Tail Events in Hedge Fund Returns

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Abstract

Hedge funds are popular investment tools as they often claim to provide significant alpha. However, investors may reconsider their decision before investing in hedge funds for the occasional huge losses. Exacerbating the problem is that those "tail events" tend to be correlated in time. From a statistical point of view, this paper examines the volatility clustering issue of several popular hedge fund strategies. We find that most of the hedge fund strategies exhibit considerably more autoccorelation than the S&P 500 index when looking at negative "tail events" and their tails are heavier than the S&P 500 index on a monthly basis.

1 Introduction

Nowadays, people invest in hedge funds as they offer a relatively high risk-adjusted return using a well-diversified portfolio. Hedge fund investors usually monitor some specific risk measures including but not limiting to the Maximum Drawdown (MDD) and the Sharpe ratio to evaluate the overall risk management capability of hedge funds. Specifically, investors hope hedge funds could contribute to their portfolio alpha after adjusted for risk. Due to the scarcity of extreme events in the financial market, there are a limited amount of data that can be used to back-testing those risk measures. Consequently, the capability of those risk measures to describe "tail events" are usually questionable. Using the Credit Suisse hedge fund index and its subindices, this research investigates the "tail events" autocorrelation of several popular hedge fund strategies.

A widely used evaluation framework for interval forecasting was constructed by Christoffersen (1998). Specifically, it evaluates an interval forecast by two criteria: 1. The occurrence of "tail events" should be independent. 2. The frequency of "tail events" should be equal to the quantile. According to the previous criteria, two hypothesis tests were designed under the name of Christoffersen Independent Test and Christoffersen Unconditional Coverage Test. Under the Christoffersen Independent Test framework, the alternative hypothesis is a first-order Markov process. Consequently, the capability to detect higher order dependence is limited. To address this issue, Pajhede (2015) extends the Christoffersen Independent Test by replacing the first-order Markov process with a k'th-order Markov process.

Using hedge fund returns, this paper conducts an empirical study on their "tail events" autocorrelation under the aforementioned Christoffersen Independent Test and Pajhede Independent Test. Specifically, we transform the original series of returns into binary hit-sequences by whether it exceeds the empirical quantile range: returns that exceed a certain quantile range will be regraded as "tail events". We then perform the hypothesis tests on those transformed hit-sequences in order to examine the autocorrelation of those "tail events".

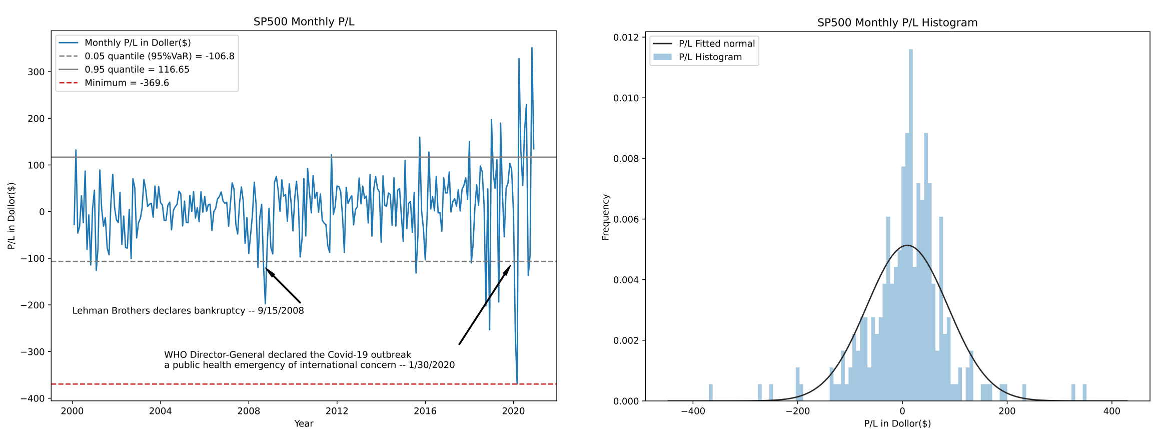
2 Preliminary Data Analysis

To perform empirical analysis, we collect the returns of the Credit Suisse hedge fund indices from January 2000 to December 2020. The indices include an aggregate index and thirteen subindices that represents different investment strategies. The S&P 500 index of the same period was introduced as a reference portfolio.

2.1 Preliminary Data Analysis for S&P 500 Index

During this period, we observe an excess kurtosis of 5.15 on the S&P 500 index monthly return while the excess kurtosis on the S&P 500 index daily return is 31.10. Obviously, the distribution of daily return exhibit much heavier tails.

Figure 1(a) shows the monthly profit and loss (P/L) of the S&P 500 index of this period. This figure visualizes the connection between the global events and market volatility. From the figure we can tell that the market volatility tend to be significantly influenced by global events. Figure 1(b) shows the histogram of the S&P 500 index monthly P/L. The data exhibits negative skewness(-0.26).



(a) SP500 Monthly P/L

(b) SP500 Monthly P/L Histogram

Figure 1: SP500 Monthly

2.2 Preliminary Data Analysis for Credit Suisse HF Index

In the following table contains the excess kurtosis for the Credit Suisse hedge fund indices including the aggregate index and thirteen subindices. For the Equity Market Neutral index and the Convertible Arbitrage index, we found that the worst cases of the "tail events" severally influence the portfolio performance. In November 2008, the Equity Market Neutral index dropped -152.54 in index value (-40%) whereas the rest of the monthly changes are between -10 and 10 in index value. Even without this extreme event, its kurtosis is still 2.56. The result is similar for the Convertible Arbitrage, with a kurtosis of 2.41 without considering the worst drop. In contrast, we found that Manage Future has negative excess kurtosis. Further analysis is conducted in the following sections.

Index Name	Excess Kurtosis[1]	Index Name	Excess Kurtosis[1]
Aggregate Index	8.82	Event Driven Multi-Strategy	23.89
Equity Market Neutral	203.87(2.56)	Event Driven Risk Arbitrage	16.18
Dedicated Short Bias	0.71	Fixed Income Arbitrage	32.42
Emerging Markets	6.56	Global Macro	6.96
Convertible Arbitrage	14.25(2.41)	Long/Short Equity	3.39
Event Driven	24.00	Managed Futures	-0.37
Event Driven Distressed	21.15	Multi-Strategy	12.31

[1] The excess kurtosis of a normal distribution is 0.

3 Method

Under the original back-testing by Christoffersen (1998) and Pajhede (2015), the independent tests is designed to evaluate the soundness of an interval forecast by evaluate the independent coverage criteria of the hit-sequence. We follow a similar framework to evaluate the dependence of "tail events" by constructing a quantile-based hit-sequence. Specifically, to detect "tail events" in P/L data, we construct two interval forecasts based on the quantile range: (0.05 quantile, $+\infty$) for one-sided interval forecasting, (0.025 quantile, 0.0975 quantile) for two-sided interval forecasting. In each case, any given P/L that exceeds that range will be regarded as a "tail event". Besides, unlike in Christoffersen (1998), this study takes all the P/L data as in-sample data to calculate the quantile for "tail events" detection. Consequently, those changes inherently guaranteed that the whole hit-sequence will pass the unconditional coverage test. This modification allow us to easily construct a hit-sequence from the original P/L data whereas make it harder to perform a meaningful unconditional coverage test on this hit-sequence.

3.1 Hit-sequence

For a given P/L series $X_1, X_2, ..., X_t$, the hit-sequence $I_1, I_2, ..., I_t$ is defined as the indicator function of whether that loss exceeds the quantile range:

$$I_t = \begin{cases} 1 & X_t \text{ exceed quantile} \\ 0 & X_t \text{ does not exceed quantile} \end{cases}$$

According to Christoffersen (1998), a good interval forecast should satisfies:

1. The unconditional coverage criteria: The unconditional probability of the occurrence of a "tail event" must be equal to the coverage rate p:

$$P(I_t = 1) = p$$

2. The independent coverage criteria: The probability conditional on all given information at time $t-1$, $\mathcal{F}_{t-1}$, of the occurrence of a "tail event" must be constant:

$$P(I_t = 1|\mathcal{F}_{t-1}) = P(I_t = 1)$$

3.2 Independent Tests

Let N_{ij} be the number of days in which state j occur at time t and state i occurred at time $t-1$. The following table represents all possible outcomes for the hit-sequence:

I_{t-1} \ I_t	$I_t = 0$	$I_t = 1$
$\mathcal{F}_{t-1} = 0$	N_{00}	N_{01}
$\mathcal{F}_{t-1} = 1$	N_{10}	N_{11}

Under the definition of the Christoffersen independent test, we have:

$$N_{ij} = \sum_{t=1}^{T} \mathbf{1}\left(\mathcal{F}_{t-1} = i\right) \mathbf{1}\left(I_t = j\right), \quad \mathcal{F}_{t-1} = \mathbf{1}\left(I_{t-1} > 0\right)$$

The Pajhede independent test modified the definition of $\mathcal{F}_{t-1}$ to incorporate higher-order dependence:

$$\mathcal{F}_{t-1} = \mathbf{1}\left(\sum_{i=1}^{k} I_{t-i} > 0\right)$$

From above we can derive the transition probability matrix given by:

$$\Pi = \begin{bmatrix} 1 - P_s & P_s \\ 1 - P_e & P_e \end{bmatrix}$$

$\hat{P}_s$ refers to the steady probability when $I_t = 1$ given $\mathcal{F}_{t-1} = 0$ and $\hat{P}_e$ refers to the excited probability when $I_t = 1$ given $\mathcal{F}_{t-1} = 1$. Under the null hypothesis, we expect those two probabilities are equal:

$$P_s = P_e = P,$$

Accordingly, the estimators of above probabilities can be found:

$$\hat{P}_s = \frac{N_{01}}{N_{01} + N_{00}}, \quad \hat{P}_e = \frac{N_{11}}{N_{10} + N_{11}}, \quad \hat{P} = \frac{N_{01} + N_{11}}{N_{01} + N_{00} + N_{10} + N_{11}}$$

The likelihood ratio test statistic is given by:

$$LR = -2 \log \left(\frac{(1 - \hat{P})^{N_{00}+N_{10}} \hat{P}^{N_{01}+N_{11}}}{(1 - \hat{P}_s)^{N_{00}} (\hat{P}_s)^{N_{01}} (1 - \hat{P}_e)^{N_{10}} (\hat{P}_e)^{N_{11}}} \right)$$

From Hoel (1954) we know that this likelihood ratio test for independence follows a χ^2 distribution with one degree of freedom.

4 Results and Analysis

Table 1: Two-sided Test Results

Name	C	$P, K = 1$	$P, K = 3$	$P, K = 5$	Excess Kurtosis[1]
Aggregate Index	8.98	8.98	9.45	8.63	8.82
Convertible Arbitrage	0.06	0.06	0.45	3.26	14.25(2.41)
Dedicated Short Bias	1.38	1.38	0.97	2.68	0.71
Emerging Markets	4.43	4.43	4.87	7.27	6.56
Equity Market Neutral	0.06	0.06	0.29	0.95	203.87(2.56)
Event Driven	8.98	8.98	5.38	4.64	24.00
Event Driven Distressed	14.06	14.06	5.94	8.34	21.15
Event Driven Multi-Strategy	14.06	14.06	18.51	13.55	23.89
Event Driven Risk Arbitrage	8.98	8.98	5.65	4.84	16.18
Fixed Income Arbitrage	20.02	20.02	21.11	20.24	32.42
Global Macro	14.06	14.06	9.45	8.34	6.96
Long/Short Equity	14.91	14.91	11.26	7.09	3.39
Managed Futures	0.10	0.10	2.10	3.73	−0.37
Multi-Strategy	8.98	8.98	9.45	8.34	12.31
S&P 500 Monthly	18.91	18.91	26.32	27.23	5.15

[1] The excess kurtosis of a normal distribution is 0.

Table 2: One-sided Test Results

Name	C	$P, K = 1$	$P, K = 3$	$P, K = 5$	Excess Kurtosis
Aggregate Index	9.60	9.60	7.15	3.91	8.82
Convertible Arbitrage	1.99	1.99	0.88	0.64	14.25(2.41)
Dedicated Short Bias	1.14	1.14	0.26	0.13	0.71
Emerging Markets	1.99	1.99	2.85	0.77	6.56
Equity Market Neutral	5.25	5.25	2.85	2.27	203.87(2.56)
Event Driven	9.60	9.60	6.35	5.65	24.00
Event Driven Distressed	9.60	9.60	10.42	9.47	21.15
Event Driven Multi-Strategy	9.60	9.60	10.42	9.77	23.89
Event Driven Risk Arbitrage	1.99	1.99	0.88	0.01	16.18
Fixed Income Arbitrage	5.25	5.25	0.97	0.02	32.42
Global Macro	5.25	5.25	14.42	10.40	6.96
Long/Short Equity	5.25	5.25	1.25	0.13	3.39
Managed Futures	0.15	0.15	0.01	0.00	-0.37
Multi-Strategy	14.91	14.91	10.83	10.08	12.31
S&P 500 Monthly	1.99	1.99	5.29	5.04	5.15

The results for the two-sided test and one-sided test are shown in the above tables. C refers to the test statistics of Christoffersen (1998) independent tests and P refers to that of Pajhede (2015) independent tests. Generally speaking, a lower magnitude means the his-sequence is more likely to be independent and the corresponding "tail events" in the return series demonstrate less autocorrelation.

4.1 Results for Two-sided Tests

For two-sided test results, under the 95% CI we could not reject the null hypothesis that the "tail events" are independent for the Convertible Arbitrage, Dedicated Short Bias, Equity Market Neutral, and Managed Futures.

4.2 Results for One-sided Tests

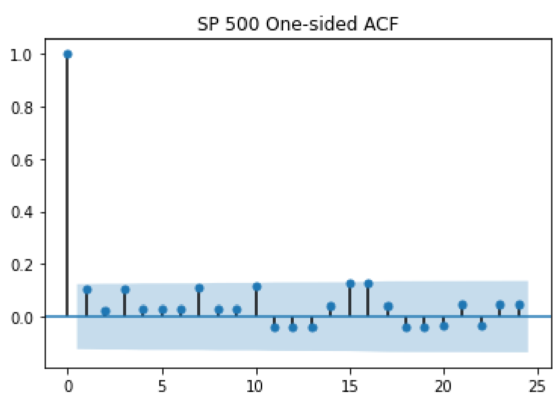
For one-sided test results, under the 95% CI we could not reject the null hypothesis that the negative "tail events" are independent for the Convertible Arbitrage, Dedicated Short Bias, Emerging Markets, Event Driven Risk Arbitrage, S&P 500 index, and Managed Futures. The overlapping categories for one-sided tests and two-sided tests are the Convertible Arbitrage, Dedicated Short Bias, and Managed Futures.

4.3 Results Analysis

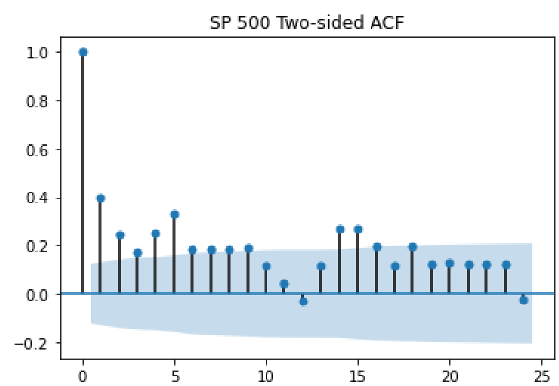
From both tables, we notice that the test statistics declined from the two-sided test to the one-sided test for both hedge fund data and the S&P 500 index in general. It is also worth noticing that the S&P 500 index seems to be more correlated than the hedge funds under the two-sided test but the inverse under the one-sided test. It shows that the hedge funds have more correlated negative "tail events" than the S&P 500 index.

In a study that investigate volatility clustering, Jacobsen and Dannenburg (2003) reported that volatility clustering is both present in high-frequency data and monthly data for the S&P 500 index. This conclusion is also supported by our two-sided test statistics: we could reject the null hypothesis under the 95% CI $(0, 3.841)$ that the "tail events" in the S&P 500 monthly return are independent. However, the results of the one-sided test conclude a different result. Under the 95% CI we could not reject the null hypothesis that the S&P 500 index month negative "tail events" are correlated.

As supportive evidence, figure 2(a) and figure 2(b) visualize the Autocorrelation Function (ACF) for the hit-sequence constructed using the one-sided interval forecast criteria and two-sided interval forecast criteria for the S&P 500 index, respectively. We observe a significant level of autocorrelation under the 95% CI for two-sided hit-sequence while no significant autocorrelation exists in the one-sided hit-sequence.



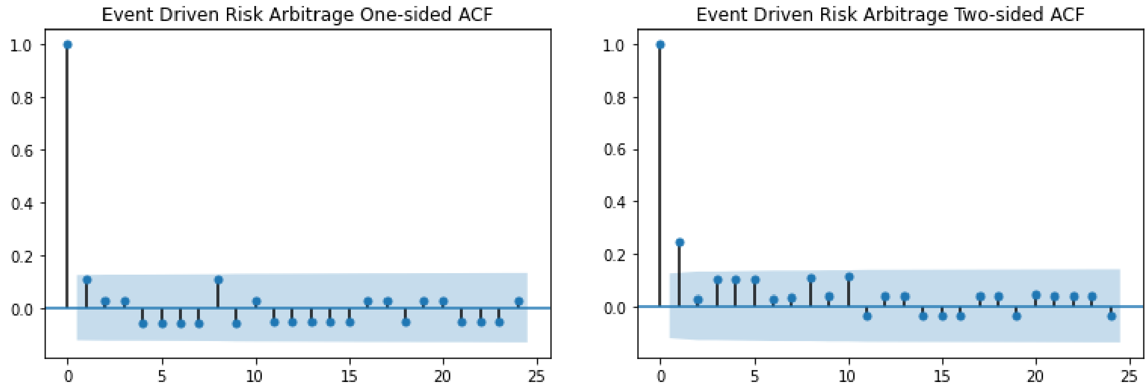
(a) SP500 One-sided ACF



(b) SP500 Two-sided ACF

Figure 2: SP500 ACF

Similarly, the ACF figures for the Event Driven Risk Arbitrage are shown below as an example of an hedge fund strategy that are dependent under two-sided test while independent under one-sided test. Besides, the two-sided test statistics for the S&P 500 index and the Event Driven Risk Arbitrage are 18.91 and 8.98, respectively. When comparing the two-sided figures for both we can easily find that the hit-sequence of the S&P 500 index exhibit more autocorrelation. Generally speaking, hit-sequences with higher test statistics usually shows more autocorrelation in their ACF figures.



(a) Event Driven Risk Arbitrage One-sided ACF (b) Event Driven Risk Arbitrage Two-sided ACF

Figure 3: Event Driven Risk Arbitrage ACF

A possible explanation for the aforementioned phenomenon is that the market tend to overreact to new information and may move to the opposite direction to compensate. Therefore, we may observe a series of large positive moves and negative moves rather than a series of large positive moves or negative moves in a short time period. However, the hedge funds are run by sophisticated investors so they are more likely to hold their opinions even when the market moves to an unfavorable direction.

We observed that the S&P 500 index has lighter tails than the hedge fund returns based on their excess kurtosis. An interesting category of hedge funds is the Managed Future which tails are even lighter than the normal distribution.

When comparing the test statistics of different k values, the hedge funds and the S&P 500 index show different characteristics: the S&P 500 index seems to exhibit a significant level of higher-order dependence under the one-sided test and two-sided test while in general hedge funds do not.

5 Conclusion

The main goal of the current study was to study the dependence and structure of the “tail events” for hedge fund returns. We have introduced the independent test from Christoffersen (1998) and Pajhede (2015) to perform an empirical study based on the Credit Suisse hedge fund index. For general extreme events, we find that the hedge fund returns exhibit heavier tails than the S&P 500 index, but their “tail events” tend to be more independent over time. However for investors who care about risk control, it seems like their negative “tail events” are more correlated than the S&P 500 index although both of them declined on one-sided condition. In other words, from the risk aspect hedge funds investors are more likely to face a series of correlated bad months as well as larger uncorrelated tail events comparatively.

In conclusion, although hedge funds usually claim they could bring investors high risk-adjusted returns, the data from the Credit Suisse hedge fund index shows a different picture: hedge funds tend to have more negative "tail events" in sequence than the S&P 500 index and also more extreme events in general. Even worse than this is that the lockup period and other buyback limitations may prevent investors to quickly react to unfavorable situations.

For further research, this paper suggests to incorporate certain common bias of hedge fund index. Investors should consider the effect of the survivorship bias. Which means the index tends to ignore hedge funds that have already gone out of the market. Moreover, the backfill bias could be an issue as well, which means hedge funds with good performance are more likely to be added to the index. Even without the incorporation of such effects, this numerical study reveals some potential problems investors may face when investing in hedge funds. Since both biases underestimate the risk and overestimate the return, investors should seriously consider investing in hedge funds and hedge funds should only open to sophisticated investors with sufficient knowledge and high level of risk tolerance.

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