The Relationship between Discovery Methods and System Properties on Exoplanets Distance

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I. Introduction

Exoplanets are planets that orbit stars outside our solar system. Unlike the planets in our solar system that orbit the Sun, exoplanets revolve around other stars in the Milky Way galaxy and potentially beyond. The NASA Exoplanet Archive is a comprehensive astronomical database dedicated to the collection and dissemination of data related to exoplanets and their host stars. The distance of the discovered exoplanets is a very important physical parameter as it permits us to convert observed properties to the actual system properties.

This project aims to investigate whether the mean exoplanet distances are different between the discovery methods while controlling for the other properties of the exoplanet and its host star. Namely, the exoplanet's orbital period, the host star's mass, and its Gaia magnitude (brightness). In this project I also investigate whether there are interactions between these predictors.

First, the data of all exoplanets discovered between 1992 and 2023 were obtained from NASA Exoplanet Archive and analyzed using R (R Core Team, 2023). There were multiple entries for each exoplanet (total sample=35115) since multiple methods/studies were done on each of the exoplanets to determine its properties. To avoid having duplicated entries for each planet, I chose to select only the entry for each planet that has the least average error in the distance (the response) and if there were ties, the most recent entry was kept. I also kept only the first discovered planet from each star system if there were multiple planets in one star system. After this filtering process, 3916 unique exoplanets remained. Due to missingness in data, the following variables; discovery method, the exoplanet's orbital period, distance, the host star's mass, and their Gaia magnitude (brightness). The orbital period had an outlier value with orbital period of 402000000 days. Upon inspection, it was revealed that this measurement had a very high error (+470000000,- 100000000) so I excluded it from the dataset. After these final filtrations, the number was reduced again to the final sample size, n=3350. Finally, for the discovery method variable, I grouped all the timing variations methods (eclipse timing variations, pulsation timing variations, and transit timing variations) into one group. I also grouped the following methods: astrometry, orbital brightness

modulation, imaging into an “other” group since they have low numbers compared to the radial velocity and transit method.

II. Statistical Procedures Used

An initial assessment of the relations between the variables using ggpairs plot from the GGally package (Schloerke et. al, 2021) (shown in Figure 1), indicated that there is a need to take the log of the response (distance) as well as the following predictors: exoplanet’s orbital period, and host star’s mass. A table containing a summary statistics of the final dataset is found in the appendix which was generated using the summary function in R (Table 1). Another ggpair plot of the logged distance and the other variables was produced (Figure 2).

An enhanced strip chart (Greenwood, 2023) between the logged distance and the discovery method (Figure 3) was also produced to investigate the relation between them. From Figure 3 and Table 1, it is clear that the discovery method observations are unbalanced with much larger number of exoplanets discovered by the radial velocity and transit methods (n=2611 and n=710, respectively) compared to the exoplanets discovered by the timing variations and the “other” category (n=15 and n=14, respectively).

For the initial model, I fit an additive linear model with all the predictors $\mu\{\log(D) \mid discvMthd, \log(OrbPrd), \log(StarMass), Mag\} = discvMthd + \log(OrbPrd) + \log(StarMass) + Mag$. Figures 3 and 4 show the diagnostic plot generated using the resid_panel function from the ggResidpanel package (Goode and Rey, 2023) and an effects plot using the effects package (Fox and Weisberg, 2019), respectively. From the data collection, there is weak evidence against the independence assumption since the distance to the different exoplanets in different star systems should not be related. From the residuals plot (upper-left panel of Fig.3), there is no clear curvature to the residual distribution, thus providing weak evidence against the linearity assumption. This is also supported by the symmetrical distribution of the residuals around the model line of all the other quantitative predictors as well as the smoothing lines seen in Figure 4. Furthermore, the residuals plot show mostly consistent vertical spread of the residuals around the zero line. But fitted values between 0 and 1.25 show a lower vertical spread, this is also evident from the location-scale plot (lower-left panel of Fig.3). Consequently, these plots provide weak to moderate evidence against the constant variance assumption. The Q-Q plot (upper-

right panel of Fig.3) suggests moderate evidence against the normality assumption since the it shows heavy tails. Finally, there are no influential points as evident from the residual-leverage plot (lower-right panel of Fig.3). In addition to the diagnostics, multicollinearity between the predictors were investigated using the vif function from the car package (Fox and Weisberg, 2019). The GVIF calculations showed no extreme multicollinearity as all GVIFs were < 5. The $(\text{GVIF})^{\frac{1}{2\text{df}}}$ values show that the most impacted predictor was the magnitude with its standard error inflated by 1.81 times compared to if it did not share any information with the other predictors. Since my sample is large, I concluded that the violations of constant variance and normality is not an issue, and I proceeded to assess my first research question. A Tukey-Kramer pairwise comparison test was applied to the different discovery methods using the emmeans package. This showed that by controlling log(orbital period), log(star mass), and magnitude, the discovery methods: radial velocity and transit, other and radial velocity, other and transit are detectably different at the family-wise 5% significance level.

To investigate the second research question, a model was all interactions up to the four-way interaction was considered $\mu\{log\ (D) \mid discvMthd\colon \log(OrbPrd)\colon \log(StarMass)\colon Mag\} = discvMthd * \log(OrbPrd) * \log(StarMass) * Mag$. A backward step-wise process using type II F-tests from the car package was used to test for the interactions as well as to refine the model, the final model arrived at by this process is the following model:$\mu\{log\ (D) \mid discvMthd\colon \log(OrbPrd)\colon \log(StarMass)\colon Mag\} = discvMthd * \log(OrbPrd) * \log(StarMass) * Mag - discvMthd : \log(OrbPrd)\colon \log(StarMass)\colon Mag - discvMthd : \log(StarMass)\colon Mag - discvMthd : \log(OrbPrd)\colon \log(StarMass) - discvMthd : \log(OrbPrd)\colon Mag$

After the model refinement process, another set of diagnostic plots (Figure 6) was produced to assess any change regarding the assumptions of the model. Comparing Figures 4 and 6, it is clear that no change happened to the assumptions assessment compared to the initial additive model. Additionally, another set of effect plots were produced to visualize the different model components and their impact on the logged distance (Figure 7).

III. Summary of Statistical Findings

After controlling for log(orbital period), log(star mass), and magnitude, the Tukey-Kramer pairwise comparison show that the discovery methods: radial velocity and transit, as well as the "other" method and any other discovery method are detectably different at the family-wise 5% significance level. Furthermore, It is estimated that the mean log distance of exoplanets discovered by the transit method is 0.1 (95% CI: 0.04 to 0.17) larger than the exoplanets discovered by the radial method, after controlling for log(orbital period), log(star mass), and magnitude.

The step-down testing approach showed strong evidence against the null hypothesis of no interaction between log(orbital period), log(star mass), and magnitude ($F_{1,3330}$=5.911,p-value=0.02) so I concluded that this interaction term should be kept and that this would be the final model. Table 2 contains the point estimates for all final model coefficients.

The effects plot in Figure 7 show that for the timing and the "other" discovery methods, the mean log distance deceases as the log orbital period of the planet increases. While for the radial velocity and transit methods, the log distance increases as the as the log orbital period of the planet increases. On the other hand, the figure shows that for the interaction between discovery methods and both the log star mass and magnitude, the log distance increases as both these predictors increase, however, the rate of the increase for the timing variation method appears to be notably lower than all other methods. Finally, the three way interaction between the log orbital period, log star mass, and magnitude, shows an interesting trend where the slope of the relationship between log distance and log orbital period depends on both the logged star mass and the magnitude.

IV. Scope of Inference

There is no random assignment of the exoplanets to any of the predictor variables nor random sampling of the exoplanets since there are selection bias due to the techniques of each discovery methods and the limitations of the instruments. For these reasons, we cannot declare any casual relationship between any of the predictors and the response and we can apply the results of this study to this sample of 3350exoplanets which was discovered between 1992 and 2023 and not missing measurements of the discovery method, the exoplanet's orbital period, distance, the host star's mass, and the Gaia magnitude (brightness). The study is thus useful to understand how the discovery method as well as the other various system properties relate to the distance of the exoplanet. The

Tukey-Kramer pairwise comparison show that the discovery methods: radial velocity and transit, as well as the "other" method and any other discovery method are detectably different at the family-wise 5% significance level. While a step-back testing process found strong evidence against the null hypothesis of no interaction between log(orbital period), log(star mass), and magnitude ($F_{1,3330}$=5.911,p-value=0.02)

V. References

NASA exoplanet archive. Available at: https://exoplanetarchive.ipac.caltech.edu/index.html

R Core Team (2023). _R: A Language and Environment for Statistical Computing_. R Foundation for Statistical Computing, Vienna, Austri

Schloerke B, Cook D, Larmarange J, Briatte F, Marbach M, Thoen E, Elberg A, Crowley J (2021). _GGally: Extension to 'ggplot2'_. R package version 2.1.2 a

Greenwood M (2023). _catstats2: Upper Level Statistics for Montana State University Bobcats_. R package version 0.1

Goode K, Rey K (2023). _ggResidpanel: Panels and Interactive Versions of Diagnostic Plots using 'ggplot2'_. R package version 0.3.0.9000

John Fox and Sanford Weisberg (2019). An R Companion to Applied Regression, 3rd Edition.

Lenth R (2023). _emmeans: Estimated Marginal Means, aka Least-Squares Means_. R package version 1.8.9

Appendix

Figures and Tables:

Table 1: Summary statistics

Discovery Method	**Orbital Period (Days)**	**Star Mass (Solar)**	**Star Magnitude (Lower is brighter)**	**Distance (pc)**
Timing Variation: n =36	Min. : 0.3	Min. : 0.09	Min. : 2.364	Min. : 1.301
Transit: n=3542	1st Qu.: 4.1	1st Qu.: 0.81	1st Qu.: 10.352	1st Qu.: 111.643
Radial Velocity: n=1029	Median : 10.1	Median : 0.97	Median : 12.378	Median : 438.385
Other: n=16	Mean : 957	Mean : 0.9821	Mean : 14.866	Mean : 558.670
	3rd Qu.: 39.6	3rd Qu.: 1.1	3rd Qu.: 14.866	3rd Qu.: 860.368
	Max. : 1790000	Max. : 10.94	Max. : 19.879	Max. : 4483.05

Note that in the analysis, log(Distance), log(Orbital Period), log(Star Mass) were used instead of the un-logged variables.

Table 2: Final model estimated coefficients.

Model Component	Estimated coefficient
Intercept	0.794209
Discoverymethod(Timing Variations)	3.711513
Discoverymethod(Radial Velocity)	0.307914
Discoverymethod(Transit)	-0.569654
Log(OrbitalPeriod)	-0.028173
Log(StarMass)	2.54061
Magnitude	0.410357
Discoverymethod(Timing Variations):log(OrbitalPeriod)	0.019555
Discoverymethod(Radial Velocity):log(OrbitalPeriod)	0.100283
Discoverymethod(Transit):log(OrbitalPeriod)	0.147418
Discoverymethod(Timing Variations):log(StarMass)	-1.201455
Discoverymethod(Radial Velocity):log(StarMass)	-0.316333
Discoverymethod(Transit): log(StarMass)	0.030299
log(OrbitalPeriod): log(StarMass)	-0.061461
Discoverymethod(Timing Variations):Magnitude	-0.240776
Discoverymethod(Radial Velocity):Magnitude	-0.068805
Discoverymethod(Transit): Magnitude	0.034398
log_OrbitalPeriod:Magnitude	-0.005283
log_StarMass:Magnitude	-0.037270
log(OrbitalPeriod): log(StarMass):Magnitude	0.007131

Note that the discovery method “other” was treated as the baseline method.

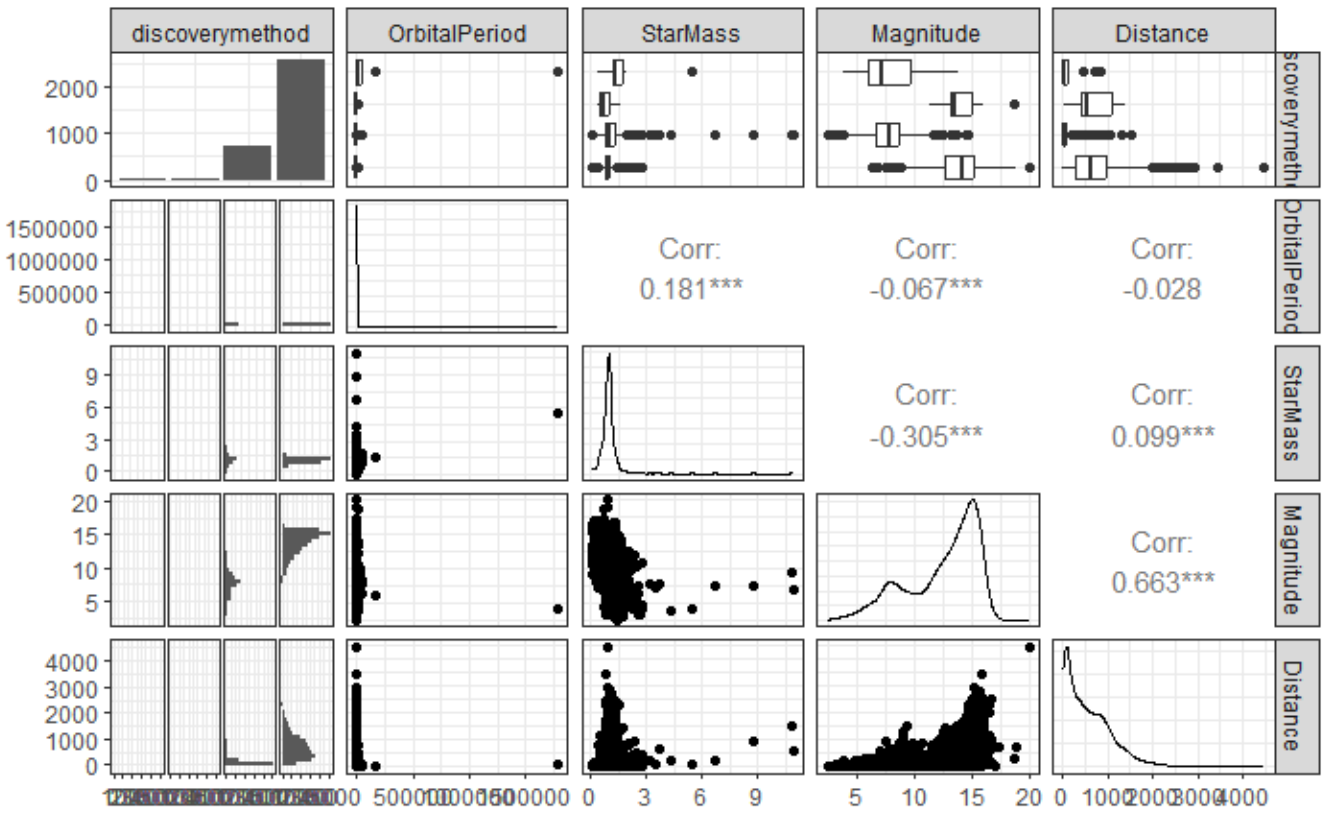


Figure 1: A ggpair plot that shows the relations between the various variables used in this analysis before taking the log transformation for the distance, exoplanet's orbital period, host star's temperature, and host star's mass.

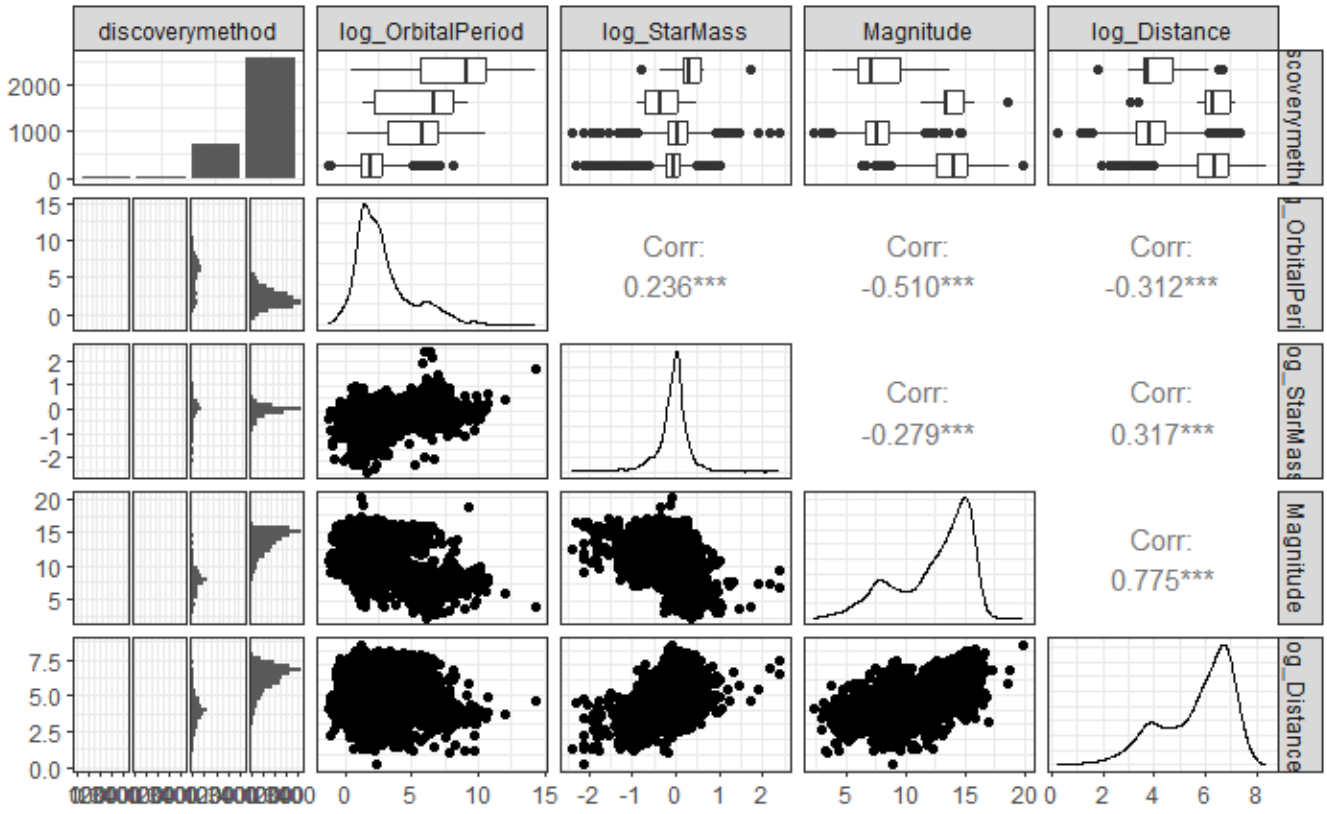


Figure 2: A ggpair plot that shows the relations between the various variables used in this analysis after taking the log transformation for the distance, exoplanet's orbital period, host star's temperature, and host star's mass.

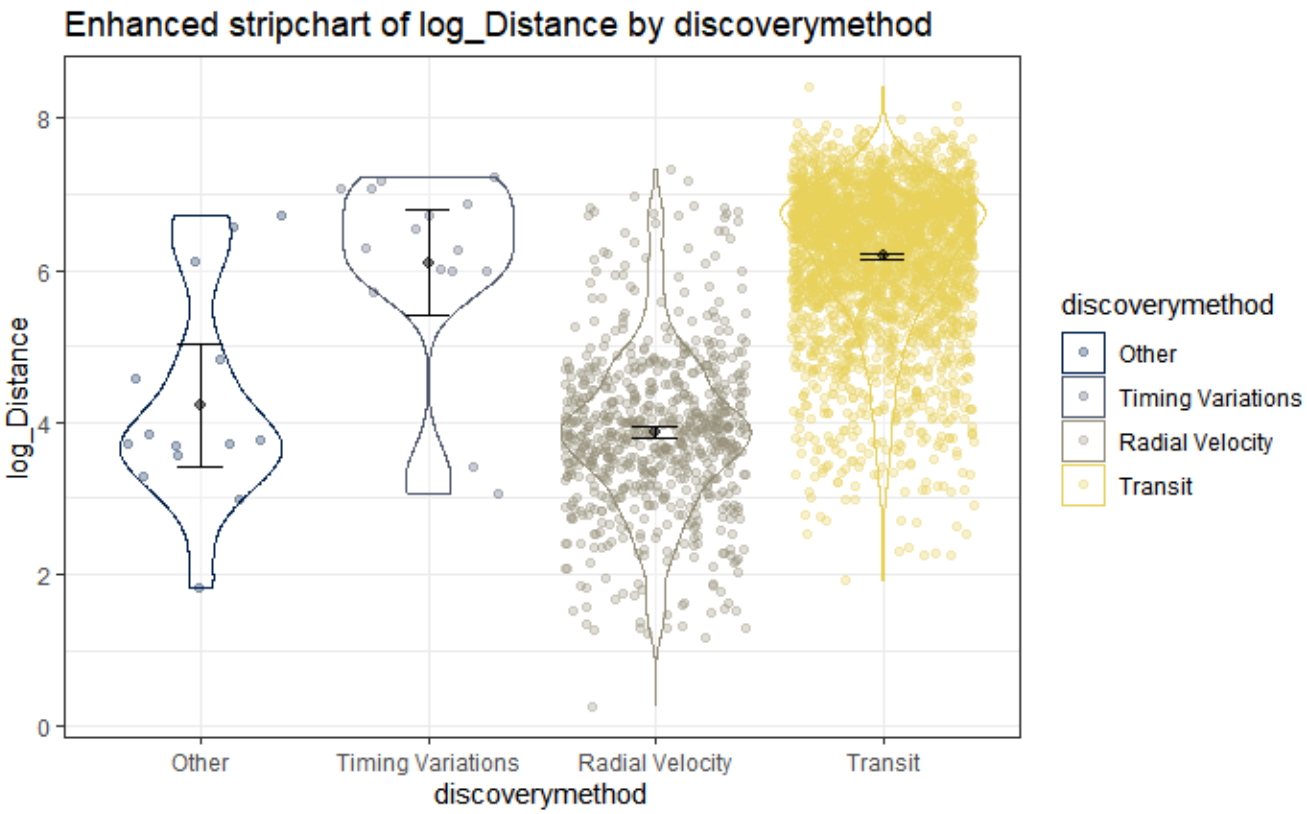


Figure 3: Stripchart between log(distance) and discovery method.

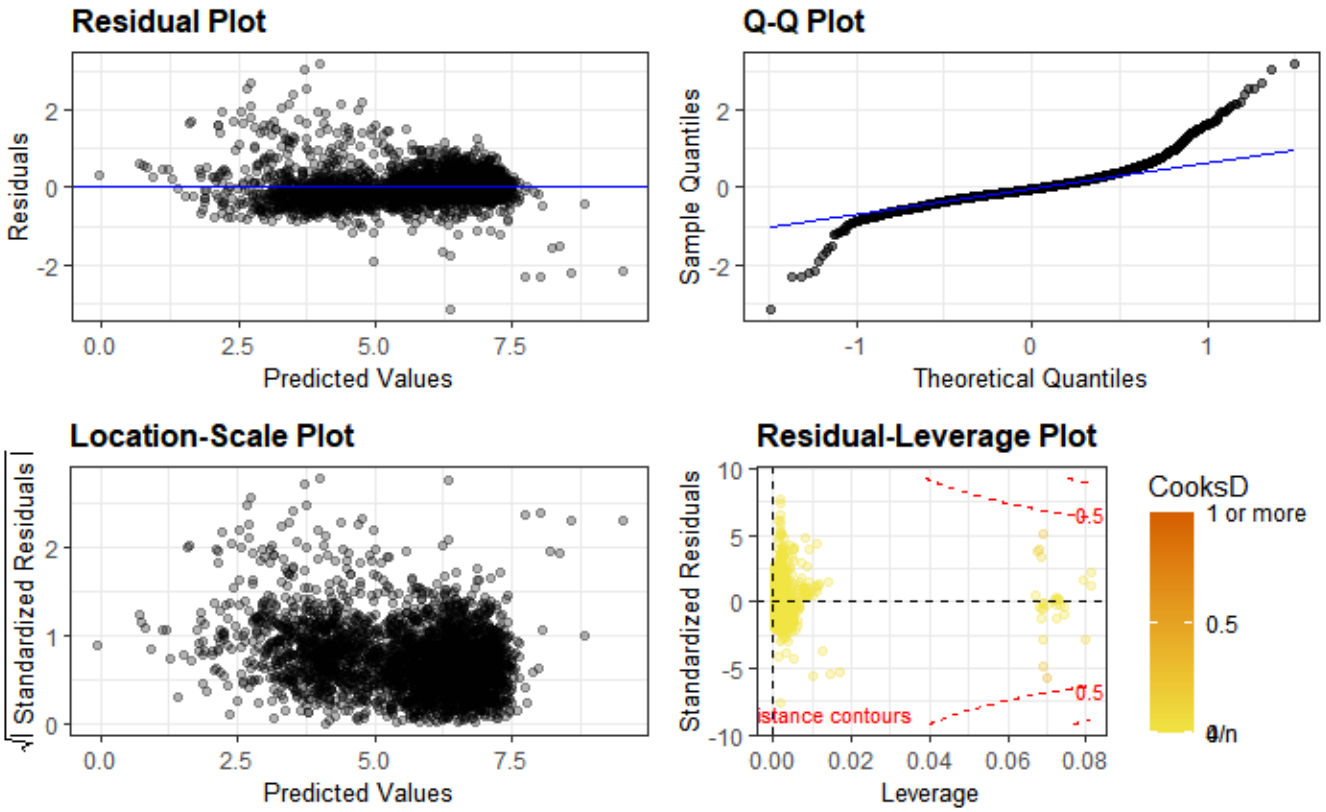


Figure 4: Diagnostic plots to assess the validity of underlying statistical assumptions for the initial linear additive model.

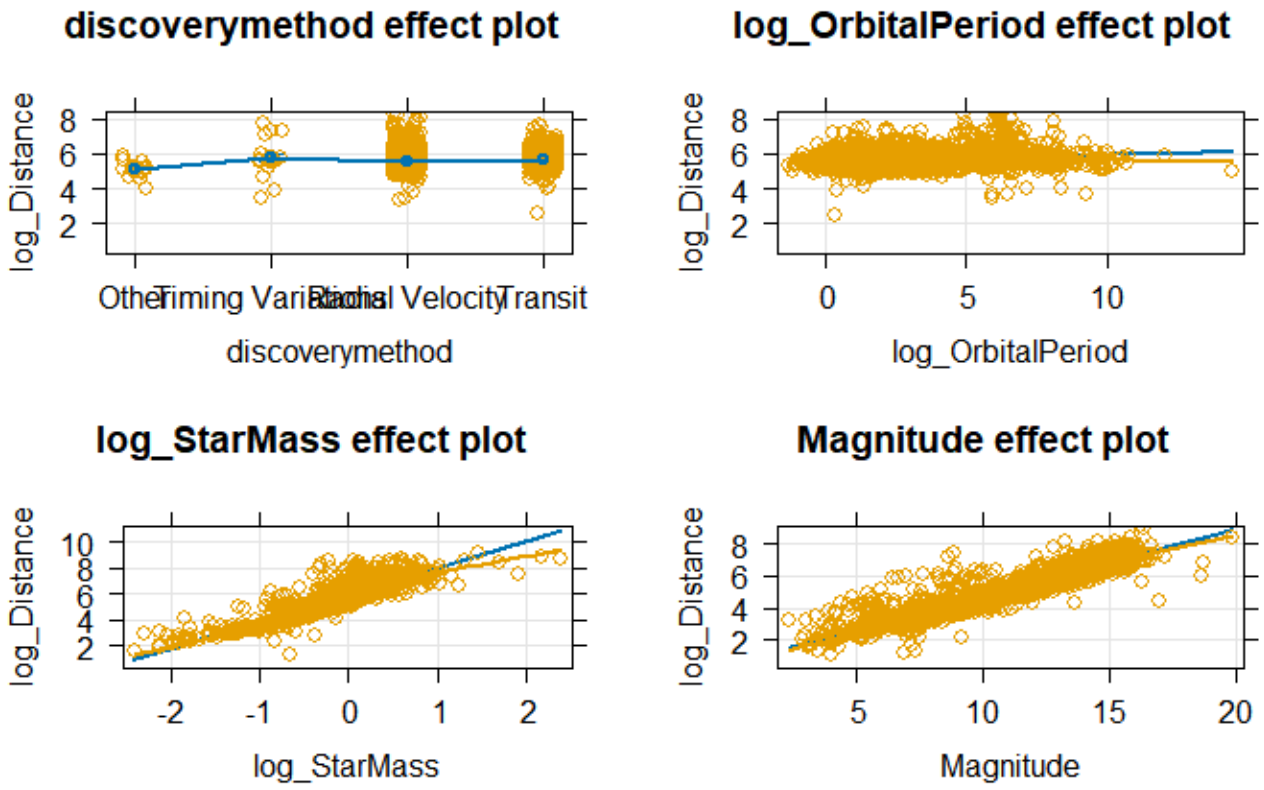


Figure 5: Effects plot examining the relationship between log(distance) and discovery method, log(exoplanet's orbital period),log(star's mass), and Gaia magnitude (brightness).

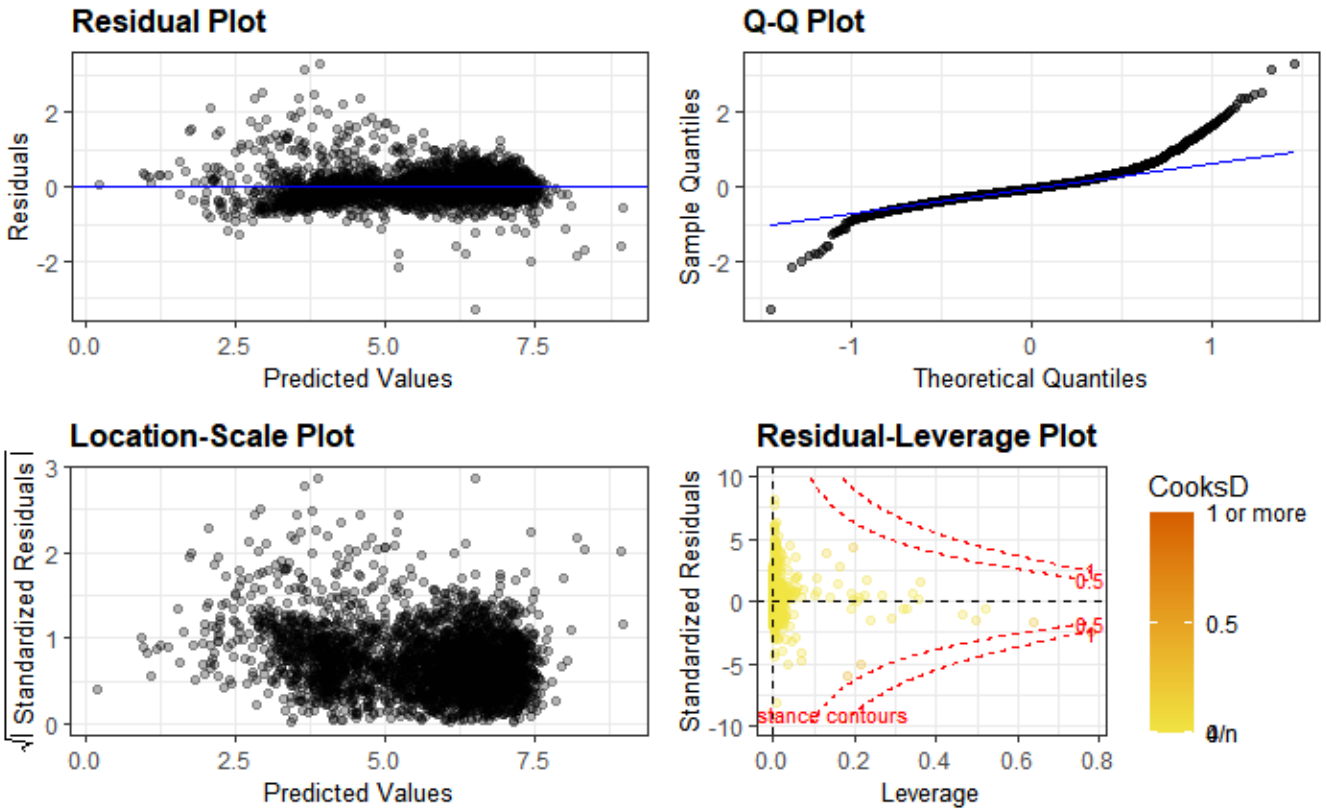


Figure 6: Diagnostic plots to assess the validity of underlying statistical assumptions for the final model selected through the step-back testing process.

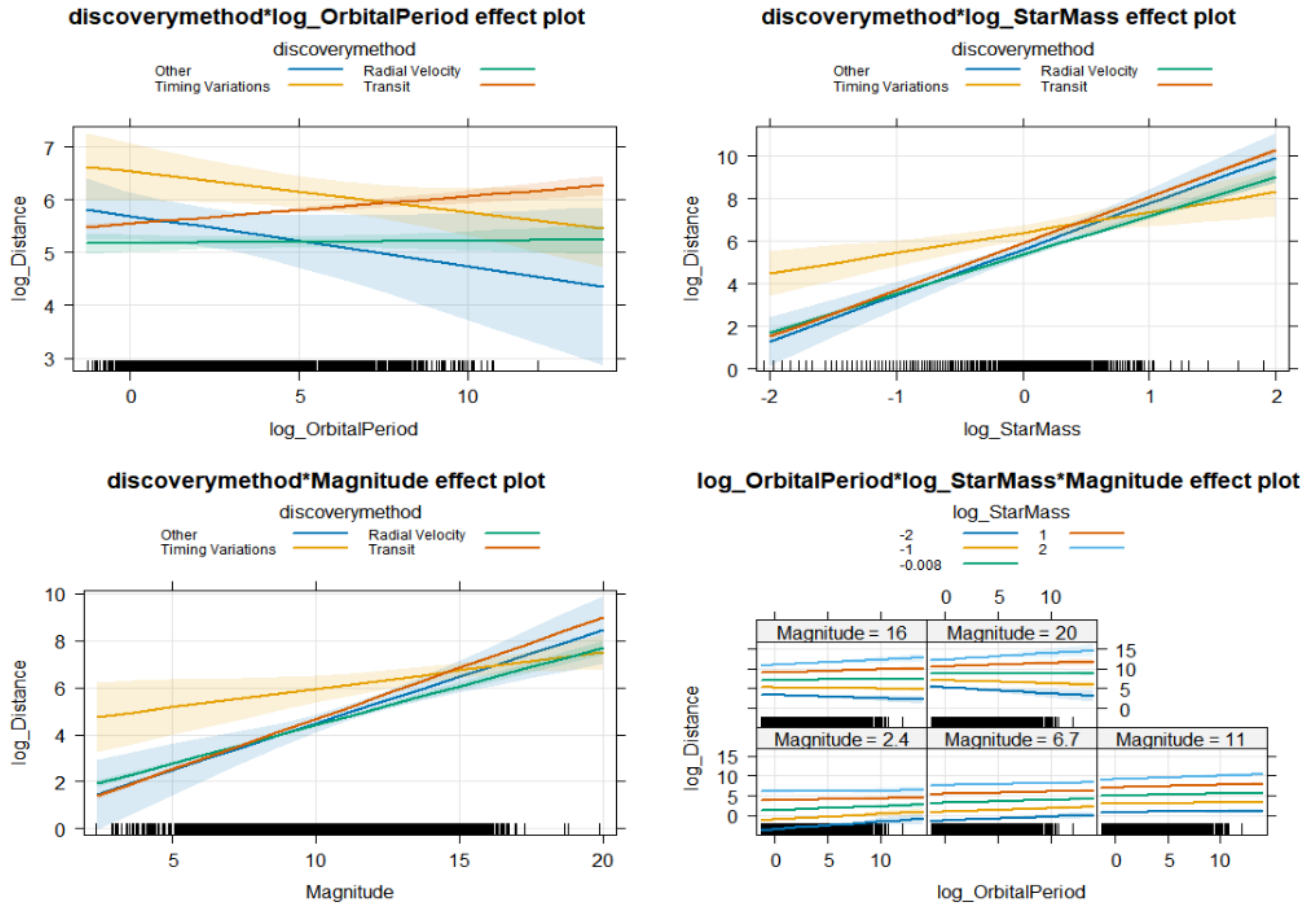


Figure 7: Effects plot examining the relationship between log(distance) and discovery method, log(exoplanet's orbital period),log(star's mass), and Gaia magnitude (brightness).

512 Project Code

Loading data

```
df <- read.csv("PS_2023.10.28_11.10.58.csv")
```

Removing planets that have a controversial flag (whether the confirmation status of a planet has been questioned in the published literature)

```
filtered_df <- df %>% filter(pl_controv_flag != 1)
```

Remove duplicates based on lowest average distance error and keep the most recent publication date in case of ties

```
filtered_df$PublicationDate <- as.Date(paste0(filtered_df$pl_pubdate, "-01")
)
filtered_df <- filtered_df %>%
  mutate(AvgDistError = (sy_disterr1 + sy_disterr1) / 2) %>%
  arrange(pl_name, AvgDistError, PublicationDate) %>%
  group_by(pl_name) %>%
  filter(AvgDistError == min(AvgDistError)) %>%
  slice_tail(n = 1) %>%
  ungroup()
filtered_df <- filtered_df %>%
  arrange(pl_name) %>%
  group_by(hostname) %>%
  slice_head(n=1) %>%
  ungroup()
```

Only keeping the columns that we are interested in

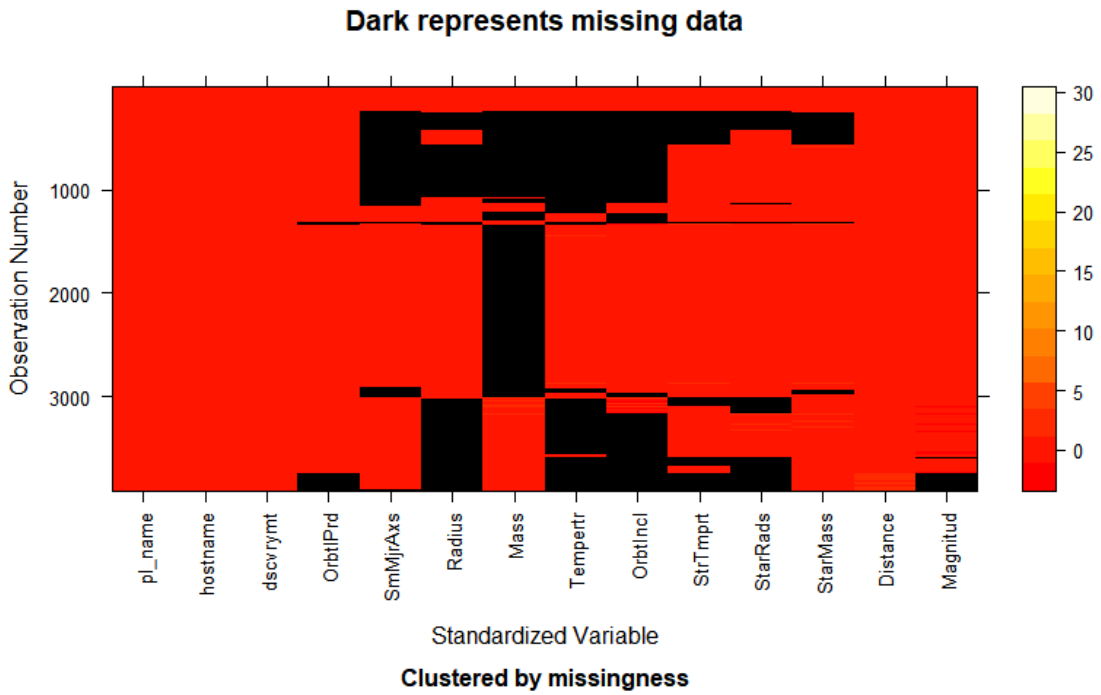
```
compact_df <- filtered_df %>% select(pl_name,hostname,discoverymethod,pl_orb
per,pl_orbsmax,pl_rade,pl_bmasse,pl_eqt,pl_orbincl,st_teff,st_rad,st_mass,sy
_dist,sy_gaiamag)
```

Renaming columns to more clear names

```
compact_df <- compact_df %>% dplyr::rename(OrbitalPeriod = pl_orbper,SemiMaj
orAxis = pl_orbsmax, Radius = pl_rade, Mass = pl_bmasse, Temperature = pl_eq
t, OrbitInclination = pl_orbincl,StarTemperature = st_teff, StarRadius = st_
rad, StarMass = st_mass, Distance = sy_dist,Magnitude = sy_gaiamag)
```

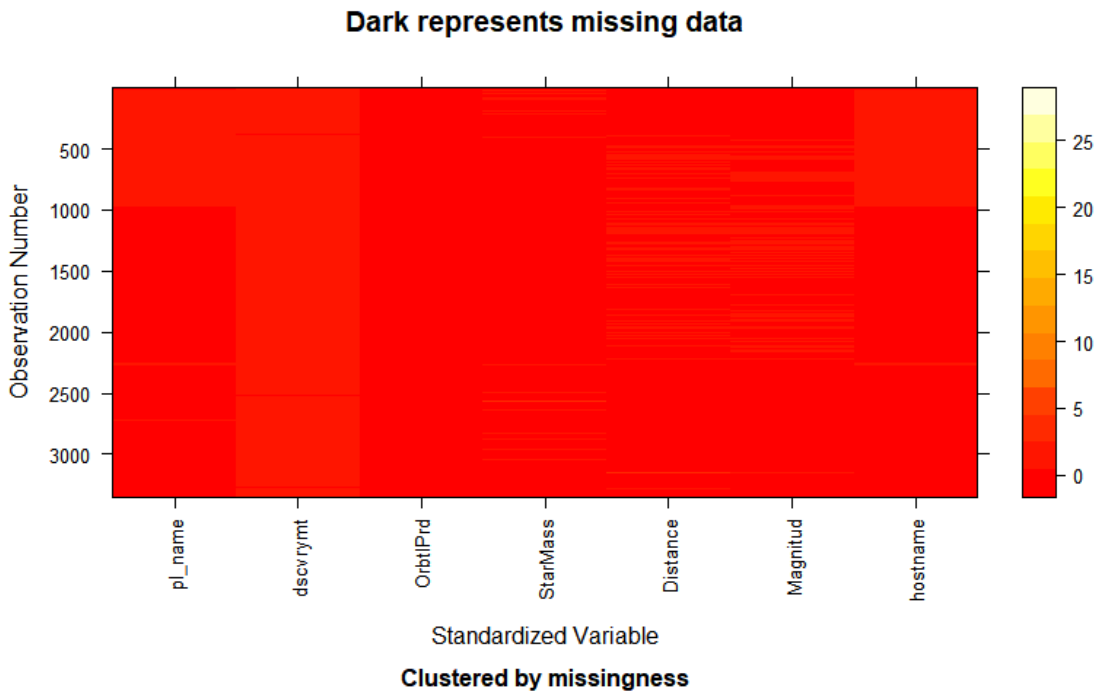
Visualizing missing data

```
#compact_df <- compact_df %>% filter(StarSpecType!="")
library(mi)
mdf <- missing_data.frame(as.data.frame(compact_df))

## NOTE: In the following pairs of variables, the missingness pattern of the
second is a subset of the first.
##  Please verify whether they are in fact logically distinct variables.
##      [,1]     [,2]
## [1,] "Radius" "Magnitude"

image(mdf)
```



Based on the missing data visualization and for the sake of simplicity of the analysis, I will only keep the following variables: planet name(for identification only), discovery method, orbital period, star temperature, star mass, magnitude and distance.

```
final_df <- compact_df %>% select(pl_name,discoverymethod,OrbitalPeriod,Star
Mass,Distance,Magnitude,hostname)
final_df <- final_df %>% drop_na(pl_name,discoverymethod,OrbitalPeriod,StarM
ass,Distance,Magnitude,hostname)
mdf <- missing_data.frame(as.data.frame(final_df))
image(mdf)
```



Converting discovery method into factors

```
final_df <- final_df %>% mutate(discoverymethod = factor(discoverymethod))
summary(final_df)
```

```
##    pl_name                              discoverymethod OrbitalPeriod
##  Length:3351        Transit                      :2613   Min.    :
0
##  Class :character   Radial Velocity              : 709   1st Qu.:
4
##  Mode  :character   Imaging                      :  10   Median :       1
0
##                     Eclipse Timing Variations    :   7   Mean   :   12092
0
##                     Transit Timing Variations    :   5   3rd Qu.:       4
0
##                     Orbital Brightness Modulation:   3   Max.   :40200000
0
##                     (Other)                      :   4
##     StarMass          Distance          Magnitude        hostname
##  Min.   : 0.0900   Min.   :   1.301   Min.   : 2.364   Length:3351
##  1st Qu.: 0.8100   1st Qu.: 111.590   1st Qu.:10.339   Class :character
##  Median : 0.9700   Median : 438.359   Median :13.390   Mode  :character
##  Mean   : 0.9818   Mean   : 558.506   Mean   :12.377
##  3rd Qu.: 1.1000   3rd Qu.: 860.380   3rd Qu.:14.866
##  Max.   :10.9400   Max.   :4483.050   Max.   :19.879
##
```

```r
tally(~discoverymethod, data = final_df)
```

```
## discoverymethod
##                     Astrometry     Eclipse Timing Variations
##                              2                             7
##                        Imaging Orbital Brightness Modulation
##                             10                             3
##   Pulsation Timing Variations                Radial Velocity
##                              2                           709
##                        Transit     Transit Timing Variations
##                           2613                             5
```

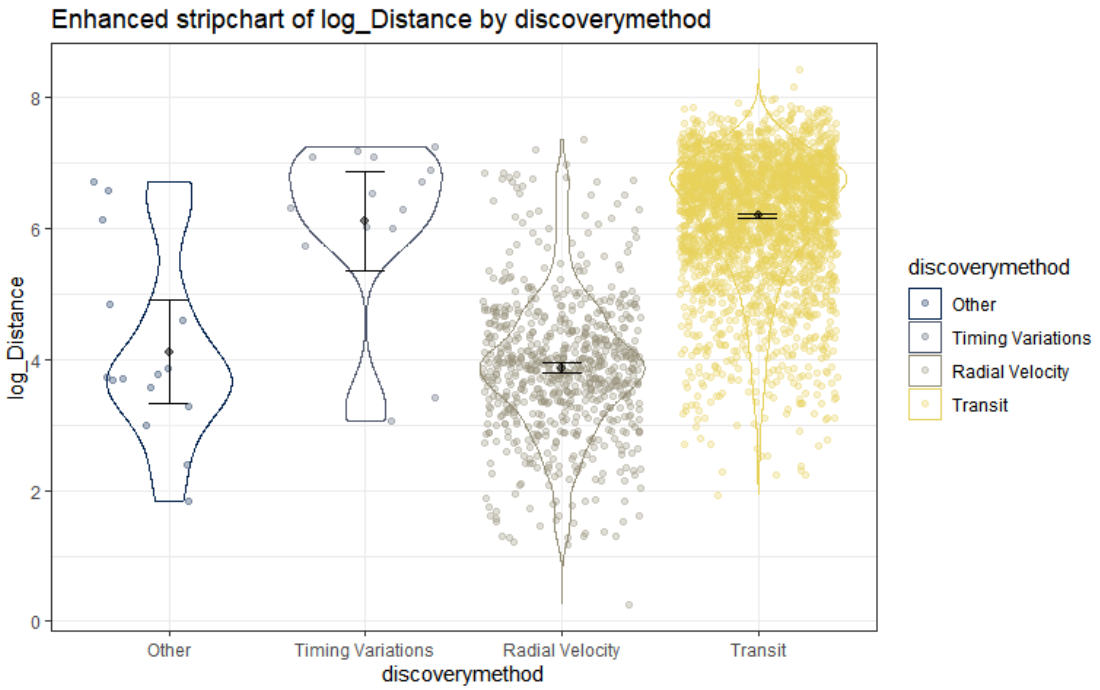
```r
final_df <- final_df %>% mutate(discoverymethod = forcats::fct_collapse(disc
overymethod,"Timing Variations"= c("Eclipse Timing Variations", "Pulsation T
iming Variations", "Transit Timing Variations"),Other = c("Astrometry", "Orb
ital Brightness Modulation","Imaging")))
tally(~discoverymethod, data = final_df)
```

```
## discoverymethod
##             Other Timing Variations   Radial Velocity           Transit
##                                  15                14               709
##                                 2613
```

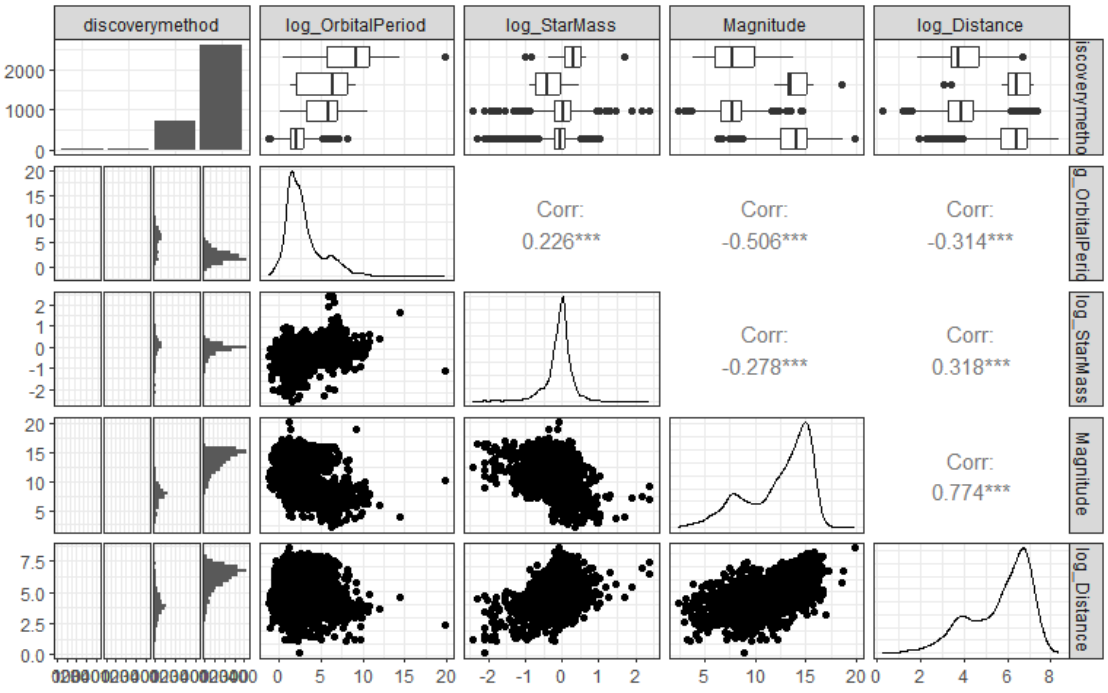
```r
summary(final_df)
```

```
##    pl_name                   discoverymethod OrbitalPeriod
##  Length:3351        Other            :  15   Min.   :        0
##  Class :character   Timing Variations:  14   1st Qu.:        4
##  Mode  :character   Radial Velocity  : 709   Median :       10
##                     Transit          :2613   Mean   :   120920
##                                              3rd Qu.:       40
##                                              Max.   :402000000
##     StarMass          Distance          Magnitude        hostname
##  Min.   : 0.0900   Min.   :   1.301   Min.   : 2.364   Length:3351
##  1st Qu.: 0.8100   1st Qu.: 111.590   1st Qu.:10.339   Class :character
##  Median : 0.9700   Median : 438.359   Median :13.390   Mode  :character
##  Mean   : 0.9818   Mean   : 558.506   Mean   :12.377
##  3rd Qu.: 1.1000   3rd Qu.: 860.380   3rd Qu.:14.866
##  Max.   :10.9400   Max.   :4483.050   Max.   :19.879
```

```r
final_df <- final_df %>%
 mutate(log_Distance = log(Distance), log_OrbitalPeriod = log(OrbitalPeriod)
, log_StarMass = log(StarMass))
enhanced_stripchart(data=final_df,log_Distance~discoverymethod)
```

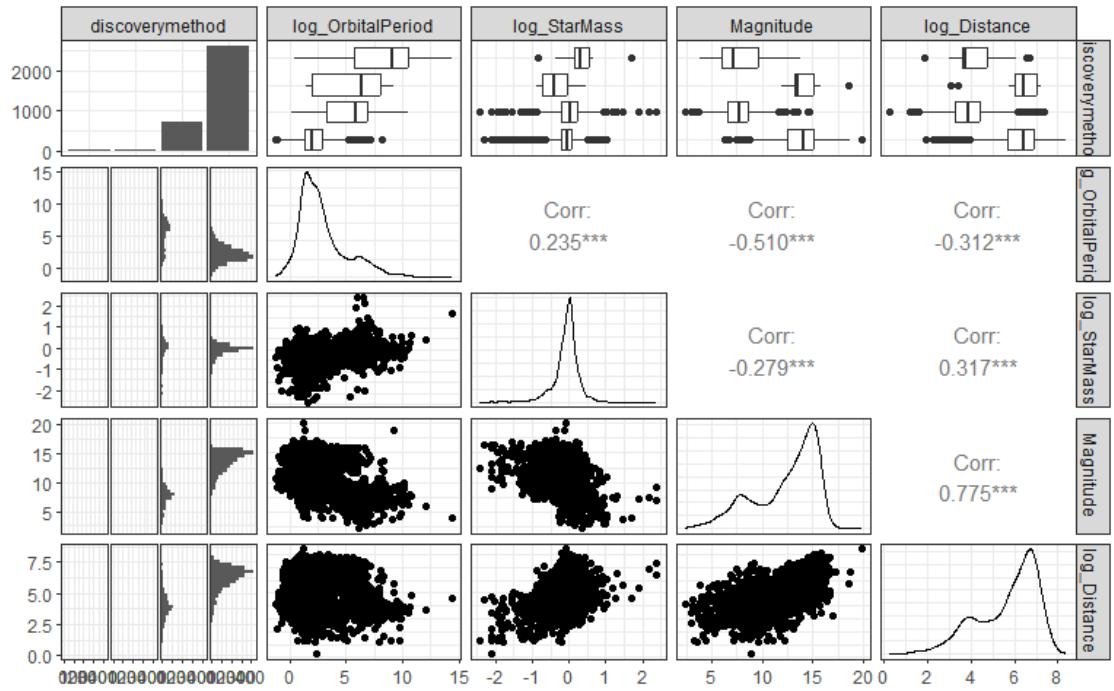



```
library(GGally)
final_df %>% select(discoverymethod,log_OrbitalPeriod,log_StarMass,Magnitude
,log_Distance) %>% ggpairs()
```



Remove the outlier in the orbital period since it has very large error

```
final_df <- final_df %>% filter(OrbitalPeriod != 402000000)
final_df %>% select(discoverymethod,log_OrbitalPeriod,log_StarMass,Magnitude
,log_Distance) %>% ggpairs()
```




```
summary(final_df)
```

```
##    pl_name                   discoverymethod OrbitalPeriod
##  Length:3350        Other            :  14   Min.   :      0.3
##  Class :character   Timing Variations:  14   1st Qu.:      4.1
##  Mode  :character   Radial Velocity  : 709   Median :     10.1
##                     Transit          :2613   Mean   :    956.1
##                                              3rd Qu.:     39.4
##                                              Max.   :1790000.0
##     StarMass         Distance          Magnitude        hostname
##  Min.   : 0.090   Min.   :   1.301   Min.   : 2.364   Length:3350
##  1st Qu.: 0.810   1st Qu.: 111.643   1st Qu.:10.352   Class :character
##  Median : 0.970   Median : 438.385   Median :13.391   Mode  :character
##  Mean   : 0.982   Mean   : 558.670   Mean   :12.378
##  3rd Qu.: 1.100   3rd Qu.: 860.386   3rd Qu.:14.866
##  Max.   :10.940   Max.   :4483.050   Max.   :19.879
##   log_Distance    log_OrbitalPeriod  log_StarMass
##  Min.   :0.2633   Min.   :-1.272    Min.   :-2.40795
##  1st Qu.:4.7153   1st Qu.: 1.406    1st Qu.:-0.21072
##  Median :6.0831   Median : 2.316    Median :-0.03046
##  Mean   :5.6845   Mean   : 2.841    Mean   :-0.08549
##  3rd Qu.:6.7574   3rd Qu.: 3.675    3rd Qu.: 0.09531
##  Max.   :8.4081   Max.   :14.398    Max.   : 2.39243
```

Initial model

```
library(lme4)
initial_model <- lm(log_Distance~(discoverymethod+log_OrbitalPeriod+log_Star
Mass+Magnitude),data=final_df)
summary(initial_model)
```

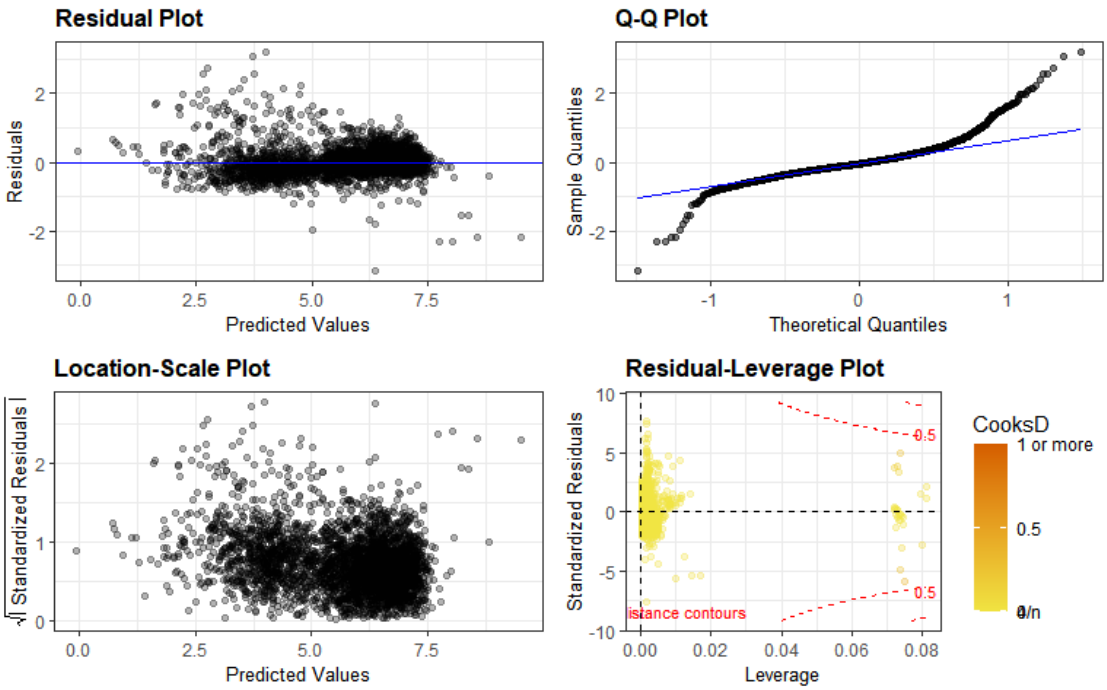
```
##
## Call:
## lm(formula = log_Distance ~ (discoverymethod + log_OrbitalPeriod +
##     log_StarMass + Magnitude), data = final_df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.1485 -0.2130 -0.0461  0.1597  3.1902
##
## Coefficients:
##                                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     -0.037353   0.120267  -0.311    0.756
## discoverymethodTiming Variations  0.736991   0.158277   4.656 3.34e-06
## discoverymethodRadial Velocity    0.544247   0.112039   4.858 1.24e-06
## discoverymethodTransit            0.648664   0.115816   5.601 2.31e-08
## log_OrbitalPeriod                 0.035486   0.004638   7.652 2.58e-14
## log_StarMass                      2.072533   0.020600 100.607  < 2e-16
## Magnitude                         0.418006   0.004193  99.685  < 2e-16
##
## Residual standard error: 0.4127 on 3343 degrees of freedom
## Multiple R-squared:  0.9104, Adjusted R-squared:  0.9102
## F-statistic:  5662 on 6 and 3343 DF,  p-value: < 2.2e-16
```

```
Anova(initial_model)
```

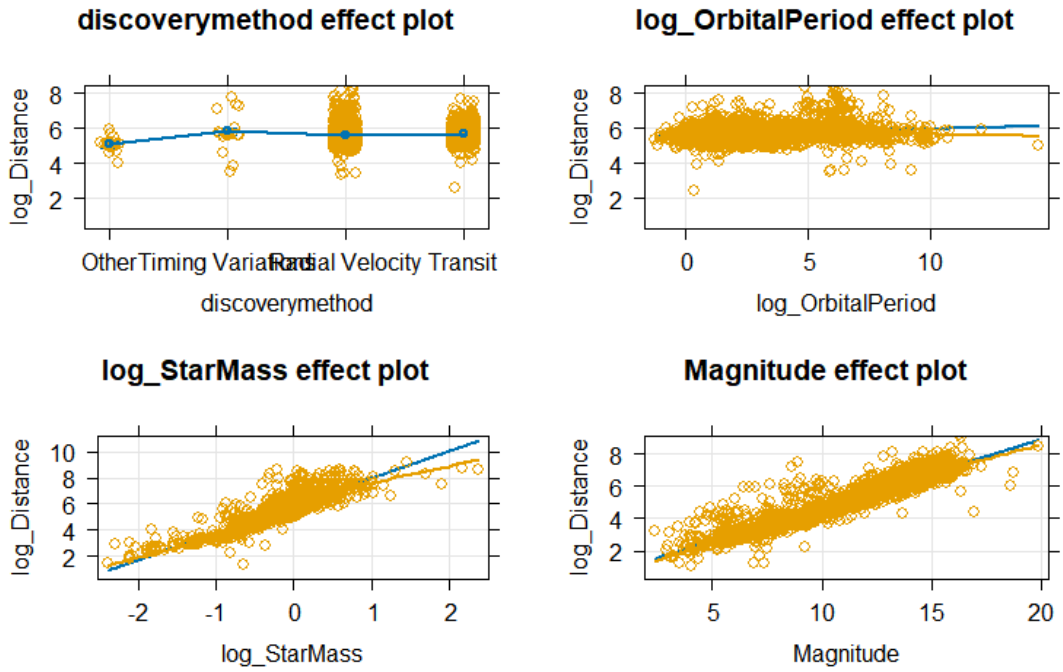
```
## Anova Table (Type II tests)
##
## Response: log_Distance
##                    Sum Sq   Df   F value    Pr(>F)
## discoverymethod      6.02    3    11.779 1.128e-07
## log_OrbitalPeriod    9.97    1    58.547 2.580e-14
## log_StarMass      1724.05    1 10121.732 < 2.2e-16
## Magnitude         1692.60    1  9937.100 < 2.2e-16
## Residuals          569.42 3343
```

Diagnostic and effects plots

```
resid_panel(initial_model, "R", alpha = 0.3)
```




```
plot(allEffects(initial_model, residuals = T), grid = T)
```



VIF Calculation

```
vif(initial_model)
```

```
##                          GVIF Df GVIF^(1/(2*Df))
## discoverymethod    4.037464  3        1.261880
```

```
## log_OrbitalPeriod 1.811232  1        1.345820
## log_StarMass      1.199062  1        1.095017
## Magnitude         3.270176  1        1.808363
```

Four-way interaction model

```
model1 <- lm(log_Distance~(discoverymethod*log_OrbitalPeriod*log_StarMass*Ma
gnitude),data=final_df)
summary(model1)

##
## Call:
## lm(formula = log_Distance ~ (discoverymethod * log_OrbitalPeriod *
##     log_StarMass * Magnitude), data = final_df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.2381 -0.2089 -0.0453  0.1532  3.3213
##
## Coefficients:
##
Estimate
## (Intercept)
8.450e-01
## discoverymethodTiming Variations
-9.366e+00
## discoverymethodRadial Velocity
7.769e-04
## discoverymethodTransit
-5.291e-01
## log_OrbitalPeriod
5.147e-02
## log_StarMass
1.136e+00
## Magnitude
4.032e-01
## discoverymethodTiming Variations:log_OrbitalPeriod
3.767e+00
## discoverymethodRadial Velocity:log_OrbitalPeriod
7.531e-02
## discoverymethodTransit:log_OrbitalPeriod
1.603e-02
## discoverymethodTiming Variations:log_StarMass
-6.078e+01
## discoverymethodRadial Velocity:log_StarMass
1.426e+00
```

```
## discoverymethodTransit:log_StarMass
1.163e+00
## log_OrbitalPeriod:log_StarMass
-7.250e-02
## discoverymethodTiming Variations:Magnitude
6.641e-01
## discoverymethodRadial Velocity:Magnitude
-2.943e-02
## discoverymethodTransit:Magnitude
3.482e-02
## log_OrbitalPeriod:Magnitude
-1.505e-02
## log_StarMass:Magnitude
1.183e-01
## discoverymethodTiming Variations:log_OrbitalPeriod:log_StarMass
1.262e+01
## discoverymethodRadial Velocity:log_OrbitalPeriod:log_StarMass
-3.571e-02
## discoverymethodTransit:log_OrbitalPeriod:log_StarMass
1.171e-01
## discoverymethodTiming Variations:log_OrbitalPeriod:Magnitude
-2.604e-01
## discoverymethodRadial Velocity:log_OrbitalPeriod:Magnitude
2.729e-03
## discoverymethodTransit:log_OrbitalPeriod:Magnitude
1.354e-02
## discoverymethodTiming Variations:log_StarMass:Magnitude
3.941e+00
## discoverymethodRadial Velocity:log_StarMass:Magnitude
-1.835e-01
## discoverymethodTransit:log_StarMass:Magnitude
-1.364e-01
## log_OrbitalPeriod:log_StarMass:Magnitude
1.309e-02
## discoverymethodTiming Variations:log_OrbitalPeriod:log_StarMass:Magnitude
-8.829e-01
## discoverymethodRadial Velocity:log_OrbitalPeriod:log_StarMass:Magnitude
-2.657e-03
## discoverymethodTransit:log_OrbitalPeriod:log_StarMass:Magnitude
-1.328e-02
##
Std. Error
## (Intercept)
3.756e+00
## discoverymethodTiming Variations
6.061e+00
```

```
## discoverymethodRadial Velocity
3.761e+00
## discoverymethodTransit
3.758e+00
## log_OrbitalPeriod
3.342e-01
## log_StarMass
5.394e+00
## Magnitude
2.746e-01
## discoverymethodTiming Variations:log_OrbitalPeriod
9.778e-01
## discoverymethodRadial Velocity:log_OrbitalPeriod
3.362e-01
## discoverymethodTransit:log_OrbitalPeriod
3.386e-01
## discoverymethodTiming Variations:log_StarMass
1.615e+01
## discoverymethodRadial Velocity:log_StarMass
5.408e+00
## discoverymethodTransit:log_StarMass
5.403e+00
## log_OrbitalPeriod:log_StarMass
2.575e-01
## discoverymethodTiming Variations:Magnitude
4.261e-01
## discoverymethodRadial Velocity:Magnitude
2.757e-01
## discoverymethodTransit:Magnitude
2.748e-01
## log_OrbitalPeriod:Magnitude
2.576e-02
## log_StarMass:Magnitude
4.246e-01
## discoverymethodTiming Variations:log_OrbitalPeriod:log_StarMass
2.695e+00
## discoverymethodRadial Velocity:log_OrbitalPeriod:log_StarMass
2.665e-01
## discoverymethodTransit:log_OrbitalPeriod:log_StarMass
3.107e-01
## discoverymethodTiming Variations:log_OrbitalPeriod:Magnitude
6.969e-02
## discoverymethodRadial Velocity:log_OrbitalPeriod:Magnitude
2.617e-02
## discoverymethodTransit:log_OrbitalPeriod:Magnitude
2.606e-02
```

```
## discoverymethodTiming Variations:log_StarMass:Magnitude
1.121e+00
## discoverymethodRadial Velocity:log_StarMass:Magnitude
4.263e-01
## discoverymethodTransit:log_StarMass:Magnitude
4.252e-01
## log_OrbitalPeriod:log_StarMass:Magnitude
3.254e-02
## discoverymethodTiming Variations:log_OrbitalPeriod:log_StarMass:Magnitude
1.913e-01
## discoverymethodRadial Velocity:log_OrbitalPeriod:log_StarMass:Magnitude
3.330e-02
## discoverymethodTransit:log_OrbitalPeriod:log_StarMass:Magnitude
3.488e-02
##
t value
## (Intercept)
0.225
## discoverymethodTiming Variations
-1.545
## discoverymethodRadial Velocity
0.000
## discoverymethodTransit
-0.141
## log_OrbitalPeriod
0.154
## log_StarMass
0.211
## Magnitude
1.468
## discoverymethodTiming Variations:log_OrbitalPeriod
3.852
## discoverymethodRadial Velocity:log_OrbitalPeriod
0.224
## discoverymethodTransit:log_OrbitalPeriod
0.047
## discoverymethodTiming Variations:log_StarMass
-3.765
## discoverymethodRadial Velocity:log_StarMass
0.264
## discoverymethodTransit:log_StarMass
0.215
## log_OrbitalPeriod:log_StarMass
-0.282
## discoverymethodTiming Variations:Magnitude
1.559
```

```
## discoverymethodRadial Velocity:Magnitude
-0.107
## discoverymethodTransit:Magnitude
0.127
## log_OrbitalPeriod:Magnitude
-0.584
## log_StarMass:Magnitude
0.279
## discoverymethodTiming Variations:log_OrbitalPeriod:log_StarMass
4.682
## discoverymethodRadial Velocity:log_OrbitalPeriod:log_StarMass
-0.134
## discoverymethodTransit:log_OrbitalPeriod:log_StarMass
0.377
## discoverymethodTiming Variations:log_OrbitalPeriod:Magnitude
-3.736
## discoverymethodRadial Velocity:log_OrbitalPeriod:Magnitude
0.104
## discoverymethodTransit:log_OrbitalPeriod:Magnitude
0.520
## discoverymethodTiming Variations:log_StarMass:Magnitude
3.514
## discoverymethodRadial Velocity:log_StarMass:Magnitude
-0.430
## discoverymethodTransit:log_StarMass:Magnitude
-0.321
## log_OrbitalPeriod:log_StarMass:Magnitude
0.402
## discoverymethodTiming Variations:log_OrbitalPeriod:log_StarMass:Magnitude
-4.615
## discoverymethodRadial Velocity:log_OrbitalPeriod:log_StarMass:Magnitude
-0.080
## discoverymethodTransit:log_OrbitalPeriod:log_StarMass:Magnitude
-0.381
##
Pr(>|t|)
## (Intercept)
0.822019
## discoverymethodTiming Variations
0.122356
## discoverymethodRadial Velocity
0.999835
## discoverymethodTransit
0.888034
## log_OrbitalPeriod
0.877618
```

```
## log_StarMass
0.833267
## Magnitude
0.142093
## discoverymethodTiming Variations:log_OrbitalPeriod
0.000119
## discoverymethodRadial Velocity:log_OrbitalPeriod
0.822770
## discoverymethodTransit:log_OrbitalPeriod
0.962252
## discoverymethodTiming Variations:log_StarMass
0.000170
## discoverymethodRadial Velocity:log_StarMass
0.792058
## discoverymethodTransit:log_StarMass
0.829588
## log_OrbitalPeriod:log_StarMass
0.778331
## discoverymethodTiming Variations:Magnitude
0.119167
## discoverymethodRadial Velocity:Magnitude
0.914992
## discoverymethodTransit:Magnitude
0.899150
## log_OrbitalPeriod:Magnitude
0.559087
## log_StarMass:Magnitude
0.780484
## discoverymethodTiming Variations:log_OrbitalPeriod:log_StarMass
2.95e-06
## discoverymethodRadial Velocity:log_OrbitalPeriod:log_StarMass
0.893382
## discoverymethodTransit:log_OrbitalPeriod:log_StarMass
0.706288
## discoverymethodTiming Variations:log_OrbitalPeriod:Magnitude
0.000190
## discoverymethodRadial Velocity:log_OrbitalPeriod:Magnitude
0.916945
## discoverymethodTransit:log_OrbitalPeriod:Magnitude
0.603373
## discoverymethodTiming Variations:log_StarMass:Magnitude
0.000447
## discoverymethodRadial Velocity:log_StarMass:Magnitude
0.666861
## discoverymethodTransit:log_StarMass:Magnitude
0.748472
```

```
## log_OrbitalPeriod:log_StarMass:Magnitude
0.687559
## discoverymethodTiming Variations:log_OrbitalPeriod:log_StarMass:Magnitude
4.07e-06
## discoverymethodRadial Velocity:log_OrbitalPeriod:log_StarMass:Magnitude
0.936410
## discoverymethodTransit:log_OrbitalPeriod:log_StarMass:Magnitude
0.703460
##
## Residual standard error: 0.4016 on 3318 degrees of freedom
## Multiple R-squared:  0.9158, Adjusted R-squared:  0.915
## F-statistic:  1164 on 31 and 3318 DF,  p-value: < 2.2e-16
```

```
Anova(model1,test.statistic = "F")
```

```
## Anova Table (Type II tests)
##
## Response: log_Distance
##                                                      Sum Sq   Df
## discoverymethod                                        5.75    3
## log_OrbitalPeriod                                      8.52    1
## log_StarMass                                        1559.32    1
## Magnitude                                           1616.01    1
## discoverymethod:log_OrbitalPeriod                      3.01    3
## discoverymethod:log_StarMass                           4.74    3
## log_OrbitalPeriod:log_StarMass                         0.08    1
## discoverymethod:Magnitude                             10.63    3
## log_OrbitalPeriod:Magnitude                            0.74    1
## log_StarMass:Magnitude                                 0.49    1
## discoverymethod:log_OrbitalPeriod:log_StarMass         0.16    3
## discoverymethod:log_OrbitalPeriod:Magnitude            0.63    3
## discoverymethod:log_StarMass:Magnitude                 0.21    3
## log_OrbitalPeriod:log_StarMass:Magnitude               0.23    1
## discoverymethod:log_OrbitalPeriod:log_StarMass:Magnitude    3.59    3
## Residuals                                            535.13 3318
##                                                        F value    Pr
(>F)
## discoverymethod                                        11.8831 9.718
e-08
## log_OrbitalPeriod                                      52.8438 4.483
e-13
## log_StarMass                                         9668.2971 < 2.2
e-16
## Magnitude                                           10019.7709 < 2.2
e-16
## discoverymethod:log_OrbitalPeriod                       6.2207 0.000
```

```
3287
## discoverymethod:log_StarMass                                    9.7977 1.967
e-06
## log_OrbitalPeriod:log_StarMass                                  0.5211 0.470
4223
## discoverymethod:Magnitude                                      21.9626 4.430
e-14
## log_OrbitalPeriod:Magnitude                                     4.5625 0.032
7519
## log_StarMass:Magnitude                                          3.0182 0.082
4289
## discoverymethod:log_OrbitalPeriod:log_StarMass                  0.3261 0.806
4929
## discoverymethod:log_OrbitalPeriod:Magnitude                     1.3104 0.269
1691
## discoverymethod:log_StarMass:Magnitude                          0.4372 0.726
3813
## log_OrbitalPeriod:log_StarMass:Magnitude                        1.3983 0.237
0933
## discoverymethod:log_OrbitalPeriod:log_StarMass:Magnitude        7.4111 6.034
e-05
## Residuals
```

Backward Step-wise Testing Process

```
model2 <- lm(log_Distance~(discoverymethod*log_OrbitalPeriod*log_StarMass*Ma
gnitude)-discoverymethod:log_OrbitalPeriod:log_StarMass:Magnitude,data=final
_df)
Anova(model2)

## Anova Table (Type II tests)
##
## Response: log_Distance
##                                                  Sum Sq   Df   F value
Pr(>F)
## discoverymethod                                    5.75    3   11.8147 1.0
73e-07
## log_OrbitalPeriod                                  8.52    1   52.5395 5.2
20e-13
## log_StarMass                                    1559.32    1 9612.6261 < 2
.2e-16
## Magnitude                                       1616.01    1 9962.0761 < 2
.2e-16
## discoverymethod:log_OrbitalPeriod                  3.01    3    6.1849 0.0
003459
## discoverymethod:log_StarMass                       4.74    3    9.7412 2.1
```

```
33e-06
## log_OrbitalPeriod:log_StarMass                    0.08    1    0.5181 0.4
717028
## discoverymethod:Magnitude                        10.63    3   21.8362 5.3
20e-14
## log_OrbitalPeriod:Magnitude                       0.74    1    4.5363 0.0
332575
## log_StarMass:Magnitude                            0.49    1    3.0008 0.0
833164
## discoverymethod:log_OrbitalPeriod:log_StarMass    0.16    3    0.3242 0.8
078556
## discoverymethod:log_OrbitalPeriod:Magnitude       0.63    3    1.3029 0.2
716865
## discoverymethod:log_StarMass:Magnitude            0.21    3    0.4347 0.7
281726
## log_OrbitalPeriod:log_StarMass:Magnitude          0.23    1    1.3902 0.2
384477
## Residuals                                       538.72 3321
```

```
model3 <- lm(log_Distance~(discoverymethod*log_OrbitalPeriod*log_StarMass*Ma
gnitude)-discoverymethod:log_OrbitalPeriod:log_StarMass:Magnitude-discoverym
ethod:log_StarMass:Magnitude,data=final_df)
Anova(model3)
```

```
## Anova Table (Type II tests)
##
## Response: log_Distance
##                                                  Sum Sq   Df   F value
Pr(>F)
## discoverymethod                                    5.75    3   11.8207 1.0
63e-07
## log_OrbitalPeriod                                  8.55    1   52.7594 4.6
74e-13
## log_StarMass                                    1559.32    1 9617.5330 < 2
.2e-16
## Magnitude                                       1616.01    1 9967.1613 < 2
.2e-16
## discoverymethod:log_OrbitalPeriod                  2.76    3    5.6772 0.0
007099
## discoverymethod:log_StarMass                       4.74    3    9.7462 2.1
18e-06
## log_OrbitalPeriod:log_StarMass                     0.02    1    0.1308 0.7
175834
## discoverymethod:Magnitude                         10.63    3   21.8473 5.2
33e-14
## log_OrbitalPeriod:Magnitude                        0.72    1    4.4274 0.0
```

```
354414
## log_StarMass:Magnitude                                0.49    1    3.0023 0.0
832377
## discoverymethod:log_OrbitalPeriod:log_StarMass    0.70    3    1.4461 0.2
273947
## discoverymethod:log_OrbitalPeriod:Magnitude       0.67    3    1.3697 0.2
501062
## log_OrbitalPeriod:log_StarMass:Magnitude          0.14    1    0.8775 0.3
489420
## Residuals                                       538.93 3324
```

```
model4 <- lm(log_Distance~(discoverymethod*log_OrbitalPeriod*log_StarMass*Ma
gnitude)-discoverymethod:log_OrbitalPeriod:log_StarMass:Magnitude-discoverym
ethod:log_StarMass:Magnitude-discoverymethod:log_OrbitalPeriod:log_StarMass,
data=final_df)
Anova(model4)
```

```
## Anova Table (Type II tests)
##
## Response: log_Distance
##                                          Sum Sq   Df   F value    Pr(
>F)
## discoverymethod                            5.75    3   11.8160 1.070e
-07
## log_OrbitalPeriod                          8.55    1   52.7382 4.723e
-13
## log_StarMass                            1559.32    1 9613.6662 < 2.2e
-16
## Magnitude                               1618.48    1 9978.3608 < 2.2e
-16
## discoverymethod:log_OrbitalPeriod          2.76    3    5.6749 0.0007
122
## discoverymethod:log_StarMass               4.74    3    9.7423 2.130e
-06
## log_OrbitalPeriod:log_StarMass             0.02    1    0.1308 0.7176
377
## discoverymethod:Magnitude                 10.68    3   21.9419 4.561e
-14
## log_OrbitalPeriod:Magnitude                0.61    1    3.7881 0.0517
013
## log_StarMass:Magnitude                     0.26    1    1.6047 0.2053
214
## discoverymethod:log_OrbitalPeriod:Magnitude    0.84    3    1.7335 0.1579
033
## log_OrbitalPeriod:log_StarMass:Magnitude       1.08    1    6.6719 0.0098
```

```
370
## Residuals                                          539.63 3327
```

```
model5 <- lm(log_Distance~(discoverymethod*log_OrbitalPeriod*log_StarMass*Ma
gnitude)-discoverymethod:log_OrbitalPeriod:log_StarMass:Magnitude-discoverym
ethod:log_StarMass:Magnitude-discoverymethod:log_OrbitalPeriod:log_StarMass-
discoverymethod:log_OrbitalPeriod:Magnitude,data=final_df)
Anova(model5)
```

```
## Anova Table (Type II tests)
##
## Response: log_Distance
##                                            Sum Sq   Df   F value    Pr(>F)
## discoverymethod                              5.75    3   11.8082 1.083e-07
## log_OrbitalPeriod                            8.55    1   52.7034 4.806e-13
## log_StarMass                              1565.62    1 9646.0949 < 2.2e-16
## Magnitude                                 1618.48    1 9971.7709 < 2.2e-16
## discoverymethod:log_OrbitalPeriod            2.76    3    5.6712 0.0007159
## discoverymethod:log_StarMass                 5.37    3   11.0304 3.327e-07
## log_OrbitalPeriod:log_StarMass               0.06    1    0.3825 0.5362854
## discoverymethod:Magnitude                   10.68    3   21.9274 4.657e-14
## log_OrbitalPeriod:Magnitude                  0.61    1    3.7856 0.0517785
## log_StarMass:Magnitude                       0.32    1    1.9519 0.1624799
## log_OrbitalPeriod:log_StarMass:Magnitude     0.98    1    6.0351 0.0140745
## Residuals                                  540.48 3330
```

```
summary(model5)
```

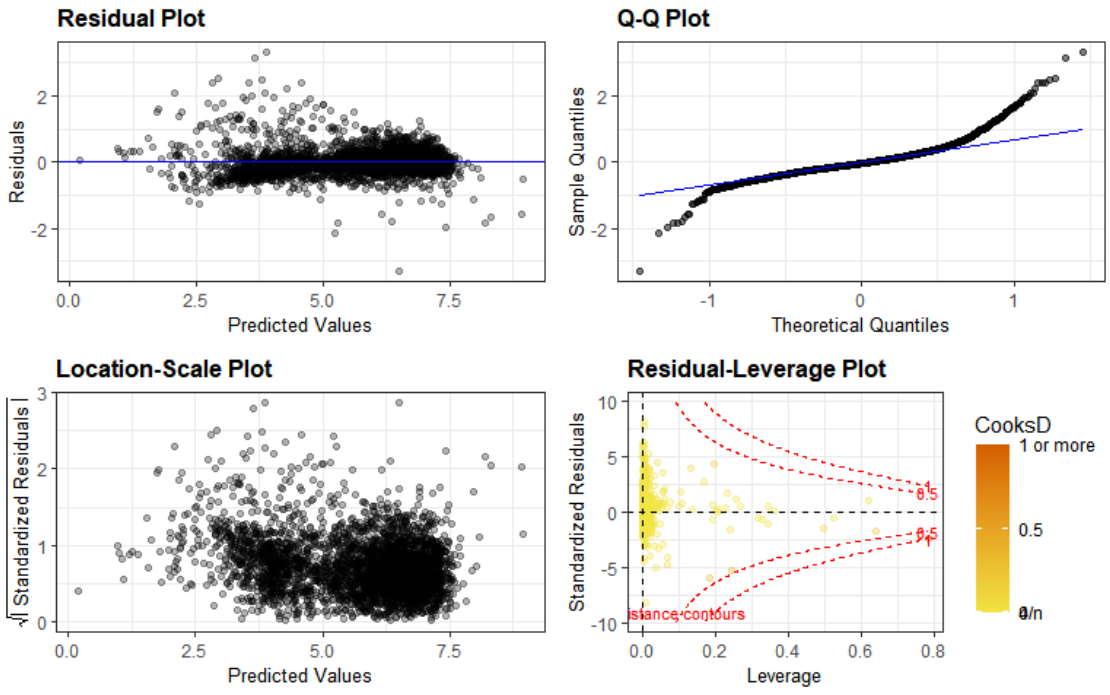
```
##
## Call:
## lm(formula = log_Distance ~ (discoverymethod * log_OrbitalPeriod *
##     log_StarMass * Magnitude) - discoverymethod:log_OrbitalPeriod:log_Sta
rMass:Magnitude -
##     discoverymethod:log_StarMass:Magnitude - discoverymethod:log_OrbitalP
eriod:log_StarMass -
##     discoverymethod:log_OrbitalPeriod:Magnitude, data = final_df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.2890 -0.2107 -0.0459  0.1571  3.3088
##
## Coefficients:
##                                                  Estimate Std. Error t
value
## (Intercept)                                      0.775387   1.124323
0.690
## discoverymethodTiming Variations                 4.061341   1.473745
```

```
2.756
## discoverymethodRadial Velocity                         0.327390   1.111157
0.295
## discoverymethodTransit                                -0.558083   1.113432
-0.501
## log_OrbitalPeriod                                     -0.025464   0.071083
-0.358
## log_StarMass                                           2.539905   0.348219
7.294
## Magnitude                                              0.411912   0.083493
4.933
## discoverymethodTiming Variations:log_OrbitalPeriod     0.038972   0.077911
0.500
## discoverymethodRadial Velocity:log_OrbitalPeriod       0.099426   0.064581
1.540
## discoverymethodTransit:log_OrbitalPeriod               0.148913   0.064101
2.323
## discoverymethodTiming Variations:log_StarMass         -1.017210   0.417950
-2.434
## discoverymethodRadial Velocity:log_StarMass           -0.314397   0.272600
-1.153
## discoverymethodTransit:log_StarMass                    0.034366   0.277715
0.124
## log_OrbitalPeriod:log_StarMass                        -0.062040   0.030891
-2.008
## discoverymethodTiming Variations:Magnitude            -0.266749   0.104292
-2.558
## discoverymethodRadial Velocity:Magnitude              -0.070464   0.082199
-0.857
## discoverymethodTransit:Magnitude                       0.033268   0.082593
0.403
## log_OrbitalPeriod:Magnitude                           -0.005542   0.002879
-1.925
## log_StarMass:Magnitude                                -0.037630   0.013729
-2.741
## log_OrbitalPeriod:log_StarMass:Magnitude               0.007201   0.002931
2.457
##                                                      Pr(>|t|)
## (Intercept)                                           0.49046
## discoverymethodTiming Variations                      0.00589
## discoverymethodRadial Velocity                        0.76829
## discoverymethodTransit                                0.61624
## log_OrbitalPeriod                                     0.72019
## log_StarMass                                         3.74e-13
## Magnitude                                            8.47e-07
## discoverymethodTiming Variations:log_OrbitalPeriod    0.61696
```

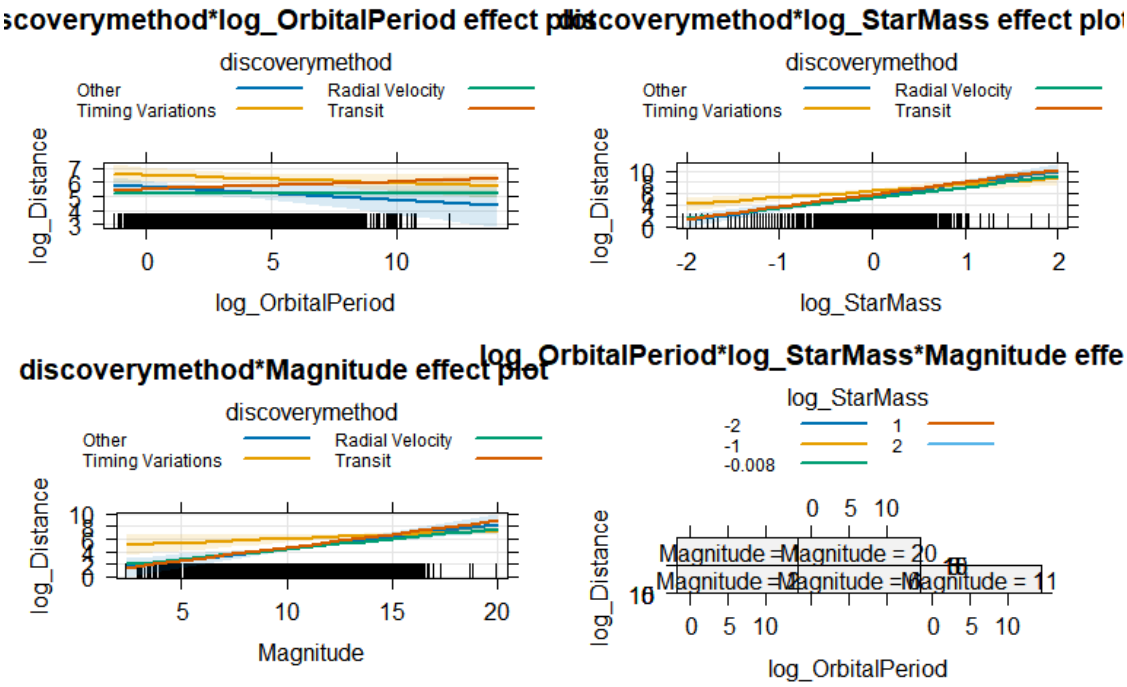
```
## discoverymethodRadial Velocity:log_OrbitalPeriod    0.12377
## discoverymethodTransit:log_OrbitalPeriod             0.02023
## discoverymethodTiming Variations:log_StarMass        0.01499
## discoverymethodRadial Velocity:log_StarMass          0.24886
## discoverymethodTransit:log_StarMass                  0.90152
## log_OrbitalPeriod:log_StarMass                       0.04469
## discoverymethodTiming Variations:Magnitude           0.01058
## discoverymethodRadial Velocity:Magnitude             0.39138
## discoverymethodTransit:Magnitude                     0.68712
## log_OrbitalPeriod:Magnitude                          0.05432
## log_StarMass:Magnitude                               0.00616
## log_OrbitalPeriod:log_StarMass:Magnitude             0.01407
##
## Residual standard error: 0.4029 on 3330 degrees of freedom
## Multiple R-squared:  0.915,  Adjusted R-squared:  0.9145
## F-statistic:  1886 on 19 and 3330 DF,  p-value: < 2.2e-16
```

Diagnostics and effects for final model

```
resid_panel(model5, "R", alpha = 0.3)
```




```
plot(allEffects(model5), grid = T,multiline=T,ci.style="bands")
```




```
#plot(allEffects(model5), grid = T,multiline=T,ci.style="bands",selection=1)
#plot(allEffects(model5), grid = T,multiline=T,ci.style="bands",selection=2)
#plot(allEffects(model5), grid = T,multiline=T,ci.style="bands",selection=3)
#plot(allEffects(model5), grid = T,multiline=T,ci.style="bands",selection=4)
```

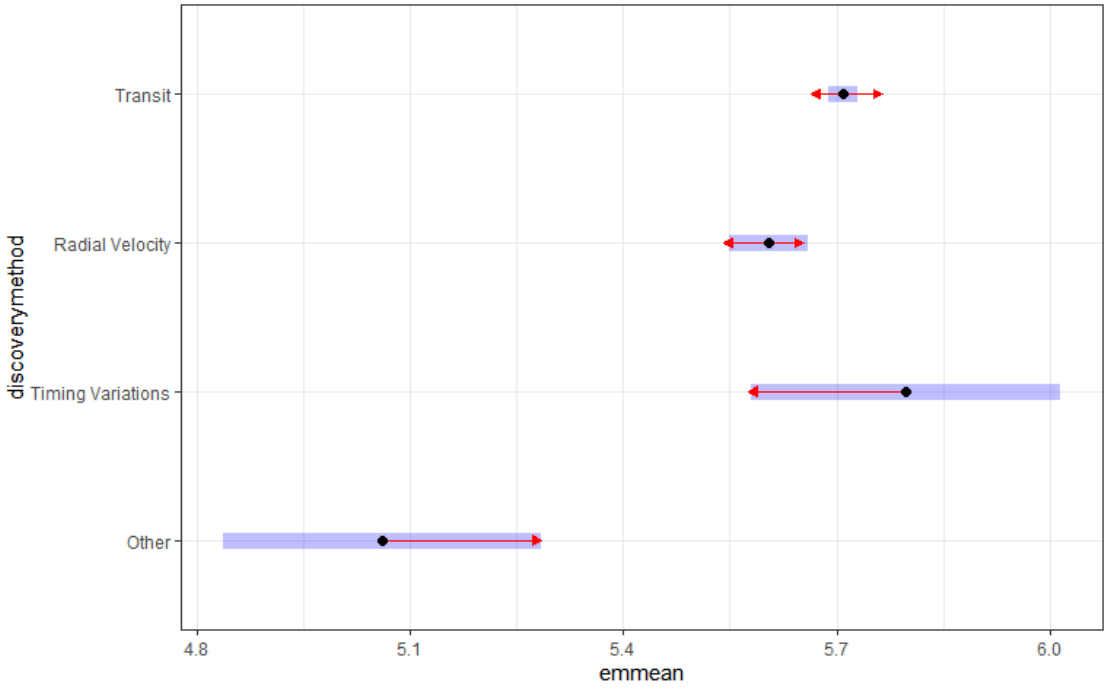
Pairwise comparison between the discovery methods in original additive model

```
library(emmeans)
res1 <- emmeans(initial_model,pairwise ~ discoverymethod,adjust="tukey")
res1
```

```
## $emmeans
##  discoverymethod   emmean      SE   df lower.CL upper.CL
##  Other              5.060 0.11389 3343    4.837    5.284
##  Timing Variations  5.797 0.11129 3343    5.579    6.015
##  Radial Velocity    5.604 0.02787 3343    5.550    5.659
##  Transit            5.709 0.01032 3343    5.689    5.729
##
## Confidence level used: 0.95
##
## $contrasts
##  contrast                              estimate     SE   df t.ratio p.value
##  Other - Timing Variations              -0.7370 0.1583 3343  -4.656  <.0001
##  Other - Radial Velocity                -0.5442 0.1120 3343  -4.858  <.0001
##  Other - Transit                        -0.6487 0.1158 3343  -5.601  <.0001
##  Timing Variations - Radial Velocity     0.1927 0.1144 3343   1.685  0.3317
##  Timing Variations - Transit             0.0883 0.1119 3343   0.789  0.8593
```

```
##  Radial Velocity - Transit            -0.1044 0.0344 3343  -3.038  0.0128
##
## P value adjustment: tukey method for comparing a family of 4 estimates
```

```
plot(res1, comparison=T)
```




```
multcomp::cld(res1,alpha=0.05, Letters=LETTERS)
```

```
##  discoverymethod   emmean      SE   df lower.CL upper.CL .group
##  Other              5.060 0.11389 3343    4.837    5.284  A
##  Radial Velocity    5.604 0.02787 3343    5.550    5.659   B
##  Transit            5.709 0.01032 3343    5.689    5.729    C
##  Timing Variations  5.797 0.11129 3343    5.579    6.015   BC
##
## Confidence level used: 0.95
## P value adjustment: tukey method for comparing a family of 4 estimates
## significance level used: alpha = 0.05
## NOTE: If two or more means share the same grouping symbol,
##       then we cannot show them to be different.
##       But we also did not show them to be the same.
```

```
model5.emms <- emmeans(initial_model,"discoverymethod")
model5.emms
```

```
##  discoverymethod   emmean      SE   df lower.CL upper.CL
##  Other              5.060 0.11389 3343    4.837    5.284
##  Timing Variations  5.797 0.11129 3343    5.579    6.015
##  Radial Velocity    5.604 0.02787 3343    5.550    5.659
```

```
##  Transit            5.709 0.01032 3343    5.689    5.729
##
## Confidence level used: 0.95
```

```
contrast(model5.emms,list(gamma1=c(0,0,-1,1))) %>% confint() %>% as.data.fra
me()
```

```
##  contrast  estimate         SE   df   lower.CL  upper.CL
##  gamma1   0.1044173 0.03436884 3343 0.03703122 0.1718034
##
## Confidence level used: 0.95
```

```
contrast(model5.emms,list(gamma1=c(-1,0,0,1))) %>% confint() %>% as.data.fra
me()
```

```
##  contrast  estimate        SE   df  lower.CL  upper.CL
##  gamma1   0.6486643 0.1158157 3343 0.4215874 0.8757411
##
## Confidence level used: 0.95
```

```
contrast(model5.emms,list(gamma1=c(-1,1,0,0))) %>% confint() %>% as.data.fra
me()
```

```
##  contrast  estimate        SE   df  lower.CL upper.CL
##  gamma1   0.7369909 0.1582775 3343 0.4266604 1.047321
##
## Confidence level used: 0.95
```

```
contrast(model5.emms,list(gamma1=c(-1,0,0,1))) %>% confint() %>% as.data.fra
me()
```

```
##  contrast  estimate        SE   df  lower.CL  upper.CL
##  gamma1   0.6486643 0.1158157 3343 0.4215874 0.8757411
##
## Confidence level used: 0.95
```