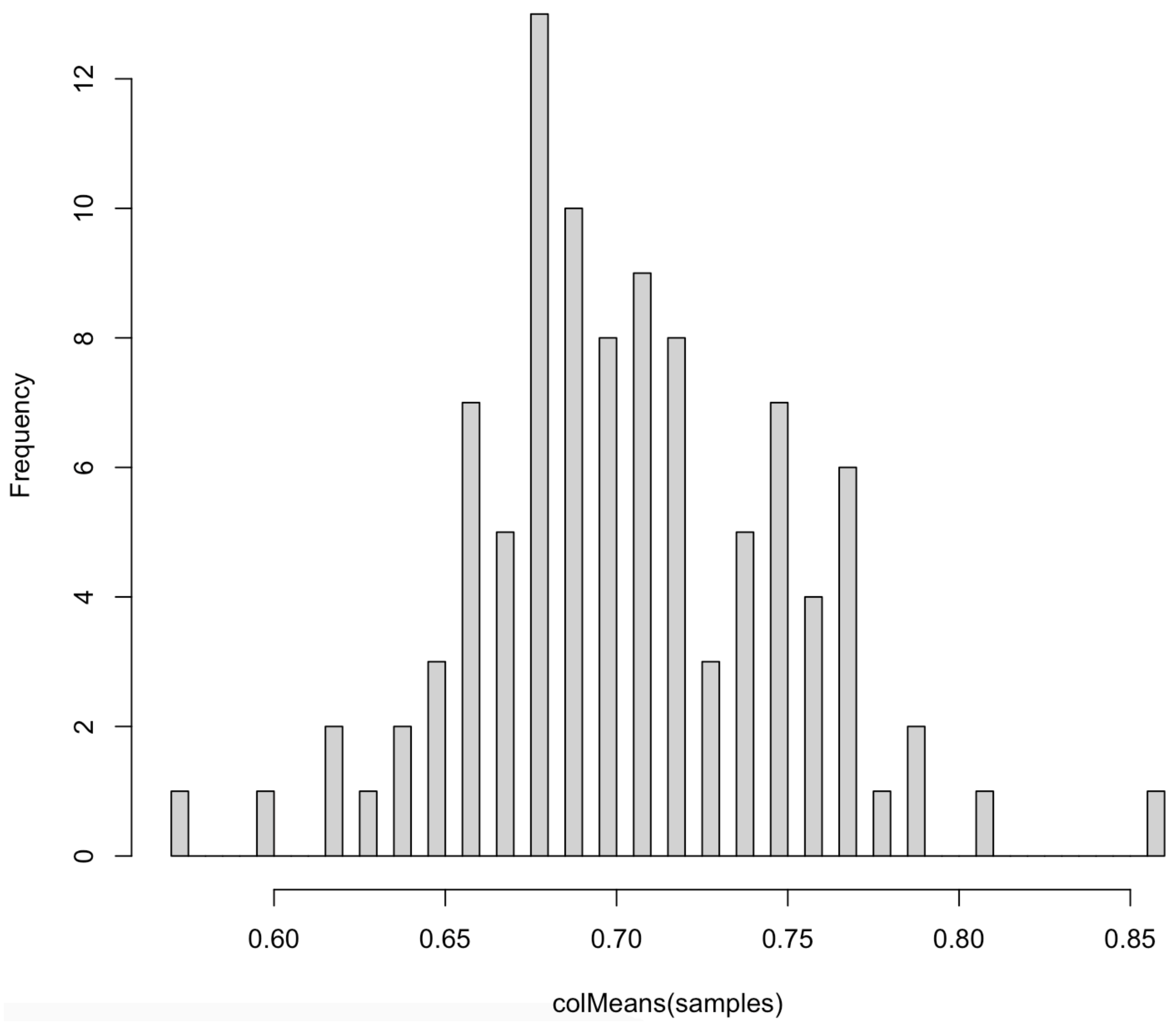
Standard Error

- Using the previous example:
    - First sample
    - $\sigma = .45$, N = 100, SE = .045
    - This means that, on average, we should expect sample means to vary from sample to sample by about 4.5%

Sampling Distributions

- The sampling distribution (collection of hypothetical samples) is normally distributed
  - 95% of hypothetical sample means fall within 1.96 standard errors
  - 99% fall within 2.576 standard errors
- Central limit theorem
  - The sum of independent random variables is normally distributed even if the variables themselves are not normally distributed

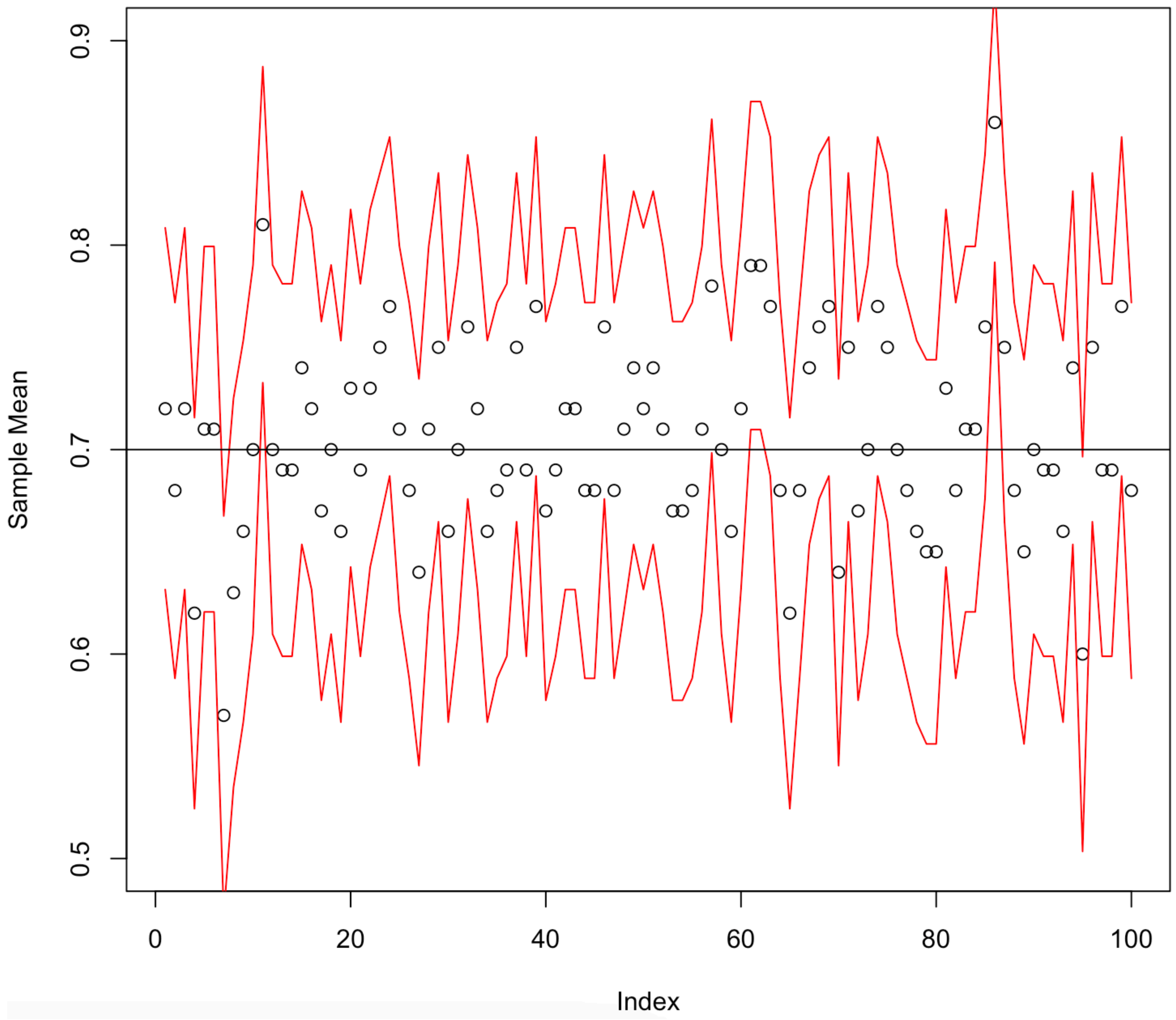
Sampling Distributions



Confidence Intervals

- Range of estimates we are sure the true population parameter lies within
  - More formally, “if confidence intervals are constructed across many separate data analyses of repeated (and possibly different) experiments, the proportion of such intervals that contain the true value of the parameter will approximately match the confidence level.”
- Confidence intervals have a lower and upper bound
- 95% confidence intervals
  - If we were to repeatedly calculate these intervals in our samples, they would contain the true population estimate 95% of the time.

Confidence Intervals



Calculating 95% CIs

- Lower 95% confidence interval: $\bar{X} - 1.96 \times \sigma/\sqrt{N}$
- Upper 95% confidence interval: $\bar{X} + 1.96 \times \sigma/\sqrt{N}$
- $\bar{X} = .74, \sigma = .45$, N = 100, SE = .045
- Lower 95% confidence interval: .65
- Upper 95% confidence interval: .83

Characterizing uncertainty in the ATE

- $SE(\hat{ATE}) = \sqrt{(\frac{Var(\hat{Y}_i(1)}{N_1} + \frac{Var(\hat{Y}_i(0)}{N_0})}$
- ATE = .2; SE = .1
- The average treatment effect is .2, with a standard error of .1
- 95% confidence intervals: (.2 - 1.96*.1) = .004; .2 + 1.96*.1 = .396