線性代數 + Julia + $\LaTeX$ 的學習筆記

(GitHub Edition)

以如下線性代數課程為主軸學習：

線性代數 台灣大學電機系 蘇柏青

本課程是線性代數的入門課程。線性代數係以「向量空間」(Vector Space)為核心概念之數學工具，擁有極廣泛之應用，非常值得理工商管等科系大學部同學深入修習，作為日後專業應用之基礎。

課程來源：**http://ocw.aca.ntu.edu.tw/ntu-ocw/index.php/ocw/cou/102S207/2**

單元 1 · Basic Concepts on Matrices and Vectors

Matrix

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix} = [a_{ij}] = M_{mn}$$

Matrix Addition

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 8 \end{bmatrix}$$

```
3×2 Array{Int64,2}:
 2  3
 4  5
 6  8
```

```
[1 2; 3 4; 5 6]+[1 1; 1 1; 1 2]
```

Scalar Multiplication

$$cA$$

$$3 \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

```
3×2 Array{Int64,2}:
  3   6
  9  12
 15  18
```

```
· 3 * [1 2; 3 4; 5 6]
```

```
3×2 Array{Int64,2}:
  3   6
  9  12
 15  18
```

```
· 3 .* [1 2; 3 4; 5 6]
```

Transpose

$$C = \begin{bmatrix} 7 & 9 \\ 18 & 31 \\ 52 & 68 \end{bmatrix} \quad \Rightarrow \quad C^T = \begin{bmatrix} 7 & 18 & 52 \\ 9 & 31 & 68 \end{bmatrix}$$

```
2×3 LinearAlgebra.Adjoint{Int64,Array{Int64,2}}:
 7  18  52
 9  31  68
```

```
· let
·     C=[7 9; 18 31; 52 68]
·     C'
· end
```

Vectors

Row Vector:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

Column Vector:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}^T$$

The ith componet of $\mathbf{v}$

$$v_i$$

```
1×4 Array{Int64,2}:
 1  2  3  4
```

```
· [ 1 2 3 4]
```

```
Int64[1,  2,  3,  4]
```

```
· [1; 2; 3; 4;]
```

```
4×1 LinearAlgebra.Adjoint{Int64,Array{Int64,2}}:
 1
 2
 3
 4
```

```
[ 1 2 3 4]'
```

Linear Combination

A *linear combination* of vectors $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k$ is a vector of the form

$$c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + \cdots + c_k\mathbf{u}_k$$

where $c_1, c_2, \ldots, c_k$ are scalars. These scalars are called the $coefficients$ of the linear combination.

Standard Vectors

The standard vectors of R^n are defined as

$$e_1 = \begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix}, e_2 = \begin{bmatrix}0\\1\\\vdots\\0\end{bmatrix}, \ldots, e_n = \begin{bmatrix}0\\0\\\vdots\\1\end{bmatrix}$$

Matrix-Vector Product

$$Av = v_1a_1 + v_2a_2 + \cdots + v_na_n$$

```
Int64[23,  53,  83]
```

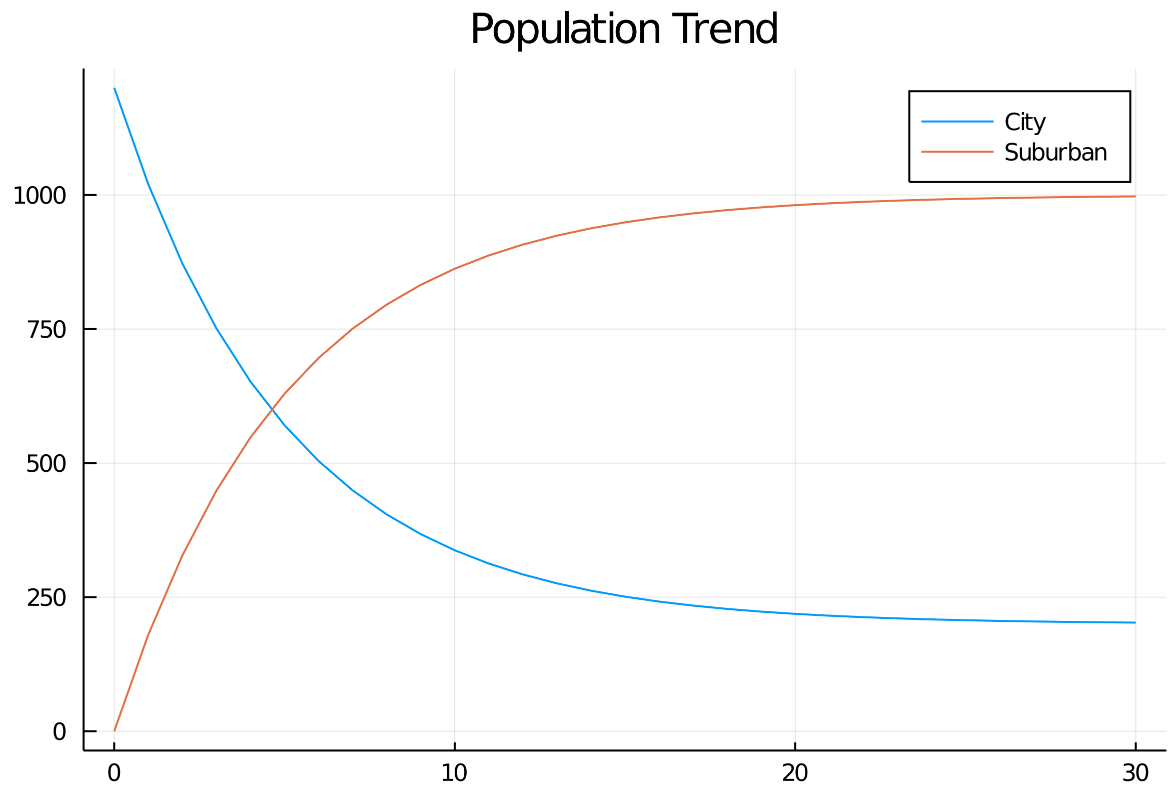
```
let
    A=[1 2; 3 4; 5 6]
    v=[7;8]
    A*v
end
```

Identity Matrix

$$I_3 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Stochastic Matrix

$$A = \begin{bmatrix}0.85 & 0.03\\0.15 & 0.97\end{bmatrix}$$




```
begin
    using Plots
    # p0 Population in year 0
    let
        A=[0.85 0.03; 0.15 0.97]
        #=
        p0=[500; 700]
        p1=A*p0
        p2=A*(p1)
        p3=A*(p2)
        p4=A*(p3)
        p5=A*(p4)
        x=0:5
        Y=hcat(p0, p1, p2, p3, p4, p5)
        plot(x, Y', title = "Population", label = ["City" "Suburban"])
        =#
        p=[1200; 000]
        x=30
        Y=p
        for i in 1:x
            p=A*p
            Y=hcat(Y, p)
        end
        plot(0:x, Y', title = "Population Trend", label = ["City" "Suburban"])
    end
end
```

單元 2 · System of Linear Equations

System of Linear Equations

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 3 & -6 & -5 \\ 2 & -1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$$

$$Ax=b$$

Solves Ax = b by (essentially) Gaussian elimination (Julia \ Operator):

$$x=A\;\backslash\;b$$

```
# \backslash \setminus
md"""
### System of Linear Equations
$$A=\begin{bmatrix} 1 & -2 & -1 \\ 3 & -6 & -5 \\ 2 & -1 & 1 \end{bmatrix}
\;\;
b=\begin{bmatrix} 3 \\ 3\\ 0 \end{bmatrix}$$
$$Ax=b$$

Solves Ax = b by (essentially) Gaussian elimination (Julia \ Operator):

$$x=A\;\backslash\;b$$
"""
```

```
Float64[-4.0, -5.0, 3.0]
```

```
# Solve System of Linear Equations
let
    A=[1 -2 -1; 3 -6 -5; 2 -1 1]
    b=[3; 3; 0]
    A \ b
end
```

Row Echelon Form & Reduced Row Echelon Form

```
md"""
### Row Echelon Form & Reduced Row Echelon Form
"""
```

```
Float64[0.403743, -1.21123, 0.112299, 1.48128, 2.0]
```

```
let
    A=[1 -3 0 2 0; 0 0 1 6 0; 0 0 0 0 1; 0 0 0 0 0]
    b=[7; 9; 2; 0]
    A \ b
end
```

參考資料

Linear Algebra

[線性代數 - 臺大開放式課程 (NTU OpenCourseWare)]

Julia

[ ] Introduction to Julia

[ ] Advanced topics

[ ] Julia for Data Science

[ ] 18.S191 Introduction to Computational Thinking

Markdown

[Markdown Cheatsheet · adam-p/markdown-here Wiki]

$\LaTeX$

[LaTeX - Mathematical Python]

[LaTeX help 1.1 - Table of Contents]

[List of mathematical symbols - Wikiwand]