

**IE523: Financial Computing**  
**Fall, 2018**  
**Programming Assignment 5: Computing the General**  
**Solution to  $\mathbf{Ax} = \mathbf{y}$**   
**Due Date: 5 October 2018**  
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I want you to write a generic solver for a set of simultaneous equations  $\mathbf{Ax} = \mathbf{y}$ , where  $\mathbf{A}$  is a known  $n \times m$  matrix and  $\mathbf{y}$  is a known  $n \times 1$  vector. You may wish to look at lesson 2 of my notes for the relevant theory. **This assignment can take some time, please get started on it ASAP.**

Here are the components your code:

1. Check if there is a solution to  $\mathbf{Ax} = \mathbf{y}$  by testing if  $\text{rank}([\mathbf{A} \mid \mathbf{y}]) = \text{rank}(\mathbf{A})$ .
2. If there is no solution – report it as such. If there is a solution, then present the general solution to  $\mathbf{Ax} = \mathbf{y}$ .

You will need *NEWMAT* for this programming exercise. You might wish to look at `linear_algebra1.cpp`, `linear_algebra2.cpp` and `linear_algebra3.cpp` on Compass to get started on this exercise. As you will see when you do this assignment (or, if you attended class regularly and listened attentively), you will need to pick  $\text{rank}(\mathbf{B})$ -many columns from a matrix  $\mathbf{B}$ , at various instants. For this, you may wish to look at the first programming assignment where you picked all  $n$ -take- $k$  combinations of a set of  $n$  objects (where  $1 \leq k \leq n$ ).

If there is a solution to  $\mathbf{Ax} = \mathbf{y}$ , you will have to compute all possible basic solutions, where you pick  $\text{rank}(\mathbf{A})$ -many columns of  $\mathbf{A}$  (call this matrix  $\mathbf{X}$ ;  $\text{rank}(\mathbf{X}) = \text{rank}(\mathbf{A})$ ) and then compute  $\mathbf{z}$  according to equation 1 of lesson 2 of my notes. You then have to insert zeros into  $\mathbf{z}$  at appropriate locations (depending on which columns of  $\mathbf{A}$  were chosen to be in  $\mathbf{X}$ ). This augmented-version of  $\mathbf{z}$  becomes one basic solution (you have several more, depending on how many candidates for  $\mathbf{X}$  you have in any given case). Following this, you stack all basic solutions into one large matrix  $\mathbf{S}$  and pick  $\text{rank}(\mathbf{S})$ -many columns of  $\mathbf{S}$  (call this matrix  $\mathbf{T}$ ;  $\text{rank}(\mathbf{T}) = \text{rank}(\mathbf{S})$ ) and present the affine combination of these columns as the general solution to  $\mathbf{Ax} = \mathbf{y}$ .

The input to your code should be read on command-line through a file. For example, the file `input1` on Compass, which is shown in figure 1 presents the relevant details for the instance shown in equation 2 in page 4 of lesson 2 of my notes. The first (second) entry in the file is the number of rows (columns) of  $\mathbf{A}$ . Following this, each element of  $\mathbf{A}$  is presented in a row-by-row fashion. Finally, the entries of  $\mathbf{y}$  are presented in a row-by-row manner. The format of the input file should be obvious with the example from equation 1 in page 4 of lesson 2 of my notes and what is shown in figure 1. I am looking for an output along the lines of what is shown in figure 2 for this example. Note that the

output in figure 2 says the general solution has the form

$$\lambda \begin{pmatrix} 1 \\ -2 \\ 3 \\ 0 \end{pmatrix} + (1 - \lambda) \begin{pmatrix} 3 \\ -3 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 - 2\lambda \\ \lambda - 3 \\ -3\lambda \\ \lambda - 1 \end{pmatrix}. \quad (1)$$

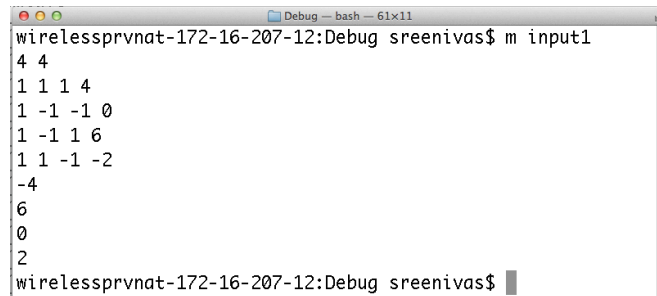
I am also including a file called `input2` on Compass that presents the relevant input for the problem in section 3.1 of lesson 2 of my notes. Figure 4 shows the sample output when my C++ code is run on `input2`. The output in figure 4 says the general solution has the form

$$\gamma_1 \begin{pmatrix} 28 \\ 14 \\ 0 \\ 15 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \gamma_2 \begin{pmatrix} 37 \\ 14 \\ 0 \\ 0 \\ -3 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \gamma_3 \begin{pmatrix} 28 \\ 0 \\ -7 \\ 15 \\ 0 \\ 7 \\ 0 \\ 0 \end{pmatrix} + (1 - \gamma_1 - \gamma_2 - \gamma_3) \begin{pmatrix} -7 \\ 0 \\ 0 \\ 15 \\ 0 \\ 0 \\ 0 \\ 7 \end{pmatrix} = \begin{pmatrix} 35\gamma_1 + 44\gamma_2 + 35\gamma_3 - 7 \\ 14\gamma_1 + 14\gamma_2 \\ -7\gamma_3 \\ 15 - 15\gamma_2 \\ -3\gamma_2 \\ 7\gamma_3 \\ 7 - 7\gamma_2 - 7\gamma_3 - 7\gamma_1 \end{pmatrix}. \quad (2)$$

The input file shown in figure 5 describes the equation that is just below equation 2 in page 4 of lesson 2 of my notes. Figure 6 shows the result when my code is run on this input file.

## Some Questions/thoughts...

**Question:** How do you reconcile the fact that while equation 1 looks similar to the general solution presented in page 7 of lesson 2 of my notes, equation 2 looks different from equation 6 in page 10 of lesson 2.



```
wirelessprvnat-172-16-207-12:Debug sreenivas$ m input1
4 4
1 1 1 4
1 -1 -1 0
1 -1 1 6
1 1 -1 -2
-4
6
0
2
wirelessprvnat-172-16-207-12:Debug sreenivas$
```

Figure 1: The format of the input file that describes the problem of equation 2 in page 4 of lesson 2 of my notes.

```

wirelessprvnat-172-16-207-12:Debug sreenivas$ ./General\ Solution input1
Solving:
1.000000  1.000000  1.000000  4.000000
1.000000 -1.000000 -1.000000  0.000000
1.000000 -1.000000  1.000000  6.000000
1.000000  1.000000 -1.000000 -2.000000

* solution =

-4.000000
6.000000
0.000000
2.000000

solution is the affine-combination of these vectors
1.000000  3.000000
-2.000000 -3.000000
-3.000000  0.000000
0.000000 -1.000000

Verification: (check each column below and y-vector above)
-4.000000 -4.000000
6.000000  6.000000
0.000000  0.000000
2.000000  2.000000

wirelessprvnat-172-16-207-12:Debug sreenivas$

```

Figure 2: Sample output when the file shown in figure 1 is presented as input. The general solution is presented as the affine combination of the two solution vectors shown in the screen.

```

wirelessprvnat-172-16-207-12:Debug sreenivas$ m input2
8 7
1 1 1 1 -2 3 7
1 -1 -1 -1 8 -3 3
1 1 -1 -1 8 1 7
1 -1 -1 -1 8 -3 3
1 1 1 -1 8 3 7
1 -1 1 1 -2 -1 3
1 1 -1 1 -2 1 7
1 -1 -1 -1 8 -3 3
57
-1
27
-1
27
29
57
-1
wirelessprvnat-172-16-207-12:Debug sreenivas$

```

Figure 3: The format of the input file that describes the problem of section 3.1 of lesson 2 of my notes.

```

wirelessprvnt-172-16-207-12:Debug sreenivas$ ./General\ Solution input2
Solving:
1.000000  1.000000  1.000000  1.000000 -2.000000  3.000000  7.000000
1.000000 -1.000000 -1.000000 -1.000000  8.000000 -3.000000  3.000000
1.000000  1.000000 -1.000000 -1.000000  8.000000  1.000000  7.000000
1.000000 -1.000000 -1.000000 -1.000000  8.000000 -3.000000  3.000000
1.000000  1.000000  1.000000 -1.000000  8.000000  3.000000  7.000000
1.000000 -1.000000  1.000000  1.000000 -2.000000 -1.000000  3.000000
1.000000  1.000000 -1.000000  1.000000 -2.000000  1.000000  7.000000
1.000000 -1.000000 -1.000000 -1.000000  8.000000 -3.000000  3.000000

* solution =

57.000000
-1.000000
27.000000
-1.000000
27.000000
29.000000
57.000000
-1.000000

solution is the affine-combination of these vectors
28.000000 37.000000 28.000000 -7.000000
14.000000 14.000000 0.000000 0.000000
-0.000000 -0.000000 -7.000000 -0.000000
15.000000 0.000000 15.000000 15.000000
0.000000 -3.000000 0.000000 0.000000
0.000000 0.000000 7.000000 0.000000
0.000000 0.000000 0.000000 7.000000

Verification: (check each column below and y-vector above)
57.000000 57.000000 57.000000 57.000000
-1.000000 -1.000000 -1.000000 -1.000000
27.000000 27.000000 27.000000 27.000000
-1.000000 -1.000000 -1.000000 -1.000000
27.000000 27.000000 27.000000 27.000000
29.000000 29.000000 29.000000 29.000000
57.000000 57.000000 57.000000 57.000000
-1.000000 -1.000000 -1.000000 -1.000000

wirelessprvnt-172-16-207-12:Debug sreenivas$ █

```

Figure 4: Sample output when the file shown in figure 3 is presented as input. The general solution is presented as the affine combination of the four solution vectors shown in the screen.

```

Ramavarapus-MacBook-Air:Debug sreenivas$ m input3
4 4
1 1 1 4
1 -1 -1 0
1 -1 1 6
1 1 -1 -2
1
0
0
0
Ramavarapus-MacBook-Air:Debug sreenivas$ █

```

Figure 5: The format of the input file that describes the problem just below equation 2 in page 4 of lesson 2 of my notes.

```
Debug -- bash -- 71x16
Ramavarapus-MacBook-Air:Debug sreenivas$ ./General\ Solution input3
Solving:
  1.000000  1.000000  1.000000  4.000000
  1.000000 -1.000000 -1.000000  0.000000
  1.000000 -1.000000  1.000000  6.000000
  1.000000  1.000000 -1.000000 -2.000000

* vector_x =

  1.000000
  0.000000
  0.000000
  0.000000

There is no solution to this equation
Ramavarapus-MacBook-Air:Debug sreenivas$
```

Figure 6: Sample output when the file shown in figure 5 is presented as input.