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University of Illinois at Urbana-Champaign
IE526 - Stochastic Calculus in Finance

# IE526 FINAL PROJECT

## AUTO-CALLABLE CONTINGENT INTEREST NOTES (LIRA-USD)

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# CHAPTER 1: INTRODUCTION

## Chapter 1: Introduction

### BACKGROUND

In this project, we evaluate the Auto-Callable Contingent Interest Notes linked to the performance of the Turkish Lira relative to the US Dollar. We begin by walking through the model's algorithm in Chapter 2, including the algorithm mesh (separated into four stages) and the instrument's payoffs (separated into eight parts). In Chapter 3, we discuss the parameters selected for our model. Lastly, in Chapter 4 we discuss our final valuation, conclusion, and briefly analyze computational errors in comparison with JPMorgan's calculation of the instrument's value.

# Chapter 2: The Algorithm

## THE EXCHANGE-RATE PDE

The exchange rate follows the geometric Brownian motion according to PS3 2(c).
The PDE is as follows:

$$\frac{\partial f}{\partial t} + (r_d - r_f)\frac{\partial f}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 f}{\partial X^2} - r_d f = 0$$

Where:

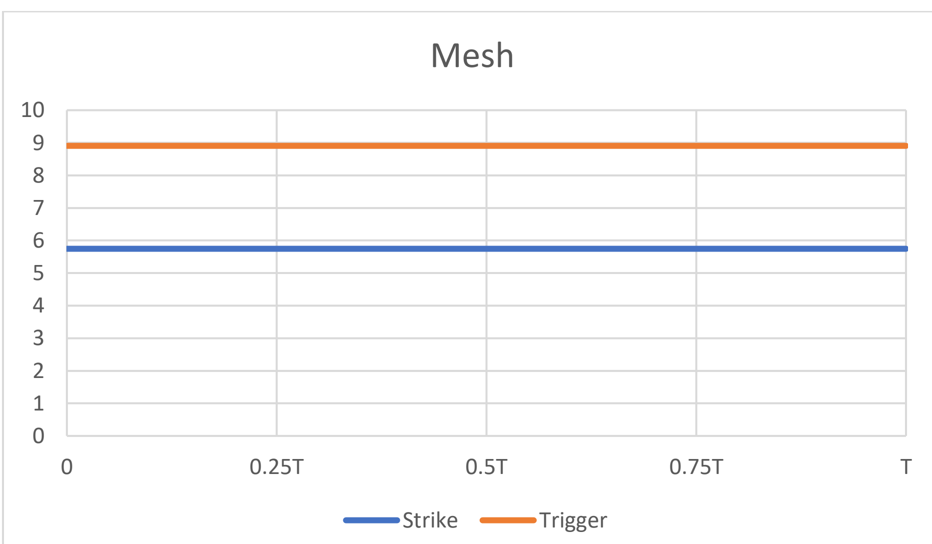
$r_d$ = domestic interest rate
$r_f$ = foreign interest rate
$X$ = FX rate
$f$ = value of the derivative

For higher convergence rate we use Crank-Nicolson Method. This method is based on the average of backward and forward difference and is of order $O(\delta\tau^2)$ and $O(\delta x^2)$. Note that this method is unconditionally stable.

We use the NEWMAT library to invert the matrix of stage K to generate stage K-1. Since it is approximately 92 days between each review day, we simply assume each of the T between each review day is 1/4 of a year.

## THE MESH

The first mesh incorporates time from 0 to 0.25T, the second mesh from 0.25T to 0.5T, the third mesh from 0.5T to 0.75T and the final fourth mesh is the time from 0.75T to T. For convenience, we ignore the possibility of previous unpaid interest payments at first; these are then added back as appropriate.



# CHAPTER 2: THE ALGORITHM

The first part is to estimate the value without potential delay payments. The source code for this is in the main.cpp file.

The terminal condition for mesh4 is the terminal payoff and the lower boundary equals to 1031.375, the upper boundary is equal to:

$$\max\left(1000 + 1000 \times \frac{Strike - S_t}{Strike}, 0\right)$$

Then we calculate the $S_0$ for mesh4.

To mesh3, if the price is lower than the strike price, the terminal value is 1031.375 (then we will add the other value together). If the terminal price is between strike rate and trigger rate, the terminal value equals to the initial value of mesh4 with the same rate plus 31.375. If the terminal price is higher than trigger price, the terminal value equals to the initial value of mesh4 with the same rate. The upper and lower boundary is as mesh4.

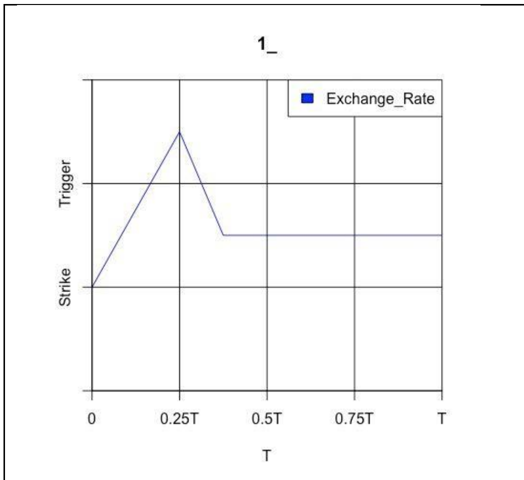
We repeat this algorithm until arrive at the initial value of mesh1.

## PAYOFFS

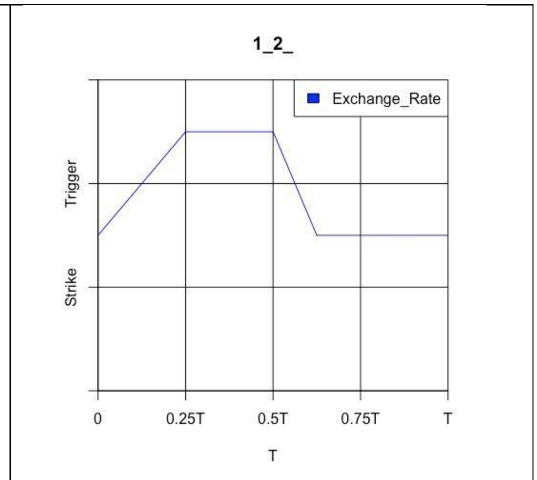
The second step is to value the potential delay in payments. The source code files are named in the following convention: 1_.cpp, 1_2_.cpp, and so on. In this naming convention, "1_" means the first payment is delayed, "1_2_" means the first and second payments are delayed, "1_3_" means the first and third payments are delayed, etc.

The figure of payoffs, broken out into eight separate parts, follows the same naming convention described above, with "1_" interpreted as the first payment delayed, and so on. The possible path of each scenario and the corresponding value is displayed on the following page; these must be added together to arrive at the final instrument value.

## FIGURE OF PAYOFFS




Value= 0.0023267

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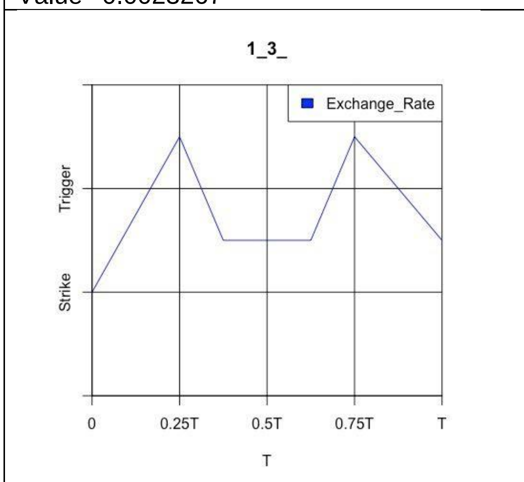
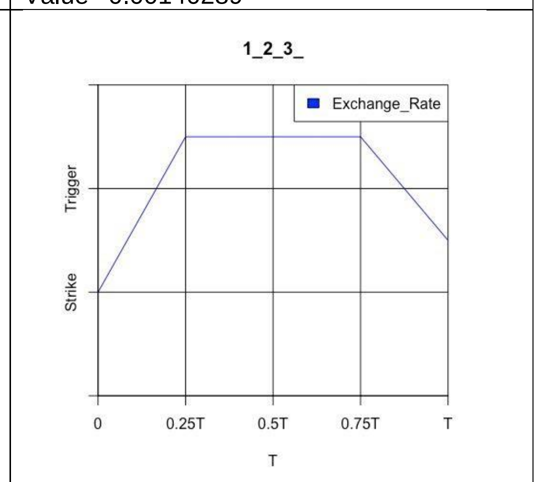
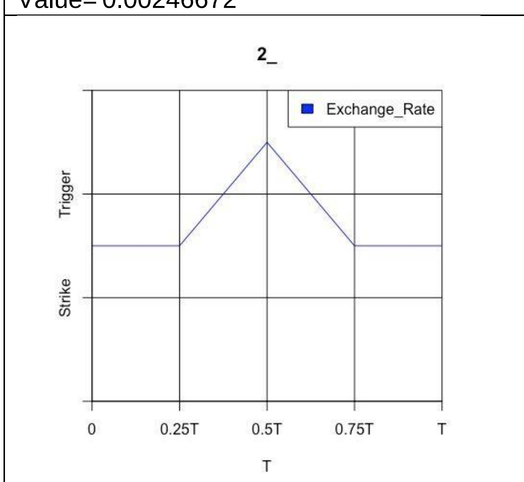


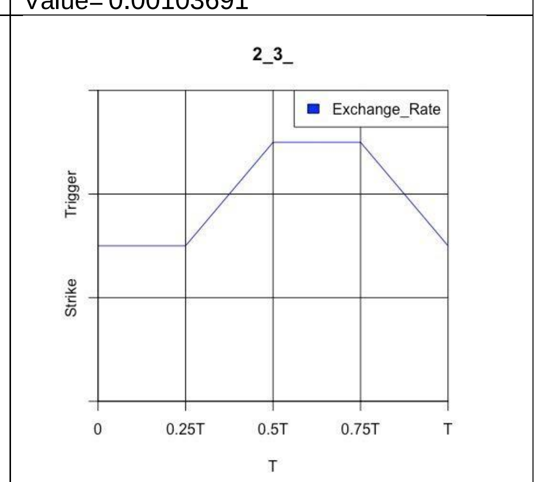

Value= 0.00246672




Value= 0.00103691




Value=10.2504




Value=10.2413

## Chapter 3: Parameters

### MODEL PARAMETERS

1. $S_0 = 5.7627 \rightarrow$ This is the price on March 22, 2019.
2. $S_{step} = 200 \rightarrow$ from 0 to 20.
3. $T_{step} = 100 \rightarrow$ for each mesh; thus we have 400 steps in total.
4. $\text{Strike} = 5.7458$
5. $\text{Trigger} = 1.55 \times \text{Strike} \rightarrow$ from materials in the SEC database.
6. $T = 0.25 \rightarrow$ our assumption.
7. $r_d = 0.02242997$
   $r_f = 0.19590 \rightarrow$ from treasury bond market; this is the 1Yr treasury note issued closest to $t = 0$ as the risk free rate
8. $\sigma = 0.2763623 \rightarrow$ obtain the FX rate for 2018 and fit the geometric Brownian motion when $[\ln(S_{t+\Delta t}) - \ln(S_t)] \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t, \sigma^2 \Delta t\right);\ \Delta t = 1\ day)$

# Chapter 4: Conclusion and Error Analysis

## CALCULATED FINAL VALUE OF INSTRUMENT

As can be seen in the following valuation today, our calculated final instrument value of $,073.634133.

| | |
|---|---|
| main | 1036.94 |
| 1_ | 2.33E-03 |
| 1_3_ | 2.47E-03 |
| 1_2_ | 0.001403 |
| 1_2_3 | 0.001037 |
| 2_ | 10.2504 |
| 2_3_ | 10.2413 |
| 3_ | 16.1952 |
| sum | 1073.634 |

## ERROR ANALYSIS

In the following points, we discuss the assumptions/calibrations in our computation of the 1,073.63 final instrument value and potential reasons why this number is different from JPMorgan's valuation of 962.30.

1. The model based on log-normal distribution, but in the real case, our return is usually with fat tail and pointed head. The historical log-return justify this flaw. We may turn to NIG or t distribution for better result.

```
> skewness(return)
[1] -2.064727
attr(,"method")
[1] "moment"
> kurtosis(return)
[1] 26.24866
attr(,"method")
[1] "excess"
```

2. The model based on fixed volatility, but the volatility is not fixed. We may use NGARCH model instead of fixed volatility.

```
> sd(return[1:100])
[1] 0.0118927
> sd(return[101:200])
[1] 0.0249546
```

3. The model is based on fixed risk-free rate, but they are changing from time to time, we may improve our model by incorporate risk-free rate of both countries as random variables.
4. The parameters we use may differ from what JPMorgan use.
5. The maturity is not exactly one year and the time between two payments is not exactly 0.25T.

# Appendix

## GITHUB REPOSITORY

[Auto-Callable-Contingent-Interest-Notes]