An impure public good model of local food systems: Aggregative games of four locals

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Abstract

This article builds a theoretical model of local food systems characterized by a public good. Local food has been flourishing across the world and attracting attention of the general public, policy makers and a wide range of academics. Most of them recognize some forms of social benefits created through local food systems. However, there exists no economic research that explicitly models the mechanism of social benefit creation. We address the issue by adopting an impure public good hypothesis as a basic framework for relating local food consumption to social benefits. In addition, to reflect its participatory nature, we use a non-linear aggregator, as opposed to the standard linear aggregator, of each contribution to the public good. These two distinguishing features necessarily lead to four types of consumers. We explain how local food systems involve economic games, prove the existence of a unique Nash equilibrium, and provide comparative statics of both the public good and the private goods. The article closes by numerically illustrating how the model works.

Keywords: *local food system, community development, impure public good, aggregative game, simulation*

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1 Introduction

New Oxford American Dictionary selected "locavore" as the word of the year in 2007, and now has an entry that reads "a person whose diet consists only or principally of locally grown or produced food." Local foods are distributed through a wide range of channels such as farmers' markets and CSA (community supported agriculture) programs. Feenstra and Campbell (1996) succinctly put it: a local food system (LFS hereafter) "is a collaborative effort to integrate agricultural production with food distribution to enhance the economic, environmental, and social well-being of a particular place." In addition to the obvious role of supplying foods, the definition clearly indicates that LFSs create social benefits. This is the most distinctive feature as opposed to conventional systems represented by large corporate intermediaries and retailers. So, any useful economic models of LFS should incorporate the mechanism of social benefit creation and account for its impacts on other variables. This article explicitly addresses the implications of social benefit creation for consumers' behavior by building an impure public good model with a non-linear aggregator.

The uniqueness of LFS has drawn attention of the general public, urban planners, policy makers, and academics in such fields as sociology, anthropology, agronomy and development studies. In economics, while some researchers have conducted empirical analysis, there is virtually no theoretical work and the overall research activity is quite limited. Most of the existing empirical research statistically characterizes local food consumers or estimates demand with associated attributes such as freshness, tastiness and localness. For example, Zepeda and Li (2006) characterize local food consumers with various factors such as economic, demographic and other lifestyle variables, while Darby, Batte, Ernst, and Roe (2008) examine willingness to pay for local production as well as farm size and freshness. None of them explicitly study the connection between local food and social benefits, but they treat localness merely as another product category or one of many attributes that affects private benefits, including nutritiousness, palatability and other psychological satisfaction.

In contrast to economics papers, even a brief survey of local food literature suffices to notice that LFSs create a wide range of social benefits that are shared by community members. Marsden, Banks, and Bristow (2000), Ross, Anderson, Goldberg, Houser, and Rogers (1999) claim that, given the activities taking place in a certain geographical area, LFSs help retain economic surplus generated through food transactions within the community. Similarly, the proximity of producers and consumers in LFSs means the disintermediation of food supply, thereby letting local farmers keep more dollars that would be otherwise captured by large corporate intermediaries (Milestad, Ahnström, & Björklund, 2011). Another social benefit is an increase in food security, i.e. sufficient access to healthy food. Because food production is both physically and socially close to consumers, it is considered relatively easier to arrange food access for people in need (DeLind & Howard, 2008; Kantor, 2001). For example, in the U.S., farmers' markets are often affiliated with food and nutrition programs, which try to give community members more access to quality foods (Thilmany & Watson, 2004). Due to the rise of environmental concerns, globalized food systems are criticized for contributing to carbon footprints. Therefore, LFS with shortened travel distance is viewed as a natural solution to the issue (Brown, 2003; Lea, 2005). Regarding food-related education, availability of local food is considered advantageous to effectively teaching the curricula (Matteson & Heuer, 2008). Most farm-to-school programs argue that tasty fruits and vegetable help promote healthy eating habits, and knowing the origin of food they actually eat gives deeper insight into food systems (Martinez et al., 2010). Most importantly, there seems to be a strong consensus in literature that LFSs heightened interaction among producers and consumers instills a sense of community, strengthens ties between citizens and assists community development (Grey, 2000; Hendrickson & Heffernan, 2002; Kirwan, 2004; C. Morris & Evans, 2004; Venn et al., 2006).

In order to theorize the mechanism of social benefit creation, we adopt an impure public good hypothesis that allows the consumption of a single economic good to generate both private benefits and social benefits. That is, social benefits are public goods and characterized by non-excludability and non-rivalry of their consumption (Cornes & Sandler, 1996). In this regard, as found in Thilmany, Bond, and Bond (2008) who include public good 'attributes' of local food consumption in their regression analysis, it is not sufficient to incorporate such psychological private benefits derived from serving social benefits. Instead, consumers should directly derive utility from consuming public goods and face the inherent tradeoff caused by non-excludability and non-rivalry. Looking to the two defining features of local food consumption, private benefits and social benefits, it is natural to liken local food to an impure public good. As Kotchen (2006) claims, the impure public good model is widely applicable to many social phenomena, raging from green electricity to ice cream that is produced by a company which donates a portion of its profit to a charitable cause. The other distinguishing feature of our model is the use of a non-linear production function of the public good, i.e. an aggregator of individual contributions to the public good. While the standard public good models use a liner aggregator, e.g. Bergstrom, Blume, and Varian (1986), the context of LFS does not seem to rationalize such a relationship between individual contributions because our public good is generated through active participation in the interaction among community members, to which each member's contribution is not a perfect substitute. By the combination of the impure public good and the non-linear aggregator, this article answer the question about how to mathematically capture the distinctive features of LFS that other social scientists delineate. In addition, the rigorous formalization provides an approach to addressing the issue of "local trap" (Born & Purcell, 2006), which means blindly promoting LFS without critical evaluations between costs and benefits.

We start the model building with specifying the fundamental assumptions and supporting them with associated literature. For the utility function, we use the characteristics approach to consumer behavior developed by Lancaster (1971) and Gorman (1980). The economic problem is formulated through three sources of utility: a private composite characteristic, a private food-related characteristic, and a public characteristic. Explained later, among the many types of social benefits, we focus on community development. As a result, the public characteristic reflects the quality of the community, which is assessed by such factors as safety, friendliness and caring culture. The private food-related characteristic formalizes the aforementioned private benefits created through purchasing and consuming foods. As standard, the private composite characteristic is a catch-all variable that captures any other utility source. Given the aggregated natures, we assume that they are normal goods. To attain these characteristics, four goods are made available: a composite good, direct donation to community, conventional food and local food. Then, we introduce an important function that captures the most distinctive mechanism of LFS, converting individual food purchase activities to community development. In the process, local food is a substitute for direct donation to the community and therefore we let a single coefficient represent relative productivity in generating the public characteristic. Looking to the food-related private characteristic, conventional food and local food are close substitutes. So, their relative productivity is also a key parameter to understand. It reflects the attractiveness of local food in generating food-related experiences. As a result, the parameter is empirically most interesting and immediately pertinent to policies, e.g. regulating post-harvesting practice and geographical boundaries (Wittman, Beckie, & Hergesheimer, 2012). Along with the assumptions up to this point, the rationality on expenditure leads to a further parametric restriction; consumers buy a cheaper bundle of goods in generating the same combination of the private characteristic and the public characteristic. Finally, we explain how these restrictions jointly result in the main actors of the model, four representative locals.

Having specified the underlying structure, next we illustrate the circumstance as an economic game, especially an aggregative game. We adopt Nash equilibrium for a solution concept and prove the existence of a unique Nash equilibrium in the local food game. Then, we mathematically derive the major implications of the model; i.e. the effects of exogenous shocks to community development and individual demand. We first investigate the effects on community development and characterize each parameter according to whether it has positive, negative, neither or undetermined an impact on community development. Based on this, we obtain individual responses to a shock. In the mathematical derivation, we apply the general theorems in aggregative games provided by Acemoglu and Jensen (2013).

Throughout the analysis, without specifying the utility function, the immediate mathematical outputs are necessarily general and provide only a few empirically useful predictions. However, the particular choice circumstances, food and community, allow us to use intuition about complementarity and substitutability between the goods and make reasonable speculations. That is, we do not stop modeling at the completion of mathematical derivations; besides, we also employ non-rigorous analysis to make concrete implications. We do not pretend all the strong predictions are realistic; rather, we expect a gap between the model and the real world. Instead of realism, the objective of this article is to quantitatively capture the most salient feature of local food systems and provide a sound basis on which future research and empirical applications can fill the gap by inductive inference (Sugden, 2000). Finally, in order to complement the non-rigorous component and illustrate how the model works, we conduct a computer simulation to confirm that those predictions and speculations are correct under Cobb-Douglas preferences.

2 The Model

2.1 Specification

2.1.1 Consumer's problem

A typical ith consumer solves the following utility maximization problem:

$$\begin{aligned} &\max\ u_i = U_i(X_i, Y_i, Z) \text{ subject to} \\ &x_i + p_c c_i + p_d d_i + p_l l_i = m_i,\ X_i = x_i,\ Y_i = c_i + \alpha_i l_i,\ Z = f(\mathbf{d}, \mathbf{l}; \beta) \end{aligned}$$

Arguments of the utility function

X_i	Composite private characteristic
Y_i	Food-related private characteristic
Z	Community development public characteristic

Choice variables

x_i	Composite good
c_i	Conventional food consumption
d_i	Direct donation or any other form of contribution to the community
l_i	Local food consumption
$\mathbf{d}, \mathbf{l}$	Vectors of all the consumers' d and l

Parameters

m_i	Income
α_i	Marginal private benefit of local food
β	Marginal effectiveness of local food on community development
p_c	Price of conventional food
p_d	Price of direct donation
p_l	Price of local food

Heterogeneous consumers are distinguished only through m, α, and utility function U; i.e. the other parameters, β, p_c, p_d and p_l are common. Notice that l_i enters two equations Y_i and Z, which is the fundamental mechanism of the model and often called a joint-production or impure public good model (Cornes & Sandler, 1996). $f(\mathbf{d}, \mathbf{l}; \beta)$ is the aggregator of individual contributions, which is explained later. While the existence of each price, especially p_d, will be an important empirical question, we build the model by assuming their availability or proxies.

2.1.2 Utility function

It is assumed to be twice continuously differentiable, strictly quasi-concave and strictly increasing in each argument. Furthermore, the subsequent analysis is based on the assumption that each characteristic is strictly normal, i.e. positive income effects on X, Y and Z. The normality is considered reasonable because all of the three characteristics are highly aggregated goods with no close substitutes. Since food and eating are truly essential and unique in human life, the derived benefits (Y) have few substitutes as well. In modeling economic decisions at a community level, community development must be, by definition, a good without close substitutes. For such goods, it is reasonable to assume normality (Mas-Colell, Whinston, & Green, 1995, p. 25). With $m_i > 0 \; \forall i$, the normality implies that $X_i, Y_i, Z > 0$ at everyone's choice. In particular, $Y_i > 0$ leads to the following (because we need to eat!).

Proposition 1 *A consumer always buys some conventional food or/and local food.*

2.1.3 Aggregator

$$Z = f(\mathbf{d}, \mathbf{l}; \beta) \equiv \left(\sum_j (d_j + \beta l_j)^\rho \right)^{\frac{1}{\rho}},$$

where $0 < \rho < 1$. The aggregator is a function of individual contributions $\mathbf{d}$ and $\mathbf{l}$ that generates the public characteristic Z, which is also referred to as the aggregate. A symmetric CES production function is adopted because of its relevance to a wide range of public goods models (Cornes, 1993) and advantage in analytical tractability. Since community development is more or less participatory, the aggregator is symmetric in the sense that individuals are anonymous and no weight is placed on anyone's contribution. For the same reason, $\rho < 1$ is assumed. Large contributions from a handful of wealthy activists are not regarded as equivalent to the same scale of contribution made by a number of community members. That is, each unit of contribution is not a perfect substitute, and their distribution matters to some extent. This is why $\rho = 1$ is ruled out. In contrast, for a technical reason, $\rho > 0$ is imposed. While the function is not defined at $d_i + \beta l_i = 0$ if $\rho \leq 0$, zero contribution must be allowed to be in the domain of the function simply because the community should still be formed even if some members contribute nothing. Nevertheless, the next proposition states that zero contribution is never optimal for anyone.

Proposition 2 *A consumer always buys some local food or/and makes some direct contribution to the community.*

[Proof]
Take a differential of Z while fixing $\mathbf{d}_{-i}$ and $\mathbf{l}_{-i}$:

$$dZ = \frac{\partial Z}{\partial (d_i + \beta l_i)} d(d_i + \beta l_i) = \left(\frac{Z}{d_i + \beta l_i} \right)^{1-\rho} d(d_i + \beta l_i).$$

$\rho < 1$ implies that the marginal gain in Z becomes arbitrarily large as $d_i + \beta l_i$ becomes close to zero. Thus, given $U_3 > 0$ and the fact that the prices are bounded from above, $d_i + \beta l_i = 0$ is never optimal for any consumer. $\square$

2.1.4 Marginal social benefit

As mentioned in Introduction, local food systems create a number of social benefits, which include economic impacts, food security, environmental friendliness, and education. Although these are commonly cited by local governments and civil society institutions, counter arguments against their effectiveness are often as convincing (Martinez et al., 2010).

In contrast, though least emphasized in practice, the arguments surrounding community development are relatively sound. Connelly, Markey, and Roseland (2011, p. 320) succinctly puts it: "Local food systems, given their appeal to community, health, and quality of life provide a compelling gateway to realizing community transformation." Reinforced by the fact that food is the absolutely essential commodity, LFS constitutes a portion of the social fabric where people interact with each other and reconnect. Lockie (2009, p. 193) argues that LFS allows citizens to participate in "social arrangements based on solidarity and coordinated action." Since it seems to be a consensus in literature that almost all the additional values created in LFS result from the interaction and reconnection of community members (Mount, 2012), we view it as currently the most salient and convincing feature of LFS (Grey, 2000; Hendrickson & Heffernan, 2002; Kirwan, 2004; C. Morris & Evans, 2004; Venn et al., 2006). As a consequence, we approximate the public good (Z), which is potentially quite broad, by community development. By adopting the impure public good hypothesis, which necessarily involves strategic incentives, we try to address much criticized a utopian view of LFS (C. Morris & Kirwan, 2010; Winter, 2003; Goodman, 2004; Sonnino & Marsden, 2006) dubbed "local trap" (Born & Purcell, 2006) and start to model from the consumer side a tension between social embeddedness and marketness (Hinrichs, 2000; Chiffoleau, 2009; DeLind, 2002). Mathematically said, in breaking up individual contribution into the direct donation effect and the local food effect, β plays a crucial role reflecting the very existence of the special mechanism of LFS that converts local food consumptions into community development. If β turned out to be zero, the publicness of local food consumption would be denied and, as a result, the whole model based on the impure public good hypothesis should be discarded.

2.1.5 Marginal private benefit

α captures the relative attractiveness or authenticity of local food in generating Y (Smithers & Joseph, 2010). Aside from the obvious role of feeding consumers, the food-related private benefits range from such common ones as tastiness, safety and nutritional value to such distinctive ones as anti-corporate sentiment, environmental friendliness, supporting the local economy and other what economists call warm-glow motives. Many of the psychological private benefits are reflections of the perceived social benefits discussed above. While the objectivity is the major concern when we model some benefits as public goods, it is less problematic when modeling individual satisfactions from contributing to those social benefits. Andreoni (1990) provides one of the classical analyses of public giving and warm-glow, and there are many more real-world examples (Nechyba, 2011).

Most of the tangible private benefits result from the freshness of local food. The geographical proximity directly implies shortened travel time, which in turn implies harvesting at more matured stages and less loss of nutrients (Feagan, Morris, & Krug, 2004; Lacy, 2000). As a result, tastiness and healthiness are almost universally regarded as the most valuable aspect of local food (Lea, 2005). In addition to these relatively objective benefits, there are a number of subjective values. For examples, (Smithers & Joseph, 2010) report that some consumers see LFS provide an outlet for their dissatisfaction with conventional food systems. Through participating in LFS, they express food politics against the fast nature of modern lives (Gaytán, 2004) and the dominance of large corporations, which allegedly exploit both producers and consumers, the natural environment, and other social concerns (Lacy, 2000; Griffin & Frongillo, 2003). As a result of the multifaceted and subjective nature, α_i is an individual parameter. It is also designed to be a receptor of common exogenous shocks, e.g. advertising campaigns and food-safety incidents.

2.1.6 Rationality on expenditure

In this model, $p_l > \alpha_i p_c$ is a rational restriction if a person chooses to consume conventional food. That is, conventional food is cheaper than local food for generating α_i units of Y.

So, she is assumed to have α_i such that the inequality holds for given p_c and p_l whenever $c_i > 0$. Similarly, $p_l > \beta p_d$; direct donation is cheaper than buying local food for making β units of contribution. Since all the three parameters p_l, β, p_d are common to everyone, the restriction is indeed imposed regardless of the choice. For those who consume local food, $p_l < \alpha_i p_c + \beta p_d$; purchasing local food is a cheaper way to generate a combination of α_i units of Y and β units of contribution than buying c and d separately. As a corollary, we assume $p_l > \alpha_i p_c + \beta p_d$ if a consumer chooses not to buy local food. Then, follows a crucial proposition that introduces a considerable structure to the model.

Proposition 3 *A consumer never chooses to simultaneously purchase conventional food, direct donation and local food.*

[Proof]
Suppose an interior solution (i.e. strictly positive consumption of all the goods). Then, given the numeraire x_i, the first order conditions yield:

$$p_c = MRS_{c,x} = \frac{D_c U}{D_x U} = \frac{U_2}{U_1}$$

$$p_d = MRS_{d,x} = \frac{D_d U}{D_x U} = \frac{U_3 D_d Z}{U_1}$$

$$p_l = MRS_{l,x} = \frac{D_l U}{D_x U} = \frac{\alpha_i U_2 + \beta U_3 D_d Z}{U_1}.$$

These three equations imply $p_l = \alpha_i p_c + \beta p_d$, which is a contradiction because $p_l < \alpha_i p_c + \beta p_d$ if $l_i > 0$. Hence, no one consumes all the four goods. Since x_i is a composite good whose consumption must be positive, we cannot have all the rest of three consumed. □

[Remark]
If we allow $p_l = \alpha_i p_c + \beta p_d$, then she may consume all the three goods. However, having exact such α_i is too trivial to justify additional complexity to the model. So, we preclude the equality.

2.1.7 Four representative locals

This is a good time to introduce a concrete scenario that the model is implicitly designed for so that we can more easily derive and interpret its predictions. Imagine four representative community members: namely Skeptic, Locavore, Foodie, and Activist. It follows from Proposition 1, 2 and 3 that there are only four logical combinations of the positive consumptions of c, d and l. Out of eight possible permutations, we have shown that the following four combinations are ruled out: $\{c = 0, d = 0, l = 0\}$, $\{c = 0, d > 0, l = 0\}$ by Proposition 1, $\{c > 0, d = 0, l = 0\}$ by Proposition 2, and $\{c > 0, d > 0, l > 0\}$ by Proposition 3. Hence, four representative locals:

A differential operator D is used to denote partial derivatives, e.g. $D_m \Pi \equiv \frac{\partial \Pi}{\partial m}$ and $D^2_{ms} \Pi \equiv \frac{\partial^2 \Pi}{\partial m \partial s}$. Specifically for the utility function, subscript numbers are also used, e.g. $U_1 \equiv \frac{\partial U(X,Y,Z)}{\partial X}$ and $U_{23} \equiv \frac{\partial^2 U(X,Y,Z)}{\partial Y \partial Z}$.

$$
\begin{array}{rl}
\text{Skeptic} & c > 0, d > 0 \\
\text{Locavore} & l > 0 \\
\text{Foodie} & c > 0, l > 0 \\
\text{Activist} & d > 0, l > 0 \ .
\end{array}
$$

As discussed above, each type has a specific range of α and MRS between the goods that support their choice. For instance, $p_l > \alpha_i p_c + \beta p_d$ supports Skeptic's choice. She little appreciates local food and has low α_i such that local food is too expensive for her. In contrast, Locavore highly values local food and has α_i such that $p_l < \alpha_i p_c$; local food is cheaper for Y and generates some Z. So, she does not find any reason for buying conventional food. While Foodie must have $p_l > \alpha_i p_c$, the distinction between Foodie and Activist stems from a more subtle combination of α, income and preferences. Intuitively said, Activist has a combination of α and weight on Y in her utility function such that demanded characteristic Y is sufficiently fulfilled by the consumption of local food.

2.2 Local food game

Upon the introduction of distinct four consumers, we analyze their optimization problems, interconnected through Z, by adopting the standard Nash equilibrium as a solution concept. Under the Nash conjecture, a consumer makes a choice taking the others' choices as given. Let $\mathbf{d}_{-i}$ and $\mathbf{l}_{-i}$ denote respectively the vectors of the others' donation and local food consumptions. Moreover, purely for the sake of tidiness, let $\mathbf{t}_i$ denote an individual vector of the parameters: $\mathbf{t}_i = (m_i, \alpha_i, \beta, p_c, p_d, p_l) \in \mathbf{T}_i \subseteq \mathbb{R}^6_+$. Taking $\mathbf{d}_i$, $\mathbf{l}_i$ and $\mathbf{t}_i$ as given, the consumer solves the problem and obtains for each choice variable a demand function in terms of $\mathbf{d}_i$, $\mathbf{l}_i$ and $\mathbf{t}_i$. Notice that the choice of the ith consumer is arbitrary and the argument is symmetric for every consumer. This implies that there exists circular interdependence in the choice of d and l among all the players. In other words, each consumer (called a player hereafter) is playing a local food game that is formalized as follows:

$$\Gamma = (u_i, S_i, \mathbf{t}_i)_{i \in N}$$

$$
\begin{gathered}
N = \{1, ..., i, ..., n\} \ (n \geq 3) \\
s_i \equiv d_i + \beta l_i \in S_i \subseteq \mathbb{R}_+ \ \forall i \\
\mathbf{s} \in \mathbf{S} = \prod_{j=1}^{n} S_j \subseteq \mathbb{R}^n_+ \\
\mathbf{t}_i \in \mathbf{T}_i \subseteq \mathbb{R}^6_+ \ \forall i.
\end{gathered}
$$

The game is a non-cooperative, strategic game played by n community members. The consumer's utility is now captured by a payoff function:

$$u_i = \pi(s_i, \mathbf{s}_{-i}; \mathbf{t}_i) \equiv U(X(s_i, \mathbf{s}_{-i}; \mathbf{t}_i),\ Y(s_i, \mathbf{s}_{-i}; \mathbf{t}_i),\ Z(s_i, \mathbf{s}_{-i}; \mathbf{t}_i);\ s_i, \mathbf{s}_{-i}; \mathbf{t}_i),$$

where X, Y and Z are demand functions after the independent optimization. Finally, for the sake of clarity, we recast the aggregator as a function of the vector of strategies:

$$g(\mathbf{s}) \equiv f(\mathbf{d}, \mathbf{l}) \text{ where } g : \mathbf{S} \to G \subseteq \mathbb{R}_+.$$

2.2.1 Aggregative game

Further scrutiny at the structure of the game enables us to dramatically simplify the analysis. First, define the aggregate of the others as follows:

$$Z_{-i} \equiv \left(\sum_{j \neq i} s_j^{\rho} \right)^{\frac{1}{\rho}}.$$

Then, the full aggregate can be written in terms of s_i and Z_{-i}:

$$Z = \left(\sum_{j} s_j^{\rho} \right)^{\frac{1}{\rho}} = \left(s_i^{\rho} + \sum_{j \neq i} s_j^{\rho} \right)^{\frac{1}{\rho}} = \left(s_i^{\rho} + Z_{-i}^{\rho} \right)^{\frac{1}{\rho}}. \tag{1}$$

Recall that the utility function is parameterized by $\mathbf{d}_{-i}$ and $\mathbf{l}_{-i}$ only through the aggregator. So, it is really a single number Z_{-i} that affects the ith player's demand and payoff. She is playing a game due essentially to the interdependence between s_i and Z_{-i}; only s_i and Z_{-i} are strategic determinants of her payoff. Moreover, because of the specific class of functions featured by additive separability (Gorman, 1968), it is a simple matter to invert the aggregate with respect to Z_i:

$$Z_{-i} = (Z^{\rho} - s_i^{\rho})^{\frac{1}{\rho}},$$

which is a function of $Z = g(\mathbf{s})$ and s_i as well. As a consequence, we can write each player's payoff as a function of only s_i and $g(\mathbf{s})$:

$$\begin{aligned} u_i &= \Pi(s_i, g(\mathbf{s}); \mathbf{t}_i) \\ &\equiv U(X(s_i, g(\mathbf{s}); \mathbf{t}_i),\ Y(s_i, g(\mathbf{s}); \mathbf{t}_i),\ Z(s_i, g(\mathbf{s}); \mathbf{t}_i);\ s_i, g(\mathbf{s}), \mathbf{t}_i), \end{aligned} \tag{2}$$

where $Z(s_i, g(\mathbf{s}); \mathbf{t}_i) = g(\mathbf{s}; \mathbf{t}_i)$. Notice that the dimension of the domain of the payoff function reduces by $n - 1$, which makes a tremendous difference to the analysis when n is large, significant enough even when $n = 3$. A game is called aggregative if each player's payoff is a function of only her own strategy and a sufficient statistic of every player's strategy (Cornes & Hartley, 2007). The result merits another proposition.

Proposition 4 *The local food game is aggregative:* $\Gamma = (\Pi_i, S_i, \mathbf{t}_i)_{i \in N}$ *where* $\Pi_i : S_i \times G \times \mathbf{T}_i \to \mathbb{R}$.

2.2.2 Normality

Since the normality assumption plays a crucial role in the subsequent analysis, we derive some mathematical implications in advance. Given the identity $Z = g(s_i, \mathbf{s}_{-i})$, the strict normality in Z implies:

$$D_m Z = D_s g D_m s = D_s g (D_m d^* + \beta D_m l^*) > 0,$$

in which $D_s g > 0$ immediately implies $D_m s = D_m d^* + \beta D_m l^* > 0$. Since the other two characteristics are also normal, $D_m X, D_m Y > 0$, we must have a positive income effect on the marginal payoff function $D_s\Pi$ to support $D_m s > 0$, i.e. $D^2_{ms}\Pi > 0$. Furthermore, given the compact and convex choice set and the strictly quasi-concave utility function, s_i is an interior and unique point on the appropriate subspaces $\{l = 0\}$ for Skeptic, $\{c = d = 0\}$ for Locavore, $\{d = 0\}$ for Foodie, and $\{c = 0\}$ for Activist. As a result, we apply the implicit function theorem to the first order condition $D_s\Pi = 0$ and get:

$$D^2_{ss}\Pi = -\frac{D^2_{ms}\Pi}{D_m s} < 0.$$

2.2.3 Nash equilibrium

To make use of the general proof for the existence of Nash equilibrium in this class of games, we will make sure that the required conditions are satisfied in the local food game. First, since the total expenditure on donation and local food cannot be greater than income, the following inequality must hold: $p_d d_i^* + p_l l_i^* \leq m_i$. Combined with the assumption $p_l > \beta p_d$ and the identity $s_i = d_i^* + \beta l_i^*$, we will get $s_i \leq m_i/p_d$. Thus, each strategy set can be restricted to a finite interval, $S_i = [0, m_i/p_d]$, which is clearly a compact and convex set. Next, the utility function is assumed to be twice continuously differentiable, and so is the aggregator $g(\mathbf{s})$. Given $D^2_{ss}\Pi < 0$, therefore, the payoff function Π is twice continuously differentiable and strictly concave in s_i. Applying Theorem 5 of Acemoglu and Jensen (2013), we closes this subsection with the following proposition.

Proposition 5 *The local food game has a unique Nash equilibrium.*

2.3 Comparative statics

Guaranteeing the existence of a unique Nash equilibrium, we are ready to glean insight into local food demand and decentralized community development. The equilibrium analysis follows the standard approach in aggregative games. First, we characterize the impact of each shock to the aggregate, whether one has a positive, negative, neither or undetermined impact. Throughout, we regard an increase in t as a positive shock if the payoff function exhibits increasing differences in s_i and t. Due to the normality and the concavity in the player's own strategy, the local food game is what Acemoglu and Jensen (2013) call a nice aggregative game, meeting all the required conditions. As a result, once identifying the nature of each parameter, final prediction about the impact on Z is merely an application of their theorems. Then, we move on to consumer's demand. It is conceptually correct that each player's marginal payoff and therefore strategy choice depend on the aggregate of every player's strategy, whereby there is complete circularity initiated by the shock. When analyzing the impact of a shock on one player's demand at a time, however, the marginal influence of the aggregate is only indirect, and 'aggregate-taking behavior' (Alos-Ferrer & Ania, 2005) is considered as a reasonable approximation. That is, after transmitting a shock to the aggregate through initial responses of all the players hit by the shock, each player takes the aggregate as fixed and not to be affected by the players' total responses to the shock.

2.3.1 Marginal payoff function

In order to examine whether the payoff function exhibits increasing differences in s_i and t, we use the marginal payoff function $D_s\Pi$ and see the sign of each cross-partial derivative of Π. Note, by the chain rule,

$$\frac{\partial}{\partial d} = \frac{\partial}{\partial s}\frac{\partial s}{\partial d} = \frac{\partial}{\partial s} \text{ and } \frac{\partial}{\partial l} = \frac{\partial}{\partial s}\frac{\partial s}{\partial l} = \beta\frac{\partial}{\partial s}.$$

So, we have:

$$\frac{\partial}{\partial s} = k\left(\frac{\partial}{\partial d} + \frac{\partial}{\partial l}\right) \text{ where } k > 0 \text{ such that } k \equiv \begin{cases} (1+\beta)^{-1} & \text{if } d, l > 0 \\ 1 & \text{if } d > 0, l = 0 \\ \beta^{-1} & \text{if } d = 0, l > 0 \end{cases} \tag{3}$$

Remember that the definition of the payoff function:

$$u_i = \Pi(s_i, g(\mathbf{s}), \mathbf{t}_i) \equiv U(m_i - p_c c_i^* - p_d d_i^* - p_l l_i^*,\ c_i^* + \alpha_i l_i^*,\ g(\mathbf{s});\ s_i, g(\mathbf{s}); \mathbf{t}_i),$$

where demand of each good is a function of s_i, $g(\mathbf{s})$ and $\mathbf{t}_i$, as seen in Eq(2). With these and the envelope theorem in mind, we obtain the marginal payoff function as follows:

$$\begin{aligned} D_s\Pi = \frac{dU}{ds} = \frac{\partial U}{\partial s} &= k(D_d U + D_l U) \\ &= k[(-p_d U_1 + U_3 D_s g) + (-p_l U_1 + \alpha U_2 + \beta U_3 D_s g)] \\ &= k[-(p_d + p_l)U_1 + \alpha U_2 + k^{-1} U_3 D_s g] \\ &= k[-(p_d + p_l)U_1 + \alpha U_2] + U_3 D_s g, \end{aligned} \tag{4}$$

where we may consistently set $p_d = 0$ if $d = 0$ and $p_l = \alpha = 0$ if $l = 0$.

2.3.2 Parameter characterization

In examining each parameter, we use the inequality $D_s U_1 = D^2_{ms}\Pi > 0$ obtained by noting $D_m\Pi = U_1 D_m X = U_1$ given the fact that m enters the utility function only through X as $X = x^* = m - p_c c^* - p_d d^* - p_l l^*$. To begin with, for the price of conventional food p_c,

$$D^2_{p_c s}\Pi = D_s(U_1 D_{p_c} X) = D_s(-c^* U_1) = -U_1 D_s c^* - c^* D_s U_1.$$

For the price of donation p_d,

$$D^2_{p_d s}\Pi = D_s(U_1 D_{p_d} X) = D_s(-d^* U_1) = -kU_1 D_l d^* - (kU_1 + d^* D_s U_1).$$

And for the price of local food p_l,

$$D^2_{p_l s}\Pi = D_s(U_1 D_{p_l} X) = D_s(-l^* U_1) = -kU_1 D_d l^* - (kU_1 + l^* D_s U_1).$$

The first and second terms in each derivative capture respectively a substitution effect and an income effect of the price change. Given $D^2_{ms}\Pi > 0$ and a negative income effect of the price increase, the second terms is negative. In contrast, the sign of the first term depends on whether s and c, d and l are substitutes or complements. As a result, if each pair of goods are complements, the shock is unambiguously negative. In general, however, we do not know it. Since it is much simpler to examine these for a specific type, we do so type by type.

[Skeptic]

$$D^2_{p_c s}\Pi = -U_1 D_s c^* - c^* D_s U_1 = -U_1 D_d c^* - c^* D_s U_1,$$

which is negative if conventional food and donation are complements. But we do not speculate this and leave it undetermined. Next,

$$D^2_{p_d s}\Pi = -kU_1 D_l d^* - (kU_1 + d^* D_s U_1) = -(kU_1 + d^* D_s U_1) < 0,$$

and, due to $l = 0$, we have $D^2_{p_l s}\Pi = 0$.

[Locavore]
$c = d = 0$ immediately implies $D^2_{p_c s}\Pi = D^2_{p_d s}\Pi = 0$. For p_l,

$$D^2_{p_l s}\Pi = -kU_1 D_d l^* - (kU_1 + l^* D_s U_1) = -(kU_1 + l^* D_s U_1) < 0.$$

[Foodie]

$$D^2_{p_c s}\Pi = -U_1 D_s c^* - c^* D_s U_1 = -U_1 D_l c^* - c^* D_s U_1,$$

which is undetermined because conventional food and local food are close substitutes so that $D_l c^* < 0$. For p_d and p_l, the signs are the same as for Locavore; i.e. $D^2_{p_d s}\Pi = 0$ and $D^2_{p_l s}\Pi < 0$.

[Activist]
Due to $c = 0$, $D^2_{p_c s}\Pi = 0$. For p_d and p_l,

$$D^2_{p_d s}\Pi = -kU_1 D_l d^* - (kU_1 + d^* D_s U_1) = -kU_1(1 + D_l d^*) - d^* D_s U_1$$

$$D^2_{p_l s}\Pi = -kU_1 D_d l^* - (kU_1 + l^* D_s U_1) = -kU_1(1 + D_d l^*) - l^* D_s U_1.$$

Here, we make a reasonable speculation. While donation to the community and local food consumption may be substitutes, it is unrealistic that they have strong substitutability whereby one unit increases in donation reduces more than one unit of local food or vice versa. Thus, we speculate $D_l d^* > -1$ and $D_d l^* > -1$. Hence, $D^2_{p_d s}\Pi < 0$ and $D^2_{p_l s}\Pi < 0$.

Since α and β represent the fundamental mechanism of the model, the joint production of characteristics, their impacts are more involved and strongly dependent on the underlying preference. To begin with, suppose that shocks do not alter the current marginal utility of each characteristic, U_1, U_2 and U_3. Then, it is easy to see intuitive results. By differentiating Eq(4) with U_1, U_2 and U_3 fixed, $D^2_{\alpha s}\Pi = kU_2 > 0$; an increase in the attractiveness of local food raises the marginal payoff. Similarly, $D^2_{\beta s}\Pi = -k^2[-(p_d + p_l)U_1 + \alpha U_2] = kU_3 D_s g > 0$ given the Nash equilibrium condition $D_s\Pi = k[-(p_d + p_l)U_1 + \alpha U_2] + U_3 D_s g = 0$.

Once looking beyond the first-order impact, the responses become unwieldy:

$$\begin{aligned} D^2_{\alpha s}\Pi &= kU_2(1 + D_d l^*) + l^* D_s U_2 \\ &= kU_2(1 + D_d l^*) + l^* k[-(p_d + p_l)U_{12} + \alpha U_{22}] + U_{23} D_s g \end{aligned}$$

$$\begin{aligned} D^2_{\beta s}\Pi &= D_s g[kU_3(1 + D_d l^*) + l^* D_s U_3] \\ &= D_s g \left\{kU_3(1 + D_d l^*) + l^* k[-(p_d + p_l)U_{13} + \alpha U_{23}] + U_{33} D_s g\right\}. \end{aligned}$$

The second order effects, $D_s U_2$ and $D_s U_3$, capture the impact of additional $d + \beta l$ on the current marginal utilities of Y and Z. While $U_{12} = D_m U_2 > 0$, $U_{13} = D_m U_3 > 0$, $U_{22} < 0$, $U_{33} < 0$ and $1 + D_d l^* > 0$ speculated above, we need more assumptions to be able to determine the sign of U_{23}, i.e. whether Y and Z are complements or substitutes. Even if we speculate that Y and Z are neither close substitutes nor complements and therefore the magnitude of U_{23} is small, the other signs are still mixed and undetermined. For the rest of the article, we will proceed by assuming that the second order effects are not as substantial as reversing the first-order positive effects. Under this assumption, the nature of each parameter is well aligned with our intuition; the price shocks are negative, while the shocks to α and β are positive.

Below is a summary of the results, where brackets indicate speculation. Overall, more assumptions on preferences are necessary to make unambiguous predictions. That said, grouping into four types clearly help determine the nature of several shocks and make some reasonable speculation on price.

Table 1. Parameter characterization

	$D^2_{ms}\Pi$	$D^2_{p_c s}\Pi$	$D^2_{p_d s}\Pi$	$D^2_{p_l s}\Pi$	$D^2_{\alpha s}\Pi$	$D^2_{\beta s}\Pi$
Skeptic	+	?	−	0	0	0
Locavore	+	0	0	−	(+)	(+)
Foodie	+	?	0	−	(+)	(+)
Activist	+	0	(−)	(−)	(+)	(+)

2.3.3 Impact on community development

To make a statement about the aggregated impact of each player's response on community development, we apply Theorem 6 of Acemoglu and Jensen (2013) that is based on an extensive proof. Despite the tremendous complications in the strategic decision process across all the players, it provides a powerful yet simple prediction that a positive shock to a single or multiple players increases the equilibrium aggregate. Therefore, based on the determined and speculated nature of each parameter, we have the following statement:

Proposition 6 *Community development is accelerated by an income rise, greater attractiveness of local food to individual, higher societal effectiveness of LFS, a fall in donation price, and a fall in local food price.*

In addition, their Theorem 7 allows us to claim an intuitive consequence of a new member to the community. Suppose that a newcomer immediately acts as one of the four types. Then, we have:

Proposition 7 *Having a new member is beneficial to community development.*

To see the significance of the proposition, write the pre-entry aggregate as Z_n and post-entry as Z_{n+1}. Then, by Eq(1):

$$Z_{n+1} = \left(s_{n+1}^{\rho} + \sum_n s_j^{\rho} \right)^{\frac{1}{\rho}}.$$

Upon the entry, some of the existing members likely reduce their contributions, thereby lowering Z_n^{ρ}, i.e. $\sum_n s_j^{\rho} < Z_n^{\rho}$. Despite this negative effect on the new aggregate, the proposition predicts that in total we have:

$$s_{n+1}^{\rho} + \sum_n s_j^{\rho} > Z_n^{\rho},$$

and therefore,

$$Z_{n+1} = \left(s_{n+1}^{\rho} + \sum_n s_j^{\rho} \right)^{\frac{1}{\rho}} > (Z_n^{\rho})^{\frac{1}{\rho}} = Z_n.$$

2.3.4 Impact on demand

As explained at the beginning of this section, after establishing the impact on the aggregate, each player takes Z as given. A change in the aggregate consists of everyone's response to a shock, so we may emphasize that Z is a function of $\mathbf{t}$: $\Pi(s_i^*, Z(\mathbf{t}), \mathbf{t}_i)$ and $s_i^*(Z(\mathbf{t}), \mathbf{t}_i)$ where $\mathbf{t}_i \in \mathbf{t}$. Let t denote a typical element of $\mathbf{t}$, i.e. $t \in \mathbf{t}_i$ or $t \in \mathbf{t}_j (j \neq i)$. Therefore, the total individual response to a shock is captured by a total derivative:

$$\frac{ds}{dt} = \frac{\partial s}{\partial Z}\frac{\partial Z}{\partial t} + \frac{\partial s}{\partial t} = D_Z s D_t Z + D_t s,$$

where $D_t s$ is the initial response discussed in the preceding analysis. Since $D_Z s$ can be replaced by the implicit function theorem,

$$\frac{ds}{dt} = D_Z s D_t Z + D_t s = -\frac{D_{Zs}^2 \Pi}{D_{ss}^2 \Pi} D_t Z + D_t s.$$

There we know $D_t s \geq 0$ and $D_t Z > 0$ given that we are analyzing a positive shock as well as $D_{ss}^2 \Pi < 0$. Hence, the total response depends on the sign of $D_{Zs}^2 \Pi$ and whether a shock has no direct impact on ith player. That is to say, if $t \in \{m_j, \alpha_j\}$ or t is a parameter with which ith player does not consume the associated good, then we have $D_t s = 0$ and call such a shock indirect. Using the same trick in Eq(3), we have:

$$\frac{\partial}{\partial Z} = \frac{1}{1 + k^{-1} D_s g} \left(\frac{\partial}{\partial g} + k^{-1} \frac{\partial}{\partial s} \right).$$

So, we differentiate the marginal payoff function Eq(4) with respect to Z and get:

$$D_{Zs}^2 \Pi = \frac{1}{1 + k^{-1} D_s g} \left\{ k[-(p_d + p_l) U_{13} + \alpha U_{23}] + U_{33} D_s g + k^{-1} D_{ss}^2 \Pi \right\},$$

where $U_{13} > 0$ and $U_{33}, D^2_{ss}\Pi < 0$. So, if Y and Z are substitutes, i.e. $U_{23} < 0$, then we have $D^2_{Zs}\Pi < 0$. While $U_{23} > 0$ is possible, there is in general no reason that food-related benefits and community development benefits are close complements. As a result, we speculate that the magnitude of U_{23} is small and therefore $D^2_{Zs}\Pi < 0$. In summary, we have an unambiguous negative demand response to a positive shock provided that the shock is indirect to a player in question.

Proposition 8 *If a positive shock does not directly hit a player's income or other parameters associated with the goods she consumes, then she decreases the contribution to the community.*

3 Simulation

We conduct a numerical simulation to illustrate how the theoretical model works, i.e. numerically confirms the analytical predictions. With the specified utility function, the complementarity and substitutability between each pair of goods are determined, and therefore we are able to obtain unambiguous results. Although in this article we restrict the exercise to a simple setting, it is not difficult to elaborate it by adding more complexity. This may include increasing the number of players, adding greater heterogeneity in the combination of the parameters, and introducing more sophisticated shocks than simple perturbation in a single parameter. Bear in mind that the simulation is by no means designed to describe the real world; rather, designed to capture the basic incentive structure of the model and demonstrate how the Nash equilibrium evolves. So, all the marginal changes are simulated by a small differential (by 1%), which necessarily makes some of the impacts look trivial. The key observation is on ordinal change instead of magnitude.

3.1 Locating the Nash equilibrium

It is, in general, increasingly challenging to pin down a reasonable Nash equilibrium as the number of players goes beyond two. In contrast, the local food game is an aggregative game with a unique Nash equilibrium, and both features substantially simplify the equilibrium search. While for numerical optimization the challenge is equivalent to figuring out good initial values, the uniqueness of the equilibrium assures that the goal will be located as the search goes on. The last phrase, 'as the search goes on', seems to be consistent with a standard interpretation of Nash equilibrium, a steady state reached by 'experienced' players (Osborne, 2004).

The method is simple: each player takes a turn and re-optimizes her choice by taking the aggregate as given. With the updated aggregate, the next player does the same, and the process continues until reaching a stable aggregate for which re-optimization makes little difference. Clearly, this kind of dynamic process, one optimization at a time, is technically incorrect given the simultaneous move game. But the aggregative nature of the game makes it a good approximation because no matter how many players involved, each player always makes a strategic decision based on only her own action and the aggregate, and any one player's action is buffered by aggregation. So, we start the simulation with arbitrarily small values of the aggregate and actions. It turns out that the process fairly quickly converges to the steady state, only after several refinements of each player.

3.2 Calibration

We specify the parameters, meeting all the analytical restrictions. Each player's utility function is in a Cobb-Douglas form and specified as follows:

$$\begin{array}{rl} \text{Skeptic} & u_1 = X_1^{0.4} Y_1^{0.5} Z^{0.1} \\ \text{Locavore} & u_2 = X_2^{0.4} Y_2^{0.4} Z^{0.2} \\ \text{Foodie} & u_3 = X_3^{0.4} Y_3^{0.5} Z^{0.1} \\ \text{Activist} & u_4 = X_4^{0.4} Y_4^{0.3} Z^{0.3} \end{array}$$

Note that Skeptic and Foodie have the same preference; the weight on the composite good is set common; Locavore derives more utility from community development and therefore relatively less from private food-related experience; This is even greater for Activist. For simplicity, the common income $m = 100$ is used for every players. The other parameters are set as follows: $p_c = p_d = 1$, $p_l = 1.3$ and $\alpha_1 = 0.9$, $\alpha_2 = 1.5$, $\alpha_3 = 1.2$, $\alpha_4 = 1.3$ for Skeptic, Locavore, Foodie, and Activist respectively. Finally, the aggregator is specified with $\beta = 0.2$ and $\rho = 0.5$:

$$Z = \left(\sum_j (d_j + 0.2\, l_j)^{0.5} \right)^{\frac{1}{0.5}} .$$

3.3 Results

Below is a benchmark Nash equilibrium result.

Table 2. Benchmark result

	x	c	d	l	u
Skeptic	43.9	54.9	1.2	0.0	53.2
Locavore	46.6	0.0	0.0	41.1	60.5
Foodie	43.4	25.4	0.0	24.0	52.6
Activist	49.2	0.0	7.2	33.5	58.3
		$Z = 97.9$			

Following the theoretical construct, Skeptic remains skeptical about local food and does not buy it, while Locavore is satisfied with having only local food. The overall pattern in optimization is that as a player derives more utility from community development, she has more money to spend on the composite good. As usual in numerical exercise, do not read too much cardinality in the utility values. They are basically not only ordinal for each player but also incomparable across the players. In what follows, we explore the effect of 1% increase in each parameter. If it raises Z, the shock is positive. In the case that a shock reduces Z, 1% decrease in the parameter is regarded as a positive shock.

3.3.1 p_c

While we have left its impact undetermined in the comparative statics, under the specified utility functions, the positive substitution effects dominate the negative income effects. As a result, an increase in the price acts as a positive shock to the aggregate. While there is no cardinal meaning in the value of Z, the shocks in conventional food and α have greater impact on community development than the other shocks, which we discussed in more detail later. Besides the impact to the aggregate, much is happening in individual response. The shock hits Foodie so hard that she substitutes much conventional food with local food, which causes both Locavore and Activist to free-ride and get better off (see the values of x in both players). These responses well correspond to Proposition 8; this positive shock is indirect to Locavore and Activist so they reduce their contribution. (Note: given $\beta = 0.2$, Activist's reduction in d has much greater impact to the total contribution than the increase in l.)

	Benchmark					p_c				
	x	c	d	l	u	x	c	d	l	u
Skeptic	43.9	54.9	1.2	0.0	53.2	43.9	54.4	1.2	0.0	53.1
Locavore	46.6	0.0	0.0	41.1	60.5	46.7	0.0	0.0	41.0	60.9
Foodie	43.4	25.4	0.0	24.0	52.6	43.3	17.8	0.0	29.8	52.4
Activist	49.2	0.0	7.2	33.5	58.3	49.4	0.0	6.8	33.7	59.1
	$Z = 97.9$					$Z = 101.5$				

3.3.2 p_d

The following table shows that a price shock in p_d negatively affects the aggregate, which is in accordance with $D^2_{p_d s}\Pi < 0$ speculation. Here, the negative shock is indirect to Foodie, so the result of her increase in l also confirms the theoretical prediction of Proposition 8. (Although not shown in the rounded number, Locavore also slightly increases her purchase on local food.)

	Benchmark					p_d				
	x	c	d	l	u	x	c	d	l	u
Skeptic	43.9	54.9	1.2	0.0	53.2	43.9	54.9	1.2	0.0	53.2
Locavore	46.6	0.0	0.0	41.1	60.5	46.6	0.0	0.0	41.1	60.4
Foodie	43.4	25.4	0.0	24.0	52.6	43.4	25.2	0.0	24.1	52.6
Activist	49.2	0.0	7.2	33.5	58.3	49.2	0.0	7.0	33.6	58.3
	$Z = 97.9$					$Z = 97.4$				

3.3.3 p_l

Next, as strongly implied in Table 1, a price shock in local food has negative effect on the aggregate. Besides, given that the shock is indirect to Skeptic, her response in donation also follows Proposition 8. While intuitively the other three local food buyers reduce their purchase, Activist increases the total contribution to community. This is understandable given the relatively large weight on Z in her utility function.

	Benchmark					p_l				
	x	c	d	l	u	x	c	d	l	u
Skeptic	43.9	54.9	1.2	0.0	53.2	43.9	54.9	1.3	0.0	53.0
Locavore	46.6	0.0	0.0	41.1	60.5	46.6	0.0	0.0	40.7	59.8
Foodie	43.4	25.4	0.0	24.0	52.6	43.5	30.9	0.0	19.5	52.5
Activist	49.2	0.0	7.2	33.5	58.3	49.0	0.0	7.7	33.0	57.4
	$Z = 97.9$					$Z = 94.7$				

3.3.4 α

To investigate the impact of α, we choose Foodie and raise α_3 by 1% because as seen above Foodie is most responsive to local food associated shocks. In accordance with the speculation in Table 1, the shock has a positive effect on the aggregate. As a result, similar to the shock in p_c, both Locavore and Activist reduce their contribution. In addition, though not shown in the table, Skeptic also reduces d by 0.04, which is again compatible with Proposition 8. Finally, notice that not only does the shock to Foodie benefit the other three players who take advantage of the greater contribution, but also increases her own welfare and ends up with Pareto improvement. This is not necessarily the case in the standard public good model with a linear aggregator, in which an immediate 'beneficiary' of a positive shock could be worse off due to the free-riding opponents (Cornes & Hartley, 2007).

	Benchmark					α_3				
	x	c	d	l	u	x	c	d	l	u
Skeptic	43.9	54.9	1.2	0.0	53.2	43.9	54.9	1.2	0.0	53.4
Locavore	46.6	0.0	0.0	41.1	60.5	46.7	0.0	0.0	41.0	60.9
Foodie	43.4	25.4	0.0	24.0	52.6	43.3	18.0	0.0	29.8	52.7
Activist	49.2	0.0	7.2	33.5	58.3	49.4	0.0	6.8	33.7	59.1
	$Z = 97.9$					$Z = 101.5$				

3.3.5 β

The next table shows the result for β. It also acts as a positive shock, increases the aggregate, and results in Pareto improvement (though the effects are small in numerical value). We interpret β as the societal effectiveness of LFS to community development, and as such it is the cornerstone of the model. So, it must exist but may not be either easy to influence or effective to the end, as seen in u and Z.

	Benchmark					β				
	x	c	d	l	u	x	c	d	l	u
Skeptic	43.9	54.9	1.2	0.0	53.2	43.9	54.9	1.2	0.0	53.2
Locavore	46.6	0.0	0.0	41.1	60.5	46.6	0.0	0.0	41.1	60.5
Foodie	43.4	25.4	0.0	24.0	52.6	43.4	25.2	0.0	24.1	52.6
Activist	49.2	0.0	7.2	33.5	58.3	49.2	0.0	7.1	33.6	58.5
	$Z = 97.9$					$Z = 98.4$				

3.3.6 m

The last table is about the impact of income, in particular Activist's income. As predicted, an income rise is unambiguously a positive shock on the aggregate and beneficial for all the players. However, the magnitude of welfare effects are as small as of the shock in β, and the impact on community development is smaller. The latter is, loosely said, because the rise in β necessitates an increase in local food purchase, whereas an income rise can be spent on anything that maximizes beneficiary's utility.

	Benchmark					m_4				
	x	c	d	l	u	x	c	d	l	u
Skeptic	43.9	54.9	1.2	0.0	53.2	43.9	54.9	1.2	0.0	53.2
Locavore	46.6	0.0	0.0	41.1	60.5	46.6	0.0	0.0	41.1	60.5
Foodie	43.4	25.4	0.0	24.0	52.6	43.4	25.5	0.0	23.9	52.6
Activist	49.2	0.0	7.2	33.5	58.3	49.7	0.0	7.3	33.9	58.8
	$Z = 97.9$					$Z = 98.3$				

3.4 Discussion

Interestingly, the shocks in p_c and α_3 lead to the identical behaviors in the choice of d and l, and therefore the same level of community development ($Z = 101.5$). However, the welfare effects are naturally different because a rise in p_c makes those who buy conventional food worse off. On the other hand, as mentioned, the positive shock in α results in Pareto improvement. Since there exist readily available means to influence both parameters, e.g. tax, advertising campaign and education, the policy implication likely depends on political circumstances. Although taxing or removing subsidies for conventional food is less costly, it is politically more difficult to implement and welfare reducing. In contrast, targeting α is easier to implement with superior results, but its costs need to be financed.

Another result to investigate is the effect of local food price. We have seen that only p_d and p_l turn out to be negative shocks when increased by 1%. Given that the former has a relatively small impact, we explore the latter by decreasing it by 1%.

	Benchmark					p_l				
	x	c	d	l	u	x	c	d	l	u
Skeptic	43.9	54.9	1.2	0.0	53.2	43.9	54.9	1.2	0.0	53.4
Locavore	46.6	0.0	0.0	41.1	60.5	46.7	0.0	0.0	41.4	61.2
Foodie	43.4	25.4	0.0	24.0	52.6	43.3	17.7	0.0	30.3	52.7
Activist	49.2	0.0	7.2	33.5	58.3	49.4	0.0	6.7	34.1	59.5
	$Z = 97.9$					$Z = 102.0$				

The result is Pareto improvement and the value of Z is larger than targeting only α of Foodie. Obviously, the natures of policy instruments for α and p_l are not directly comparable, and their effects depend on how many consumers of each type exist in the community. Having said that, subsidizing local food may be as effective as those used in influencing α.

While the model is constructed based on the non-perfect substitutability of individual contributions, which necessitates $d > 0$ or $l > 0$, it is interesting to see the behavior of complete free-riders, a type with $\{c > 0, d = 0, l = 0\}$. So, we briefly present a benchmark Nash equilibrium result, which can be obtained by allowing the perfect substitutability (i.e. $\rho = 1$). Bear in mind, however, that since all the functions are continuous in the domain of each parameter and $0 < \rho \leq 1$, there is no meaningful distinction between $\rho = 1$ and the value arbitrarily close to 1. In the end, we can still conceptualize that day-to-day kindness and politeness make slight contributions to the community ($d > 0$), whereby it is a good approximation that no one who lives in society completely free-rides. Thus, precluding complete free-riders does not strongly restrict the model but rather provides a useful mathematical framework.

Table 3. Complete free-riding

	x	c	d	l	u
Skeptic	44.4	55.6	0.0	0.0	48.2
Locavore	47.0	0.0	0.0	40.7	48.5
Foodie	44.4	55.6	0.0	0.0	48.2
Activist	43.3	0.0	18.4	29.5	38.3

Clearly, despite the difference in α, Skeptic and Foodie identically behave and completely free-ride on the other two's contributions, especially Activist's direct donation, thereby costing her much utility. With this result, comparative statics do not provide any insight but only slightly modify each number in the same arrangement. Although the result still depends on calibration, it may indicate that the non-perfect substitutability has some bearing on LFS, and the value of ρ representing some social structure is well away from 1 so that Foodie buys local food and Skeptic makes a contribution to the community she lives in.

4 Concluding remarks

We have built a theoretical model of local food systems by formalizing the observation that it plays a dual role in supply chain and social fabric. In order to capture some of the most salient features of LFS, we have adopted the impure public good hypothesis and the non-linear aggregator of contributions to the public good. Given the fact that various social benefits are involved in LFS, it is considered reasonable to liken local food to an impure public good. Focusing on a particular type of social benefit, community development, we highlight the participatory nature of LFS and justify the use of the non-linear aggregator. These two distinguishing assumptions are the corner stones of the model, giving rise to four types of consumers and permitting a large degree of analytical tractability. The introduction of four representative consumers not only helps ease the quantitative analysis but also provide concrete ideas behind the equations and a framework for empirical work. Another attempt for empirical applicability is the demonstration of the model through computer simulation, which confirms all the analytical predictions.

There are several points that may direct the future research. One of the important empirical questions is what factors and how much they affect α. As discussed above, possibly quite a few factors affect individual valuation of local food, and most of the determinants can be influenced by both policy and incidental shocks. So, as opposed to the existing empirical works, we may estimate local food demand not directly with those factors but indirectly through α.

We have adopted a relatively simple functional form for the aggregator, just enough for the essential incentive structure in the consumer's problem. It has only the basic restriction $0 < \rho < 1$ and the direction of change in ρ as a community becomes more participatory. If we are to add some degree of cardinality to Z and/or prepare for empirical work, it is necessary to give it more structure. Currently, if ρ is close to 0 and n reflects some realistic size of community, Z behaves poorly; it easily explodes to positive infinity or becomes arbitrarily small depending on whether input values are greater or less than 1 (think about the product obtained by taking $\rho \to 0$). To mitigate the misbehavior, first, we can make the magnitude of Z consistent across $0 < \rho < 1$ by replacing the CES function with the generalized mean:

$$Z = \left(n^{-1} \sum_j (d_j + \beta l_j)^\rho \right)^{\frac{1}{\rho}},$$

which becomes the arithmetic mean if $\rho = 1$ and the geometric mean if $\rho \to 0$. However, mathematically the arithmetic mean always return the largest Z regardless of the data set, so there is still no useful connection between ρ and Z. One approach to establishing the interaction between two variables is to add a penalty coefficient as follows:

$$Z = r(1-\rho) \left(n^{-1} \sum_j (d_j + \beta l_j)^\rho \right)^{\frac{1}{\rho}},$$

where $r > 0$ is an exogenous scaling factor to get a desirable range of Z. In this way, for a given set of non-uniform contributions, neither $\rho = 1$ nor 0 is a maximizer. Since the coefficient and the sum parts are oppositely affected by ρ, there exists the 'socially optimal' ρ somewhere in the open unit interval, depending on the dispersion of the data. Since ρ reflects how participatory the community is, it can be interpreted as a policy variable that local governments indirectly influences through various policy instruments such as changing the number of social events, open public spaces and community gardens.

Community development is often used ambiguously, and we too have treated it merely as a public 'good', from which community members derive utility. However, it will be necessary to reduce the ambiguity if we proceed to analyze in more detail what specific benefits are involved in this public good and how they are related to private goods. For example, while institutional purchase, e.g. school, hospitals, and jails, is an important part of the picture of LFS (Gregoire, Arendt, & Strohbehn, 2005; Joshi & Azuma, 2009; Matts & Colasanti, 2013; Grace, 2010; Sachs & Feenstra, 2008; Colasanti, Matts, & Hamm, 2012), we may learn much from modeling the connection between institutions' benefits from local food and the other participants in the system. That is, we may influence the preference of Skeptic, Foodie, Locavore and convert them to Activist by informing shoppers in a farmer's market of how LFS help those institutions and therefore their community as a whole.

Finally, looking to the production side of the systems, the decentralized community development may have implication for producers' decisions. We think that the level of community development must have some or strong connection to social capital as a capital in the economist's sense. That is, while a vendor in a farmer's market as a consumer of community development enjoys the interaction with customers, she as a producer may also benefit from the increased social capital, thereby increasing her production. Local food systems and social capital are still untapped areas by economists, but there seem to exist a promising link and rewarding potentials in practical applications.

Appendix

Code for the simulation

Tested in *MATLAB* R2015a.

```
function s004()
% Economic parameters
n=4;
m=ones(n,1)*100;
pc=1; pd=1; pl=1.3;
b=0.2;
a=[0.9 1.5 1.2 1.3]';

% Cobb-Douglas
k3=[0.1 0.2 0.1 0.3];
k2=[0.5 0.4 0.5 0.3];
k1=1-k2-k3;

% Others
r=0.5; % rho
iter=n; % #iteration = #players works
    well

% Store original values
m0=m; pc0=pc; pd0=pd; pl0=pl; b0=b; a0=a;

%%%%%%%%%%%%%%%%%%%%
h('Benchmark');
%%%%%%%%%%%%%%%%%%%%
% Initialization
u=zeros(n,1);
X=zeros(n,1);
Y=zeros(n,1);
cdl=zeros(n,3);

find_NE
x=m-cdl*[pc pd pl]';
Z1=Z
result1=[m x cdl -u]

%%%%%%%%%%%%%%%%%%%%
h('Price of conventional');
%%%%%%%%%%%%%%%%%%%%
pc=pc0*1.01; % 1% up
initialize
find_NE
x=m-cdl*[pc pd pl]';
Z2=Z
result2=[m x cdl -u]
pc=pc0;
%%%%%%%%%%%%%%%%%%%%
h('Price of donation');
%%%%%%%%%%%%%%%%%%%%
pd=pd0*1.01; % 1% up
initialize
find_NE
x=m-cdl*[pc pd pl]';
Z3=Z
result3=[m x cdl -u]
pd=pd0;

%%%%%%%%%%%%%%%%%%%%
h('Price of local');
%%%%%%%%%%%%%%%%%%%%
% pl=pl0*1.01; % 1% up
pl=pl0*0.99; % 1% down
initialize
find_NE
x=m-cdl*[pc pd pl]';
Z4=Z
result4=[m x cdl -u]
pl=pl0;

%%%%%%%%%%%%%%%%%%%%
h('Alpha');
%%%%%%%%%%%%%%%%%%%%
a(3)=a0(3)*1.01; % 1% up
initialize
find_NE
x=m-cdl*[pc pd pl]';
Z5=Z
result5=[m x cdl -u]
a(3)=a0(3);

%%%%%%%%%%%%%%%%%%%%
h('Beta');
%%%%%%%%%%%%%%%%%%%%
b=b0*1.01; % 1% up
initialize
find_NE
x=m-cdl*[pc pd pl]';
Z6=Z
result6=[m x cdl -u]
b=b0;

%%%%%%%%%%%%%%%%%%%%
```

```
h('Income');
%%%%%%%%%%%%%%%%%%%%
m(4)=m0(4)*1.01; % 1% up
initialize
find_NE
x=m-cdl*[pc pd pl]';
Z7=Z
result7=[m x cdl -u]
m(4)=m0(4);

%%%%%%%%%%%%%%%%%%%%
% Functions
%%%%%%%%%%%%%%%%%%%%
function find_NE() % Locate the NE
  for cnt=1:iter
    for i=1:n
      lb=[0 0 0];
      ub=[m(i)/pc m(i)/pd m(i)/pl];
      s0=lb; % not sensitive
      options=optimset('Algorithm','
          interior-point','Display','off
          ');
      [cdl(i,:),u(i)]=fmincon(@umax,s0
          ,[],[],[],[],lb,ub,[],options);
    end
  end

  function uu=umax(var) % Individual
      optimization
    cdl(i,:)=var;
    c=var(1);
    d=var(2);
    l=var(3);
    X=m(i)-pc*c-pd*d-pl*l;
    Y=c+a(i)*l;
    Z=aggregator(cdl(:,2)+b*cdl(:,3));
    uu=-((X^k1(i))*(Y^k2(i))*(Z^k3(i)));
  end
end

function ZZ=aggregator(s) % s: n-vector
  ZZ=sum(s.^r)^(1/r);
end

function initialize()
  u=zeros(n,1);
  X=zeros(n,1);
  Y=zeros(n,1);
  cdl=zeros(n,3);
end

function h(hmsg) % Generate a heading
  disp(sprintf(strcat('\n\n','***',hmsg
      ,'***')) )
end
end
```

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