Machine learning for optimizing complex site-specific management

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November 2, 2019

Abstract

Despite the promise of precision agriculture for increasing the productivity by implementing site-specific management, farmers remain skeptical and its utilization rate is lower than expected. A major cause is a lack of concrete approaches to higher profitability. When involving many variables in both controlled management and monitored environment, optimal site-specific management for such high-dimensional cropping systems is considerably more complex than the traditional low-dimensional cases widely studied in the existing literature, calling for a paradigm shift in optimization of site-specific management. We propose an algorithmic approach that enables farmers to efficiently learn their own site-specific management through on-farm experiments. We test its performance in two simulated scenarios—one of medium complexity with 150 management variables and one of high complexity with 864 management variables. Results show that, relative to uniform management, site-specific management learned from 5-year experiments generates \$43/ha higher profits with 25 kg/ha less nitrogen fertilizer in the first scenario and \$40/ha higher profits with 55 kg/ha less nitrogen fertilizer in the second scenario. Thus, complex site-specific management can be learned very efficiently and be more profitable and environmentally sustainable than uniform management.

Keywords: Machine learning, Bayesian optimization, APSIM, precision agriculture, site-specific management, on-farm experiments

1 Introduction

Modern agriculture is faced with some of the most challenging problems in the 21st century such as food security, farm profitability and environmental sustainability, all of which require increasing agricultural productivity. To increase the productivity, it is crucial to exploit advanced farming (also referred as *precision agriculture*) technologies such as yield monitors, remote sensing, and variable rate application, from which site-specific management (SSM) emerges as an effective management strategy [1–4]. This is because SSM can optimize a production system at the subfield level, which amounts to finer-scale optimization than at the field level. Therefore, for both individual profitability and collective societal benefits, SSM has been advocated to farmers over the past two decades [5].

Despite the potential benefits for farmers, the adoption of SSM has been slower than expected [6, 7], which is attributed to a lack of relative advantage over the current management strategies

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[8], particularly a lack of greater profitability [9, 10]. In principle, those advantages of SSM can be realized by exploiting advanced technologies (e.g., adjusting fertilizer application rates across the field according to the varying soil conditions inferred from satellite imagery). In reality, however, these technologies are quite sophisticated and difficult to exploit. Indeed, many farmers express their concerns about the complexity of these technologies [11] and have yet to be convinced of the value of SSM [12–15]. This lack of actionable procedures or decision support systems for SSM is noted as a serious problem in the literature [16].

The vast majority of SSM research investigates only a handful of management types in particular research environments. Commonly explored types include nitrogen fertilizer [17–24], irrigation water [25, 26], and sowing density [27, 28]. Besides the fact that other types of management (e.g., tillage, spraying, and harvest) are also important, in practice, what really makes farm management difficult is the total number of decisions farmers have to make. Note that the total number of decisions increases exponentially as types of management increases, because each type of management typically involves decisions on its amount, frequency, and timing. Since crop yield and hence the overall effectiveness of SSM are determined by the totality of these management decisions and environmental factors [29, 30], independent studies of few management types involving a small number of decisions provide only partial knowledge and can be misleading as complex systems typically involve significant nonlinearity [31, 32].

Besides the small number of management decisions investigated in the existing research, farmers are also concerned about the generalizability of results obtained under the particular research environments, because such results may not be representative of their unique environments [33]. Each farm is unique and operates with different resources (e.g., machinery) in different environments (e.g., soil and weather). Consequently, results from studying "representative" cases are useful for only farmers who face similar managerial and environmental conditions [34, 35]. This problem is exacerbated in modern agriculture where the high dimensionality created by advanced sensor technologies makes it even more difficult for selected cases to be representative, the problem called "the curse of dimensionality" in mathematics [36]. To have a sense of how particular a typical study setting is, refer to Lo et al. [24] who studied nitrogen fertilizer SSM at the university extension research site in Nebraska that has been "under annual summer corn or soybean production without any tillage and any stover removal" with irrigation water applied through "a center pivot with sprayhead sprinklers positioned every other interrow at a height of 0.6 m above ground" using "GrowSmart Precision Variable Rate Irrigation system". The problem is not how particular existing studies are, as they serve different research purposes. Rather, the problem is a lack of studies that generalize the effects of environmental factors and jointly examine many management variables for the purpose of optimizing SSM in practice.

To address these issues (few management variables only in particular environments) and offer an actionable procedure for SSM, we propose an algorithmic approach that enables each farmer to efficiently learn their own SSM through on-farm experiments implemented via existing advanced farming technologies. We emphasize the significance of on-farm experimentation [33, 37–39] to deal with both problems. First, on-farm experimentation allows farmers to adaptively design experiments and efficiently navigate a high-dimensional variable space [40]. Second, on-farm experimentation inherently allows each farmer to collect data from their own environment and circumvents the representativeness issue. Particularly for SSM, use of field-scale experimentation is important to deal with spatial, infield variability [35, 41–43]. Notice that our approach is different from the most common type of machine learning (i.e., supervised learning), which constructs an empirical model that assumes specific variables and estimates their associated parameters using a large observational dataset. The constructed model is supposed to be representative of users' production systems and therefore capable of indicating their optimal choices. However, owing to the very nature of observa-

tional data, the dataset likely contains insufficient variation in the high-dimensional management space because observed management choices mostly follow the standard recommendations from the existing low-dimensional existing studies. In other words, purposeful experimentation is crucial to include unconventional management choices and discover unexpected optimal choices. Consequently, it is highly unlikely for such supervised learning models to be able to indicate optimal choices for each farmer. Our approach is, instead, a machine learning algorithm that allows each farmer to construct their own unique model. It is versatile enough to be used for learning optimal SSM through on-farm experiments in a wide range of farming scenarios. The algorithm is based on Bayesian optimization [40], and capable of handling an arbitrary number of management and environmental variables and adapt to unexpected interaction effects. Moreover, if the farmer has historical data, large or small, it can be incorporated as prior knowledge.

In the remainder of the paper, we first mathematically formulate the farmer's problem as profit maximization and then, we describe our machine learning algorithm to solve it. We test the algorithm's performance and versatility in two simulated environments, with either medium- or high-complexity. To highlight the generality of the approach, we report results on a per-hectare basis so that they can be easily scaled. Results suggest that complex SSM can be learned very efficiently through on-farm experiments within a few years, and it can be more profitable and more environmentally sustainable than uniform management.

2 Materials and methods

2.1 Farmer's problem

Imagine a farmer who has access to precision agriculture equipment for SSM but currently does not implement it due to a lack of knowledge, a typical story about SSM [16]. Suppose that the farmer is now interested in learning SSM through on-farm experiments. To conduct field-scale experiments, the farmer divides an entire field into a grid of sites of equal size according to the capacity of the variable rate technologies and yield monitor. That is, in each site, the farmer can uniquely identify and collect a pair of data $(\boldsymbol{x}, y)$—applying management $\boldsymbol{x}$ and observing the corresponding yield y. Let M denote the total number of sites. Site $s \in \{1, 2, \dots, M\}$ is characterized by a state variable $\boldsymbol{z}_s$, to which management x_s is applied. Then, a site-specific profit function is

$$\pi(\boldsymbol{x}_s; \boldsymbol{z}_s) = py(\boldsymbol{x}_s; \boldsymbol{z}_s) - \boldsymbol{c} \cdot \boldsymbol{x}_s,$$

where $y(\boldsymbol{x}_s; \boldsymbol{z}_s)$ is a site-specific yield function, p is an output price, and $\boldsymbol{c}$ is a vector of input prices. Notice that this is technically a partial profit as we subtract only the costs for modeled management $\boldsymbol{x}_s$. Nonetheless, it is immaterial because our analysis is based on the difference between the profits of the SSM and the uniform management.

For conventional low-dimensional yield functions, it is common to use simple concave functions such as quadratic [44–46], negative exponential [47, 48], and piecewise linear [49, 50]. These simple functions may serve well for answering isolated questions about the optimality of a single management variable under homogeneous conditions. However, in high-dimensional cropping systems, the yield function is a fundamental source of the challenge because its uncertainty increases with the number of variables entering the function $y(\cdot\,;\cdot)$.

Note that each site need not be recognized as distinct or, equivalently, each $\boldsymbol{z}_s$ need not be distinct. A simple consequence of this assumption is that adjacent sites $\{s_1, s_2, \dots\}$ may have the same value $\boldsymbol{z}_{s_1} = \boldsymbol{z}_{s_2} = \dots$ and form a homogeneous "zone", which receives the same management. This is the framework commonly used to study SSM, particularly zone delineation, in the literature [26, 51–57]. Our formulation is more general and contains the existing one as a special case.

Having each site-specific profit defined, a field-level profit is simply the sum of the site-specific profits:

$$\sum_{s=1}^{M} \pi(\boldsymbol{x}_s; \boldsymbol{z}_s) = \sum_{s=1}^{M} py(\boldsymbol{x}_s; \boldsymbol{z}_s) - \boldsymbol{c} \cdot \boldsymbol{x}_s.$$

The farmer's objective is to learn optimal SSM $\boldsymbol{x}_s^*$ for all $s \in \{1, 2, \ldots, M\}$:

$$(\boldsymbol{x}_1^*, \ldots, \boldsymbol{x}_M^*) = \underset{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_M}{\operatorname{argmax}} \sum_{s=1}^{M} \pi(\boldsymbol{x}_s; \boldsymbol{z}_s).$$

In contrast, under uniform management, a single management $\boldsymbol{x}$ is applied to every site s. Therefore, a field-level profit function is:

$$\sum_{s=1}^{M} \pi(\boldsymbol{x}; \boldsymbol{z}_s) = \sum_{s=1}^{M} py(\boldsymbol{x}; \boldsymbol{z}_s) - \boldsymbol{c} \cdot \boldsymbol{x},$$

and the optimal uniform management $\bar{\boldsymbol{x}}^*$ is:

$$\bar{\boldsymbol{x}}^* = \underset{\boldsymbol{x}}{\operatorname{argmax}} \sum_{s=1}^{M} \pi(\boldsymbol{x}; \boldsymbol{z}_s).$$

2.2 Solution algorithm

We construct an algorithm based on Bayesian optimization (BO) [58, 59], a class of numerical optimization techniques to find the global optimum of an unknown function. As with many other numerical optimization techniques, BO navigates the search space by examining one point at a time until it locates an acceptable point and halts. Since BO tries to optimize an unknown function, it needs a surrogate model to guide its search. For this purpose, a Gaussian process (GP) statistical model is a standard choice in the literature.

BO has two features that makes it suitable for agricultural experiments [40]. First, GP as a nonparametric Bayesian model is sufficiently flexible so that it can adapt to cases in which the objective function takes a complex shape. In high-dimensional SSM, this complexity will likely happen due to strong interactions among the many variables involved. Second, BO is in general known for its sample efficiency, which means that BO can locate a good enough point with relatively a small number of examinations. Since agricultural experiments take time before obtaining results, typically a year, sample efficiency is a desirable feature.

We assume that a single observation is collected from each site annually, giving M samples in each year. However, depending on the size of a site, a farmer will likely collect more than one observation from each site. If this is the case, the average value of all observations from the site may be treated as the sample for that site. While the basic BO sequentially processes one sample at a time, we modify it using the "batch expected improvement" acquisition function proposed by Saikai et al. [40] in order to process M samples at a time. Each year, the algorithm proceeds as follows:

1. Prescribe $\boldsymbol{x}_s$ for each site s by maximizing the acquisition function $\alpha(\boldsymbol{x}; \boldsymbol{z}_s)$;
2. Observe a yield $y(\boldsymbol{x}_s; \boldsymbol{z}_s)$ for each s;
3. Compute the corresponding $\pi(\boldsymbol{x}_s; \boldsymbol{z}_s)$ for each s
4. Update the GP with $\{(\boldsymbol{x}_s, \boldsymbol{z}_s, \pi_s)\}_{s=1}^{M}$ and the samples from the preceding years.

After completing the planned number of years of experiments, a candidate for $\boldsymbol{x}_s^*$ for each s can be obtained by maximizing the mean function of the learned GP with fixed $\boldsymbol{z}_s$. Below is the complete algorithm.

Algorithm 1 Batch Bayesian optimization for site-specific management

1: **require:** T, M, $\mathcal{S}$, GP, α
2: **for** $t \in \{1, 2, \ldots, T\}$ **do**
3: $\quad \mathcal{I} \leftarrow \{\ \}$
4: $\quad \widehat{GP} \leftarrow GP$
5: $\quad$ **for** $s \in \{1, 2, \ldots, M\}$ **do**
6: $\quad\quad \boldsymbol{x}_s \leftarrow \mathrm{argmax}_{\boldsymbol{x}}\, \alpha(\boldsymbol{x}; \boldsymbol{z}_s)$
7: $\quad\quad \mathcal{I} \leftarrow \mathcal{I} \cup \{(\boldsymbol{x}_s, \boldsymbol{z}_s, \underline{\pi})\}$ where $\underline{\pi} = \min\{\mathcal{S}_\pi\}$
8: $\quad\quad$ Update $\widehat{GP}$ with $\mathcal{S} \cup \mathcal{I}$
9: $\quad$ **for** $s \in \{1, 2, \ldots, M\}$ **do**
10: $\quad\quad y_s \leftarrow Oracle(\boldsymbol{x}_s; \boldsymbol{z}_s)$
11: $\quad\quad \pi_s \leftarrow p y_s - \boldsymbol{c} \cdot \boldsymbol{x}_s$
12: $\quad\quad \mathcal{S} \leftarrow \mathcal{S} \cup \{(\boldsymbol{x}_s, \boldsymbol{z}_s, \pi_s)\}$
13: $\quad$ Update GP with $\mathcal{S}$
14: **return** $M \times T$ number of samples

In terms of notation, T is the total number of years used for experimentation, $\mathcal{S}$ is a set of samples, $\min\{\mathcal{S}_\pi\}$ implies the minimum realized profit, $\alpha(\,\cdot\,; \boldsymbol{z}_s)$ is an acquisition function for site s defined based on the Gaussian process GP, and $Oracle(\boldsymbol{x}; \boldsymbol{z})$ returns an observed yield when $\boldsymbol{x}$ is applied to a site characterized by $\boldsymbol{z}$. Notice that in Line 2-8 an interim $\widehat{GP}$ is updated with a hypothetical observation ($\underline{\pi}$) so that we can collect a batch of M samples while using the sequential sampling algorithm. As a small detail, in Line 5, site s is chosen in a random order to avoid a systematic bias arising from how sites are numbered. Another detail is that, when updating GP, we fit the hyperparameters of the GP only to observed data (Line 13) and not to hypothetical data (Line 8).

2.3 Simulation experiments

To construct simulation environments, we make use of the Agricultural Production Systems sIMulator (APSIM), an advanced simulator of cropping systems [60] widely used for various purposes, including generating synthetic datasets [61–66]. In each environment, we run the algorithm to learn optimal SSM over T years and compare the profit resulting from implementing the learned SSM against the benchmark profit resulting from uniform management. This analysis assumes that uniform management follows university extension recommendations.

Depending on specific scenarios, we can customize the algorithm in many ways. An interesting modification is to incorporate observational data collected prior to beginning on-farm experimentation, the case for many farmers. When using uniform management as a benchmark for comparison, a natural dataset incorporated is the data from implementing uniform management, as it represents the existing knowledge. Here, we initialize the GP embedded in the algorithm as follows: let $\{(\bar{\boldsymbol{x}}, \boldsymbol{z}_s, \bar{\pi}_s)\}_{s=1}^M$ be a set of the uniform management ($\bar{\boldsymbol{x}}$), site characteristics ($\boldsymbol{z}_s$), and the corresponding profits ($\bar{\pi}_s$) and then, we fit the GP to these M data points before the algorithm starts a learning process. Note that the use of the uniform management for both benchmark and prior knowledge is a useful case for illustration. In practical applications, farmers may use any

benchmark management of interest (either uniform or not) and any existing dataset. Finally, since the algorithm itself involves some randomness, we conduct Monte Carlo simulations and present averaged results over the Monte Carlo samples.

Though the algorithm's applicability is by no means restricted to the scenarios described in this section, we rely on APSIM simulator and construct illustrative test beds within its capability. We simulate a maize production system in Ames, Iowa with the weather data for 2013, which is the most recent year available in APSIM. In terms of management variables, we follow Saikai et al. [40] and identify six variables $\boldsymbol{x} = (x^1, \ldots, x^6)$ in the APSIM maize module:

- x^1 : sowing density (seeds/m^2)
- x^2 : sowing depth (mm)
- x^3 : row spacing (m)
- x^4 : N fertilizer amount before sowing (kg/ha)
- x^5 : N fertilizer amount at sowing (kg/ha)
- x^6 : N fertilizer amount for top dressing (kg/ha)

Based on Iowa State University extension recommendations, we specify uniform management ($\bar{\boldsymbol{x}}$) as:

$$(\bar{x}^1, \bar{x}^2, \bar{x}^3, \bar{x}^4, \bar{x}^5, \bar{x}^6) = (8, 50, 0.76, 67, 67, 67).$$

$\bar{x}^1 = 8$, $\bar{x}^2 = 50$, and $\bar{x}^3 = 0.76$ follow from Farnham [67] and Elmore [68]. The recommended total nitrogen amount is identified by using the Corn Nitrogen Rate Calculator [69], which gives $\bar{x}^4 + \bar{x}^5 + \bar{x}^6 = 201$, which we evenly split into $\bar{x}^4 = \bar{x}^5 = \bar{x}^6 = 67$. Finally, for calculating profits, the output price is $p = \$0.177$/kg [70], and input costs are $c^1 = \$3.64$/1000 seeds and $c^4 = c^5 = c^6 = \$1.29$/kg [71]. We assume no cost for sowing depth and row spacing, which implies the cost vector $\boldsymbol{c} = (c^1, 0, 0, c^4, c^5, c^6)$.

Based on these same sources, when the algorithm searches for the optimal management, the search space is restricted to the following:

- $x^1 \in [6.0, 10.0]$ (seeds/m^2)
- $x^2 \in [25, 150]$ (mm)
- $x^3 \in [0.4, 1.0]$ (m)
- $x^4, x^5, x^6 \in [0, 200]$ (kg/ha)

Finally, data points resulting from the benchmark uniform management $\{(\bar{\boldsymbol{x}}, \boldsymbol{z}_s, \bar{\pi}_s)\}_{s=1}^M$ are the only existing dataset incorporated prior to beginning on-farm experimentation. Since initial uniform management provides no variation in $\boldsymbol{x}$, to build up smoothly, in the first year, the algorithm randomly chooses $\boldsymbol{x}$ for each s from the range defined by $\pm 50\%$ of the uniform management.

2.3.1 Scenario A (medium complexity)

This scenario assumes that a square field is divided into a grid of 25 sites ($M = 25$). All the sites are distinct, each characterized by a state vector $\boldsymbol{z}_s = (z_s^1, z_s^2)$ where z_s^1 is plant available water capacity (mm) and z_s^2 is organic carbon (%). We set $z_s^1 \in \{231, 259, 288, 317, 346\}$ and $z_s^2 \in \{2.56, 2.88, 3.2, 3.52, 3.84\}$ ($\pm 10\%$ or 20% from the mid-point values of the default values in the APSIM soil module used).

231, 2.56	259, 2.56	288, 2.56	317, 2.56	346, 2.56
231, 2.88	259, 2.88	288, 2.88	317, 2.88	346, 2.88
231, 3.2	259, 3.2	288, 3.2	317, 3.2	346, 3.2
231, 3.52	259, 3.52	288, 3.52	317, 3.52	346, 3.52
231, 3.84	259, 3.84	288, 3.84	317, 3.84	346, 3.84

Figure 1: Simulated maize field divided into a grid of 25 distinct sites. The first number in each grid indicates plant available water capacity (mm) and the second indicates organic carbon (%) at that site.

2.3.2 Scenario B (high complexity)

This scenario imagines that the farmer possesses more precise equipment that can operate at a more granular scale, and so divides a field into more granular sites: $16 \times 9 = 144$ sites. We also assume that the farmer has conducted more exhaustive soil tests, measuring four state variables (z^1, z^2, z^3, z^4) in each site:

- z^1: plant available water capacity (mm)
- z^2: organic carbon (%)
- z^3: initial nitrate-N (kg/ha)
- z^4: initial ammonium-N (kg/ha)

The addition of z^3 and z^4 is because of their significance for nitrogen management [72] and availability in APSIM. Notice that nitrogen is also supplied by soil organic matter (z^1) through N-mineralization, creating stronger interactions among management and environmental variables [73], so that SSM in scenario B is more complex than in scenario A. We generate a state vector for each site in a random but spatially correlated fashion, i.e., a random walk (see Appendices for details). Below are summary statistics of the generated state vectors for the 144 sites.

- z^1: mean = 296, std = 32, min = 199, max = 365
- z^2: mean = 3.19, std = 0.31, min = 2.59, max = 3.90
- z^3: mean = 9.1, std = 0.98, min = 7.0, max = 11.5
- z^4: mean = 10.6, std = 1.6, min = 7.7, max = 14.1

Instead of reporting four numbers at each site, we illustrate the in-field variability using a yield map arising from applying the uniform management to the generated field. Since each site receives the same management, the variability in yield indicates the variability in the underlying growing conditions.

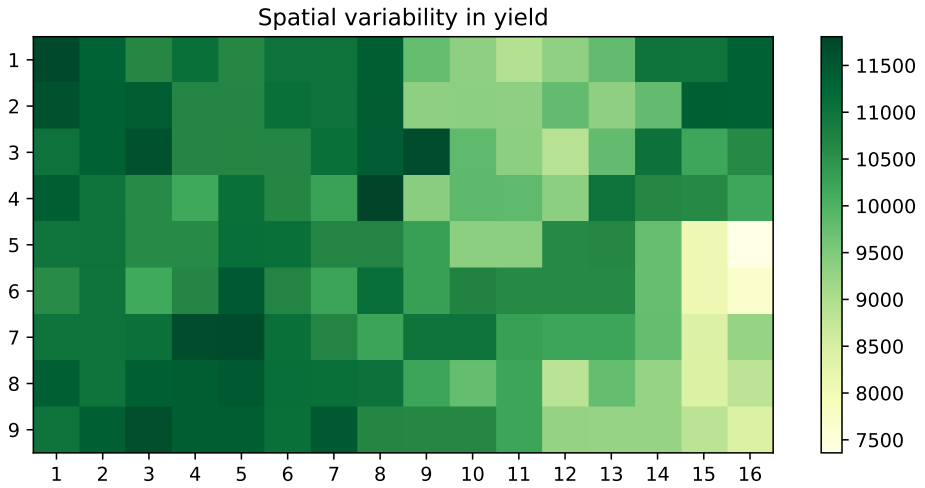

Figure 2: Yield map (kg/ha) resulting from the uniform management in scenario B. Axis ticks indicate site coordinates.

3 Results

3.1 Scenario A (medium complexity)

Table 1 and Figure 3 report field-level profits ($/ha) from implementing the SSM learned after conducting experiments for T years. Specifically, the value for each $T \in \{1, 2, \ldots, 10\}$ is the profit if the farmer terminates the experiments after T years and implements the learned SSM without further improvement.

Years (T)	1	2	3	4	5	6	7	8	9	10
Learned	1103	1237	1266	1274	1277	1280	1283	1284	1285	1285
Uniform	1234	1234	1234	1234	1234	1234	1234	1234	1234	1234
Difference	-131	3	32	40	43	46	49	50	51	51
										($/ha)

Table 1: Field-level profits ($/ha) from implementing the learned SSM and uniform management in scenario A.

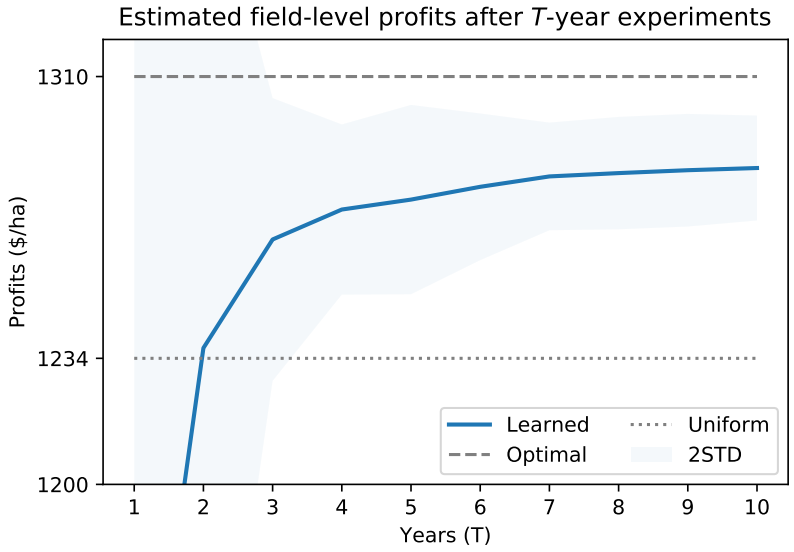

Figure 3: Learning curve of the algorithm with profits plotted against years of experiments in scenario A. The shaded areas indicate two standard deviations around each mean field-level profit over the 100 Monte Carlo samples.

The shaded areas indicate two standard deviations around each mean field-level profit over the 100 Monte Carlo samples. The dashed line indicates the profits from implementing the optimal SSM, while the dotted line indicates the profits from uniform management.

Since field-level profit is the sum of the site-specific profits, we next examine profit at each site. In Figure 4, profits when using the learned SSM uses $T = 5$, as the learning mostly levels off and the deviation from the mean prediction becomes small after four or five years. The following heatmaps illustrate the learned SSM and uniform management.

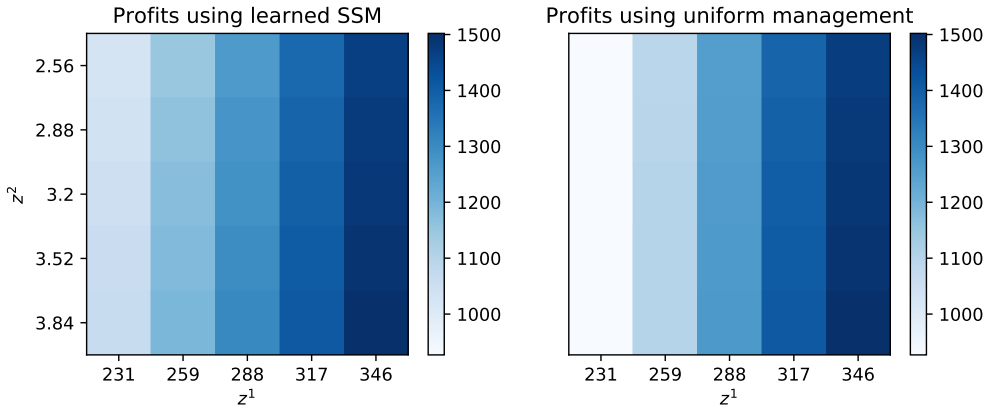

Figure 4: Site-specific profits for the learned SSM after 5 years and uniform management in scenario A

As indicated in the panel for uniform management, plant available water capacity (z^1) has a much stronger influence on yield and profit than organic carbon (z^2). However, as both variables increase, the site becomes more fertile (though hard to see for organic carbon z^2). Since panels for both management systems look quite similar, to highlight their difference, Figure 5 illustrates the difference at each site.

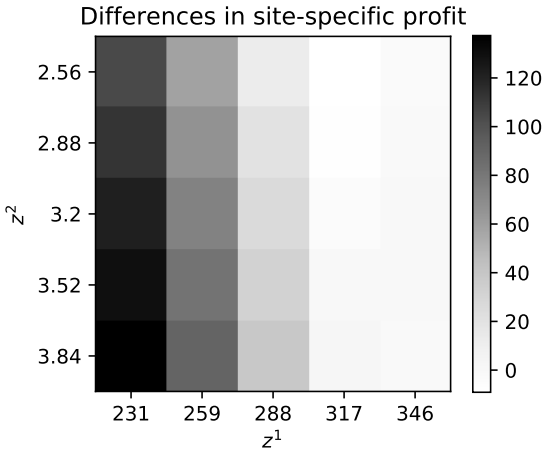

Figure 5: Differences in site-specific profit ($/ha) for SSM versus uniform management in scenario A. The maximum difference is $138/ha at site (231,3.84) and the minimum difference is $-9.4/ha at site (317,2.56).

Finally, Figure 6 reports the learned SSM $(x^1, \ldots, x^6)$ after 5 years.

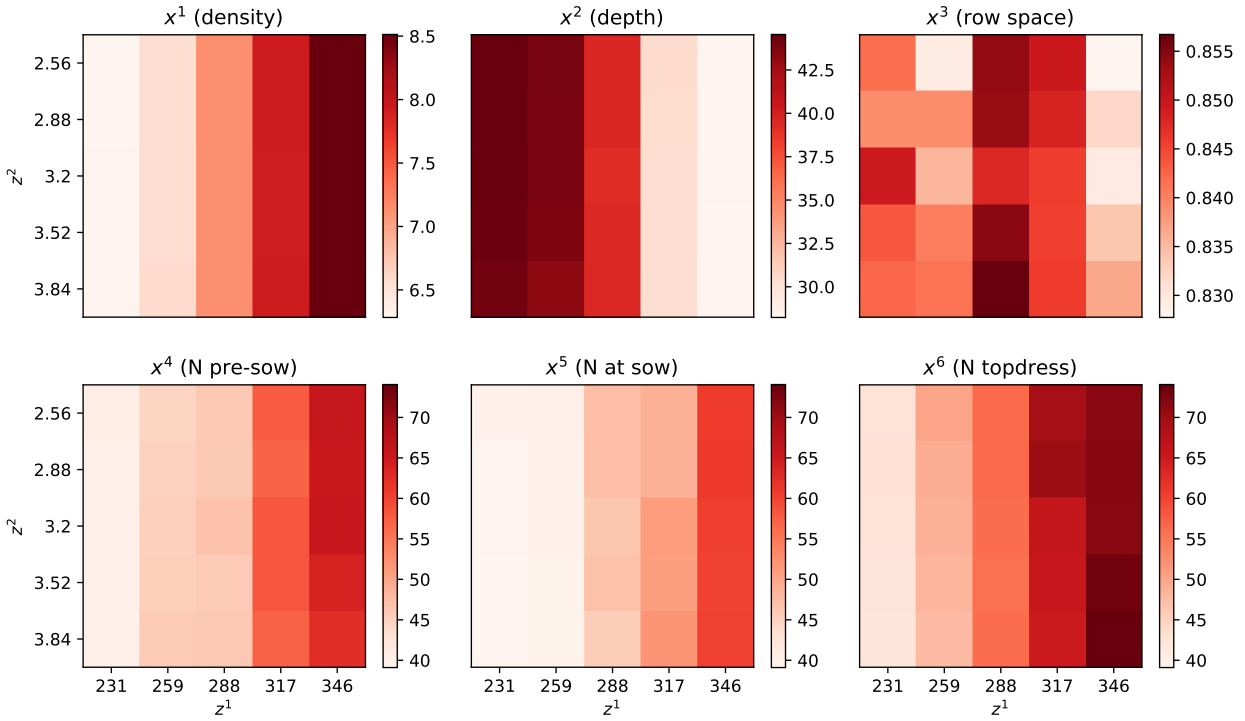

Figure 6: Learned SSM after 5 years in scenario A. The average sowing density is 7.3 seeds/m^2, and the average amount of total nitrogen is 156 kg/ha.

The average sowing density is

$$\frac{1}{25}\sum_{s=1}^{25} x_s^1 = 7.3 \text{ seeds/m}^2,$$

and the average amount of total nitrogen fertilizer is

$$\frac{1}{25}\sum_{s=1}^{25}\sum_{i=4}^{6} x_s^i = 156 \text{ kg/ha}.$$

As a result, \$43/ha higher profit is achieved by using 0.7 fewer seeds/m^2 and 45 kg/ha less nitrogen than uniform management. To further emphasize the generality and robustness of our algorithmic approach, results from other years than 2013 are also provided in Appendices.

3.2 Scenario B (high complexity)

Table 2 and Figure 7 report field-level profits (\$/ha) from implementing the SSM learned after conducting experiments for T years. Again, the value for each $T \in \{1, 2, \ldots, 10\}$ is the profit if the farmer terminates the experiments after T years and implements the learned SSM without further improvement.

Years (T)	1	2	3	4	5	6	7	8	9	10
Learned	1319	1324	1331	1333	1335	1335	1336	1337	1338	1339
Uniform	1295	1295	1295	1295	1295	1295	1295	1295	1295	1295
Difference	24	29	36	38	40	40	41	42	43	44
										(\$/ha)

Table 2: Field-level profits (\$/ha) from implementing the learned SSM and uniform management in scenario B.

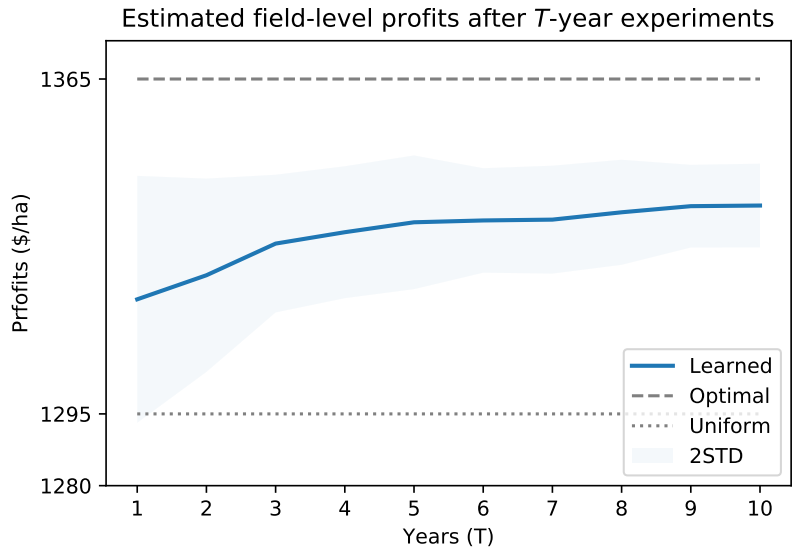

Figure 7: Learning curve of the algorithm with profits plotted against years of experiments in scenario B. The shaded areas indicate two standard deviations around each mean field-level profit over the 100 Monte Carlo samples.

The shaded areas indicate two standard deviations around each mean field-level profit over the Monte Carlo samples. The dashed line indicates the profits from the optimal SSM, while the dotted line indicates the profits from the uniform management.

The shaded areas indicate two standard deviations around each mean field-level profit over the 100 Monte Carlo samples. Again, the dashed line indicates the profits from implementing the optimal SSM, while the dotted line indicates the profits from uniform management.

The following heatmaps (Figure 8) compare the site-specific profits from the learned SSM at $T = 5$ and uniform management.

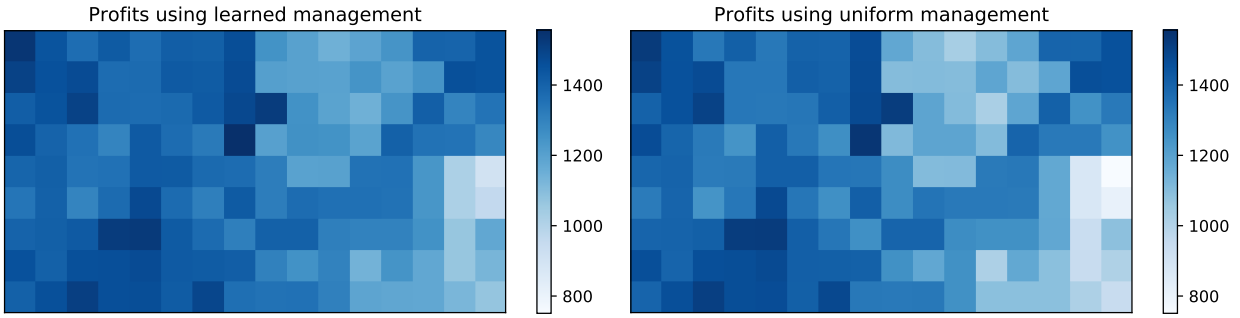

Figure 8: Site-specific profits for the learned SSM after 5 years and uniform management in scenario B

As in scenario A, the difference between the two management systems is difficult to discern. The SSM, however, has higher profits (darker colors) in low-yielding sites. Figure 9 illustrates the difference at each site.

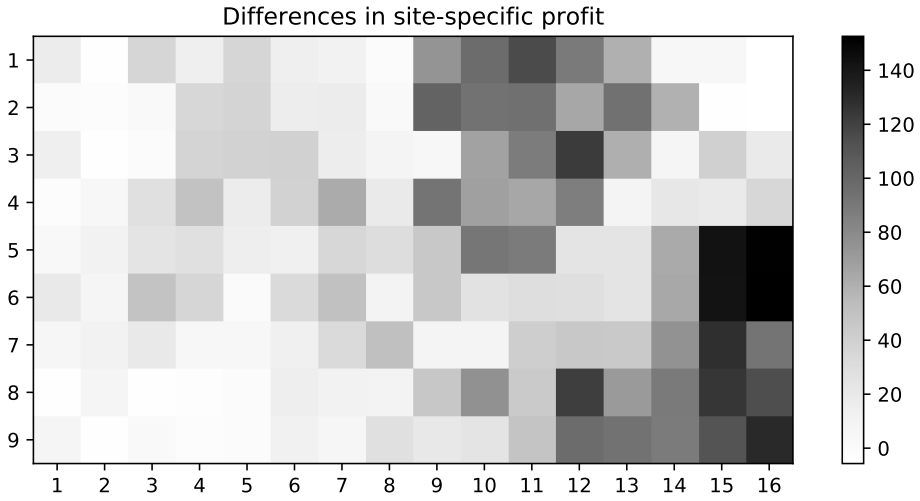

Figure 9: Differences in site-specific profit ($/ha) for SSM versus uniform management in scenario B. The maximum difference is $153/ha at site (16,6) and the minimum difference is $-5.7/ha at site (8,1).

Finally, Figure 10 illustrates the learned SSM $(x^1, \ldots, x^6)$ after 5 years.

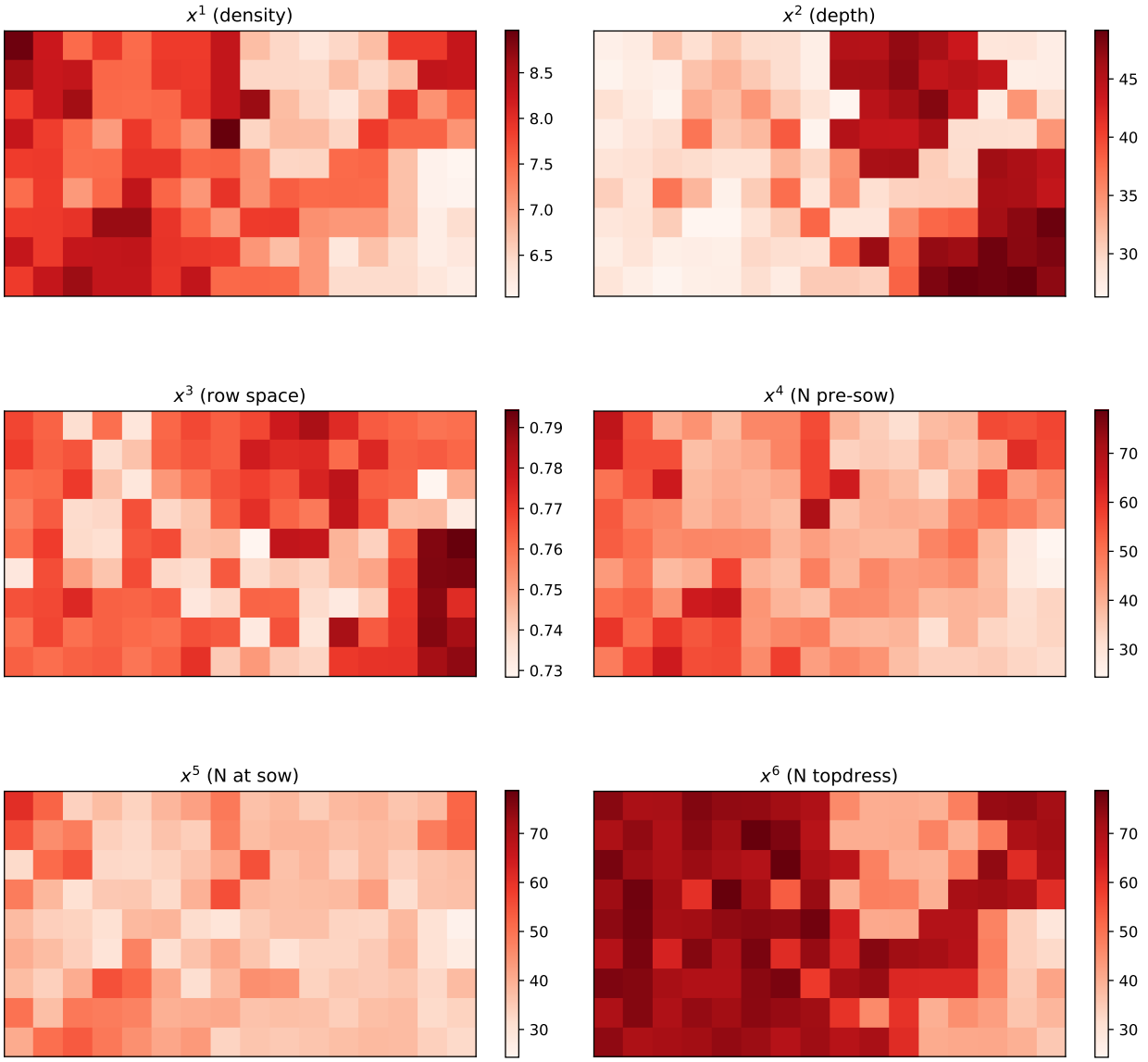

Figure 10: Learned SSM after 5 years in scenario B. The average sowing density is 7.4 seeds/m^2, and the average amount of total nitrogen is 146 kg/ha.

The average sowing density is

$$\frac{1}{144}\sum_{s=1}^{144} x_s^1 = 7.4 \text{ seeds/m}^2,$$

and the average amount of total nitrogen fertilizer is

$$\frac{1}{144}\sum_{s=1}^{144}\sum_{i=4}^{6} x_s^i = 146 \text{ kg/ha}.$$

As a result, \$40/ha higher profit is achieved by using 0.6 fewer seeds/m^2 and 55 kg/ha less nitrogen than uniform management.

4 Discussion

The results for field-level profit in Tables 1 and 2 and Figures 3 and 7 are well aligned with our intuition. When the algorithm starts with little existing data to incorporate, the algorithm has difficulty identifying good management. This is particularly the case in scenario A, in which the profit from the learned SSM after one year is far below that for uniform management. In contrast to scenario A, good management is found in scenario B even after one year because, although most sites are technically distinct, some sites are similar and provide mutual information. As a result, with a greater number of sites in the field, more information is collected each year. Despite the low performance in the first year in scenario A, the algorithm quickly learns and its performance surpasses the performance with uniform management after two years. In both scenarios, the algorithm continues to learn and widen the performance gap. With 5 years of learning, the estimated profit reaches \$1,274 or 97% of the maximum possible profit (\$1,310) in scenario A and \$1,335 or 98% of the maximum possible profit (\$1,365) in scenario B.

The random sampling used in year 1 creates the large shaded area formed by two standard deviations around the mean predictions over the first few years in scenario A (Figure 3), indicating that imprecise prediction of mean profits. However, performance dramatically improves after four years, and thereafter its spread around the mean profits continues to shrink. Combined with the increasing mean profits, this is a desirable feature because it implies that no matter how the algorithm starts off, after several years, the algorithm consistently learns good SSM. Scenario B exhibits far less impreciseness due to the larger sample size used right from the beginning (Figure 7).

As seen in Figures 4,5,8, and 9, the higher field-level profits for the learned SSM are due mainly to their higher profits from the low-yielding sites (e.g., sites with $z^1 \in \{231, 259\}$ in scenario A and sites around (11,2) and (15,7) in scenario B). These results imply that uniform management is excessively tailored to the high-yielding conditions—$z^1 \in \{317, 346\}$ in scenario A and the left half of the field in scenario B—leading to the decrease in profitability in the low-yielding sites where it is optimal to put less inputs. Overall, albeit not necessarily true in other environments, in these particular scenarios, the algorithm discovers that it is profitable to put more inputs in the high-yielding sites and less in the low-yielding sites as indicated in panels for x^1, x^4, x^5, and x^6 in Figure 6 and 10.

We dismiss the patchy look of row spacing (x^3) in Figures 6 and 10 as an artifact of numerical optimization, which strictly distinguishes two values whenever one results in even a minuscule amount greater than the other. Indeed, the color bar for x^3 has a very small range (0.830–0.855 in scenario A and 0.73–0.79 in scenario B), indicating little practical significance for management.

In both scenarios, the learned SSM is evidently more efficient than the benchmark uniform management. After five years, the learned SSM generates \$43/ha higher profits with 25 kg/ha less nitrogen in scenario A and \$40/ha higher profits with 55 kg/ha less nitrogen in scenario B. In terms of the total yield, the learned management produces 210 kg/ha less maize in scenario A and 278 kg/ha less maize in scenario B. While the SSM optimization is guided by profit maximization, it turns out to be environmentally more sustainable as well because both costs of fertilizer (i.e., to profitability and to the environment) are aligned so that less is better. However, higher yield does not necessarily coincide with higher profit, though yield increases with more inputs, substantially higher costs can reduce profit.

Despite the promising results, there are several clarifications and limitations to note before real-world implementations, as well as future research needs.

- This analysis assumes no costs for switching management from site to site, which can be

unrealistic for some inputs and management types. For example, varying types of fertilizer, seed treatments or hybrids may require equipment modifications, multiple field passes or additional labor. As another example, changing seeding rates too frequently may put an excessive strain on and damage an electric motor. If these are costs to consider, optimization of SSM will be even harder due to the switching frequencies of management as another set of control variables to optimize.

- The algorithm assumes choice variables are continuous, even though choices of continuous variables can be constrained for various practical reasons. For example, though the algorithm may recommend fertilizer application rates that differ by less than 0.1 kg/ha, such small differences are impossible to implement practically with current equipment. Real-world implementation will require modifying the algorithm to convert continuous choice variables into appropriate discrete variables.
- This approach is limited to one-shot optimization in which the farmer makes all the management decisions at the beginning of the year and waits to see results at the end of the year. In agriculture, many management choices are sequentially made throughout the year. To handle these more realistic situations requires dynamic models with information feedback and learning that take place both within and across years.
- The algorithm assumes the rationality and risk neutrality of the farmer when the acquisition function prescribes the next sampling choices. To be more realistic and useful for real-world applications, future work needs to incorporate research from behavioral economics into the design of the acquisition function.
- All the simulations use a single season's weather and stable site characteristics that prevail over T years. In addition, it assumes no spatial interaction between input choices across sites (e.g., fertilizer use on one site does not affect adjacent sites). These assumptions are excessively strong in many real-world circumstances, and so future work focuses on developing spatiotemporally explicit models that handle dynamics over space and time.

5 Conclusions

We have proposed an algorithmic approach to optimizing complex site-specific management with many management and environmental variables. The proposed algorithm enables individual farmers to efficiently learn their own site-specific management through on-farm experiments. We have demonstrated its performance using simulated environments. The results have provided a positive answer to both the learnability of complex site-specific management and the higher profitability possible relative to uniform management. Overall, the results suggest that on-farm experimentation implemented with precision agriculture equipment can help farmers and society realize the benefits of precision agriculture—more profitable management, greater food security, and improved environmental sustainability.

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Appendices

APSIM configuration

As a basis, we use the Continuous Maize module in APSIM. Then, we modify its default settings as follows. To simulate maize production in Ames, Iowa.

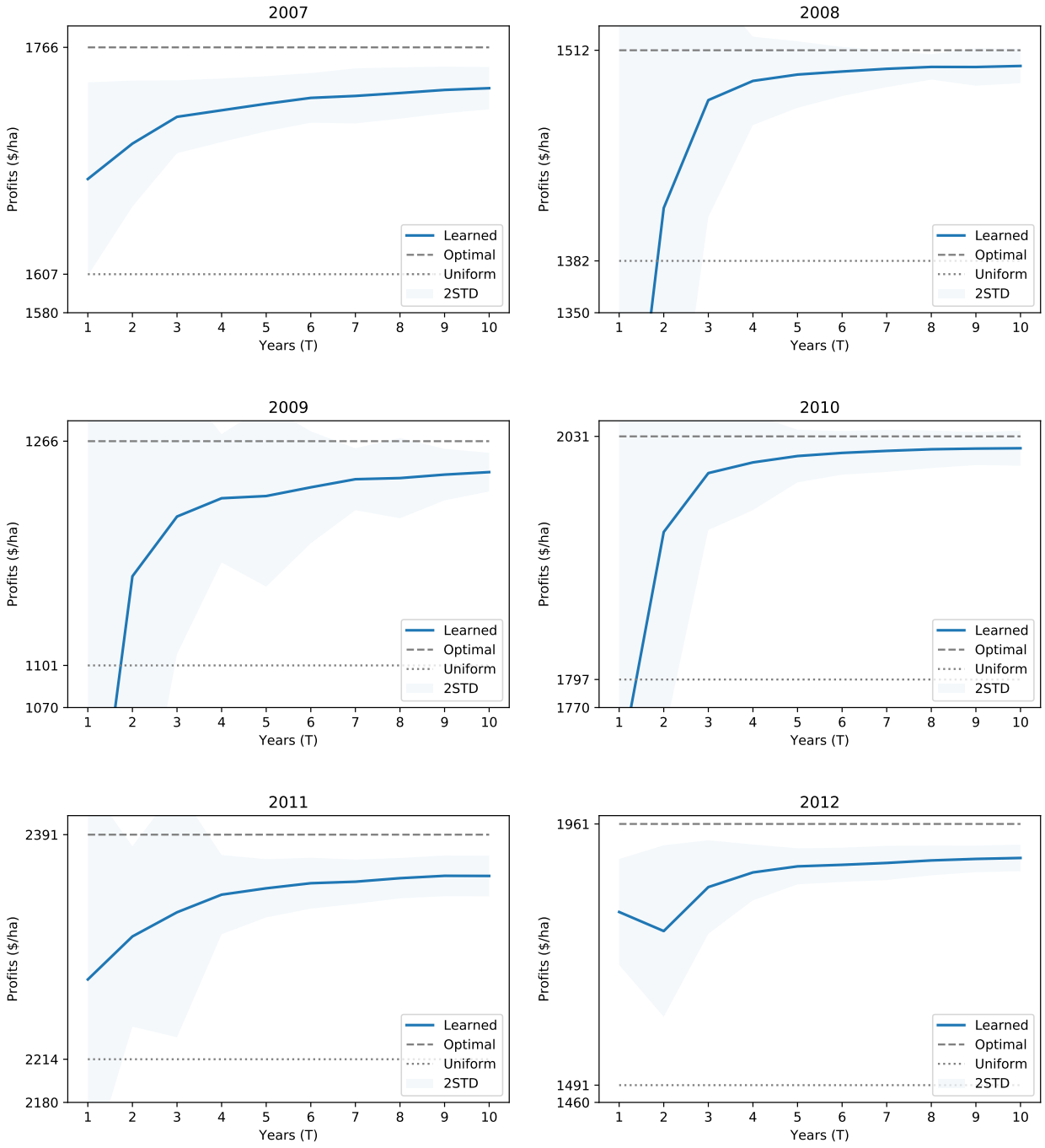
- Metfile: USA_Iowa_Ames.met
- Calendar: Jan 1, 2013 - Dec 31, 2013
- Cultivar: Pioneer 3394
- Sowing window START data: 15-apr
- Sowing window END data: 2-may
- Soil: Iowa Nicollet soil series
- Initial nitrogen: 0 kg/ha for both NO3 and NH4 for scenario A
- Initial water: 80% filled from top

Sensitivity analysis

In addition to two simulated environments with medium- and high-complexity, to further emphasize the generality and robustness of our algorithmic approach, we conducted simulation experiments in different years than 2013. Since output price, input prices, and weather are all dependent on a particular year, the differences in year provide different environments for profit maximization. The price information for each year was obtained from the same sources [70, 71].

Year	2007	2008	2009	2010	2011	2012
Output price ($/kg)	0.17	0.16	0.14	0.21	0.24	0.27
Seed price ($/1000 seeds)	1.82	2.10	3.13	3.44	3.25	3.40
Nitrogen price ($/kg)	0.69	1.02	1.51	0.73	1.13	1.40

Note that all sensitivity analysis was conducted under the environments with medium complexity, because of the significantly greater computational resources required in environments with high complexity. While there were considerable variations in both the growing and economic conditions across the different years, overall, the algorithm is quite versatile and able to learn good SSM within a few years in every environment. Similar to Figure 3, for each environment, we plot estimated field-level profits after T-year experiments.



Constructing scenario B

To generate a state vector $z = (z^1, z^2, z^3, z^4)$ for each site, we need to choose which site s and what values for $(z_s^1, z_s^2, z_s^3, z_s^4)$. For both purposes, we use random walk. Start from the mid site $(9, 5)$ with the initial values $(288, 3.2, 10, 10)$ assigned. Then, with probability of $1/3$, randomly either move right, move left, or stay. Similarly, with probability of $1/3$, randomly either move up, move down, or stay. This gives us the next site to consider. If the move means hitting a boundary, it stays at the site. Once moving into the new site, see if the site has already been assigned a state

vector. If not, with probability of 1/3, randomly perturb the state vector at the originating site by either -5%, 0%, or 5%. Continue the process until all sites are assigned a state vector.

Gaussian process

Gaussian process is a Bayesian nonparametric model, and its behavior is largely dependent on a choice of kernel and its hyperparameters [74]. A kernel is a function that returns a similarity measure $k(x, x')$ between two points x and x'. We use the Matérn kernel—a popular class of isotropic stationary kernels.

$$k_\nu(x, x') = \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{2\nu} \frac{d}{\rho} \right)^\nu B_\nu \left(\sqrt{2\nu} \frac{d}{\rho} \right),$$

where Γ is the gamma function, B_ν is the modified Bessel function of the second kind, and d is a metric often induced by the Euclidean norm, i.e. $d = \|x - x'\|$. The Matérn kernel is characterized by two hyperparameters ν and ρ, which control, respectively, the smoothness and the scaling of distance. As standard in applied work, we do not estimate but rather handpick ν and write as Matérn_ν or $k_\nu(x, x')$. To simplify the notation, let r denote the scaled distance, $r = d/\rho$. An important property of the Matérn kernel is that when $\nu = p + 1/2, p \in \mathbb{N}$, it can be written as a product of an exponential and a polynomial of order p:

$$k_{p+1/2}(x, x') = \sigma^2 \exp\left(-\sqrt{2p+1} r \right) \frac{p!}{(2p)!} \sum_{i=0}^{p} \frac{(p+i)!}{i!(p-i)!} (2\sqrt{2p+1} r)^{p-i}.$$

Common choices of ν are $1/2, 3/2, 5/2$ and ∞, with each of which the kernel reduces to, respectively,

$$\begin{aligned}
k_{1/2}(x, x') &= \sigma^2 \exp(-r) \\
k_{3/2}(x, x') &= \sigma^2 \exp\left(-\sqrt{3} r \right) \left(1 + \sqrt{3} r \right) \\
k_{5/2}(x, x') &= \sigma^2 \exp\left(-\sqrt{5} r \right) \left(1 + \sqrt{5} r + \frac{5}{3} r^2 \right) \\
k_\infty(x, x') &= \lim_{\nu \to \infty} k_\nu(x, x') = \sigma^2 \exp\left(-\frac{1}{2} r^2 \right).
\end{aligned}$$

Matérn_∞ is also known as squared exponential kernel or radial basis function. Following Snoek et al. [75] and Stein [76], we avoid squared exponential and use Matérn with $\nu = 3/2$ for our algorithm. The following figure plots $k_\nu(x, x')$ with $\sigma^2 = \rho = 1$ for $\nu \in \{1/2, 3/2, \infty\}$.

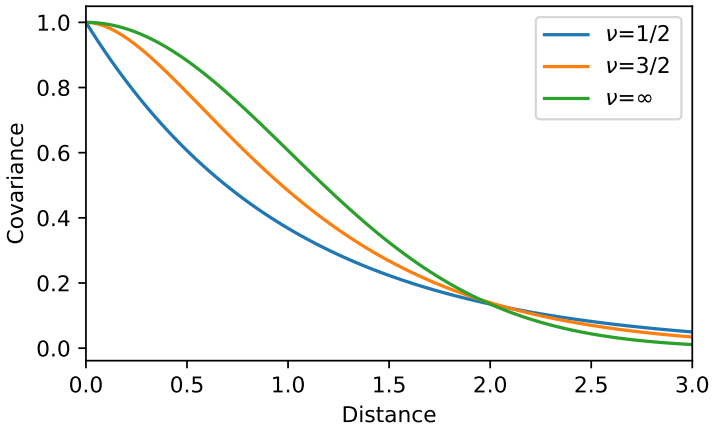


Figure 11: Matérn$_\nu$ kernels

Expected improvement acquisition function

In Bayesian optimization, an algorithm prescribes the next sampling point x based on how we value the mean and variance at x estimated by the accompanying GP. Specifically, the recommendation x_t for the next round t is determined by maximizing an acquisition function $\alpha(x|D_{t-1})$:

$$x_t = \underset{x}{\operatorname{argmax}}\, \alpha(x|D_{t-1}),$$

where D_{t-1} is the data used to fit the GP at round $t-1$. The acquisition function is a reflection of the underlying utility of the next sample or our preference in selecting the next sampling point. It is heuristic and designed to trade off exploration of the search space and exploitation of the current promising areas. There are a number of acquisitions functions proposed in the literature. One of the popular acquisition functions is called expected improvement, which is constructed based on the following intuitive idea. Let y^* be the maximum value observed up until round $t-1$, i.e. $y^* = \max\{y_1, \ldots, y_{t-1}\}$. Then, we may define "improvement" at point x at round t to be

$$\max\{0, GP(x) - y^*\},$$

which is random as $GP(x)$ is a random function. Thus, the expected improvement acquisition function is defined to be:

$$\alpha_{EI}(x|D_{t-1}) = \mathbb{E}[\max\{0, GP(x) - y^*\}|D_{t-1}].$$

When using Gaussian process, at each point x in the domain, we have $GP(x) \sim \mathcal{N}(\mu(x), \sigma(x))$, which allows the expected improvement to have a closed form [77, 78]:

$$\alpha_{EI}(x|D_{t-1}) = \begin{cases} (\mu(x) - y^*)\Phi\left(\frac{\mu(x)-y^*}{\sigma(x)}\right) + \sigma(x)\phi\left(\frac{\mu(x)-y^*}{\sigma(x)}\right) & \text{if } \sigma(x) > 0 \\ 0 & \text{if } \sigma(x) = 0 \end{cases},$$

where Φ is the standard normal cumulative distribution function and ϕ is the standard normal probability density function.