DIP assignment 5

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1 Ans1

We have

$$g_1 = f_1 + h_2 * f_2 \tag{1}$$

$$g_2 = h_1 * f_1 + f_2 \tag{2}$$

where $g_1, g_2, f_1, f_2, h_1, h_2$ are matrices of appropriate dimensions. We know g_1, g_2, h_1, h_2. Using g_1, g_2 dimensions we can find out the dimensions of f_1, f_2. Now let f_1, f_2 be matrices of those dimensions whose all entries are real variables. Now to find the value of those variables take discrete Fourier transform on both equations to get

$$DFT(g_1) = DFT(f_1) + DFT(h_2).DFT(f_2)$$

$$DFT(g_2) = DFT(f_2) + DFT(h_1).DFT(f_1)$$

while simplifying we have made use of the property of Discrete Fourier transform is linear and Discrete Fourier transform of convolution of two functions is same as point-wise product of the discrete Fourier transform of individual functions. We would also have to perform appropriate zero padding on the matrices so that after taking Discrete fourier transform we get matrcies of same dimensions.

We can now do simple algebraic manipulation in above equations to get

$$DFT(f_1) = \frac{DFT(g_2)DFT(h_2) - DFT(g_1)}{DFT(h_1)DFT(h_2) - \mathbf{1}}$$

$$DFT(f_2) = \frac{DFT(g_1)DFT(h_1) - DFT(g_2)}{DFT(h_1)DFT(h_2) - \mathbf{1}}$$

Where **1** represents an image with same dimensions as $DFT(h_1)DFT(h_2)$ and all entries as 1. Also note that the division is point-wise and because of zero padding that we did all matrices are of same dimensions and hence there is no ambiguity in pointwise division.

Then after computing $DFT(f_1), DFT(f_2)$ f_2,f_2 can be computed as $f_1 = DFT^{-1}(DFT(f_1)), f_2 = DFT^{-1}(DFT(f_2))$.

The problems are

1. If the value of $DFT(h_1)DFT(h_2)$ becomes 1 at some pixel than at that pixel the value of $DFT(f_1), DFT(f_1)$ isn't calculable and hence this approach wont work exactly in that case we would have to assume some value for that frequency to calculate its inverse discrete Fourier transform

2. Even if its close to one than also there are high chances of having huge errors due to some errors in measurement of h_1, h_2, g_1, g_2 would be magnified by division by some small quantity.

3. This is most likely to happen for the lower frequencies as the blur kernel is a low pass filter and hence $DFT(h_)), DFT(h_2)$ will have higher values for lower frequencies and hence for lower frequencies $DFT(h_1).DFT(h_2)$ is more likely to be close to 1 than for higher frequencies. This would lead to loss of information about low frequencies