Image restoration using learned gradient descent

Iaroslav Koshelev

22/09/2020

0 Abstract

Being an ill-posed problem, image restoration opts to restore high-quality image from a low-quality one, assuming that given low-quality input was produced from a known degradation model. A lot of great restoration methods were proposed for the case when linear degradation operator and i.i.d. Gaussian likelihood are assumed. However such methods are known to not generalize well and show a sub-par performance on real data, for which the true degradation model is neither linear, nor even exactly known.

Modern machine learning advances allow us to overcome this issue and learn a restoration model to the real data. The main drawback of such approaches is overfitting, since to learn an inverse mapping between low-quality and high-quality samples they completely rely on data and do not utilize may be limited, but existing knowledge of how degradation was performed.

In this project I am going to test some ideas, related to learned gradient descent based image restoration ans synthesis, which I have in mind for already a long time. I will assume both linear and non-linear known restoration problems and try to do a research on how a known degradation model may be incorporated in learned gradient based restoration procedure.

1 Introduction

Let us firstly assume the most general case, when a low-quality image y produced from a high-quality image x by some known degradation $\phi(x)$ and some signal-dependent noise model:

$$y = \phi(x) + \eta(x),\ \eta \sim \mathcal{D}_x \tag{1}$$

We may not know the signal-dependent noise model $\eta(x) \sim \mathcal{D}_x$exactly, so our degradation model consists of a known deterministic part $\phi(x)$ and unknown stochastic one $\eta(x)$. From Bayes theorem we can obtain MAP solution for this problem as:

$$\hat{x} = \arg\min_x log(p(y|x)) + log(p(x)) = \arg\min_x L(y,x) + \varphi(x), \tag{2}$$

where we denoted a data fidelity term as $L(y,x)$ and image prior term as $\varphi(x)$, and assume that both are differentiable. There are several gradient based optimization approaches, which are designed specifically to solve such problems. They include gradient descent, but if we assume distribution $\eta(x)$ to be Gaussian (hence likelihood term as L2 norm), then regularized Gauss–Newton [3] (or Levenberg–Marquardt as more advanced) are also at hand.

Let us discuss an optimization via gradient descent and a method of how to make it learning-based (great again). The classical gradient descent algorithm provides the following solution to our problem:

$$x_{t+1} = x_t + \gamma_t \nabla_x (L(y,x) + \varphi(x))|_{x=x_t}. \tag{3}$$

Here each update step is performed in the direction of gradient, computed for both lieklihood and prior terms $\nabla_x (L(y,x) + \varphi(x))|_{x=x_t} = \nabla_x L(y,x)|_{x=x_t} + \nabla_x \varphi(x)|_{x=x_t}$, and step size γ_t is obtained using for example line search. The idea behind the learned gradient descent [1, 2] may be described in the following way. Existing update step direction and step size are replaced altogether with a result of some convolution neural network $\mathcal{N}_\omega$ with parameters ω, which we feed with the maximum amount of information we have, i. e. with current estimate x_t and gradient of likelihood $\nabla_x L(y,x)|_{x=x_t}$:

$$x_{t+1} = x_t + \mathcal{N}_\omega (\nabla_x L(y,x)|_{x=x_t}, x_t). \tag{4}$$

Restoration then is provided in recurrent fashion and may be seen as an unrolled inference of recurrent neural network (RNN) $\mathcal{N}_w$ with an additional input. This is the method, which I am going to implement, based on the paper [5], where authors propose to use it for solving linear restoration problems and test it on core image restoration problems like denoising and super-resolution. I will try to reproduce their results and to experiment with some problems, where degradation is known, but nonlinear, for example semantic image synthesis [4] (degradation model is just a semantic segmentation neural network). If there will be enough time, I will also try to work on learned regularized Gauss-Newton method and conduct some experiments with it.

All codes of the project are open and available at my GitHub repo.

References

[1] J. Adler and O. Öktem. "Learned Primal-Dual Reconstruction". In: *IEEE Transactions on Medical Imaging* 37.6 (2018), pp. 1322–1332.

[2] Jonas Adler and Ozan Öktem. "Solving ill-posed inverse problems using iterative deep neural networks". In: *Inverse Problems* 33.12 (Nov. 2017), p. 124007. DOI: `10.1088/1361-6420/aa9581`. URL: `https://doi.org/10.1088%2F1361-6420%2Faa9581`.

[3] Martin Burger and Barbara Kaltenbacher. "Regularizing Newton–Kaczmarz Methods for Nonlinear Ill-Posed Problems". In: *SIAM Journal on Numerical Analysis* 44.1 (2006), pp. 153–182. DOI: `10.1137/040613779`. eprint: `https://doi.org/10.1137/040613779`. URL: `https://doi.org/10.1137/040613779`.

[4] Taesung Park et al. "Semantic Image Synthesis with Spatially-Adaptive Normalization". In: *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*. 2019.

[5] Patrick Putzky and Max Welling. *Recurrent Inference Machines for Solving Inverse Problems*. 2017. arXiv: `1706.04008 [cs.NE]`.