線性代數 + Julia + $\LaTeX$ + GitHub 的學習筆記

(GitHub Edition)

整個過程將以如下「線性代數」課程為主軸學習：

線性代數 台灣大學電機系 蘇柏青

本課程是線性代數的入門課程。線性代數係以「向量空間」(Vector Space)為核心概念之數學工具，擁有極廣泛之應用，非常值得理工商管等科系大學部同學深入修習，作為日後專業應用之基礎。

課程來源：**http://ocw.aca.ntu.edu.tw/ntu-ocw/index.php/ocw/cou/102S207/2**

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# 線性代數 + Julia + $$\LaTeX$$ + GitHub 的學習筆記
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整個過程將以如下「線性代數」課程為主軸學習：

### 線性代數 台灣大學電機系 蘇柏青

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目前進度： 單元 3 · Gaussian Elimination

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### 目前進度： 單元 3 · Gaussian Elimination
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單元 1 · Basic Concepts on Matrices and Vectors

Matrix

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix} = [a_{ij}] = M_{mn}$$

Matrix Addition

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 8 \end{bmatrix}$$

```
3×2 Array{Int64,2}:
 2  3
 4  5
 6  8
```

```
· [1 2; 3 4; 5 6]+[1 1; 1 1; 1 2]
```

Scalar Multiplication

$$cA$$

$$3 \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

```
3×2 Array{Int64,2}:
  3   6
  9  12
 15  18
```

```
· 3 * [1 2; 3 4; 5 6]
```

```
3×2 Array{Int64,2}:
  3   6
  9  12
 15  18
```

```
· 3 .* [1 2; 3 4; 5 6]
```

Transpose

$$C = \begin{bmatrix} 7 & 9 \\ 18 & 31 \\ 52 & 68 \end{bmatrix} \Rightarrow C^T = \begin{bmatrix} 7 & 18 & 52 \\ 9 & 31 & 68 \end{bmatrix}$$

```
2×3 LinearAlgebra.Adjoint{Int64,Array{Int64,2}}:
 7  18  52
 9  31  68
```

```
· let
·     C=[7 9; 18 31; 52 68]
·     C'
· end
```

Vectors

Row Vector:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

Column Vector:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}^T$$

The ith componet of $\mathbf{v}$

$$v_i$$

```
1×4 Array{Int64,2}:
 1  2  3  4
```

```
· [ 1 2 3 4]
```

```
Int64[1, 2, 3, 4]
```

```
· [1; 2; 3; 4;]
```

```
4×1 LinearAlgebra.Adjoint{Int64,Array{Int64,2}}:
 1
 2
 3
 4
```

```
· [ 1 2 3 4]'
```

Linear Combination

A *linear combination* of vectors $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k$ is a vector of the form

$$c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + \cdots + c_k\mathbf{u}_k$$

where $c_1, c_2, \ldots, c_k$ are scalars. These scalars are called the *coefficients* of the linear combination.

Standard Vectors

The standard vectors of R^n are defined as

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \ldots, e_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

Matrix-Vector Product

$$Av = v_1a_1 + v_2a_2 + \cdots + v_na_n$$

```
Int64[23, 53, 83]
```

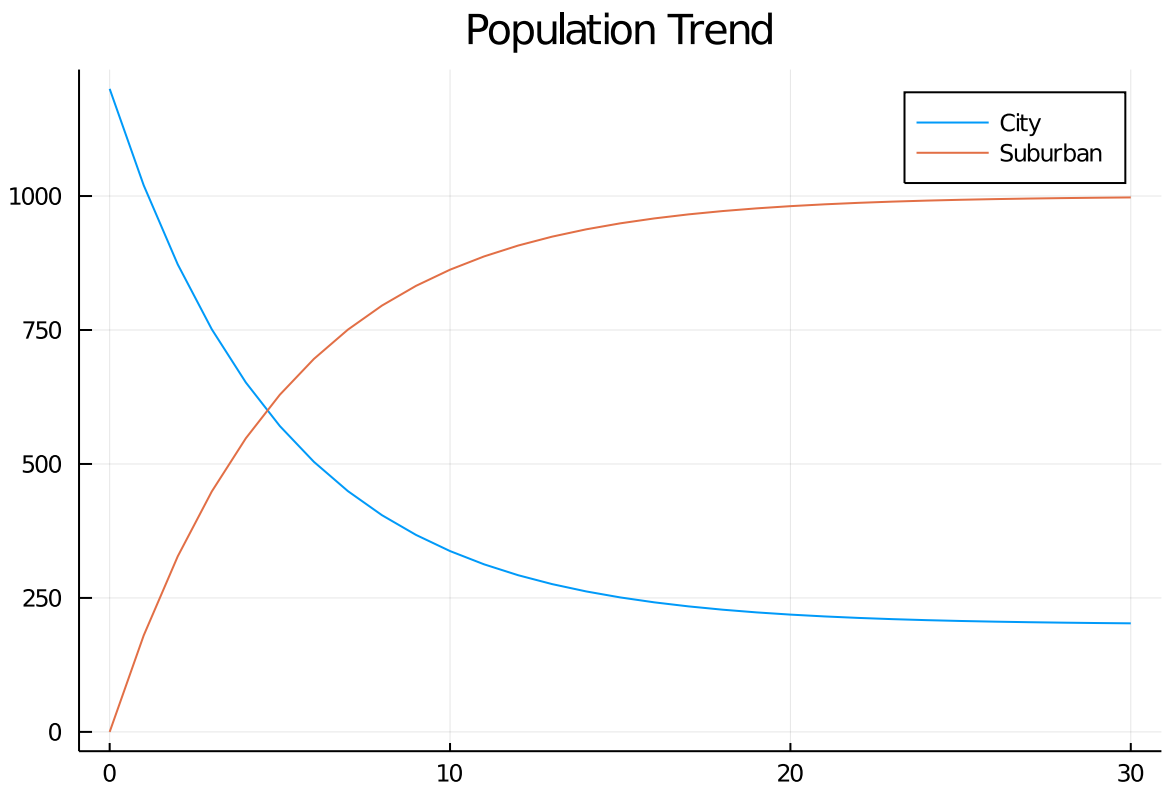
```
let
    A=[1 2; 3 4; 5 6]
    v=[7;8]
    A*v
end
```

Identity Matrix

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Stochastic Matrix

$$A = \begin{bmatrix} 0.85 & 0.03 \\ 0.15 & 0.97 \end{bmatrix}$$




```
begin
    using Plots
    # p0 Population in year 0
    let
        A=[0.85 0.03; 0.15 0.97]
        #=
        p0=[500; 700]
        p1=A*p0
        p2=A*(p1)
        p3=A*(p2)
        p4=A*(p3)
        p5=A*(p4)
        x=0:5
        Y=hcat(p0, p1, p2, p3, p4, p5)
        plot(x, Y', title = "Population", label = ["City" "Suburban"])
        =#
        p=[1200; 000]
```

```
        x=30
        Y=p
        for i in 1:x
            p=A*p
            Y=hcat(Y, p)
        end
        plot(0:x, Y', title = "Population Trend", label = ["City" "Suburban"])
    end
end
```

單元 2 · System of Linear Equations

System of Linear Equations

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 3 & -6 & -5 \\ 2 & -1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$$

$$Ax = b$$

Solves Ax = b by (essentially) Gaussian elimination (Julia \ Operator):

$$x = A \setminus b$$

```
Float64[-4.0,  -5.0,  3.0]
```

```
# Solve System of Linear Equations
let
    A=[1 -2 -1; 3 -6 -5; 2 -1 1]
    b=[3; 3; 0]
    A \ b
end
```

Row Echelon Form & Reduced Row Echelon Form

```
Float64[0.403743,  -1.21123,  0.112299,  1.48128,  2.0]
```

```
let
    A=[1 -3 0 2 0; 0 0 1 6 0; 0 0 0 0 1; 0 0 0 0 0]
    b=[7; 9; 2; 0]
    A \ b
end
```

單元 3 · Gaussian Elimination

實作參考：

Gaussian-elimination.pdf

Numerical Analysis by Julia Series 1 — Gauss Elimination | by Treee July | Medium

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## 單元 3 · Gaussian Elimination
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實作參考：

[Gaussian-elimination.pdf](http://web.mit.edu/18.06/www/Spring17/Gaussian-elimination.pdf)

[Numerical Analysis by Julia Series 1 – Gauss Elimination | by Treee July | Medium]
(https://medium.com/@julytreee/numerical-analysis-by-julia-series-1-gauss-elimination-68e902a43c7e)
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```

參考資料

Linear Algebra

[ ] **線性代數 - 臺大開放式課程 (NTU OpenCourseWare)**

Julia

[ ] **Introduction to Julia**

[ ] Advanced topics

[ ] **Julia for Data Science**

[ ] **18.S191 Introduction to Computational Thinking**

Markdown

Markdown Cheatsheet · adam-p/markdown-here Wiki

LaTeX

LaTeX - Mathematical Python

LaTeX help 1.1 - Table of Contents

List of mathematical symbols - Wikiwand

GitHub

[ ] **Hello World · GitHub Guides**

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## 參考資料
### Linear Algebra

[ ] [線性代數 - 臺大開放式課程 (NTU OpenCourseWare)](http://ocw.aca.ntu.edu.tw/ntu-ocw/index.php/ocw/cou/102S207/3)

### Julia

[ ] [Introduction to Julia](https://juliaacademy.com/courses/enrolled/375479)

[ ] Advanced topics

[ ] [Julia for Data Science](https://juliaacademy.com/courses/enrolled/937702)
```

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- [ ] [18.S191 Introduction to Computational Thinking](https://computationalthinking.mit.edu/Fall20/)

### Markdown
[Markdown Cheatsheet · adam-p/markdown-here Wiki](https://github.com/adam-p/markdown-here/wiki/Markdown-Cheatsheet)

### $$\LaTeX$$
[LaTeX - Mathematical Python](https://www.math.ubc.ca/~pwalls/math-python/jupyter/latex/)

[LaTeX help 1.1 - Table of Contents](http://www.emerson.emory.edu/services/latex/latex_toc.html)

[List of mathematical symbols - Wikiwand](https://www.wikiwand.com/en/List_of_mathematical_symbols)

### GitHub

[ ] [Hello World · GitHub Guides](https://guides.github.com/activities/hello-world/)

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