

Dataset

stocknet

Stocknet tracks Two-year price movements from 2012-09-04 to 2017-09-01 of 88 stocks are selected to target, coming from all the 8 stocks in the Conglomerates sector and the top 10 stocks in capital size in each of the other 8 sectors.

The preprocessing steps are as follows:

1. For each stock, extract movement percent (daily return of close price):

$$r_t = (P_t - P_{t-1})/P_{t-1}$$

2. Remove stocks that has missing days (81 stocks left)

3. Missing values are handled using linear interpolation

The final dataset has shape [1257, 81] (1257 days, 81 stocks).

S & P 500 (self made)

The collected dataset from <https://www.kaggle.com/datasets/camnugent/sandp500/data>

contains 2013-02-08 to 2018-02-07 of SP500 constituent stocks

1. For each stock, extract movement percent (daily return of close price):

$$r_t = (P_t - P_{t-1})/P_{t-1}$$

2. Remove stocks that has missing days (470 stocks left)

3. Missing values are handled using linear interpolation

The final dataset has shape (491, 4278) (4278 days, 491 stocks).

Benchmarks

1. [Diffusion Factor Models](#)
2. [Stationary Bootstrap](#)

Experimental Setting

For Factor DM, we follow the default training/sampling setting: batch size 32, learning rate 1e-4, 600 epochs.

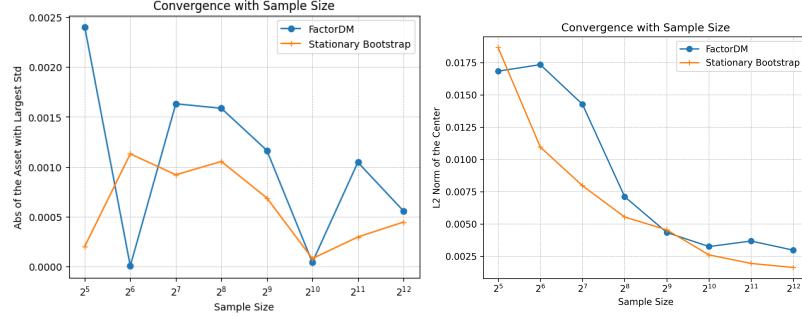
cosine noise scheduler, 200 sampling step.



For stationary bootstrap, we set the block size = 8 ($p = 0.125$), we stop until we got [1257, 81], when sampling we randomly select 1 cross-section with replacement.

Then we sample scenarios and overall we get [sample_size, 81] samples from each benchmark, and we compare the generated samples with the training data using various metrics. We mainly focus on distribution similarity and marginal statistics of each asset. The sampling size is decided by following process.

1. Generate 4096 samples from each method
2. Slice it as [32, 64, 128, 256, 512, 1024, 2048, 4096]
3. Choose the size that gives a stable mean of the distance to the training set.



Distance of all stocks(left) and largest variance stock(right) with the training set

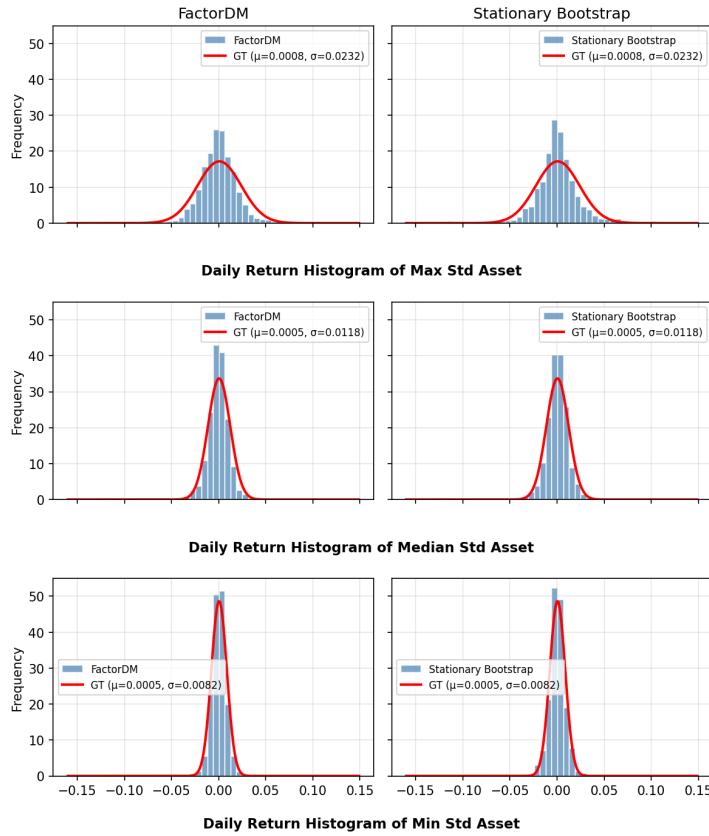
Based on the above results, we choose sample size = 2048 for both methods. This balances stability and efficiency

Evaluation Results

We evaluate the quality of generated scenarios by comparing their distribution similarity and marginal statistics with the training data. The results are summarized in the table below.

Method	MMD↓	Cov↓	ES↓	Mean↓	Std↓	Skew↓	Kurt↓
FactorDM	0.0412	0.1943	0.0979±0.05	-0.4953±3.00	-0.1006±0.03	-1.3566±3.26	-0.3632±0.28
Stationary Bootstrap	0.0000	0.0785	0.0257±0.04	0.3907±1.56	-0.0082±0.03	2.2752±17.37	-0.0423±0.23

Result on Distribution Comparing

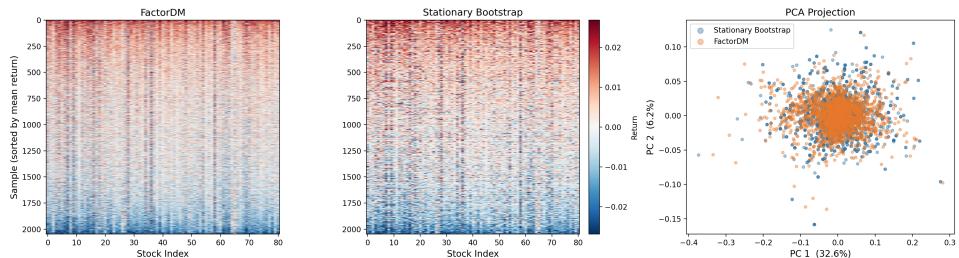


Diversity and Novelty

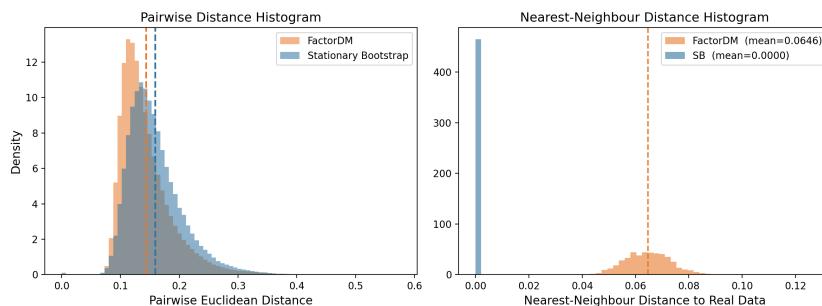
We further compare sample diversity and novelty between the two methods.

Diversity is measured by the pairwise Euclidean distance distribution within each sample set.

Novelty is measured by each generated sample's nearest-neighbour distance to the training set — larger distance indicates the sample is further from any observed data.



Heatmap of generated returns sorted by mean return (left: FactorDM, middle: Stationary Bootstrap) and PCA projection fitted on real data (right)



Pairwise distance distribution (left) and nearest-neighbour distance to real data (right)

Stationary Bootstrap samples are exact resamples of the training set (NN distance = 0), so its diversity simply reflects the original data distribution.

FactorDM generates genuinely novel scenarios (mean NN distance = 0.065) at the cost of lower diversity and reduced tail coverage in PCA space.

Downstream Task

The generated scenarios can be used for Risk Management(Stress Testing) or Portfolio Optimization.

Risk Management

We assume a fixed portfolio and evaluate the risk metrics of the portfolio under the generated scenarios.

For each method, we compute portfolio returns $r_p = X \cdot w$ from the generated scenarios, then calculate risk metrics directly from the portfolio return distribution.

We test two portfolio strategies:

Equal-Weight Portfolio ($w_i = 1/d$):

Method	VaR5%	ES5%	Vol	MaxLoss
Historical (GT)	-0.0120	-0.0168	0.0072	-0.0385
FactorDM	-0.0114	-0.0164	0.0066	-0.0350
Stationary Bootstrap	-0.0109	-0.0158	0.0070	-0.0367

Minimum Variance Portfolio ($\min_w w^\top \Sigma w$, s.t. $\sum w_i = 1, w_i \geq 0$, 20 non-zero weights):

Method	VaR5%	ES5%	Vol	MaxLoss
Historical (GT)	-0.0080	-0.0120	0.0054	-0.0349
FactorDM	-0.0072	-0.0107	0.0047	-0.0321
Stationary Bootstrap	-0.0074	-0.0112	0.0052	-0.0237

Cross-Sectional Risk Metrics

Temporal Risk (Stationary Bootstrap only)

Only for stationary bootstrap. we evaluate on the total project on $T = [5, 10, 20]$ days.

Equal-Weight Portfolio:

Window	Method	VaR5%	ES5%	Vol	MaxLoss
T=5	Historical (GT)	-0.0232	-0.0348	0.0155	-0.0962
T=5	Stationary Bootstrap	-0.0239	-0.0315	0.0154	-0.0566
T=10	Historical (GT)	-0.0285	-0.0441	0.0207	-0.0959

Window	Method	VaR _{5%}	ES _{5%}	Vol	MaxLoss
T=10	Stationary Bootstrap	-0.0311	-0.0410	0.0214	-0.0723
T=20	Historical (GT)	-0.0335	-0.0529	0.0271	-0.0905
T=20	Stationary Bootstrap	-0.0431	-0.0526	0.0293	-0.0828

Minimum Variance Portfolio:

Window	Method	VaR _{5%}	ES _{5%}	Vol	MaxLoss
T=5	Historical (GT)	-0.0189	-0.0258	0.0117	-0.0652
T=5	Stationary Bootstrap	-0.0172	-0.0233	0.0119	-0.0432
T=10	Historical (GT)	-0.0239	-0.0345	0.0165	-0.0612
T=10	Stationary Bootstrap	-0.0228	-0.0313	0.0169	-0.0536
T=20	Historical (GT)	-0.0284	-0.0387	0.0228	-0.0600
T=20	Stationary Bootstrap	-0.0320	-0.0430	0.0238	-0.0696

Temporal Risk Metrics

Portfolio Optimization

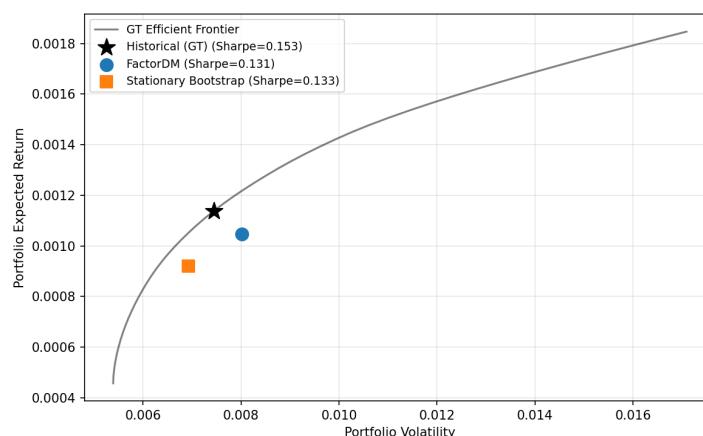
We use each method's generated scenarios to estimate μ and Σ , solve for the max-Sharpe portfolio

$$(\max_w \frac{\mu^\top w}{\sqrt{w^\top \Sigma w}}, \text{ s.t. } \sum w_i = 1, w_i \geq 0),$$

then evaluate the resulting portfolio on historical (GT) data.

Efficient frontier is calculated by minimizing volatility while the target return is at least t for a range of t.

t minimum is the return under minimum variance portfolio, and t maximum is the maximum return among all assets.



Max-Sharpe portfolios evaluated on GT, with GT efficient frontier as reference

Appendix: Metrics

Distribution Similarity

MMD (Maximum Mean Discrepancy) measures the distance between two distributions using a Gaussian RBF kernel. Lower is better.

$$\text{MMD}^2 = \frac{1}{n(n-1)} \sum_{i \neq j} k(x_i, x_j) + \frac{1}{m(m-1)} \sum_{i \neq j} k(y_i, y_j) - \frac{2}{nm} \sum_{i,j} k(x_i, y_j)$$

where $k(a, b) = \exp\left(-\frac{\|a-b\|^2}{2\sigma^2}\right)$, σ is the median pairwise distance, n is the number of generated samples, and m is the number of training samples.

Cov Error (Covariance Error) is the relative Frobenius norm error between covariance matrices:

$$\text{Cov Error} = \frac{\|\Sigma_{\text{sample}} - \Sigma_{\text{train}}\|_F}{\|\Sigma_{\text{train}}\|_F}$$

where $\Sigma_{\text{sample}}, \Sigma_{\text{train}} \in \mathbb{R}^{d \times d}$ are the covariance matrices of d assets.

Marginal Statistics

- **Left ES (Expected Shortfall / CVaR)** measures the left tail behavior of the scenarios
 $\text{ES}_\alpha = \mathbb{E}[X \mid X \leq \text{VaR}_\alpha]$
where VaR_α is the α -quantile. We use $\alpha = 5\%$ and only consider left tail.
- **Mean** summarizes the central tendency of the scenarios $\bar{x} = \frac{1}{n} \sum_i x_i$
- **Std** measures volatility $\sigma = \sqrt{\frac{1}{n} \sum_i (x_i - \bar{x})^2}$
- **Skew** measures asymmetry of the distribution $\gamma_1 = \mathbb{E}\left[\left(\frac{x-\mu}{\sigma}\right)^3\right]$
- **Kurt (Excess Kurtosis)** measures tail heaviness and peakedness relative to a normal distribution; higher values indicate more extreme outliers/heavier tails.
 $\gamma_2 = \mathbb{E}\left[\left(\frac{x-\mu}{\sigma}\right)^4\right] - 3$

All marginal statistics are computed asset-wise and we record the relative value comparing to GT, their mean and std across all assets are reported.

Risk Metrics

Given a portfolio with weights $w \in \mathbb{R}^d$, the portfolio return under scenario x is $r_p = w^\top x$.

VaR (Value at Risk) is the α -quantile of the portfolio return distribution. It represents the worst loss at confidence level $1 - \alpha$.

$$\text{VaR}_\alpha = \inf\{l : P(r_p \leq -l) \leq \alpha\}$$

We use $\alpha = 5\%$ (left tail).

ES (Expected Shortfall / CVaR) is the expected loss given that the loss exceeds VaR. It captures the average severity of tail losses.

$$\text{ES}_\alpha = \mathbb{E}[r_p \mid r_p \leq \text{VaR}_\alpha]$$

Volatility (Vol) is the standard deviation of portfolio returns $\sigma_{r_p} = \sqrt{\text{Var}(r_p)}$.

Max Loss is the worst single-scenario portfolio return $\min_i r_{p,i}$.